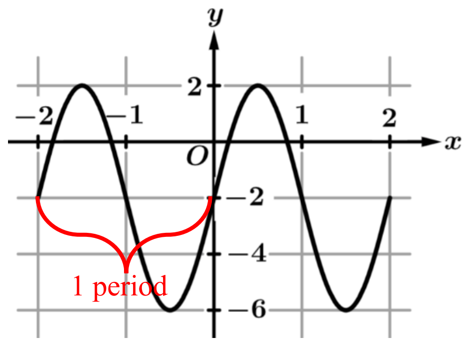
In previous units, we have looked at different methods to construct models of various functions. Now, we will apply those same techniques to create function models for sinusoidal functions.

We have already seen that a sinusoidal function can be written in the form $f(\theta)=a\sin(b(\theta+c))+d$ (or in a similar form with cosine). To construct a model, we will need to connect information that is given verbally, graphically, numerically, and algebraically to the properties of a sinusoidal function graph.

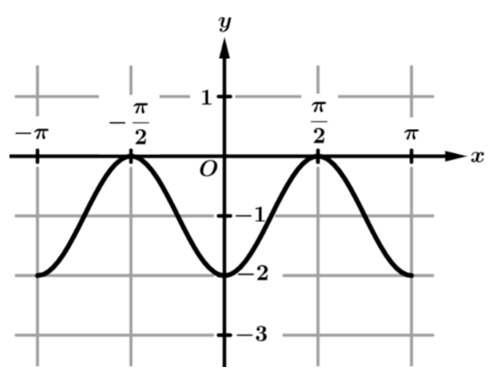
Data Modeling from Graphical Information



Graph of f

Example 1: Let f be a sinusoidal function. The graph of $y=f(x)$ is given in the xy-plane. What is the period of f?

(A) 1 (B) 2 (C) 4 (D) 8



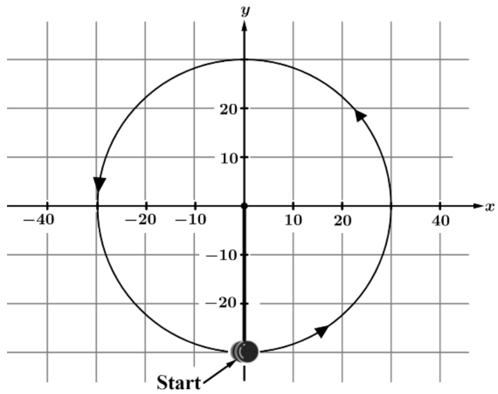
Graph of g

Example 2: Let g be a sinusoidal function where $g(x)=\cos(2(x+c))-1$. The graph of $y=g(x)$ is given in the xy-plane. Which of the following values could represent the constant c?

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

$\cos(x)$ has a maximum value of 1 at $x=0$ and $g\left(-\frac{\pi}{2}\right)=0$ is a max so $2\left(-\frac{\pi}{2}+c\right)=0$ $\quad -\frac{\pi}{2}+c=0$ $\quad c=\frac{\pi}{2}$

Modeling Data from Verbal Information



Example 3: A yo-yo that is attached to a 30-inch-long string is rotated at a constant rate. The figure provides a representation of the yo-yo in the xy-plane with the direction of rotation indicated. At time $t = 0$ seconds, the yo-yo begins to rotate. The yo-yo is at the "Start" position in the figure. At time $t = 5$ seconds, the yo-yo has completed 20 rotations and the yo-yo is in the same position as it was at time $t = 0$. A sinusoidal function is used to model the x-coordinate of the position of the yo-yo as a function of time t in seconds. Which of the following functions is an appropriate model for this situation?

(A) $f(t) = 30\sin\left(\frac{t}{4}\right)$

(B) $f(t) = 30\sin(4t)$

(C) $f(t) = 30\sin\left(\frac{\pi t}{2}\right)$

(D) $f(t) = 30\sin(8\pi t)$

$\frac{5\text{ sec}}{20\text{ rotations}} \Rightarrow \frac{1}{4}$ second per rotation is the period

$f(0) = 0 \qquad f\left(\frac{1}{4}\right) = 0$

$f\left(\frac{1}{4}\right) = 30\sin\left(8\pi \cdot \frac{1}{4}\right) = 30\sin(2\pi) = 0$

Example 4: A clock is mounted on the wall, and the point P is located at the tip of the minute hand of the clock. The function h represents the height of point P from the ground at time t, where $h(t) = a\sin(b(t+c)) + d$. In the xy-plane, the points $h(0) = 70$ and $h(30) = 52$ represent a maximum value and a minimum value, respectively, on the graph of h. What are the values of a and d?

$\text{midline} = \frac{70 + 52}{2} = \frac{122}{2} = 61 = d \qquad \text{amplitude} = 70 - 61 = 9 = a$

Modeling Data from Analytical Information (Equations)

Example 5: The average monthly temperature, in degrees Fahrenheit, for a town in central Indiana can be modeled by the sinusoidal function $T(m) = 25.7\sin\left(\frac{\pi}{6}(m-4)\right) + 61.2$, for $1 \le m \le 12$ months. Based on the model, which of the following is true?

(A) The maximum average monthly temperature is 61.2 degrees Fahrenheit. maximum $= 25.7 + 61.2 = 86.9$

(B) The maximum average monthly temperature occurs at $m = 1$ month.

(C) The minimum average monthly temperature is 35.5 degrees Fahrenheit.

(D) The minimum average monthly temperature occurs at $m = 7$ months.

$T(1)$ is a minimum average monthy temperature because $\sin(x)$ is a minimum at $x = -\frac{\pi}{2}$

$\left(\frac{\pi}{6}(1-4)\right) = \left(\frac{\pi}{6}(-3)\right) = -\frac{\pi}{2}$ $\quad T(1) = 25.7\sin\left(\frac{\pi}{6}(1-4)\right) + 61.2 = 25.7(-1) + 61.2 = 35.5$

$T(7)$ is a maximum average monthy temperature because $\sin(x)$ is a maximum at $x = \frac{\pi}{2}$. $\quad \frac{\pi}{6}(7-4) = \frac{\pi}{2}$

Modeling Data from Numerical Information (Tables)

t	1	3	4	6	7	8	11	12
$N(t)$	11.4	9.7	8.2	5.2	5.0	6.2	10.5	11.3

Example 6: The function N gives the number of nighttime hours on the first day of a given month, where $1 \le t \le 12$. The table gives values of N at selected values of t. A sinusoidal regression $y = a\sin(bt+c)+d$ with 16 iterations is used to model these data. What is the maximum number of nighttime hours predicted by the sinusoidal function model, to the nearest hour?

(A) 5 (B) 8 (C) 11 (D) 12

$a + d \approx 11$