**Graph of f**

1. A portion of the graph of the function f is shown in the xy -plane, where f is the result of multiplicative transformation of the graph of $y = \tan x$. What is the period of f ?

The period is from $-\pi$ to π or 2π .

2. The function h is given by $h(x) = 7 - 3 \tan(4x)$. What is the period of h ?

The period = $\left| \frac{\pi}{b} \right| = \frac{\pi}{4}$

3. The function g is given by $g(\theta) = \tan\left(\frac{\theta}{3}\right)$. What is the period of g ?

The period = $\left| \frac{\pi}{b} \right| = \frac{\pi}{1/3} = 3\pi$

4. The function k is given by $k(\theta) = \tan(\pi\theta)$. What is the period of k ?

The period = $\left| \frac{\pi}{b} \right| = \frac{\pi}{\pi} = 1$

5. The function p is given by $p(\theta) = \tan\left(\frac{\pi\theta}{4}\right)$. What is the period of p ?

The period = $\left| \frac{\pi}{b} \right| = \frac{\pi}{\pi/4} = 4$

6. The function j is given by $j(\theta) = \tan(2\theta)$. Which of the following gives the vertical asymptotes to the graph of j ?

- (A) $\frac{\pi}{2} + \pi k$, where k is an integer
- (B) $\pi + 2\pi k$, where k is an integer
- (C) $\frac{\pi}{4} + \frac{\pi}{2}k$, where k is an integer
- (D) $\frac{\pi}{4} + \pi k$, where k is an integer

$$y = \tan \theta \text{ has vertical asymptotes } \theta = \frac{\pi}{2} + \pi k$$

$$y = \tan(2\theta) \text{ has vertical asymptotes } 2\theta = \frac{\pi}{2} + \pi k \Rightarrow \theta = \frac{\pi}{4} + \frac{\pi}{2}k$$

7. The function f is given by $f(\theta) = \tan\left(\frac{1}{4}\theta\right)$. Which of the following gives the vertical asymptotes to the graph of f ?

- (A) $\frac{\pi}{8} + \frac{\pi}{4}k$, where k is an integer
- (B) $\frac{\pi}{2} + \pi k$, where k is an integer
- (C) $2\pi + 4\pi k$, where k is an integer
- (D) $4\pi + 8\pi k$, where k is an integer

$$y = \tan \theta \text{ has vertical asymptotes } \theta = \frac{\pi}{2} + \pi k$$

$$y = \tan\left(\frac{1}{4}\theta\right) \text{ has vertical asymptotes } \frac{1}{4}\theta = \frac{\pi}{2} + \pi k \Rightarrow \theta = 2\pi + 4\pi k$$

8. The function g is given by $g(\theta) = \tan(\pi\theta)$. Which of the following gives the vertical asymptotes to the graph of g ?

- (A) $\frac{\pi}{2} + \pi k$, where k is an integer
- (B) $\pi + 2\pi k$, where k is an integer
- (C) $\frac{1}{2} + k$, where k is an integer
- (D) $\frac{1}{2} + 2k$, where k is an integer

$$y = \tan \theta \text{ has vertical asymptotes } \theta = \frac{\pi}{2} + \pi k$$

$$y = \tan(\pi\theta) \text{ has vertical asymptotes } \pi\theta = \frac{\pi}{2} + \pi k \Rightarrow \theta = \frac{1}{2} + k$$

