x	-4	-3	-2	-1	0	1	2	3	4	8	9
$f(x)$	0	1	3	-5	-1	7	-3	5	2	-2	-6

Selected values of the continuous function $f(x)$ are shown in the table above. Use the values in the table to answer the following.

1. Let $g(x) = 3f(x+2) - 1$.

(a) Find $g(1)$.

$g(1) = 3f(1+2) - 1$

$= 3f(3) - 1 = 3(5) - 1 = 14$

(b) Find $g(-2)$.

$g(-2) = 3f(-2+2) - 1$

$= 3f(0) - 1 = 3(-1) - 1 = -4$

(c) If $g(k) = -7$, find k.

$g(k) = 3f(k+2) - 1 = -7$

$f(k+2) = \frac{-6}{3} = -2$

$k + 2 = 8 \Rightarrow k = 6$

2. Let $h(x) = 5 - f(2x)$.

(a) Find $h(2)$.

$h(2) = 5 - f(2 \cdot 2)$

$= 5 - f(4) = 5 - 2 = 3$

(b) Find $h(0)$.

$h(0) = 5 - f(2 \cdot 0)$

$= 5 - f(0) = 5 - (-1) = 6$

(c) Find $h^{-1}(4)$.

$h\left(\underbrace{h^{-1}(4)}_{u}\right) = 4$

$h(u) = 5 - f(2u)$

$4 = 5 - f(2u)$

$f(2u) = 1 \Rightarrow 2u = -3$

$u = -\frac{3}{2} \Rightarrow h^{-1}(4) = -\frac{3}{2}$

3. Let $p(x)$ be the function that results from applying three transformations to the graph of f in this order: a horizontal dilation by a factor of 3, a reflection over the x axis, and a vertical translation by -4 units.

(a) Find $p(3)$.

$p(x) = -f\left(\frac{1}{3}x\right) - 4$

$p(3) = -f(1) - 4$

$= -7 - 4 = -11$

(b) Find $p(-6)$.

$p(x) = -f\left(\frac{1}{3}x\right) - 4$

$p(-6) = -f(-2) - 4$

$= -3 - 4 = -7$

(c) If $p(x) = f(x)$, find x.

$p(x) = -f\left(\frac{1}{3}x\right) - 4 = f(x)$

$-4 = f(x) + f\left(\frac{1}{3}x\right) \Rightarrow x = -3$

$f(-3) + f(-1)$

$= 1 + (-5) = -4$

Solutions by Ted Gott tedg20776@gmail.com

x	-4	-3	-2	-1	0	1	2	3	4	8	9
$f(x)$	0	1	3	-5	-1	7	-3	5	2	-2	-6

4. Let $m(x) = af(bx) + c$, where a, b, and c are positive constants. The graph of m can be constructed by applying three transformations to the graph of f in this order: a horizontal dilation by a factor of $\frac{1}{2}$, a vertical dilation by a factor of $\frac{1}{2}$, and a vertical translation by 3 units.

$$m(x) = \frac{1}{2}f(2x) + 3$$

(a) Find $m(-2)$.

$m(-2) = \frac{1}{2}f\big(2(-2)\big) + 3$

$= \frac{1}{2}f(-4) + 3 = \frac{1}{2}(0) + 3 = 3$

(b) Find $m(4)$.

$m(4) = \frac{1}{2}f\big(2(4)\big) + 3$

$= \frac{1}{2}f(8) + 3 = \frac{1}{2}(-2) + 3 = 2$

(c) If $m(k) = 0$, find k.

$m(k) = \frac{1}{2}f(2k) + 3 = 0$

$f(2k) = -6 \Rightarrow 2k = 9 \Rightarrow k = \frac{9}{2}$

x	-3	-1	0	1	3	4	6	9
$g(x)$	-4	2	3	6	1	-1	-5	-2

Selected values of the continuous function $g(x)$ are shown in the table above. Use the values in the table to answer the following.

5. Let $h(x) = -2g(x-3) - 5$.

(a) Find $h(0)$.

$h(0) = -2g(-3) - 5$

$= -2(-4) - 5 = 3$

(b) Find $h(3)$.

$h(3) = -2g(0) - 5$

$= -2(3) - 5 = -11$

(c) If $h(k) = 5$, find k.

$h(k) = -2g(k-3) - 5 = 5$

$-2g(k-3) = 10$

$g(k-3) = -5 \Rightarrow k - 3 = 6$

$k = 9$

6. Let $n(x) = 2 + g\left(\frac{x}{3}\right)$.

(a) Find $n(3)$.

$n(3) = 2 + g\left(\frac{3}{3}\right) = 2 + g(1)$

$= 2 + 6 = 8$

(b) Find $n(-3)$.

