Recall: A rational function is a function that is a ratio of two polynomial functions (think fractions).

$$f(x) = \frac{2x - 3}{x^2 - 4x - 45}$$

We explored polynomial inequalities previously (Topic 1.5), and we can utilize some of those concepts to help us understand more about rational functions like $f(x)$.

Two Important Traits of a Rational Function

Let $f(x) = \dfrac{g(x)}{h(x)}$ be a rational function where $g(x)$ and $h(x)$ have no factors in common. Then, we know …

1. $f(x)$ has zeros when $g(x) = 0$.

2. $f(x)$ is undefined when $h(x) = 0$.

Note: When solving rational inequalities, we need to identify **BOTH** the **zeros** and **undefined values**!

Solving Rational Inequalities

1. Make sure the inequality has **0** on the other side!

2. Make sure $f(x) = \dfrac{g(x)}{h(x)}$ (Make sure you have a single rational function)

3. Set $g(x) = 0$ and $h(x) = 0$ to find values to include on the sign chart. (Make sure to factor!)

4. Create a sign chart with all values from **Step 3**.

5. Be careful to mark the values where $h(x) = 0$ so that we **NEVER** include those values in our solution.

6. **Test values** in each interval to see if the values in the interval are _____ or _____.

7. **Interpret** the sign chart to answer the given inequality from the problem.

NOTE: Be sure to write your answer in **interval notation** and think about the **endpoints**!

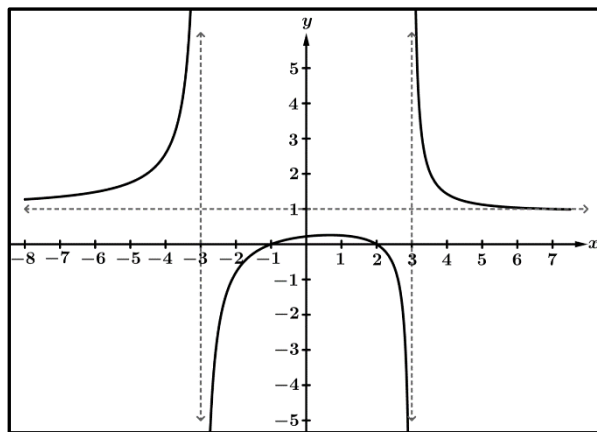**Example 1**: Solve $\dfrac{x - 2}{(x + 6)(x - 3)} \geq 0$

Example 2: Solve $\dfrac{x^2 - 4}{x^2 - 10x + 25} < 0$

Example 3: Solve $\dfrac{2}{x-3} > 0$

Example 4: Solve $\dfrac{4x+8}{x+5} \leq 0$

Example 5: Solve $\dfrac{(x-1)(x+2)^2}{x-2} \geq 0$

Example 6: Solve $\dfrac{1}{(x-1)^2} \leq 0$

Example 7: The graph of the rational function f is shown in the figure above. Use the graph to solve the following inequalities.

a) $f(x) \leq 0$

b) $f(x) > 0$

c) $f(x) \geq 1$