Homework2

February 11, 2025

1 CSC 380 Homework 2

[1]:
```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
```

1.1 Question 1

[2]:
```
data = pd.read_csv("olympics.csv")
data
```

[2]:

	ISO country code	Country name	2011 GDP	2010 population	Female count	Male count	Gold medals	Silver medals	Bronze medals
0	USA	US	1.509400e+13	309349000	271	260	46	29	29
1	CHN	China	7.298100e+12	1338300000	208	163	38	27	23
2	JPN	Japan	5.867150e+12	127451000	162	141	7	14	17
3	DEU	Germany	3.570560e+12	81777000	176				
4	FRA	France	2.773030e+12	64895000	148				
5	BRA	Brazil	2.476650e+12	194946000	128				
6	GBR	UK	2.431590e+12	62232000	269				
7	ITA	Italy	2.194750e+12	60483000	122				
8	RUS	Russia	1.857770e+12	141750000	227				
9	IND	India	1.847980e+12	1224615000	23				
10	JAM	Jamaica	1.506977e+10	2702000	25				
11	GEO	Georgia	1.436657e+10	4452000	6				
12	PRK	North Korea	1.228000e+10	24589122	40				
13	ARM	Armenia	1.024779e+10	3092000	4				
14	MNG	Mongolia	8.557530e+09	2756000	13				
15	BHS	Bahamas	7.787514e+09	343000	11				
16	MDA	Moldova	7.000319e+09	3562000	10				
17	TJK	Tajikistan	6.522200e+09	6879000	3				
18	MNE	Montenegro	4.550463e+09	632000	16				
19	GRD	Grenada	8.160541e+08	104000	4				

3	219	11	19	14
4	187	11	11	12
5	138	3	5	9
6	287	29	17	19
7	159	8	9	11
8	208	24	26	32
9	60	0	2	4
10	25	4	4	4
11	29	1	3	3
12	15	4	0	2
13	21	0	1	2
14	16	0	2	3
15	15	1	0	0
16	12	0	0	2
17	13	0	0	1
18	18	0	1	0
19	6	1	0	0

[3]:
```
medal_data = data[["Country name", "Gold medals", "Silver medals", "Bronze␣
↪medals"]]
medal_data
```

[3]:

	Country name	Gold medals	Silver medals	Bronze medals
0	US	46	29	29
1	China	38	27	23
2	Japan	7	14	17
3	Germany	11	19	14
4	France	11	11	12
5	Brazil	3	5	9
6	UK	29	17	19
7	Italy	8	9	11
8	Russia	24	26	32
9	India	0	2	4
10	Jamaica	4	4	4
11	Georgia	1	3	3
12	North Korea	4	0	2
13	Armenia	0	1	2
14	Mongolia	0	2	3
15	Bahamas	1	0	0
16	Moldova	0	0	2
17	Tajikistan	0	0	1
18	Montenegro	0	1	0
19	Grenada	1	0	0

