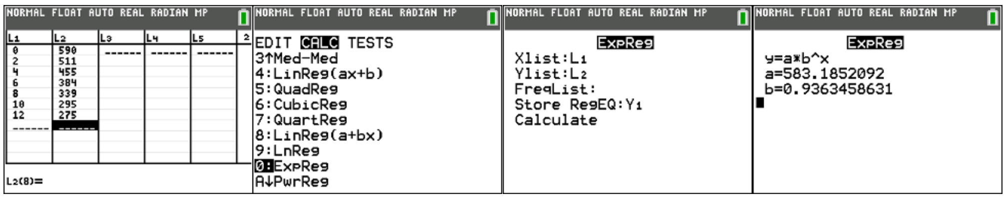
**Worksheet A**: (Topic 2.5) Exponential Data Modeling Name: Solutions

| $t$ (years since 2006) | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $D(t)$ (deaths in thousands) | 590 | 511 | 455 | 384 | 339 | 295 | 275 |

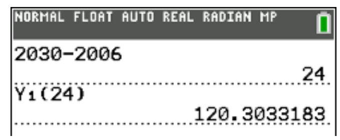
1. The number of worldwide deaths of children under five years old due to malaria has been decreasing in recent years. The table above gives the yearly number of death (in thousands) for children under five years old due to malaria for several years since 2006.

a) Use the data in the table to create an exponential function model of the form $y = ab^x$, where $y$ represents the number of deaths (in thousands) $x$ years since 2006.




$D(t) = ab^t$ where $a = 583.1852\ldots$ and $b = 0.9363\ldots$ are the values stored in calculator.

b) Based on the exponential regression model found in part a, approximately how many children under five years old are predicted to die from malaria in the year 2030? $t = 2030 - 2006 = 24$

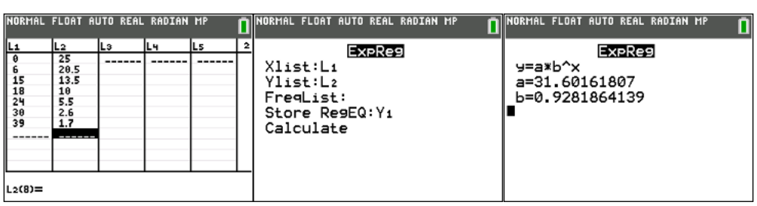



$D(24) = ab^{24} = 120.3033\ldots$ About 120,303 children under five years old are predicted to die from malaria in the year 2030.

| $m$ (months since Jan. 2009) | 0 | 6 | 15 | 18 | 24 | 30 | 39 |
|---|---|---|---|---|---|---|---|
| $V(m)$ (daily visitors in millions) | 25 | 20.5 | 13.5 | 10 | 5.5 | 2.6 | 1.7 |

2. Before Facebook, we (old people) had MySpace, the first social networking site that reached a global audience. At its peak in 2008, MySpace was averaging over 100 million daily visitors. However, as Facebook rose in popularity, MySpace became less and less popular. The table shows the number of daily visitors to MySpace (in millions) $m$ months since January 2009.

a) Use the data in the table to create an exponential regression model of the form $y = ab^x$, where $y$ represents the number of daily visitors to Myspace (in millions) $x$ months since January 2009.




$V(m) = ab^m$ where $a = 31.6016\ldots$ and $b = 0.9281\ldots$ are the values stored in calculator.

b) Based on the model found in part a, how many daily visitors did MySpace have after 20 months?
$V(20) = ab^{20} = 7.1189\ldots$ million daily visitors after 20 months.

c) Let $y = ab^t$ be an equivalent form of the model found in part a, where $t$ represents the number of years since January 2009. Find the values of $a$ and $b$. From part a, let $V(m) = a_1(b_1)^m$ Let $V(t) = a_2(b_2)^t$

$t = \frac{m}{12}$ $a_2(b_2)^{m/12} = a_1(b_1)^m$ $a_2 = a_1 = 31.6016\ldots$

$(b_2)^{m/12} = \left((b_2)^{1/12}\right)^m = (b_1)^m \Rightarrow (b_2)^{1/12} = b_1 \Rightarrow b_2 = (b_1)^{12} = 0.4089\ldots$

3. After an alien spacecraft lands in the middle of Times Square in New York City, video of the event starts spreading rapidly across news sites and social media sites. The number of people in the world that have seen video of the alien landing can be modeled by the exponential function $y = 1391(1.07)^m$, where $m$ is the number of minutes since the videos were first posted online and $0 \leq m \leq 180$.

a) According to this model, how many people have seen the alien video after two hours?

$p(120) = 1391(1.07)^{120} = 4{,}670{,}683.6405$ ...people

b) Let $y = 1391(b)^s$ be an equivalent form of the exponential model $y = 1391(1.07)^m$, where $s$ represents the number of seconds since the videos were first posted online. Find the value of $b$.

