	AP Precalculus Notes	Name:
	Topic 1.1: Change in Tandem	
	Created by Bryan Passwater Speedway High School BryanPasswater1@gmail.com	Mathematical Practices/Skills Highlighted **2.A** **Identify information** from multiple representations. **2.B** **Construct equivalent** representations of functions. **3.A** **Describe** the **characteristics** of a function.

A _______________ is a mathematical relation that maps a set of input values to a set of output values such that each input value is mapped to exactly ___ output value.

Note: In previous courses, you may have used the "Vertical Line Test" to determine if a graph is a function or not. However, in AP Precalculus, we will use the statement "**Each input has exactly one output**" when explaining why a given relation is a function…we must connect the inputs and outputs.

FUN FACT!

Later in this course, we will learn why "the vertical line test" is not a valid explanation for a function. In Unit 3, we will discover **polar functions**. Polar graphs often FAIL the vertical line test, but they ARE actually functions!

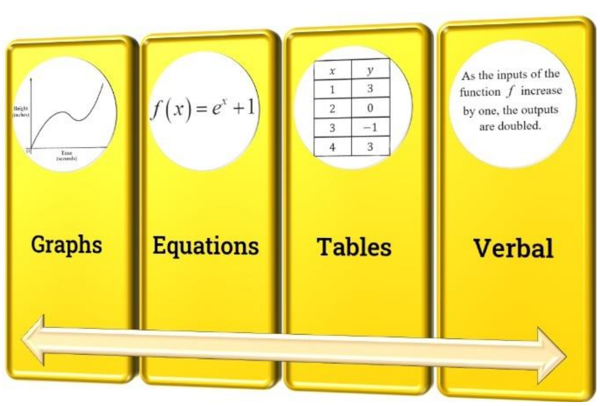
The set of input values of a function is called the ______________, represented by the ___________________ variable.

The set of output values of a function is called the ____________, represented by the __________________ variable.

A function f is **positive** when the graph of f lies above the x-axis, i.e. the outputs $(y\text{-values})$ are greater than zero.

A function f is **negative** when the graph of f lies below the x-axis, i.e. the outputs $(y\text{-values})$ are less than zero.

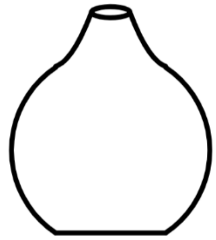
Multiple Representations



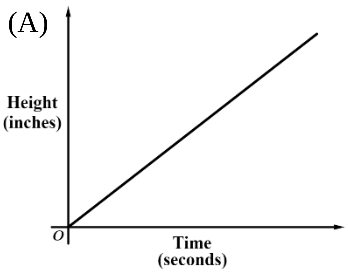
Throughout this course, we will learn and study concepts utilizing the four mathematical representations: **graphical**, **analytical** (equations), **numerical** (tables), and **verbal**.

To be successful in AP Precalculus, we need to be able to **identify important information** from each representation and **construct equivalent representations** of functions.

This may sound scary and/or confusing at first, but we will have lots of opportunities to practice throughout the course – eventually, these ideas will become part of our natural thought process when thinking about mathematical concepts!



Example 1: The figure shows an empty vase. At time $t = 0$, water begins pouring into the vase at a constant rate until it is full. Which of the following could depict this situation, where time, in seconds, is the independent variable, and the height of the water in the vase, in inches, is the dependent variable?

(A)


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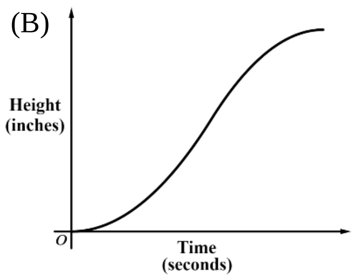

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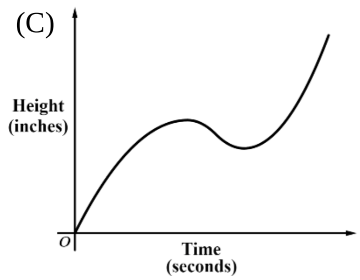

