MODERN ALGEBRA 2

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1. Syllabus

MATH 6140 (Modern Algebra 2), instructed by Dr. Richard Green[1]. Website: `https://math.colorado.edu/~rmg/6140/`.

1.1. **Prerequisites.**

Obviously, Graduate Algebra 1[2]. Here're exams from MATH 6130 Fall 2017 the last time Dr. Green instructed: ring theory[3], further topics group theory[4], and basic group theory[5].

I also talked a bit with Dr. Thiem about preparing for modules, field theory, Galois theory, etc. Our consensus: one must *know* linear algebra. (One can never know enough linear algebra!)

The undergraduate handbook[6] for MA337 the University of Warwick suggests:

> MA106 Linear Algebra, familiarity with elementary group theory and the ring theory part of MA249 Algebra II: Groups and Rings is desirable.

1.2. **Prelim Exam Outline. Modules and linear algebra**

- foundations
- canonical examples
  - finite dimensional vector spaces
  - linear transformations
  - matrix representations
- modules
  - lattice of submodules
  - quotient modules
- homomorphism theorems
- structure of finitely generated modules
- language of categories and functors
- minimal polynomial of a transformation
- Cayley-Hamilton theorem over a commutative ring
- direct sums and products
- free, projective, injective modules
- duality
- multilinear forms
- determinants
- canonical forms

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[1] `https://math.colorado.edu/~rmg/`
[2] `alg1`
[3] `http://math.colorado.edu/~rmg/6130/p6130f.pdf`
[4] `http://math.colorado.edu/~rmg/6130/p6130b.pdf`
[5] `http://math.colorado.edu/~rmg/6130/p6130a.pdf`
[6] `https://warwick.ac.uk/fac/sci/maths/undergrad/ughandbook/year3/ma377/`

    - Jordan
    - rational
    - primary rational
- invariant factors
- elementary divisors
- localization of modules

Field theory

- foundations
- canonical examples
    - finite fields
- field extensions
    - algebraic
    - transcendental
    - cyclotomic and cyclic
    - transcendence degree
    - algebraic closure
- Greek construction problems
    - impossibility proofs
    - e.g., trisecting angles
- Fundamental theorem of Galois theory
    - Galois correspondence
- splitting fields
    - separability
    - normality
- Galois groups of extensions/polynomials
    - solvable and nilpotent groups
- solvable and radical extensions
    - the insolvability of the quintic
- Fundamental Theorem of Algebra
- Frobenius endomorphism

Additional Algebra

- applications of Zorn's lemma
- tensor products
    - algebras
- chain conditions
    - Artinian and Noetherian rings and modules
    - Hilbert basis theorem
- Nullstellensatz

1.3. **Week 0.** Course with Richard Green[7]. Git repo for assignments: `http://github.com/coltongrainger/fy19alg2`.

Changes I hope to make in contrast with the algebra 1[8] notes:

- more
    - reflection

[7] `https://math.colorado.edu/~rmg/`
[8] `alg1`

    - narrative
    - references
- less
    - mathjax bloat
    - rote memorization
    - agony