$n(-3) = 2 + g\left(\frac{-3}{3}\right)$

$= 2 + g(-1) = 2 + 2 = 4$

(c) Find $n^{-1}(4)$

$n\Big(\underbrace{n^{-1}(4)}_{u}\Big) = 4$

$n(u) = 2 + g\left(\frac{u}{3}\right)$

$4 = 2 + g\left(\frac{u}{3}\right) \Rightarrow g\left(\frac{u}{3}\right) = 2$

$g(-1) = 2 \Rightarrow \frac{u}{3} = -1 \Rightarrow u = -3$

$n^{-1}(4) = -3$

x	-3	-1	0	1	3	4	6	9
$g(x)$	-4	2	3	6	1	-1	-5	-2

7. Let $p(x)$ be the function that results from applying three transformations to the graph of g in this order: a horizontal dilation by a factor of $\frac{1}{2}$, a reflection over the y axis, and a vertical translation by 1 unit.

$p(x) = g(-2x) + 1$

(a) Find $p(-2)$.

$p(-2) = g(4) + 1 = -1 + 1 = 0$

(b) Find the average rate of change of p over the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

$$\frac{p\left(\frac{1}{2}\right) - p\left(-\frac{1}{2}\right)}{\frac{1}{2} - \left(-\frac{1}{2}\right)} = p\left(\frac{1}{2}\right) - p\left(-\frac{1}{2}\right)$$

$[g(-1) + 1] - [g(1) + 1] = [2 + 1] - [6 + 1] = 3 - 7 = -4$

8. Let $s(x) = ag(bx) + c$, where a, b, and c are positive constants. The graph of s can be constructed by applying three transformations to the graph of g in this order: a horizontal dilation by a factor of 3, a vertical dilation by a factor of 4, and a vertical translation by -5 units.

(a) Find $s(3)$.

(b) Find $s(-9)$.

(c) If $s(k) = -9$, find k.

$$s(x) = 4g\left(\frac{1}{3}x\right) - 5$$

(a) Find $s(3)$.

$s(3) = 4g\left(\frac{1}{3}3\right) - 5$

$= 4(6) - 5 = 19$

(b) Find $s(-9)$.

$s(-9) = 4g(-3) - 5$

$= 4(-4) - 5 = -16 - 5 = -21$

(c) If $s(k) = -9$, find k.

$s(k) = 4g\left(\frac{k}{3}\right) - 5 = -9$

$4g\left(\frac{k}{3}\right) = -4 \Rightarrow g\left(\frac{k}{3}\right) = -1$

$\frac{k}{3} = 4 \Rightarrow k = 12$

x	-5	-2	-1	2	3	4	6	12	15
$h(x)$	6	1	0	-3	-2	2	8	11	9

Selected values of the continuous function $h(x)$ are shown in the table above. Use the values in the table to answer the following.

9. Let $h(x) = 6f(x + 2) - 3$.

(a) Find $f(4)$.

$h(2) = 6f(2 + 2) - 3$

$-3 = 6f(4) - 3$

$6f(4) = 0 \Rightarrow f(4) = 0$

(b) Find $f(0)$.

$h(-2) = 6f(-2 + 2) - 3$

$1 = 6f(0) - 3$

$6f(0) = 4 \Rightarrow f(0) = \frac{4}{6} = \frac{2}{3}$

(c) If $f(k) = 2$, find k.

$h(k - 2) = 6f(k) - 3$

$h(k - 2) = 6(2) - 3$

$h(k - 2) = 9$

$k - 2 = 15 \Rightarrow k = 17$

x	-5	-2	-1	2	3	4	6	12	15
$h(x)$	6	1	0	-3	-2	2	8	11	9

10. Let $h(x) = -2g\left(\frac{x}{2}\right)$.

(a) Find $g(6)$.

$h(12) = -2g\left(\frac{12}{2}\right) \Rightarrow 11 = -2g(6)$

$g(6) = -\frac{11}{2}$

(b) If $g(x) = 1$, find x.

$h(2x) = -2g\left(\frac{2x}{2}\right) = -2g(x) = -2(1) = -2$

$h(2x) = -2 \Rightarrow 2x = 3 \Rightarrow x = \frac{3}{2}$

(c) Put the following in order from least to greatest: $g(-1)$, $g(1)$, $g(2)$. $g(2) < g(-1) < g(1)$

$h(-2) = -2g\left(\frac{-2}{2}\right)$

$1 = -2g(-1)$

$g(-1) = -\frac{1}{2}$

$h(2) = -2g\left(\frac{2}{2}\right)$

$-3 = -2g(1)$

$g(1) = \frac{3}{2}$

$h(4) = -2g\left(\frac{4}{2}\right)$

$2 = -2g(2)$

$g(2) = -1$

11. Let $h(x)$ be the function that results from applying three transformations to the graph of j in this order: a horizontal dilation by a factor of $\frac{1}{3}$, a vertical dilation by a factor of 2, and a vertical translation by -4 units.

$h(x) = 2j(3x) - 4$

(a) Find $j(6)$.

