	AP Precalculus Notes	Name:
	Topic 1.2: Rates of Change	
	Created by Bryan Passwater Speedway High School BryanPasswater1@gmail.com	**Mathematical Practices/Skills Highlighted** **2.A** **Identify information** from multiple representations. **3.A** **Describe** the **characteristics** of a function.

In Topic 1.1, we learned that the phrase "**rate of change**" is synonymous with the word "**slope**". So, a positive rate of change indicates that a function has a positive slope (the function is increasing). And a negative rate of change indicates that a function has a negative slope (the function is decreasing).

Rate of Change at a Point

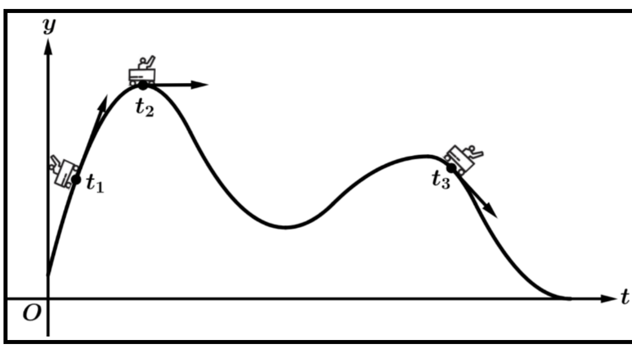
In AP Precalculus, we will define the **rate of change** of a function **at a point** as the rate at which the output values would change if the input values were to change at that point.

Later, in (AP) Calculus, you will call this the "**instantaneous rate of change**". The concept of a function changing instantaneously is called a **derivative** and is quite challenging to understand without first learning the foundational ideas found in AP Precalculus.

In AP Precalculus, we are unable to find the rate of change of a function at a given point (this will require calculus). But, we need to determine if the **rate of change at a given point** is positive, negative, or zero. We will also need to be able to **compare the rates of change at two distinct points**.

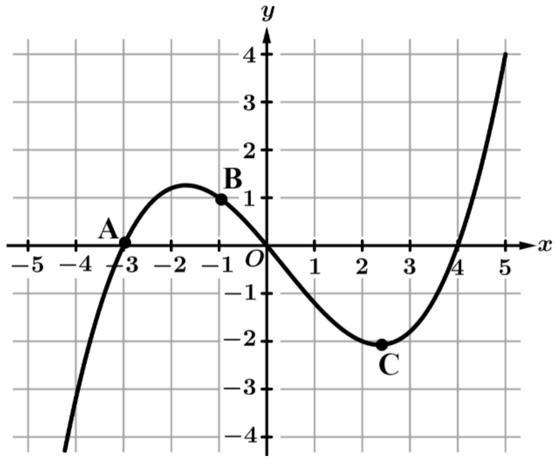
When considering rates of change at a given point, it may be helpful to think about the graph as a roller coaster. If the car you are riding is "**going up**", then the function has a **positive rate of change** at that point. If you are "**going down**", the function has a **negative rate of change** at that point.

At the instant (point) when you are at the top of a peak or at the bottom of a valley, you are not moving up or down. At the single instant (point), you are changing between going up and going down. At these points, the rate of change is neither positive nor negative; the **rate of change is zero**!



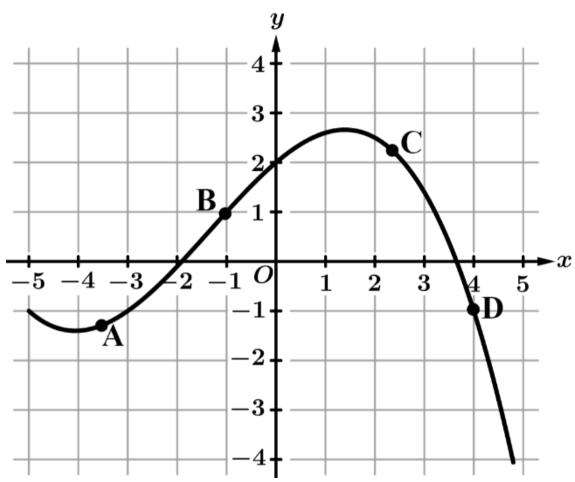
- At t_1, the rate of change of f is **positive**.
- At t_2, the rate of change of f is **zero**.
- At t_3, the rate of change of f is **negative**.

