

## Notes: (Topic 1.8) Rational Functions and Zeros [Solutions](#)

**Recall:** A rational function is a function that is a ratio of two polynomial functions (think fractions).

$$f(x) = \frac{2x - 3}{x^2 - 4x - 45}$$

We explored polynomial inequalities previously (Topic 1.5), and we can utilize some of those concepts to help us understand more about rational functions like  $f(x)$ .

### Two Important Traits of a Rational Function

Let  $f(x) = \frac{g(x)}{h(x)}$  be a rational function where  $g(x)$  and  $h(x)$  have no factors in common. Then, we know ...

1.  $f(x)$  has zeros when  $g(x) = 0$ .
2.  $f(x)$  is undefined when  $h(x) = 0$ .

**Note:** When solving rational inequalities, we need to identify **BOTH** the **zeros** and **undefined values**!

### Solving Rational Inequalities

1. Make sure the inequality has **0** on the other side!
2. Make sure  $f(x) = \frac{g(x)}{h(x)}$  (Make sure you have a single rational function)
3. Set  $g(x) = 0$  and  $h(x) = 0$  to find values to include on the sign chart. (Make sure to factor!)
4. Create a sign chart with all values from **Step 3**.
5. Be careful to mark the values where  $h(x) = 0$  so that we **NEVER** include those values in our solution.
6. **Test values** in each interval to see if the values in the interval are + or -.
7. **Interpret** the sign chart to answer the given inequality from the problem.

**NOTE:** Be sure to write your answer in **interval notation** and think about the **endpoints**!

**Example 1:** Solve  $\frac{x - 2}{(x + 6)(x - 3)} \geq 0$

$$\begin{array}{ccccccc} & \frac{-}{-} & \frac{-}{+} & \frac{+}{-} & \frac{++}{+} \\ < - & \frac{\cap}{\cup} & - & \frac{\cap}{\cup} & - & \frac{\cap}{\cup} & - \rightarrow \\ & -6 & 2 & 3 & & & \end{array}$$

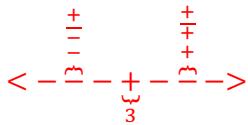
$(-6, 2]$  and  $(3, \infty)$  or  $-6 < x \leq 2$  and  $x > 3$

**Example 2:** Solve  $\frac{x^2 - 4}{x^2 - 10x + 25} < 0$

$$\begin{array}{ccccccc} \frac{x^2 - 4}{x^2 - 10x + 25} & = & \frac{(x + 2)(x - 2)}{(x - 5)(x - 5)} \\ & & \frac{\frac{x^2 - 4}{-}}{\frac{x^2 - 10x + 25}{--}} & \frac{+}{-} & \frac{++}{+} & \frac{++}{+} \\ & & & \frac{\cap}{\cup} & - & \frac{\cap}{\cup} & - \rightarrow \\ & & & -2 & 2 & 5 & \end{array}$$

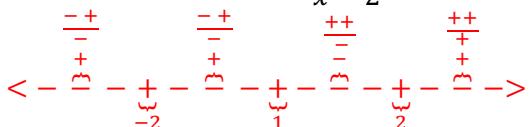
$(-2, 2)$  or  $-2 < x < 2$

**Example 3:** Solve  $\frac{2}{x-3} > 0$



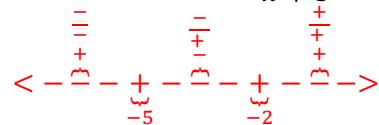
$(3, \infty)$  or  $x > 3$

**Example 5:** Solve  $\frac{(x-1)(x+2)^2}{x-2} \geq 0$



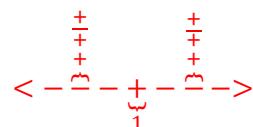
$(-\infty, 1]$  and  $(2, \infty)$  or  $x \leq 1$  and  $x > 2$

**Example 4:** Solve  $\frac{4x+8}{x+5} \leq 0$

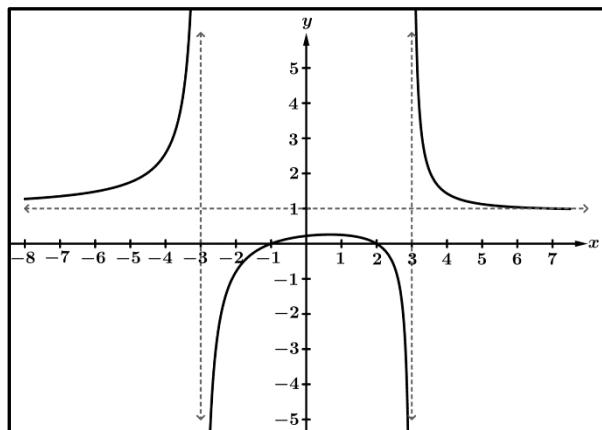


$(-5, -2]$  or  $-5 < x \leq -2$

**Example 6:** Solve  $\frac{1}{(x-1)^2} \leq 0$



Never  $\leq 0$



**Example 7:** The graph of the rational function  $f$  is shown in the figure above. Use the graph to solve the following inequalities.

a)  $f(x) \leq 0$

$(-3, -1]$  and  $[2, 3)$  or  
 $-3 < x \leq -1$  and  $2 \leq x < 3$

b)  $f(x) > 0$

$(-\infty, -3)$  and  $(-1, 2)$  and  $(3, \infty)$   
 $x < -3, -1 < x < 2$  and  $x > 3$

c)  $f(x) \geq 1$

$(-\infty, -3)$  and  $(3, \infty)$   
 $x < -3$  and  $x > 3$