Previously, we looked at inverse functions and finding the inverse of a given function, including exponential and logarithmic functions of the form $y = b^x$ and $y = \log_b x$.

Now, we will look at finding inverses of exponential and logarithmic functions in general form.

General Form of Exponential and Logarithmic Functions

Exponential Function: $f(x) = ab^{(x+h)} + k$ **Logarithmic Function**: $f(x) = a\log_b(x+h) + k$

We can find the inverse of $y = f(x)$ by determining the inverse operations to reverse the mapping.

Finding Inverses of Logarithmic and Exponential Functions

1. Switch the roles of x and y in the equation.
2. Use inverse operations to solve for y in the inverse relation.
3. Write the final inverse function using correct notation.

Example 1: Find the inverse of the function $f(x) = 3(2)^{x+4} - 15$.

$$x = 3(2)^{y+4} - 15$$
$$x + 15 = 3(2)^{y+4}$$
$$\frac{x+15}{3} = (2)^{y+4}$$
$$\log_2\left(\frac{x+15}{3}\right) = y + 4$$
$$\log_2\left(\frac{x+15}{3}\right) - 4 = y$$
$$f^{-1}(x) = \log_2\left(\frac{x+15}{3}\right) - 4$$

Example 2: Find the inverse of the function $g(x) = -3\log_5(x-4) + 6$.

$$x = -3\log_5(y-4) + 6$$
$$x - 6 = -3\log_5(y-4)$$
$$\frac{6-x}{3} = \log_5(y-4)$$
$$5^{\frac{6-x}{3}} = y - 4$$
$$y = 5^{\frac{6-x}{3}} + 4$$
$$g^{-1}(x) = 5^{\frac{6-x}{3}} + 4$$

Example 3: Let $h(x) = 3^{x+1} - 2$ and let $k(x) = h^{-1}(x)$. For what value(s) of x does $k(x) = 1$?

$h(1) = 3^{1+1} - 2 = 9 - 2 = 7$ $k(7) = 1$

Solutions by Ted Gott tedg20776@gmail.com

<u>Solving Inequalities Involving Logarithmic and Exponential Functions</u>

Previously, we solved inequalities of polynomial and rational functions. To do this, we first found any x-coordinates where the functions had zeros, holes, or vertical asymptotes. Then, we used a sign chart to determine intervals where the function was positive and negative.

We can solve inequalities involving exponentials and logarithms in the same way…except it is actually much simpler when working with exponential and logarithms (believe it or not)!

Example 4: Let $f(x)=\log\left(x^2-4x+7\right)-\log(x+3)$. What are all the values of x for which $f(x)>0$?

Domain: $\log(x+3)$ term only defined when

$x+3>0 \Rightarrow x>-3$

$f(x)=\log\left(\dfrac{x^2-4x+7}{x+3}\right)>0$

$\dfrac{x^2-4x+7}{x+3}>1$

$x^2-4x+7>x+3$

$x^2-5x+4>0$

$(x-1)(x-4)>0$

$$\underset{-3}{\underbrace{+}}\overbrace{----}^{\overset{(-)(-)}{+}}\underset{1}{\underbrace{+}}\overbrace{----}^{\overset{(+)(-)}{-}}\underset{4}{\underbrace{+}}\overbrace{----}^{\overset{(+)(+)}{+}}>$$

$f(x)>0$ when $-3<x<1$ or $x>4$

Example 5: Let $g(x)=\ln\left(x^2+x-5\right)$ and let $h(x)=\ln(x+4)$. What are all values of x for which $g(x)<h(x)$?

Domain: $\ln(x+4)$ term only defined when

$x+4>0 \Rightarrow x>-4$

$\ln(x^2+x-5)$ term only defined when

$(x^2+x-5)>0 \Rightarrow x^2+x-5=0$

$x=\dfrac{-1\pm\sqrt{1^2-4(1)(-5)}}{2}=\dfrac{-1\pm\sqrt{21}}{2}$

$(x^2+x-5)>0 \Rightarrow x<\dfrac{-1-\sqrt{21}}{2}\approx -2.7912\ldots$ or

$x>\dfrac{-1+\sqrt{21}}{2}\approx 1.7912\ldots$

$g(x)<h(x) \Rightarrow g(x)-h(x)<0$

$\ln(x^2+x-5)-\ln(x+4)=\ln\left(\dfrac{x^2+x-5}{x+4}\right)<0$

$\dfrac{x^2+x-5}{x+4}<1 \qquad x^2+x-5<x+4$

$x^2-9<0 \qquad (x+3)(x-3)<0$

$$\underset{-4}{\underbrace{+}}\overbrace{---}^{\overset{(-)(-)}{+}}\underset{-3}{\underbrace{+}}\overbrace{-\ \underset{-2.791}{\underbrace{+}}}^{\overset{(+)(-)}{-}}-\overbrace{\underset{1.791}{\underbrace{+}}\ -}^{\overset{(+)(-)}{-}}\underset{3}{\underbrace{+}}\overbrace{----}^{\overset{(+)(+)}{+}}>$$

$g(x)<h(x)$ when $-3<x<\dfrac{-1-\sqrt{21}}{2}$

or $\dfrac{-1+\sqrt{21}}{2}<x<3$

Example 6: Let $k(x)=1-4(2)^{x-3}$. What are all the values of x for which $k(x)\le -31$?

$1-4(2)^{x-3}\le -31$

$-4(2)^{x-3}\le -32$

$(2)^{x-3}\ge 8$

$x-3\ge 3$

$x\ge 6$

$k(x)\le -31$ when $x\ge 6$