



$t$ (in seconds)	0.1	0.5	0.9	1.5	1.9	2.3	2.6
$H(t)$ (in meters)	1.4	5.7	8.4	9.6	8.4	5.6	2.5

1. Justin Tucker, the kicker for the Baltimore Ravens, is considered one of the greatest kickers in NFL history. On a recent kickoff, the height of the ball, in meters, was measured for selected times. This data is shown in the table above.

a) Based on this situation and the data presented in the table, would a linear, quadratic, or cubic function be most appropriate to model this data? Give a reason for your answer.

quadratic because  $H(t)$  values increase to a maximum then decrease and are roughly symmetric about  $t = 1.5$ .

b) Find the appropriate regression function to model these data.

$$H(t) = (-4.9147 \dots)x^2 + (13.7138 \dots)x + 0.0695 \dots$$

NORMAL FLOAT AUTO REAL RADIAN MP	
L1	L2
0.1	1.4
0.5	5.7
0.9	8.4
1.5	9.6
1.9	8.4
2.3	5.6
2.6	2.5

NORMAL FLOAT AUTO REAL RADIAN MP	
QuadReg	
$y=ax^2+bx+c$	
$a=-4.914786841$	
$b=13.71385178$	
$c=0.069558117$	

c) Using the model found in part b, what is the predicted height of the football, in meters, at time  $t = 1.3$  seconds?

$$H(1.3) = 9.5915 \dots$$

NORMAL FLOAT AUTO REAL RADIAN MP	
$Y_1(1.3)$	
9.591575667	



$t$ (months)	0	1	13.5	19	26.5	31.5	35
$A(t)$ (in \$)	87.58	124.15	164.61	185.97	152.60	122.42	90.98

2. The price of Amazon stock fluctuates over time. On December 6, 2019, one share of Amazon stock was \$87.58. The table above gives the price of one share of Amazon stock, in dollars, for selected times  $t$ , the number of months after December 6, 2019.

a) Based on the data presented in the table, would a linear, quadratic, or cubic function be most appropriate to model this data? Give a reason for your answer.

quadratic because  $A(t)$  values increase to a maximum then decrease and are roughly symmetric about  $t = 19$ .

b) Find the appropriate regression function to model these data.

$$A(t) = (-0.2686 \dots)x^2 + (9.1615 \dots)x + 99.9721 \dots$$

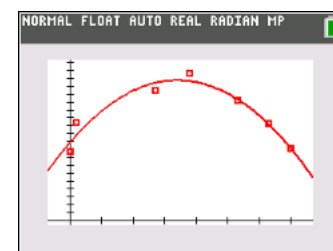
NORMAL FLOAT AUTO REAL RADIAN MP					
L1	L2	L3	L4	L5	2
0	87.58				
1	124.15				
13.5	164.61				
19	185.97				
26.5	152.6				
31.5	122.42				
35	90.98				

NORMAL FLOAT AUTO REAL RADIAN MP	
QuadReg	
$y=ax^2+bx+c$	
$a=-0.2686194348$	
$b=9.161544778$	
$c=99.97216471$	

c) Using the model found in part b, what is the predicted price of one share of Amazon stock when  $t = 16$  months (April 6, 2021)?  $A(16) = 177.7903 \dots$

d) The actual price for one share of Amazon stock on April 6, 2021 ( $t = 16$ ) was \$168.61. What is the residual for this value?  $168.61 - A(16) = -9.1803 \dots$



NORMAL FLOAT AUTO REAL RADIAN MP	
$Y_1(16)$	
177.7903058	

NORMAL FLOAT AUTO REAL RADIAN MP	
$168.61 - Y_1(16)$	
-9.180305844	



$t$ (years)	0	1	2	3	5
$R(t)$ (in %)	1.5	2.5	1.9	0.5	4.5

3. The Federal Funds Rate is the targeted interest rate that commercial banks use when lending or borrowing money for each other. This rate is set by the US Federal Reserve and had a large impact on the entire population. When the Federal Funds Rate increases, the interest rates banks charge for personal loans, car loans, home mortgages, etc... will also increase. The table above gives the Federal Funds Rate, in percent, for selected times  $t$ , where  $t$  is the number of years since 2018.

