Directions: Evaluate the following expressions.

1. $\cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$

2. $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = -\dfrac{\pi}{4}$

3. $\tan^{-1}(1) = \dfrac{\pi}{4}$

4. $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) = \dfrac{5\pi}{6}$

5. $\sin^{-1}\left(-\dfrac{1}{2}\right) = -\dfrac{\pi}{6}$

6. $\tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right) = -\dfrac{\pi}{6}$

Directions: Solve the following equations.

7. $\sin^{-1}(x) = \cos^{-1}(0)$

$\sin^{-1}(x) = \dfrac{\pi}{2}$ $x = \sin\left(\dfrac{\pi}{2}\right) = 1$

8. $2\sin^{-1}(x) = \cos^{-1}\left(-\dfrac{1}{2}\right)$

$2\sin^{-1}(x) = \dfrac{2\pi}{3}$ $\sin^{-1}(x) = \dfrac{\pi}{3}$

$x = \sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$

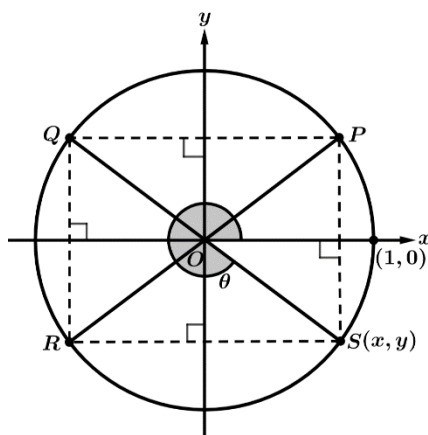9. $\tan^{-1}(\pi x) = \sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$

$\tan^{-1}(x\pi) = -\dfrac{\pi}{4}$ $\pi x = \tan\left(-\dfrac{\pi}{4}\right) = -1$

$x = -\dfrac{1}{\pi}$

10. $3\sin^{-1}\left(\dfrac{x}{2}\right) = \cos^{-1}(-1)$

$3\sin^{-1}\left(\dfrac{x}{2}\right) = \pi$ $\sin^{-1}\left(\dfrac{x}{2}\right) = \dfrac{\pi}{3}$

$\dfrac{x}{2} = \sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$ $x = \sqrt{3}$

11. The angle θ is in standard position. The terminal ray intersects the unit circle at point S, whose coordinates are (x, y). The points $P, Q,$ and R are the result of the terminal ray being reflected over the y-axis, the origin, and the x-axis, respectively. For each of the following expressions, determine which labeled point intersects the terminal ray of the given angles.

a) $\sin^{-1}(-y)$ *P*

Range: $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

What rotation (angle) has a sine value (second coordinate) of $-y$? Which labeled point intersects the terminal ray of this angle? *P*
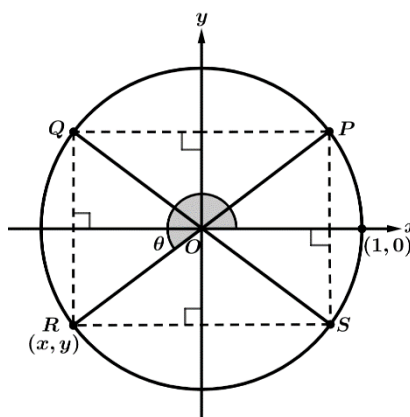
b) $\cos^{-1}(-x)$ *Q*

Range: $[0, \pi]$

What rotation (angle) has a cosine value (first coordinate) of $-x$? Which labeled point intersects the terminal ray of this angle? *Q*

c) $\tan^{-1}\left(\dfrac{y}{x}\right)$ *S*

Range: $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

What rotation (angle) has a slope $\dfrac{y}{x}$? Which labeled point intersects the terminal ray of this angle? *S*

12. The angle θ is in standard position. The terminal ray intersects the unit circle at point R, whose coordinates are (x, y). The points $P, Q,$ and S are the result of the terminal ray being reflected over the y-axis, the origin, and the x-axis respectively. For each of the following expressions, determine which labeled point intersects the terminal ray of the given angles. Be careful to consider the range!

a) $\sin^{-1}(y)$ *S*

b) $\cos^{-1}(x)$ *Q*

c) $\tan^{-1}\left(\dfrac{y}{x}\right)$ *P*

d) $\sin^{-1}(-y)$ *P*

e) $\cos^{-1}(-x)$ *P*

f) $\tan^{-1}\left(-\dfrac{y}{x}\right)$ *S*

Worksheet A: Topic 3.9   Inverse Trigonometric Functions   Created by Bryan Passwater

Solutions by Ted Gott tedg20776@gmail.com