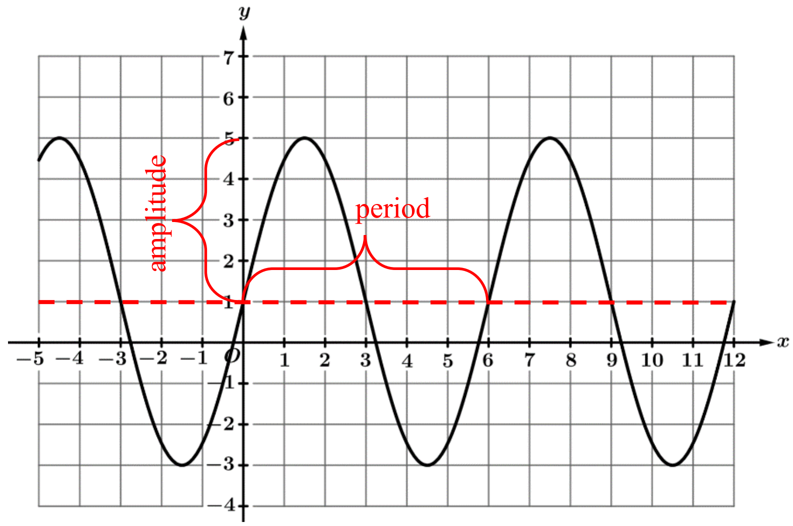


Graph of f

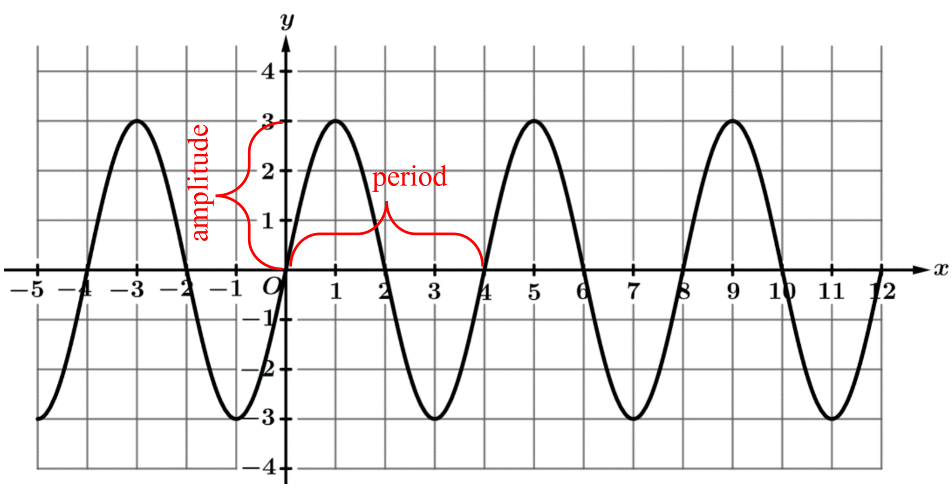
1. The figure shows the graph of a sinusoidal function f. what are the values of the period and amplitude of f?

(A) The period is 3, and the amplitude is 4.

(B) The period is 3, and the amplitude is 8.

(C) The period is 6, and the amplitude is 4.

(D) The period is 6, and the amplitude is 8.



Graph of h

2. The figure shows the graph of a sinusoidal function h. what are the values of the period and amplitude of h?

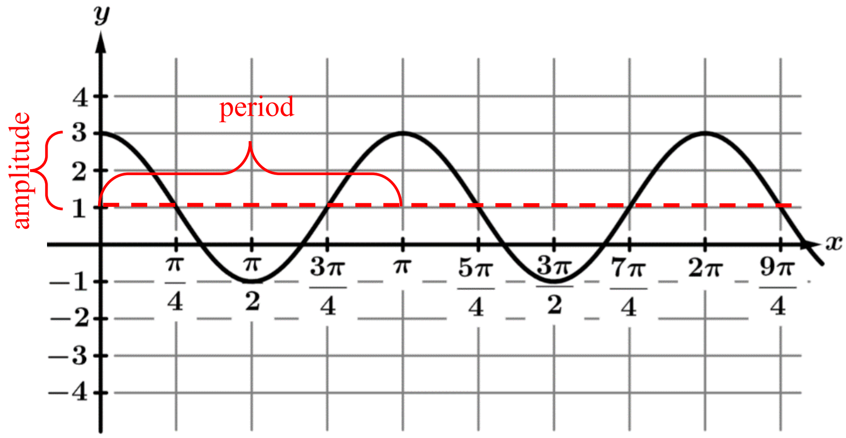
(A) The period is 2, and the amplitude is 3.

