Now that we have learned and explored how polar coordinates work, it is only natural that we would begin to explore the concept of polar functions and their graphs.

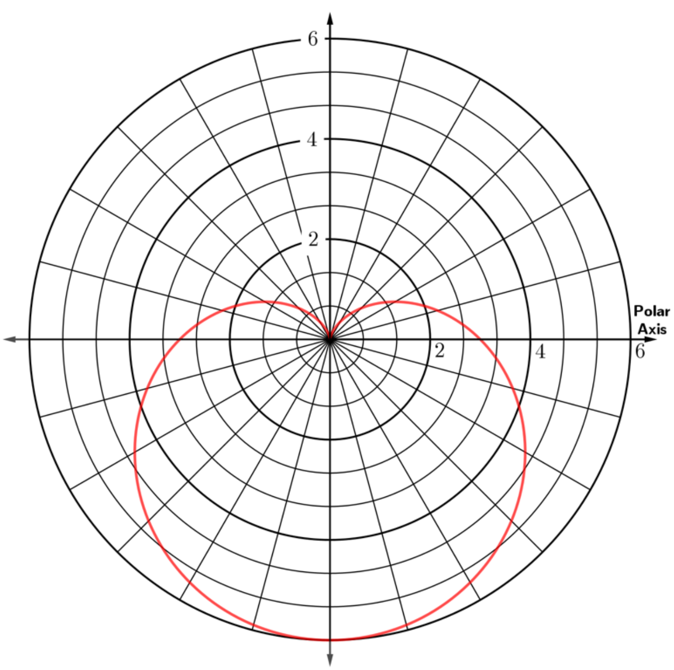
For polar functions, the inputs are given by θ (independent variable), and the outputs are given by r (dependent variable). On the AP Precalculus Exam, questions may use the phrasing below to introduce a polar function.

"The graph of the polar function $r = f(\theta)$, where $f(\theta) = 3 - 3\sin(\theta)$, is shown in the polar coordinate system for $0 \leq \theta \leq 2\pi$."

Graphing Polar Functions

Creating a table of values is always a good strategy when attempting to graph a new or unknown function. We will utilize this strategy to help us create a graph of a polar function.

Example 1: Let $r = f(\theta)$, where $f(\theta) = 3 - 3\sin(\theta)$, be a polar function in the polar coordinate system for $0 \leq \theta \leq 2\pi$. Sketch the graph of $r = f(\theta)$ on the axes below.



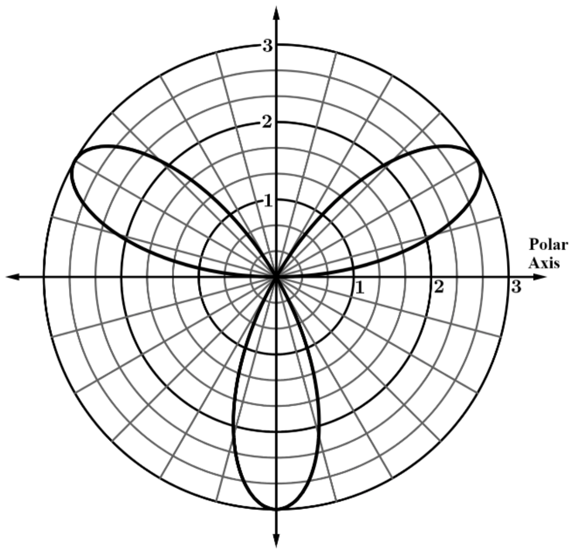
θ	$r = 3 - 3\sin(\theta)$
0	3
$\frac{\pi}{6}$	1.5
$\frac{\pi}{4}$	0.8787
$\frac{\pi}{3}$	0.4019
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	0.4019
$\frac{3\pi}{4}$	0.8787
$\frac{5\pi}{6}$	1.5
π	3

θ	$r = 3 - 3\sin(\theta)$
$\frac{7\pi}{6}$	4.5
$\frac{5\pi}{4}$	5.1213
$\frac{4\pi}{3}$	5.5981
$\frac{3\pi}{2}$	6
$\frac{5\pi}{3}$	5.5981
$\frac{7\pi}{4}$	5.1213
$\frac{11\pi}{6}$	4.5
2π	3

Note: It is very common for polar functions to display symmetry. We can use our understanding of symmetries to help us construct polar graphs quickly and easily.

On the AP Precalculus Exam, polar functions will **only** appear in multiple choice questions. This means that students will **NOT** be required to sketch a polar function by hand on the AP Exam. However, students will be expected to understand polar functions, their graphs, and what portion of the graphs exist with various domain restrictions.

