Worksheet B: (Topic 1.11) Polynomial and Rational Functions Name: Solutions

Directions: Determine if the following rational functions have a horizontal asymptote, a slant asymptote, or neither.

1. $f(x) = \dfrac{2x^2 - 3x + 5}{5x^2 - 6}$

Horizontal $y = \frac{2}{5}$ because the quotient of the leading terms $\frac{2x^2}{5x^2} = \frac{2}{5}$.

2. $r(x) = \dfrac{2x^2 + 4x + 7}{6 - 5x}$

Slant because the quotient of the leading terms $\frac{2x^2}{-5x} = -\frac{2}{5}x$ is linear.

3. $h(x) = \dfrac{x^3 - 2x + 5}{3x - 4}$

Neither because the quotient of the leading terms $\frac{x^3}{3x} = \frac{1}{3}x^2$ is not linear.

4. $k(x) = \dfrac{x^4 - 3x^2 + x - 9}{2x^3 - x + 7}$

Slant because the quotient of the leading terms $\frac{x^4}{2x^3} = \frac{1}{2}x$ is linear.

5. $g(x) = \dfrac{x^2 - x - 1}{x^3 + x^2 - 2}$

Horizontal $y = 0$ because the quotient of the leading terms $\frac{x^2}{x^3} = \frac{1}{x}$ is has a constant numerator and nonconstant denominator.

6. $y = \dfrac{(x-2)^2(3x^2 + 2)}{x(x+1)(x-5)}$

Slant because the quotient of the leading terms $\frac{3x^4}{x^3} = 3x$ is linear.

Directions: For each rational function below, use long division to find the equation of the slant asymptote.

7. $f(x) = \dfrac{x^2 - 6x + 7}{x - 1}$ Slant asymptote: $y = x - 5$

$$\begin{array}{r|l} & x - 5 \\ \hline x - 1 & x^2 - 6x + 7 \\ & \underline{x^2 - x} \\ & -5x + 7 \\ & \underline{-5x + 5} \\ & 2 \end{array}$$

8. $g(x) = \dfrac{2x^2 - x + 4}{x + 3}$ Slant asymptote: $y = 2x - 7$

$$\begin{array}{r|l} & 2x - 7 \\ \hline x + 3 & 2x^2 - x + 4 \\ & \underline{2x^2 + 6x} \\ & -7x + 4 \\ & \underline{-7x - 21} \\ & 25 \end{array}$$

9. $h(x) = \dfrac{x^3 - 4x^2 + 3x - 1}{x^2 - 2x + 5}$

Slant asymptote: $y = x - 2$

$$\begin{array}{r|l} & x - 2 \\ \hline x^2 - 2x + 5 & x^3 - 4x^2 + 3x - 1 \\ & \underline{x^3 - 2x^2 + 5x} \\ & -2x^2 - 2x - 1 \\ & \underline{-2x^2 + 4x - 10} \\ & -6x + 9 \end{array}$$

10. $k(x) = \dfrac{2x^3 - x^2 + 1}{x^2 + x + 1}$ Slant asymptote: $y = 2x - 3$

$$\begin{array}{r|l} & 2x - 3 \\ \hline x^2 + x + 1 & 2x^3 - x^2 + 0x + 1 \\ & \underline{2x^3 + 2x^2 + 2x} \\ & -3x^2 - 2x + 1 \\ & \underline{-3x^2 - 3x - 3} \\ & x + 4 \end{array}$$