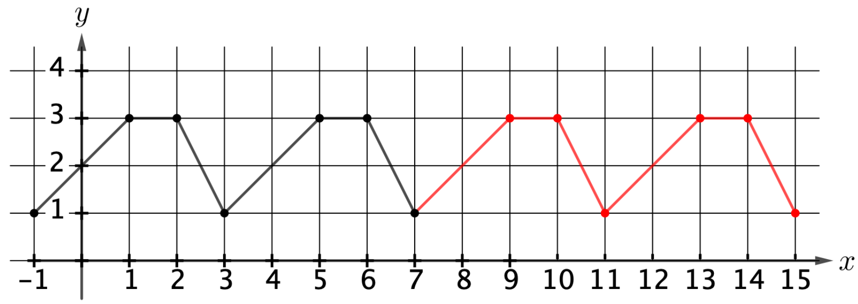
Notes: (Topic 3.1) Periodic Phenomena Solutions

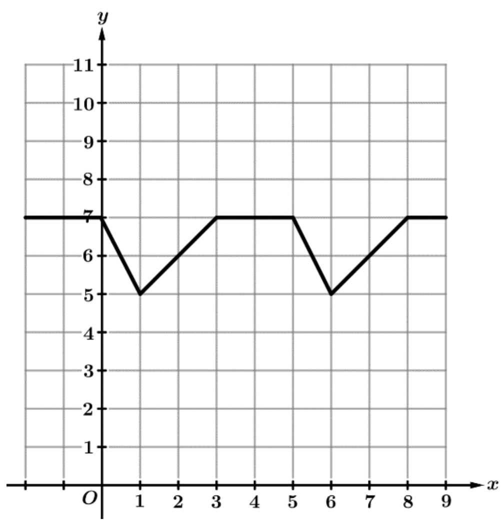
A **periodic** relationship between two variables occurs when the output values demonstrate a repeating pattern over successive equal-length intervals.

The **period** of a periodic function is the length of the x-values that is takes for the function to complete one cycle.



Graph of $f(x)$

Example 1: The function f is periodic with period 4. A portion of the graph of f is shown above. Draw two additional periods for the graph of f on the axes above.



Graph of $g(x)$

Example 2: A portion of the graph of the periodic function g is shown above. What is the least possible value of the period of g? The least possible value of the period is 5.

Example 3: Using the function g and the period found in **Example 2,** find the following values:

a) $g(14) = g\left(4 + \underbrace{2(5)}_{\text{2 periods}}\right) = g(4) = 7$

b) $g(72) = g\left(2 + \underbrace{14(5)}_{\text{14 periods}}\right) = g(2) = 6$

c) $g(-17) = g\left(3 - \underbrace{4(5)}_{\text{4 periods}}\right) = g(3) = 7$

x	1	3	4	7
$h(x)$	-2	0	3	2

Example 4: The graph of h is periodic with a period of 5. Values of h are shown at selected values of x. Find the following values

a) $h(-2) = h\left(3 - \underbrace{5}_{1\text{ period}}\right) = h(3) = 0$

b) $h(6) = h\left(1 + \underbrace{5}_{1\text{ period}}\right) = h(1) = -2$

c) $h(h(9))$ $\quad h(9) = h\left(4 + \underbrace{5}_{1\text{ period}}\right) = h(4) = 3$

$h(h(9)) = h(3) = 0$

d) $h(5k - 3)$, where k is an integer.

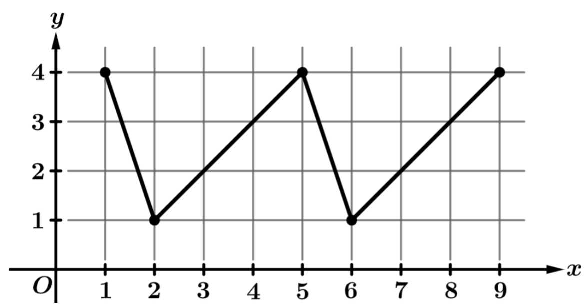
$h(5k-3) = h\left(-3 + \underbrace{5k}_{k\text{ periods}}\right) = h(-3)$

$= h\left(7 - \underbrace{5(2)}_{2\text{ periods}}\right) = h(7) = 2$

It is useful to have notation to model this phenomena. A periodic function starts with a basic pattern and translates the pattern translate repeatedly over its domain. As a result, we are able to consider a periodic function as **a horizontal translation of itself**.

In **Example 4**, the period of h was 5, meaning that we could translate the graph of h horizontally 5 units and end up with the same graph. This means that the function h satisfies the following equation for all values of x:

$$h(x) = h(x - 5).$$

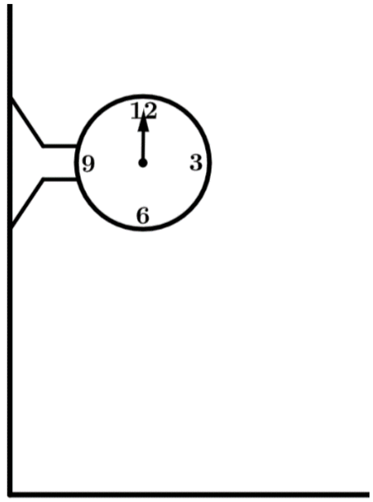


Graph of f

Example 5: The graph of f is periodic with a domain of all real numbers. Two full periods of f are shown. Find all input values of f that yield an output value of 1.

The period of f is 4. $f(2) = 1$ so, any input value $x = 2 + 4k$ where k is an integer will yield an output value of 1.

AP Exam: FRQ 3 Task Model



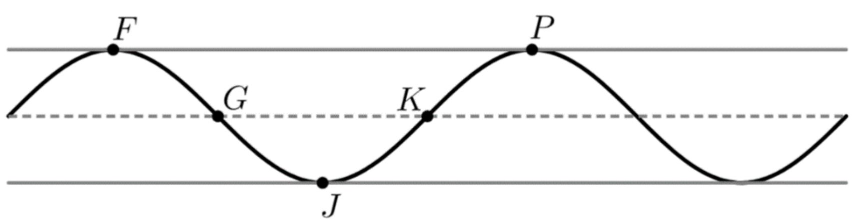
Note: Figure NOT drawn to scale

Example 6: The figure shows a large clock mounted to a vertical wall. The clock has an 8-inch-long moving minute hand. The center of the clock is 120 inches directly above the floor. At time $t = 0$ minutes, the minute hand is pointed directly up at the 12. However, the clock is not working properly, and the minute hand is moving twice as fast as it should. Thus, the next time the minute hand points directly up to the 12 is at time $t = 30$ minutes. As the minute hand moves, the distance between the endpoint of the minute hand and the floor periodically decreases and increases.

The periodic function h models the distance, in inches, between the endpoint of the minute hand from the floor and the floor as a function of time t in minutes.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, $F, G, J, K,$ and P are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: $F, G, J, K,$ and P.



$F:(0,128)$ $G:(7.5,120)$ $J:(15,112)$ $K:(22.5,120)$ $P:(30,128)$

(B) Refer to the graph of h in part (A). The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

(j) On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing over the interval (t_1, t_2). On the interval (t_1, t_2) the graph of h appears to be concave up, so the rate of change of h is increasing.