1. The figure shows the graph of the polar function $r = f(\theta)$, for $0 \leq \theta \leq 2\pi$, in the polar coordinate system. Which of the following could be an expression for $f(\theta)$?
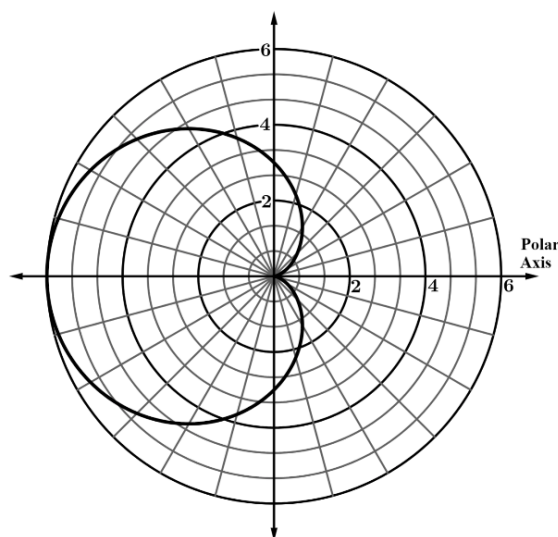
(A) $2 + 4\sin\theta$ (B) $2 - 4\sin\theta$ (C) $2 + 4\cos\theta$ (D) $2 - 4\cos\theta$

$f(0) = 2$ from the graph which eliminates (C) and (D) maybe. $f\left(\frac{\pi}{2}\right) = 6$ which eliminates (B).

2. The figure shows the graph of the polar function $r = f(\theta)$, for $0 \leq \theta \leq 2\pi$, in the polar coordinate system. Which of the following could be an expression for $f(\theta)$?

(A) $3 + 3\sin\theta$ (B) $3 - 3\sin\theta$ (C) $3 + 3\cos\theta$ (D) $3 - 3\cos\theta$

$f(0) = 0$ from the graph which eliminates (A), (B) and (C).

3. The figure shows the graph of the polar function $r = f(\theta)$, for $0 \le \theta \le 2\pi$, in the polar coordinate system. Which of the following could be an expression for $f(\theta)$?
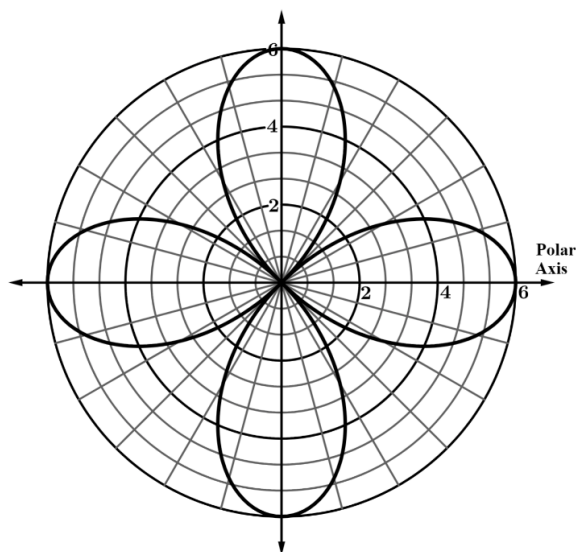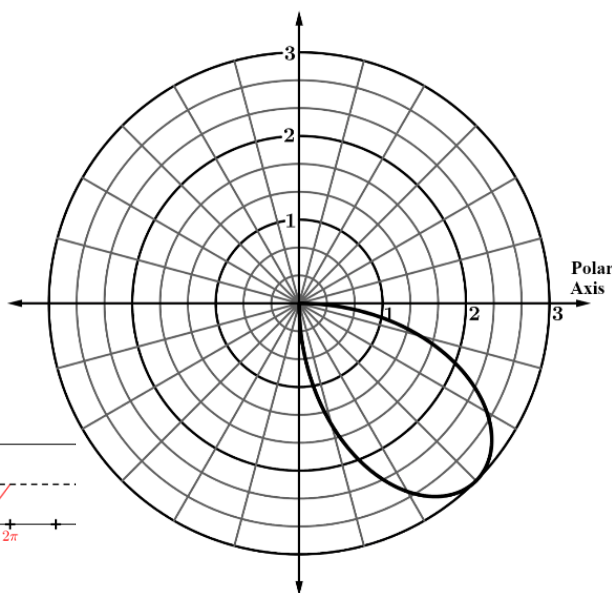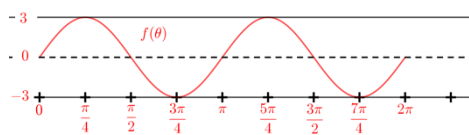
(A) $6\cos(2\theta)$ (B) $6\cos(4\theta)$ (C) $6\sin(2\theta)$ (D) $6\sin(4\theta)$