$h(2) = 2j(3(2)) - 4$

$-3 = 2j(6) - 4$

$1 = 2j(6)$

$j(6) = \frac{1}{2}$

(b) Find $j(-3)$.

$h(-1) = 2j(3(-1)) - 4$

$0 = 2j(-3) - 4$

$4 = 2j(-3)$

$j(-3) = 2$

12. Let $w(x) = 2h(x-3) + 1$

(a) Find $w(-2) \cdot h(6)$.

$w(-2) = 2h(-2-3) + 1$

$w(-2) = 2h(-5) + 1$

$w(-2) = 2(6) + 1 = 13$

$w(-2) \cdot h(6) = 13 \cdot 8 = 104$

(b) Find $w(h(-5)) = -3$

$w(h(-5)) = w(6)$

$w(6) = 2h(3) + 1$

$= 2(-2) + 1 = -3$

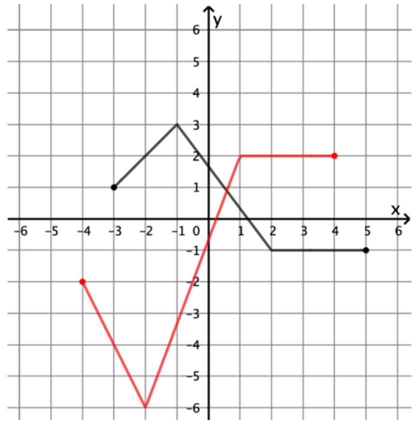
(c) Find $w(w(2)) = w(1) = 3$.

$w(2) = 2h(-1) + 1$

$w(2) = 2(0) + 1 = 1$

$w(1) = 2h(-2) + 1$

$w(1) = 2(1) + 1 = 3$



Transformation of

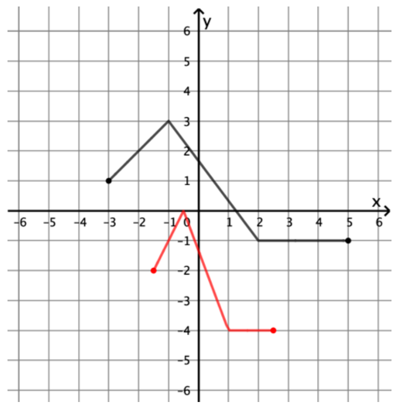
Key points on $f(x)$

$x_g + 1 = x_f \quad y_g = -2y_f$

$f(x)$	$g(x)$
$(-3,1)$	$(-4,-2)$
$(-1,3)$	$(-2,-6)$
$(2,-1)$	$(1,2)$
$(5,-1)$	$(4,2)$

The graph of $f(x)$ is shown in the figure above and consists of three line segments.

13. Let $g(x) = -2f(x+1)$. Sketch the graph of $g(x)$ on the same axes as $f(x)$ above.



Transformation of

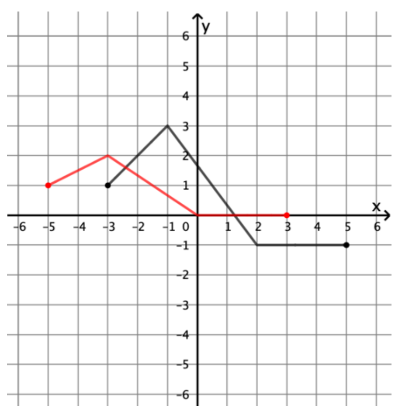
Key points on $f(x)$

$x_h = \frac{1}{2}x_f \quad y_h = y_f - 3$

$f(x)$	$h(x)$
$(-3,1)$	$(-1.5,-2)$
$(-1,3)$	$(-0.5,0)$
$(2,-1)$	$(1,-4)$
$(5,-1)$	$(2.5,-4)$

The graph of $f(x)$ is shown in the figure above and consists of three line segments.

14. Let $h(x) = f(2x) - 3$. Sketch the graph of $h(x)$ on the same axes as $f(x)$ above.



Transformation of

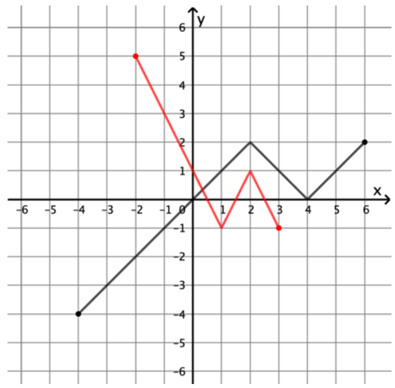
Key points on $f(x)$

$x_k = x_f - 2 \quad y_k = \frac{y_f + 1}{2}$

$f(x)$	$k(x)$
$(-3,1)$	$(-5,1)$
$(-1,3)$	$(-3,2)$
$(2,-1)$	$(0,0)$
$(5,-1)$	$(3,0)$

The graph of $f(x)$ is shown in the figure above and consists of three line segments.

