

Notes: (Topic 1.6) Polynomial Functions and End Behavior

End Behavior: The end behavior of a function describes how a function _____ as it moves infinitely to the right and left.

In other words, the end behavior is what happens to the values of _____ as x increases or decreases without bound.

End Behavior and Limit Notation

Left End Behavior: $\lim_{x \rightarrow -\infty} f(x)$

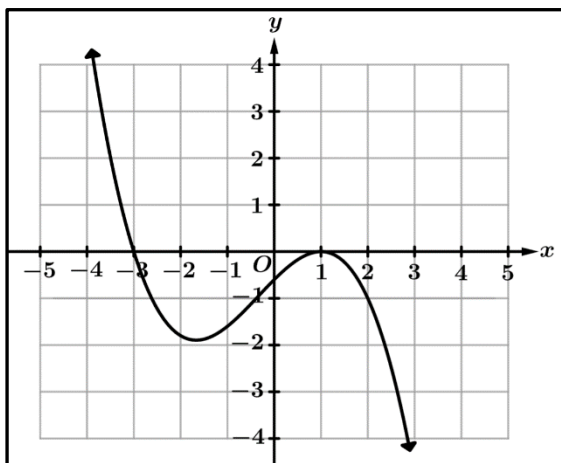
As the x values decrease without bound, the y values of $f(x)$...

Right End Behavior: $\lim_{x \rightarrow \infty} f(x)$

As the x values increase without bound, the y values of $f(x)$...

Example 1: Describe the end behavior of the following polynomials verbally and using limit notation.

a)



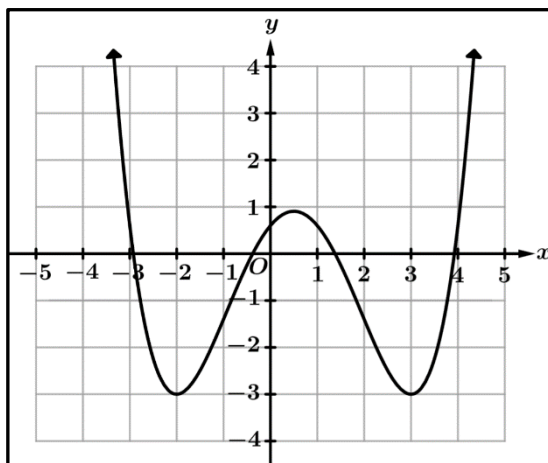
Left Behavior Verbally:

Left Behavior Limit Statement:

Right Behavior Verbally:

Right Behavior Limit Statement:

b)



Left Behavior Verbally:

Left Behavior Limit Statement:

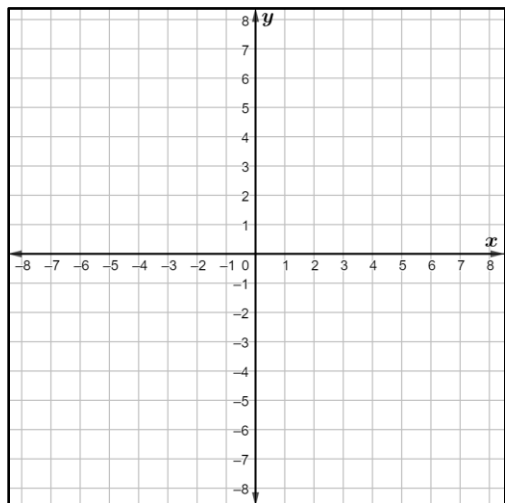
Right Behavior Verbally:

Right Behavior Limit Statement:

Example 2: Sketch a polynomial function with the following end behaviors.

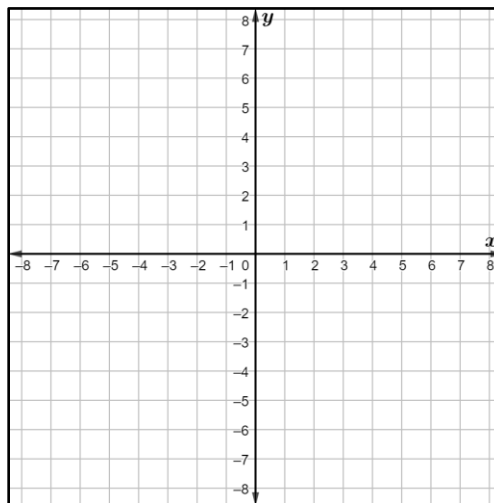
a) $\lim_{x \rightarrow -\infty} f(x) = -\infty$

$\lim_{x \rightarrow \infty} f(x) = -\infty$



b) $\lim_{x \rightarrow -\infty} g(x) = -\infty$

$\lim_{x \rightarrow \infty} g(x) = +\infty$



Polynomial End Behavior

For polynomial equations, it is easiest to find the end behavior of the _____ side first.

The **RIGHT** side:

1. Goes _____ if the leading coefficient is _____.
2. Goes _____ if the leading coefficient is _____.

The **LEFT** side:

1. Goes in the _____ direction as the right if the degree is _____.
2. Goes in the _____ direction as the right if the degree is _____.

Example 3: Determine the end behavior for the following polynomials. Limit notation is not necessary.

a) $f(x) = 4x^5$

L: R:

b) $g(x) = \frac{1}{2}x^4$

L: R:

c) $y = -2(x + 3)^6$

L: R:

d) $h(x) = 3 - x^5$

L: R:

e) $k(x) = 8x^2 + 4 - x^5$

L: R:

f) $m(x) = 2x(x - 1)(6 - x)$

L: R:

Example 4: Write limit statements for the end behavior of the following polynomials.

a) $f(x) = -3x^4$

b) $g(x) = 5x^3 + 2x^2 - 7$

Example 5: Use a graphing calculator to determine the end behavior for the following functions.

a) $f(x) = x^2 - 3x + 1$

b) $g(x) = \frac{2x - 3}{x + 1}$

c) $h(x) = -\frac{10}{x^2}$