

Recall: A rational function is a function that is a ratio of two polynomial functions (think fractions).

$$f(x) = \frac{2x - 3}{x^2 - 4x - 45}$$

We explored polynomial inequalities previously (Topic 1.5), and we can utilize some of those concepts to help us understand more about rational functions like $f(x)$.

Two Important Traits of a Rational Function

Let $f(x) = \frac{g(x)}{h(x)}$ be a rational function where $g(x)$ and $h(x)$ have no factors in common. Then, we know ...

1. $f(x)$ has zeros when $g(x) = 0$.
2. $f(x)$ is undefined when $h(x) = 0$.

Note: When solving rational inequalities, we need to identify **BOTH** the **zeros** and **undefined values**!

Solving Rational Inequalities

1. Make sure the inequality has **0** on the other side!
2. Make sure $f(x) = \frac{g(x)}{h(x)}$ (Make sure you have a single rational function)
3. Set $g(x) = 0$ and $h(x) = 0$ to find values to include on the sign chart. (Make sure to factor!)
4. Create a sign chart with all values from **Step 3**.
5. Be careful to mark the values where $h(x) = 0$ so that we **NEVER** include those values in our solution.
6. **Test values** in each interval to see if the values in the interval are + or -.
7. **Interpret** the sign chart to answer the given inequality from the problem.

NOTE: Be sure to write your answer in **interval notation** and think about the **endpoints**!

Example 1: Solve $\frac{x - 2}{(x + 6)(x - 3)} \geq 0$

$$\begin{array}{ccccccc} & \frac{-}{-} & & \frac{+}{+} & & \frac{+}{-} & & \frac{+}{+} \\ & \frac{-}{-} & & \frac{+}{+} & & \frac{+}{-} & & \frac{+}{+} \\ < - & \frac{-}{-} & - & \frac{+}{+} & - & \frac{+}{-} & - & \frac{+}{+} & - > \\ & \underbrace{-6} & & \underbrace{2} & & \underbrace{3} & & & \end{array}$$

$(-6, 2]$ and $(3, \infty)$ or $-6 < x \leq 2$ and $x > 3$

Example 2: Solve $\frac{x^2 - 4}{x^2 - 10x + 25} < 0$

$$\begin{array}{ccccccc} & \frac{-}{-} & & \frac{+}{+} & & \frac{+}{-} & & \frac{+}{+} \\ & \frac{-}{-} & & \frac{+}{+} & & \frac{+}{-} & & \frac{+}{+} \\ < - & \frac{-}{-} & - & \frac{+}{+} & - & \frac{+}{-} & - & \frac{+}{+} & - > \\ & \underbrace{-2} & & \underbrace{2} & & \underbrace{5} & & & \end{array}$$

$(-2, 2)$ or $-2 < x < 2$

Example 3: Solve $\frac{2}{x-3} > 0$

$$< \overbrace{-}^{+} \overbrace{-}^{+} \underbrace{-}_{-3} \overbrace{-}^{+} \overbrace{-}^{+} >$$

$(3, \infty)$ or $x > 3$

Example 4: Solve $\frac{4x+8}{x+5} \leq 0$

$$< \overbrace{-}^{+} \overbrace{-}^{+} \underbrace{-}_{-5} \underbrace{-}_{-2} \overbrace{-}^{+} \overbrace{-}^{+} >$$

$(-5, -2]$ or $-5 < x \leq -2$

Example 5: Solve $\frac{(x-1)(x+2)^2}{x-2} \geq 0$

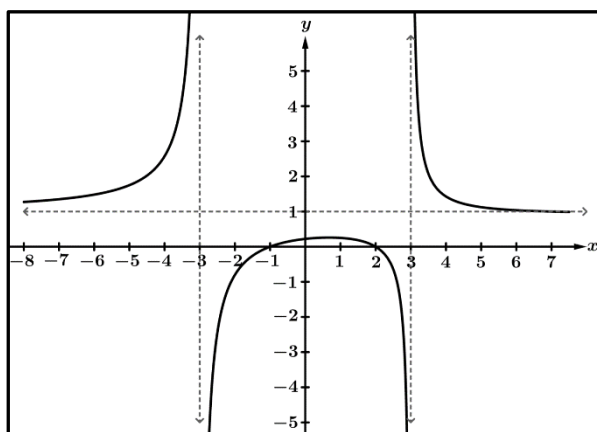
$$< \overbrace{-}^{++} \overbrace{-}^{++} \underbrace{-}_{-2} \underbrace{-}_{-1} \underbrace{-}_{-1} \underbrace{-}_{-2} \overbrace{-}^{++} \overbrace{-}^{++} >$$

$(-\infty, 1]$ and $(2, \infty)$ or $x \leq 1$ and $x > 2$

Example 6: Solve $\frac{1}{(x-1)^2} \leq 0$

$$< \overbrace{-}^{+} \overbrace{-}^{+} \underbrace{-}_{-1} \overbrace{-}^{+} \overbrace{-}^{+} >$$

Never ≤ 0



Example 7: The graph of the rational function f is shown in the figure above. Use the graph to solve the following inequalities.

a) $f(x) \leq 0$

$(-3, -1]$ and $[2, 3)$ or
 $-3 < x \leq -1$ and $2 \leq x < 3$

b) $f(x) > 0$

$(-\infty, -3)$ and $(-1, 2)$ and $(3, \infty)$
 $x < -3$, $-1 < x < 2$ and $x > 3$

c) $f(x) \geq 1$

$(-\infty, -3)$ and $(3, \infty)$
 $x < -3$ and $x > 3$