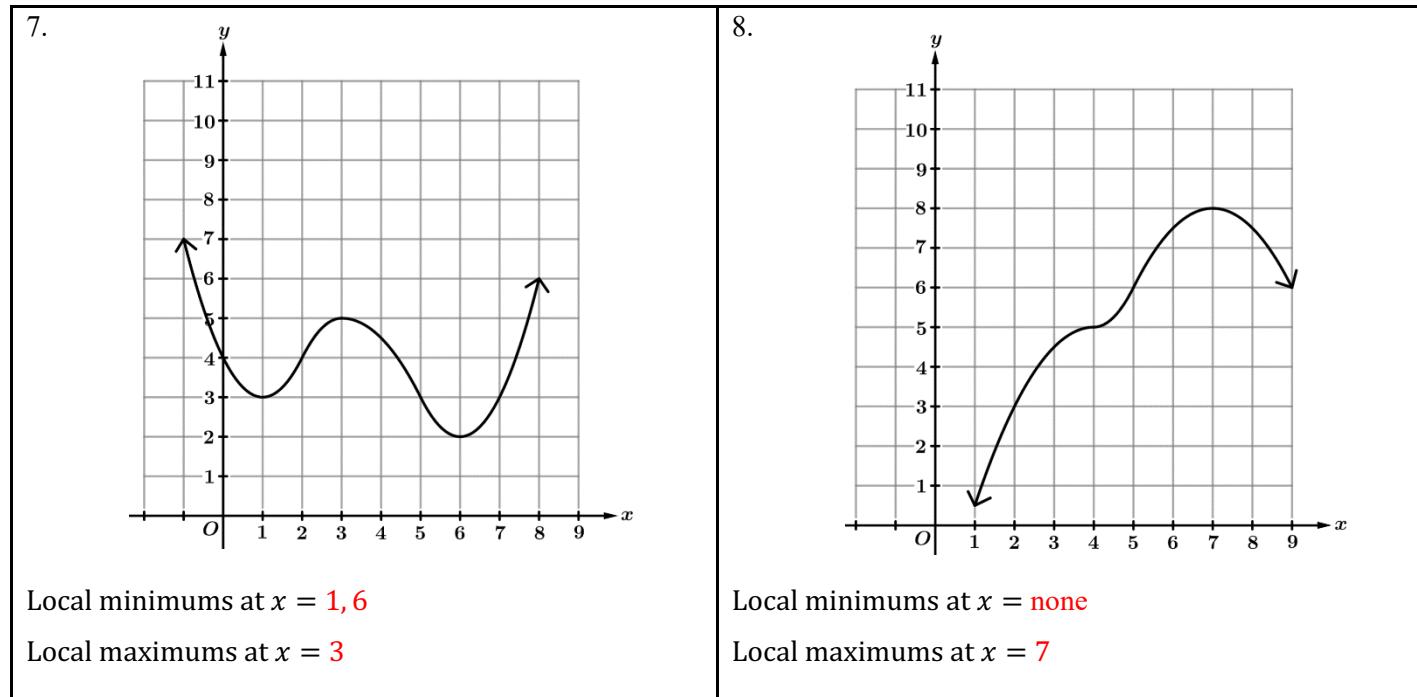


Worksheet A: (Topic 1.4) Polynomial Functions and Rates of Change **Solutions**

Directions: For each of the following, determine if the given function is a polynomial. If the function is a polynomial, indicate the degree.

