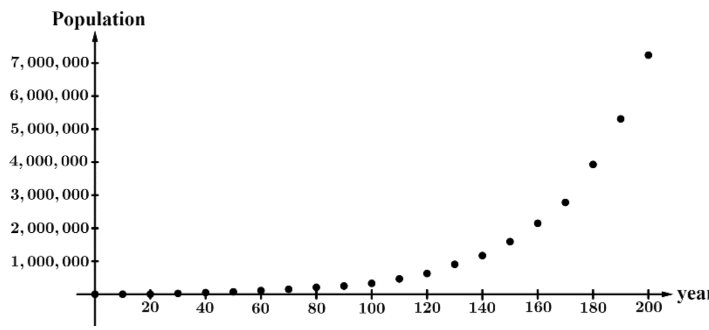
Notes: (Topic 2.15) Semi-log Plots Solutions

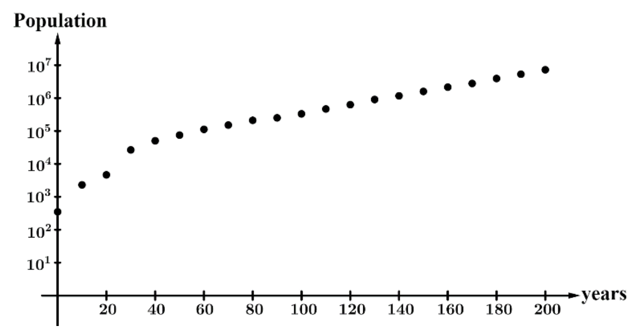
Previously (in Topic 2.9 – Logarithmic Expressions), we looked at the idea of using a logarithmically scaled axis to help us better understand data that is exponential. We now return to this idea to examine it in a little more detail through the use of **Semi-log Plots**.

Semi-Log Plots

In a semi-log plot, one of the axes is logarithmically scaled. In AP Precalculus, we will only be scaling the vertical (y) axis.

With a semi-log plot where the y-axis logarithmically scaled, **exponential functions will appear linear.**



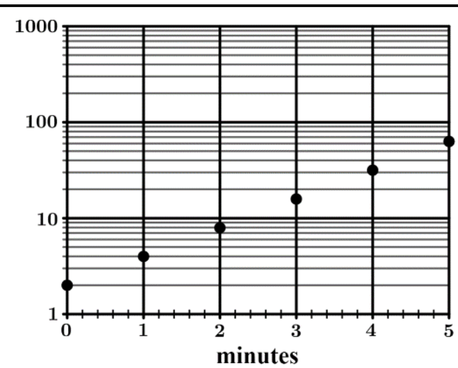


Previously, we looked at the two graphs displayed above displaying the population of English Americans in the (current) United States from 1620 – 1820, where $t = 0$ represents the year 1620.

The graph on the **left** shows the population using a normal scale on the vertical axis.

The graph on the **right** is a semi-log plot where the vertical axis has been logarithmically scaled.

Example 1: Use the features of the semi-log plot above to justify why an exponential model is appropriate for the population of English Americans in the (current) United States from 1620 – 1820. The semi-log plot above appears to be linear after 40 years (1660) so the exponential model is appropriate.



Example 2: After Mr. Passwater tells another one of his hilarious math jokes, it begins to spread around the school. The number of people P that have heard the joke after t minutes is graphed on the semi-log plot above where the vertical axis has been logarithmically scaled. Which of the following functions could be a model for P?

(A) $P(t) = 2 + 2t$ (B) $P(t) = 2 + 2^t$ (C) $P(t) = 2 + \log_2 t$ (D) $P(t) = 2(2)^t$

The line on the semi-log plot has equation $\log P = b + mt \Rightarrow P(t) = 10^{b+mt} = 10^b \cdot (10^m)^t$ which could be $2(2)^t$ if $b = m$ and $10^b = 2$.

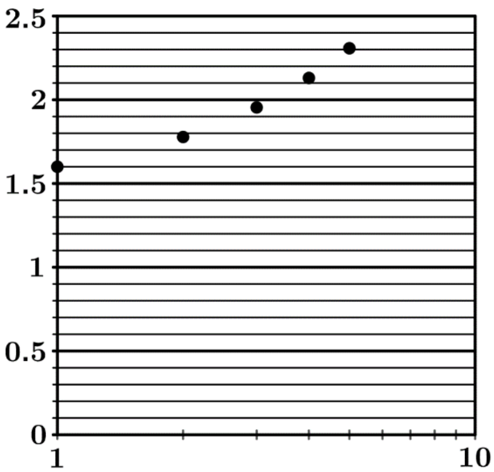
Important Note About Semi-log Plots

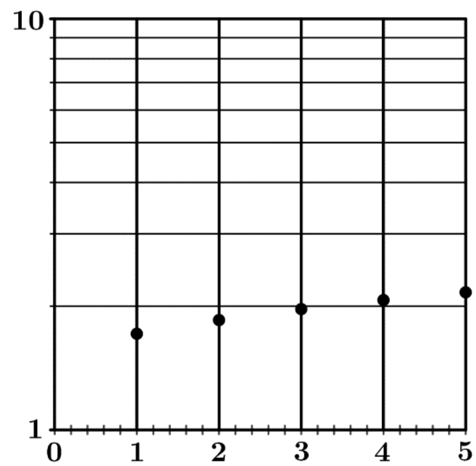
When we "logarithmically scale" the vertical axis for a semi-log plot, we are NOT changing the actual y-values of the data!

