

Directions: Write limit statements for the end behavior of the following logarithmic functions.

1. $f(x) = 3\log_2 x$

$a > 0$ and $b > 1$

Left: $\lim_{x \rightarrow 0^+} f(x) = -\infty$

Right: $\lim_{x \rightarrow +\infty} f(x) = +\infty$

2. $g(x) = -2\log x$

$a < 0$ and $b > 1$

Left: $\lim_{x \rightarrow 0^+} g(x) = +\infty$

Right: $\lim_{x \rightarrow +\infty} g(x) = -\infty$

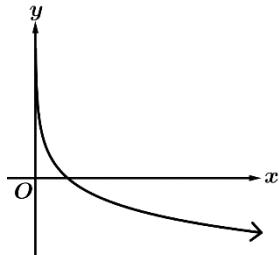
3. $h(x) = \frac{3}{4}\log_\pi x$

$a > 0$ and $b > 1$

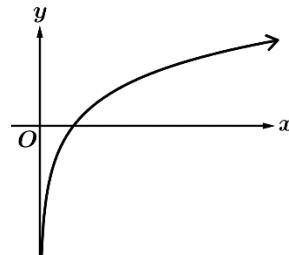
Left: $\lim_{x \rightarrow 0^+} h(x) = -\infty$

Right: $\lim_{x \rightarrow +\infty} h(x) = +\infty$

Directions: The graphs of k and m are shown below. Use the graphs to answer the following.



Graph of $k(x)$



Graph of $m(x)$

4. The graph of k is

- (A) increasing at an increasing rate.
- (B) increasing at a decreasing rate.
- (C) decreasing at an increasing rate. Concave up
- (D) decreasing at a decreasing rate.

5. Which of the following pairs of limit statements correctly describes the end behavior of k ?

- (A) $\lim_{x \rightarrow 0^+} k(x) = -\infty$ and $\lim_{x \rightarrow \infty} k(x) = -\infty$
- (B) $\lim_{x \rightarrow 0^+} k(x) = -\infty$ and $\lim_{x \rightarrow \infty} k(x) = \infty$
- (C) $\lim_{x \rightarrow 0^+} k(x) = \infty$ and $\lim_{x \rightarrow \infty} k(x) = -\infty$
- (D) $\lim_{x \rightarrow 0^+} k(x) = \infty$ and $\lim_{x \rightarrow \infty} k(x) = \infty$

6. Which of the following could be the equation for k ?

- | | |
|-------------------------|------------------------------------------|
| (A) $k(x) = -2\log_4 x$ | (B) $k(x) = 2\log_4 x$ |
| (C) $k(x) = -4(2)^x$ | (D) $k(x) = 4\left(\frac{1}{2}\right)^x$ |

$a < 0$ and $b > 1$

8. The graph of m is

- (A) increasing at an increasing rate.
- (B) increasing at a decreasing rate. Concave down
- (C) decreasing at an increasing rate.
- (D) decreasing at a decreasing rate.

9. Which of the following pairs of limit statements correctly describes the end behavior of m ?

- (A) $\lim_{x \rightarrow 0^+} m(x) = -\infty$ and $\lim_{x \rightarrow \infty} m(x) = -\infty$
- (B) $\lim_{x \rightarrow 0^+} m(x) = -\infty$ and $\lim_{x \rightarrow \infty} m(x) = \infty$
- (C) $\lim_{x \rightarrow 0^+} m(x) = \infty$ and $\lim_{x \rightarrow \infty} m(x) = -\infty$
- (D) $\lim_{x \rightarrow 0^+} m(x) = \infty$ and $\lim_{x \rightarrow \infty} m(x) = \infty$

10. Which of the following could be the equation for m ?

- | | |
|-------------------------|------------------------|
| (A) $m(x) = -3\log_8 x$ | (B) $m(x) = 3\log_8 x$ |
| (C) $m(x) = -3(8)^x$ | (D) $m(x) = 3(8)^x$ |

$a > 0$ and $b > 1$

7. Which of the following equations could be k^{-1} ?
- (A) $k^{-1}(x) = \left(\frac{1}{2}\right)^x$ (B) $k^{-1}(x) = -(2)^x$
 (C) $k^{-1}(x) = \frac{-1}{2\log_4 x}$ (D) $k^{-1}(x) = -2\log_4 x$

From 6. $x = -2\log_4 y$ $\log_4 y = -\frac{x}{2}$
 $y = (4^{-1/2})^x = \left(\frac{1}{2}\right)^x$

11. Which of the following equations could be m^{-1} ?
- (A) $m^{-1}(x) = \left(\frac{1}{2}\right)^x$ (B) $m^{-1}(x) = -(2)^x$
 (C) $m^{-1}(x) = 2^x$ (D) $m^{-1}(x) = \frac{1}{3\log_8 x}$

From 10. $x = 3\log_8 y$ $\log_8 y = \frac{x}{3}$
 $y = (8^{1/3})^x = 2^x$

Directions: Selected values of the several logarithmic functions are shown in the tables below. For each table, find the value of the constant k .

12.

x	$f(x)$
0.3	2
3	5
30	8
k	11
3000	14

x values are changing by multiplying by 10 over equal-length y value intervals, so
 $k = 30(10) = 300$
 The next value of
 $x = 10k = 300(10) = 3000$

13.

x	$g(x)$
$\frac{3}{4}$	1
3	2
k	3
48	4

x values are changing by multiplying by 4 over equal-length y value intervals, so
 $k = 3(4) = 12$
 The next value of
 $x = 4k = 12(4) = 48$

14.

x	$h(x)$
$12k$	$k - 1$
$6k$	k
$3k$	$k + 1$
6	$k + 2$
3	$k + 3$

x values are changing by multiplying by $\frac{1}{2}$ over equal-length y value intervals, so
 $\frac{1}{2}(3k) = 6 \Rightarrow k = 4$
 The x values form a sequence of $12k = 12(4) = 48, 24, 12, 6, 3$

15.

x	$l(x)$
3^7	4
3^5	6
27	8
3	10
k	12

x values are changing by multiplying by $\frac{1}{3^2} = \frac{1}{9}$ over equal-length y value intervals, so
 $k = 3\left(\frac{1}{9}\right) = \frac{1}{3}$

16. Let $f(x) = 3\log_5(x+4)$.

a) Find the domain and range of the function f . Domain: $x + 4 > 0 \Rightarrow x > -4$ $a > 0$ and $b > 1 \Rightarrow$ Range is $(-\infty, \infty)$

b) If $g(x) = -2f(x-3)$, find the domain and range of g . Domain: $x > -1$ Range is $(-\infty, \infty)$

g is a horizontal shift of $f(x)$ of 3 so the domain of $g(x)$ is $x > -4 + 3 = -1$. There is also a vertical dilation of -2 of $f(x)$ which will reflect $f(x)$ over the x -axis which will still have a range of $(-\infty, \infty)$

c) If $k(x) = f(2x) + 7$, find the domain and range of k . Domain: $x > -2$ Range is $(-\infty, \infty)$

k is a horizontal dilation of $f(x)$ by a factor of $\frac{1}{2}$ so domain is $x > -4\left(\frac{1}{2}\right) = -2$ and the vertical shift of $f(x)$ does not affect the range.