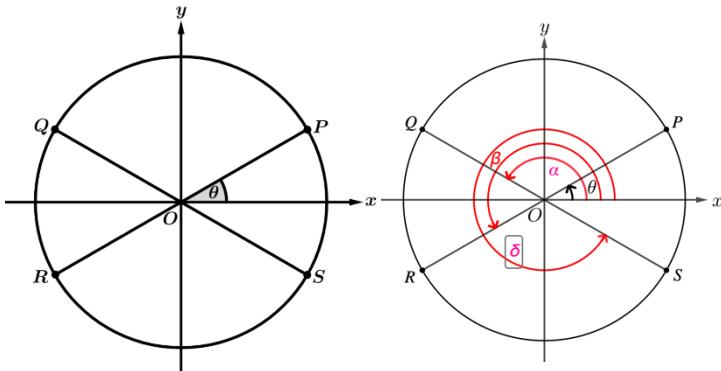


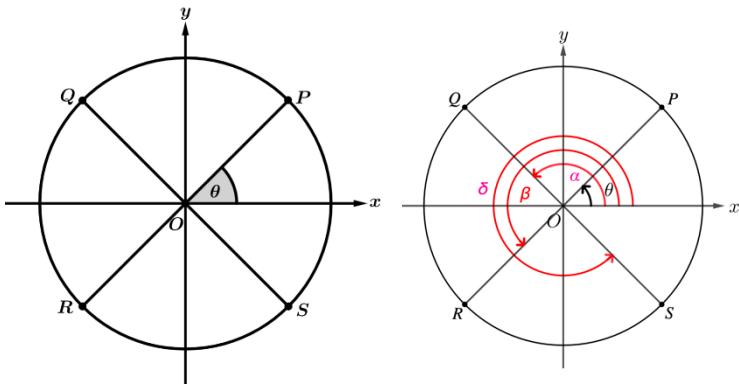
1. The figure shows a circle centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P . The measure of angle θ is $\frac{\pi}{3}$. Find the measures of the angles in standard position whose terminal ray intersects the circle at points Q, R , and S .

$$\begin{aligned} Q: \alpha &= \pi - \frac{\pi}{3} = \frac{2\pi}{3} & \text{or} & \quad \alpha = 2\left(\frac{\pi}{3}\right) = \frac{2\pi}{3} & R: \beta &= \pi + \frac{\pi}{3} = \frac{4\pi}{3} & \text{or} & \quad \beta = 4\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} & \frac{2\pi}{3} \\ S: \delta &= 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} & \text{or} & \quad \delta = 5\left(\frac{\pi}{3}\right) = \frac{5\pi}{3} \end{aligned}$$



2. The figure shows a circle centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P . The measure of angle θ is $\frac{\pi}{6}$. Find the measures of the angles in standard position whose terminal ray intersects the circle at points Q, R , and S .

$$\begin{aligned} Q: \alpha &= \pi - \frac{\pi}{6} = \frac{5\pi}{6} & R: \beta &= \pi + \frac{\pi}{6} = \frac{7\pi}{6} & S: \delta &= 2\pi - \frac{\pi}{6} = \frac{11\pi}{6} \end{aligned}$$



3. The figure shows a circle centered at the origin with an angle of measure θ in standard position. The terminal ray of the angle intersects the circle at point P . The measure of angle θ is $\frac{\pi}{4}$. Find the measures of the angles in standard position whose terminal ray intersects the circle at points Q, R , and S .

$$Q: \alpha = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad R: \beta = \pi + \frac{\pi}{4} = \frac{5\pi}{4} \quad S: \delta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

4. Let θ be an angle in standard position whose terminal ray intersects a circle centered at the origin at point P . If point P is in quadrant II, which of the following could be θ ?

