**Graph of  $f$** 

1. The sinusoidal function  $f(x)$  is shown in the figure above. Find the period, frequency, amplitude, and midline for the graph of  $f(x)$ .

**Period:**

$4$

**Frequency:**

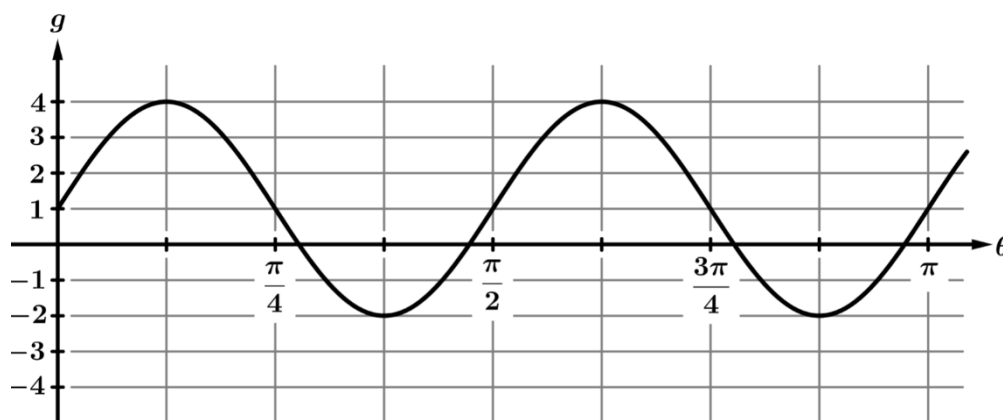
$\frac{1}{4}$

**Midline:**

$$y = \frac{3 + (-1)}{2} = 1$$

**Amplitude:**

$3 - (1) = 2$

**Graph of  $g$** 

2. The sinusoidal function  $g(\theta)$  is shown in the figure above. Find the period, frequency, amplitude, and midline for the graph of  $g(\theta)$ .

**Period:**

$\frac{\pi}{2}$

**Frequency:**

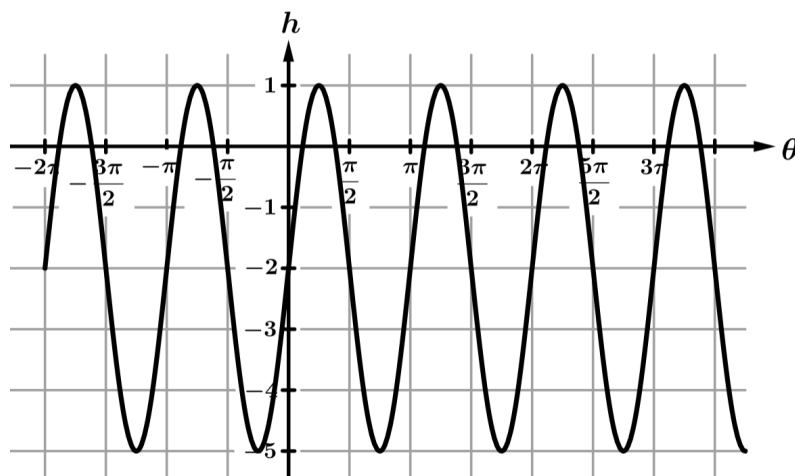
$\frac{2}{\pi}$

**Midline:**

$$y = \frac{4 + (-2)}{2} = 1$$

**Amplitude:**

$4 - (1) = 3$



Graph of  $h$

3. The sinusoidal function  $h(\theta)$  is shown in the figure above. Find the period, frequency, amplitude, and midline for the graph of  $h(\theta)$ .

**Period:**

$$\pi$$

**Frequency:**

$$\frac{1}{\pi}$$

**Midline:**

$$y = \frac{1 + (-5)}{2} = -2$$

**Amplitude:**

$$1 - (-2) = 3$$

4. The sinusoidal function  $g(\theta)$  has a maximum at the point  $(0, 24)$ . The first minimum after reaching this maximum value occurs at the point  $(3\pi, 10)$ . Find the period, frequency, amplitude, and midline for the graph of  $g(\theta)$ .

