Notes: (Topic 1.4) Polynomial Functions and Rates of Change

Polynomial Functions

A polynomial function is any function representation equivalent to the analytical form:

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0,$$

where n is a positive integer, a_i is a real number for each i from 1 to n, and a_n is nonzero.

Leading Term: __________ **Degree**: _________ **Leading Coefficient**: _______

Example 1: Find the leading coefficient and degree of the following polynomial functions.

a) $f(x) = 3x^4 + 2x - 7$

Leading Coefficient: ______

Degree: ______

b) $y = 12x - 7x^3 + 11$

Leading Coefficient: ______

Degree: ______

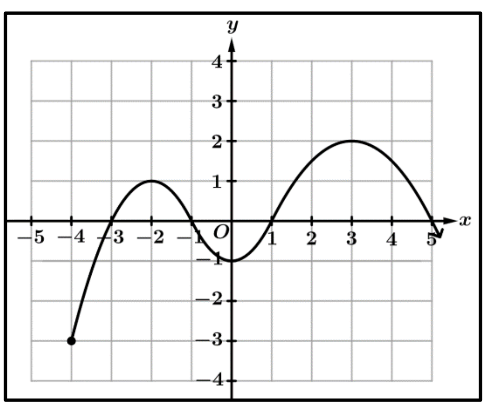
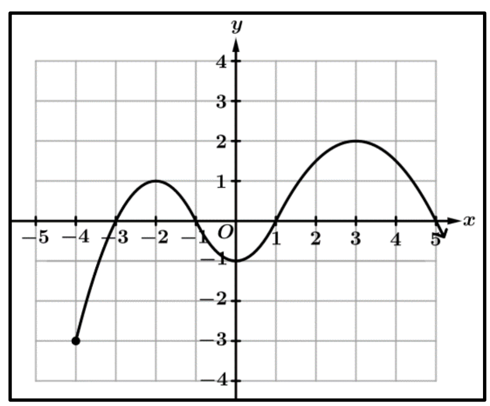
c) $g(x) = 4$

Leading Coefficient: ______

Degree: ______

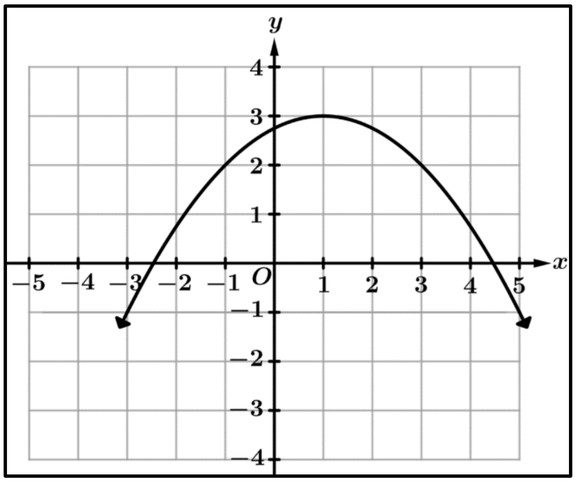
Extrema

The extrema of a graph are the minimums and maximums of a function. There are _____ types of extrema.

Relative Extrema (Local)	**Absolute Extrema (Global)**
A polynomial has a relative minimum or relative maximum where the polynomial switches between decreasing and increasing (or at an endpoint if the polynomial has a restricted domain) .	Of all local maxima, the greatest is called the absolute maximum. The least of all local minima is called the absolute minimum.
 Local minimums at $x =$ ____________________ Local maximums at $x =$ ____________________	 Absolute maximum = _______at $x =$ ___________ Absolute minimum = _______at $x =$ ___________

Example 2: Find and classify each type of extrema for the functions below or write N/A .

a)



