Worksheet B: (Topic 2.9) Logarithmic Expressions Name: Solutions

Directions: For problems 1 – 3, write each exponential equation in logarithmic form.

1. $e^{x+2} = 7$
   $\ln(e^{x+2}) = \ln(7)$
   $x + 2 = \ln(7)$
   $x = \ln(7) - 2$

2. $4 = 5^{-3x}$
   $\log_5 4 = -3x$
   $-\frac{1}{3}\log_5 4 = x$

3. $10^x = x + 3$
   $\log 10^x = \log(x + 3)$
   $x = \log(x + 3)$

Directions: For problems 4 – 6, write each logarithmic equation in exponential form.

4. $\ln(3x) = -2$
   $e^{\ln(3x)} = e^{-2}$
   $3x = e^{-2}$
   $x = \frac{e^{-2}}{3} = \frac{1}{3e^2}$

5. $1 = \log(x)$
   $10^1 = 10^{\log(x)}$
   $10 = x$

6. $\log_7(e^x) = 3$
   $7^{\log_7(e^x)} = 7^3$
   $e^x = 7^3$

Directions: Evaluate the following expressions without a calculator.

7. $\log_2 16 = 4$
   $2^4 = 16$

8. $\ln e^{-3} = -3$

9. $\log\frac{1}{1000} = -3$
   $\frac{1}{1000} = 10^{-3}$

10. $\log_3 27 = 3$
    $3^3 = 27$

11. $\log_5 125 = 3$
    $5^3 = 125$

12. $\log_{27} 3 = \frac{1}{3}$
    $\sqrt[3]{27} = 27^{\frac{1}{3}} = 3$

13. $\ln e^{12} = 12$

14. $\log 1 = 0$

15. $\log_{16} 4 = \frac{1}{2}$
    $\sqrt{16} = 16^{\frac{1}{2}} = 4$

16. $\log_9 3 = \frac{1}{2}$
    $\sqrt{9} = 9^{\frac{1}{2}} = 3$

17. $\log_{49} 7 = \frac{1}{2}$
    $\sqrt{49} = 49^{\frac{1}{2}} = 7$

18. $\ln\sqrt{e} = \frac{1}{2}$

19. $\log_8\frac{1}{64} = -2$
    $8^{-2} = \frac{1}{8^2} = \frac{1}{64}$

20. $\log\sqrt{10} = \frac{1}{2}$

21. $\log_8\frac{1}{2} = -\frac{1}{3}$
    $8^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

22. $\ln\frac{1}{e^5} = -5$
    $\frac{1}{e^5} = e^{-5}$

23. $\ln e^{\pi} = \pi$

24. $\log_{16}\frac{1}{4} = -\frac{1}{2}$
    $16^{-\frac{1}{2}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$

25. $\log_{36}\frac{1}{6} = -\frac{1}{2}$
    $36^{-\frac{1}{2}} = \frac{1}{\sqrt{36}} = \frac{1}{6}$

26. $\log_2\frac{1}{32} = -5$
    $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

x	-11	-6	-2	0	1	4	8
$f(x)$	4	8	3	-2	-6	0	-9
$g(x)$	8	5	4	1	-2	-3	-6

x	4	8	3	-2	-6	0	-9
$f^{-1}(x)$	-11	-6	-2	0	1	4	8

x	8	5	4	1	-2	-3	-6
$g^{-1}(x)$	-11	-6	-2	0	1	4	8

Selected values of the continuous functions f and g are shown above. Use the table to find the following, if possible.

27. $f(g(0)) = f(1) = -6$

28. $g(f(-6)) = g(8) - 6$

29. $f^{-1}(0) = 4$

30. $g^{-1}(-2) = 1$

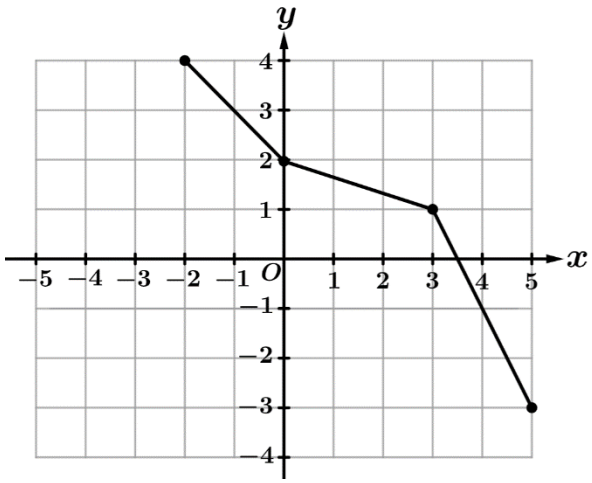
31. $g^{-1}(-6) = 8$

32. $f^{-1}(1)$ not defined because $f(x) \neq 1$

33. $f(g^{-1}(5)) = f(-6) = 8$

34. $g(f^{-1}(3)) = g(-2) = 4$

35. $f^{-1}(g^{-1}(4)) = f^{-1}(-2) = 0$



Graph of h

The graph of the piecewise-linear function h is shown in the figure. Let k be the inverse function of h.

36. $k(3) = -1$ because $h(-1) = 3$

37. $k(-1) = 4$ because $h(4) = -1$

38. $k(0) = 3.5$ because $h(3.5) = 0$

39. What is the minimum value of k?
-2 because $[-2,5]$, the domain of $h(x)$, is the range of $h^{-1}(x) = k(x)$.

40. What is the domain of k?
$[-3,4]$ because the domain of k is the range of h.