**Period:**

$$\underbrace{3\pi - 0 = 3\pi}_{\frac{1}{2} \text{ period}} \Rightarrow 6\pi$$

**Frequency:**

$$\frac{1}{6\pi}$$

**Midline:**

$$y = \frac{24 + (10)}{2} = 17$$

**Amplitude:**

$$24 - 17 = 7$$

5. The sinusoidal function  $h(\theta)$  has a minimum at the point  $\left(\frac{\pi}{4}, 6\right)$ . The first maximum after reaching this minimum value occurs at the point  $\left(\frac{5\pi}{4}, 16\right)$ . Find the period, frequency, amplitude, and midline for the graph of  $h(\theta)$ .

**Period:**

$$\underbrace{\frac{5\pi}{4} - \frac{\pi}{4} = \pi}_{\frac{1}{2} \text{ period}} \Rightarrow 2\pi$$

**Frequency:**

$$\frac{1}{2\pi}$$

**Midline:**

$$y = \frac{16 + (6)}{2} = 11$$

**Amplitude:**

$$16 - 11 = 5$$

6. The sinusoidal function  $k(\theta)$  has a minimum at the point  $\left(-\frac{\pi}{2}, -4\right)$ . The first maximum after reaching this minimum value occurs at the point  $(0, 8)$ . Find the period, frequency, amplitude, and midline for the graph of  $k(\theta)$ .

**Period:**

$$\underbrace{0 - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}}_{\frac{1}{2} \text{ period}} \Rightarrow \pi$$

**Frequency:**

$$\frac{1}{\pi}$$

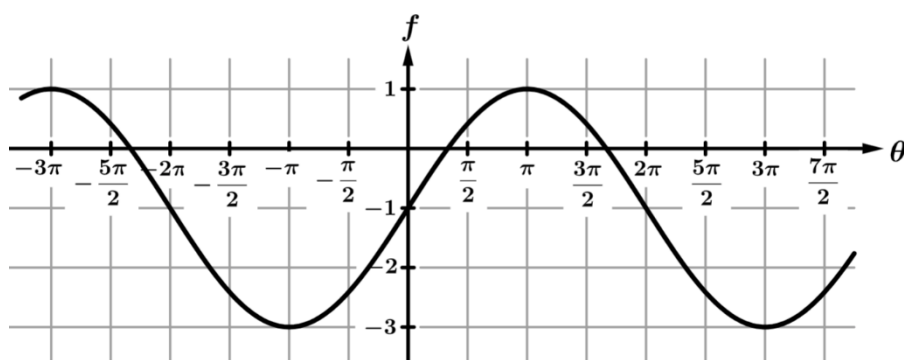
**Midline:**

$$y = \frac{8 + (-4)}{2} = 2$$

**Amplitude:**

$$8 - 2 = 6$$

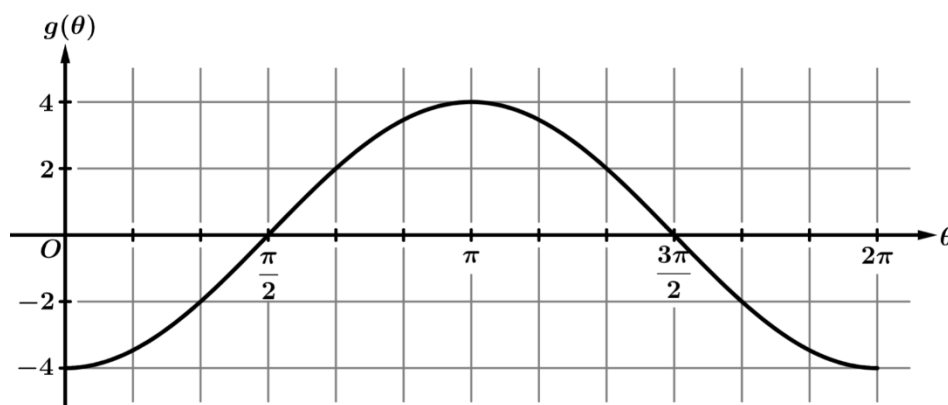
**Directions:** Use the graph of the sinusoidal function  $f(\theta)$  to answer problems 7 – 10.



