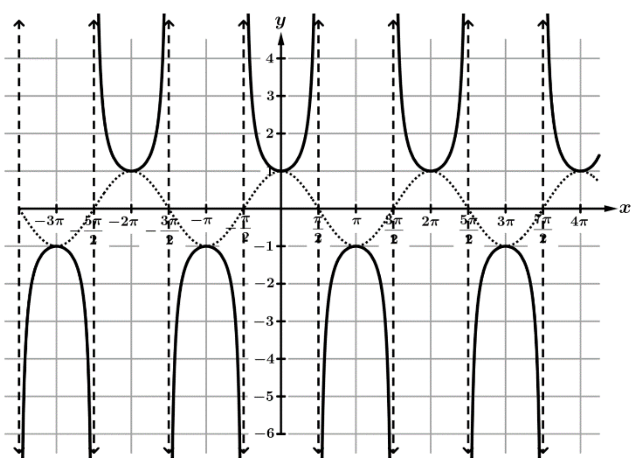
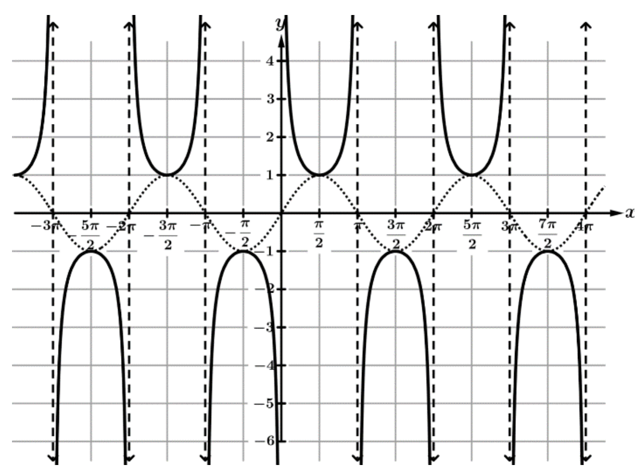
Notes: (Topic 3.11) The Secant, Cosecant, and Cotangent Functions Solutions

Just when you thought these trigonometric functions couldn't get more exciting…today, we get to learn about three additional trig functions!

The good news is that, while these are three new functions, they correspond to the sine, cosine, and tangent functions we already know and love. So, these new functions should not be too difficult for us to incorporate into our existing knowledge and quickly understand the ideas behind them.

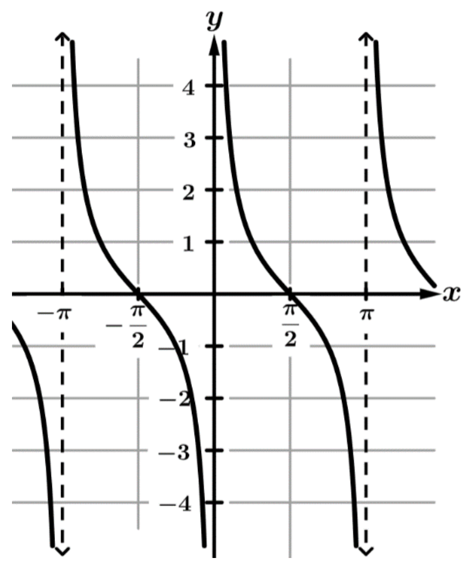
The Reciprocal Functions: Secant, Cosecant, and Cotangent		
Secant	**Cosecant**	**Cotangent**
$\sec x = \dfrac{1}{\cos x}$, where $\cos x \neq 0$	$\csc x = \dfrac{1}{\sin x}$, where $\sin x \neq 0$	$\cot x = \dfrac{1}{\tan x}$, where $\tan x \neq 0$
Secant is the reciprocal of the cosine function.	Cosecant is the reciprocal of the sine function.	$\cot x = \dfrac{\cos x}{\sin x}$, where $\sin x \neq 0$

Note: The graphs of each of these reciprocal functions will include vertical asymptotes whenever the denominator is 0.

$y = \sec x$	$y = \csc x$
	
Note: The graph of $y = \sec x$ has vertical asymptotes where $\cos x = 0$: $x = \dfrac{\pi}{2} + \pi k$, where k is an integer	**Note:** The graph of $y = \csc x$ has vertical asymptotes where $\sin x = 0$: $x = \pi k$, where k is an integer

The ranges of $y = \sec x$ and $y = \csc x$ are both $(-\infty, -1] \cup [1, \infty)$.

