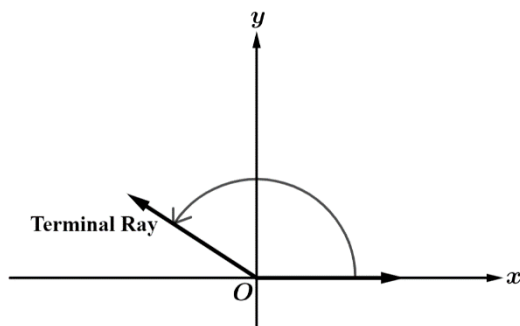


Reminder: In the coordinate plane, an angle is in the **standard position** when its vertex is at the origin and one ray of the angle lies on the positive x -axis. The **terminal ray** is the second ray of an angle in standard position.



The figure above shows an angle θ in the standard position.

Positive and Negative Angles

Positive and negative angle measures simply indicate the direction of the rotation from the positive x -axis.

Positive angles are in the counterclockwise direction.

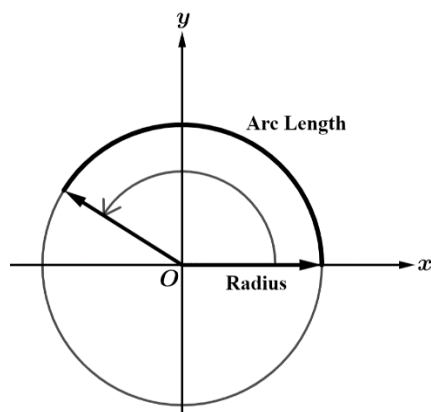
Negative angles are in the clockwise direction.

The figure above indicates a positive angle because the direction of rotation is counterclockwise.

Since we are using the coordinate plane to define angles, we can consider angles to have periodic behavior. Angles in standard position that share a terminal ray differ by an integer number of revolutions.

Measuring Angles

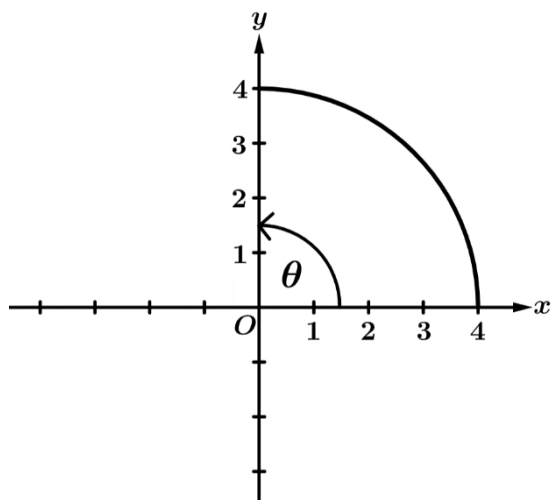
The way we will measure angles throughout this course, and in future courses, may seem a bit strange at first.



The measure of the angle indicated in the figure on the left is $\frac{\text{arc length}}{\text{radius}}$ radians.

The **radian** measure of an angle in standard position is the ratio of the length of the arc of a circle centered at the origin created by the angle to the radius of that of that same circle.

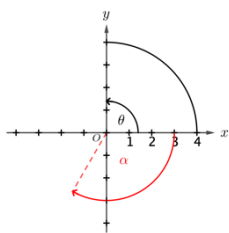
Reminder: The circumference of a circle is given by $C = 2\pi r$. We can find the arc length by considering a ratio of a circle's circumference.