(B) The period is 2, and the amplitude is 6.

(C) The period is 4, and the amplitude is 3.

(D) The period is 4, and the amplitude is 6.

Solutions by Ted Gott tedg20776@gmail.com



Graph of *k*

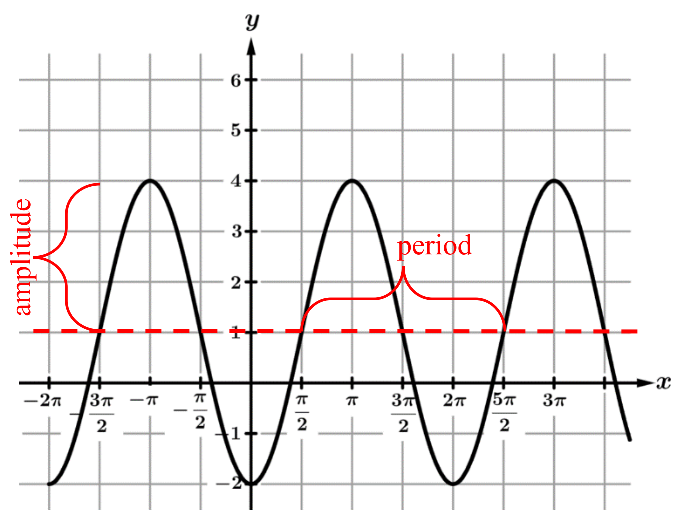
3. The figure shows the graph of a sinusoidal function k. what are the values of the period and amplitude of k?

(A) The period is $\frac{\pi}{2}$, and the amplitude is 2.

(B) The period is $\frac{\pi}{2}$, and the amplitude is 4.

(C) The period is π, and the amplitude is 2.

(D) The period is π, and the amplitude is 4.



Graph of *f*

4. The figure shows the graph of a trigonometric function f. Which of the following could be an expression for $f(x)$?

(A) $3\cos(x)+1$

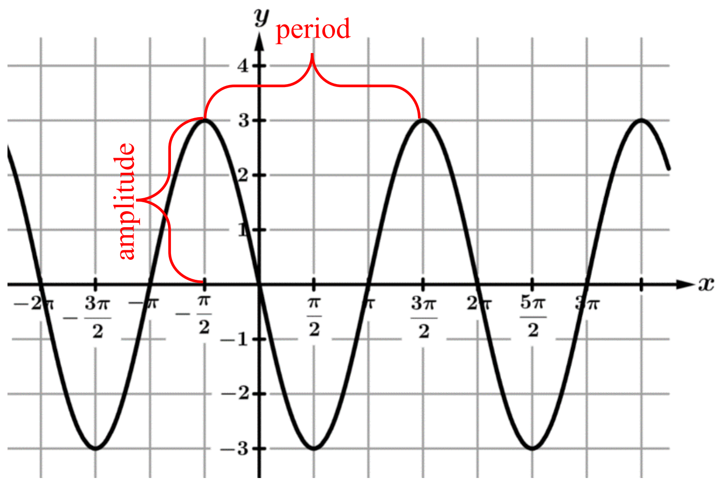
(B) $3\cos\left(x-\frac{\pi}{2}\right)+1$

(C) $3\sin\left(x-\frac{3\pi}{2}\right)+1$

(D) $3\sin\left(x-\frac{5\pi}{2}\right)+1$

Looks like a sine curve shifted to the right. Amplitude 3, vertical shift 1

period $2\pi = \frac{2\pi}{b} \Rightarrow b = 1$ horizontal shift is either $\frac{\pi}{2}$ or $\frac{5\pi}{2}$