$f(0) = 6$ from the graph which eliminates (C) and (D). $f\left(\dfrac{\pi}{4}\right) = 0$

$6\cos\left(2 \cdot \dfrac{\pi}{4}\right) = 6\cos\left(\dfrac{\pi}{2}\right) = 0$ $6\cos\left(4 \cdot \dfrac{\pi}{4}\right) = 6\cos(\pi) = -6$ which eliminates (B).

On the interval $\left[\dfrac{3\pi}{4}, \pi\right]$, we only get half the loop.

On the interval $\left[\dfrac{3\pi}{2}, 2\pi\right]$ $f(\theta) \le 0$ and $r < 0$, reflect through the origin into the 2nd quadrant.
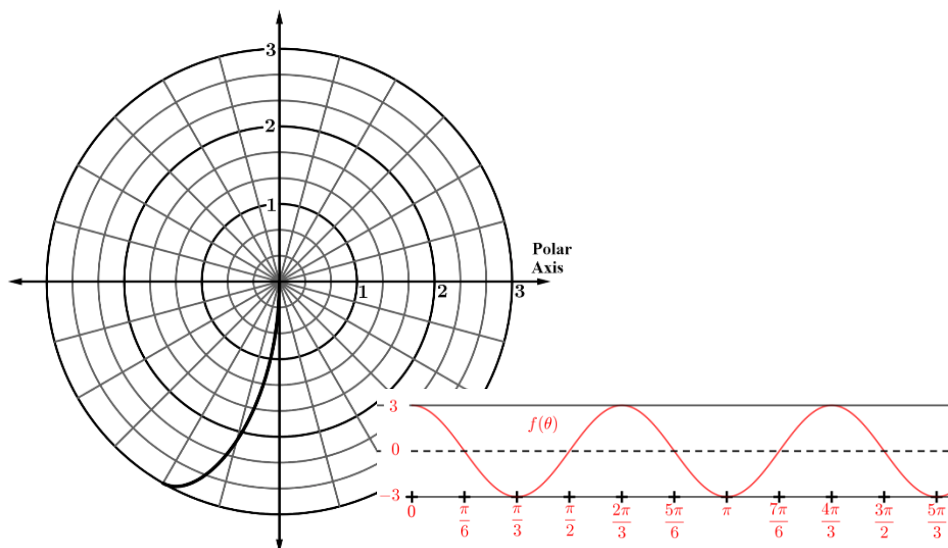

4. A portion of the graph of the polar function $r = f(\theta)$, where $f(\theta) = 3\sin(2\theta)$, is shown in the polar coordinate system for $a \le \theta \le b$. If $0 \le a < b < 2\pi$, which of the following could be the values for a and b?

(A) $a = \dfrac{\pi}{4}$ and $b = \dfrac{\pi}{2}$ (B) $a = \dfrac{\pi}{2}$ and $b = \pi$ (C) $a = \dfrac{3\pi}{4}$ and $b = \pi$ (D) $a = \dfrac{3\pi}{2}$ and $b = 2\pi$

The graph of $f(\theta)$ above (sketched by hand) on the interval $\left[\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ is positive.

On the interval $\left[\dfrac{\pi}{2}, \pi\right]$, $f(\theta) \le 0$, ($r < 0$, reflection through origin). $f\left(\dfrac{3\pi}{4}\right) = -3$.
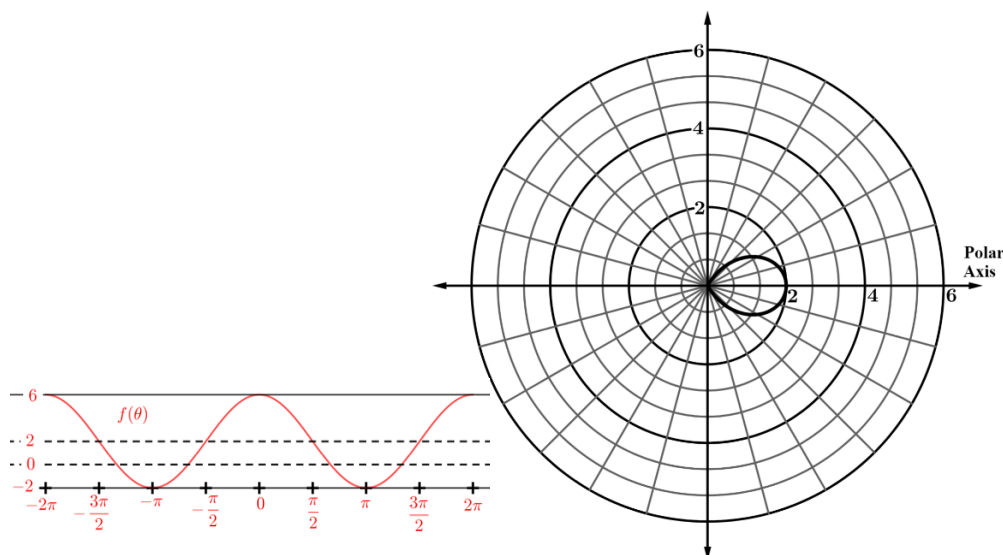
Worksheet A: Topic 3.14   Polar Function Graphs   Created by Bryan Passwater
Solutions by Ted Gott tedg20776@gmail.com

