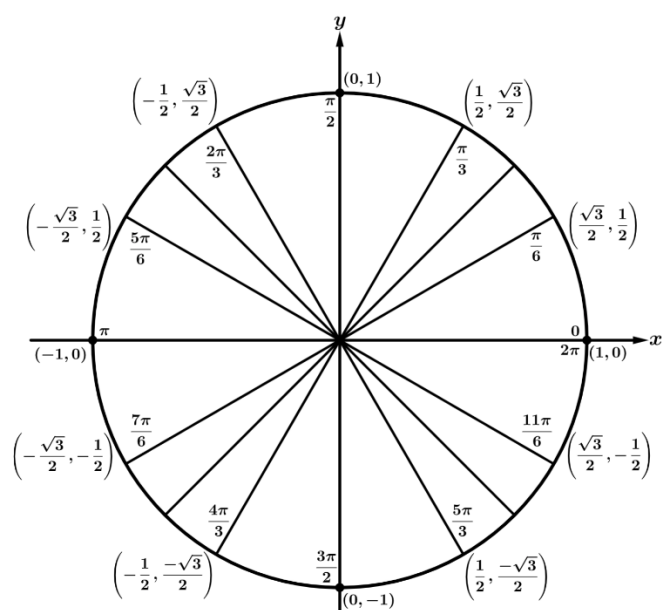**Notes**: (Topics 3.4 – 3.5) Sine and Cosine Function Graphs and Sinusoidal Functions Solutions

So far in Unit 3, we have learned how to define sine and cosine values for angles in standard position, how to measure angles in radians, and how to use the unit circle to evaluate sine and cosine functions at given angles.

Now, we will put all of these ideas together and discover that they can help us develop and understand how to represent sine and cosine functions graphically.
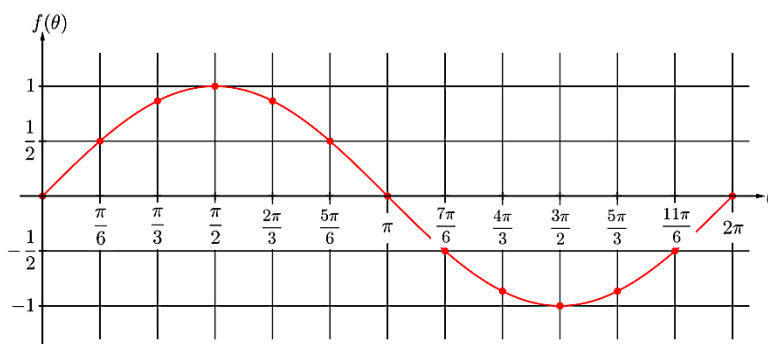
Let's start by considering the function $f(\theta) = \sin\theta$. Since we know that angle measures in standard position are periodic, we can expect $f(\theta)$ to be periodic. Also, since θ is our input-value, we will use the horizontal axis for our angle measures and the vertical axis for our output-values $\left(f(\theta)\right)$.

The angle measures from the unit circle give us several convenient input-values to use on the horizontal axis.

θ	$\dfrac{\pi}{6}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$	$\dfrac{2\pi}{3}$	$\dfrac{5\pi}{6}$	π
$\sin\theta$	$\dfrac{1}{2}$	$\dfrac{\sqrt{3}}{2}$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{2}$	0

θ	$\dfrac{7\pi}{6}$	$\dfrac{4\pi}{3}$	$\dfrac{3\pi}{2}$	$\dfrac{5\pi}{3}$	$\dfrac{11\pi}{6}$	2π
$\sin\theta$	$-\dfrac{1}{2}$	$-\dfrac{\sqrt{3}}{2}$	-1	$-\dfrac{\sqrt{3}}{2}$	$-\dfrac{1}{2}$	0

	Properties of the graph for $f(\theta) = \sin\theta$	
	Midline The midline is halfway between the maximum and minimum values. **Midline:** $y = 0$	**Amplitude** The amplitude is the distance from the midline to the maximum (or minimum). **Amplitude:** $a = 1$
	Period: $P = 2\pi$	**Frequency** The frequency is the reciprocal of the period. **Frequency:** $\dfrac{1}{2\pi}$
	The graph of $f(\theta) = \sin\theta$ oscillates between concave down and concave up.	

Amplitude = 1

Midline $y = 0$

Period $P = 2\pi$

Similarly, we can use our knowledge of the unit circle to develop a graph of the function $g(\theta) = \cos\theta$.