15. Let $f(x) = 2k(x-2) - 1$. Sketch the graph of $k(x)$ on the same axes as $f(x)$ above.



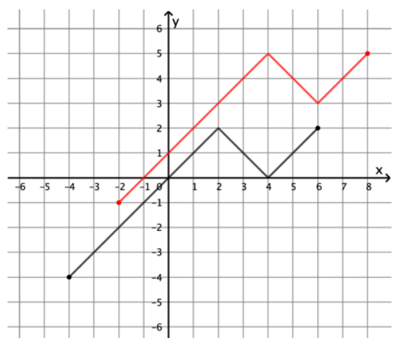
Transformation of
Key points on $f(x)$

$x_g = \frac{1}{2}x_f \quad y_g = -y_f + 1$

$f(x)$	$g(x)$
$(-4,-4)$	$(-2,5)$
$(2,2)$	$(1,-1)$
$(4,0)$	$(2,1)$
$(6,2)$	$(3,-1)$

The graph of $f(x)$ is shown in the figure above and consists of three line segments.

16. Let $g(x) = 1 - f(2x)$. Sketch the graph of $g(x)$ on the same axes as $f(x)$ above.



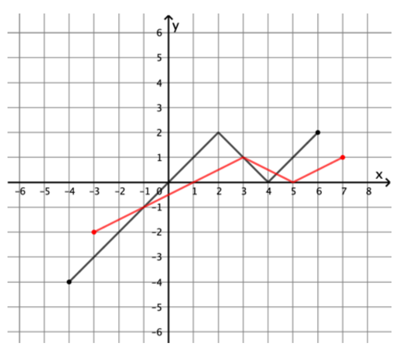
Transformation of
Key points on $f(x)$

$x_h = x_f + 2 \quad y_h = y_f + 3$

$f(x)$	$h(x)$
$(-4,-4)$	$(-2,-1)$
$(2,2)$	$(4,5)$
$(4,0)$	$(6,3)$
$(6,2)$	$(8,5)$

The graph of $f(x)$ is shown in the figure above and consists of three line segments.

17. Let $h(x) = f(x-2) + 3$. Sketch the graph of $h(x)$ on the same axes as $f(x)$ above.



Transformation of
Key points on $f(x)$

$x_k = x_f + 1 \quad y_k = \frac{1}{2}y_f$

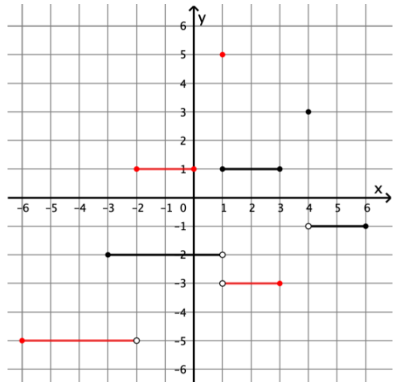
$f(x)$	$k(x)$
$(-4,-4)$	$(-3,-2)$
$(2,2)$	$(3,1)$
$(4,0)$	$(5,0)$
$(6,2)$	$(7,1)$

The graph of $f(x)$ is shown in the figure above and consists of three line segments.

18. Let $f(x) = 2k(x+1)$. Sketch the graph of $k(x)$ on the same axes as $f(x)$ above.

Transformation of
Key points on $f(x)$
$x_g = x_f - 3 \quad y_g = 2y_f - 1$

$f(x)$	$g(x)$
$(-3,-2)$	$(-6,-5)$
$(1,-2)$	$(-2,-5)$
$(1,1)$	$(-2,1)$
$(3,1)$	$(0,1)$



Transformation of
Key points on $f(x)$
$x_g = x_f - 3 \quad y_g = 2y_f - 1$

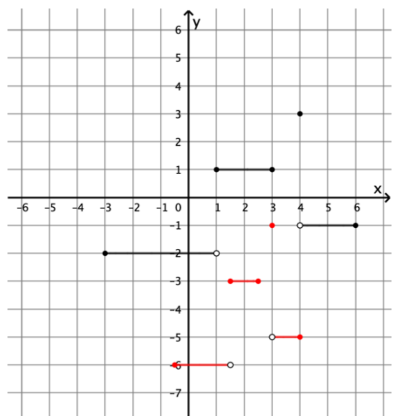
$f(x)$	$g(x)$
$(4,3)$	$(1,5)$
$(4,-1)$	$(1,-3)$
$(6,-1)$	$(3,-3)$

The graph of $f(x)$ is shown in the figure above and consists of three linear pieces and a point at $(4,\ 3)$.

