An inverse relation will "**undo**" a given relation. **Every** inverse relation can be found by _____ **each** x and y value.

In some situations, this process is intuitive, but other times this may not be as obvious. In these cases, it is important that we understand the _____ of inverse relations as they will help us tackle a variety of problems.

Let's look at this **numerically** (tables), **graphically** (pictures) and **analytically** (equations):

Numerical (Tables)

Example 1: Find the inverse relation of the given table.

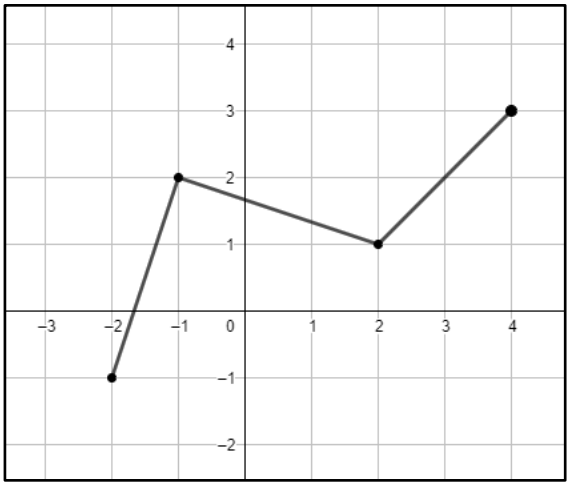
x	1	3	4	6
y	-1	2	0	2

x				
y				

Note: The original table _____ a function because each _____ has exactly one _____. However, the inverse is _____ a function because _____.

Graphical (Pictures)

Example 2: Sketch the inverse relation of the given graph.



Steps to Sketch an Inverse Graph
(Linear Pieces)

1. List the **key** points in a **table**
2. Create a **new** table for the inverse by **switching** the x and y values.
3. Plot the **new** points for the inverse and **sketch** the inverse graph.

Example 2B: Is the original relation a function? Is the inverse relation a function?

Graphical Property of Inverses: The graphs of inverses are _____ over the line _____.

Analytical (Equations)

Example 3: Find the inverse for $f(x) = 3x - 7$ (**Reminder**: $f(x) = y$)

Steps to Find an Inverse Equation

1. **Switch** the x and y values.
2. **Solve** for y (Get y by itself!)
3. If original equation was in $f(x) =$ form, write the inverse equation as $f^{-1}(x) =$

Notation: The inverse function of $f(x)$ is written as _____.

Finding Inverse Functions of Rational Functions (with multiple $x's$)	
Step 1	Switch the x and y values.
Step 2	Multiply/Distribute both sides by the denominator of the rational expression to eliminate the fraction.
Step 3	Move all terms that include the variable y to the left side of the equation and move all terms that do not include the variable y to the right side of the equation.
Step 4	Factor out an y from the terms on the left side of the equation.
Step 5	Divide both sides by the terms remaining on the left side after y was factored out.
Step 6	Rewrite the equation with proper inverse notation $\left(f^{-1}(x)\right)$

Example 4: Find the inverse functions for each of the following.

a) $f(x) = \dfrac{x-2}{x+3}$

b) $g(x) = \dfrac{2x+1}{x-3}$

More on Graphing Inverses

We know that to find an inverse, we simply switch the _____ and _____ values.

Graphically, this means that a function and its inverse will be reflections over the line _____.

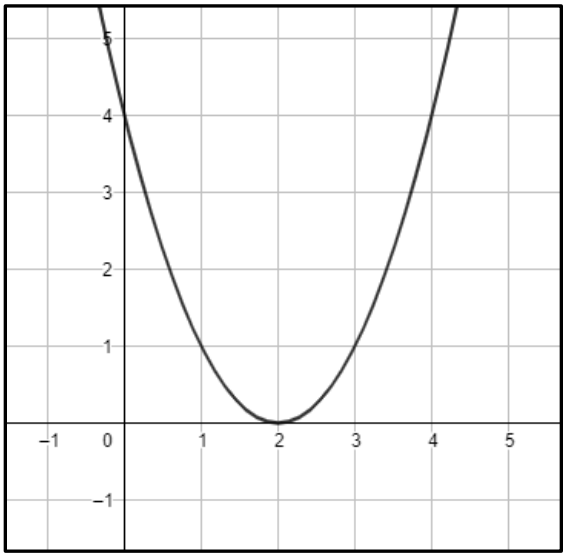
If a graph is not made up of simply line segments, sketching the inverse can be challenging. To do this, we use the fact that a function and its inverse are reflections over the line $y = x$ to help us sketch the inverse graph.

Steps to Sketch an Inverse Graph (Nonlinear pieces)

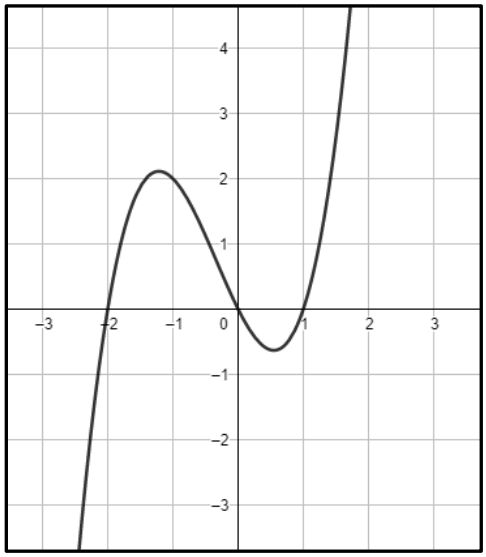
1. Sketch the line _____ on the graph.
2. Mark any points from the original graph that are already _____ the line. These points will stay the same!
3. Select a few additional points on the original graph and find their inverse points.
   - You can find the inverse points by switching the x and y values, **OR**
   - Graphically reflecting each point over the line $y = x$ by drawing a line _____ from the point to the line $y = x$ and extending it an equal distance on the other side of the line.
4. Sketch the inverse graph by connecting the new points in a similar pattern to the original function.

Example 5: Sketch the inverse of the functions below.

a)



b)



What do Inverse Functions do?

If you plug a number into a function, it generally will output a new (different) number. For example, if we plug in $x = 2$ into the function $f(x) = 3x - 1$, we get _____.

Now, If we plug that **answer** into the inverse function, the answer should be _____.

This is because an inverse function essentially will "undo" a function. So, if we plug _____ into a function, and then plug the output into the inverse function, we should end up with _____ again.

Inverse Functions

Two functions $f(x)$ and $g(x)$ are inverses if and only if $f\big(g(x)\big) = x$ and $g\big(f(x)\big) = x$.

To show that two functions are inverses, we must show that the _____ **BOTH** equal _____.

Example 6: Let $f(x)$ and $g(x)$ be the functions below. Determine if $f(x)$ and $g(x)$ are inverses.

$f(x) = 2x - 3$ $\qquad$ $g(x) = \frac{1}{2}x + \frac{3}{2}$

Step 1: Find $f\big(g(x)\big)$

Step 2: Find $g\big(f(x)\big)$

Note: Sometimes, we must use a restricted domain to ensure two functions are inverses.

Example 7: Show that $h(x)$ and $k(x)$ below are inverses where $x \geq 10$.

$h(x) = x^2 + 10$ $\qquad$ $k(x) = \sqrt{x - 10}$

Example 8: Show that $n(x)$ and $p(x)$ below are inverses where $x \neq 0$ and $x \neq 4$.

$n(x) = \frac{6}{x - 4}$ $\qquad$ $p(x) = \frac{6}{x} + 4$