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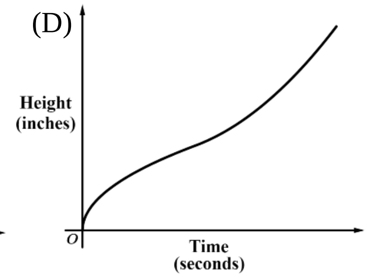
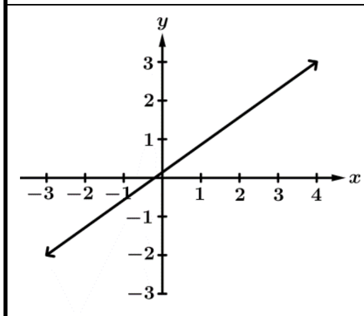
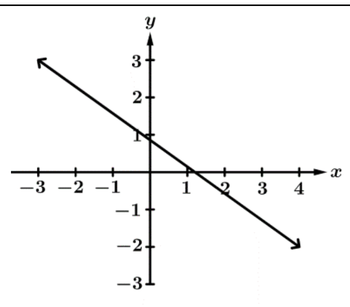
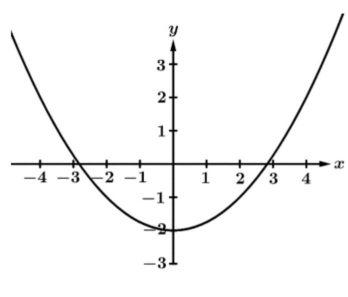
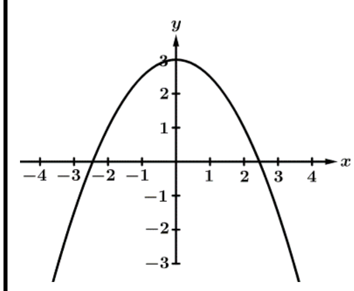
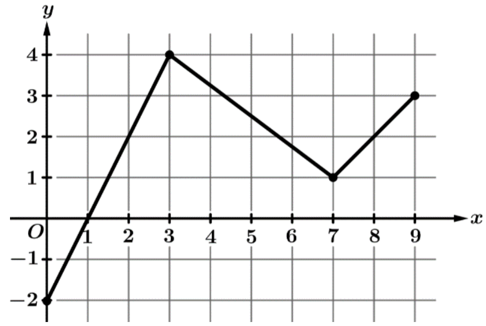


Graphical Behavior of Functions			
Increasing	**Decreasing**	**Concave Up**	**Concave Down**
As the input values increase, the output values always increase: If $a < b$, then $f(a) < f(b)$.	As the input values increase, the output values always decrease: If $a < b$, then $f(a) > f(b)$.	The rate of change is increasing.	The rate of change is decreasing.
			

Before we move ahead, it is important that we clarify a few things about the notation and vocabulary that we will use throughout the course.

First, when describing features on a graph, we generally write the intervals where the graph displays those features. We will always use the **<u>input</u>** (x) variable when writing intervals.



Graph of f

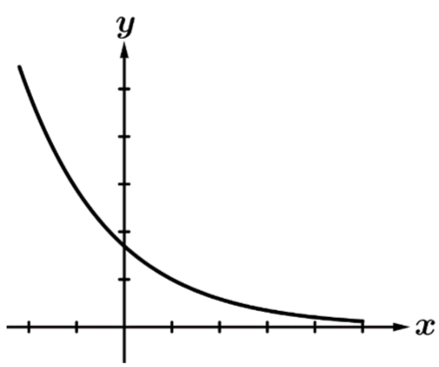
Example 2: The figure shows the graph of the piecewise function f over the interval $0 \le x \le 9$. What are all the intervals where the graph of f is increasing?

The second thing that needs to be clarified is the phrase "**rate of change**." This phrase will be utilized repeatedly throughout this course (and in AP Calculus), so it is important that we understand its meaning.

Essentially, "**rate of change**" simply means "**slope**". Anytime we see the phrase "rate of change" in this course, we can generally replace it with the word "slope" to help us better comprehend what is being said.

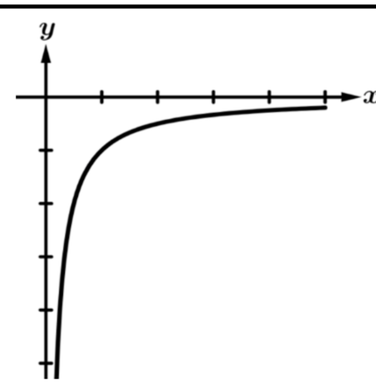
WORDS MATTER!

We must read very carefully in this course. Stating that "f is increasing" is very different than stating that "the rate of change of f is increasing".



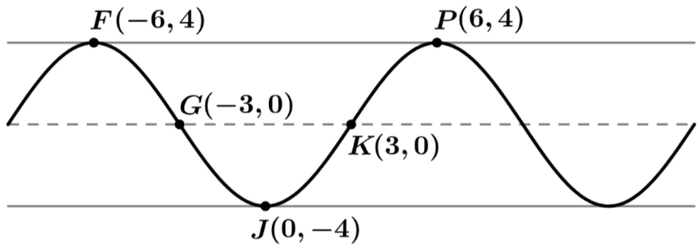
Graph of f

- f is positive because the outputs $(y\text{-values})$ are all positive, i.e. the graph of f lies above the x-axis.
- f is decreasing because the outputs $(y\text{-values})$ decrease as the inputs $(x\text{-values})$ increase, i.e. the graph of f "goes down" as we move to the right.
- The rate of change of f is increasing because the graph of f is concave up.



Graph of g

- g is negative because the outputs $(y\text{-values})$ are all negative, i.e. the graph of g lies below the x-axis.
- g is increasing because the outputs $(y\text{-values})$ increase as the inputs $(x\text{-values})$ increase, i.e. the graph of g "goes up" as we move to the right.
- The rate of change of g is decreasing because the graph of g is concave down.



