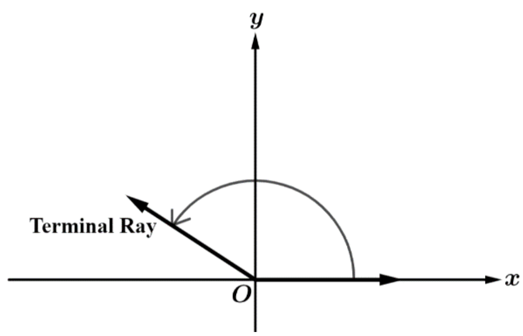
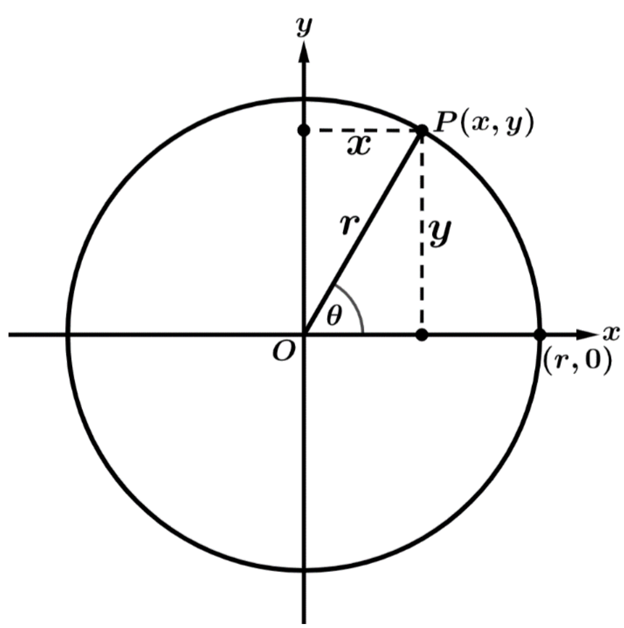
In this course, we are going to approach angles in a way that may be different than what you have seen in the past. We will approach angles in a generalized method that allows us to view them through a periodic lens. To do this, we must first make clear a few definitions.

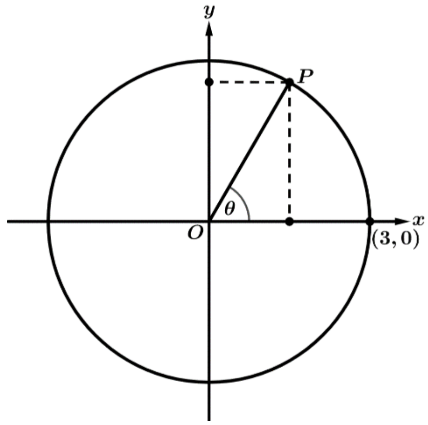
Definition 1: <u>Standard Position</u> – In the coordinate plane, an angle is in the standard position when its vertex is at the origin and one ray of the angle lies on the positive x-axis.

Definition 2: <u>Terminal Ray</u>—The terminal ray is the second ray of an angle in standard position.



The figure above shows an angle in the standard position.

<table>
<tr><th colspan="2">Sine, Cosine, and Tangent</th></tr>
<tr><td colspan="2">Given an angle in standard position and a circle centered at the origin, there is a point P, where the terminal ray intersects the circle.</td></tr>
<tr><td rowspan="3">
</td><td>$\sin\theta$ is the <u>ratio</u> of the vertical displacement of P from the x-axis to the distance between the origin and point P.
$$\sin\theta = \frac{y}{r}$$</td></tr>
<tr><td>$\cos\theta$ is the <u>ratio</u> of the horizontal displacement of P from the y-axis to the distance between the origin and point P.
$$\cos\theta = \frac{x}{r}$$</td></tr>
<tr><td>$\tan\theta$ is the slope of the terminal ray – the <u>ratio</u> of the vertical displacement to the horizontal displacement of P.
$$\tan\theta = \frac{y}{x} \quad \text{or} \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$</td></tr>
</table>