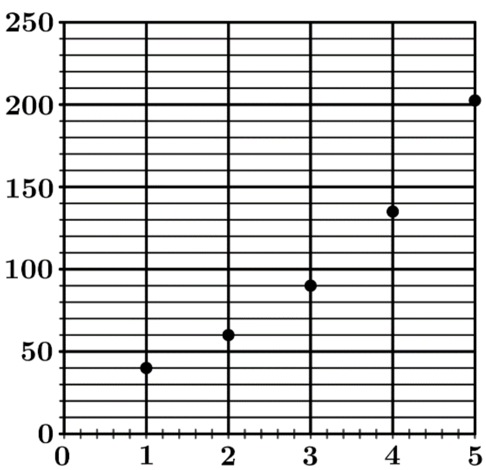
To "logarithmically scale" the vertical axis means that equally-spaced values on the y-axis are proportional, whereas equally-spaced values on the x-axis are linear.

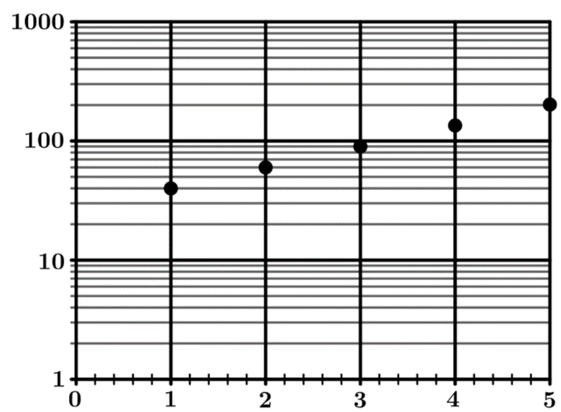
x	1	2	3	4	5
$f(x)$	40	60	90	135	203

Example 3: The table above gives selected values for the function f. Which of the following graphs could represent these data in a semi-log plot, where the vertical axis is logarithmically scaled?

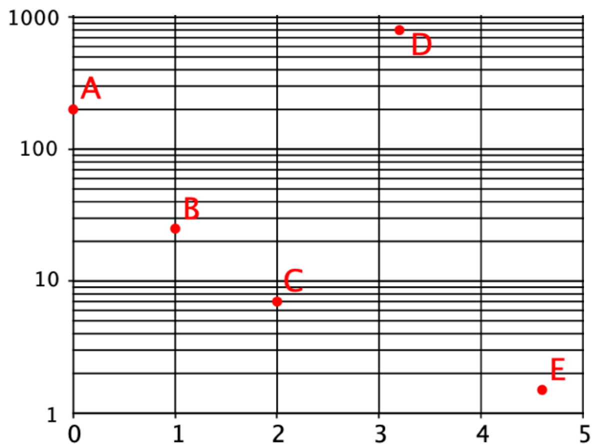
(A)


(B)


(C)


(D)


The equally spaced values on the y-axis are proportional or powers of 10.



Example 4: Plot the following points on the same coordinate plane above.

A$(0, 200)$ **B**$(1, 25)$ **C**$(2, 7)$ **D**$(3.2,\ 800)$ **E**$(4.6,\ 1.5)$

As we have seen, if we graph an exponential function on a semi-log plot, the graph will appear linear. In these cases, we can create a linear model for the graph on the semi-log plot.

Linear Models for a Semi-log Plot

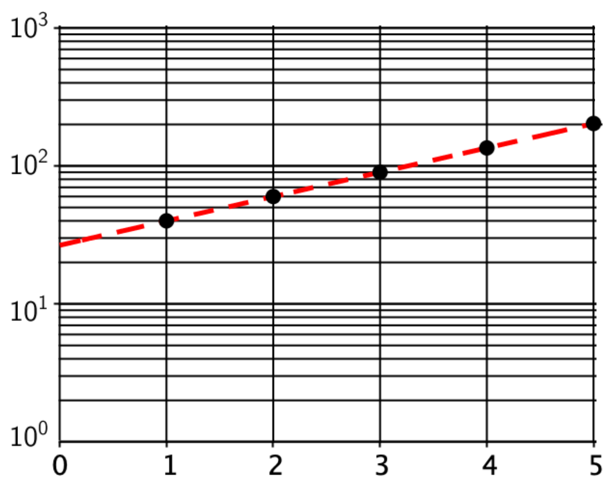
Given the exponential model $y = ab^x$, the corresponding linear model for the semi-log plot is given by

$$y = (\log_n b)x + \log_n a,$$

where $n > 0$ and $n \neq 1$.

Note #1: The slope of our linear function is $\log_n b$ and the y-intercept is $\log_n a$.

Note #2: The base n corresponds to the base used for the scaling of the vertical axis.



Example 5: The semi-log plot above corresponds to the data table for **Example 3**.

a) Write an equation for the linear model for the semi-log plot of the form $y = (\log_n b)x + \log_n a$.

$\text{AROC} = \dfrac{\log 60 - \log 40}{2 - 1} = 0.17609\ldots$ $y = 0.17609\ldots x + \log a$ $\log a = \log 60 - 0.17609\ldots(2) = 1.4259\ldots$

$y = 0.17609\ldots x + 1.4259\ldots$

b) Using the linear model from part a, write the equation of the exponential model $y = ab^x$ for this data.

$a = 10^{1.4259\ldots} = 26.\overline{666}$ $b = 10^{0.17609} = 1.5$ $y = \dfrac{80}{3}(1.5)^x$