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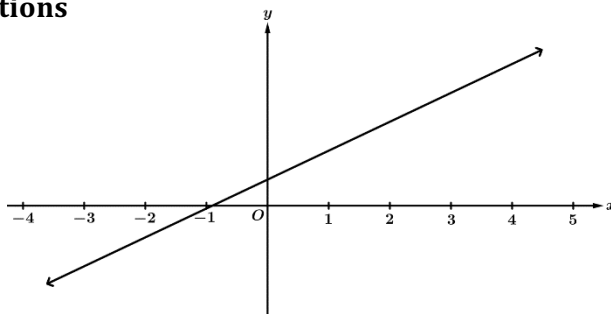**Linear Functions**

For any linear function, the average rate of change over any length input − value interval is constant.

**Note**: We often think of this as the ___slope___ of the line.



| $x$ | $f(x)$ |
|-----|--------|
| 1 | 4 |
| 2 | 7 |
| 4 | 10 |
| 8 | 13 |

**Example 1**:  The table above gives selected values of the function $f(x)$.  Explain why $f(x)$ is not a linear function.

$f(x)$ is not a linear function because the rate of change (slope) is not constant.

On [1,2]: slope $= \dfrac{7-4}{2-1} = 3$    On [2,4]: slope $= \dfrac{10-7}{4-2} = \dfrac{3}{2}$    On [4,8]: slope $= \dfrac{13-10}{8-4} = \dfrac{3}{4}$

**Example 2**:  Consider the quadratic function  $g(x) = x^2$.  Complete the table of values for $g(x)$ over the consecutive equal length input value intervals below.  Then complete the table for the average rates of change of $g(x)$ for each consecutive interval of equal length input values.

| $x$ | $g(x)$ |
|-----|--------|
| −3 | 9 |
| −1 | 1 |
| 1 | 1 |
| 3 | 9 |
| 5 | 25 |

$\dfrac{1-9}{-1-^-3} = -\dfrac{8}{2} = -4$

$\dfrac{9-1}{3-1} = \dfrac{8}{2} = 4$

| Interval | Avg. rate of change |
|----------|---------------------|
| [−3, −1] | −4 |
| [−1, 1] | 0 |
| [1, 3] | 4 |
| [3, 5] | 8 |

$\dfrac{1-1}{1-^-1} = \dfrac{0}{2} = 0$

$\dfrac{25-9}{5-2} = \dfrac{16}{2} = 8$

What do you notice about the average rates of change of $g(x)$ over consecutive equal length intervals?

The average rates of change of $g(x)$ over consecutive intervals of length 2 are increasing a constant rate of 4, so rates of change of $g(x)$ are given by a linear function over consecutive intervals of length 2.

**Example 3**: Selected values of various functions are given in the tables below. For each table, determine if the function could be linear, quadratic, or neither.

a)

| $x$ | $f(x)$ |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 3 | 4 |
| 4 | 9 |

$\frac{1}{1}=1$

$\frac{3}{1}=3$

$\frac{5}{1}=5$

Quadratic: the average rates of change form a linear pattern.

b)

| $x$ | $g(x)$ |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 5 | 4 |
| 10 | 9 |

$\frac{1}{1}=1$

$\frac{3}{3}=1$

$\frac{5}{5}=1$

Linear: the average rates of change are constant.

c)

| $x$ | $h(x)$ |
|---|---|
| 1 | −1 |
| 3 | 1 |
| 5 | 2 |
| 7 | 2 |

$\frac{2}{2}=1$

$\frac{1}{2}$

$\frac{0}{2}=0$

Quadratic: the average rates of change decrease at a constant rate of $\frac{1}{2}$.

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## More On Concavity

**Concave Up**: The average rate of change over equal length input value intervals is __increasing__ for all small length intervals.

**Concave Down**: The average rate of change over equal length input value intervals is ____decreasing____ for all small length intervals.

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**Example 4**: Selected values of the functions $k$, $m$, and $p$ are given in the tables below. For each function, determine if the function could be concave up, concave down, or neither over its domain.

a)

| $x$ | $k(x)$ |
|---|---|
| 1 | 4 |
| 1.1 | 1 |
| 1.2 | −1 |
| 1.3 | −2 |

$\frac{-3}{0.1}=-30$

$\frac{-2}{0.1}=-20$

$\frac{-1}{0.1}=-10$

Concave up because the rates of change are increasing.

b)

| $x$ | $m(x)$ |
|---|---|
| 1 | 1 |
| 1.1 | 4 |
| 1.2 | 7 |
| 1.3 | 10 |

$\frac{3}{0.1}=30$

$\frac{3}{0.1}=30$

$\frac{3}{0.1}=30$

Neither because the rate of change is constant.

c)

| $x$ | $p(x)$ |
|---|---|
| 1 | 1 |
| 1.1 | 7 |
| 1.2 | 11 |
| 1.3 | 13 |

$\frac{6}{0.1}=60$

$\frac{4}{0.1}=40$

$\frac{2}{0.1}=20$

Concave down because the rates of change are decreasing.