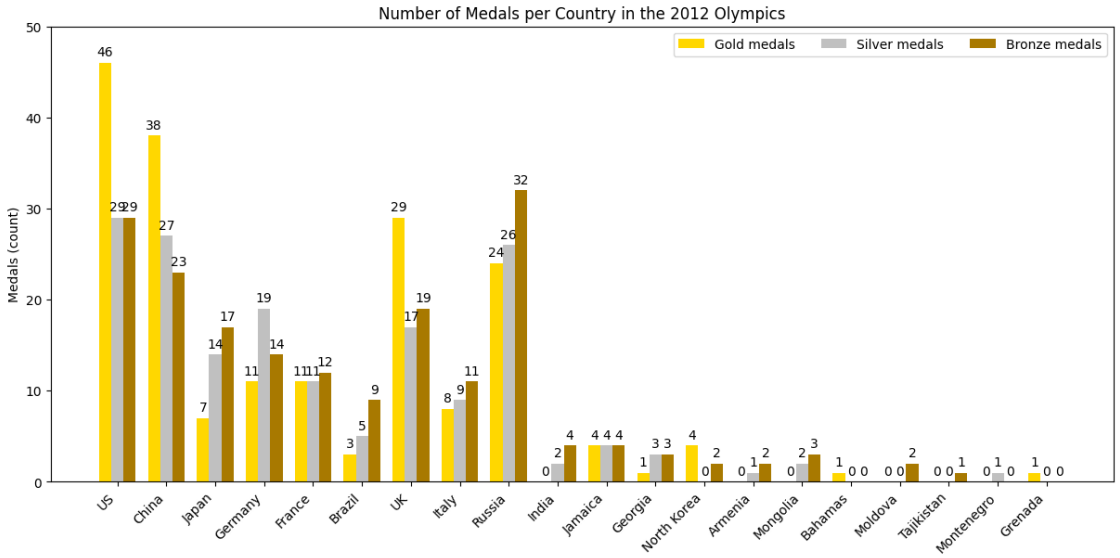
[4]:
```
x = np.arange(len(data["Country name"]))
width = 0.25
multiplier = 0
```

```
fig, ax = plt.subplots(figsize=(12, 6.0), layout="constrained")

medal_labels = ["Gold medals", "Silver medals", "Bronze medals"]
colors = ["gold", "silver", "xkcd:bronze"]
for color, medal in zip(colors, medal_labels):
    offset = width * multiplier
    rects = ax.bar(x+offset, medal_data[medal], width, label=medal, color=color)
    ax.bar_label(rects, padding=3)
    multiplier += 1

ax.set_ylabel("Medals (count)")
ax.set_title("Number of Medals per Country in the 2012 Olympics")
ax.set_xticks(x + width, data["Country name"], rotation=45, ha="right")
ax.legend(loc="upper right", ncols=3)
ax.set_ylim(0, 50)

plt.show()
```



This visualization it is easier to tell the relative distribution of the gold, silver, and bronze medals per country.

However it is more difficult to determine which country has more medals than another country very quickly.

1.2 Question 2

Given two fair six-sided dice there are 36 possible outcomes for the two rolls.

For the two rolls there are only 5 possible outcomes where the sum of the two rolls adds to 8: 2&6,

3&5, 4&4, 5&3, and 6&2.

Thus the probability that the two dice's outcome sums to 8 is:

$$P(E) = \frac{5}{36} = 0.13\overline{8} \approx 13.9\%$$

1.3 Question 3

[5]:
```
np.random.seed(2025)
```

[6]:
```
n = 1000
res_dice1 = np.random.randint(1, 6+1, size=n)
res_dice2 = np.random.randint(1, 6+1, size=n)
# res = [(res_dice1[i], res_dice2[i]) for i in range(len(res_dice1))]
res = res_dice1 + res_dice2
index = np.where(res==8)[0]
prob = np.size(res[index])/n
prob
```

[6]:
```
0.15
```

In this simulation the empirical frequency of E is 15%. Of the 1000 "rolls" of the dice 15% of the time the sum of the die was 8.

1.4 Question 4

[7]:
```
np.random.seed(2025)
```

[8]:
```
def prob_E(n):
    res_dice1 = np.random.randint(1, 6+1, size=n)
    res_dice2 = np.random.randint(1, 6+1, size=n)

    res = res_dice1 + res_dice2
    index = np.where(res==8)[0]
    prob = np.size(res[index])/n
    return prob
```

[9]:
```
n_rolls = 1000
for i in range(10):
    prob = prob_E(n_rolls)
    print(prob)
```

```
0.15
0.147
0.143
0.135
0.152
0.125
```

```
0.152
0.15
0.147
0.144
```

The numbers differ because each trial is a random process. It is not guaranteed that the final outcome will be 13.9% just that on average, and with a larger number of trials/"rolls" the probability will approach that fixed value. But for any individual trial it is not guaranteed to be exactly 13.9%.