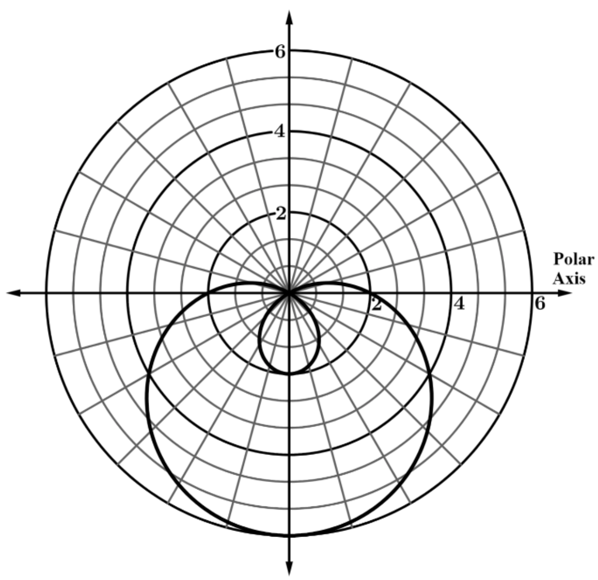
When working with multiple choice questions involving polar functions, it is advantageous to evaluate the function at several values of θ. Utilizing the multiple-choice options can be helpful when determining which values of θ to consider.



Example 2: The figure shows the graph of the polar function $r = f(\theta)$, for $0 \le \theta \le 2\pi$, in the polar coordinate system. Which of the following could be an expression for $f(\theta)$?

(A) $3\sin(3\theta)$ (B) $-3\sin(3\theta)$ (C) $3\cos(3\theta)$ (D) $-3\cos(3\theta)$

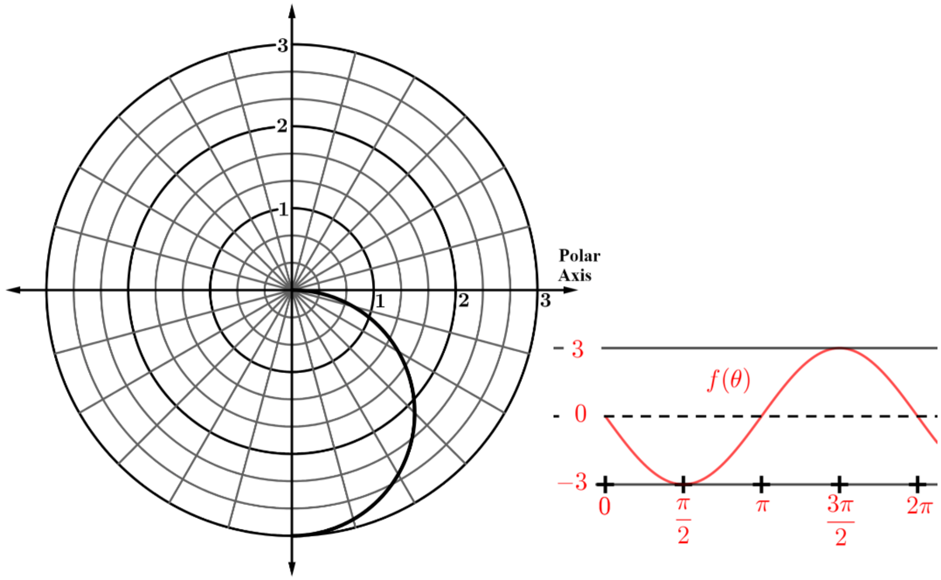
$f(0) = 0$ from the graph which eliminates (C) and (D). $f\left(\frac{\pi}{6}\right) = 3$ which eliminates (B).



Example 3: The figure shows the graph of the polar function $r = f(\theta)$, for $0 \le \theta \le 2\pi$, in the polar coordinate system. Which of the following could be an expression for $f(\theta)$?

(A) $2+4\sin(\theta)$ (B) $2-4\sin(\theta)$ (C) $2+4\cos(\theta)$ (D) $2-4\cos(\theta)$

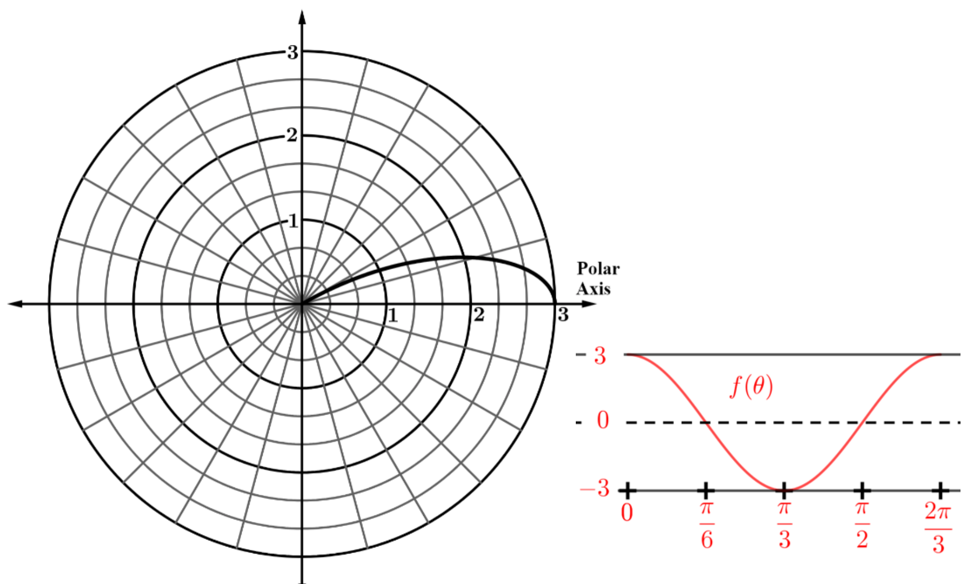
$f(0) = 2$ from the graph which eliminates (C) and (D) maybe. $f\left(\frac{\pi}{2}\right) = -2$ which eliminates (A).



