Notes: (Topic 2.9) Logarithmic Expressions Solutions

Important Concept: Every operation has **exactly one** inverse (opposite) operation. We learn inverse operations because these will allow us to solve equations involving various operations.

Operation	**Example**	**Inverse Operation**
Addition	$x + 3 = 7$	Subtraction
Multiplication	$3x = 7$	Division
Cubing	$x^3 = 7$	Cube Root
Exponentiation	$3^x = 7$	????

Recently, we learned about exponential functions and their properties. We would like to solve equations involving exponential functions, which creates the need of an inverse operation related to exponentiation.

Big Idea: Just as we can write rational numbers in decimal form and fraction form $\left(0.5 = \frac{1}{2}\right)$, we can also write equations in __exponential__ or __logarithmic__ form. These forms create equivalent statements that represent the same claim. We are not changing the problem when we convert back and forth, we are simply ***re-writing*** the equation in an equivalent form.

Exponential Form	**Logarithmic Form**	**Notes**
$b^a = c$	$\log_b c = a$	

Notation: If the base of a log expression is 10, this is called the "common logarithm" and is written without a base. For example, $\log 3$ is understood to have a base of 10 -- $\log 3 = \log_{10} 3$.

Example 1: Convert the following equations from exponential form to logarithmic (log) form.

a) $3^2 = 9$ b) $10^x = y$ c) $r^t = w$ d) $y = e^{3x}$

a) $\log_3 9 = 2$ b) $\log y = x$ c) $\log_r w = t$ d) $\ln y = 3x$

Example 2: Convert the following equations from logarithmic (log) form to exponential form.

a) $\log_7 x = 2$ b) $x = \log_3 y$ c) $\log y = 2$ d) $\ln c = 4$

a) $7^2 = x$ b) $3^x = y$ c) $10^2 = y$ d) $e^4 = c$

Evaluating Expressions Involving Logarithms

Sometimes, we can evaluate log expressions without the use of a calculator by utilizing its equivalent exponential form.

Example 3: Evaluate the following expressions without a calculator.

a) $\log_2 8 = 3$
$2^3 = 8$

b) $\log_5 25 = 2$
$5^2 = 25$

c) $\log_2 32 = 5$
$2^5 = 32$

d) $\log 100 = 2$
$10^2 = 100$

e) $\log_{16} 4 = \frac{1}{2}$
$2^3 = 8$

f) $\log_3 1 = 0$
$3^0 = 1$

g) $\log 10 = 1$
$10^1 = 10$

h) $\log_6 \frac{1}{36} = -2$
$6^{-2} = \frac{1}{36}$

Example 4: Use a calculator to evaluate the following expressions. Include at least four places after the decimal point.

a) $\log 9 = 0.9542 \ldots$

b) $\log 6125 = 3.7871 \ldots$

c) $\log_2 10 = 3.3219 \ldots$

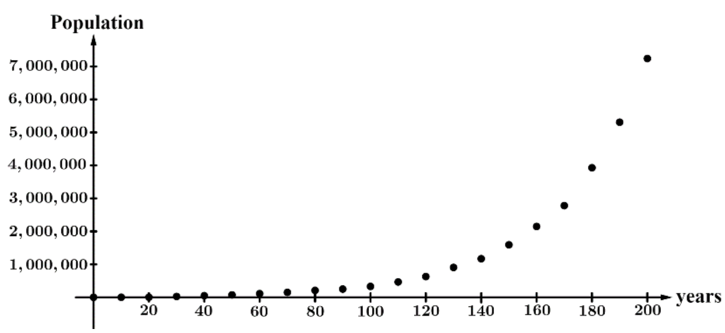
d) $\log_7 135 = 2.5208 \ldots$

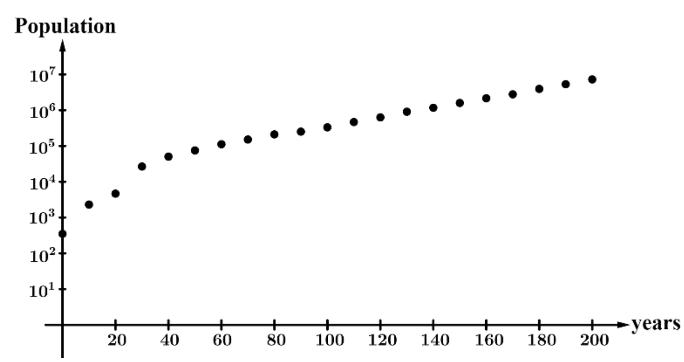
One useful purpose for logarithms is when dealing with exponential data. Exponential data can be very challenging to display graphically because the values may start out very small, but they grow quickly. It is difficult to create a display that effectively shows the smaller values along with the extremely large values together.