Example 3: The graph of $h(t)$ and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented. The t-coordinate of F is t_1, and the t-coordinate of G is t_2.

On the AP Exam, FRQ3 will appear exactly like this example!

(i) On the interval (t_1, t_2), which of the following is true about h?

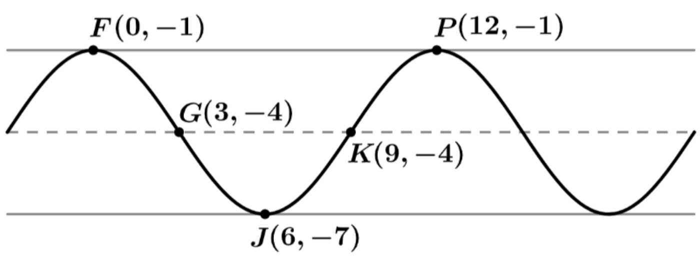
a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).



Example 4: The graph of $h(t)$ and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P are labeled on the graph. No scale is indicated, and no axes are presented. The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h?

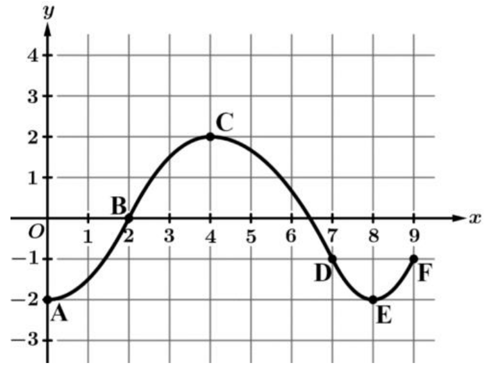
a. h is positive and increasing.

b. h is positive and decreasing.

c. h is negative and increasing.

d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).



Graph of k

The figure shows the graph of the function k for the interval $0 \le x \le 9,$ as well as the six labeled points: A, B, C, D, E, and F. Use the graph of k for the following examples.

Example 5: On which of the following intervals is k negative and decreasing?

(A) the interval from A to B
(B) the interval from B to C
(C) the interval from D to E
(D) the interval from E to F

Example 6: Which of the following statements about the rate of change of k is true?

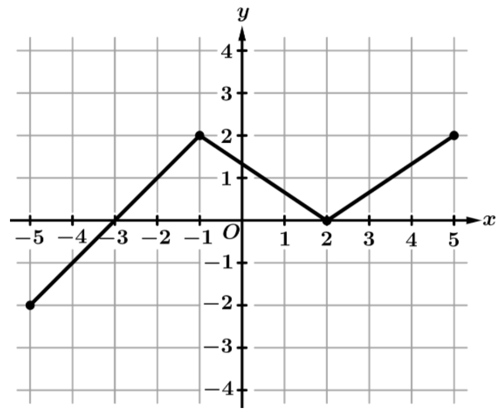
(A) The rate of change of k is negative on the interval from A to B.
(B) The rate of change of k is negative on the interval from B to C.
(C) The rate of change of k is positive on the interval from D to E.
(D) The rate of change of k is positive on the interval from E to F.

Example 7: Which of the following statements about the rate of change of k is true?

(A) The rate of change of k is decreasing on the interval from A to B.
(B) The rate of change of k is increasing on the interval from B to C.
(C) The rate of change of k is increasing on the interval from D to E.
(D) The rate of change of k is decreasing on the interval from E to F.

Example 8: On which of the following intervals is k increasing and the graph of k concave down?

(A) the interval from A to B
(B) the interval from B to C
(C) the interval from C to D
(D) the interval from D to E

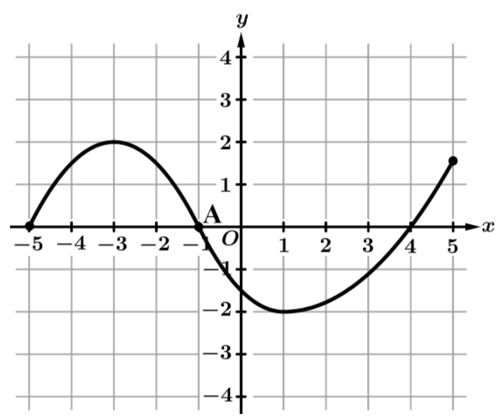


Graph of f

The figure shows the graph of the function f on the interval $-5 \le x \le 5$.

Example 9: On what intervals is f decreasing?

Example 10: On what intervals is f both negative and increasing?



Graph of g

The figure shows the graph of the function g on the interval $-5 \le x \le 5$. Point A is located at $(-1, 0)$ and is the only point where the graph of g changes concavity.

Example 11: On what intervals is g decreasing and the graph of g concave up?

Example 12: On what intervals is the rate of change of g positive and decreasing?