As with all functions, the inverse of a trigonometric function is the result of switching the input (x) and output (y) values of the function. As a result, the output value of an inverse trigonometric function will be an angle measure.

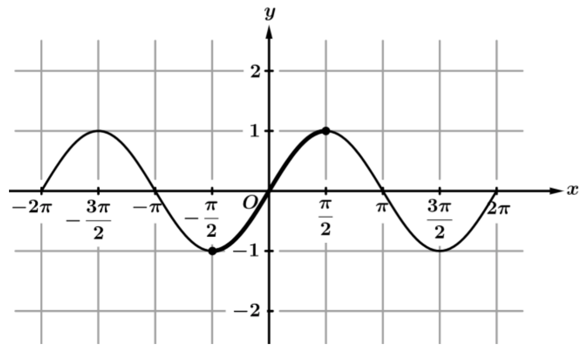
Notation: We can represent inverse trigonometric functions in two different ways: $\sin^{-1}(x)$ or $\arcsin(x)$. With either notation, we would say "arcsine of x" when reading it aloud.

Example 1: Write the statement $\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$ in an equivalent form using arcsine notation.

$\arcsin\left(\frac{1}{2}\right)=\frac{\pi}{6}$ or $\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

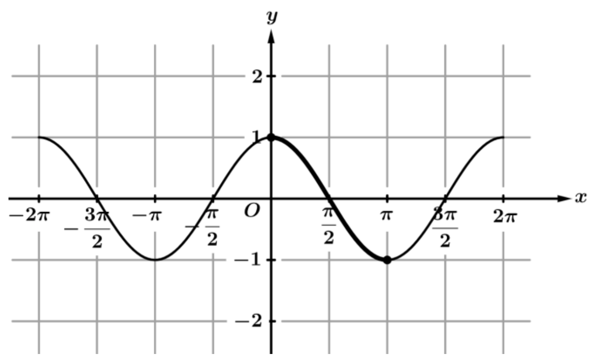
Restricted Domains of Inverse Trigonometric Functions

Because trig functions are periodic, we must restrict their domains to create their corresponding inverse functions.



$y=\sin(x)$

In order to make the inverse a function, we will restrict the function $y=\sin(x)$ to the domain $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, as highlighted in the figure above.

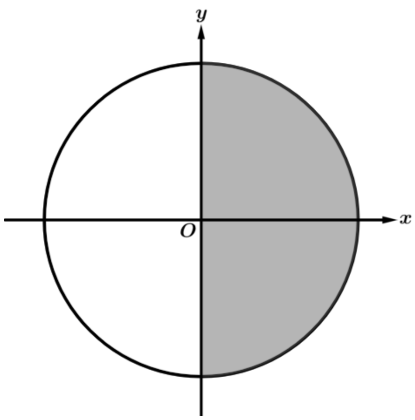
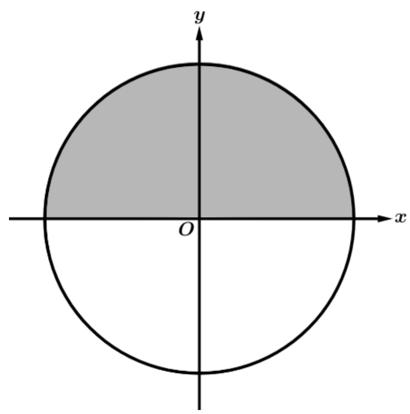
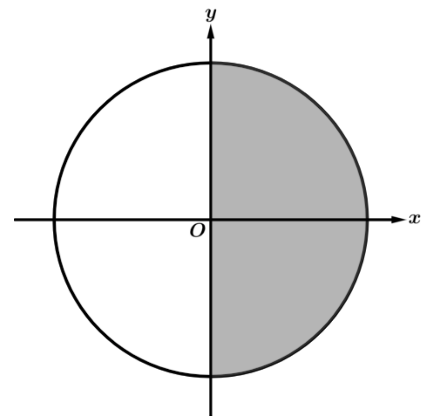


$y=\cos(x)$

Similarly, we will restrict our cosine function for the same purposes. However, for $y=\cos(x)$, we will restrict our domain to $[0,\pi]$.

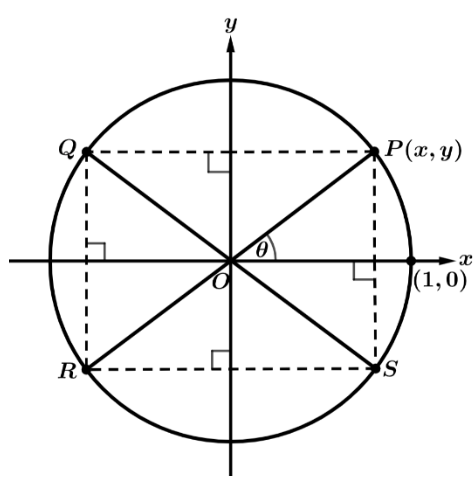
For the tangent function, we will restrict the domain of $y=\tan(x)$ to $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, due to the vertical asymptotes at $x=-\frac{\pi}{2}$ and $x=\frac{\pi}{2}$.

Important Note About Inverse Trig Functions: It is important to always remember and consider the domain restrictions when working with inverse trigonometric functions and values.

$\sin^{-1}(x)$	$\cos^{-1}(x)$	$\tan^{-1}(x)$
		
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	**Range:** $[0, \pi]$	**Range:** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Example 2: Evaluate the following expressions.

a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ c) $\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$



Example 3: The angle θ is in standard position. The terminal ray intersects the unit circle at point P, whose coordinates are (x, y). The points Q, R, and S are the result of the terminal ray being reflected over the y-axis, the origin, and the x-axis respectively. For each of the following expressions, determine which labeled point intersects the terminal ray of the given angles.

a) $\cos^{-1}(x)$ P b) $\sin^{-1}(-y)$ S c) $\cos^{-1}(-x)$ Q d) $\tan^{-1}\left(-\frac{y}{x}\right)$ S