Graph of *g*

5. The figure shows the graph of a trigonometric function g. Which of the following could be an expression for $g(x)$?

(A) $3\cos\left(x-\dfrac{\pi}{2}\right)$

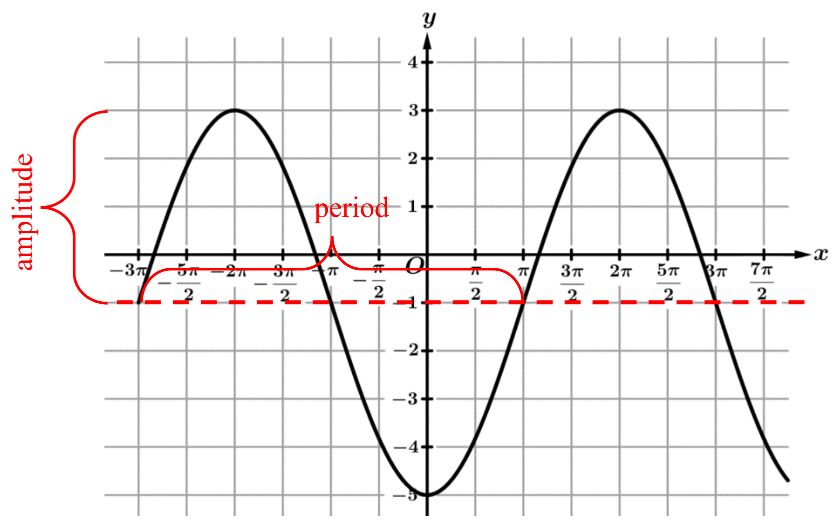
(B) $3\cos\left(x+\dfrac{\pi}{2}\right)$

(C) $3\sin\left(x-2\pi\right)$

(D) $-3\sin\left(x-\pi\right)$

Looks like a cosine curve shifted to the left. Amplitude 3, no vertical shift

period $2\pi = \dfrac{2\pi}{b} \Rightarrow b = 1$ horizontal shift is $-\dfrac{\pi}{2}$



Graph of *h*

6. The figure shows the graph of a trigonometric function h. Which of the following could be an expression for $h(x)$?

(A) $-4\cos\left(\dfrac{1}{2}(x+\pi)\right)-1$

(B) $-4\cos\left(\dfrac{1}{2}(x-2\pi)\right)-1$

(C) $4\sin\left(\dfrac{1}{2}(x+\pi)\right)-1$

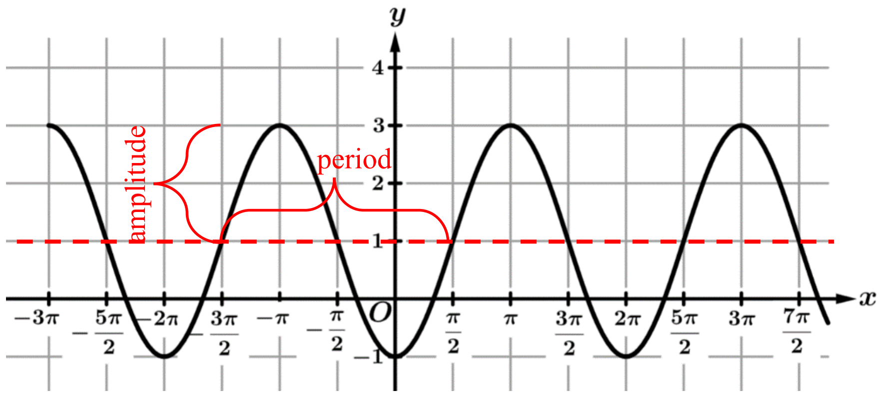
Looks like a sine curve shifted to the left. Amplitude 4, vertical shift -1

period $4\pi = \dfrac{2\pi}{b} \Rightarrow b = \dfrac{1}{2}$ horizontal shift is -3π

$4\sin\left(\dfrac{1}{2}(x+3\pi)\right)-1$ which is not one of the choices but lets look at 1 period

to the right $4\sin\left(\dfrac{1}{2}(x+3\pi-4\pi)\right)-1 = 4\sin\left(\dfrac{1}{2}(x-\pi)\right)-1$

