

Notes: (Topic 1.12) Transformations of Functions Solutions

An important aspect of understanding functions is the concept of transformations. Throughout this course (as well as past and future courses), we will study a variety of functions and their graphs. All of these functions can be viewed through the perspective of transformations. Transformations will weave through everything we learn in AP Precalculus, and they provide a way for us to connect the topics we learn throughout the course.

Below is a table of transformations, along with how each transformation affects a graph.

Transformations of Functions			
Vertical Translation	Horizontal Translation	Vertical Dilation	Horizontal Dilation
$g(x) = f(x) + k$ <p>f has a vertical translation of k units</p> <p>$g(x) = f(x) - 2$</p>	$g(x) = f(x + h)$ <p>f has a horizontal translation of $-h$ units</p> <p>$g(x) = f(x + 2)$</p>	$g(x) = af(x)$ <p>f has a vertical dilation by a factor of a units</p> <p>$g(x) = -2f(x)$</p> <p>Note: If $a < 0$, f is reflected over the x-axis.</p>	$g(x) = f(bx)$ <p>f has a horizontal dilation by a factor of $\left \frac{1}{b}\right$ units</p> <p>$g(x) = f(-x)$</p> <p>Note: If $b < 0$, f is reflected over the y-axis</p>

A Few Important Notes and Tips on Transformations

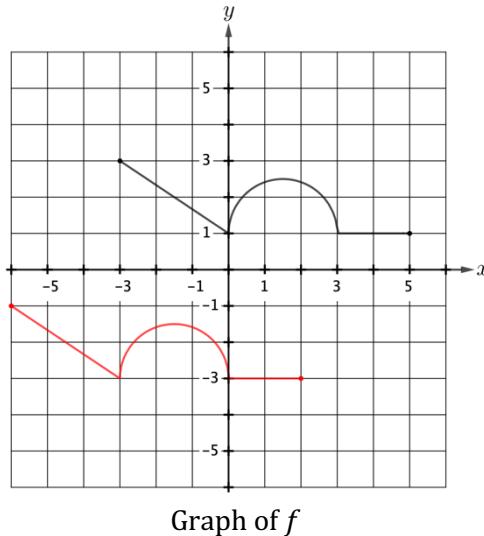
1. Any transformations that affect the x values will occur inside the parenthesis with the x variable.
2. Any transformations that affect the x values will do the opposite of what it looks like to the x values.
3. Any transformations that affect the y values will occur outside the parenthesis.
4. Any transformations that affect the y values will do exactly what it looks like to the y values.

Example 1: Let g be a function that is a transformation of the function f such that $g(x) = 2f(x - 3) + 1$. Describe the transformations of the function f that result with the function g .

The function f is horizontally translated to the right 3 units and vertically dilated by a factor of 2 then vertically translated 1 unit up.

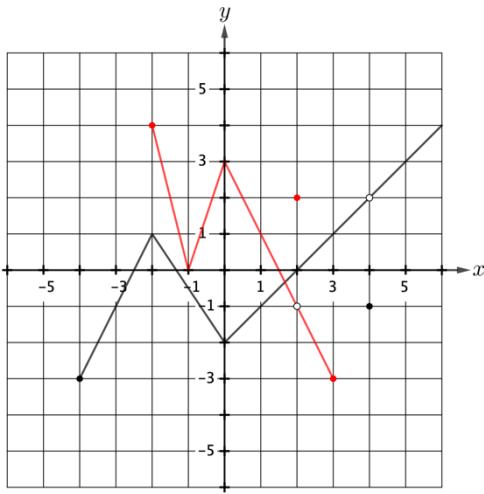
Example 2: Let n be a function that is a transformation of the function m such that $n(x) = -4m(2x)$. Describe the transformations of the function m that result with the function n .

The function m is horizontally dilated by a factor of $\frac{1}{2}$ and vertically dilated by a factor of 4 then reflected over the x -axis.



Graph of f

Example 3: The graph of $y = f(x)$, consisting of two line segments and a semicircle, is shown for $-3 \leq x \leq 5$. Sketch a graph of g on the same axes above where $g(x) = f(x + 3) - 4$.



Graph of h

Example 4: The graph of $y = h(x)$ has a hole at $x = 4$ and consists of three linear segments. Sketch a graph of k on the same axes above where $k(x) = -h(2x) + 1$.

Example 5: The domain of f from Example 3 is $-3 \leq x \leq 5$ and the range of f is $1 \leq y \leq 3$. Find the domain and range of g , where $g(x) = 2f(x + 3) - 4$.

$$(x_g + 3) = x_f \Rightarrow (x_g + 3) = -3 \Rightarrow x_g = -6 \quad (x_g + 3) = x_f \Rightarrow (x_g + 3) = 5 \Rightarrow x_g = 2$$

The domain of g is $-6 \leq x \leq 2$.

$$2y_f - 4 = y_g \Rightarrow 2(1) - 4 = -2 = y_g \quad 2y_f - 4 = y_g \Rightarrow 2(3) - 4 = 2 = y_g \quad \text{The range of } g \text{ is } -2 \leq y \leq 2.$$

x	-2	0	2	4	6
$p(x)$	1	-1	0	3	7

Example 6: The table above gives values for a function p at selected values of x . Let $h(x) = 3p(2x) - 1$. Find the following values.

$$(a) h(-1) = 3p(2(-1)) - 1 =$$

$$3p(-2) - 1 = 3(1) - 1 = 2$$

$$(b) h(2) = 3p(2(2)) - 1$$

$$= 3p(4) - 1 = 3(3) - 1 = 8$$

$$(c) h(0) = 3p(2(0)) - 1$$

$$= 3p(0) - 1 = 3(-1) - 1 = -4$$

Example 7: The function g is constructed by applying three transformations to the graph of f in this order: a horizontal dilation by a factor of 3, a vertical dilation by a factor of 4, and a vertical translation by -7 units. If $g(x) = af(bx) + c$, find the values of a , b , and c .

$$a = 4 \quad b = \frac{1}{3} \quad c = -7$$

x	-12	-6	-3	0	3
$h(x)$	1	-1	0	3	7

Example 8: The table above gives values for a function h at selected values of x . Let $k(x) = ah(bx) + c$, where a , b , and c are positive constants. In the xy -plane, the graph of k is constructed by applying three transformations to the graph of h in this order: a horizontal dilation by a factor of $\frac{1}{2}$, a vertical dilation by a factor of 3, and a vertical translation by 1 unit. Find the value of $k(-6)$.

$$k(x) = 3h(2x) + 1 \quad k(-6) = 3h(2(-6)) + 1 = 3h(-12) + 1 = 3(1) + 1 = 4$$

Example 9: The function p is given by $p(x) = 3x - 2$. The graph of r is the image of the graph of p after a horizontal translation of 4 to the graph of p . If $r(x) = ax + b$, find the values of a and b .

$$r(x) = p(x - 4) = 3(x - 4) - 2 = 3x - 12 - 2 = 3x - 14 \quad a = 3, b = -14$$