(A) $\frac{\pi}{4}$ (B) $\frac{5\pi}{6}$ (C) $\frac{4\pi}{3}$ (D) $\frac{7\pi}{4}$

Quadrant II $\frac{\pi}{2} < \theta < \pi$ $\frac{\pi}{4} < \frac{\pi}{2}$ $\frac{\pi}{2} < \left(\frac{5\pi}{6} = \pi - \frac{\pi}{6}\right) < \pi$ $\frac{4\pi}{3} > \pi$ $\frac{7\pi}{4} > \pi$

5. Let θ be an angle in standard position whose terminal ray intersects a circle centered at the origin at point P . If point P is in quadrant III, which of the following could be θ ?

(A) $\frac{\pi}{3}$ (B) $\frac{3\pi}{2}$ (C) $\frac{5\pi}{4}$ (D) $\frac{11\pi}{6}$

Quadrant III $\pi < \theta < \frac{3\pi}{2}$ $\frac{\pi}{3} < \frac{\pi}{2}$ $\frac{3\pi}{2} < \frac{3\pi}{2}$ $\pi < \left(\frac{5\pi}{4} = \pi + \frac{\pi}{4}\right) < \frac{3\pi}{2}$ $\frac{11\pi}{6} > \frac{3\pi}{2}$

6. Let θ be an angle in standard position whose terminal ray intersects a circle centered at the origin at point P . If point P is in quadrant IV, which of the following could be θ ?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{2}$ (C) $\frac{5\pi}{6}$ (D) $\frac{5\pi}{3}$

Quadrant IV $\frac{3\pi}{2} < \theta < 2\pi$ $\frac{\pi}{6} < \frac{\pi}{2}$ $\frac{\pi}{2} < \frac{3\pi}{2}$ $\frac{5\pi}{6} < \pi$ $\frac{3\pi}{2} < \left(\frac{5\pi}{3} = 2\pi - \frac{\pi}{3}\right) < 2\pi$

7. Let θ be an angle in standard position whose terminal ray intersects a circle centered at the origin at point P . If point P is in quadrant I, which of the following could be θ ?

- (A) $-\frac{\pi}{6}$ (B) $\frac{11\pi}{6}$ (C) $-\frac{5\pi}{3}$ (D) $\frac{5\pi}{3}$

Quadrant I $0 < \theta < \frac{\pi}{2}$ $-\frac{\pi}{6} > -\frac{3\pi}{2}$ $\frac{11\pi}{6} > \frac{\pi}{2}$
 $-2\pi < \theta < -\frac{3\pi}{2}$ $-2\pi < \left(-\frac{5\pi}{3} = -2\pi + \frac{\pi}{3}\right) < -\frac{3\pi}{2}$ $\frac{5\pi}{3} > \frac{\pi}{2}$

8. Let θ be an angle in standard position whose terminal ray intersects a circle centered at the origin at point P . If point P is in quadrant II, which of the following could be θ ?

- (A) π (B) $\frac{2\pi}{3}$ (C) $\frac{7\pi}{6}$ (D) $\frac{7\pi}{4}$

Quadrant II $\frac{\pi}{2} < \theta < \pi$ $\pi < \frac{2\pi}{3} = \pi - \frac{\pi}{3} < \pi$ $\frac{7\pi}{6} > \pi$ $\frac{7\pi}{4} > \pi$

9. Let θ be an angle in standard position whose terminal ray intersects a circle centered at the origin at point P . If point P is in quadrant III, which of the following could be θ ?

- (A) $\frac{4\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{3}$

Quadrant III $\pi < \theta < \frac{3\pi}{2}$ $\pi < \left(\frac{4\pi}{3} = \pi + \frac{\pi}{3}\right) < \frac{3\pi}{2}$ $\frac{\pi}{2} < \pi$ $\frac{3\pi}{4} < \pi$ $\frac{5\pi}{3} > \frac{3\pi}{2}$

10. Let θ be an angle in standard position whose terminal ray intersects a circle centered at the origin at point P . If point P is in quadrant IV, which of the following could be θ ?