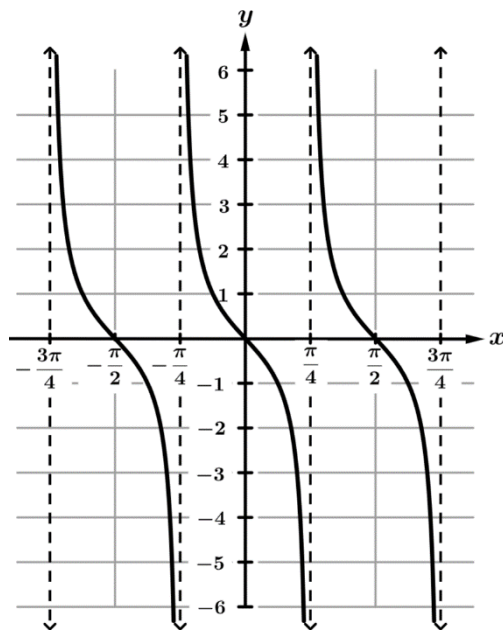
9. The function h is given by $h(\theta) = \tan\left(\theta - \frac{\pi}{2}\right)$. Which of the following gives the vertical asymptotes to the graph of h ?

- (A) $\frac{\pi}{2} + \pi k$, where k is an integer
- (B) $\frac{\pi}{2} + \frac{3\pi}{2}k$, where k is an integer
- (C) πk , where k is an integer
- (D) $\pi + \frac{3\pi}{2}k$, where k is an integer

$$y = \tan \theta \text{ has vertical asymptotes } \theta = \frac{\pi}{2} + \pi k$$

$$y = \tan\left(\theta - \frac{\pi}{2}\right) \text{ has vertical asymptotes}$$

$$\theta - \frac{\pi}{2} = \frac{\pi}{2} + \pi k \quad \theta = \frac{\pi}{2} + \frac{\pi}{2} + \pi k = \pi + \pi k = \pi k$$

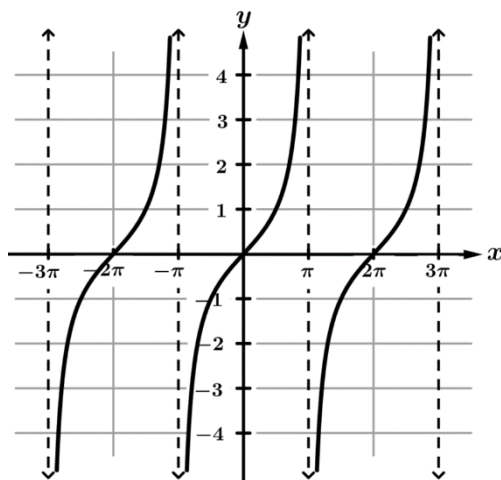


Graph of f

10. The graph of the function f is given in the xy -plane. If $f(x) = a \tan(bx)$, where a and b are constants, which of the following is true?

- (A) $a > 0$ and $b > 1$ (B) $a > 0$ and $0 < b < 1$ (C) $a < 0$ and $b > 1$ (D) $a < 0$ and $b < 1$

The period = $\left| \frac{\pi}{b} \right| = \frac{\pi}{2} \Rightarrow b = 2 > 1$ $a < 0$ because the tangent graph is reflected over the x -axis.

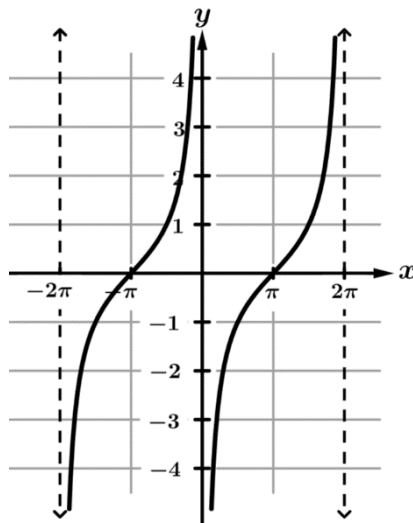


Graph of g

11. The graph of the function g is given in the xy -plane. If $g(x) = a \tan(bx)$, where a and b are constants, which of the following is true?

- (A) $a > 0$ and $b > 1$ (B) $a > 0$ and $b < 1$ (C) $a < 0$ and $b > 1$ (D) $a < 0$ and $b < 1$

The period = $\left| \frac{\pi}{b} \right| = 2\pi \Rightarrow b = \frac{1}{2} < 1$ $a > 0$ because the tangent graph is not reflected over the x -axis.

