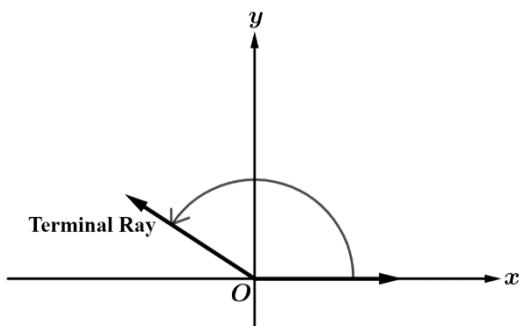


In this course, we are going to approach angles in a way that may be different than what you have seen in the past. We will approach angles in a generalized method that allows us to view them through a periodic lens. To do this, we must first make clear a few definitions.

Definition 1: Standard Position – In the coordinate plane, an angle is in the standard position when its vertex is at the origin and one ray of the angle lies on the positive x -axis.

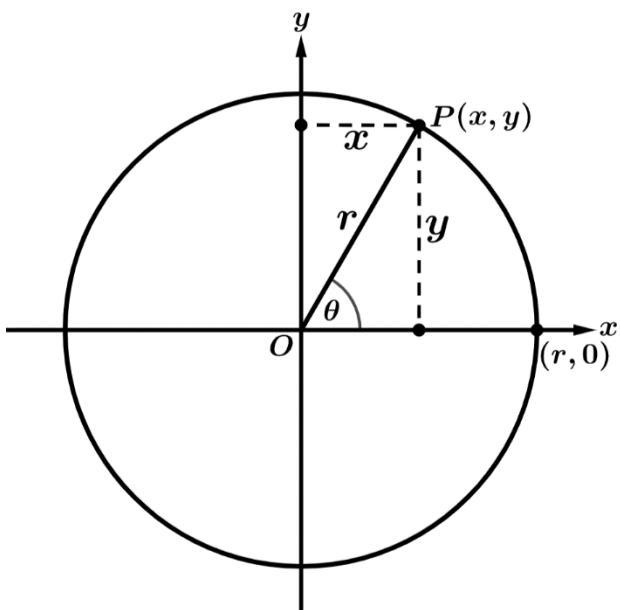
Definition 2: Terminal Ray—The terminal ray is the second ray of an angle in standard position.



The figure above shows an angle in the standard position.

Sine, Cosine, and Tangent

Given an angle in standard position and a circle centered at the origin, there is a point P , where the terminal ray intersects the circle.



$\sin \theta$ is the **ratio** of the **vertical displacement of P** from the **x -axis** to the **distance between the origin and point P** .

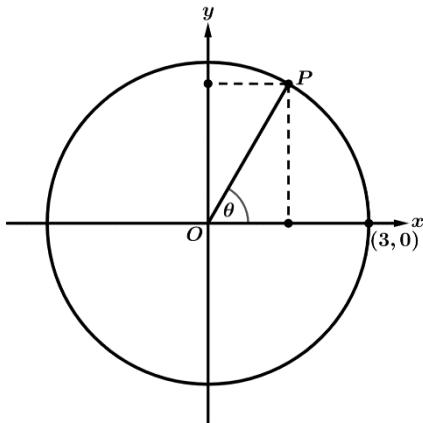
$$\sin \theta = \frac{y}{r}$$

$\cos \theta$ is the **ratio** of the **horizontal displacement of P** from the **y -axis** to the **distance between the origin and point P** .

$$\cos \theta = \frac{x}{r}$$

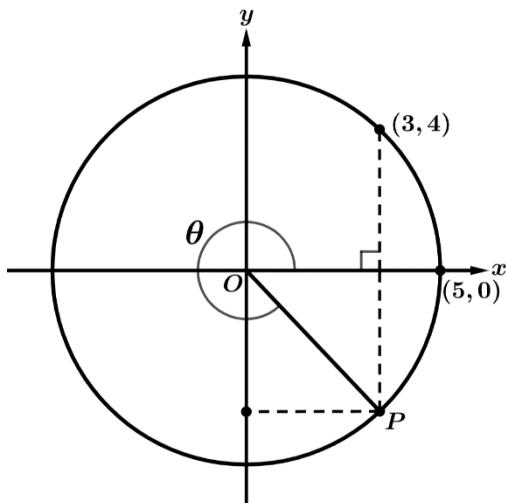
$\tan \theta$ is the **slope** of the terminal ray – the **ratio** of the **vertical displacement** to the **horizontal displacement** of P .

$$\tan \theta = \frac{y}{x} \quad \text{or} \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$



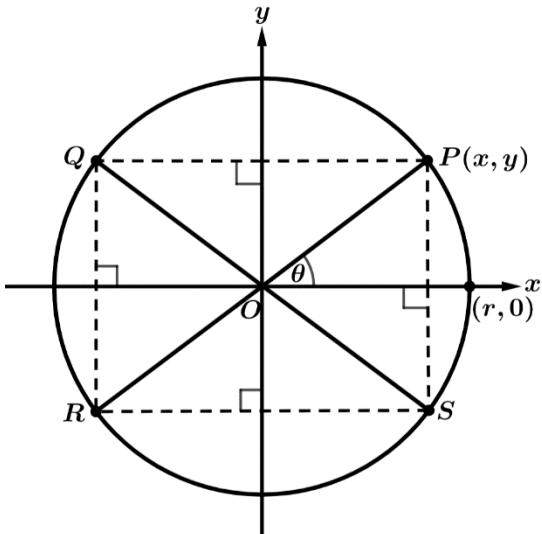
Example 1: The figure shows a circle centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P . The coordinates of P are (x, y) . Which of the following is true about the cosine of θ ?

- (A) $\cos \theta = \frac{x}{3}$, because it is the ratio of the horizontal displacement of P from the y -axis to the distance between the origin and P .
- (B) $\cos \theta = \frac{y}{3}$, because it is the ratio of the vertical displacement of P from the x -axis to the distance between the origin and P .
- (C) $\cos \theta = \frac{y}{x}$, because it is the ratio of the vertical displacement to the horizontal displacement of P .
- (D) $\cos \theta = \frac{3}{x}$, because it is the ratio of the distance between the origin and P to the horizontal displacement of P from the y -axis.



Example 2: The figure shows a circle with radius 5 centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P . Find the following values.

a) $\sin \theta =$ b) $\cos \theta =$ c) $\tan \theta =$



The figure shows a circle with radius r centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P . The coordinates of P are (x, y) . The points Q , R , and S are the result of reflecting point P across the y -axis, the origin, and the x -axis respectively.

Example 3: Find the coordinates of the points Q , R , and S in terms of x and y .

$$\text{Point } Q = (\quad , \quad)$$

$$\text{Point } R = (\quad , \quad)$$

$$\text{Point } S = (\quad , \quad)$$

Example 4: Find the sine of the angle whose terminal ray intersects the circle at point S .

Example 5: Find the cosine of the angle whose terminal ray intersects the circle at point Q .

Example 6: Find the tangent of the angle whose terminal ray intersects the circle at point R .

Example 7: Let θ be an angle in standard position whose terminal ray coincides with the line $y = -3x$ in quadrant II. Find the following values.

a) $\sin \theta =$

b) $\cos \theta =$

c) $\tan \theta =$