The properties of exponents that you learned in algebra can also be used to manipulate/rearrange exponential functions into an equivalent form.

Review of Important Exponent Rules		
Product Property	**Power Property**	**Negative Exponent Property**
$b^m b^n = b^{m+n}$	$\left(b^m\right)^n = b^{mn}$	$b^{-n} = \dfrac{1}{b^n}$
Examples: $x^5 x^3 = x^{5+3} = x^8$	**Examples:** $\left(x^5\right)^3 = x^{5(3)} = x^{15}$	**Examples:** $x^{-3} = \dfrac{1}{x^3}$
$2^x 2^3 = 2^{x+3}$	$\left(2^x\right)^3 = 2^{3x}$	$2^{-x} = \dfrac{1}{2^x}$

These same exponent rules can be used when working with horizontal transformations of exponential functions.

Example 1: Determine the horizontal transformations of each of the following exponential functions.

a) $f(x) = 4^{x+2}$

$y = 4^x$ is shifted left 2 units

b) $g(x) = 2^{3x}$

$y = 2^x$ is horizontally dilated by a factor of $\frac{1}{3}$

c) $h(x) = 9^{x/2}$

$y = 9^x$ is horizontally dilated by a factor of 2

d) $k(x) = 5^{x-1}$

$y = 5^x$ is shifted right 1 unit

Now, let's reexamine the exponential function from **Example 1a**, and see if we can use our exponent properties to rearrange the function into an equivalent form.

We can use the **Product Property** (in reverse) to show that $4^{x+2} = 4^x 4^2 = 16\left(4^x\right)$.

This means that $f(x) = 4^{x+2}$ is equivalent to writing $f(x) = 16(4)^x$.

When we rewrite the exponential function in this way, there is no long a horizontal translation, but now we have a vertical dilation by a factor of 16.

Important Idea: Every **horizontal translation** of an exponential function $\left(f(x) = b^{x+h}\right)$ is equivalent to a **vertical dilation** of the exponential function $\left(f(x) = ab^x\right)$ where $a = b^h$.

Example 2: Each of the following exponential functions has a horizontal translation. For each, write an equivalent representation that has a vertical dilation and no horizontal translation $\left(f(x) = ab^x\right)$.

a) $f(x) = 2^{x+3}$

$f(x) = 2^3 2^x = 8(2^x)$

b) $g(x) = 3^{x-2}$

$g(x) = 3^{-2} 3^x = \frac{1}{9}(3^x)$

c) $k(x) = 4(3)^{x+2}$

$k(x) = 4(3^2)(3^x) = 36(3^x)$

We can also use the **Power Property of Exponents** to show that every horizontal dilation of an exponential function, $\left(f(x)=b^{cx}\right)$, is equivalent to changing the base of the exponential function $\left(f(x)=\left(b^c\right)^x\right)$.

Example 3: Which of the following functions is an equivalent form of the function $y=9^{2x}$?

(A) $f(x)=3^x$

(B) $f(x)=3\cdot9^x$

(C) $f(x)=18^x$

(D) $f(x)=81^x=(9^2)^x=9^{2x}$

Example 4: Which of the following functions is an equivalent form of the function $y=9\cdot4^x$?

(A) $f(x)=3\cdot16^{x/2}$

(B) $f(x)=3\cdot16^{2x}$

(C) $f(x)=9\cdot16^{x/2}=9\left(16^{1/2}\right)^x=9\cdot4^x$

(D) $f(x)=9\cdot16^{2x}$

Exponential Function Manipulation