(D) $4\sin\left(\frac{1}{2}(x-\pi)\right)-1$



Graph of k

7. The figure shows the graph of a trigonometric function k. Which of the following could be an expression for $k(x)$?

(A) $2\cos(x)+1$

(B) $-2\cos(x-\pi)+1$

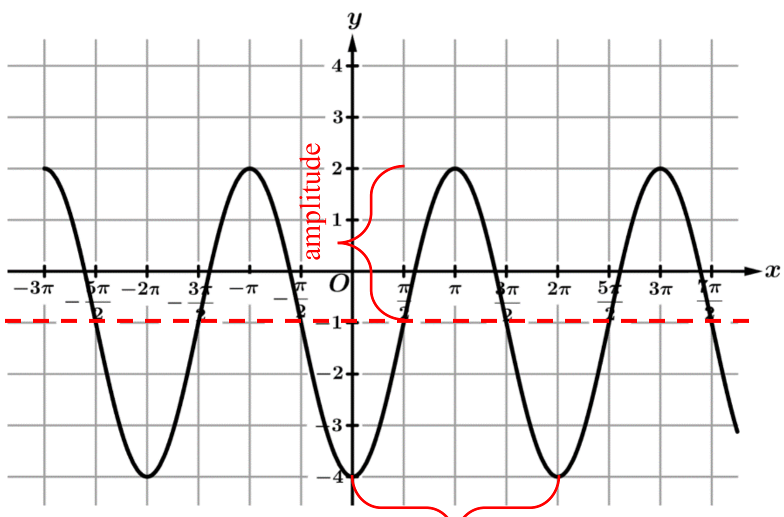
(C) $2\sin\left(x+\frac{\pi}{2}\right)+1$

(D) $-2\sin\left(x-\frac{3\pi}{2}\right)+1$

Looks like a sine curve shifted to the left. Amplitude 2, vertical shift 1

period $2\pi = \frac{2\pi}{b} \Rightarrow b = 1$ horizontal shift is $-\frac{3\pi}{2}$

$2\sin\left(\frac{1}{2}\left(x+\frac{3\pi}{2}\right)\right)+1$ which is not one of the choices but look at the sine curve shifted to the right $\frac{3\pi}{2}$ with a reflection over $y = 1$ $-2\sin\left(\frac{1}{2}\left(x-\frac{3\pi}{2}\right)\right)+1$



Graph of p period

8. The figure shows the graph of a trigonometric function p. Which of the following could be an expression for $p(x)$?

(A) $-3\cos(x-\pi)-1$

(B) $-3\cos(x-2\pi)-1$

(C) $3\sin\left(x-\frac{3\pi}{2}\right)-1$

Looks like a cosine curve with a reflection over $y = 1$. Amplitude 3, vertical shift -1

period $2\pi = \frac{2\pi}{b} \Rightarrow b = 1$ no horizontal shift

$-3\cos(x)-1$ which is not one of the choices but look at the cosine curve shifted to the right 2π $-3\cos(x-2\pi)-1$

(D) $-3\sin\left(x-\frac{\pi}{2}\right)-1$

9. In a particular city, the amount of daylight hours is modeled by the function D, defined by $D(t)=2.715\cos(0.017t)+12.250$ for $0\le t\le 365$ days. Based on the model, which of the following is true?

(A) The maximum amount of daylight hours is 12.250 hours.

(B) The maximum amount of daylight hours occurs at $t=0$ days. $0.017t = 0 \Rightarrow t = 0$

(C) The minimum amount of daylight hours is 12.250 hours.

(D) The minimum amount of daylight hours occurs at $t=0$ days.

10. For a given city, the average high temperature, in ℃, in a given month can be modeled by the function C, defined by $C(t)=3.9\cos(0.475(t-1))+14.1$ for $0\le t\le 12$ months. Based on the model, which of the following is true?

(A) The maximum average high temperature is 14.1 ℃.

(B) The maximum average high temperature occurs at $t=0$ months.

(C) The minimum average high temperature is 10.2 ℃. $\overbrace{14.1}^{\text{midline}} - \overbrace{3.9}^{\text{amplitude}} = 10.2$

(D) The minimum average high temperature occurs at $t=1$ months.