The fastest roller coaster in the world is the Formula Rossa in Abu Dhabi (United Arab Emirates). Built in 2010, the coaster reaches a maximum speed of 149.1 miles per hour! For comparison, the average speed for the 2008 Indy 500 was only 143.6 miles per hour!



Graph of f

Example 1: The figure shows the graph of a function f in the xy-plane with three labeled points. Order the rates of change of f at the three labeled points from least to greatest.



Graph of g

Example 2: The figure shows the graph of a function g in the xy-plane with four labeled points. Of the following points, at which is the rate of change of g the least?

(A) A

(B) B

(C) C

(D) D

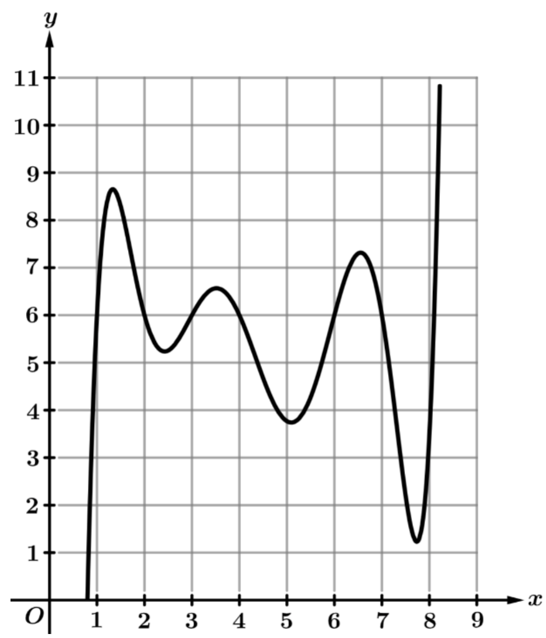
Another important concept that we will use throughout AP Precalculus (and AP Calculus) is the average rate of change of a function over an interval.

Average Rate of Change (AROC)

The average rate of change of a function over an interval is the constant rate of change that yields the same change in the output values as the function yielded on that interval. It is the ratio of the change in the output values to the change in the input values over that interval.

We will look more closely at the average rate of change of a function in Topic 1.3, including how to calculate the average rate of change over a given interval.

In Topic 1.2, we will approach average rates of change in a way that is very similar to how we approached the rate of change at a given point – we will determine if the average rate of change over an interval is positive, negative, or zero. And we will compare the average rates of change over two distinct intervals.



Graph of h

Example 3: The figure shows the graph of the function $y = h(x)$. Of the following, on which interval is the average rate of change of h greatest?

(A) $2 \le x \le 3$

(B) $3 \le x \le 5$

(C) $5 \le x \le 7$

(D) $7 \le x \le 8$

Example 4: The graph of the function $y = f(x)$ in the xy-plane always has a negative rate of change. Which of the following definitions for the variables x and y would best model the function f?

(A) $x =$ the age, in years, of a young child; $y =$ the height, in inches, of the young child
(B) $x =$ the total number of points scored in a basketball game; $y =$ the time remaining, in seconds, in the game.
(C) $x =$ the time, in seconds, since a ball was thrown straight up in the air; $y =$ the height, in feet, of the ball
(D) $x =$ the radius, in meters, of a circle; $y =$ the area, in square inches, of the circle

Example 5: The function k is defined by $k(x) = 3.16 + 4.2x - 0.85x^2$ for $-10 \le x \le 10$. Which of the following statements about k is correct?

(A) k has a positive rate of change over the interval $-10 \le x \le 8.348$.
(B) k has a positive rate of change over the interval $-0.663 \le x \le 5.605$.
(C) k has a negative rate of change over the interval $-10 \le x \le 2.470$.
(D) k has a negative rate of change over the interval $2.470 \le x \le 10$.

Birthyear	1800	1850	1900	1950	2000
Life Expectancy	41.24	46.10	53.63	70.65	81.83

Example 6: The table gives the life expectancy of US females born in a given year over successive 50-year intervals. Over which of the following intervals is the average rate of change in life expectancy the greatest?

(A) from 1800 to 1850
(B) from 1850 to 1900
(C) from 1900 to 1950
(D) from 1950 to 2000