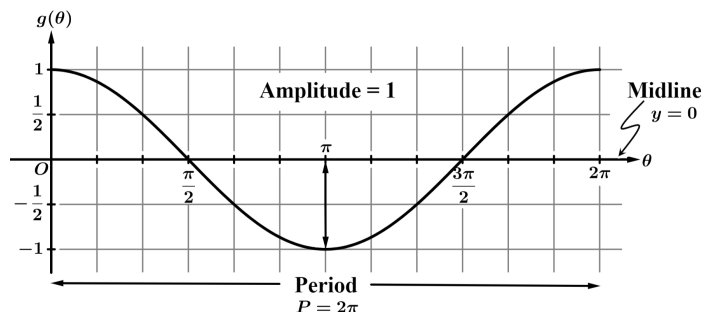| θ	$\dfrac{\pi}{6}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$	$\dfrac{2\pi}{3}$	$\dfrac{5\pi}{6}$	π
$\cos\theta$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{2}$	0	$-\dfrac{1}{2}$	$-\dfrac{\sqrt{3}}{2}$	-1

θ	$\dfrac{7\pi}{6}$	$\dfrac{4\pi}{3}$	$\dfrac{3\pi}{2}$	$\dfrac{5\pi}{3}$	$\dfrac{11\pi}{6}$	2π
$\cos\theta$	$-\dfrac{\sqrt{3}}{2}$	$-\dfrac{1}{2}$	0	$\dfrac{1}{2}$	$\dfrac{\sqrt{3}}{2}$	1

Properties of the graph for $g(\theta) = \cos\theta$

| **Midline** The midline is halfway between the maximum and minimum values. **Midline:** $y = 0$	**Amplitude** The amplitude is the distance from the midline to the maximum (or minimum). **Amplitude:** $a = 1$
Period: $P = 2\pi$	**Frequency** The frequency is the reciprocal of the period. **Frequency:** $\dfrac{1}{2\pi}$

The graph of $g(\theta) = \cos\theta$ oscillates between concave down and concave up.

Sinusoidal Functions

A **sinusoidal function** is any function that involves additive and multiplicative transformations of $f(\theta) = \sin\theta$.

The **sine** and **cosine** functions are **both** sinusoidal functions because $g(\theta) = \cos\theta = \sin\left(\theta + \dfrac{\pi}{2}\right)$.
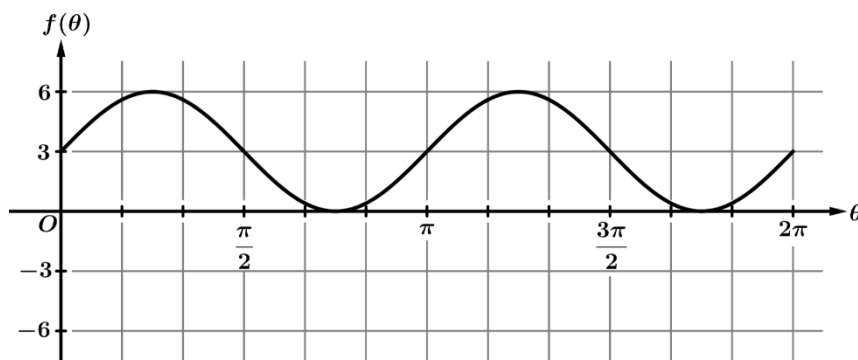
Graph of f

Example 1: Two periods of the sinusoidal function $f(\theta)$ are shown in the figure above. Find the period, frequency, amplitude, and midline for the graph of $f(\theta)$.
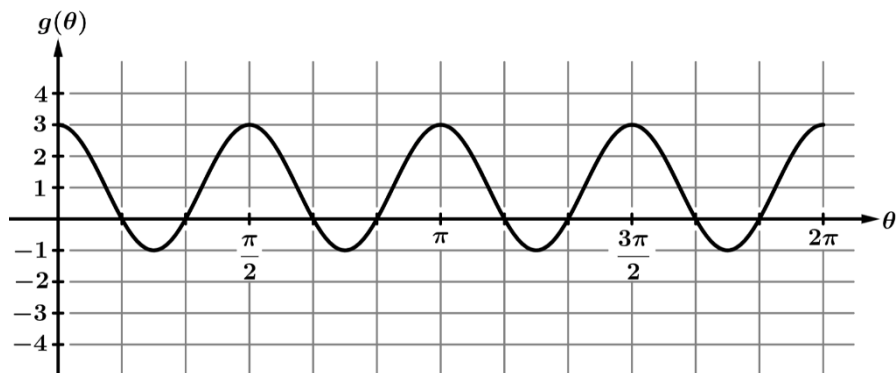
Period:

