Notes: (Topic 1.4) Polynomial Functions and Rates of Change Solutions

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Polynomial Functions

A polynomial function is any function representation equivalent to the analytical form:

$$p(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0,$$

where n is a positive integer, a_i is a real number for each i from 1 to n, and a_n is nonzero.

Leading Term: a_nx^n **Degree**: n **Leading Coefficient**: a_n

Example 1: Find the leading coefficient and degree of the following polynomial functions.

a) $f(x) = 3x^4 + 2x - 7$

Leading Coefficient: 3

Degree: 4

b) $y = 12x - 7x^3 + 11$

Leading Coefficient: -7

Degree: 3

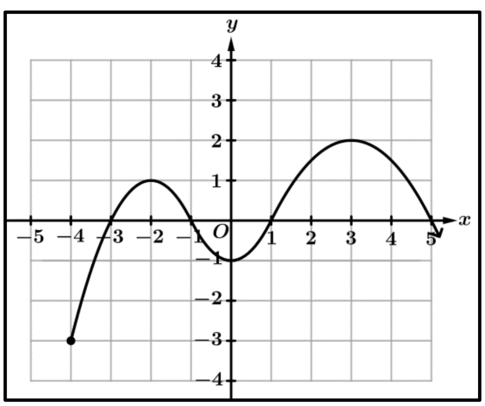
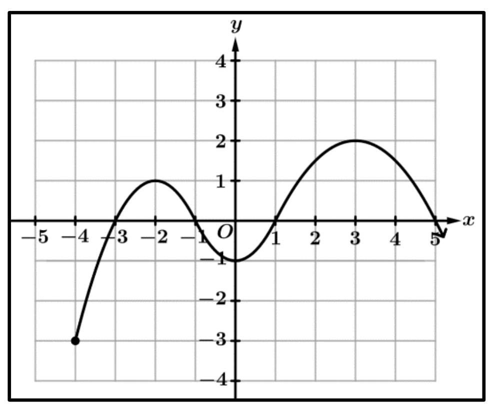
c) $g(x) = 4 = 4x^0$

Leading Coefficient: 4

Degree: 0

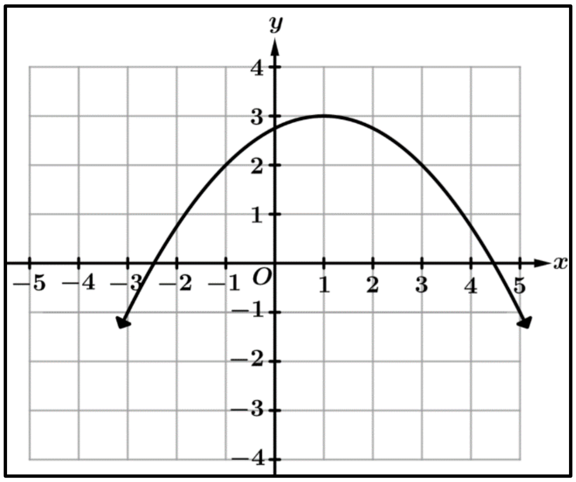
Extrema

The extrema of a graph are the minimums and maximums of a function. There are 2 types of extrema.

Relative Extrema (Local)	**Absolute Extrema (Global)**
A polynomial has a relative minimum or relative maximum where the polynomial switches between decreasing and increasing (or at an endpoint if the polynomial has a restricted domain) .	Of all local maxima, the greatest is called the absolute maximum. The least of all local minima is called the absolute minimum.
 Local minimums at $x =$ $-4, 0$ Local maximums at $x =$ $-2, 3$	 Absolute maximum = 2 at $x =$ 3 Absolute minimum = None at $x =$ ____ As x increases without bound, the output values decrease without bound.

Example 2: Find and classify each type of extrema for the functions below or write N/A .

a)



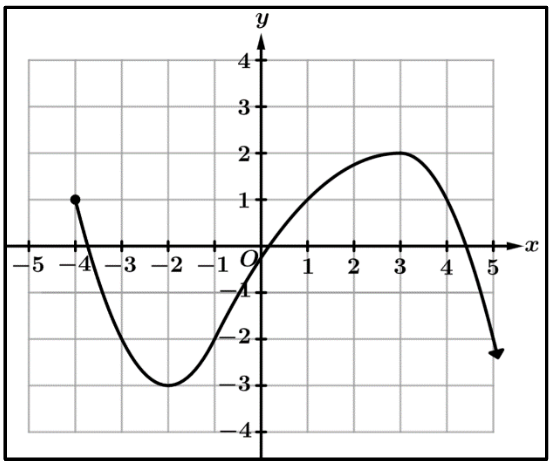
Relative Minimum at $x =$ N/A

Relative Maximum at $x =$ 1

Absolute Minimum $=$ N/A at $x =$ ______

Absolute Maximum $=$ 3 at $x =$ 1

b)



