Notes: (Topic 2.1) Change in Arithmetic and Geometric Sequences

A ________________ is a function from the __________ numbers to the ________ numbers.

This means that we are only able to "plug" in whole numbers (0, 1, 2, 3, …) into a sequence but we can get any real number as the output.

As a result, when we graph a sequence, we will have points but we cannot "connect" them together to form a line or curve.

Example 1: Consider the sequence defined by $a_n = 4n - 3$. Find a_1 and a_7.

In this course, we will study two important types of sequences: **<u>arithmetic sequences</u>** and **<u>geometric sequences</u>**.

<table>
<tr><th colspan="3">Arithmetic Sequences</th></tr>
<tr><th>Property of Successive Terms</th><th>Formulas/Equations</th><th>Notes</th></tr>
<tr>
<td>Successive terms have a common difference, or constant rate of change.</td>
<td>$a_n = a_0 + dn$
or
$a_n = a_k + d(n - k)$

where $a_0 =$
$d =$
$a_k =$</td>
<td>Arithmetic sequences behave like linear functions, except they are not continuous.

Increasing arithmetic sequences increase equally each step. (slope always stays the same!)</td>
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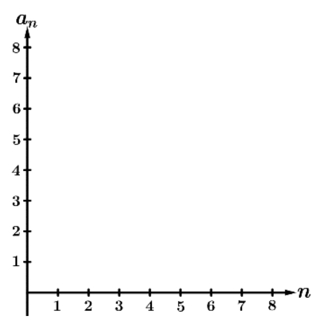
<tr><td colspan="3">Example

$a_n = 3n + 1$

</td></tr>
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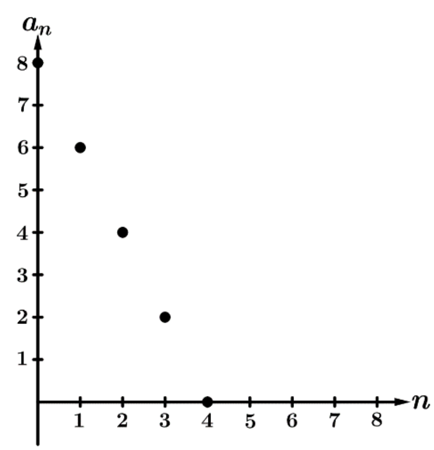
Example 2: For each of the following, determine if the sequence could be arithmetic. If yes, identify the common difference.

a) $s_n = n^2 - 3$ b) $s_n = 6 - 2n$ c) $-7, -2, 3, 8, 13, \ldots$ d) $1, -2, 3, -4, 5, \ldots$

Example 3: Let a_n be an arithmetic sequence with $a_3 = 8$ and $d = -3$. Find an expression for a_n, and use the expression to find a_{12}.

Example 4: Let a_n be an arithmetic sequence with $a_2 = 7$ and $a_6 = 9$. Find an expression for a_n, and use the expression to find a_{24}.



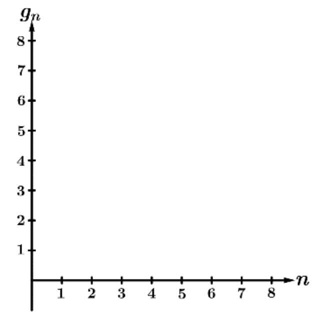
Example 5: Several terms of the arithmetic sequence a_n are shown above. Find an expression for a_n and use the expression to find a_{17}.

Geometric Sequences

Property of Successive Terms	Formulas/Equations	Notes
Successive terms have a **common ratio**, or constant proportional change.	$g_n = g_0 r^n$ **or** $g_n = g_k r^{(n-k)}$ where $g_0 =$ $r =$ $g_k =$	Geometric sequences behave like **exponential functions**, except they are not continuous. Increasing geometric sequences increase by a larger amount each step. (% increase always stays the same!)

Example

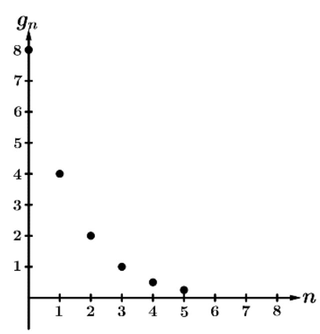
$g_n = 8\left(\frac{1}{2}\right)^n$



Example 6: For each of the following, determine if the sequence could be geometric. If yes, identify the common ratio.

a) $s_n = 3n^2$ b) $s_n = 4(2)^{n-1}$ c) $1, 3, 2, 6, 4, 12, 8, 24, \ldots$ d) $16, -8, 4, -2, 1, \ldots$

Example 7: Let g_n be a geometric sequence with $g_1 = 12$ and $r = 2$. Find an expression for g_n, and use the expression to find g_4.



Example 8: Several terms of the geometric sequence g_n are shown above. Find an expression for g_n and use the expression to find g_{10}.