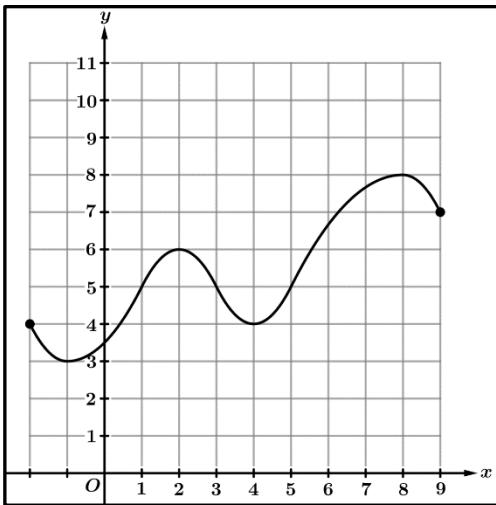
1. $f(x) = 5x^4 - 2x^3 + 7x + 1$ Polynomial: Yes or No If yes, degree: 4	2. $g(x) = 3x^2 - 4^x + 8$ Polynomial: Yes or No, has a 4^x term If yes, degree:	3. $h(x) = x^5 - 4x^{-2} + 5$ Polynomial: Yes or No, has a negative power If yes, degree:
4. $k(x) = \frac{1}{3}x^5 - 2x^3 + 4x$ Polynomial: Yes or No If yes, degree: 5	5. $p(x) = \pi x^2 - x^3 + ex$ Polynomial: Yes or No If yes, degree: 3	6. $m(x) = (4 - 3x^2)(x^2 + x - 5)$ Polynomial: Yes or No If yes, degree: 4 when expression is expanded

Directions: For each of the following polynomial graphs, determine any x -values where the graph has a local extrema. If the graph does not have a specific local extrema, write “none” in the appropriate space.



Directions: For each of the following polynomial graphs, determine any x -values where the graph has a relative extrema. If the graph does not have a specific relative extrema, write “none” in the appropriate space.

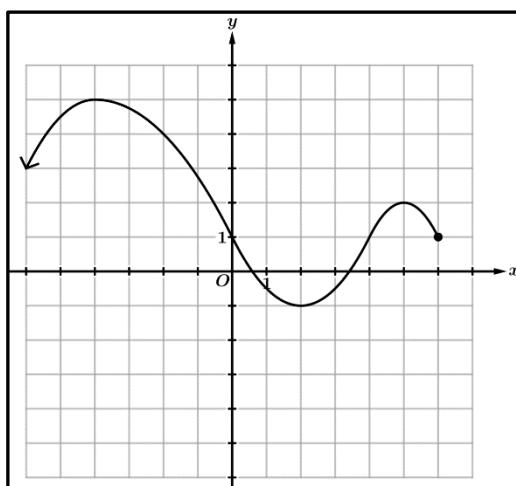
9.



Relative minimum at $x = -1, 4, 9$

Relative maximum at $x = -2, 2, 8$

10.

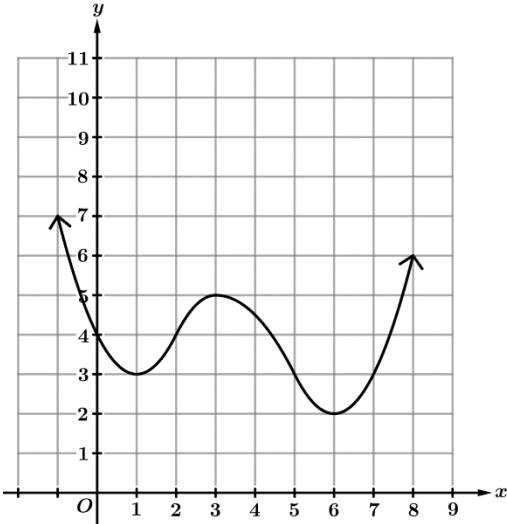


Relative minimum at $x = 2, 6$

Relative maximum at $x = -4, 5$

Directions: For each of the following polynomial graphs, determine the absolute minimum and absolute maximum. If the graph does not have a specific absolute extrema, write “none” in the appropriate space.