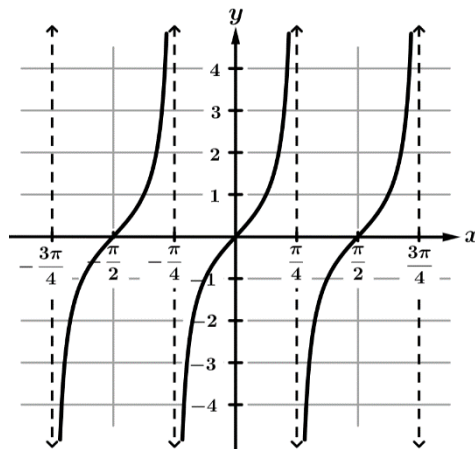


Graph of h

12. The graph of the function h is given in the xy -plane. If $h(x) = \tan(b(x+c))$, where b and c are constants, what are the values of b and c ?

- (A) $b = \frac{1}{2}$ and $c = -\frac{\pi}{2}$ (B) $b = \frac{1}{2}$ and $c = -\pi$ (C) $b = 2$ and $c = -\frac{\pi}{2}$ (D) $b = 2$ and $c = -\pi$

The period = $\left|\frac{\pi}{b}\right| = 2\pi \Rightarrow b = \frac{1}{2}$ There is a shift to the right of π .



Graph of k

13. The graph of the function k is given in the xy -plane. Which of the following could be the expression for $k(x)$?

- (A) $k(x) = -\tan(4x)$ (B) $k(x) = \tan\left(\frac{x}{2}\right)$ (C) $k(x) = \tan(2x)$ (D) $k(x) = \tan\left(x + \frac{\pi}{4}\right)$

The period = $\left|\frac{\pi}{b}\right| = \frac{\pi}{2} \Rightarrow b = 2$ There is no shift.

14. In the xy -plane, an angle θ is in standard position. The slope of the terminal ray of angle θ is -1 . Which of the following could be the measure of angle θ ?

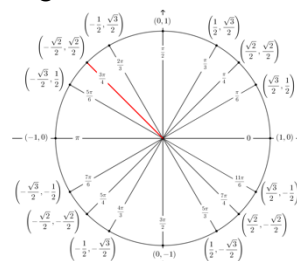
(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$

(D) $\frac{3\pi}{2}$

$$\tan \theta = \frac{y}{x} = -1 \Rightarrow y = -x \quad \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$



15. In the xy -plane, an angle θ is in standard position. The slope of the terminal ray of angle θ is undefined. Which of the following could be the measure of angle θ ?

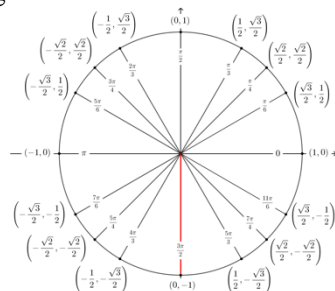
(A) $\frac{\pi}{4}$

(B) π

(C) $\frac{3\pi}{2}$

(D) 2π

$$\tan \theta = \frac{y}{x} = \infty \Rightarrow x = 0 \quad (0, -1)$$



16. In the xy -plane, an angle θ is in standard position. The slope of the terminal ray of angle θ is $\sqrt{3}$. Which of the following could be the measure of angle θ ?

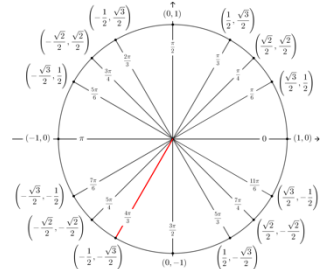
(A) $\frac{\pi}{6}$

(B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{3}$

(D) $\frac{11\pi}{6}$

$$\tan \theta = \frac{y}{x} = \sqrt{3} \Rightarrow y = \sqrt{3}x \quad \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$



17. In the xy -plane, an angle θ is in standard position. The slope of the terminal ray of angle θ is 0 . Which of the following could be the measure of angle θ ?

