Notes: (Topic 2.5) Exponential Function Context and Data Modeling Solutions

Sometimes data from an exponential function does not display a proportional growth pattern. This is due to a vertical translation of the function.

However, if we can add/subtract a constant from the output values (y) to reveal a proportional growth pattern, then the initial function was also exponential.

Example 1: Selected values from several exponential functions are given in the tables below. For each, find the constant value that can be added/subtracted to the output values to reveal a proportional growth pattern. Then, write an equation to model the function.

a)

x	$f(x)$	$f(x)-1$	
0	7	6	$\times 2$
1	13	12	$\times 2$
2	25	24	$\times 2$
3	49	48	$\times 2$
4	97	96	

$f(x)-1=6(2)^x$
$f(x)=6(2)^x+1$

b)

x	$g(x)$	$g(x)+2$	
0	2	4	$\times 3$
1	10	12	$\times 3$
2	34	36	$\times 3$
3	106	108	$\times 3$
4	322	324	

$g(x)+2=4(3)^x$
$g(x)=4(3)^x-2$

c)

x	$h(x)$	$h(x)+1$	
0	63	64	$\div 2$
1	31	32	$\div 2$
2	15	16	$\div 2$
3	7	8	$\div 2$
4	3	4	

$h(x)+1=64(2^{-1})^x=64(2)^{-x}$
$h(x)=64(2)^{-x}-1$

Modeling Exponential Functions from Data

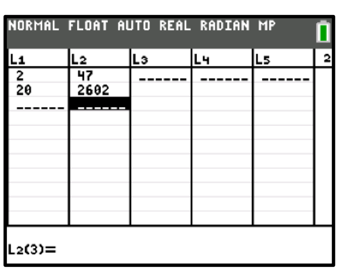
An exponential model can be constructed if we are given either

1. The **common ratio** and the **initial value** , or
2. **Two input-output pairs** (we can find the initial value and the base/common ratio with these two pairs)

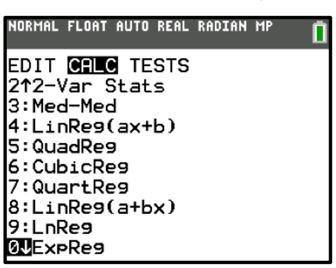
Example 2: For $0 \le t \le 30$ minutes, the number of fans inside a large sports stadium can be modeled by the exponential function $F(t)=a(b)^t$ where F represents the number of fans inside the stadium t minutes after the doors open. There are 47 people in the stadium 2 minutes after the doors open, and there are 2602 fans inside the stadium 20 minutes after the doors open.
Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $F(t)$. $F(2)=a(b)^2=47$ $\quad F(20)=a(b)^{20}=2602$

Using the TI-84 to Create an Exponential Regression Model

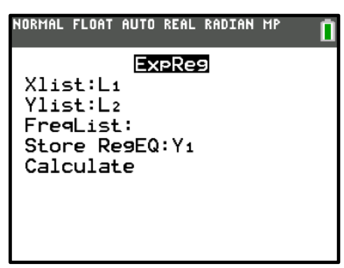
Enter data in L1 and L2



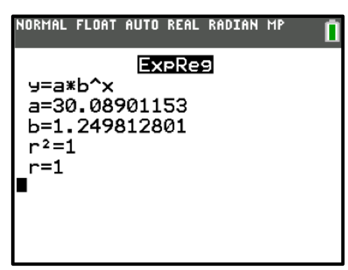
Select 0: ExpReg



Enter L1, L2 and Y_1



Get regression equation

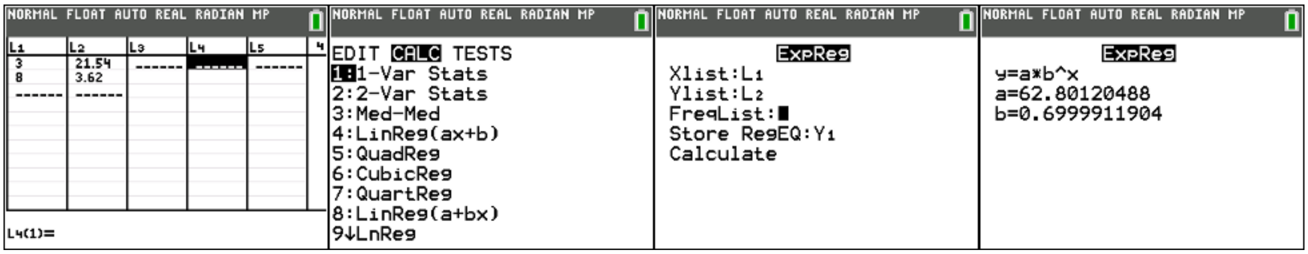


x	3	8
$g(x)$	21.54	3.62

Example 3: Selected values of the exponential function g are shown in the table above, where $g(x) = ab^x$.

(a) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $g(x)$. $g(3) = 21.54 = ab^3$ $\quad$ $g(8) = 3.62 = ab^8$

(b) Find the values of a and b. $a = 62.8012 \ldots$ $\quad b = 0.6999 \ldots$



The Natural Base: e
$e \approx 2.718$ The natural number e is often used as the base in exponential functions that model contextual scenarios.

Example 4: The number of bacteria B in a particular culture can be modeled by the exponential function $B(t) = 17e^{0.31t}$, where t is measured in hours.

(a) Let $A(t) = a \cdot b^t$ be an equivalent expression for $B(t)$. Find the value of b.
$b = e^{0.31} \Rightarrow b = 1.3634 \ldots$

(b) Find the average rate of change of $B(t)$ over the interval from $t = 1$ to $t = 7$ hours.

$$\text{AROC} = \frac{B(7) - B(1)}{7 - 1} = 20.9521 \ldots$$

(c) Find the number of bacteria B predicted by the model after 15 hours. $B(15) = 1777.9447 \ldots$

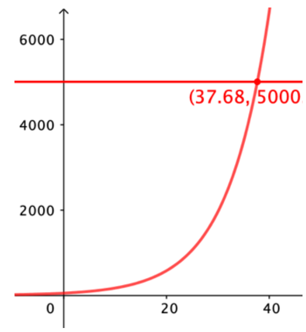
Example 5: After 50 deer were introduced to a large wooded area, the population of deer in the wooded area can be modeled by the exponential function $D(t) = a \cdot b^t$, where t represents the number of years since deer were first introduced to the wooded area. The population of deer increases by 13% each year.

(a) Find the values of a and b, and use these values to write an expression for $D(t)$.

$D(0) = a \cdot b^0 = 50 \Rightarrow a = 50$ $\qquad$ $D(1) = 50b^1 = 50(1.13) \Rightarrow b = 1.13$

(b) According to the model found in part (a), what is the population of deer in the large wooded area after 6 years?

$D(t) = 50 \cdot (1.13)^t$ $\qquad$ $D(6) = 50 \cdot (1.13)^6 = 104.0975 \ldots$ deer



(c) Find the time t, when the population of deer in the wooded area is expected to reach 5000?

$D(t) = 50 \cdot (1.13)^t = 5000$ $\qquad$ $t = 37.68008 \ldots$

(d) Find an equivalent expression for $D(t)$ where t is measured in months (instead of years). $D(t) = 50 \cdot (1.13)^{t/12}$