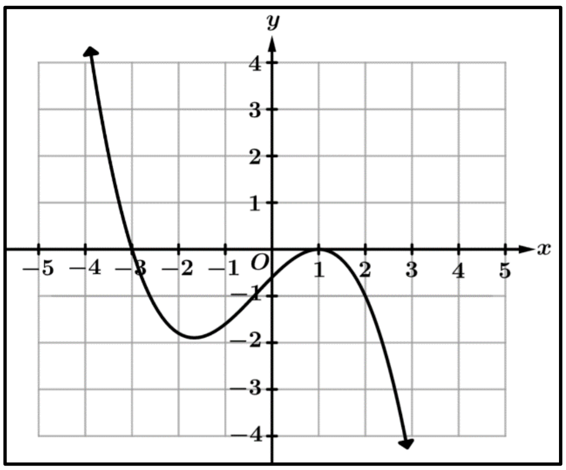
End Behavior: The end behavior of a function describes how a function ___behaves___ as it moves infinitely to the right and left.

In other words, the end behavior is what happens to the values of $f(x)$ as x increases or decreases without bound.

End Behavior and Limit Notation	
Left End Behavior: $\lim_{x\to-\infty} f(x)$	**Right End Behavior**: $\lim_{x\to\infty} f(x)$
As the $x-$values decrease without bound, the $y-$values of $f(x)$ ...	As the $x-$values increase without bound, the $y-$values of $f(x)$ ...

Example 1: Describe the end behavior of the following polynomials verbally and using limit notation.

a)



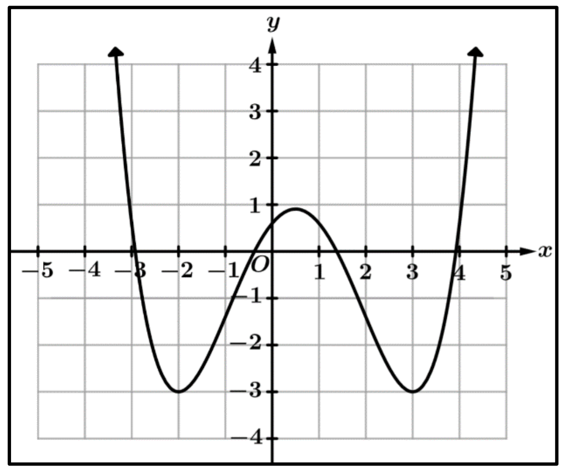
Left Behavior Verbally:
As the $x-$values decrease without bound, the $y-$values of $f(x)$ increase without bound.

Left Behavior Limit Statement:
$\lim_{x\to-\infty} f(x) = \infty$

Right Behavior Verbally:
As the $x-$values increase without bound, the $y-$values of $f(x)$ decrease without bound.

Right Behavior Limit Statement:
$\lim_{x\to\infty} f(x) = -\infty$

b)



Left Behavior Verbally:
As the $x-$values decrease without bound, the $y-$values of $f(x)$ increase without bound.

Left Behavior Limit Statement:
$\lim_{x\to-\infty} f(x) = \infty$

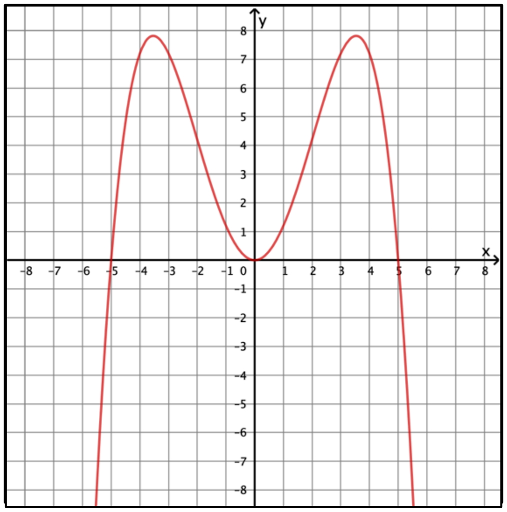
Right Behavior Verbally:
As the $x-$values increase without bound, the $y-$values of $f(x)$ increase without bound.

Right Behavior Limit Statement:
$\lim_{x\to\infty} f(x) = \infty$

Example 2: Sketch a polynomial function with the following end behaviors. Sketches will vary.