11. In Myrtle Beach, South Carolina, the height of the tide, in feet (ft), is modeled by the function H, defined by $H(t)=2.1\sin\left(\frac{\pi}{6}(t-1)\right)+2.6$ for $0\le t\le 12$ hours. Based on this model, which of the following is true?

(A) The maximum height of the tide is 4.7 feet. $\overbrace{2.6}^{\text{midline}} + \overbrace{2.1}^{\text{amplitude}} = 4.7$

(B) The maximum height of the tide occurs at $t=1$ hours.

(C) The minimum height of the tide is 2.6 feet.

(D) The minimum height of the tide occurs at $t=1$ hours.

12. The population of trout in a particular pond can be modeled by the function F, defined by $F(t)=3000-1200\cos(2\pi t)$ for $0\le t\le 10$ years. Based on the model, which of the following is true?

(A) The maximum number of trout is 3000.

(B) The maximum number of trout occurs at $t=0$ years.

(C) The minimum number of trout is 1200.

$\text{minimum} = 3000 - 1200$

$\text{when } \cos t = 1 \Rightarrow (2\pi t) = 0 \Rightarrow t = 0$

(D) The minimum number of trout occurs at $t = 0$ years.

13. The function f is defined by $f(x) = a\sin(b(x+c)) + d$, for constants $a, b, c,$ and d. In the xy-plane, the points $(4, 1)$ and $(8, 5)$ represent a minimum value and a maximum value, respectively, on the graph of f. What are the values of a and d?

(A) $a = 2$ and $d = 1$

$d = \dfrac{5+1}{2} = 3 \qquad a = 5 - 3 = 2$

(B) $a = 2$ and $d = 3$

(C) $a = 4$ and $d = 1$

(D) $a = 4$ and $d = 3$

14. The function g is defined by $g(x) = a\sin(b(x+c)) + d$, for constants $a, b, c,$ and d. In the xy-plane, the points $(0, -4)$ and $(2\pi, 8)$ represent a minimum value and a maximum value, respectively, on the graph of g. What are the values of a and d?

(A) $a = 3$ and $d = 2$

$d = \dfrac{8+(-4)}{2} = 2 \qquad a = 8 - 2 = 6$

(B) $a = 3$ and $d = 4$

(C) $a = 6$ and $d = 2$

(D) $a = 6$ and $d = 4$

15. The function h is defined by $h(x) = a\sin(b(x+c)) + d$, for constants $a, b, c,$ and d. In the xy-plane, the points $(0, 0)$ and $(2\pi, 6)$ represent a minimum value and a maximum value, respectively, on the graph of h. What is the value of b?

(A) $b = \dfrac{1}{2}$ (B) $b = 1$ (C) $b = 2$ (D) $b = 4\pi$

$\dfrac{1}{2}$ a period $= 2\pi \Rightarrow$ period $= 4\pi \qquad 4\pi = \dfrac{2\pi}{b} \Rightarrow b = \dfrac{2\pi}{4\pi} = \dfrac{1}{2}$

16. The function k is defined by $k(x) = a\cos(bx) + d$, for constants $a, b, c,$ and d. In the xy-plane, the points $(0, -4)$ and $(4, 4)$ represent a minimum value and a maximum value, respectively, on the graph of k. What are the values of a and b?

(A) $a = 4$ and $b = \dfrac{\pi}{4}$

$d = \dfrac{4+(-4)}{2} = 0 \qquad |a| = 4 - 0 = 4$ reflection $\Rightarrow a = -4$

$\dfrac{1}{2}$ a period $= 4 \Rightarrow$ period $= 8 \qquad 8 = \dfrac{2\pi}{b} \Rightarrow b = \dfrac{2\pi}{8} = \dfrac{1}{4}\pi$