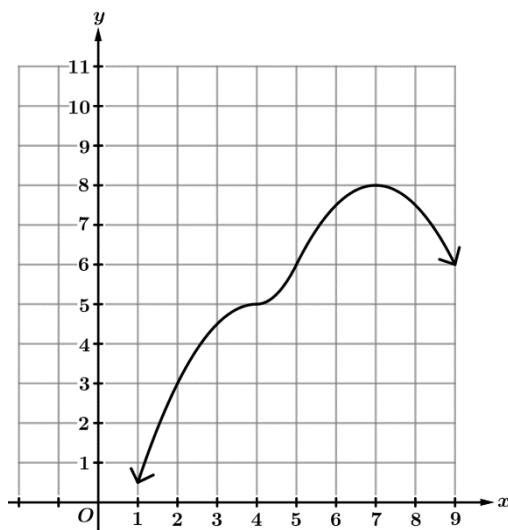
11.



Absolute minimum = 2 at $x = 6$

Absolute maximum = none at $x = \underline{\hspace{2cm}}$

12.

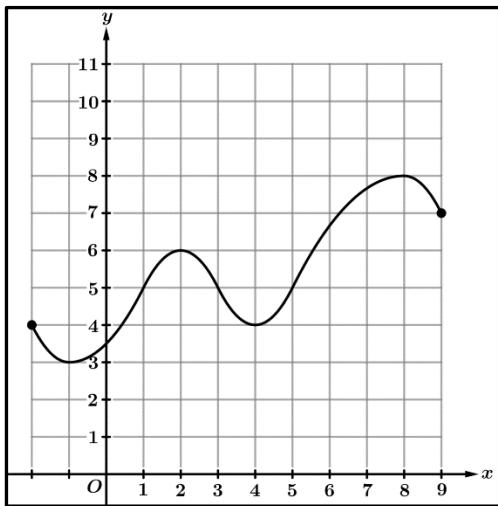


Absolute minimum = none at $x = \underline{\hspace{2cm}}$

Absolute maximum = 8 at $x = 7$

Directions: For each of the following polynomial graphs, determine the global minimum and global maximum. If the graph does not have a specific global extrema, write “none” in the appropriate space.

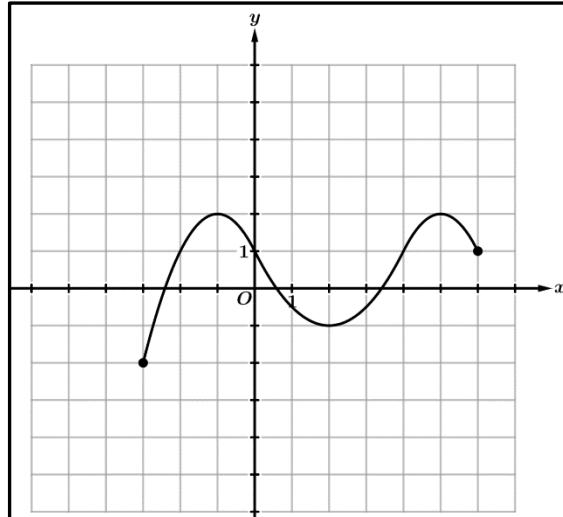
13.



Global minimum = 3 at $x = -1$

Global maximum = 8 at $x = 8$

14.



Global minimum = -2 at $x = -3$

Global maximum = 2 at $x = -1$ and 5

Directions: For each of the following, determine if the given polynomial must have a global minimum, global maximum, or neither. Explain your reasoning.

15. $f(x) = x^4 - 5x^3 + x + 6$

global minimum, even degree,
positive leading coefficient, and
 $\lim_{x \rightarrow \pm\infty} f(x) = \infty$

16. $y = -2x^3 - x^2 + 8x$

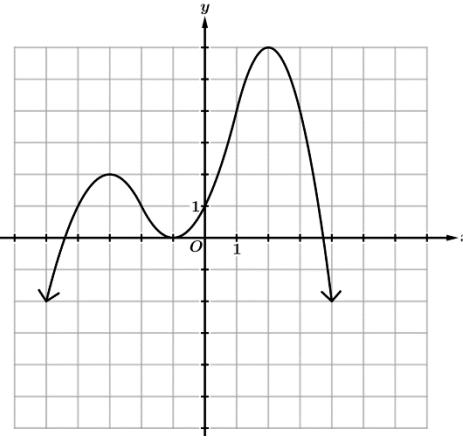
neither, odd degree

17. $g(x) = -x^6 + x^3 + 4x^2 + 1$

global maximum, even
degree,
negative leading
coefficient, and
 $\lim_{x \rightarrow \pm\infty} f(x) = -\infty$