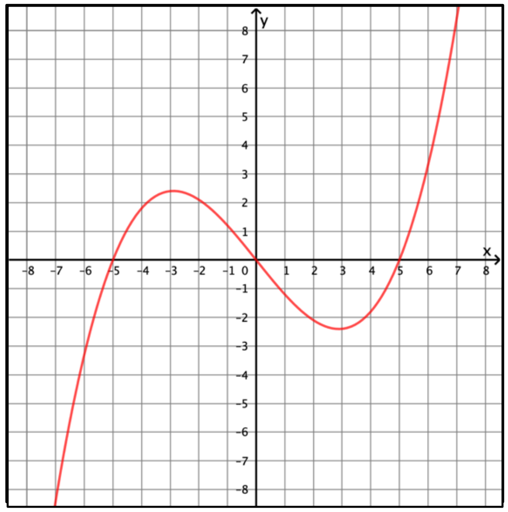
a) $\lim_{x\to-\infty} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = -\infty$



b) $\lim_{x\to-\infty} g(x) = -\infty$

$\lim_{x\to\infty} g(x) = +\infty$



Polynomial End Behavior

For polynomial equations, it is easiest to find the end behavior of the ___right___ side first.

The **RIGHT** side:

1. Goes __to ∞__ if the leading coefficient is ___positive___.
2. Goes __to $-\infty$__ if the leading coefficient is ___negative___.

The **LEFT** side:

1. Goes in the ___same___ direction as the right if the degree is __even__.
2. Goes in the ___opposite___ direction as the right if the degree is __odd__.

Example 3: Determine the end behavior for the following polynomials. Limit notation is not necessary.

a) $f(x) = 4x^5$

L: $-\infty$ R: ∞

Odd degree, positive leading coefficient

b) $g(x) = \frac{1}{2}x^4$

L: ∞ R: ∞

even degree, positive leading coefficient

c) $y = -2(x+3)^6$

L: $-\infty$ R: $-\infty$

even degree, negative leading coefficient

d) $h(x) = 3 - x^5$

L: ∞ R: $-\infty$

Odd degree, negative leading coefficient

e) $k(x) = 8x^2 + 4 - x^5$

L: ∞ R: $-\infty$

odd degree, negative leading coefficient

f) $m(x) = 2x(x-1)(6-x)$

L: ∞ R: $-\infty$

odd degree, negative leading coefficient

Example 4: Write limit statements for the end behavior of the following polynomials.

a) $f(x) = -3x^4$

$\lim_{x\to-\infty} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = -\infty$

b) $g(x) = 5x^3 + 2x^2 - 7$

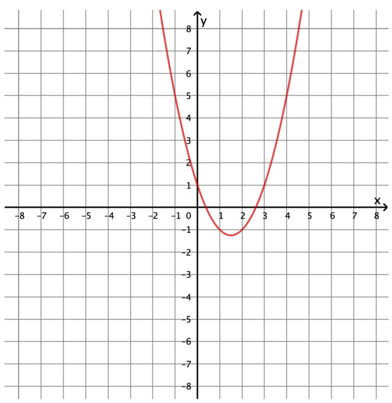
$\lim_{x\to-\infty} g(x) = -\infty$

$\lim_{x\to\infty} g(x) = \infty$

Example 5: Use a graphing calculator to determine the end behavior for the following functions.

a) $f(x) = x^2 - 3x + 1$ b) $g(x) = \dfrac{2x-3}{x+1}$ c) $h(x) = -\dfrac{10}{x^2}$

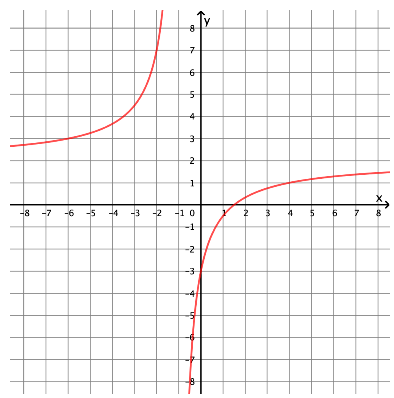
a) $f(x) = x^2 - 3x + 1$



$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = \infty$

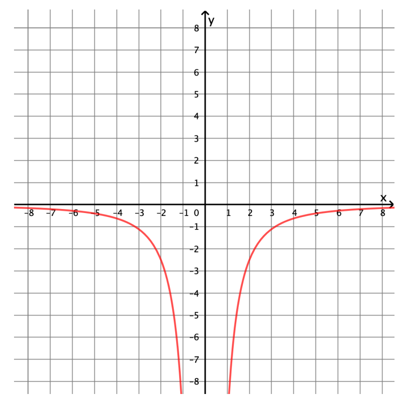
b) $g(x) = \dfrac{2x-3}{x+1}$



$\lim_{x\to-\infty} g(x) = 2$

$\lim_{x\to\infty} g(x) = 2$

c) $h(x) = -\dfrac{10}{x^2}$



$\lim_{x\to-\infty} h(x) = 0$

$\lim_{x\to\infty} h(x) = 0$