(B) $a = 4$ and $b = 8$

(C) $a = -4$ and $b = \dfrac{\pi}{4}$

(D) $a = -4$ and $b = 8$

Starting Position For Gear

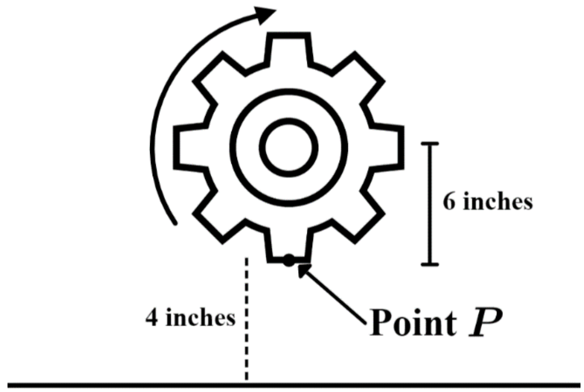
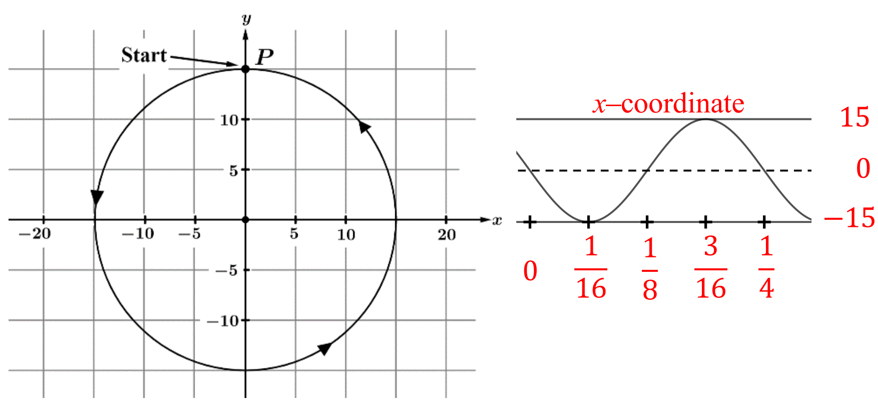


Figure NOT drawn to scale

17. Point P is located at the end of a large gear. At time $t = 0$ seconds, the point P is directly below the center of the gear and is 4 inches above a level surface, as shown in the figure. Point P is 6 inches from the center of the gear. The height of point P, in inches, above the level surface periodically increases and decreases as the gear rotates at a constant speed in the clockwise direction. The gear completes 1 revolution in 2 seconds. Which of the following could be an expression for $h(t)$, the height, in inches, of point P above the level ground at time t seconds?

(A) $6-4\cos(2t)$ (B) $10-6\cos(2t)$ (C) $6-4\cos(\pi t)$ (D) $10-6\cos(\pi t)$

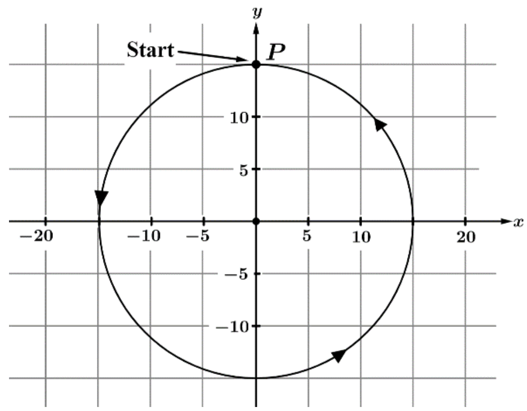
$a = \frac{max - min}{2} = \frac{16-4}{2} = 6$ $d = \frac{16+4}{2} = 10$ period $= 2 = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{2} = \pi$

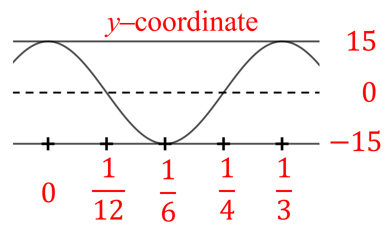


18. A game show uses a large spinner of radius 15 feet that rotates at a constant rate to determine which prizes contestants win. The figure above provides a representation of the wheel in the xy-plane with the direction of rotation indicated. At time $t = 0$ seconds, the spinner begins to rotate. Point P on the wheel is at the "Start" position in the figure. At time $t = 5$ seconds, 20 rotations of the spinner have been completed, and P is in the same position as it was at time $t = 0$. A sinusoidal function is used to model the x-coordinate of the position of P as a function of time t in seconds. Which of the following functions is an appropriate model for this situation?