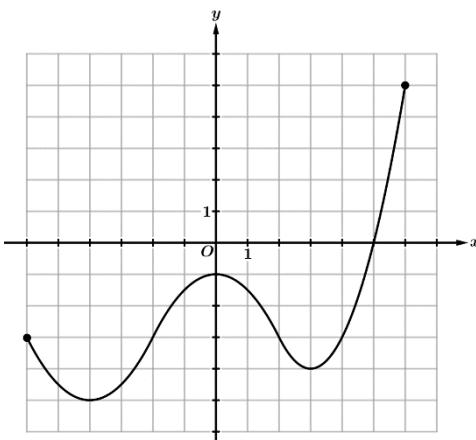
Directions: For the following polynomial graphs, determine any x -values where the function has a point of inflection.

18.



$x = -2, 1$

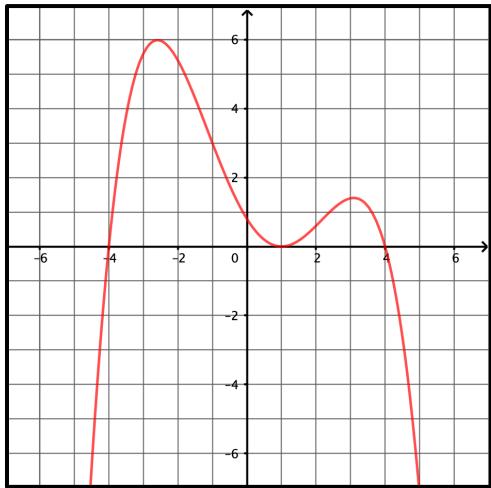
19.



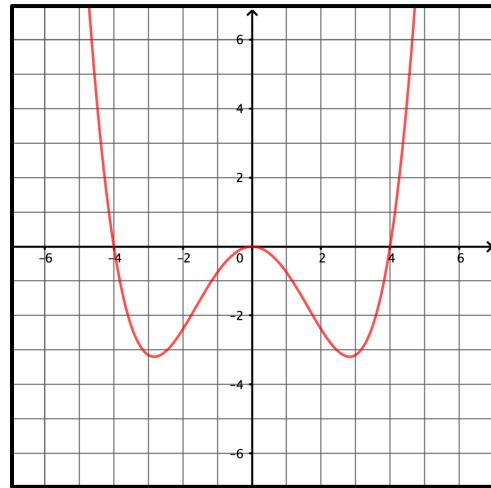
$x = -2, 2$

Directions: Sketch a polynomial function on each axis provided that has the following properties and the domain $(-\infty, \infty)$. Sketches will vary.

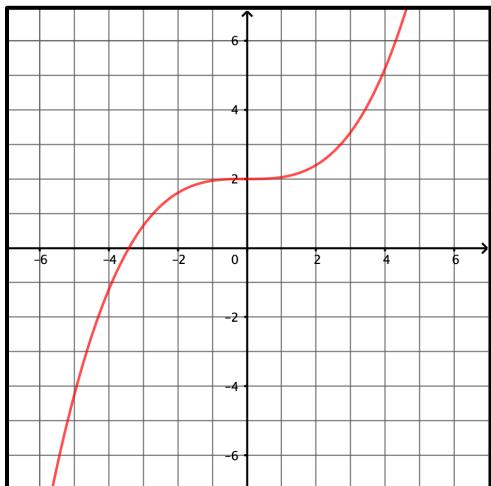
20. $f(x)$ has two points of inflection, one absolute maximum, and no absolute minimum.