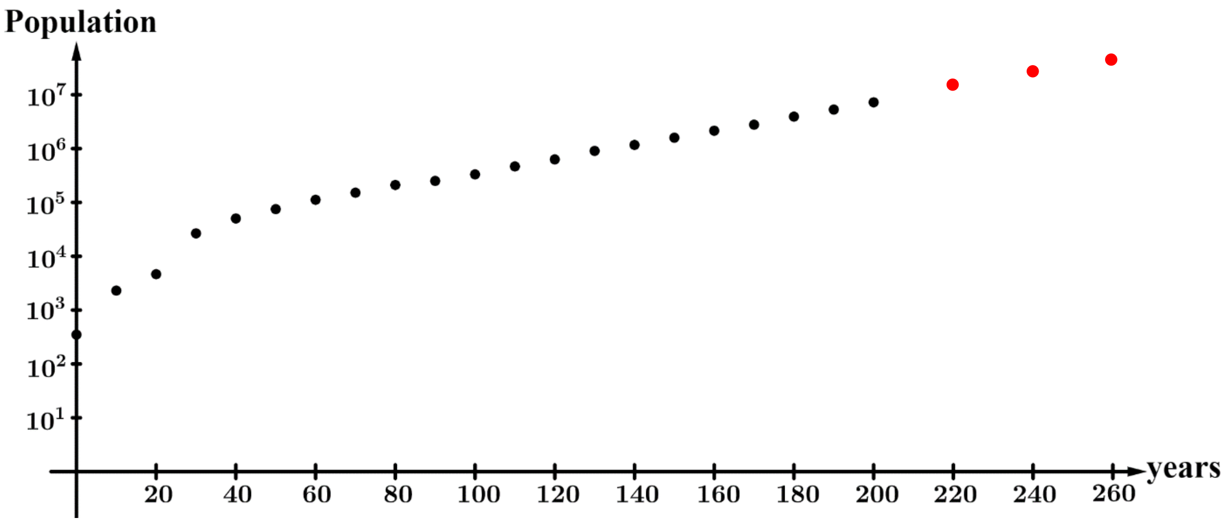
The population of English Americans in the (current) United States since 1620 has grown exponentially. The estimated population is shown below from 1620 – 1820, where $t = 0$ represents the year 1620.

The graph on the **left** shows the population using a normal scale on the vertical axis. Because the population expanded into the millions, the data points representing the years 1620 – 1700 are almost indistinguishable. It would be difficult to notice any differences in population during those beginning years.

The graph on the **right** uses a logarithmic scale (base 10) for the vertical axis. Using a logarithmic scale allows us to view all the data points relative to each other.



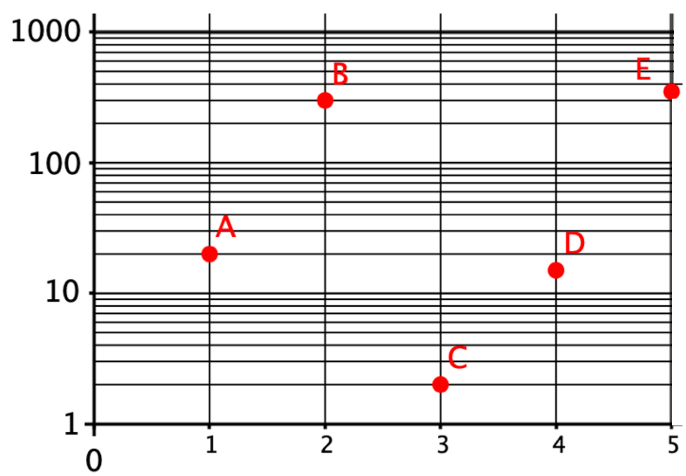




Example 5: The population data of English Americans in the current United States for the years 1840, 1860, and 1880 are given below. Plot these three additional data points on the graph above.

Year	1840	1860	1880
Population	17,069,453	31,443,321	50,189,209
log(Population)	7.2322 ...	7.4975 ...	7.7006 ...

One common type of graph with a logarithmically scaled vertical axis includes gridlines as seen below. This type of coordinate grid is likely to appear on the AP Precalculus Exam.



Example 6: Plot the following points on the same coordinate plane above.

$\mathbf{A}(1, 20)$ $\mathbf{B}(2, 300)$ $\mathbf{C}(3, 2)$ $\mathbf{D}(4, 15)$ $\mathbf{E}(5, 350)$