(A) $f(t) = -15\sin(4t)$ (B) $f(t) = -15\sin(8\pi t)$ (C) $f(t) = 15\sin\left(\frac{1}{4}t\right)$ (D) $f(t) = 15\sin(8\pi t)$

$a = -15$ because reflection $\quad$ $\text{period} = \frac{5 \text{ sec}}{20 \text{ rotations}} = \frac{1}{4} = \frac{2\pi}{b} \Rightarrow b = 8\pi$





19. A game show uses a large spinner of radius 15 feet that rotates at a constant rate to determine which prizes contestants win. The figure above provides a representation of the wheel in the xy-plane with the direction of rotation indicated. At time $t = 0$ seconds, the spinner begins to rotate. Point P on the wheel is at the "Start" position in the figure. At time $t = 10$ seconds, 30 rotations of the spinner have been completed, and P is in the same position as it was at time $t = 0$. A sinusoidal function is used to model the y-coordinate of the position of P as a function of time t in seconds. Which of the following functions is an appropriate model for this situation?

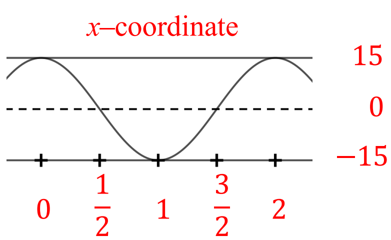
(A) $f(t) = 15\cos(3t)$

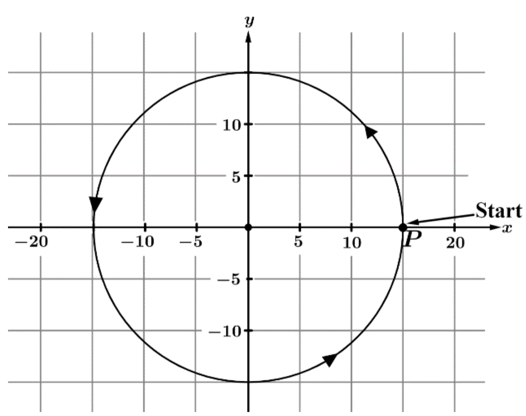
(B) $f(t) = 15\cos(6\pi t)$

(C) $f(t) = 15\cos\left(\frac{2\pi}{3}t\right)$

(D) $f(t) = 15\cos\left(\frac{1}{3}t\right)$

$a = 15$ $\quad$ $\text{period} = \frac{10 \text{ sec}}{30 \text{ rotations}} = \frac{1}{3} = \frac{2\pi}{b} \Rightarrow b = 6\pi$





20. A game show uses a large spinner of radius 15 feet that rotates at a constant rate to determine which prizes contestants win. The figure above provides a representation of the wheel in the xy-plane with the direction of rotation indicated. At time $t = 0$ seconds, the spinner begins to rotate. Point P on the wheel is at the "Start" position in the figure. At time $t = 8$ seconds, 4 rotations of the spinner have been completed, and P is in the same position as it was at time $t = 0$. A sinusoidal function is used to model the x-coordinate of the position of P as a function of time t in seconds. Which of the following functions is an appropriate model for this situation?

(A) $f(t)=15\cos\left(\frac{1}{2}t\right)$ (B) $f(t)=15\cos(2t)$ (C) $f(t)=15\cos(\pi t)$ (D) $f(t)=15\cos(4\pi t)$

$a = 15$ $\quad$ period $= \dfrac{8 \text{ sec}}{4 \text{ rotations}} = 2 = \dfrac{2\pi}{b} \Rightarrow b = \pi$

t	1	3	4	7	9	11
$F(t)$	29.1	42.3	50.6	88.4	76.8	46.9

21. The average high temperature, in degrees Fahrenheit (°F), in a city for a given month is modeled by the function F, where $0 \le t \le 12$ months. The table gives values for the function F at selected values of t. A sinusoidal regression $y = a\sin(bx+c)+d$ is used to model these data. Based on the sinusoidal model, what was the average high temperature, to the nearest degree (°F), at time $t = 6$ months.