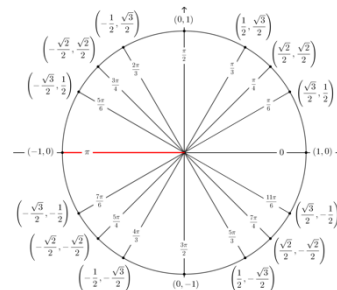
(A) $\frac{\pi}{2}$

(B) $\frac{3\pi}{4}$

(C) π

(D) $\frac{3\pi}{2}$

$$\tan \theta = \frac{y}{x} = 0 \Rightarrow y = 0 \quad (-1, 0)$$



18. In the xy -plane, an angle θ is in standard position with a measure of $\theta = \frac{\pi}{6}$. Which of the following is true?

(A) The terminal ray of the angle θ has a slope of $\frac{1}{2}$.

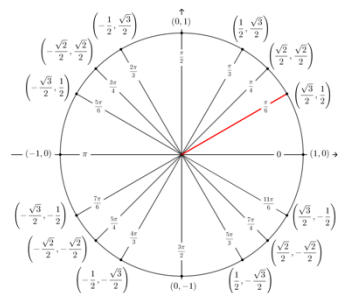
(B) The terminal ray of the angle θ has a slope of $\frac{1}{\sqrt{3}}$.

(C) The terminal ray of the angle θ has a slope of $\frac{\sqrt{3}}{2}$.

(D) The terminal ray of the angle θ has a slope of $\sqrt{3}$.

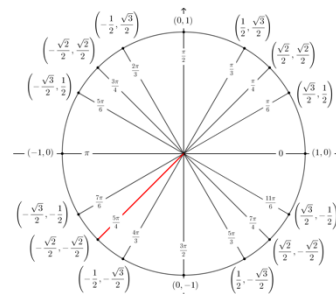
$$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\tan \theta = \frac{y}{x} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$



19. In the xy -plane, an angle θ is in standard position with a measure of $\theta = \frac{5\pi}{4}$. Which of the following is true?

- (A) The terminal ray of the angle θ has a slope of $-\frac{\sqrt{2}}{2}$. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
- (B) The terminal ray of the angle θ has a slope of $\frac{\sqrt{2}}{2}$. $\tan \theta = \frac{y}{x} = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} = -1$
- (C) The terminal ray of the angle θ has a slope of -1 .
- (D) The terminal ray of the angle θ has a slope of 1 .

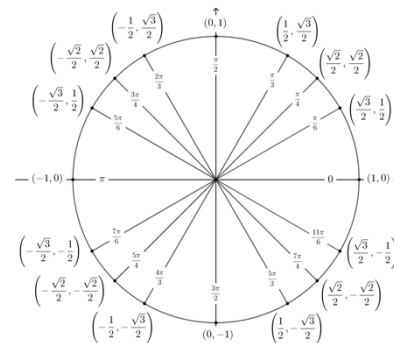


Directions: Find the exact values of the following expressions if possible.

$$20. \tan\left(\frac{7\pi}{6}\right) = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$21. \tan\left(\frac{3\pi}{4}\right) = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$$

$$22. \tan\left(\frac{2\pi}{3}\right) = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$$



$$23. \tan\left(\frac{5\pi}{3}\right) = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

$$24. \tan\left(\frac{11\pi}{6}\right) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$25. \tan(2\pi) = \frac{0}{1} = 0$$

$$26. \tan\left(-\frac{\pi}{2}\right) = \frac{-1}{0} = \infty \text{ DNE}$$

$$27. \tan\left(\frac{5\pi}{4}\right) = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$$

$$28. \tan\left(\frac{7\pi}{3}\right) = \tan\left(2\pi + \frac{\pi}{3}\right) = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$