$y = \cot x$



Note: The graph of $y = \cot x$ has vertical asymptotes where $\sin x = 0$: $x = \pi k$, where k is an integer

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Example 1: Let $f(x) = 3\sec(2x)$. Which of the following is a vertical asymptote on the graph of f?

(A) $x = \pi$ (B) $x = \dfrac{\pi}{2}$ (C) $x = \dfrac{\pi}{4}$ (D) $x = 0$

$\sec(2x) = \dfrac{1}{\cos(2x)}$ has a vertical asymptote when $\cos(2x) = 0 \Rightarrow 2x = \dfrac{\pi}{2} \Rightarrow x = \dfrac{\pi}{4}$

Example 2: Let $g(x) = 4 - 2\csc(\pi x)$. Which of the following is a vertical asymptote on the graph of g?

(A) $x = \dfrac{\pi}{2}$ (B) $x = \pi$ (C) $x = \dfrac{1}{2}$ (D) $x = 1$

$\csc(\pi x) = \dfrac{1}{\sin(\pi x)}$ has a vertical asymptote when $\sin(\pi x) = 0 \Rightarrow \pi x = \pi k \Rightarrow x = k$

Example 3: Let $h(\theta) = 3\csc\left(\dfrac{\theta}{2}\right)$. Which of the following gives the range of h?

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $(-\infty, -2] \cup [2, \infty)$ (C) $(-\infty, -3] \cup [3, \infty)$ (D) $[-3, 3]$

The range for $\csc(x)$ is $(-\infty, -1] \cup [1, \infty) \Rightarrow \begin{matrix} -\infty < \csc\left(\dfrac{\theta}{2}\right) \le -1 \\ 1 \le \csc\left(\dfrac{\theta}{2}\right) < \infty \end{matrix} \Rightarrow \begin{matrix} -\infty < 3\csc\left(\dfrac{\theta}{2}\right) \le -3 \\ 3 \le 3\csc\left(\dfrac{\theta}{2}\right) < \infty \end{matrix}$

Example 4: Let $k(x) = -5\cot(2\pi x)$. Which of the following is a vertical asymptote on the graph of k?

(A) $x = \dfrac{1}{4}$ (B) $x = \dfrac{1}{2}$ (C) $x = \dfrac{\pi}{2}$ (D) $x = 2\pi$

$2\pi x = \pi k$ $\quad x = \dfrac{\pi}{2\pi}k = \dfrac{1}{2}k$

Solving Equations Involving Secant, Cosecant, and Cotangent

Just as we were able to solve equations involving sine, cosine, and tangent functions, we can similarly solve equations involving their reciprocals. To do this, it is generally best to isolate the trig function first in the equation. Once the trig function is isolated, we can use the reciprocal properties to rewrite the equation in terms of sine, cosine, or tangent. This will allow us to use our previous knowledge to find the appropriate solutions.

Example 5: Let $f(x) = 4\csc(x) + 3$ and $g(x) = 11$. In the xy-plane, what are the x-coordinates of the points of intersection of the graphs of f and g for $0 \le x < 2\pi$?

$4\csc(x) + 3 = 11$ $\quad 4\csc(x) = 8$ $\quad \csc(x) = 2$ $\quad \dfrac{1}{\sin(x)} = 2$ $\quad \sin(x) = \dfrac{1}{2}$ $\quad x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

Example 6: Let $h(x) = 3 - \frac{1}{2}\sec x$ and $k(x) = 4$. In the xy-plane, what are the x-coordinates of the points of intersection of the graphs of h and k for $0 \le x < 2\pi$?

$3 - \dfrac{1}{2}\sec(x) = 4$ $\quad -\dfrac{1}{2}\sec(x) = 1$ $\quad \sec(x) = -2$ $\quad \dfrac{1}{\cos(x)} = -2$ $\quad \cos(x) = -\dfrac{1}{2}$ $\quad x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

Example 7: Let $p(x) = 3 - 1.7\cot(0.5x - 1)$. In the xy-plane, what are the x-coordinates of the points of where $p(x) = 2$ for $0 \le x < 2\pi$?

$3 - 1.7\cot(0.5x - 1) = 2$ $\quad x = 4.0781\ldots$