Notes: (Topics 1.13 – 1.14) Modeling Functions and Applications Solutions

The idea of modeling (also called **regression**) is a major component of the AP Precalculus curriculum and will be revisited throughout the course. **These topics will utilize the graphing calculators heavily and often include context.**

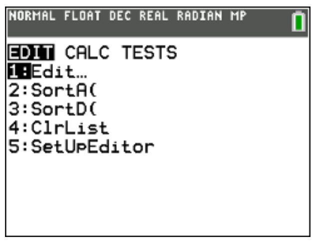
In Unit 1, we will work with regression models for **polynomial functions** (linear, quadratic, cubic, and quartic) and for **rational functions.**

Building Regression Models on the Graphing Calculator

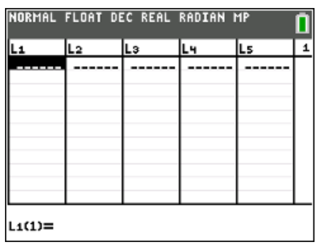
Building a regression model on the graphing requires two steps:

1. Entering the data to be modeled
2. Selecting the regression model

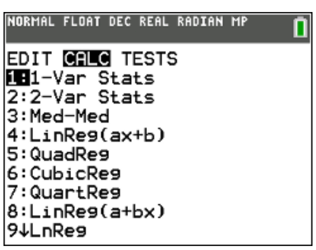
Step 1: Press the **"stat"** button on the TI – 84 and select **"1: Edit…"** from the menu.



Enter the data into the lists with L1 $= x$ and L2 $= y$

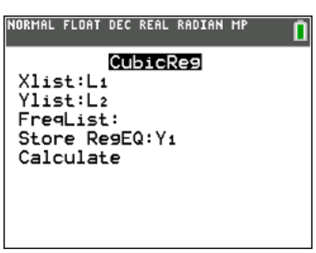


Step 2: In the **"stat"** menu, arrow to the right to the **"CALC"** menu.



Select the desired regression model.

Note: There are 2 linear regression options: **4: LinReg (ax + b)** and **8: LinReg (a + bx)**. These two options are essentially equivalent – consider option 8 (AP Stats)



Use **L1** for **Xlist:** and **L2** for **Ylist:**
(**L1** and **L2** are in blue above the **"1"** and **"2"** buttons: Press **"2nd"** and then **"1"** or **"2"**)

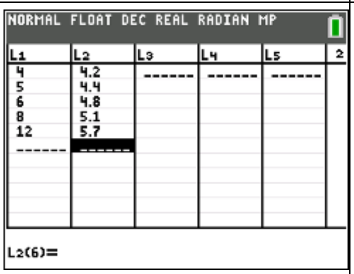
Tip: For **Store RegEQ:**, enter **Y1**. (Press **"alpha"** and then **"trace"** to select **Y1**)

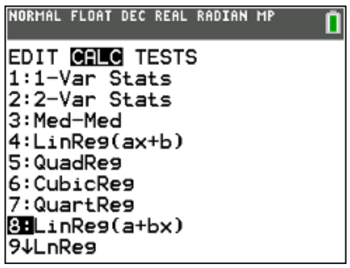
t (age in weeks)	4	5	6	8	12
$W(t)$ (weight in kg)	4.2	4.4	4.8	5.1	5.7

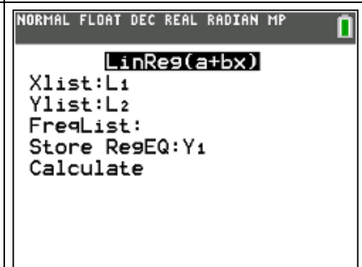
Example 1: The age (in weeks) and weight (in kilograms) of 5 randomly selected babies from a particular pediatrician's office are listed in the table above.
A linear regression $y = a + bx$ can be used to model these data, where y is the predicted weight of a baby (in kg) that is x weeks old.

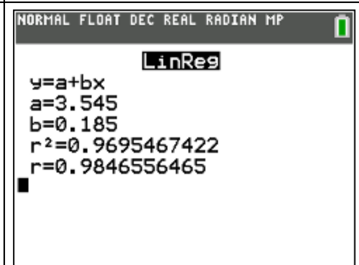
a) Write the equation of the linear model for these data.