(A) 70 (B) 76 (C) 81 (D) 85

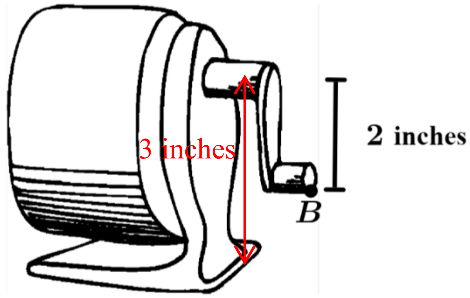
The average high temperature is $y(6) = 81.3397\ldots$

t	1	6	13	18	24	29
$H(t)$	48.2	37.1	24.5	28.5	43.0	50.4

22. The height, in feet, of a remote-controlled drone can be modeled by the function H, for $0 \le t \le 30$ seconds. The table gives values for the function H at selected values of t. A sinusoidal regression $y = a\sin(bx+c)+d$ is used to model these data. Based on the sinusoidal model, what is the height of the drone, to the nearest tenth of a foot, at time $t = 21$ seconds.

(A) 32.7 (B) 35.4 (C) 35.8 (D) 39.5

$y(21) = 35.4108\ldots$ 3 interations (TI − 84 default) $y = 13.1182\ldots\sin(9.1975..x + 1.9806\ldots) + 37.4252\ldots$

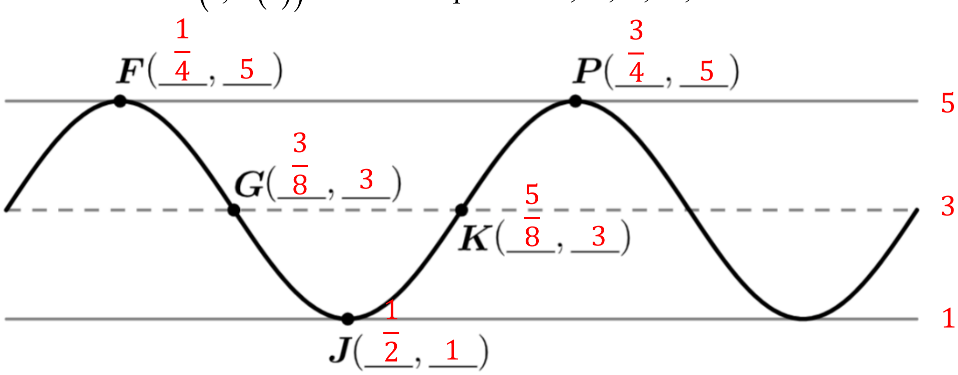


The figure shows a pencil sharpener on a level surface. Point B is located at the end of a handle that is 2 inches from the center of rotation. The handle rotates in a clockwise direction and completes 2 rotations every second. At time $t = 0$ seconds, point B is located directly below the center of rotation. The center of rotation is 3 inches above the level surface on which the pencil sharpener sits. As the handle rotates, the distance between point B and the level surface periodically increases and decreases.

The sinusoidal function h models the distance, in inches, between point B and the level surface as a function of time t in seconds.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, $F, G, J, K,$ and P are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates $(t, h(t))$ for the five points: $F, G, J, K,$ and P.



$\frac{1 \text{ sec}}{2 \text{ rotations}} = \frac{1}{2}\frac{\text{sec}}{\text{rotation}}$ period $= \frac{1}{2}$ seconds Each quarter turn takes $\frac{1}{8}$ seconds

(B) The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of the constants $a, b, c,$ and d.

$\boxed{a = 2}$ period $= \frac{1}{2} = \frac{2\pi}{b} \Rightarrow \boxed{b = 4\pi}$ $4\pi\left(\frac{3}{8} + c\right) = \pi$ which is when sine is at the midline, G

$\left(\frac{3}{8} + c\right) = \frac{\pi}{4\pi} = \frac{1}{4}$ $c = \frac{1}{4} - \frac{3}{8} = \boxed{-\frac{1}{8} = c}$ $\boxed{d = 3}$ midline

(C) Refer to the graph of h in part (A). The t-coordinate of J is t_1, and the t-coordinate of K is t_2.

(j) On the interval (t_1, t_2), which of the following is true about h?

a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing over the interval (t_1, t_2).

The rate of change of h is increasing because the graph is concave up.