**Graph of  $f$**

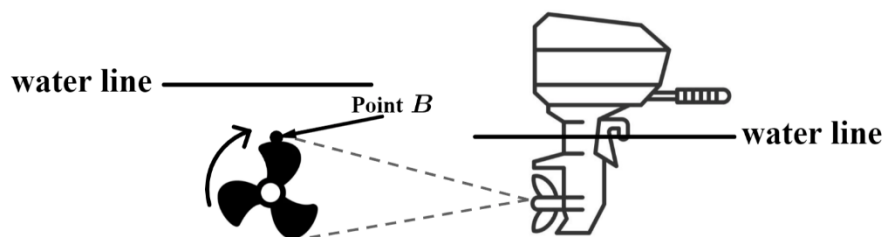
7. Which of the following best describes the behavior of  $f$  over the interval  $0 < \theta < \pi$ ?
- (A)  $f$  is increasing at an increasing rate.
  - (B)  $f$  is increasing at a decreasing rate. **Increasing and concave down**
  - (C)  $f$  is decreasing at an increasing rate.
  - (D)  $f$  is decreasing at a decreasing rate.
8. Which of the following best describes the behavior of  $f$  over the interval  $-2\pi < \theta < -\pi$ ?
- (A)  $f$  is increasing and the graph of  $f$  is concave up.
  - (B)  $f$  is increasing and the graph of  $f$  is concave down.
  - (C)  $f$  is decreasing and the graph of  $f$  is concave up.
  - (D)  $f$  is decreasing and the graph of  $f$  is concave down.
9. Which of the following best describes the behavior of  $f$  over the interval  $2\pi < \theta < 3\pi$ ?
- (A)  $f$  is positive and increasing.
  - (B)  $f$  is positive and decreasing.
  - (C)  $f$  is negative and increasing.
  - (D)  $f$  is negative and decreasing.
10. Which of the following best describes the rate of change of  $f$  over the interval  $-\pi < \theta < 0$ ?
- (A) The rate of change of  $f$  is positive and increasing.  **$f$  is concave up and is increasing.**
  - (B) The rate of change of  $f$  is positive and decreasing.
  - (C) The rate of change of  $f$  is negative and increasing.
  - (D) The rate of change of  $f$  is negative and decreasing.

**Directions:** Use the graph of the sinusoidal function  $g(\theta)$  to answer problems 11 – 14.



**Graph of  $g$**

11. Which of the following best describes the behavior of  $g$  over the interval  $0 < \theta < \frac{\pi}{2}$ ?
- (A)  $g$  is positive and increasing.
  - (B)  $g$  is positive and decreasing.
  - (C)  $g$  is negative and increasing.
  - (D)  $g$  is negative and decreasing.
12. Which of the following best describes the behavior of  $g$  over the interval  $\frac{\pi}{2} < \theta < \pi$ ?
- (A)  $g$  is increasing at an increasing rate.
  - (B)  $g$  is increasing at a decreasing rate. g is concave down
  - (C)  $g$  is decreasing at an increasing rate.
  - (D)  $g$  is decreasing at a decreasing rate.
13. Which of the following best describes the rate of change of  $g$  over the interval  $\pi < \theta < \frac{3\pi}{2}$ ?
- (A) The rate of change of  $g$  is positive and increasing.
  - (B) The rate of change of  $g$  is positive and decreasing.
  - (C) The rate of change of  $g$  is negative and increasing.
  - (D) The rate of change of  $g$  is negative and decreasing. g is decreasing and concave down.
14. Which of the following best describes the behavior of  $g$  over the interval  $\frac{3\pi}{2} < \theta < 2\pi$ ?
- (A)  $g$  is increasing and the graph of  $g$  is concave up.
  - (B)  $g$  is increasing and the graph of  $g$  is concave down.
  - (C)  $g$  is decreasing and the graph of  $g$  is concave up.
  - (D)  $g$  is decreasing and the graph of  $g$  is concave down.



