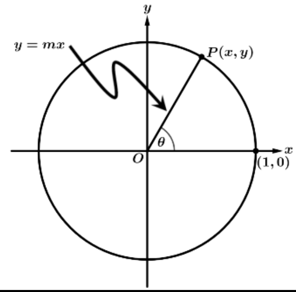
**Notes**: (Topic 3.8) The Tangent Function

**Recall**: We first introduced the tangent of an angle in Topic 3.2. We defined the tangent of an angle as the slope of the terminal ray, or as the ratio of the angle's sine to its cosine values.



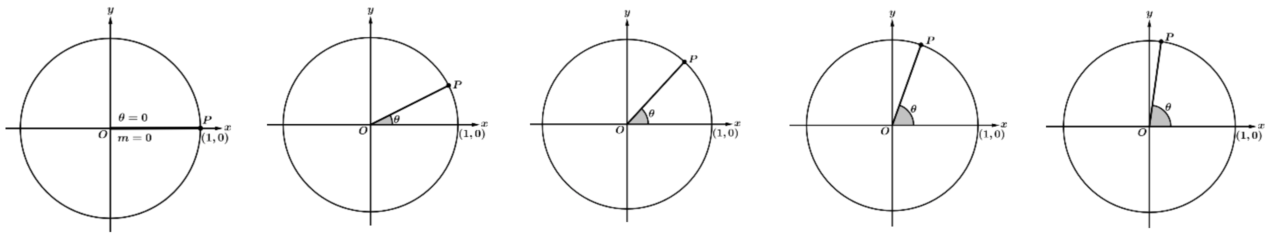

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = m$$

$$\tan\theta = \frac{y}{x}$$

As we did previously with sine and cosine, we can use our understanding of the tangent function to develop its graph.

Let's look at the slope of the terminal ray as our angle $\theta$ increases from $0$ to $\frac{\pi}{2}$.

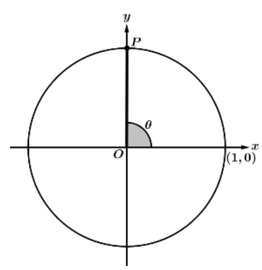



When $\theta = 0$, the slope of the terminal ray is $0$ (first circle above). As $\theta$ increases, the slope also increases.

As $\theta$ approaches $\frac{\pi}{2}$, the slopes get steeper and steeper (more and more positive).

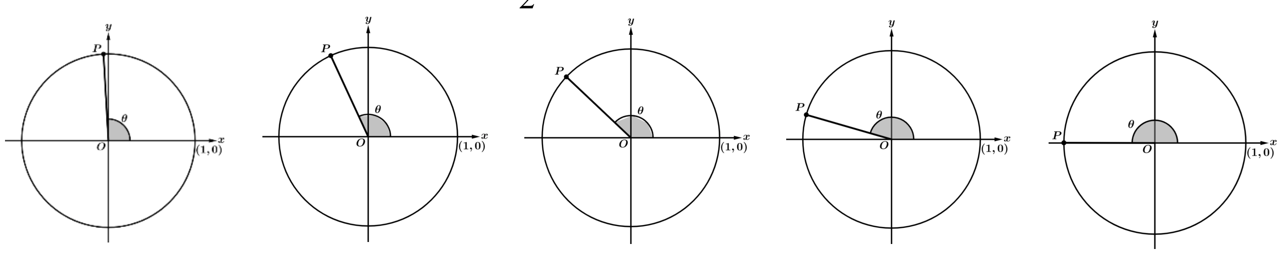
And, when $\theta = \frac{\pi}{2}$, the terminal ray is now vertical.

The slope of a vertical line is ___________________.