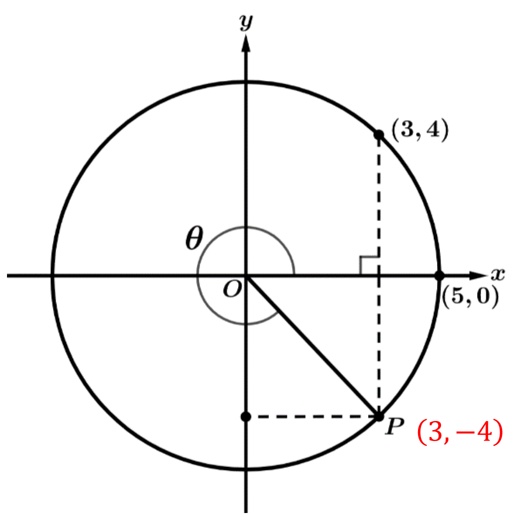
Example 1: The figure shows a circle centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P. The coordinates of P are (x, y). Which of the following is true about the cosine of θ?

(A) $\cos\theta = \frac{x}{3}$, because it is the ratio of the horizontal displacement of P from the y-axis to the distance between the origin and P.

(B) $\cos\theta = \frac{y}{3}$, because it is the ratio of the vertical displacement of P from the x-axis to the distance between the origin and P.

(C) $\cos\theta = \frac{y}{x}$, because it is the ratio of the vertical displacement to the horizontal displacement of P.

(D) $\cos\theta = \frac{3}{x}$, because it is the ratio of the distance between the origin and P to the horizontal displacement of P from the y-axis.

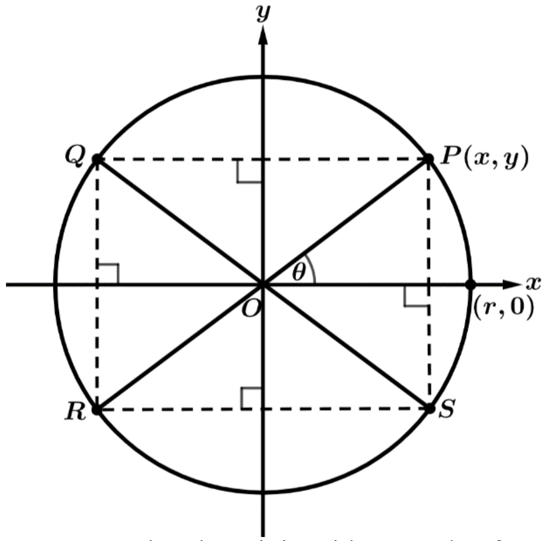


Example 2: The figure shows a circle with radius 5 centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P. Find the following values.

a) $\sin\theta = \frac{-4}{5} = -\frac{4}{5}$

b) $\cos\theta = \frac{3}{5}$

c) $\tan\theta = -\frac{4}{3}$



The figure shows a circle with radius r centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P. The coordinates of P are (x, y). The points $Q, R,$ and S are the result of reflecting point P across the y-axis, the origin, and the x-axis respectively.

Example 3: Find the coordinates of the points $Q, R,$ and S in terms of x and y.

Point $Q = (-x, y)$ Point $R = (-x, -y)$ Point $S = (x, -y)$

Example 4: Find the sine of the angle whose terminal ray intersects the circle at point S.

$\sin\theta = -\frac{y}{r}$

Example 5: Find the cosine of the angle whose terminal ray intersects the circle at point Q.

$\cos\theta = -\frac{x}{r}$

Example 6: Find the tangent of the angle whose terminal ray intersects the circle at point R.

$\tan\theta = \frac{-y}{-x} = \frac{y}{x}$

Example 7: Let θ be an angle in standard position whose terminal ray coincides with the line $y = -3x$ in quadrant II. Find the following values.

$x^2 + y^2 = r^2$

$x^2 + (-3x)^2 = r^2$

$x^2 + 9x^2 = r^2$

$10x^2 = r^2$ $x^2 = \frac{r^2}{10}$

$x = -\frac{r}{\sqrt{10}}$ 2^{nd} quadrant

a) $\sin\theta = \frac{y}{r} = \frac{-3x}{r} = \frac{-3\left(-\frac{r}{\sqrt{10}}\right)}{r}$

$= \frac{3}{\sqrt{10}}$

b) $\cos\theta = \frac{x}{r} = \frac{-\frac{r}{\sqrt{10}}}{r} = -\frac{1}{\sqrt{10}}$

c) $\tan\theta = slope = -3$