π

Frequency:

$\dfrac{1}{\pi}$

Midline:

$y = 3$

Amplitude:

3

Graph of g

Example 2: Several periods of the sinusoidal function $g(\theta)$ are shown in the figure above. Find the period, frequency, amplitude, and midline for the graph of $g(\theta)$.

Period:

$\dfrac{\pi}{2}$

Frequency:

$\dfrac{2}{\pi}$

Midline:

$y = 1$

Amplitude:

2

Example 3: The sinusoidal function $h(\theta)$ has a maximum at the point $(\pi, 8)$. The first minimum after reaching this maximum value occurs at the point $(3\pi, -2)$. Find the period, frequency, amplitude, and midline for the graph of $h(\theta)$.

Period:

$\underbrace{3\pi - \pi = 2\pi}_{\frac{1}{2}\text{ period}} \Rightarrow 4\pi$
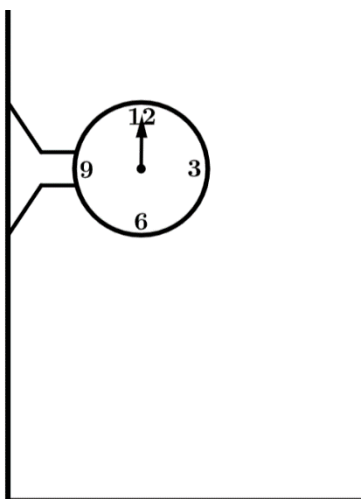
Frequency:

$\dfrac{1}{4\pi}$

Midline:

$y = \dfrac{8 + (-2)}{2} = 3$

Amplitude:

$8 - 3 = 5$

Sine and Cosine Function Graphs
Solutions by Ted Gott tedg20776@gmail.com

Created by Bryan Passwater

Note: Figure NOT drawn to scale

Example 4: The figure shows a large clock mounted to a vertical wall. The clock has an 8-inch-long moving minute hand. The center of the clock is 120 inches directly above the floor. At time $t = 0$ minutes, the minute hand is pointed directly up at the 12. However, the clock is not working properly, and the minute hand is moving twice as fast as it should. Thus, the next time the minute hand points directly up to the 12 is at time $t = 30$ minutes. As the minute hand moves, the distance between the endpoint of the minute hand and the floor periodically decreases and increases.

The periodic function h models the distance, in inches, between the endpoint of the minute hand from the floor and the floor as a function of time t in minutes.

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, $F, G, J, K,$ and P are labeled on the graph. No scale is indicated, and no axes are presented.

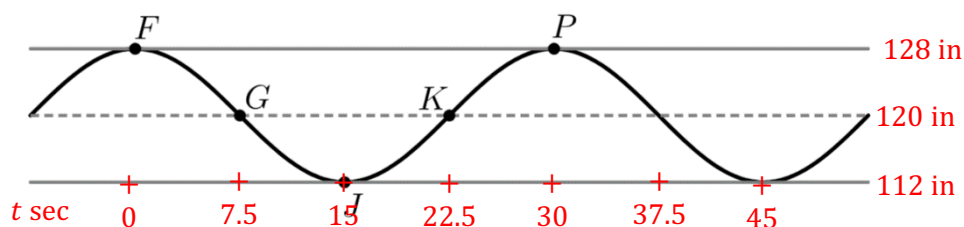Determine possible coordinates $(t, h(t))$ for the five points: $F, G, J, K,$ and P.

$F: (0,128)$

$G: (7.5,120)$

$J: (15,112)$

$K: (22.5,120)$

$P: (30,128)$

(B) Find the period, frequency, amplitude, and midline for the graph of h.

Period:	Frequency:	Midline:	Amplitude:
30	$\dfrac{1}{30}$	$y = 120$	8

(C) Find two intervals for which the graph of h is both decreasing and concave up.

The graph of h is both decreasing and concave up on the intervals $(7.5,15)$ and $(37.5,45)$.

Sine and Cosine Function Graphs