- a) The data in the table can be modeled by the cubic regression function  $y = ax^3 + bx^2 + cx + d$ . Write the equation of this cubic regression function.  
 $y = (0.2937 \dots)x^3 + (-1.8994 \dots)x^2 + (2.7593 \dots)x + 1.4676 \dots$

NORMAL FLOAT AU	
L1	L2
0	1.5
1	2.5
2	1.9
3	0.5
5	4.5

NORMAL FLOAT AUTO REAL RADIAN MP	
CubicReg	
$y=ax^3+bx^2+cx+d$	
$a=0.2937106918$	
$b=-1.899460916$	
$c=2.759389039$	
$d=1.467654987$	

- b) Based on the model found in part a, what is the predicted Federal Funds Rate for the year 2026 ( $t = 8$ )?

$$R(8) = 52.3571 \dots$$

NORMAL FLOAT AUTO REAL RADIAN MP	
Y1(8)	
52.35714286	

- c) The highest Federal Fund Rate in US history was 20% in 1980. Based on this information and the answer found in part b, do you think the cubic regression model found in part a is useful in predicting rates into the future? Explain your reasoning. The answer found in part (b) the model for  $R(t)$  is not very useful in predicting rates into the future because the value 52.357 ... % is too large considering the highest rate ever was only 20%.



$t$ (years)	0	10	22	30	37
$N(t)$ (in quadrillion BTUs)	12.4	21.8	20.4	19.3	22.6

4. The function  $N$  represents the total natural gas consumption in the United States, in quadrillions of BTUs, in a given year. The table above gives values of  $N$  for selected values of  $t$ , measured in years since 1960.

- a) The data in the table can be modeled by the cubic regression function  $y = ax^3 + bx^2 + cx + d$ . Write the equation of this cubic regression function.

$$y = (0.00155 \dots)x^3 + (-0.0978 \dots)x^2 + (1.7617 \dots)x + 12.4012 \dots$$

NORMAL FLOAT AU	
L1	L2
0	12.4
10	21.8
22	20.4
30	19.3
37	22.6

NORMAL FLOAT AUTO REAL RADIAN MP	
CubicReg	
$y=ax^3+bx^2+cx+d$	
$a=0.001558017$	
$b=-0.0978085936$	
$c=1.761711452$	
$d=12.40120117$	

- b) Based on the model found in part a, what was the predicted natural gas consumption for the US, in quadrillions of BTUs, for the year 1967 ( $t = 7$ )?

$$N(7) = 20.4749 \dots$$

NORMAL FLOAT AUTO REAL RADIAN MP	
Y1(7)	
20.47496009	



$t$ (years)	0	4	7	13	17	22	25
$F(t)$ (in %)	23	28	30.5	36	42	47	50

5. In 1988, only 24% of primary care doctors in the UK were women. Since that time, the percentage of female primary care doctors has consistently been increasing. The function  $F$  represents the percent of primary care doctors that were women in a given year, where  $t = 0$  represents the year 1988. The table above gives values for  $F$  at selected values of  $t$ .

The function  $F$  can be modeled by the linear regression function  $y = at + b$ , where  $a$  and  $b$  are constants.

a) Based on the data presented in the table, explain why a linear regression model for  $F$  is appropriate.

Linear because there is roughly a constant rate of change of about 1.  $AROC = \frac{50-23}{25} = \frac{27}{25} = 1.08$

L1	L2	L3
0	23	1.25
4	28	0.8333
7	30.5	0.9167
13	36	1.5
17	42	1
22	47	1
25	50	-----

Column  $L_3$  is calculated using the LIST OPS command.

$L_3 = \frac{\Delta \text{list}(L_2)}{\Delta \text{list}(L_1)}$  which is the AROC over each interval.

NORMAL	FLOAT	AUTO	REAL	RADIAN	MP
NAMES OPS MATH					
1:SortA(					
2:SortD(					
3:dim(					
4:Fill(					
5:seq(					
6:cumSum(					
7:ΔList(					
8:Select(					
9:Augment(					

b) Find the equation of the linear regression model  $y = at + b$ .

$y = 1.0783 \dots t + 23.0858 \dots$

NORMAL	FLOAT	AUTO	REAL	RADIAN	MP
LinReg					
$y = ax + b$					
$a = 1.078396739$					
$b = 23.08586957$					

c) Using the model found in part b, what is the predicted percentage of female primary care doctors in the UK for the year 2021 ( $t = 33$ )?  $F(33) = 58.6729 \dots$

d) In 2021, the women accounted for 55% of all primary care doctors in the UK. What is the residual for this value?

$55 - F(33) = -3.6729 \dots$

NORMAL	FLOAT	AUTO	REAL	RADIAN	MP
$Y_1(33)$					
58.67296196					
$55 - Y_1(33)$					
-3.672961957					