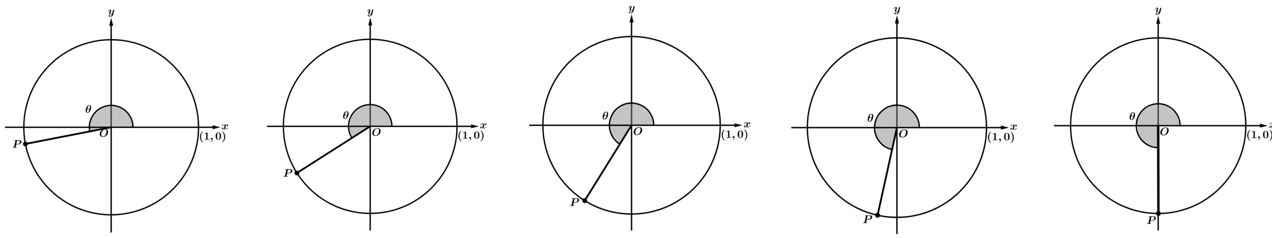

We can see now that when we say the slope is **<u>undefined</u>**, it is because it is infinite: $\lim_{\theta \to \frac{\pi}{2}^-} \tan(\theta) = +\infty$

Now, let's investigate what happens between $\theta = \frac{\pi}{2}$ and $\theta = \pi$.

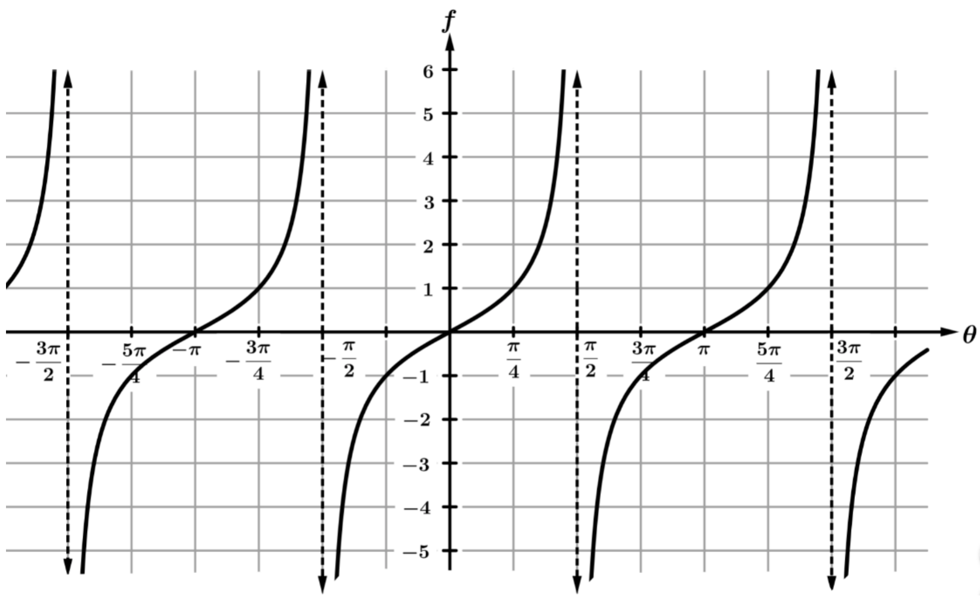


From $\theta = \frac{\pi}{2}$ to $\theta = \pi$, we can see the slopes transition from $-\infty$ to $0$.

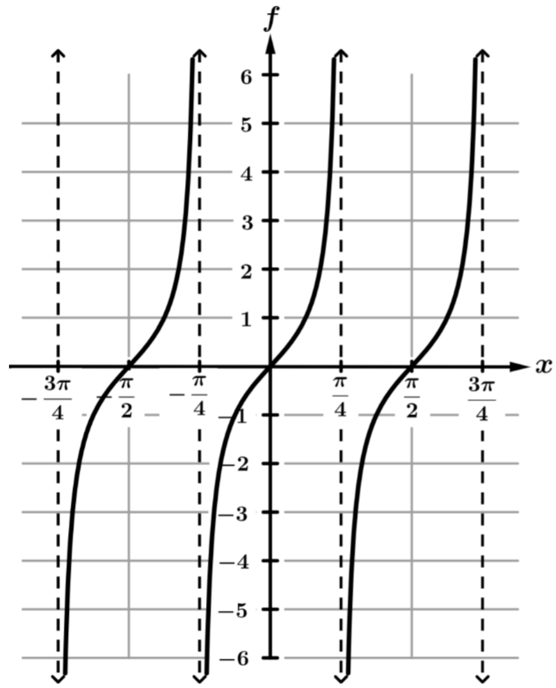
As we continue from $\theta = \pi$ to $\theta = \dfrac{3\pi}{2}$, we can see the slopes are now positive again and progress from a slope of $0$ when $\theta = \pi$ to slopes that increase without bound as $\theta$ approaches $\dfrac{3\pi}{2}$ $\left(\text{slopes approach } +\infty\right)$.