19. Let $g(x) = 2f(x+3) - 1$. Sketch the graph of $g(x)$ on the same axes as $f(x)$ above.

Transformation of
Key points on $f(x)$
$x_h = \frac{1}{2}x_f + 1$
$y_h = y_f - 4$

$f(x)$	$h(x)$
$(-3,-2)$	$(-0.5,-6)$
$(1,-2)$	$(1.5,-6)$
$(1,1)$	$(1.5,-3)$
$(3,1)$	$(2.5,-3)$



Transformation of
Key points on $f(x)$
$x_h = \frac{1}{2}x_f + 1$
$y_h = y_f - 4$

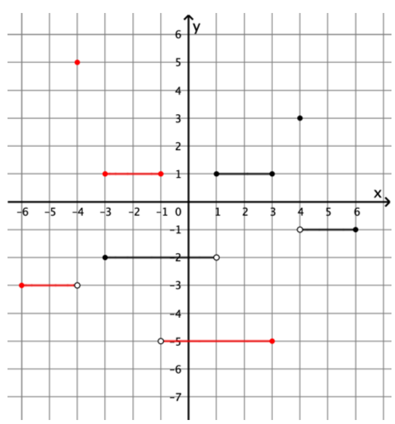
$f(x)$	$h(x)$
$(4,3)$	$(3,-1)$
$(4,-1)$	$(3,-5)$
$(6,-1)$	$(4,-5)$

The graph of $f(x)$ is shown in the figure above and consists of three linear pieces and a point at $(4,\ 3)$.

20. Let $h(x) = f(2x-2) - 4$. Sketch the graph of $h(x)$ on the same axes as $f(x)$ above.

Transformation of
Key points on $f(x)$
$x_k = -x_f$
$y_k = 2y_f - 1$

$f(x)$	$k(x)$
$(-3,-2)$	$(3,-5)$
$(1,-2)$	$(-1,-5)$
$(1,1)$	$(-1,1)$
$(3,1)$	$(-3,1)$



Transformation of
Key points on $f(x)$
$x_k = -x_f$
$y_k = 2y_f - 1$

$f(x)$	$k(x)$
$(4,3)$	$(-4,5)$
$(4,-1)$	$(-4,-3)$
$(6,-1)$	$(-6,-3)$

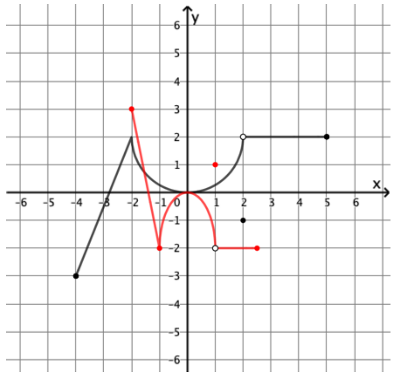
The graph of $f(x)$ is shown in the figure above and consists of three linear pieces and a point at $(4,\ 3)$.

21. Let $k(x)$ be the function that results from applying three transformations to the graph of f in this order: a vertical dilation by a factor of 2, a reflection over the y axis, and a vertical translation by -1 unit. Sketch the graph of $k(x)$ on the same axes as $f(x)$ above.

Transformation of
Key points on $f(x)$

$x_g = \frac{1}{2}x_f \quad y_g = -y_f$

$f(x)$	$g(x)$
$(-4,-3)$	$(-2,3)$
$(-2,2)$	$(-1,-2)$
$(0,0)$	$(0,0)$
$(2,2)$	$(1,-2)$
$(5,2)$	$(2.5,-2)$
$(2,-1)$	$(1,1)$



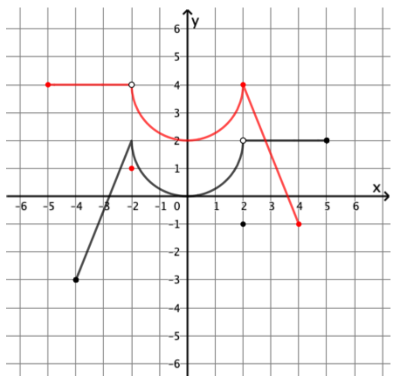
The graph of $f(x)$ is shown in the figure above and consists of two linear pieces, a semi-circle, and a point at $(2,-1)$.

22. Let $g(x) = -f(2x)$. Sketch the graph of $g(x)$ on the same axes as $f(x)$ above.

Transformation of
Key points on $f(x)$

$x_h = -x_f \quad y_h = y_f + 2$

$f(x)$	$h(x)$
$(-4,-3)$	$(4,-1)$
$(-2,2)$	$(2,4)$
$(0,0)$	$(0,2)$
$(2,2)$	$(-2,4)$
$(5,2)$	$(-5,4)$
$(2,-1)$	$(-2,1)$



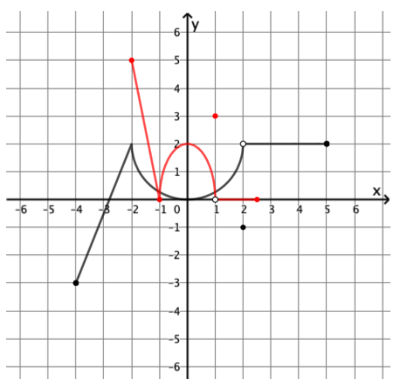
The graph of $f(x)$ is shown in the figure above and consists of two linear pieces, a semi-circle, and a point at $(2,-1)$.

