x	1	2	3	4	5
$f(x)$	5	10	20	40	80

× 2   × 2   × 2   × 2

1. Let f be an increasing function defined for all values of x. The table gives values for $f(x)$ at selected values of x.

(i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, or exponential function.

     Exponential

(ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f.

    The best model for f is exponential because over equal-length input-value intervals the output values of a function change proportionally.

x	1	3	5	7	9
$f(x)$	17	10	5	2	1

−7   −5   −3   1

−2   −2   −2

2. Let f be a continuous function. The table gives values for $f(x)$ at selected values of x.

(i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, or exponential function.

    Quadratic

(ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f. For a quadratic function, since the average rates of change over consecutive equal-length input-value intervals can be given by a linear function, these average rates of change for a quadratic function are changing at a constant rate or the second differences are constant.

| x	1	2	3	4	5
$f(x)$	8	12	18	27	81/2

× 1.5   × 1.5   × 1.5   × 1.5
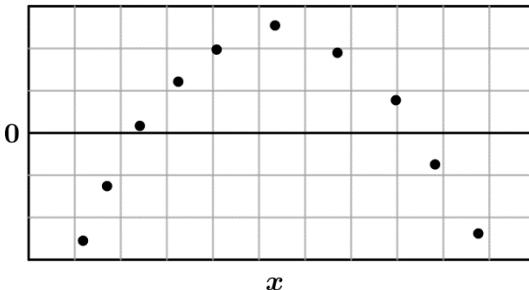
3. Let f be a continuous function. The table gives values for $f(x)$ at selected values of x.

(i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, or exponential function.

    Exponential

(ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f.

The best model for f is exponential because over equal-length input-value intervals the output values of a function change proportionally.

x
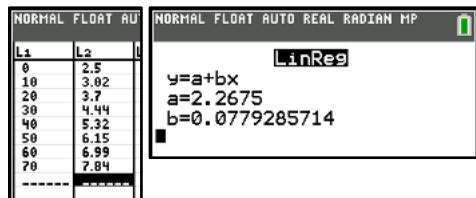
4. A quadratic regression was used to model a data set. The residual plot for the model is above. Which of the following statements about the appropriateness of the model is correct?

(A) A quadratic regression model is appropriate because the residuals do not show a linear pattern.
(B) A quadratic regression model is appropriate because the residuals show a quadratic pattern.
(C) A quadratic regression model is not appropriate because the residuals do not show a linear pattern.
(D) A quadratic regression model is not appropriate because the residuals show a quadratic pattern.

Years Since 1950	0	10	20	30	40	50	60	70
Total World Population (in billions)	2.5	3.02	3.7	4.44	5.32	6.15	6.99	7.84

5. Over the years 1950 – 2020, the total world population can be modeled by a linear function. Selected values for the total world population P, in billions, are given in the table above, where t represents the number of years since 1950.
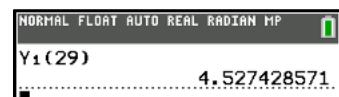
a) Use the regression capabilities on your calculator to find a linear model of the form $y = ax + b$ for the world population (in billions) x years since 1950.

$P(t) = a + bt$ where a and b are the values stored in the calculator.

b) According to the model found in part a, what was the world population in 1979, the year Mr. Passwater was born?
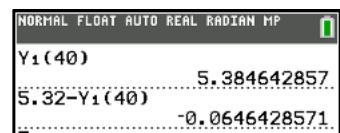
$1979 \Rightarrow t = 1979 - 1950 = 29$ $P(29) = a + b(29) = 4.5274 \ldots$ billion
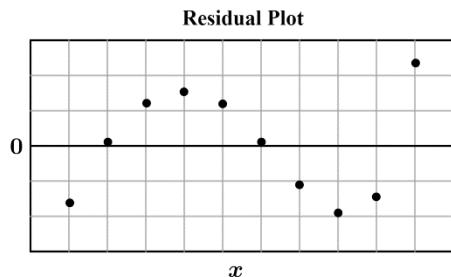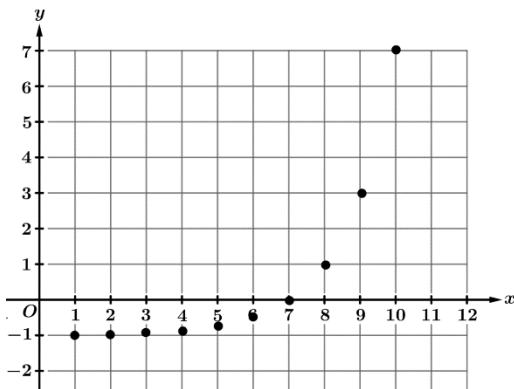
c) What is the residual of the total world population for the year 1990? Did our model underestimate or overestimate the total world population for the year 1990?

