x	$f(x)$
1	2
6	10

1. Selected values of the logarithmic function f are given in the table above, where $f(x) = a + b\ln x$.

a) Use the data to write two equations that can be used to find the values for constants a and b in the expression for $f(x)$.

$f(1) = a + b\ln 1 = 2$ $\quad$ $f(6) = a + b\ln 6 = 10$

b) Find the values of a and b.

$a + b\ln 1 = a + b(0) = a = 2$ $\quad$ $2 + b\ln 6 = 10$

$b\ln 6 = 8 \qquad b = \dfrac{8}{\ln 6} = 4.4648 \ldots$

x	$g(x)$
3	11
10	14.5

2. Selected values of the logarithmic function g are given in the table above, where $g(x) = a + b\ln x$.

a) Use the data to write two equations that can be used to find the values for constants a and b in the expression for $g(x)$.

$g(3) = a + b\ln 3 = 11$ $\quad$ $g(10) = a + b\ln 10 = 14.5$

b) Find the values of a and b.

$$\begin{array}{r} a + b\ln 10 = 14.5 \\ \underline{-a - b\ln 3 = -11} \\ b\ln 10 - b\ln 3 \;\; = 3.5 \end{array}$$

$b(\ln 10 - \ln 3) \;\; = 3.5$

$b = \dfrac{3.5}{\ln \frac{10}{3}} = 2.90704 \ldots$

$a = 11 - b\ln 3 = 7.8062 \ldots$

x	$h(x)$
1	-2
16	3

3. Selected values of the logarithmic function h are given in the table above, where $h(x) = a + b\log_4 x$.

a) Use the data to write two equations that can be used to find the values for constants a and b in the expression for $h(x)$.

$h(1) = a + b\log_4 1 = -2$ $\quad$ $h(16) = a + b\log_4 16 = 3$

b) Find the values of a and b.

$a + b\log_4 1 = a + b(0) = a = -2$ $\quad$ $-2 + b\log_4 16 = -2 + b\log_4 4^2 = -2 + 2b = 3$

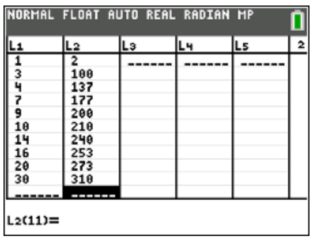
$-2 + 2b = 3 \quad 2b = 5 \qquad b = \dfrac{5}{2}$

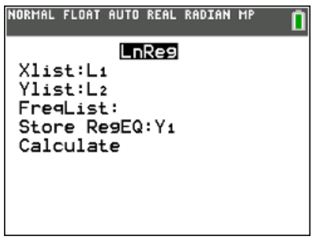
Solutions by Ted Gott tedg20776@gmail.com

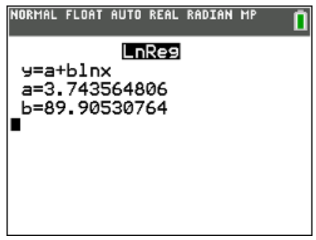
Days	1	3	4	7	9	10	14	16	20	30
No. of Downloads (in thousands)	2	100	137	177	200	210	240	253	273	310

4. After Mr. Passwater drops his latest diss track, it begins to get downloaded rapidly. The number of downloads (in thousands) t days after his diss track drops can be modeled by the logarithmic function $D(t)$.
The table above shows the total number of downloads at selected values of t.

a) Use the regression capabilities on your graphing calculator to find a logarithmic function model $D(t) = a + b\ln t$, where D represents the total number of downloads, in thousands, t days after Mr. Passwater drops his diss track.





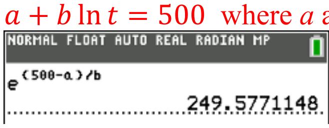


$D(t) = 3.7435\ldots + 89.9053\ldots\ln t$

b) Using the model found in part a, how many days will it take for Mr. Passwater to reach 500,000 total downloads?

$D(t) = \underbrace{3.7435\ldots}_{a} + \underbrace{89.9053\ldots}_{b}\ln t = 500$

$a + b\ln t = 500$ where a and b are stored values in the calculator.



$\ln t = \frac{500 - a}{b} =$

$t = e^{\frac{500-a}{b}} = 249.5771\ldots$

5. The most common method to measure the magnitude is the Richter scale, developed by Charles Richter in 1935. The Richter scale gives output values (magnitude of the earthquake) based on the maximum ground displacement measured by a seismograph that is a given distance away from the epicenter of the earthquake. However, the Richter scale modeled based on data specific to Southern California and is not always a reliable way to measure earthquakes. Several improved models have been introduced since 1935, including the Lillie Empirical Formula. If a seismograph is positioned 200 km away from the epicenter of an earthquake, the Lillie Empirical Formula can be modeled by

$$M_L = a + b\log x,$$

where x represents the maximum ground displacement measured by the seismograph measured in microns (μm).

For a seismograph positioned 200 km away from the epicenter, an earthquake of magnitude $M_L = 5.2$ will create a maximum ground displacement of 21 μm, and an earthquake of magnitude $M_L = 6.1$ will create a maximum displacement of 180 μm.

a) Write two equations that can be used to find the values for constants a and b in the expression for M_L.

$M_L(21) = a + b\log 21 = 5.2$ $\quad\quad$ $M_L(180) = a + b\log 180 = 6.1$

b) Find the values of a and b.

$$\begin{array}{r} a + b\log 180 = 6.1 \\ -a - b\log 21 = -5.2 \\ \hline b\log 180 - b\log 21 \quad = 0.9 \end{array}$$

$b(\log 180 - \log 21) \quad = 0.9$

$b = \dfrac{0.9}{\log\frac{180}{21}} = 0.9645\ldots$

$a = 5.2 - b\log 21 = 3.9246\ldots$

c) The largest recorded earthquake in history was the Great Chilean Earthquake on May 22, 1960. This earthquake had a magnitude of $M_L = 9.5$! Using the model found in part b, what was the maximum ground displacement , in microns, measured by a seismograph positioned 200 km from the epicenter?

$M_L(x) = a + b\log x = 9.5$ $\quad\quad$ $\log x = \dfrac{9.5 - a}{b}$ $\quad\quad$ $x = 10^{\frac{9.5-a}{b}} = 602754.1216\ldots\,\mu m$