Example 4: A portion of the graph of the polar function $r = f(\theta)$, where $f(\theta) = -3\sin\theta$, is shown in the polar coordinate system for $a \le \theta \le b$. If $0 \le a < b < 2\pi$, which of the following could be the values for a and b?

(A) $a = 0$ and $b = \frac{\pi}{2}$ (B) $a = \frac{\pi}{2}$ and $b = \pi$ (C) $a = 0$ and $b = \pi$ (D) $a = \pi$ and $b = 2\pi$

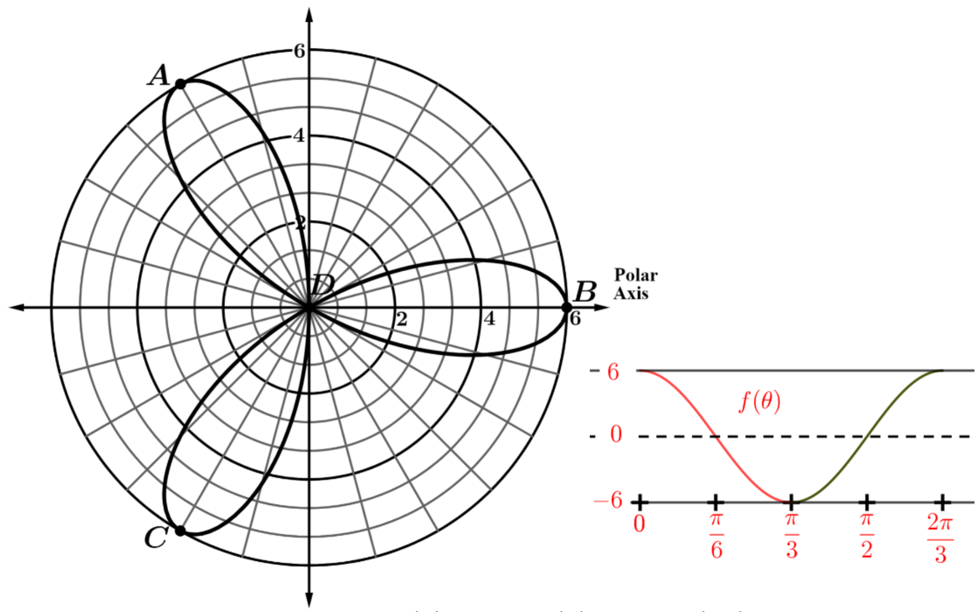
The graph of $f(\theta)$ above (sketched by hand)on the interval $\left[0, \frac{\pi}{2}\right]$ is negative ($r < 0$, reflection through origin) and decreasing. On the interval $\left[\frac{\pi}{2}, \pi\right]$, $f\left(\frac{\pi}{2}\right) = -3$ and increasing, becoming less negative.



Example 5: A portion of the grpah of the polar function $r = f(\theta)$, where $f(\theta) = 3\cos(3\theta)$, is shown in the polar coordinate system for $a \le \theta \le b$. If $0 \le a < b < 2\pi$, which of the following could be the values for a and b?

(A) $a = \pi$ and $b = \frac{3\pi}{2}$ (B) $a = 0$ and $b = \frac{\pi}{2}$ (C) $a = 0$ and $b = \frac{\pi}{3}$ (D) $a = 0$ and $b = \frac{\pi}{6}$

One period of $f(\theta)$ is sketched above above. On $\left[0, \frac{\pi}{6}\right]$, $f(\theta)$ is positive and decreasing. This the only interval where this is true.



Example 6: The figure shows the graph of the polar function $r = f(\theta)$ where $f(\theta) = 6\cos(3\theta)$ in the polar coordinate system for $0 \le \theta \le 2\pi$. There are four points labeled A, B, C, and D. If the domain of f is restricted to $\frac{\pi}{3} \le \theta \le \frac{2\pi}{3}$, the portion of the given graph that remains consists of two pieces. One of those pieces is the portion of the graph in Quadrant III from C to D. Which of the following describes the other remaining piece?

(A) The portion of the graph in Quadrant I from D to B

(B) The portion of the graph in Quadrant I from B to D

(C) The portion of the graph in Quadrant II from D to A

(D) The portion of the graph in Quadrant III from D to C

The portion of the graph from C to D is the part of the graph on the interval $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ since the polar curve is returning to the origin, $f\left(\frac{\pi}{2}\right) = 0$. The remaining portion is the part on the interval $\left[\frac{\pi}{2}, \frac{2\pi}{3}\right]$. This part of the graph starts at D and $f(\theta)$, the value of r is positive and increasing, ending at A.