- (A) $\frac{7\pi}{6}$ (B) $\frac{7\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{3\pi}{2}$

Quadrant IV $\frac{3\pi}{2} < \theta < 2\pi$ $\frac{7\pi}{6} < \frac{3\pi}{2}$ $\frac{3\pi}{2} < \left(\frac{7\pi}{4} = 2\pi - \frac{\pi}{4}\right) < 2\pi$ $\frac{3\pi}{4} < \frac{3\pi}{2}$ $\frac{3\pi}{2} < \frac{3\pi}{2}$

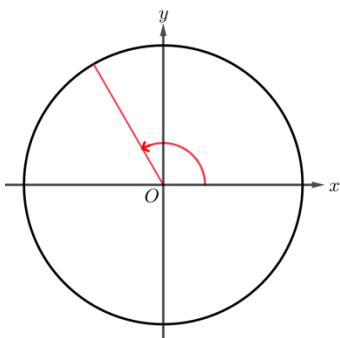
11. Let θ be an angle in standard position whose terminal ray intersects a circle centered at the origin at point P . If point P is in quadrant I, which of the following could be θ ?

- (A) $\frac{13\pi}{6}$ (B) $\frac{13\pi}{4}$ (C) $\frac{11\pi}{3}$ (D) $\frac{11\pi}{2}$

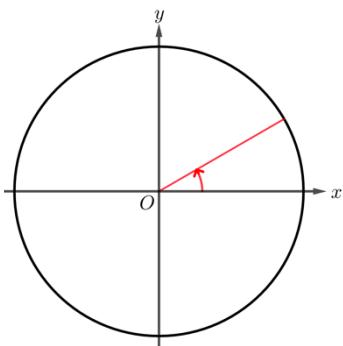
Quadrant I $0 < \theta < \frac{\pi}{2}$ $\left(\frac{13\pi}{6} = \underbrace{\frac{2\pi}{1}_{\text{period}} + \frac{\pi}{6}}\right) \Rightarrow \frac{\pi}{6} < \frac{\pi}{2}$ $\left(\frac{13\pi}{4} = \underbrace{\frac{2\pi}{1}_{\text{period}} + \frac{5\pi}{4}}\right) \Rightarrow \frac{5\pi}{4} > \frac{\pi}{2}$
 $\left(\frac{11\pi}{3} = \underbrace{\frac{2\pi}{1}_{\text{period}} + \frac{5\pi}{3}}\right) \Rightarrow \frac{5\pi}{3} > \frac{\pi}{2}$ $\left(\frac{11\pi}{2} = \underbrace{\frac{4\pi}{2}_{\text{periods}} + \frac{3\pi}{2}}\right) \Rightarrow \frac{3\pi}{2} > \frac{\pi}{2}$

Directions: For problems 12 – 20, sketch the following angles in standard position on the axes below.

