1. Let $f(x)=\sec\left(\frac{1}{2}x\right)$. Which of the following is a vertical asymptote on the graph of f?

(A) $x=0$ (B) $x=\frac{\pi}{4}$ (C) $x=\frac{\pi}{2}$ (D) $x=\pi$

$\sec\left(\frac{1}{2}x\right)=\frac{1}{\cos\left(\frac{1}{2}x\right)}$ has a vertical asymptote when $\cos\left(\frac{1}{2}x\right)=0\Rightarrow\frac{1}{2}x=\frac{\pi}{2}\Rightarrow x=\pi$

2. In the xy-plane, the graph of which of the following functions has a vertical asymptote at $x=\frac{\pi}{2}$?

(A) $f(x)=\csc(x)$ (B) $f(x)=\csc(2x)$ (C) $f(x)=\sec\left(x-\frac{\pi}{2}\right)$ (D) $f(x)=\sec\left(\frac{1}{2}x\right)$

$\sec x$ has vertical asymptotes $x=\frac{\pi}{2}+\pi k$ $\csc x$ has vertical asymptotes $x=\pi k$

$\csc(2x)$ has vertical asymptotes $2x=\pi\Rightarrow x=\frac{\pi}{2}$

$\sec\left(x-\frac{\pi}{2}\right)$ has vertical asymptotes $x-\frac{\pi}{2}=\frac{\pi}{2}\Rightarrow x=\pi$ $\sec\left(\frac{1}{2}x\right)$ has vertical asymptotes $\frac{1}{2}x=\frac{\pi}{2}\Rightarrow x=\pi$

3. In the xy-plane, the graph of which of the following functions has a vertical asymptote at $x=2$?

(A) $f(x)=\csc\left(\frac{1}{2}x\right)$ (B) $f(x)=\csc(2x)$ (C) $f(x)=\sec(\pi x)$ (D) $f(x)=\sec\left(\frac{\pi}{4}x\right)$

$\sec x$ has vertical asymptotes $x=\frac{\pi}{2}+\pi k$ $\csc x$ has vertical asymptotes $x=\pi k$

$\csc\left(\frac{1}{2}x\right)$ has vertical asymptotes $\frac{1}{2}x=\pi k\Rightarrow x=2\pi k$ $\csc(2x)$ has vertical asymptotes $2x=\pi k\Rightarrow x=\frac{\pi}{2}k$

$\sec(\pi x)$ has vertical asymptotes $\pi x=\frac{\pi}{2}\Rightarrow x=\frac{1}{2}$ $\sec\left(\frac{\pi}{4}x\right)$ has vertical asymptotes $\frac{\pi}{4}x=\frac{\pi}{2}\Rightarrow x=2$

4. Let $g(x)=\cot(2x)$. Which of the following is a vertical asymptote on the graph of g?

(A) $x=\frac{\pi}{6}$ (B) $x=\frac{\pi}{4}$ (C) $x=\frac{\pi}{3}$ (D) $x=\frac{\pi}{2}$

$\cot x$ has vertical asymptotes $x=\pi k$ $\cot(2x)$ has vertical asymptotes $2x=\pi k\Rightarrow x=\frac{\pi}{2}k$

5. Let $h(x)=3\sec(4x)+1$. Which of the following statements about the graph of h is correct?

(A) The graph of h has vertical asymptotes when $x=\frac{\pi}{8}+\frac{\pi}{4}k$, where k is an integer.

(B) The graph of h has vertical asymptotes when $x=\frac{\pi}{4}+\frac{\pi}{2}k$, where k is an integer.

(C) The graph of h has vertical asymptotes when $x=\frac{\pi}{2}+\frac{\pi}{4}k$, where k is an integer.

(D) The graph of h has vertical asymptotes when $x=2\pi+4\pi k$, where k is an integer.

$\sec x$ has vertical asymptotes $x=\frac{\pi}{2}+\pi k$

$4x=\frac{\pi}{2}+\pi k$

$x=\frac{\pi}{8}+\frac{\pi}{4}k$

6. Let $h(x)=5\sec\left(\frac{1}{2}x\right)$. Which of the following gives the range of h?

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $(-\infty,-5]\cup[5,\infty)$ (C) $(-\infty,-2]\cup[2,\infty)$ (D) $[-5,5]$

The range for $\sec(x)$ is $(-\infty,-1]\cup[1,\infty)\Rightarrow$ $-\infty<\sec\left(\frac{1}{2}x\right)\le -1$, $1\le\sec\left(\frac{1}{2}x\right)<\infty$ $\Rightarrow$ $-\infty<5\sec\left(\frac{1}{2}x\right)\le -5$, $5\le 5\sec\left(\frac{1}{2}x\right)<\infty$

7. Let $k(x)=4\csc(2x)-1$. Which of the following gives the range of k?

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $(-\infty,-4]\cup[4,\infty)$ (C) $(-\infty,-5]\cup[3,\infty)$ (D) $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$

The range for $\csc(x)$ is $(-\infty,-1]\cup[1,\infty)\Rightarrow$ $-\infty<4\csc(2x)\le -4$, $4\le 4\csc(2x)<\infty$ $\Rightarrow$ $-\infty<4\csc(2x)-1\le -5$, $3\le 4\csc(2x)-1<\infty$

8. Let $f(x)=2\sec(x)-5$ and $g(x)=-1$. In the xy-plane, what are the x-coordinates of the points of intersection of the graphs of f and g for $0\le x<2\pi$?

$2\sec(x)-5=-1$ $2\sec(x)=4$ $\sec(x)=2$ $\frac{1}{\cos(x)}=2$ $\cos(x)=\frac{1}{2}$ $x=\frac{\pi}{3},\frac{5\pi}{3}$

9. Let $h(x)=2-3\csc x$ and $k(x)=5$. In the xy-plane, what are the x-coordinates of the points of intersection of the graphs of h and k for $0\le x<2\pi$?

$2-3\csc(x)=5$ $-3\csc(x)=3$ $\csc(x)=-1$ $\frac{1}{\sin(x)}=-1$ $\sin(x)=-1$ $x=\frac{3\pi}{2}$

10. Let $m(x)=3\csc^2 x-2$ and $p(x)=2$. In the xy-plane, what are the x-coordinates of the points of intersection of the graphs of m and p for $0\le x<2\pi$?

$3\csc^2 x-2=2$ $3\csc^2 x=4$ $\csc^2 x=\frac{4}{3}$ $\csc x=\pm\frac{2}{\sqrt{3}}$ $\frac{1}{\sin(x)}=\pm\frac{2}{\sqrt{3}}$

$\sin(x)=\pm\frac{\sqrt{3}}{2}$ $x=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$

11. Let $f(x)=2+3.1\cot(0.3x+5)$. In the xy-plane, what are the x-coordinates of the points of where $f(x)=-6$ for $0\le x<2\pi$?

$2+3.1\cot(0.3x+5)=-6$ $x=3.0450\ldots$