$1990 \Rightarrow t = 1990 - 1950 = 40$ $P(40) = a + b(40) = 5.3846 \ldots$ billion
Residual $= 5.32 - 5.3846 \ldots = -0.0646 \ldots$ which means the model overestimated.

Worksheet A: Topic 2.6   Competing Function Model Validation   Created by Bryan Passwater
Solutions by Ted Gott tedg20776@gmail.com
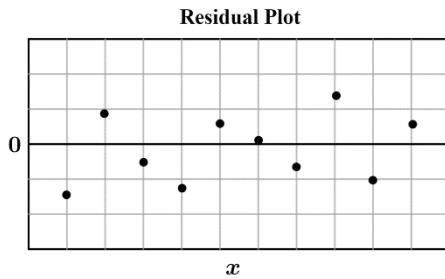
6. A regression model was created for the data in the graph above (left). The residual plot for the model is given above (right). Which of the following statements about the regression model is best?
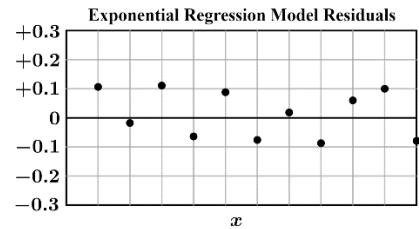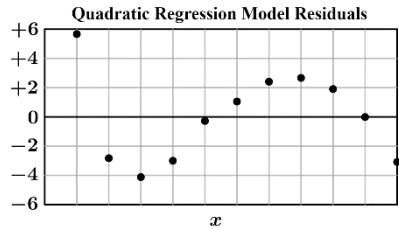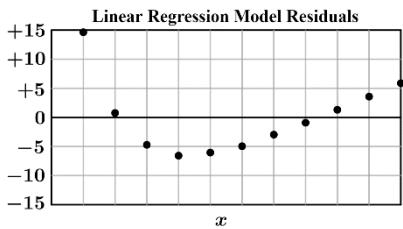
(A) A quadratic regression model was used and the model is appropriate.

(B) A quadratic regression model was used and the model is not appropriate.

(C) An exponential regression model was used and the model is appropriate.

(D) An exponential regression model was used and the model is not appropriate.

The residual plot shows that whatever regression was used, it is not appropriate. From the scatterplot it looks like an exponential regression model should have been used and the residual plot would have a random pattern.

7. Mr. Passwater used a set of data to create a quadratic regression model. The residual plot for his model is shown above. Based on the residual plot above, which of the following conclusions is correct?

(A) The residual plot has no apparent pattern, so the quadratic model was appropriate.

(B) The residual plot has no apparent pattern, so the quadratic model was not appropriate.

(A) The residual plot displays a pattern, so the quadratic model was appropriate.

(A) The residual plot displays a pattern, so the quadratic model was not appropriate.

Linear Regression Model Residuals

Quadratic Regression Model Residuals

Exponential Regression Model Residuals

8. A set of data was used to create a linear, a quadratic, and an exponential regression model. The residual plots for the three models are shown above. Based on the three residual plots, which of the following could be an appropriate model for the data?

(A) $y = 3 + 2x$ (B) $y = x^2 + 2x + 3$ (C) $y = 3(2)^x$ (D) $y = 3 + 2\log x$

From the scatterplot it looks like an exponential regression model could be an appropriate model because the residual plot shows a random pattern.

9. Mr. Passwater loves to invest his money in mutual funds. Over the past twenty years, he has closely tracked how his account grows and noticed that each year his account grows by approximately 10.4%. If Mr. Passwater wants to find a function that models the amount of money in his account over time, should he use a linear, quadratic, or exponential model? Give a reason for your answer.

He should use an exponential model because the value in the account is changing proportionally, 1.104, per year.

10. After Mr. Passwater creates his model from question 9, he uses the model to create a residual plot in order to check the appropriateness of his model. If his model was appropriate, what should he expect to see when looking at the residual plot?

He should expect to see no pattern in the residual plot.