1.5 Question 5

[10]:
```
n_rolls = 10000
for i in range(10):
    prob = prob_E(n_rolls)
    print(prob)
```

```
0.1392
0.1376
0.1438
0.137
0.1413
0.1417
0.1376
0.1417
0.131
0.1356
```

The reported values of each trial are much closer to the true value of 13.9%. Thus the variation in final values per trial is lower. While both "experiments" final results centered around 13.9% this verion, with 10000 rolls per trial, has a smaller standard deviation. This is because as the number of rolls increases the percentage will approach the theoretical value.

1.6 Question 6

$$A : \text{The sum of the two die is 6}$$

$$B : \text{The second die is even}$$

The inclusion exclusion principle is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The probability of event A is $\frac{5}{36}$ as there are five possible events of the 36 total after two dice rolls where the dice sum to 6.

The probability of event B is $\frac{1}{2}$ as there are three possible events of the 6 total after one dice roll where the die is even.

The probability of the dice summing to 6 and the second die being even is a bit trickier. There are five events where the dice sum to 6, namely: (1, 5), (2, 4), (3, 3), (4, 2), (5, 1). Of those five events

two of them meet the condition of event B (2, 4) and (4, 2). Thus the probability of event A and event B happenning simultaneously is $\frac{2}{36} = \frac{1}{18}$

Thus the final probability of $P(A \cup B)$ is:

$$P(A \cup B) = \frac{5}{36} + \frac{18}{36} - \frac{2}{36} = \frac{21}{36} = \frac{7}{12} = 58.\overline{3}\%$$

1.7 Question 7

[11]:
```
grades = np.array(200*["Freshman"] + 180*["Sophomore"] + 160*["Junior"] +␣
↪140*["Senior"] +\
                  800*["Freshman"] + 820*["Sophomore"] + 840*["Junior"] +␣
↪860*["Senior"])

majors = np.array(200*["CS Major"] + 180*["CS Major"] + 160*["CS Major"] +␣
↪140*["CS Major"] +\
                  800*["Non-CS Major"] + 820*["Non-CS Major"] + 840*["Non-CS␣
↪Major"] + 860*["Non-CS Major"])
```

[12]:
```
table = pd.crosstab(majors, grades, rownames=["Major"], colnames=["Grade"],␣
↪margins=True, margins_name="Total")
order = ["Freshman", "Sophomore", "Junior", "Senior", "Total"]
table = table[order]
table
```

[12]:

Grade	Freshman	Sophomore	Junior	Senior	Total
Major					
CS Major	200	180	160	140	680
Non-CS Major	800	820	840	860	3320
Total	1000	1000	1000	1000	4000

[13]:
```
prob_table = table/4000
prob_table
```

[13]:

Grade	Freshman	Sophomore	Junior	Senior	Total
Major					
CS Major	0.05	0.045	0.04	0.035	0.17
Non-CS Major	0.20	0.205	0.21	0.215	0.83
Total	0.25	0.250	0.25	0.250	1.00

These 7 new entries show the relative distributions of the majors and the grades, as well as the total probability of "selecting a student at random and they are a student." For example the grades had the same number of students so the probability of selecting a student at random and them being a particular grade is 1/4 or .25 as shown in the probability table. The row marginals show the probability of selecting a student and they are a CS Major 0.17 or they are a Non-CS Major, 0.83. Lastly the probability of selecting a student and they are a student is 1.0 which is indicated in the lower right cell in the probability table.

Given that a student is a senior the probability that they are a CS Major is:

$$P(\text{CS}|\text{Senior}) = \frac{0.035}{0.25} = 0.14 = 14\%$$

Given that the student is a senior actually decreases our belief that the student is a CS Major. This is because the probability of them being a CS Major given that they are a senior is 14% while the total probability of any student being a CS Major is 17%.