Example 1: Consider the angle θ created by the quarter circle of radius 4 above. Find the measure of θ in radians.

$$\text{arc length of a quarter circle} = \frac{\text{arc length of the whole circle}}{4} = \frac{\overbrace{2\pi(4)}^{\text{circumference}}}{4} = 2\pi \quad \theta = \frac{\text{arc length}}{r} = \frac{2\pi}{4} = \frac{\pi}{2}$$

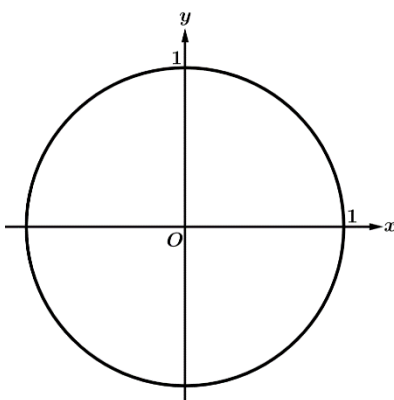
Example 2: Find the measure of the angle formed by a third of a circle with radius 3 centered at the origin in the clockwise direction.



$$\begin{aligned} \text{arc length of a third of a circle} &= \frac{\text{arc length of the whole circle}}{3} = \frac{\overbrace{2\pi(3)}^{\text{circumference}}}{3} = 2\pi \\ \alpha &= \frac{\text{arc length}}{r} = -\frac{2\pi}{3} \end{aligned}$$

The Unit Circle

The **unit circle** is a circle, centered at the origin, with radius 1.



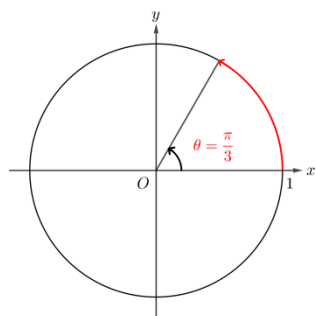
The Unit Circle

For a unit circle, the radian measure is the same as the length of the arc length formed by the angle.

The concept of the unit circle will become pivotal throughout the rest of this course because of its incredible usefulness when working with trigonometric values.

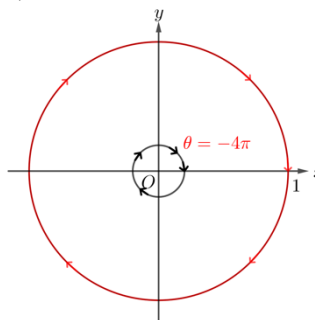
Example 3: For each of the following, sketch a picture of the arc length described based on a unit circle. Then find the measure of the angle corresponding to the given arc length.

a) $\frac{1}{6}$ of a circle in the counterclockwise direction



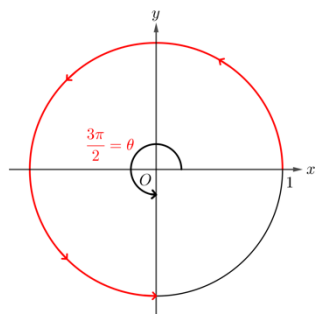
$$\theta = \frac{2\pi}{6} = \frac{\pi}{3}$$