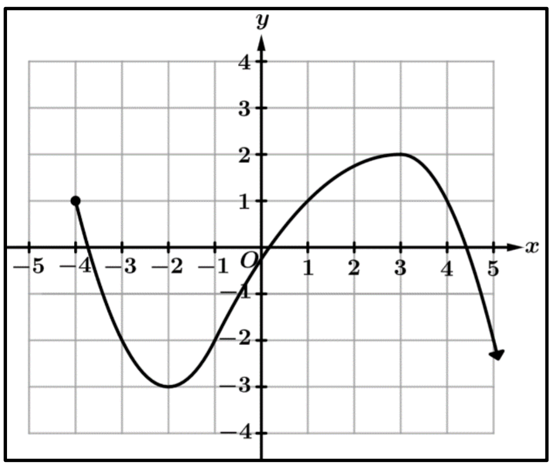
Relative Minimum at $x =$__________

Relative Maximum at $x =$__________

Absolute Minimum $=$_________ at $x =$_____

Absolute Maximum $=$_________ at $x =$_____

b)



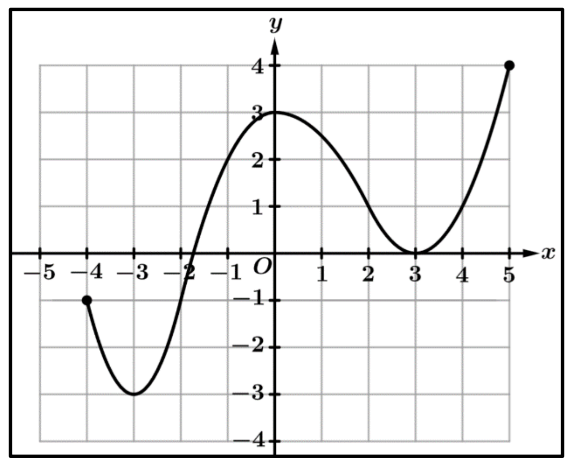
Relative Minimum at $x =$__________

Relative Maximum at $x =$__________

Absolute Minimum $=$_________ at $x =$_____

Absolute Maximum $=$_________ at $x =$_____

c)



Relative Minimum at $x =$__________

Relative Maximum at $x =$__________

Absolute Minimum $=$_________ at $x =$_____

Absolute Maximum $=$_________ at $x =$_____

Fun Facts About Polynomials

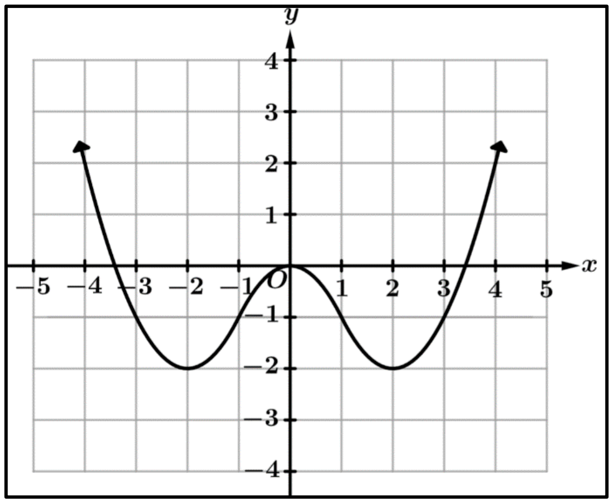
1. Between 2 real zeros of a polynomial, there must be at least one ____________ _________________ or ___________ _________________.

2. Polynomials of _________ degree must have either a ______________ ___________________ or a ______________ ___________________.

Points of Inflection

A **point of inflection** occurs when a function changes from concave up to concave down or from concave down to concave up.

At a point of inflection, the rate of change of a function changes from increasing to decreasing or from decreasing to increasing.



Example 3: The graph of $g(x)$ is shown in the figure above. Use the graph of g to answer the following.

a) Find any values of x where g has a point of inflection.

b) For each of the following intervals, determine if the rate of change of g is increasing or decreasing. Explain your reasoning for each answer using features of the graph of $g(x)$.

i. $(3,\ 4)$ ii. $(-4, -3)$ iii. $(-1,\ 1)$ iv. $(1,\ 2)$

Example 4: For $0 \le t \le 3$, the number of cars in a parking lot at time t hours can be modeled by the function $C(t) = -1.37t^5 + 4.218t^4 - 0.357t^2 + 3$. Based on this model, at what time t does the number of cars in the parking lot change from increasing to decreasing?