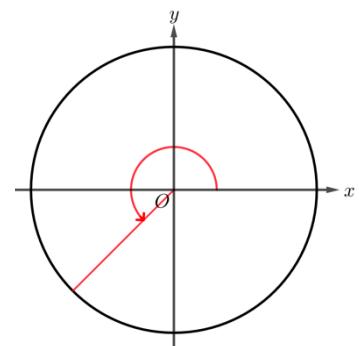
12. $\theta = \frac{2\pi}{3}$



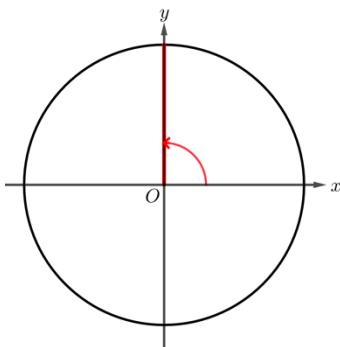
13. $\theta = \frac{\pi}{6}$



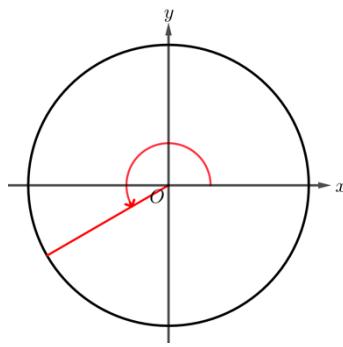
14. $\theta = \frac{5\pi}{4}$



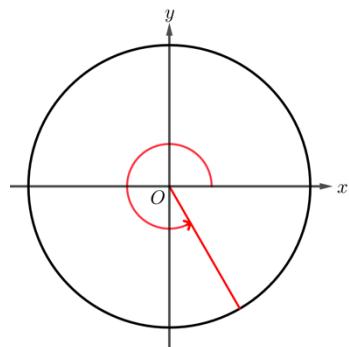
15. $\theta = \frac{\pi}{2}$



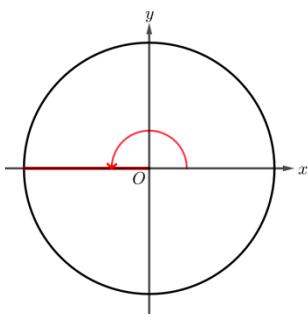
16. $\theta = \frac{7\pi}{6}$



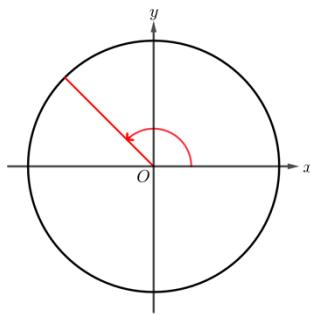
17. $\theta = \frac{5\pi}{3}$



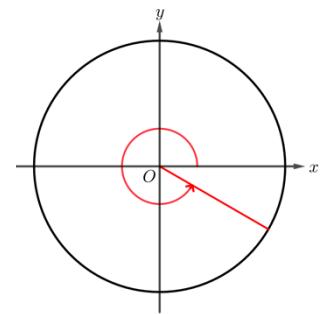
18. $\theta = \pi$



19. $\theta = \frac{3\pi}{4}$

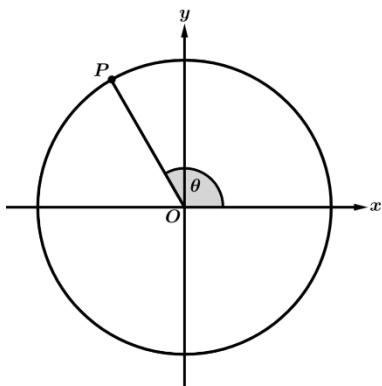


20. $\theta = \frac{11\pi}{6}$



Directions: For problems 21 – 36, determine which of the given answers could be the measure of angle θ in the figure.

21.

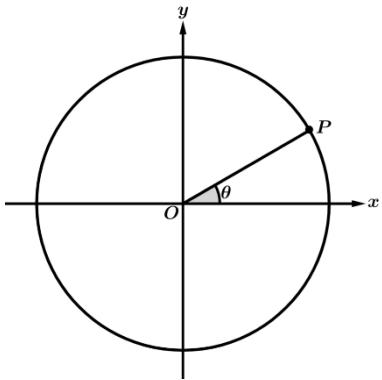


- (A) $\frac{5\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

Quadrant II $\frac{\pi}{2} < \theta < \pi$ $\frac{7\pi}{6} > \pi$ $\frac{4\pi}{3} > \pi$

$\frac{\pi}{2} < \frac{5\pi}{6} < \pi$ $\frac{\pi}{2} < \frac{2\pi}{3} < \pi$ but $\frac{5\pi}{6} > \frac{2\pi}{3}$

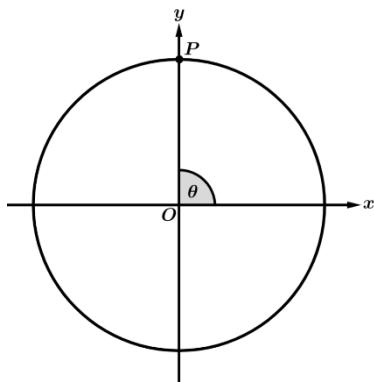
22.



- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

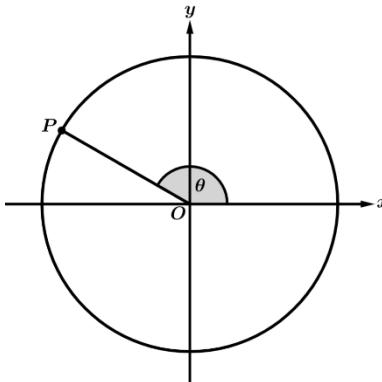
$\frac{\pi}{6} < \frac{\pi}{4} < \frac{\pi}{3} < \frac{\pi}{2}$ $\frac{\pi}{4} = \frac{1}{2} \cdot \frac{\pi}{2}$ so it cuts the rotation in half

23.



- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) $\frac{3\pi}{2}$

24.

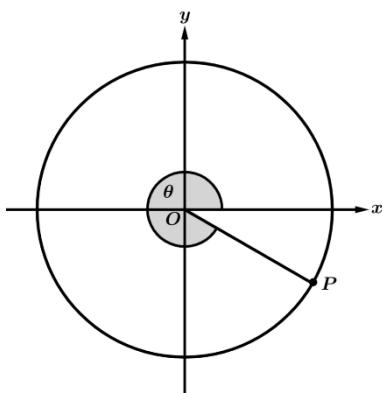


- (A) $\frac{\pi}{6}$ (B) $\frac{2\pi}{3}$ (C) $\frac{5\pi}{6}$ (D) $\frac{7\pi}{6}$

Quadrant II $\frac{\pi}{2} < \theta < \pi$ $\frac{\pi}{6} < \frac{\pi}{2}$ $\frac{7\pi}{6} > \pi$

$\frac{\pi}{2} < \frac{5\pi}{6} < \pi$ $\frac{\pi}{2} < \frac{2\pi}{3} < \pi$ and $\frac{5\pi}{6} > \frac{2\pi}{3}$

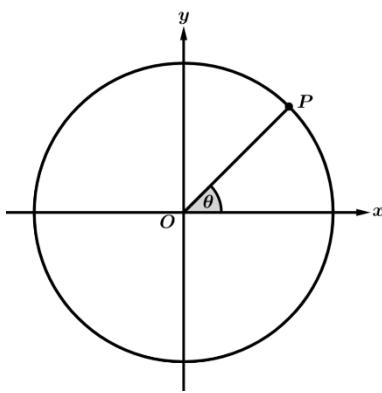
25.



- (A) $\frac{\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $\frac{5\pi}{3}$ (D) $\frac{11\pi}{6}$

Quadrant IV $\frac{3\pi}{2} < \theta < 2\pi$ $\frac{\pi}{6} < \frac{3\pi}{2}$ $\frac{7\pi}{6} < \frac{3\pi}{2}$
 $\frac{3\pi}{2} < \frac{5\pi}{3} < 2\pi$ $\frac{3\pi}{2} < \frac{11\pi}{6} < 2\pi$ but $\frac{5\pi}{3} < \frac{11\pi}{6}$

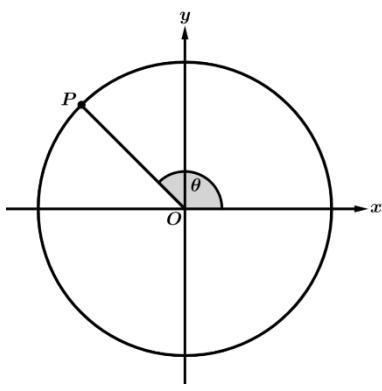
26.



- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

$\frac{\pi}{6} < \frac{\pi}{4} < \frac{\pi}{3} < \frac{\pi}{2}$ $\frac{\pi}{4} = \frac{1}{2} \cdot \frac{\pi}{2}$ so it cuts the rotation in half

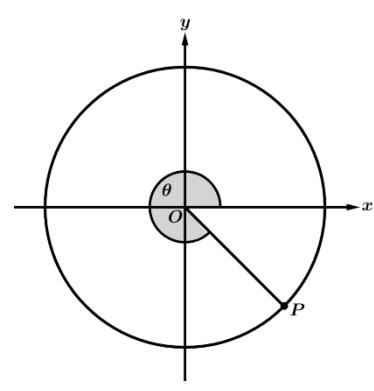
27.



- (A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) $\frac{5\pi}{4}$

$\frac{3\pi}{4}$ cuts the rotation from $\frac{\pi}{2}$ to π in half

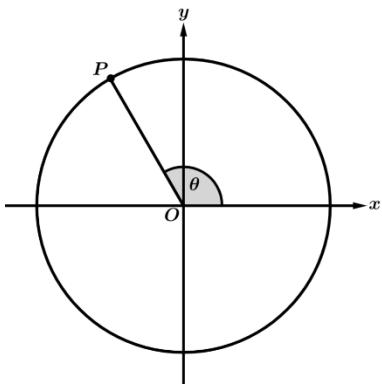
28.



- (A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

$\frac{7\pi}{4}$ cuts the rotation from $\frac{3\pi}{2}$ to 2π in half

29.

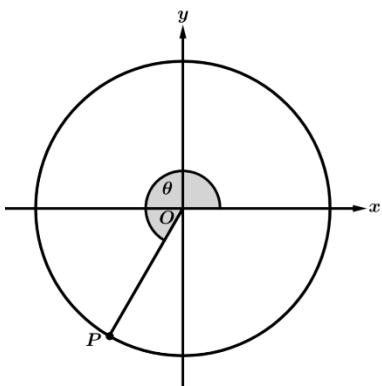


- (A) $\frac{5\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

Quadrant II $\frac{\pi}{2} < \theta < \pi$ $\frac{7\pi}{6} > \pi$ $\frac{4\pi}{3} > \pi$

$\frac{\pi}{2} < \frac{5\pi}{6} < \pi$ $\frac{\pi}{2} < \frac{2\pi}{3} < \pi$ but $\frac{5\pi}{6} > \frac{2\pi}{3}$

30.

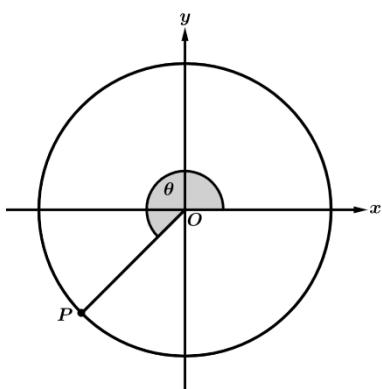


- (A) $\frac{5\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

Quadrant III $\pi < \theta < \frac{3\pi}{2}$ $\frac{5\pi}{6} < \pi$ $\frac{2\pi}{3} < \pi$

$\pi < \frac{7\pi}{6} < \frac{3\pi}{2}$ $\pi < \frac{4\pi}{3} < \frac{3\pi}{2}$ but $\frac{4\pi}{3} > \frac{7\pi}{6}$

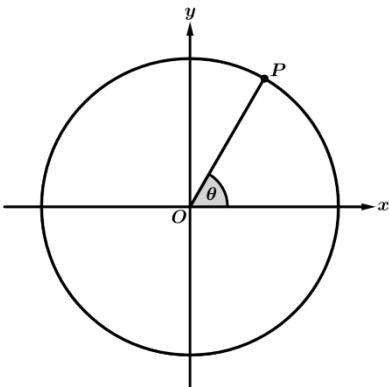
31.



