Worksheet A: (Topic 2.12) Logarithmic Function Manipulation Name: Solutions

Directions: For each of the following, write an equivalent expression using a single logarithm.

1. $\log_3 4 + \log_3 z$
$= \log_3(4z)$

2. $\ln x - \ln w$
$= \ln\left(\frac{x}{w}\right)$

3. $\log_{10} 3 - \log_{10} x - \log_{10} 5$
$= \log_{10}\left(\frac{3}{5x}\right)$

4. $\log_4 6 + 3\log_4 x$
$= \log_4(6x^3)$

5. $5\ln x + 2\ln y - \ln w$
$= \ln\left(\frac{x^5y^2}{w}\right)$

6. $x\log 2 - 3\log x - \frac{1}{2}\log y$
$= \log\left(\frac{2^x}{x^3y^{1/2}}\right) = \log\left(\frac{2^x}{x^3\sqrt{y}}\right)$

7. Which of the following expressions is equivalent to $\log_5\left(\frac{3y^4}{z^2}\right)$, where y and z are positive constants?

(A) $\log_5 3 + \log_5 4y - \log_5 2z$

(B) $3 + 4\log_5 y - 2\log_5 z$

(C) $\log_5 3 + 4\log_5 y - 2\log_5 z$

(D) $\frac{\log_5 3 \cdot \log_5 y^4}{\log_5 z^2}$

8. Which of the following expressions is equivalent to $\ln\left(\frac{5e^2}{\sqrt{x}}\right)$, where x is a positive constant?

(A) $2 + \ln 5 - \frac{1}{2}\ln x$ $\quad \ln 5 + 2\underbrace{\ln e}_{1} - \frac{1}{2}\ln x$

(B) $2\ln 5 - \frac{1}{2}\ln x$

(C) $2 + \ln 5 - 2\ln x$

(D) $2\ln 5 - 2\ln x$

9. Which of the following is equivalent to $3\log x - 2\log y + \frac{1}{2}\log 16$, where x and y are positive constants?

(A) $\log\left(\frac{x^3 + 4}{y^2}\right)$ (B) $\log\left(\frac{x^3}{4y^2}\right)$ (C) $\log\left(\frac{4x^3}{y^2}\right)$ (D) $\log\left(\frac{8x^3}{y^2}\right)$

$\log x^3 - \log y^2 + \log 16^{1/2} = \log x^3 - \log y^2 + \log 4$

10. Which of the following expressions is equivalent to $\log_5 x$, where x is a positive constant?

(A) $5\log x$ (B) $\dfrac{\log 5}{\log x}$ (C) $\dfrac{\log x}{\log 5}$ (D) $\log x - \log 5$

$y = \log_5 x \Rightarrow 5^y = x \quad \log 5^y = \log x \quad y \log 5 = \log x \quad y = \dfrac{\log x}{\log 5}$

11. Which of the following expressions is equivalent to $\log_3 7$?

(A) $\ln 7 - \ln 3$ (B) $\dfrac{\log_7 10}{\log_3 10}$ (C) $\dfrac{\log_{10} 3}{\log_{10} 7}$ (D) $\dfrac{\ln 7}{\ln 3}$

$y = \log_3 7 \Rightarrow 3^y = 7 \quad \ln 3^y = \ln 7 \quad y \ln 3 = \ln 7 \quad y = \dfrac{\ln 7}{\ln 3}$

12. Let $f(x) = \log_6 x$. Which of the following is equivalent to $f(w) - 3 \cdot f(z)$, where w and z are positive constants?

(A) $\log_6 (w - z^3)$ (B) $\log_6 \left(\dfrac{w}{z^3}\right)$ (C) $\log_6 \left(\dfrac{w}{z}\right)^3$ (D) $\log_6 \left(\dfrac{w}{3z}\right)$ (E) $\log_6 (wz^3)$

13. Let $f(x) = \log(x)$ and let $g(x) = \log(4x)$. Which of the following statements about the graphs of f and g is correct?

(A) The graph of g is a horizontal translation of the graph of f by -4 units.

(B) The graph of g is a horizontal dilation of the graph of f by a factor of $\frac{1}{4}$.

(C) The graph of g is a vertical translation of the graph of f by 4 units.

(D) The graph of g is a vertical dilation of the graph of f by a factor of 4.

1.12.A.4 The function $g(x) = f(bx)$, where $b \neq 0$, is a multiplicative transformation of the function f that results in a horizontal dilation of the graph of f by a factor of $\left|\frac{1}{b}\right|$. If $b < 0$, the transformation involves a reflection over the y-axis.

14. Let $h(x) = \ln(x)$ and let $k(x) = \ln(8x)$. Which of the following statements about the graphs of h and k is correct?

(A) The graph of k is a horizontal dilation of the graph of h by a factor of 8.

(B) The graph of h is a horizontal dilation of the graph of k by a factor of 8.

(C) The graph of k is a vertical translation of the graph of h by -8 units.

(D) The graph of h is a vertical translation of the graph of k by -8 units.

The graph of k is a horizontal dilation of h by a factor of $\frac{1}{8}$. $h(x) = k\left(\frac{1}{8}(8x)\right)$ is a horizontal dilation of the graph of k by a factor of 8.