b) 2 full revolutions in the clockwise direction



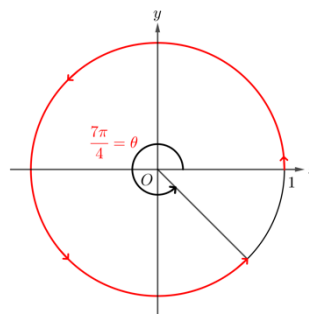
$$\theta = 2(-2\pi) = -4\pi$$

c) $\frac{3}{4}$ of a circle in the counterclockwise direction

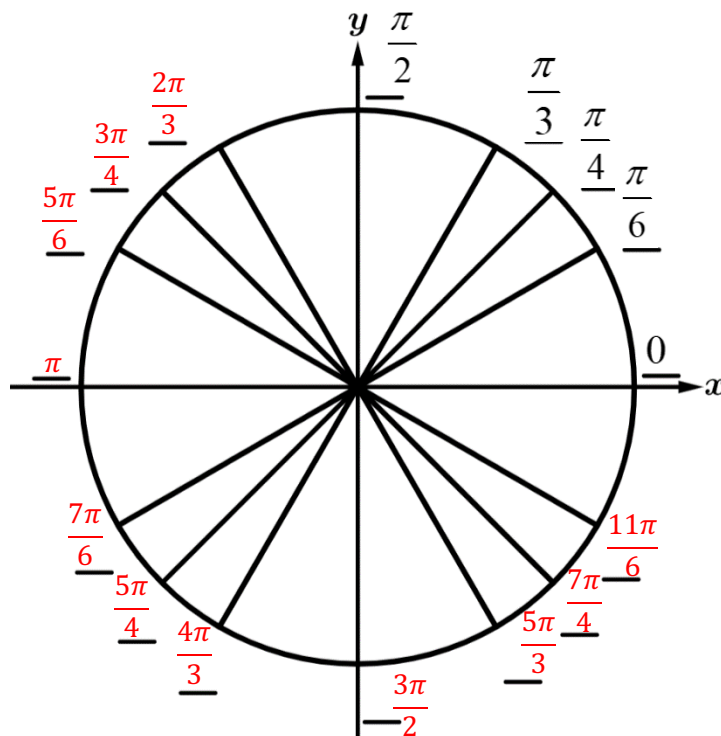


$$\theta = \frac{3}{4}(2\pi) = \frac{3\pi}{2}$$

d) $\frac{7}{8}$ of a circle in the counterclockwise direction

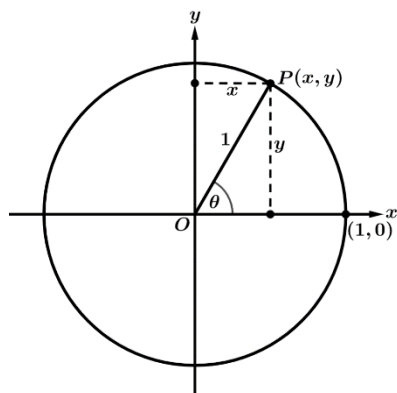


$$\theta = \frac{7}{8}(2\pi) = \frac{7\pi}{4}$$



Example 4: The circle above has markings at the important angles that will be used repeatedly throughout this course. The radian measures in the first quadrant are given for the circle. Use these angles and symmetry to complete the remaining labeled angles on the circle.

Sine, Cosine, and Tangent on the Unit Circle



For a unit circle, the radius is 1. As a result, the sine and cosine values of an angle θ in standard position are simplified.

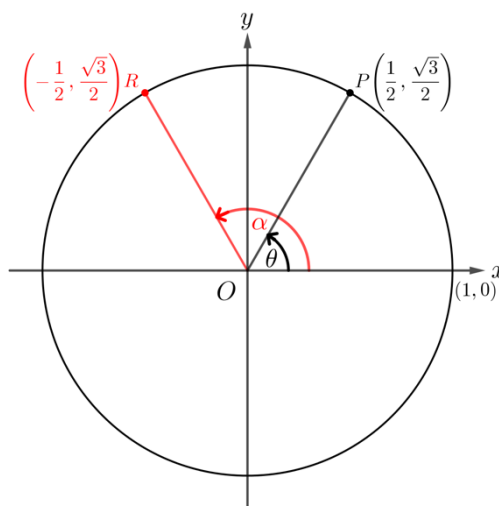
The sine of the angle θ is the y -coordinate of the point P .

$$\sin \theta = y$$

The cosine of the angle θ is the x -coordinate of the point P .

$$\cos \theta = x$$

Note: $\tan \theta = \frac{y}{x}$ or $\tan \theta = \frac{\sin \theta}{\cos \theta}$



Example 5: The figure shows a circle centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P . The coordinates of P are $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Find the following values.

a) $\sin \theta = \frac{\sqrt{3}}{2}$

b) $\cos \theta = \frac{1}{2}$

c) $\tan \theta = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

Example 6: The point R is the result after point P is reflected over the y -axis. Let α be the angle in standard position whose terminal ray intersects the circle above point R . Find the following values.

a) $\sin \alpha = \frac{\sqrt{3}}{2}$

b) $\cos \alpha = -\frac{1}{2}$

c) $\tan \alpha = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$