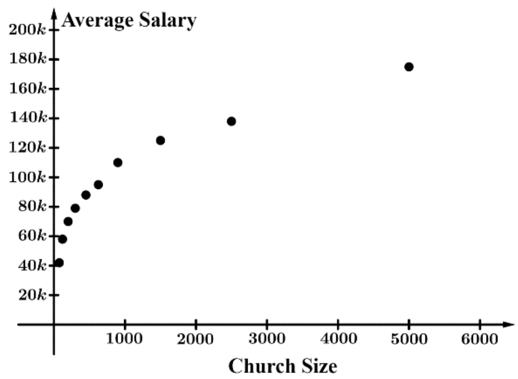


The average salary for a church pastor across the US increases as the size of the church increases. The graph above shows the average pastor salaries for various church sizes.

We can clearly see a pattern in the data, but the models we have used previously are not appropriate for this data. The grow above displays a large rate of change initially, but as the church size increases, we see the rate of change gradually decrease. The data above appears to be logarithmic.

Just as we have seen with other types of functions (linear, quadratic, exponential, …), many real-world scenarios follow a logarithmic pattern. Outside the salary data in the graph above, logarithmic patterns are found in the way we measure earthquakes, sound, acidity, and many other contexts. As a result, we have a need to create logarithmic models for these types of data.

Logarithmic Function Models

For a given set of data, if the input-values appear to change proportionally over equal-length output-value intervals, then a logarithmic model is appropriate.

A logarithmic function model has the form $y = a + b\ln x$ or $y = a + b\log x$, where a and b are constants and $b \neq 0$.

x	$L(x)$
2	3
5	7

Example 1: Selected values of the logarithmic function L are given in the table above, where $L(x) = a + b\ln x$.

a) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $L(x)$.

$L(2) = a + b\ln 2 = 3$ $\qquad$ $L(5) = a + b\ln 5 = 7$

b) Find the values of a and b.

$$\begin{array}{r} a + b\ln 5 = 7 \\ \underline{-a - b\ln 2 = -3} \\ b\ln 5 - b\ln 2 = 4 \end{array}$$

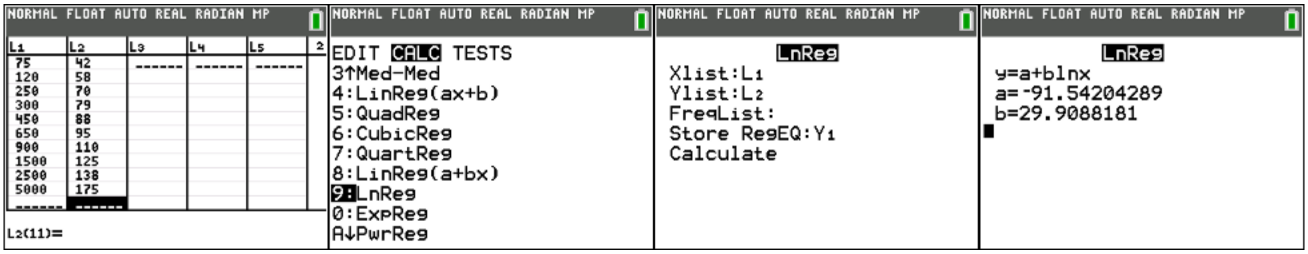
$$b(\ln 5 - \ln 2) = 4$$
$$b = \frac{4}{\ln 5 - \ln 2} = 4.3654\ldots$$
$$a = 3 - b\ln 2 = -0.0258\ldots$$

Solutions by Ted Gott tedg20776@gmail.com

Church Size	75	120	250	300	450	650	900	1500	2500	5000
Avg. Salary (in 1000's of dollars)	42	58	70	79	88	95	110	125	138	175

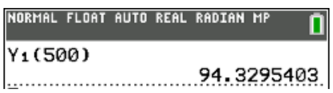
Example 2: The table above gives the data from the graph at the beginning of the notes related church size and average salaries for pastors in the United States.

a) Use the regression capabilities of your graphing calculator to find a logarithmic function model for the data above of the form $S(p) = a + b\ln p$, where p represents the church size and S is the average pastor salary in thousands of dollars.



$S(p) = a + b\ln p$ where the values for a and b have been stored in the calculator.

b) Using the model found in part a, what is the predicted annual salary, in thousands of dollars, for a pastor whose church size is 500 people?



$S(500) = 94.3295 \ldots$ thousands of dollars

Example 3: The most common, and preferred, method to measure sound intensity levels is in decibels (dB). The sound intensity level β (in decibels) of a sound having intensity I (in watts per meter squared) is modeled by the function

$$\beta(I) = a + b\log(I),$$

where a and b are constants.

The sounds of noisy traffic has intensity 1×10^{-5} watts per meter squared and corresponds to 70 decibels. A loud concert has intensity of 1 and corresponds to 120 decibels.

a) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $\beta(I)$.

$\beta(1\times10^{-5}) = a + b\log(1\times10^{-5}) = a + b\cdot(-5) = 70$

$\beta(1) = a + b\log(1) = a + b\cdot(0) = 120 \Rightarrow a = 120$

Fun Fact

The word decibel comes from two pieces:

1. deci- refers to the fact that the level of sound found is multiplied by 10 as part of the formula.

2. -bel references Alexander Graham Bell, the inventor of the telephone and the person decibels are named after.

b) Find the values of a and b.

$a = 120 \qquad 120 - 5b = 70 \Rightarrow b = \dfrac{70-120}{-5} = 10$

c) The human eardrum can burst when sounds reach an intensity level of 1×10^{4} watts per meter squared. According to the model found above, what is the predicted number of decibels required to burst a human eardrum?

$\beta(1\times10^{4}) = 120 + 10\log(1\times10^{4}) = 120 + (10)(4) = 160$ decibels