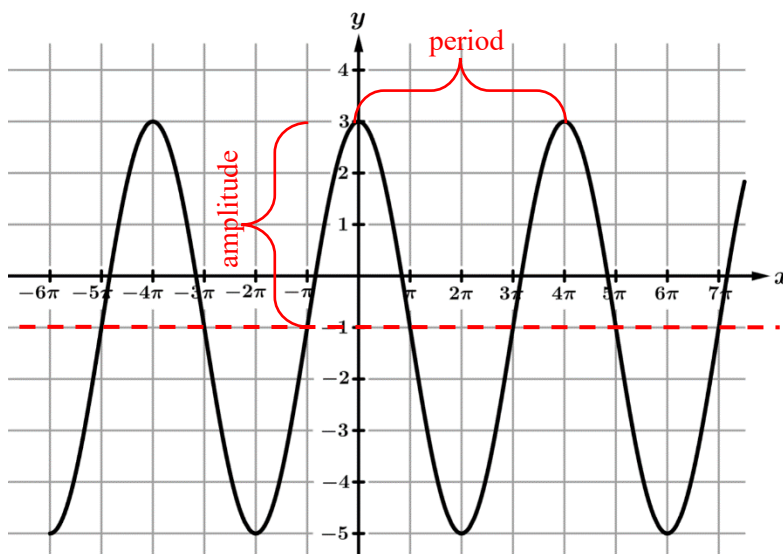
**Graph of  $h$** 

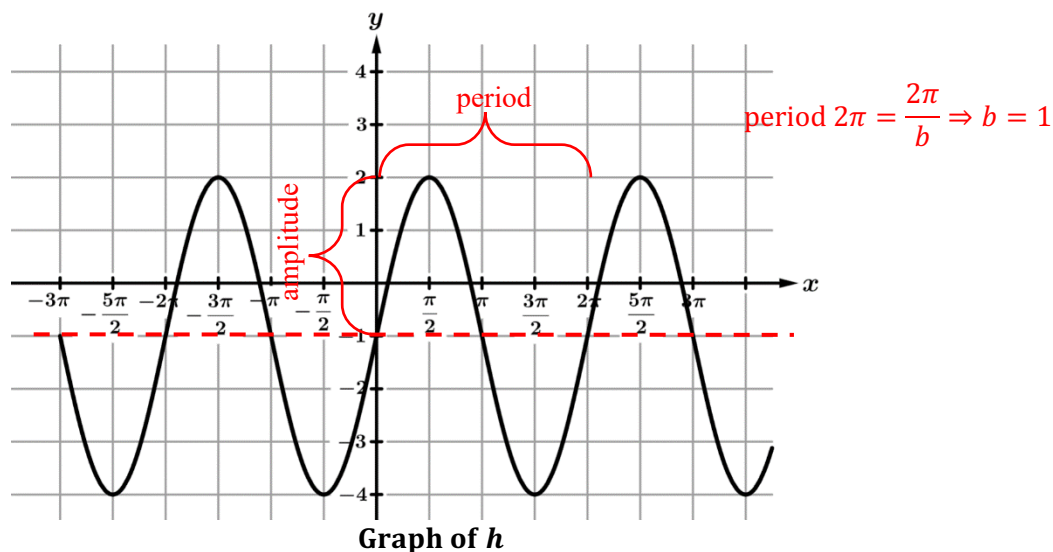
1. The graph of the sinusoidal function  $h$  is shown in the figure above. The function  $h$  can be written as  $h(\theta) = a \sin(b\theta) + d$ . Find the values of the constants  $a$ ,  $b$ , and  $d$ .

A sine curve reflected over the midline  $y = 1$ . Amplitude 3  $\boxed{a = -3}$  because of the reflection, vertical shift  $\boxed{1 = d}$   
 period  $\pi = \frac{2\pi}{b} \Rightarrow \boxed{b = 2}$

**Graph of  $f$** 

2. The graph of the sinusoidal function  $f$  is shown in the figure above. The function  $f$  can be written as  $f(\theta) = a \cos(b\theta) + d$ . Find the values of the constants  $a$ ,  $b$ , and  $d$ .

Amplitude 4  $\boxed{a = 4}$  vertical shift  $\boxed{-1 = d}$  period  $4\pi = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{4\pi} \boxed{b = \frac{1}{2}}$

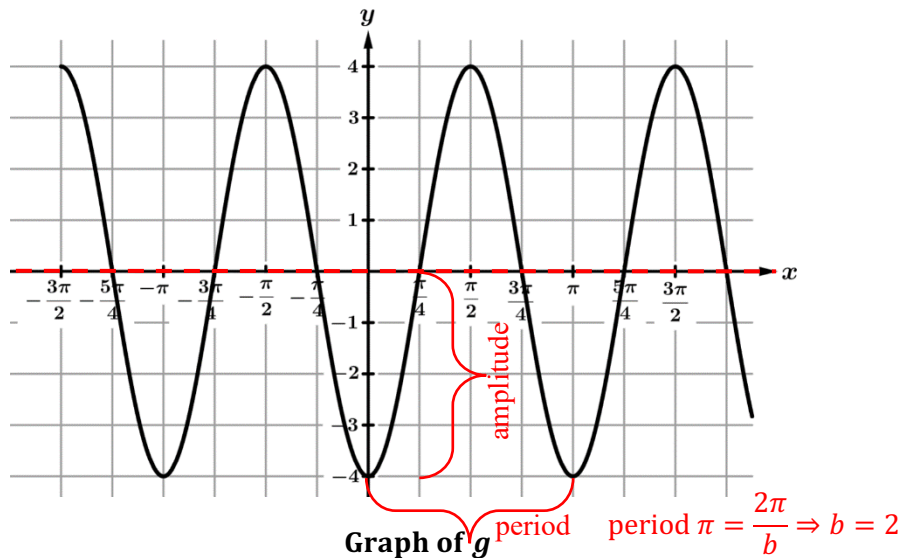


3. The figure shows the graph of a trigonometric function  $h$ . Which of the following could be an expression for  $h(x)$