23. Let $h(x) = f(-x) + 2$. Sketch the graph of $h(x)$ on the same axes as $f(x)$ above.

Transformation of
Key points on $f(x)$

$x_k = \frac{1}{2}x_f \quad y_k = -y_f + 2$

$f(x)$	$k(x)$
$(-4,-3)$	$(-2,5)$
$(-2,2)$	$(-1,0)$
$(0,0)$	$(0,2)$
$(2,2)$	$(1,0)$
$(5,2)$	$(2.5,0)$
$(2,-1)$	$(1,3)$

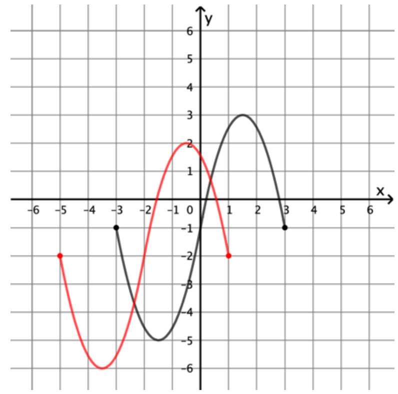


The graph of $f(x)$ is shown in the figure above and consists of two linear pieces, a semi-circle, and a point at $(2,-1)$.

24. Let $k(x)$ be the function that results from applying three transformations to the graph of f in this order: a horizontal dilation by a factor of $\frac{1}{2}$, a reflection over the x axis, and a vertical translation by 2 units. Sketch the graph of $k(x)$ on the same axes as $f(x)$ above.

Transformation of
Key points on $f(x)$
$x_g = x_f - 2 \quad y_g = y_f - 1$

$f(x)$	$g(x)$
$(-3,-1)$	$(-5,-2)$
$(-1.5,-5)$	$(-3.5,-6)$
$(0,-1)$	$(-2,-2)$
$(1.5,3)$	$(-0.5,2)$
$(3,-1)$	$(1,-2)$

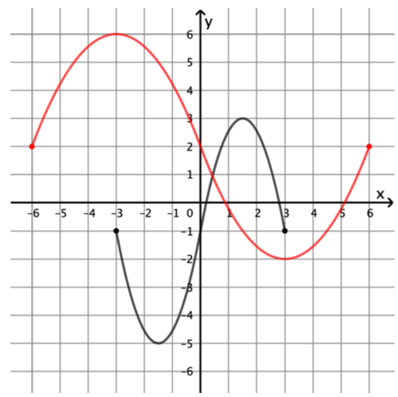


The graph of $f(x)$ is shown in the figure above and has the domain $[-3,\ 3]$ and the range $[-5,\ 3]$.

25. Let $g(x) = f(x+2) - 1$. Sketch the graph of $g(x)$ on the same axes as $f(x)$ above.

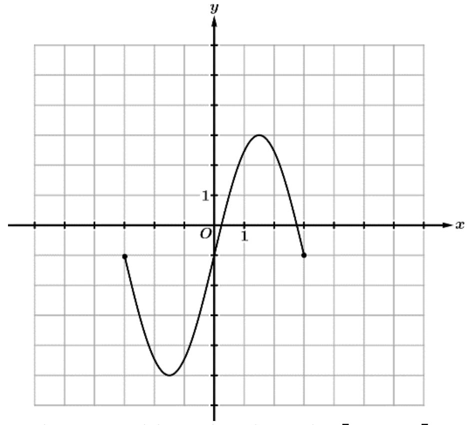
Transformation of
Key points on $f(x)$
$x_h = 2x_f \quad y_h = 1 - y_f$

$f(x)$	$h(x)$
$(-3,-1)$	$(-6,2)$
$(-1.5,-5)$	$(-3,6)$
$(0,-1)$	$(0,2)$
$(1.5,3)$	$(3,-2)$
$(3,-1)$	$(6,2)$



The graph of $f(x)$ is shown in the figure above and has the domain $[-3,\ 3]$ and the range $[-5,\ 3]$.

26. Let $h(x) = 1 - f\left(\frac{x}{2}\right)$. Sketch the graph of $h(x)$ on the same axes as $f(x)$ above.



The graph of $f(x)$ is shown in the figure above and has the domain $[-3,\ 3]$ and the range $[-5,\ 3]$.

27. Let $k(x) = -3f(2x) + 1$.

Find the domain and range of $k(x)$.

Domain: $\left[-\frac{3}{2}, \frac{3}{2}\right]$ Range: $[-8,16]$

$f(a) = -5 \Rightarrow k\left(\frac{a}{2}\right) = -3(-5) + 1 = 16$

$f(b) = 3 \Rightarrow k\left(\frac{b}{2}\right) = -3(3) + 1 = -8$

28. Let $p(x) = \frac{1}{2}f(x+3) - 4$.

Find the domain and range of $p(x)$.

