Just as we can graph previous functions, we can also graph logarithmic functions.

General Form of a Logarithmic Function
$f(x) = a\log_b x$
Where $b > 0$, $b \neq 1$, and $a \neq 0$

Because exponential functions and logarithmic functions are **inverses** of each other, we would expect the characteristics of the input-output values of an exponential function to become the characteristics of the output-input values of a logarithmic function.

Recall: For exponential functions, over equal-width input-value intervals, the output values change multiplicatively.

x	$f(x)$
1	2
3	4
5	8
7	16

Notice how the input values (x) are changing additively (equally spaced), while the output values $(f(x))$ are changing multiplicatively (multiplying by 2 each time).

This is indicative of an exponential function.

For logarithmic functions, these two properties of the input and output values are reversed.

x	$g(x)$
2	1
4	3
8	5
16	7

Notice how the input values (x) are changing multiplicatively (multiplying by 2 each time), while the output values $(g(x))$ are changing additively (equally spaced).

This is indicative of a logarithmic function.

Example 1: Selected values for several functions are shown below. For each, determine if the given function could be logarithmic, exponential, or neither.

a)

x	$h(x)$
1	16
2	8
3	4
4	2

Exponential: x values equally spaced and y values multiplying by $\frac{1}{2}$ each time.

b)

x	$k(x)$
10	10
30	20
90	30
270	40

Logarithmic: y values equally spaced and x values multiplying by 3 each time.

c)

x	$p(x)$
5	1
50	2
500	4
5000	8

Neither: x values and y values multiplying by 10 and 2, respectively, each time.

d)

x	$l(x)$
4	-1
8	-4
16	-7
32	-10

Logarithmic: y values equally spaced and x values multiplying by 2 each time.

Exponential and Logarithmic Functions as Inverses

If $f(x) = b^x$ and $g(x) = \log_b x$, then f and g are inverse functions, and

$$f(g(x)) = g(f(x)) = x.$$

Recall: Two important properties of inverse functions:

1. Graphs of inverse functions are reflections of each other over the line $y = x$.

2. If the point (x_1, y_1) is on the graph of f, then the point (y_1, x_1) must be on the graph of f^{-1}.

Example 2: Let $f(x) = 3^x$ and $g(x) = \log_3 x$. Show that f and g are inverse functions by showing that $f(g(x)) = g(f(x)) = x$.

$f(g(x)) = f(\log_3 x) = 3^{\log_3 x} = x$

$g(f(x)) = g(3^x) = \log_3(3^x) = x$

Example 3: A portion of the graph of the exponential function $k(x) = 2^x$ is shown on the graph below. Sketch a picture of $k^{-1}(x) = \log_2 x$ on the same coordinate grid.
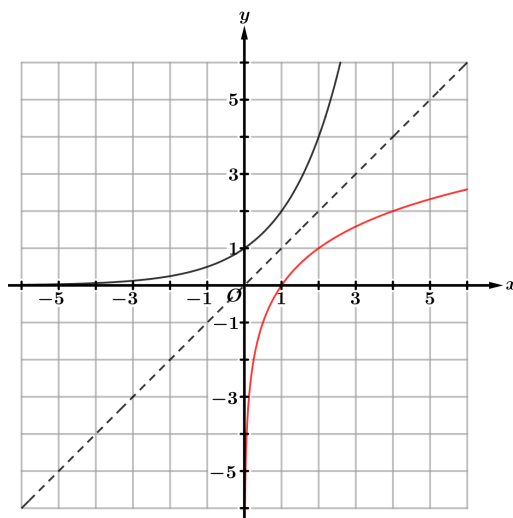
Example 4: The exponential function $h(x) = a^x$ contains the points $(2, 3)$ and $(6, 27)$. Find the average rate of change of $y = \log_a x$ over the interval $[3, 27]$.

$y = \log_a x$ is the inverse of $h(x)$, so $y = \log_a x$ contains the points $(3,2)$ and $(27,6)$.

$AROC = \dfrac{\log_a 27 - \log_a 3}{27 - 3} = \dfrac{6 - 2}{24} = \dfrac{4}{24} = \dfrac{1}{6}$