$p(s) = 1391(b)^s \quad m = \frac{s}{60} \qquad 1391(b)^s = 1391(1.07)^m = 1391(1.07)^{\frac{s}{60}} = 1391\left(1.07^{\frac{1}{60}}\right)^s$

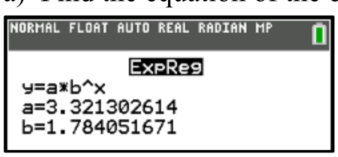
$(b)^s = \left(1.07^{1/60}\right)^s \Rightarrow b = 1.07^{1/60} = 1.0011$ ...

c) What is an appropriate domain for the model found in part b? $0 \leq m \leq 180 \Rightarrow 0 \leq s \leq 10800$

| $x$ | $f(x)$ |
|---|---|
| 1 | 6 |
| 4 | 33 |
| 6 | 107 |
| 7 | 191 |
| 9 | 613 |

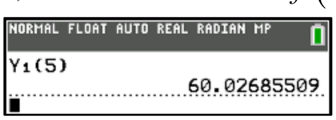
4. The table above gives values for a function $f$ at selected values of $x$.

a) Find the equation of the exponential regression $y = ab^x$ to model these data.




$f(x) = ab^x$ where $a = 3.3213$ ... and $b = 1.78405$ ... are the values stored in calculator.

b) What is the value of $f(5)$ predicted by the exponential function model found in part a?



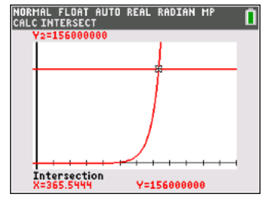

$f(x) = 60.0268$ ...

5. After Mr. Passwater finally relents and agrees to make a TikTok account, where he only posts videos of him telling math jokes. As you can imagine, the number of people that follow his account rapidly grows! The number of followers for Mr. Passwater's TikTok account is modeled by the function $F$. The number of followers is expected to increase by 31.4% **each week**. At time $t = 0$ days, Mr. Passwater had 100 followers.

a) If $t$ is measured in days, what is an expression for $F(t)$? (**Note**: There are seven days in a week)

$F(t) = 100(1.314)^{t/7}$

b) The most popular account on TikTok has 156 million followers. According to the model found in part a, how many days will it take for Mr. Passwater to have the most followers on TikTok?




$F(t) = 100(1.314)^{t/7} = 156{,}000{,}000 \Rightarrow t = 365.5444$ ... days or about a year.

6. The number of bacteria found in a petri dish can be modeled by the function $B(t) = ke^t$, where $k$ is a constant and $t$ is measured in hours. At time $t = 2.5$, there were 5,100 bacteria in the petri dish. According to this model, what is the number of bacteria in the petri dish after 90 minutes?

$B(2.5) = 5100 = ke^{2.5} \Rightarrow k = \frac{5100}{e^{2.5}}$ $\quad B(1.5) = \frac{5100}{e^{2.5}} e^{1.5} = 1876.1851$ ... bacteria.

7. Mr. Passwater decides to buy a brand new 2024 MINI Cooper convertible for \$39,645. The value of his car is expected to depreciate (decrease) by 17% each year for the first five years. Let $V$ represent the value of Mr. Passwater's MINI Cooper at time $t$ years after purchase.

a) Write an expression for $V(t)$ where $V(t) = ab^t$.

$V(t) = 39645(0.83)^t$

b) Let $V(m) = ak^m$ be an equivalent expression for $V(m)$ where $m$ is the number of months after purchase. Find the value of $k$.

$V(m) = 39645(k)^m \quad t = \frac{m}{12} \quad 39645(k)^m = 39645(0.83)^t = 39645(0.83)^{m/12} = 39645\left(0.83^{1/12}\right)^m$

$(k)^m = \left(0.83^{1/12}\right)^m \Rightarrow k = 0.83^{1/12} = 0.9845$ ...

| $x$ | 1 | 7 |
|---|---|---|
| $f(x)$ | 3.4 | 9.1 |

8. Selected values of the exponential function $f$ are shown in the table above, where $f(x) = ab^x$.

a) Use the given data to write two equations that can be used to find the values for constants $a$ and $b$ in the expression for $f(x)$.

$f(1) = ab^1 = 3.4 \quad f(7) = ab^7 = 9.1$

b) Find the values of $a$ and $b$.

$f(1) = ab^1 = 3.4 \qquad f(7) = ab^7 = 9.1$

$a = \frac{3.4}{b} \qquad \frac{3.4}{b}b^7 = 3.4b^6 = 9.1$

$b^6 = \frac{9.1}{3.4}$

$a = \frac{3.4}{b} = 2.8854 \ldots \quad b = \left(\frac{9.1}{3.4}\right)^{1/6} = 1.1783 \ldots$

**Directions**: Selected values from several exponential functions are given in the tables below. For each, find the constant value that can be added/subtracted to the output values to reveal a proportional growth pattern. Then, write an equation to model the function.

9.

| $x$ | $g(x)$ | $g(x)+1$ |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 2 | 3 |
| 2 | 8 | 9 |
| 3 | 26 | 27 |
| 4 | 80 | 81 |

$g(x) + 1 = 3^x$

$g(x) = 3^x - 1$

10.

| $x$ | $h(x)$ | $h(x)-2$ |
|---|---|---|
| 0 | 34 | 32 |
| 1 | 18 | 16 |
| 2 | 10 | 8 |
| 3 | 6 | 4 |
| 4 | 4 | 2 |

$h(x) - 2 = 32\left(\frac{1}{2}\right)^x$

$h(x) = 32\left(\frac{1}{2}\right)^x + 2$

11.

| $x$ | $k(x)$ | $k(x)+1$ |
|---|---|---|
| 0 | $-4$ | $-3$ |
| 1 | $-7$ | $-6$ |
| 2 | $-13$ | $-12$ |
| 3 | $-25$ | $-24$ |
| 4 | $-49$ | $-48$ |

$k(x) + 1 = -3(2)^x$

$k(x) = -3(2)^x - 1$