Domain: $[-3-3, 3-3] = [-6,0]$ Range: $\left[-\frac{13}{2}, -\frac{5}{2}\right]$

$f(a) = -5 \Rightarrow p(a+3) = \frac{1}{2}(-5) - 4 = -\frac{13}{2}$

$f(b) = 3 \Rightarrow p(a+3) = \frac{1}{2}(3) - 4 = -\frac{5}{2}$

29. The graph of $f(x)$ has zeros at $x = -2,\ 0,$ and 3. Find the zeros of the following functions.

(a) $g(x) = 2f(x-4)$

$x = -2,\ 0,$ and 3
shifted right 4
$x - 4 = -2 \Rightarrow x = 2$
$x - 4 = 0 \Rightarrow x = 4$
$x - 4 = 3 \Rightarrow x = 7$
$\boxed{x = 2,\ 4, \text{ and } 7}$

(b) $h(x) = -\frac{1}{3}f(2x)$

$x = -2,\ 0,$ and 3
horizontal dilation $\frac{1}{2}$
$2x = -2 \Rightarrow x = -1$
$2x = 0 \Rightarrow x = 0$
$2x = 3 \Rightarrow x = \frac{3}{2}$
$\boxed{x = -1,\ 0, \text{ and } \frac{3}{2}}$

(c) $k(x) = -5f(3x-2)$

$x = -2,\ 0,$ and 3
horizontal dilation $\frac{1}{3}$
shifted right $\frac{2}{3}$
$3x - 2 = -2 \Rightarrow x = 0$
$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$
$3x - 2 = 3 \Rightarrow x = \frac{5}{3}$
$\boxed{x = 0,\ \frac{2}{3}, \text{ and } \frac{5}{3}}$

30. The graph of $f(x)$ has the vertical asymptote $x = -2$ and horizontal asymptote $y = 3$. Find the vertical and horizontal asymptotes of the following functions.

(a) $g(x) = 2f(x+1) - 3$

$x + 1 = -2 \Rightarrow x = -3$
vertical asymptote $x = -3$
$2(y = 3) - 3 = 3$
horizontal asymptote $y = 3$

(b) $h(x) = 4 - 3f\left(\frac{x}{5}\right)$

$\frac{x}{5} = -2 \Rightarrow x = -10$
vertical asymptote $x = -10$
$4 - 3(y = 3) = -5$
horizontal asymptote $y = -5$

(c) $k(x) = \frac{1}{2}f(4-2x) + 3$

$4 - 2x = -2 \Rightarrow x = 3$
vertical asymptote $x = 3$
$\frac{1}{2}(y = 3) + 3 = \frac{3}{2} + 3 = \frac{9}{2}$
horizontal asymptote $y = \frac{9}{2}$

31. The graph of $f(x)$ is continuous where $\lim\limits_{x\to-\infty} f(x) = 4$ and $\lim\limits_{x\to\infty} f(x) = -\infty$.

(a) If $g(x) = -2f(x+7) + 5$, find $\lim\limits_{x\to-\infty} g(x)$ and $\lim\limits_{x\to\infty} g(x)$.

$\lim\limits_{x\to-\infty} g(x) = \lim\limits_{x\to-\infty} [-2f(x+7)+5] = -2 \lim\limits_{x+7\to-\infty} [f(x+7)] + 5 = -2(4) + 5 = -3$
$\lim\limits_{x\to\infty} g(x) = \lim\limits_{x\to\infty} [-2f(x+7)+5] = -2 \lim\limits_{x+7\to\infty} [(x+7)] + 5 = -2(-\infty) = \infty$

(b) If $h(x) = -f(-x)$, find $\lim\limits_{x\to-\infty} h(x)$ and $\lim\limits_{x\to\infty} h(x)$.

$\lim\limits_{x\to-\infty} h(x) = -\lim\limits_{-x\to\infty} f(-x) = -(-\infty) = \infty$

$\lim\limits_{x\to\infty} h(x) = -\lim\limits_{-x\to-\infty} f(-x) = -4$

32. The graph of $f(x)$ has the vertical asymptote $x = 5$ and horizontal asymptote $y = -3$. Find the vertical and horizontal asymptotes of the following functions that result from transforming the graph of f.

(a) The graph of g results from applying the following transformations to graph of f in this order: vertical dilation by a factor of 3, reflection over the x axis, reflection over the y axis, and a horizontal translation by -3 units.

$\lim_{x\to 5} f(x) = \infty \qquad g(x) = -3f(-(x+3)) \qquad -(x+3) = 5 \Rightarrow x = -8$

$\lim_{x\to -8} g(x) = \lim_{x\to -8}[-3f(-(x+3))] = -3\lim_{x\to -8}[f(-(x+3))] = -3\lim_{u\to 5}\left[f\left(\underbrace{-(x+3)}_{u}\right)\right] = -3(\infty) = -\infty$

$\lim_{x\to\pm\infty} f(x) = -3 \qquad \lim_{x\to\pm\infty} g(x) = \lim_{x\to\pm\infty}[-3f(-(x+3))] = -3\lim_{x\to\pm\infty}[f(-(x+3))] = -3(-3) = 9$

$g(x)$ has a vertical asymptote $x = -8$ and a horizontal asymptote $y = 9$.