- (A) $\frac{3\pi}{4}$ (B) $\frac{7\pi}{6}$ (C) $\frac{5\pi}{4}$ (D) $\frac{4\pi}{3}$

$\frac{5\pi}{4}$ cuts the rotation from π to $\frac{3\pi}{2}$ in half

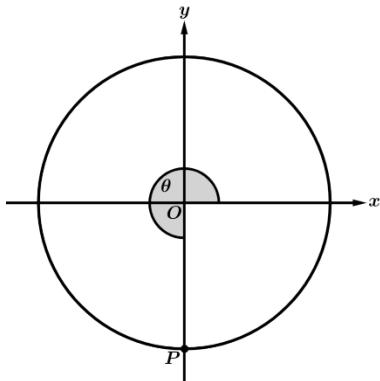
32.



- (A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

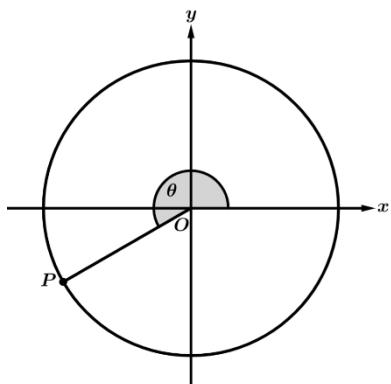
$\frac{\pi}{6} < \frac{\pi}{4} < \frac{\pi}{3} < \frac{\pi}{2}$ $\frac{\pi}{4} = \frac{1}{2} \cdot \frac{\pi}{2}$ so it cuts the rotation in half

33.



- (A) $\frac{\pi}{2}$ (B) π (C) $\frac{3\pi}{4}$ (D) $\frac{3\pi}{2}$

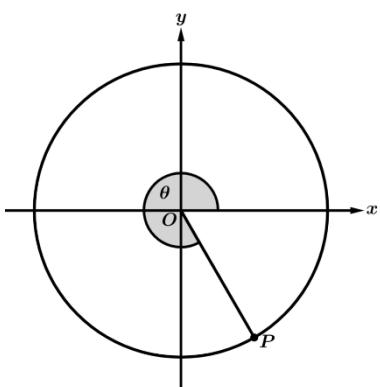
34.



- (A) $\frac{5\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{4\pi}{3}$

Quadrant III $\pi < \theta < \frac{3\pi}{2}$ $\frac{5\pi}{6} < \pi$ $\frac{2\pi}{3} < \pi$
 $\pi < \frac{7\pi}{6} < \frac{3\pi}{2}$ $\pi < \frac{4\pi}{3} < \frac{3\pi}{2}$ but $\frac{4\pi}{3} > \frac{7\pi}{6}$

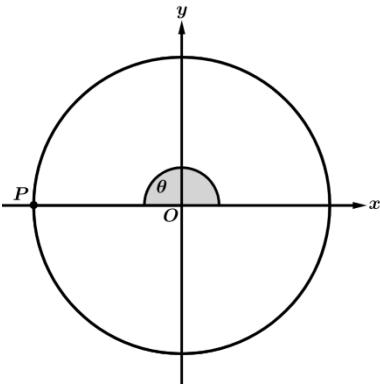
35.



- (A) $\frac{4\pi}{3}$ (B) $\frac{5\pi}{3}$ (C) $\frac{7\pi}{4}$ (D) $\frac{11\pi}{6}$

Quadrant IV $\frac{3\pi}{2} < \theta < 2\pi$ $\frac{\pi}{6} < \frac{3\pi}{2}$ $\frac{7\pi}{6} < \frac{3\pi}{2}$
 $\frac{3\pi}{2} < \frac{5\pi}{3} < 2\pi$ $\frac{3\pi}{2} < \frac{11\pi}{6} < 2\pi$ but $\frac{5\pi}{3} < \frac{11\pi}{6}$

36.



- (A) $\frac{\pi}{2}$ (B) π (C) $\frac{3\pi}{2}$ (D) 2π