Worksheet A: (Topic 2.7) Composition of Functions   Name: Solutions

Directions: Use the given functions below to evaluate the following, if possible.

$f(x) = 4x - 5$ $g(x) = x^2 - 2x + 4$ $h(x) = 3(2)^x$ $k(x) = 3 - 2x$

1. $f(g(1)) = f(1^2 - 2 \cdot 1 + 4)$
$= f(3) = 4(3) - 5 = 7$

2. $g(f(0)) = g(-5)$
$= (-5)^2 - 2(-5) + 4 = 39$

3. $h(k(2)) = h(3 - 2 \cdot 2) = h(-1)$
$= 3(2)^{-1} = \frac{3}{2}$

4. $f(f(-1)) = f(4(-1) - 5)$
$= f(-9) = 4(-9) - 5 = -41$

5. $h(h(0)) = h(3(2)^0) = h(3)$
$= 3(2)^3 = 24$

6. $(g \circ k)(4) = g(3 - 2 \cdot 4)$
$= g(-5) = 39$
See #2

7. $k(f(x)) = k(4x - 5)$
$= 3 - 2(4x - 5) = 13 - 8x$

8. $(f \circ g)(x) = f(x^2 - 2x + 4)$
$= 4(x^2 - 2x + 4) - 5$
$= 4x^2 - 8x + 16 - 5$
$= 4x^2 - 8x + 11$

9. $g(f(x)) = g(4x - 5)$
$= (4x - 5)^2 - 2(4x - 5) + 4$
$= 16x^2 - 40x + 25 - 8x + 10 + 4$
$= 16x^2 - 48x + 39$

x	4	5	6	7	8
$f(x)$	135	45	15	5	5/3

10. Let f be a function defined for all real numbers. The table gives values for $f(x)$ at selected values of x. The function g is given by $g(x) = \dfrac{x^2 - 3}{7x + 23}$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(6)$ as a decimal approximation or indicate that it is not defined.

$h(6) = g(f(6)) = g(15) = \dfrac{15^2 - 3}{7 \cdot 15 + 23} = \dfrac{222}{128} = 1.7343\ldots$

(ii) Find all values of x for which $f(x) = 5$, or indicate there are no such values.

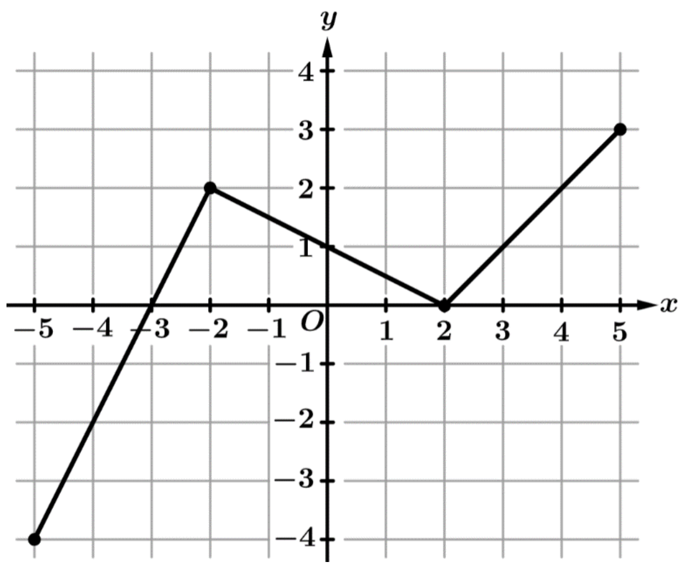
$f(x) = 5 \rightarrow x = 7$

x	4	5	6	7	8
$f(x)$	135	45	15	5	5/3

$\div 3$ $\div 3$ $\div 3$ $\div 3$

(C) (i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, exponential, or logarithmic function. exponential

(ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f. The best model for f is exponential because over equal-length input-value intervals the output values of a function change proportionally.



Graph of f

11. The function f is defined for $-5 \le x \le 5$, and consists of three line segments, as shown in the figure. The function g is given by $g(x) = 1.57x^3 - 2.07x^2 + 5.62$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation or indicate that it is not defined.

$h(3) = g(f(3)) = g(1) = 1.57(1)^3 - 2.07(1)^2 + 5.62 = 5.12$

(ii) Find all values of x for which $f(x) = 2$, or indicate there are no such values.

$f(x) = 2 \rightarrow x = -2, 4$

x	2	5	8	11	14
$f(x)$	-1	6	11	14	15

7 5 3 1

2 2 2

12. The domain of f consists of the five real numbers 2, 5, 8, 11, and 14. The table defines the function f for these values. The function g is given by $g(x) = 1.3(0.9)^x$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(11)$, as a decimal approximation, or indicate that it is not defined.

