	AP Precalculus Notes	**Name:**
	Topic 2.7: Composition of Functions	
	Created by Bryan Passwater Speedway High School BryanPasswater1@gmail.com	**Mathematical Practices/Skills Highlighted** **2.A** **Identify information** from multiple representations. **2.B** **Construct equivalent** representations of functions. **1.C** **Construct new functions** using compositions.

Consider the function $f(x) = 2x - 3$. Evaluate the following:

a) $f(5) = 2(5) - 3 = 7$

b) $f(5x - 1) = 2(5x - 1) - 3 = 10x - 2 - 3 = 10x - 5$

Composite Functions

Sometimes in math, we input a value of x into a function and then use the output (answer) to plug into another function—the output of the first function becomes the input for the second function. When we use the output (y) of one function as the input of a function, this is called a **composition of functions**.

Example 1: Let $g(x) = 3x + 1$. Let $g(2) = c$. Find $g(c)$. $g(2) = 3(2) + 1 = 7 = c$ $g(c) = 3(7) + 1 = 22$

Notation for Composite Functions

Let f and g be functions. There are 2 ways to notate a composition of functions.

1. $f(g(x))$
2. $(f \circ g)(x)$

Important Note: In general compositions are NOT commutative. So, generally, $f(g(x)) \neq g(f(x))$

Evaluating a Composition of Functions

When working with composite functions, we first evaluate the inner (right) function. Then, we use that output as the input for the outer (left) function.

Example 2: The functions f and g are defined by $f(x) = 3x - 5$ and $g(x) = 2x + 1$. Evaluate the following.

a) $f(g(3)) = f(7) = 16$
$g(3) = 2(3) + 1 = 7$
$f(7) = 3(7) - 5 = 16$

b) $g(f(3)) = g(4) = 9$
$f(3) = 3(3) - 5 = 4$
$g(4) = 2(4) + 1 = 9$

c) $(f \circ f)(2) = f(1) = -2$
$f(2) = 3(2) - 5 = 1$
$f(1) = 3(1) - 5 = -2$

When working with compositions of functions, the input does not necessarily need to be a numeric value. We can also simply find the composition of two functions without evaluating any specific input value.

Example 3: Using the equations for f and g from **Example 2**, find expressions for the following.

a) $f(g(x)) = f(2x+1) = 3(2x+1) - 5$
$= 6x + 3 - 5 = 6x - 2$

b) $(g \circ f)(x) = g(3x-5) = 2(3x-5) + 1$
$= 6x - 10 + 1 = 6x - 9$

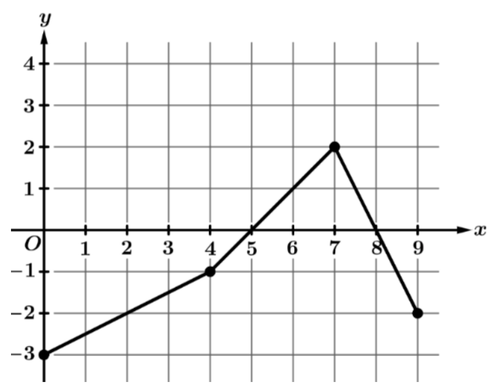
Example 4: The functions $h, k, p,$ and m are given below. Use these functions to find the following.

$$h(x) = 2x - 3 \qquad k(x) = x^2 + 4x + 5 \qquad m(x) = 3x + 2$$

a) $k(h(4)) = k(5) = 50$
$h(4) = 2(4) - 3 = 8 - 3 = 5$
$k(5) = 5^2 + 4 \cdot 5 + 5 = 50$

b) $(h \circ m)(-2) = h(-4) = -11$
$m(-2) = 3(-2) + 2 = -4$
$h(-4) = 2(-4) - 3 = -11$

c) $(k \circ m)(x) = k(3x+2)$
$= (3x+2)^2 + 4(3x+2) + 5$
$= 9x^2 + 12x + 4 + 12x + 8 + 5$
$= 9x^2 + 24x + 17$

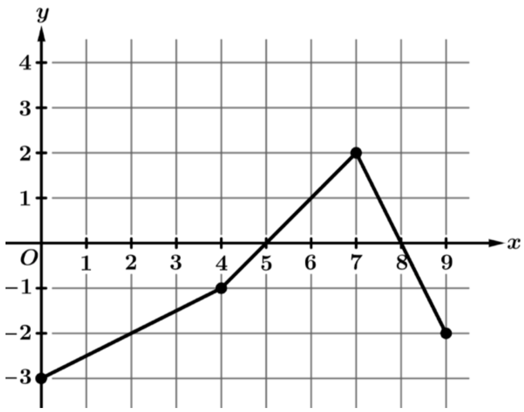


Graph of f

Example 5: The figure shows the graph of the function f on its domain of $0 \le x \le 9$. The function g is given by $g(x) = 1.2x^3 - 4.7x + 9.62$.

(A) (i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(6)$ as a decimal approximation or indicate that it is not defined.

$h(6) = g(f(6)) = g(1) = 1.2 - 4.7 + 9.62 = 6.12$



Graph of f

x	-4	-1	2	6	7
$g(x)$	π	3	3	-2	4

$$h(x) = \begin{cases} |-7+3x|, & x < 1 \\ -4, & x = 1 \\ -x, & x > 1 \end{cases}$$

$k(x)$ is a quadratic function with a horizontal translation of 2 followed by a vertical translation of -3.

Example 6: The graph of the function f is given above along with a table of selected values for the function g, an equation of the piecewise function h and a verbal description of the function k. Use this information to evaluate the following, if possible.

a) $f\big(g(7)\big) = f(4) = -1$
$g(7) = 4 \qquad f(4) = -1$

b) $(g \circ f)(7) = g(2) = 3$
$f(7) = 2 \qquad g(2) = 3$

c) $(h \circ k)(0) = h(1) = -4$
$k(x) = (x-2)^2 - 3$
$k(0) = (-2)^2 - 3 = 4 - 3 = 1$
$h(1) = -4$

d) $f\big(f(8)\big) = f(0) = -3$
$f(8) = 0 \qquad f(0) = -3$

e) $(g \circ h)(1) = g(-4) = \pi$
$h(1) = -4 \qquad g(-4) = \pi$

f) $(g \circ k)(-1) = g(6) = -2$
$k(x) = (x-2)^2 - 3$
$k(-1) = (-3)^2 - 3 = 9 - 3 = 6$
$g(6) = -2$

x	-2	-1	0	1	2
$f(x)$	0	-2	-1	2	1
$g(x)$	1	2	-1	0	-2

Example 7: The table gives values for the functions f and g at selected values of x. Functions f and g are defined for all real numbers. Let h be the function defined by $h(x) = f\big(g(x)\big)$. What is the value of $h(1)$?

(A) -2 (B) -1 (C) 0 (D) 2

$$h(1) = f\big(g(1)\big) = f(0) = -1$$

Example 8: The function f is given by $f(x) = x^2 - 3x + 7$, and the function g is given by $g(x) = 3x + 5$. Which of the following is an expression for $f\big(g(x)\big)$?

(A) $3x^2 - 9x + 26$
(B) $9x^2 + 21x + 17$
(C) $9x^2 + 27x + 32$
(D) $3x^3 - 4x^2 + 6x + 35$

$$f\big(g(x)\big) = f(3x+5) = (3x+5)^2 - 3(3x+5) + 7$$
$$= 9x^2 + 30x + 25 - 9x - 15 + 7 = 9x^2 + 21x + 17$$