5. A portion of the graph of the polar function $r=f(\theta)$, where $f(\theta)=3\cos(3\theta)$, is shown in the polar coordinate system for $a\le\theta\le b$. If $0\le a<b<2\pi$, which of the following could be the values for a and b?

(A) $a=0$ and $b=\dfrac{\pi}{6}$ (B) $a=\dfrac{\pi}{6}$ and $b=\dfrac{\pi}{3}$ (C) $a=\dfrac{\pi}{3}$ and $b=\dfrac{\pi}{2}$ (D) $a=\dfrac{5\pi}{4}$ and $b=\dfrac{3\pi}{2}$

On the interval $\left[\frac{\pi}{3},\frac{\pi}{2}\right]$, $f\left(\frac{\pi}{3}\right)=-3$ and the curve is in Q3 heading to the origin $f\left(\frac{\pi}{2}\right)=0.$

6. A portion of the graph of the polar function $r=f(\theta)$, where $f(\theta)=2+4\cos\theta$, is shown in the polar coordinate system for $a\le\theta\le b$. If $-2\pi\le a<b<2\pi$, which of the following could be the values for a and b?
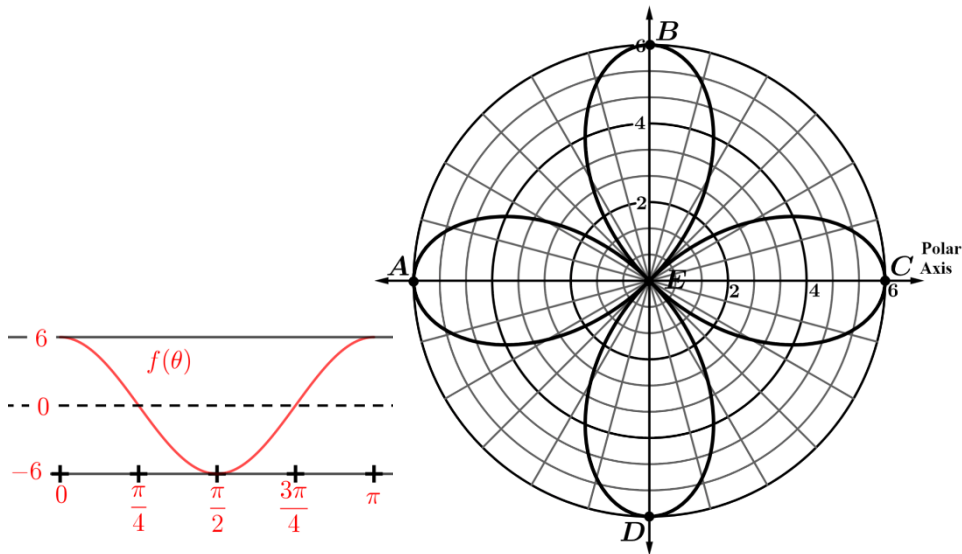
(A) $a=-\dfrac{2\pi}{3}$ and $b=\dfrac{2\pi}{3}$

(B) $a=-\dfrac{\pi}{3}$ and $b=\dfrac{\pi}{3}$

(C) $a=\dfrac{\pi}{2}$ and $b=\dfrac{3\pi}{2}$

(D) $a=\dfrac{2\pi}{3}$ and $b=\dfrac{4\pi}{3}$

The graph of $f(\theta)$ is above. $f(\theta)=0\Rightarrow\cos\theta=-\dfrac{1}{2}\Rightarrow\theta=-\dfrac{4\pi}{3},-\dfrac{2\pi}{3},\dfrac{2\pi}{3},\dfrac{4\pi}{3}.$