**Key Concept:** Each time that the terminal ray is vertical, the slopes are undefined $\left(\text{they approach } \pm\infty\right)$. When we graph the tangent function, this means that we will see _______________ ______________________ when $\theta = \dfrac{\pi}{2}$ and $\theta = \dfrac{3\pi}{2}$. Each half-turn around the circle, will create another vertical asymptote!




| **Important Features for the Graph of the Tangent Function** | | | |
|---|---|---|---|
| The graph of $y = f(\theta) = a\tan\left(b(\theta + c)\right) + d$ has the following properties: | | | |
| Vertical dilation of the graph of $y = \tan(\theta)$ by a factor of $\lvert a \rvert$. | Period = $\left\lvert \dfrac{\pi}{b} \right\rvert$. | Phase shift of $-c$ units. (Horizontal Translation) | Vertical Translation of the graph of $y = \tan(\theta)$ by $d$ units. |




**Graph of *f***

**Example 1:** A portion of the function $f$ is shown above. The function $f$ can be written as $f(x) = a\tan(bx) + d$, where $a, b,$ and $d$ are constants. Find the values of $a, b,$ and $d$.

**Example 2:** The function $h$ is defined by $h(x) = 4\tan(2x) + 5$. What is the period of $h$?

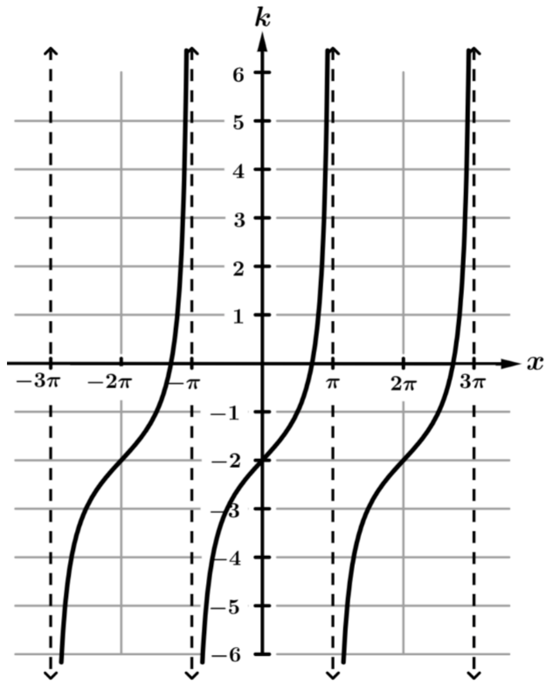
**Example 3:** Let $g$ be the function defined by $g(x) = 2\tan\left(\frac{x}{3}\right) - 1$. Which of the following gives the vertical asymptotes to the graph of $g$?

(A) $x = \frac{\pi}{2} + \pi k$, where $k$ is a integer.

(B) $x = \frac{\pi}{6} + \frac{\pi}{3}k$, where $k$ is a integer.

(C) $x = \frac{3\pi}{2} + 3\pi k$, where $k$ is a constant.

(D) $x = \pi + 2\pi k$, where $k$ is a integer.




**Graph of $k$**

**Example 4:** A portion of the function $k$ is shown above. Which of the following could be the expression for $k$?

(A) $\tan\left(\frac{x}{2}\right)-2$ (B) $\tan\left(\frac{x}{2}\right)+2$ (C) $\tan(2x)-2$ (D) $\tan(2x)+2$

**Example 5:** Find the values of the following expressions using the unit circle.

a) $\tan\left(\frac{\pi}{6}\right)$ b) $\tan\left(\frac{\pi}{4}\right)$ c) $\tan\left(\frac{\pi}{3}\right)$

d) $\tan\left(\frac{5\pi}{6}\right)$ e) $\tan\left(\frac{4\pi}{3}\right)$ f) $\tan\left(\frac{7\pi}{4}\right)$

g) $\tan\left(\frac{3\pi}{2}\right)$ h) $\tan(\pi)$ i) $\tan\left(\frac{\pi}{2}\right)$