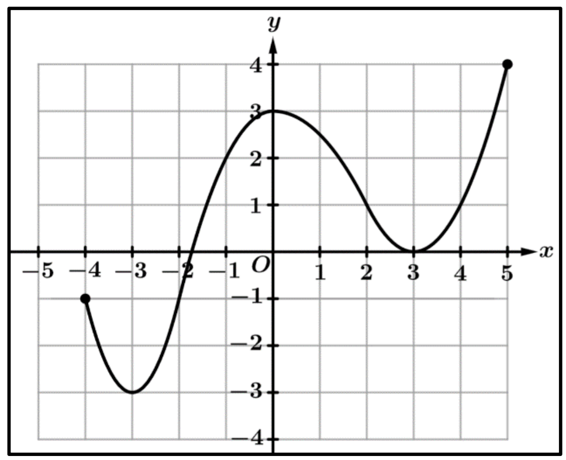
Relative Minimum at $x =$ -2

Relative Maximum at $x =$ $-4, 3$

Absolute Minimum $=$ N/A at $x =$ ______

Absolute Maximum $=$ 2 at $x =$ 3

c)



Relative Minimum at $x =$ $-3,3$

Relative Maximum at $x =$ $-4,0,5$

Absolute Minimum $=$ -3 at $x =$ -3

Absolute Maximum $=$ 4 at $x =$ 5

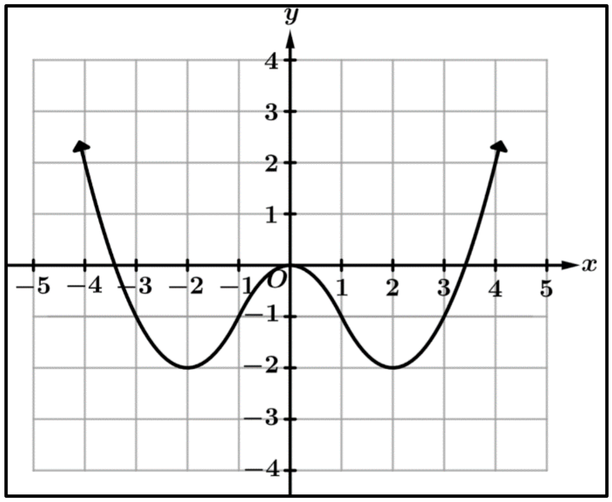
Fun Facts About Polynomials

1. Between 2 real zeros of a polynomial, there must be at least one local maximum or local minimum.

2. Polynomials of even degree must have either a global maximum or a global minimum.

Points of Inflection

A point of inflection occurs when a function changes from concave up to concave down or from concave down to concave up.
At a point of inflection, the rate of change of a function changes from increasing to decreasing or from decreasing to increasing.



Example 3: The graph of $g(x)$ is shown in the figure above. Use the graph of g to answer the following.

a) Find any values of x where g has a point of inflection.

$x = -1$ and $x = 1$

b) For each of the following intervals, determine if the rate of change of g is increasing or decreasing. Explain your reasoning for each answer using features of the graph of $g(x)$.

i. $(3,\ 4)$
The rate of change of $g(x)$ is increasing because $g(x)$ is concave up and the graph is getting steeper as the output values are increasing.

ii. $(-4, -3)$
The rate of change of $g(x)$ is increasing because $g(x)$ is concave up and the graph is getting less steep as the output values are decreasing. Negative slopes are increasing as the absolute values are decreasing.

iii. $(-1,\ 1)$
The rate of change of $g(x)$ is decreasing because $g(x)$ is concave down and graph is getting less steep as the output values are increasing and getting steeper as the output values are decreasing. Negative slopes are decreasing as the absolute values are increasing.

iv. $(1,\ 2)$
The rate of change of $g(x)$ is increasing because $g(x)$ is concave up and the graph is getting less steep as the output values are decreasing. Negative slopes are increasing as the absolute values are decreasing.

Example 4: For $0 \le t \le 3$, the number of cars in a parking lot at time t hours can be modeled by the function $C(t) = -1.37t^5 + 4.218t^4 - 0.357t^2 + 3$. Based on this model, at what time t does the number of cars in the parking lot change from increasing to decreasing?

At time $t = 2.4456\ldots$ hours, there is a maximum number of cars in the lot and the number of cars in the parking lot changes from increasing to decreasing.