Interval where $f(\theta)=0$ at both ends $\left[-\dfrac{2\pi}{3},\dfrac{2\pi}{3}\right]$ and $\left[\dfrac{2\pi}{3},\dfrac{4\pi}{3}\right].$

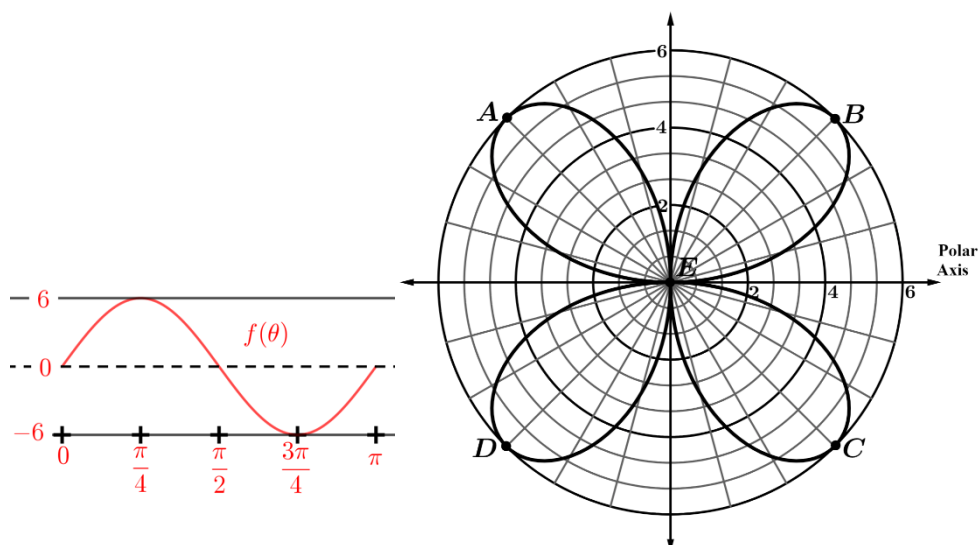$f(0)=6$ is at the midpont of $\left[-\dfrac{2\pi}{3},\dfrac{2\pi}{3}\right]$ which eliminates (A).

$f(\pi)=-2$ is at the midpont of $\left[\dfrac{2\pi}{3},\dfrac{4\pi}{3}\right],$ which confirms (D).

7. The figure shows the graph of the polar function $r = f(\theta)$, where $f(\theta) = 6\cos(2\theta)$, in the polar coordinate system for $0 \leq \theta \leq 2\pi$. There are four points labeled A, B, C, D and E. If the domain of f is restricted to $\dfrac{\pi}{2} \leq \theta \leq \pi$, the portion of the given graph that remains consists of two pieces. One of those pieces is the portion of the graph in Quadrant IV from D to E. Which of the following describes the other remaining piece?

(A) The portion of the graph in Quadrant II from B to E

(B) The portion of the graph in Quadrant II from E to A

(C) The portion of the graph in Quadrant III from E to D

(D) The portion of the graph in Quadrant IV from E to C

The portion of the graph from D to E is the part of the graph on the interval $\left[\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right]$ since the polar curve is returning to the origin, $f\left(\dfrac{3\pi}{4}\right) = 0$. The remaining portion is the part on the interval $\left[\dfrac{3\pi}{4}, \pi\right]$. This part of the graph starts at E and $f(\theta)$, the value of r is positive and increasing, ending at A.

8. The figure shows the graph of the polar function $r = f(\theta)$, where $f(\theta) = 6\sin(2\theta)$, in the polar coordinate system

for $0 \leq \theta \leq 2\pi$. There are four points labeled A, B, C, D and E. If the domain of f is restricted to $\dfrac{\pi}{2} \leq \theta \leq \dfrac{3\pi}{4}$,

which of the following describes the portion of the given graph?

(A) The top portion of the graph in Quadrant II from E to A

(B) The bottom portion of the graph in Quadrant II from A to E

(C) The top portion of the graph in Quadrant IV from C to E

(D) The bottom portion of the graph in Quadrant IV from E to C

The portion of the graph from E to C is the part of the graph on the interval $\left[\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right]$ since the polar curve

starts at the origin, $f\left(\dfrac{\pi}{2}\right) = 0$. This part of the graph starts at E and $f(\theta)$, the value of r is zero and decreasing to

become negative, heading for C.