- (A)  $-3\sin(x) - 2$       (B)  $3\sin(x - \pi) - 1$       (C)  $-3\cos\left(x - \frac{3\pi}{2}\right) - 1$       (D)  $3\cos\left(x + \frac{\pi}{2}\right) - 1$

The midline is  $y = -1$  which eliminates (A). Looks like a sine curve with no horizontal shift but that is none of the choices. The amplitude is 3 and  $a = 3$  but choice (B) has a horizontal shift to the right of  $\pi$  which means there needs to be a reflection over the midline and  $a = -3$  which eliminates (B).

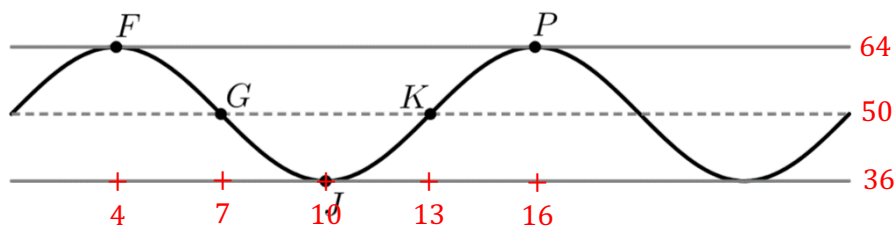
A cosine curve shifted to the right  $\frac{3\pi}{2}$  will require a reflection over the midline so  $a = -3$ .



4. The figure shows the graph of a trigonometric function  $g$ . Which of the following could be an expression for  $g(x)$

- (A)  $4\cos(2x)$       (B)  $4\cos\left(2\left(x - \frac{\pi}{4}\right)\right)$       (C)  $-4\cos\left(2\left(x - \frac{\pi}{2}\right)\right)$       (D)  $-4\cos(2(x - \pi))$

Looks like a cosine curve with no vertical shift,  $d = 0$  and no horizontal shift with a reflection over the  $x$ -axis. Choice (A) is eliminated because there is no reflection,  $a = 4 > 0$ . Amplitude is 4 with a reflection over the midline so  $a = -4$ .  $g(x) = -4\cos(2x)$  which is not one of the choices. Choice (D) has a shift of 1 period to the right.



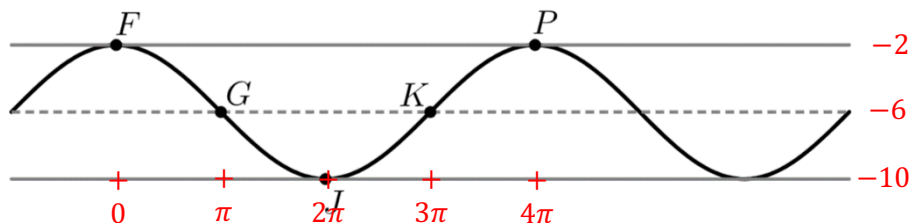
5. The graph of  $h$  and its dashed midline for two full cycles is shown. Five points,  $F$ ,  $G$ ,  $J$ ,  $K$ , and  $P$  are labeled on the graph. No scale is indicated, and no axes are presented.

The coordinates for the five points  $F$ ,  $G$ ,  $J$ ,  $K$ , and  $P$  are  $F(4, 64)$ ,  $G(7, 50)$ ,  $J(10, 36)$ ,  $K(13, 50)$ ,  $P(16, 64)$ .

The function  $h$  can be written in the form  $h(t) = a \cos(b(t+c)) + d$ . Find values of constants  $a$ ,  $b$ ,  $c$ , and  $d$ .

$$\text{amplitude } a = 64 - 50 = 14 \quad \text{period} = 16 - 4 = 12 = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{12} = \frac{\pi}{6} \quad d = 50 \quad \text{midline } y = 50$$

$$b(t+c) = 0 \quad \frac{\pi}{6}(4+c) = 0 \Rightarrow 4+c = 0 \Rightarrow c = -4$$



6. The graph of  $h$  and its dashed midline for two full cycles is shown. Five points,  $F$ ,  $G$ ,  $J$ ,  $K$ , and  $P$  are labeled on the graph. No scale is indicated, and no axes are presented.

The coordinates for the points  $F$ ,  $G$ ,  $J$ ,  $K$ , and  $P$  are  $F(0, -2)$ ,  $G(\pi, -6)$ ,  $J(2\pi, -10)$ ,  $K(3\pi, -6)$ ,  $P(4\pi, -2)$ .

The function  $h$  can be written in the form  $h(t) = a \sin(b(t+c)) + d$ . Find values of constants  $a$ ,  $b$ ,  $c$ , and  $d$ .

$$\text{amplitude } a = -2 - (-6) = 4 \quad \text{period} = 4\pi = \frac{2\pi}{b} \Rightarrow b = \frac{2\pi}{4\pi} = \frac{1}{2} \quad d = -6 \quad \text{midline } y = -6$$

The maximum value of a sine curve occurs at  $x = \frac{\pi}{2}$ . On  $h(t)$ , the maximum is  $h(0)$ .

$$b(t+c) = \frac{\pi}{2} \quad \frac{1}{2}(0+c) = \frac{\pi}{2} \Rightarrow \frac{1}{2}c = \frac{\pi}{2} \Rightarrow c = \pi$$