(b) The graph of h results from applying the following transformations to graph of f in this order: horizontal dilation by a factor of 2 and a horizontal translation by 4 units.

$h(x) = f\left(\frac{1}{2}(x-4)\right) \qquad \frac{1}{2}(x-4) = 5 \Rightarrow x - 4 = 10 \Rightarrow 14$

$\lim_{x\to 14} h(x) = \lim_{x\to 14} f\left(\underbrace{\frac{1}{2}(x-4)}_{u}\right) = \lim_{u\to 5}[f(u)] = \infty \quad h(x)$ has a vertical asymptote $x = 14$

$\lim_{x\to\infty} h(x) = \lim_{x\to\infty} f\left(\underbrace{\frac{1}{2}(x-4)}_{u}\right) = \lim_{u\to\infty}[f(u)] = -3 \quad h(x)$ has a horizontal asymptote $y = -3$

33. Let $f(x) = x^2 + 4x + 1$. Write an equation of the following functions that are transformations of $f(x)$.

(a) $g(x) = 3f(x) - 7$
$= 3(x^2 + 4x + 1) - 7$
$= (3x^2 + 12x + 3) - 7$
$= 3x^2 + 12x - 4$

(b) $h(x) = -2f(x) + 2$
$= -2(x^2 + 4x + 1) + 2$
$= (-2x^2 - 8x - 2) + 2$
$= -2x^2 - 8x$

(c) $k(x) = f(x-2)$
$= (x-2)^2 + 4(x-2) + 1$
$= (x^2 - 4x + 4) + 4x - 8 + 1$
$= x^2 - 3$

(d) $m(x) = f\left(\frac{x}{2}\right) + 3$
$= \left[\left(\frac{x}{2}\right)^2 + 4\left(\frac{x}{2}\right) + 1\right] + 3$
$= \frac{1}{4}x^2 + 2x + 4$

(e) $p(x) = 2f(x+1) - 5$
$= 2((x+1)^2 + 4(x+1) + 1) - 5$
$= 2(x^2 + 2x + 1 + 4x + 4 + 1) - 5$
$= 2(x^2 + 6x + 6) - 5$
$= 2x^2 + 12x + 12 - 5$
$= 2x^2 + 12x + 7$

(f) $s(x) = -f(-x)$
$= -[(-x)^2 + 4(-x) + 1]$
$= -[x^2 - 4x + 1]$
$= -x^2 + 4x - 1$

34. Let $f(x) = \dfrac{x-1}{(x+2)(x-3)}$. Write an equation of the following functions that are transformations of $f(x)$.

(a) $g(x) = f(x+4)$

$$= \frac{(x+4)-1}{\big((x+4)+2\big)\big((x+4)-3\big)}$$
$$= \frac{x+3}{(x+6)(x+1)}$$

(b) $h(x) = -2f\left(\dfrac{x}{3}\right)$

$$= -2\left(\frac{\left(\frac{x}{3}\right)-1}{\left(\left(\frac{x}{3}\right)+2\right)\left(\left(\frac{x}{3}\right)-3\right)}\right)$$
$$= -2\left(\frac{x-3}{(x+6)(x-9)}\right)$$
$$= \frac{6-2x}{(x+6)(x-9)}$$

(c) $k(x) = f(4-x)$

$$= \frac{(4-x)-1}{\big((4-x)+2\big)\big((4-x)-3\big)}$$
$$= \frac{3-x}{(6-x)(1-x)}$$

35. Let $f(x) = 2x^2 - 3$. Write an equation of the following functions that are transformations of $f(x)$.

(a) $g(x) = f(2x-3) + 2$

$= [2(2x-3)^2 - 3] + 2 = 2(4x^2 - 12x + 9) - 3 + 2$

$= (8x^2 - 24x + 18) - 1 = 8x^2 - 24x + 17$

(b) $h(x) = 4f(x) + 1$

$= 4[2x^2 - 3] + 1 = 8x^2 - 12 + 1 = 8x^2 - 11$

(c) $k(x)$ results when the graph of f has a horizontal dilation by a factor of 3, followed by a horizontal translation by -5 units, and a vertical translation by 2 units.

$$k(x) = f\left(\frac{1}{3}(x+5)\right) + 2 = \left[2\left(\frac{1}{3}(x+5)\right)^2 - 3\right] + 2 = \left[2\left(\frac{1}{9}(x^2+10x+25)\right) - 3\right] + 2$$
$$= \left(\frac{2}{9}(x^2+10x+25)\right) - 1 = \frac{2x^2+20x+50}{9} - 1 = \frac{2x^2+20x+50-9}{9} = \frac{2x^2+20x+41}{9}$$