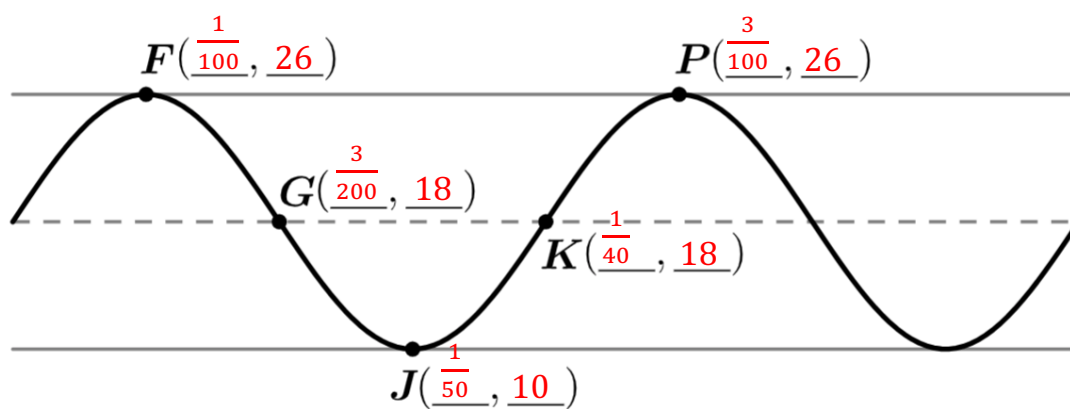
**Note:** Figure NOT drawn to scale

15. The blades of a boat motor rotate in a clockwise direction and complete 50 rotations every second. Point B is on the tip of one of the blades and is located directly above the center of the motor at time  $t = 0$  seconds, as indicated in the figure. Point B is 8 inches from the center of the motor. The center of the motor is 18 inches below the water line. As the blades of the motor rotate at a constant speed, the distance between B and the water line periodically increases and decreases.

The sinusoidal function  $h$  models the distance between point B and the water line, in inches, as a function of time  $t$  in seconds.

(A) The graph of  $h$  and its dashed midline for two full cycles is shown. Five points,  $F$ ,  $G$ ,  $J$ ,  $K$ , and  $P$  are labeled on the graph. No scale is indicated, and no axes are presented.

Determine possible coordinates  $(t, h(t))$  for the five points:  $F$ ,  $G$ ,  $J$ ,  $K$ , and  $P$ .



$$\frac{1}{4} \cdot \frac{1}{50} = \frac{1}{200}$$

$$\frac{1}{100} = \frac{2}{200}$$

$$\frac{1}{50} = \frac{4}{200}$$

$$\frac{1}{40} = \frac{5}{200}$$

$$\frac{3}{100} = \frac{6}{200}$$

$$\frac{1}{25} = \frac{8}{200}$$

At time  $t = 0$  seconds, point B is at the highest spot on the blade and 10 inches from the water line, so one rotation later, at  $t = \frac{1}{50} = 0.02$  seconds, the point B is 10 inches from the water line.

(B) Find the period, frequency, amplitude, and midline for the graph of  $h$ .

**Period:**

$$\frac{1}{50}$$

**Frequency:**

$$50$$

**Midline:**

$$y = 18$$

**Amplitude:**

$$8$$

(C) Find two intervals for which the graph of  $h$  is both decreasing and concave up.

$h$  is both decreasing and concave up on the intervals  $\left(\frac{3}{200}, \frac{1}{50}\right)$  and  $\underbrace{\left(\frac{3}{200} + \frac{1}{50}, \frac{1}{50} + \frac{1}{50}\right)}_{1 \text{ period}} = \left(\frac{7}{200}, \frac{1}{25}\right)$