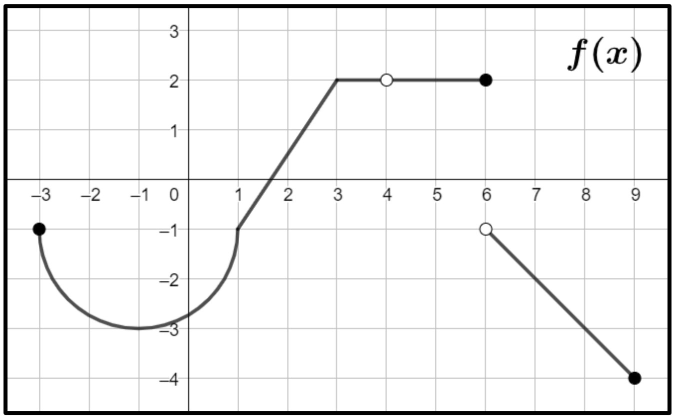
$h(11) = g(f(11)) = g(14) = 1.3(0.9)^{14} = 0.2973..$

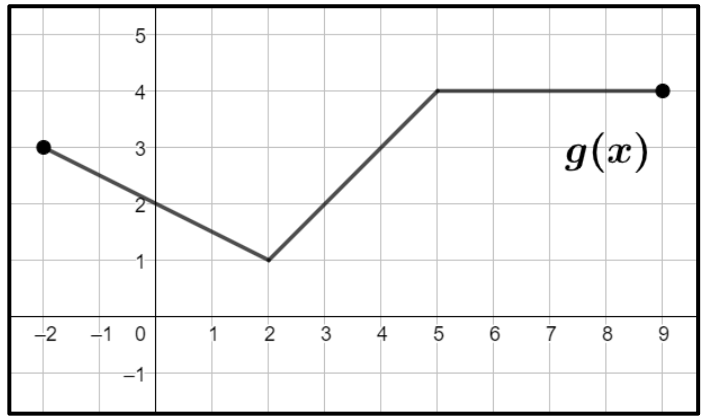
(ii) Find all values of x for which $f(x) = 8$, or indicate there are no such values.

There are no such values because 8 is not in the range of f.

(C) (i) Use the table of values of $f(x)$ to determine if f is best modeled by a linear, quadratic, exponential, or logarithmic function. Quadratic

(ii) Give a reason for your answer based on the relationship between the change in the output values of f and the change in the input values of f. For a quadratic function, since the average rates of change over consecutive equal-length input-value intervals can be given by a linear function, these average rates of change for a quadratic function are changing at a constant rate or the second differences are constant.





x	-3	-1	2	6	9
$p(x)$	$f(6)$	e	-1	1	3

$$h(x) = \begin{cases} 8\left(\frac{1}{2}\right)^x, & x < 2 \\ 1 - x^2, & x = 2 \\ 4, & x > 3 \end{cases}$$

The function m is is the result of applying three transformations to the graph of g in this order: a vertical dilation by a factor of 2, a vertical translation by -3 units, and a horizontal translation by 1 unit.

Directions: Use the given information above to evaluate the following, if possible.

13. $f(g(4)) = f(3) = 2$

14. $(g \circ f)(6) = g(2) = 1$

15. $(g \circ g)(-2) = g(3) = 2$

16. $p(f(\pi)) = p(2) = -1$

17. $(f \circ g)(8) = f(4)$ undefined

18. $(g \circ h)(0) = g\left(8\left(\frac{1}{2}\right)^0\right) = g(8)$
$= 4$

19. $h(f(6)) = h(2) = 1 - 2^2 = -3$

20. $(p \circ f)(-3) = p(-1) = e$

21. $(h \circ p)(6) = h(1) = 8\left(\frac{1}{2}\right)^1 = 4$

22. $m(h(7)) = m(4)$
$= 2g(4-1) - 3 = 2g(3) - 3$
$= 2(2) - 3 = 1$

23. $(m \circ m)(-1) =$
$= m(2g(-1-1) - 3) =$
$= m(2g(-2) - 3) = m(2(3) - 3)$
$= m(3) = 2g(3-1) - 3$
$= 2g(2) - 3 = 2(1) - 3 = -1$

24. $(p \circ f)(8) = p(-3) = f(6) = 2$

25. $f(m(1)) = f(2g(1-1) - 3)$
$= f(2g(0) - 3) = f(2(2) - 3)$
$= f(1) = -1$

26. $(m \circ f \circ p)(9) = m(f(3))$
$= m(2) = 2g(2-1) - 3$
$= 2g(1) - 3 = 2\left(\frac{3}{2}\right) - 3 = 3 - 3$
$= 0$

27. $h(h(2)) = h(-3) = 8\left(\frac{1}{2}\right)^{-3}$
$= 8(2)^3 = 64$