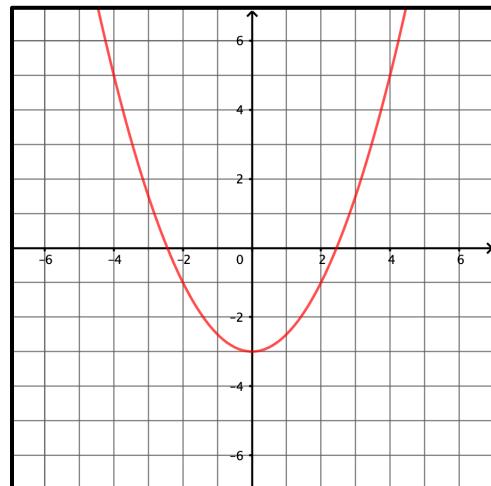
21. $g(x)$ has one local maximum, two global minima, and two points of inflection.



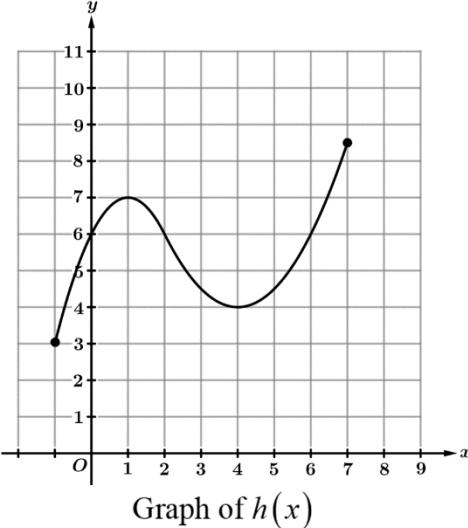
22. $m(x)$ has one point of inflection, no relative extrema, and no absolute extrema.



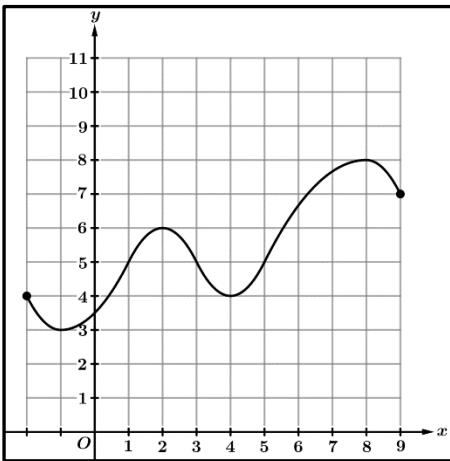
23. $k(x)$ has one absolute extremum, no points of inflection, and one local extremum.



Directions: The graph of $h(x)$ is shown below on the interval $-1 \leq x \leq 7$. Find the open intervals where the rate of change of $h(x)$ has the following properties.



24. The rate of change of $h(x)$ is positive and decreasing. **(-1,1)** because the graph of h is increasing and concave down on this interval.
25. The rate of change of $h(x)$ is negative and decreasing. **(1,2)** because the graph of h is decreasing and concave down on this interval.
26. The rate of change of $h(x)$ is positive and increasing. **(4,7)** because the graph of h is increasing and concave up on this interval.
27. The rate of change of $h(x)$ is negative and increasing. **(2,4)** because the graph of h is decreasing and concave up on this interval.



28. Consider the graph of $g(x)$ shown above. For each of the following intervals, determine if the rate of change of $g(x)$ is positive and increasing, positive and decreasing, negative and increasing, or negative and decreasing.

a. $(3, 4)$

negative and increasing because the graph of g is decreasing and concave up

b. $(1, 2)$

positive and decreasing because the graph of g is increasing and concave down

c. $(8, 9)$

negative and decreasing because the graph of g is decreasing and concave down

d. $(-1, 1)$

positive and increasing because the graph of g is increasing and concave up