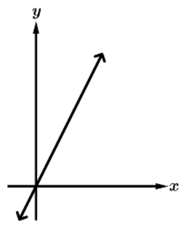
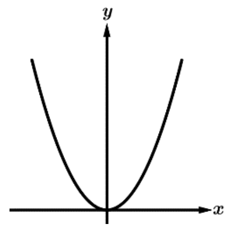
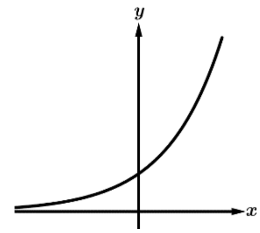
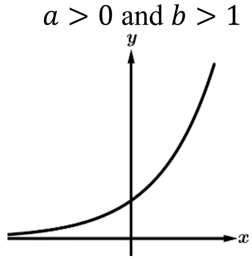
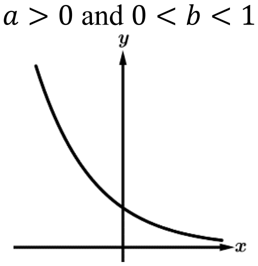
In math, we study several different types of functions. Three types of functions seem to keep showing up in every math course from Algebra 1 to Calculus. These three families of functions are also featured predominantly on the SAT and ACT!

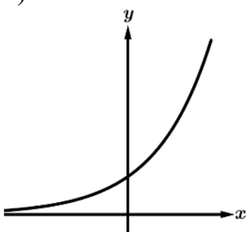
Function	$f(x) = 2x$	$g(x) = x^2$	$h(x) = 2^x$
Name	Linear	Quadratic	Exponential
Graph			

Exponential functions are some of the most important functions in the real world. They appear in many places including population growth, money (interest and debt), radioactive decay, spread of disease, etc… In addition, the SAT will include several questions that require an understanding of exponential functions.

Key Characteristics of Exponential Functions	
An exponential function has the general form $f(x) = a(b)^x, \quad b > 0$ where a and b are constants with $a \neq 0$ and $b \neq 1$.	a represents the initial amount. b represents the base or common ratio.
Exponential Growth $a > 0$ and $b > 1$ 	**Exponential Decay** $a > 0$ and $0 < b < 1$ 
Increasing vs. Decreasing Exponential functions are **always increasing** or **always decreasing**! They will never switch from one to the other, so they have **no relative (local) extrema**.	**Concave Up vs. Concave Down** Exponential functions are **always concave up** or **always concave down**! They will never switch concavity, so they have **no points of inflection**.
End Behavior For exponential functions in general form, as the input values (x) increase/decrease without bound, the output values (y) will increase/decrease without bound or they will approach zero.	**End Behavior Limit Statements** $\lim\limits_{x\to\pm\infty} ab^x = \infty$ or $\lim\limits_{x\to\pm\infty} ab^x = -\infty$ or $\lim\limits_{x\to\pm\infty} ab^x = 0$

Example 1: Write limit statements for the end behavior of the following exponential functions.

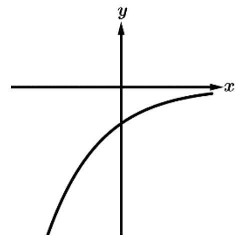
a)



Left: $\lim_{x\to-\infty} ab^x = 0$

Right: $\lim_{x\to\infty} ab^x = \infty$

b)



Left: $\lim_{x\to-\infty} ab^x = -\infty$

Right: $\lim_{x\to\infty} ab^x = 0$

c) $g(x) = 5\left(\frac{2}{3}\right)^x$

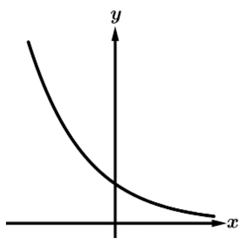
$0 < b < 1 \Rightarrow$ decay

Left: $\lim_{x\to-\infty} g(x) = \infty$

Right: $\lim_{x\to\infty} g(x) = 0$

Example 2: For each of the following, determine if the exponential function is increasing/decreasing and concave up/down.

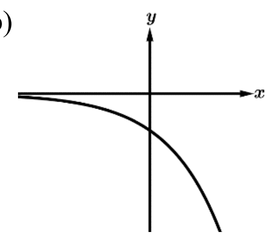
a)



Concave Up or Concave Down

Increasing or Decreasing

b)



Concave Up or Concave Down

Increasing or Decreasing

c) $h(x) = 3(4)^x$

$b > 1 \Rightarrow$ growth

Concave Up or Concave Down

Increasing or Decreasing

x	$f(x)$
1	3
6	5
11	9
16	17
21	33

2, 4, 8, 16

Geometric $\Rightarrow$ transformation of exponential function.

Example 3: Selected values of the function f are shown in the table above. Determine if f could be linear, quadratic, exponential, or none of these. $g(x)$ is exponential and $f(x) = g(x) + 1$ is a vertical translation of $g(x)$.

Students are not asked to give an expression for $f(x)$.
$f(x) = 2^{(x+4)/5} + 1$

x	$f(x)$	$g(x) = f(x) - 1$
1	3	2
6	5	$4 = 2^2$
11	9	$8 = 2^3$
16	17	$16 = 2^4$
21	33	$32 = 2^5$