Enter data in L1 and L2	Select 8: LinReg (a + bx)	Enter L1, L2 and Y_1	Get regression equation



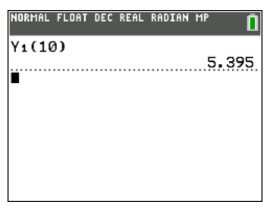






Linear Regression Model: $y =$ $3.545 + 0.185t$

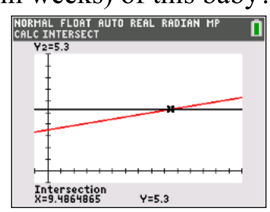
b) Using the linear model from part a), what is the predicted weight (in kilograms) of a baby that is 10 weeks old?



Tip: You can quickly find values of functions that are saved in Y_1 from the home screen on the calculator.

c) The weight of a sixth baby is 5.3 kg. Using the model from part a), what is the age (in weeks) of this baby?

$W(t) = 3.545 + 0.185t = 5.3$ $\quad 0.185t = 1.755$ $\quad t = 9.4864 \ldots$ weeks



Selecting an Appropriate Model Type

While some problems will indicate which model should be used, you will also be expected to select an appropriate model type based on a given table of values or by the context of the problem.

Linear Models: roughly constant rates of change

Quadratic Models: roughly linear rates of change or roughly symmetric with a single maximum/minimum or context involving area

Cubic Models: context involving volume

Example 2: For each of the following situations, determine whether a linear, quadratic, or cubic model would be most appropriate.

a) Balloons are filled with water in preparation for an epic water balloon battle. Each water balloon is roughly spherical. The radius of each water balloon is measured relative to the amount of water it holds.

Cubic because the context is volume.

b) Totino's pizzas are on sale at the local grocery store. The price of one pizza varies between \$1.99 - \$2.19 depending on the variety. The total number of Totino's pizzas are counted relative to the total price of the purchase.

linear because the price depends on variety and probably a constant price per topping.

c) A sprinkler is placed in a yard to water the grass. The sprinkler rotates in a circular pattern and waters all the grass between the sprinkler head and the furthest distance it reaches. The radius of the circular path is measured relative to the area watered by the sprinkler.

quadratic because the context involve area. $A = \pi r^2$

x	0	0.4	0.9	1.2	1.7	2.2	2.9	3.4
y	5	10.6	15.4	17.1	18.0	16.5	10.2	2.8

↑ center

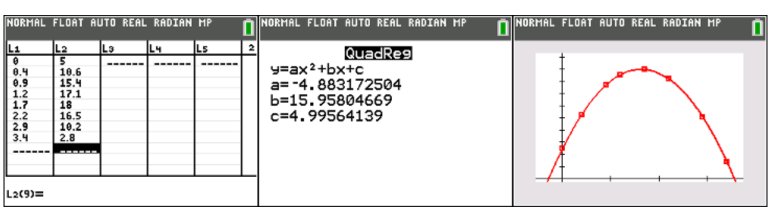
Example 3: The table above provides data for 8 ordered pairs (x, y).

a) Which function type best models the data in the table: linear, quadratic, or cubic? Explain your answer using characteristics from the data in the table.

quadratic because the y −values increase to a maximum then decrease somewhere between x −values 1.2 and 1.7. The y −values are roughly symmetric about a x −value between 1.2 and 1.7.

b) Write the equation of the regression model for the data in the table.

$y = (-4.8831\ldots)x^2 + (15.95804\ldots)x + 4.995\ldots$



Residuals (Topic 2.6)

When weType equation here. use a model to predict values, we expect our model to produce values reasonably close to the actual values, but our models are not expected to result in exact values generally. The difference between an actual value and the value predicted by a model is called a **residual.**

Residuals
Residual = Actual Value − Predicted Value
Residual $= y - \hat{y}$

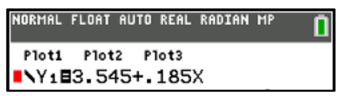
t (age in weeks)	4	5	6	8	12
W (weight in kg)	4.2	4.4	4.8	5.1	5.7

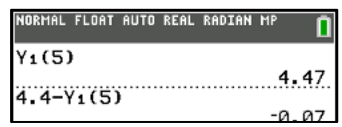
Example 4: Using the model from **Example** 1, what is the residual of the baby that is 5 weeks old? Interpret the meaning of this value in the context of this problem.

Actual Value: 4.4

Predicted Value: 4.47

Residual: $4.4 - 4.47 = -0.07$





Interpretation: The model overestimated the actual values.