The properties of exponents that you learned in algebra can also be used to manipulate/rearrange exponential functions into an equivalent form.

Review of Important Exponent Rules		
Product Property	**Power Property**	**Negative Exponent Property**
$b^m b^n = b^{m+n}$	$\left(b^m\right)^n = b^{mn}$	$b^{-n} = \dfrac{1}{b^n}$
Examples: $x^5x^3 = x^{5+3} = x^8$ $2^x2^3 = 2^{x+3}$	**Examples:** $\left(x^5\right)^3 = x^{5(3)} = x^{15}$ $\left(2^x\right)^3 = 2^{3x}$	**Examples:** $x^{-3} = \dfrac{1}{x^3}$ $2^{-x} = \dfrac{1}{2^x}$

These same exponent rules can be used when working with horizontal transformations of exponential functions.

Example 1: Determine the horizontal transformations of each of the following exponential functions.

a) $f(x) = 4^{x+2}$ b) $g(x) = 2^{3x}$ c) $h(x) = 9^{x/2}$ d) $k(x) = 5^{x-1}$

Now, let's reexamine the exponential function from **Example 1a**, and see if we can use our exponent properties to rearrange the function into an equivalent form.

We can use the **Product Property** (in reverse) to show that $4^{x+2} = 4^x4^2 = 16\left(4^x\right)$.

This means that $f(x) = 4^{x+2}$ is equivalent to writing $f(x) = 16(4)^x$.

When we rewrite the exponential function in this way, there is no long a horizontal translation, but now we have a vertical dilation by a factor of 16.

<u>**Important Idea**</u>: Every **horizontal translation** of an exponential function $\left(f(x) = b^{x+h}\right)$ is equivalent to a **vertical dilation** of the exponential function $\left(f(x) = ab^x\right)$ where $a = b^h$.

Example 2: Each of the following exponential functions has a horizontal translation. For each, write an equivalent representation that has a vertical dilation and no horizontal translation $\left(f(x) = ab^x\right)$.

a) $f(x) = 2^{x+3}$ b) $g(x) = 3^{x-2}$ c) $k(x) = 4(3)^{x+2}$

We can also use the **Power Property of Exponents** to show that every horizontal dilation of an exponential function, $\left(f\left(x\right)=b^{cx}\right)$, is equivalent to changing the base of the exponential function $\left(f\left(x\right)=\left(b^{c}\right)^{x}\right)$.

Example 3: Which of the following functions is an equivalent form of the function $y=9^{2x}$?

(A) $f\left(x\right)=3^{x}$

(B) $f\left(x\right)=3\cdot 9^{x}$

(C) $f\left(x\right)=18^{x}$

(D) $f\left(x\right)=81^{x}$

Example 4: Which of the following functions is an equivalent form of the function $y=9\cdot 4^{x}$?

(A) $f\left(x\right)=3\cdot 16^{x/2}$

(B) $f\left(x\right)=3\cdot 16^{2x}$

(C) $f\left(x\right)=9\cdot 16^{x/2}$

(D) $f\left(x\right)=9\cdot 16^{2x}$