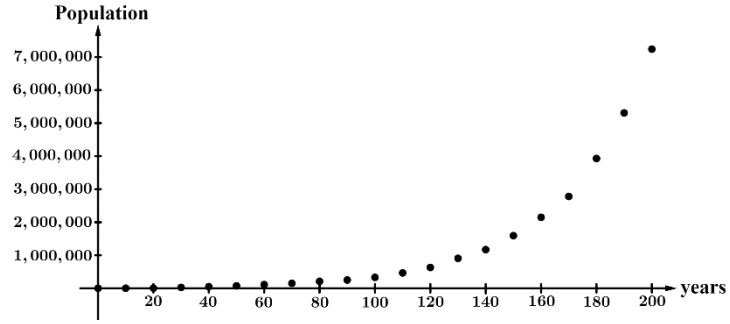
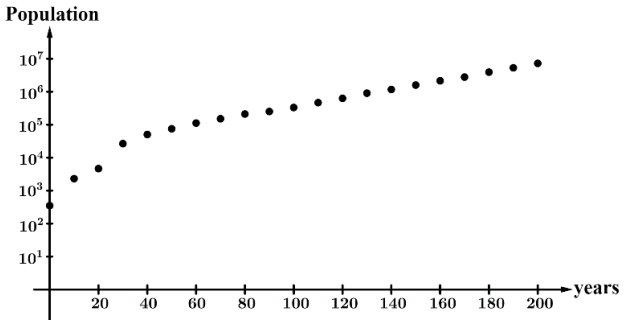
Previously (in Topic 2.9 – Logarithmic Expressions), we looked at the idea of using a logarithmically scaled axis to help us better understand data that is exponential. We now return to this idea to examine it in a little more detail through the use of **Semi-log Plots**.

Semi-Log Plots

In a semi-log plot, one of the axes is logarithmically scaled. In AP Precalculus, we will only be scaling the vertical (y) axis.

With a semi-log plot where the y-axis logarithmically scaled, **exponential functions will appear linear.**



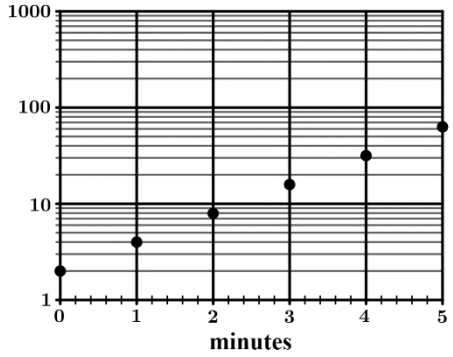


Previously, we looked at the two graphs displayed above displaying the population of English Americans in the (current) United States from 1620 – 1820, where $t = 0$ represents the year 1620.

The graph on the **left** shows the population using a normal scale on the vertical axis.

The graph on the **right** is a semi-log plot where the vertical axis has been logarithmically scaled.

Example 1: Use the features of the semi-log plot above to justify why an exponential model is appropriate for the population of English Americans in the (current) United States from 1620 – 1820.



Example 2: After Mr. Passwater tells another one of his hilarious math jokes, it begins to spread around the school. The number of people P that have heard the joke after t minutes is graphed on the semi-log plot above where the vertical axis has been logarithmically scaled. Which of the following functions could be a model for P?

(A) $P(t) = 2 + 2t$ (B) $P(t) = 2 + 2^t$ (C) $P(t) = 2 + \log_2 t$ (D) $P(t) = 2(2)^t$

Important Note About Semi-log Plots

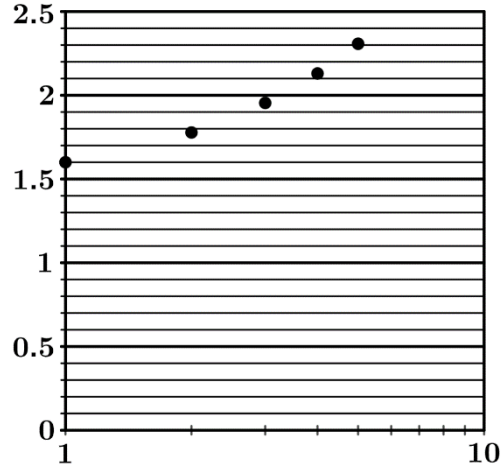
When we "logarithmically scale" the vertical axis for a semi-log plot, we are NOT changing the actual y-values of the data!

To "logarithmically scale" the vertical axis means that equally-spaced values on the y-axis are proportional, whereas equally-spaced values on the x-axis are linear.

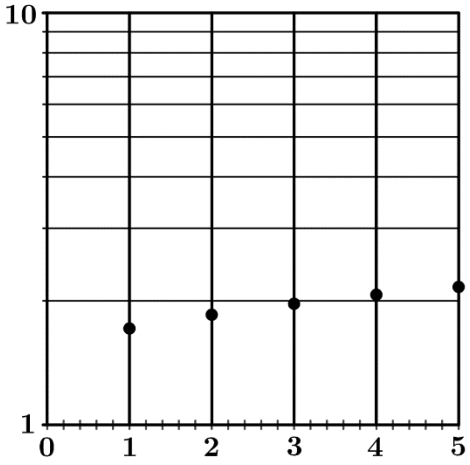
x	1	2	3	4	5
$f(x)$	40	60	90	135	203

Example 3: The table above gives selected values for the function f. Which of the following graphs could represent these data in a semi-log plot, where the vertical axis is logarithmically scaled?

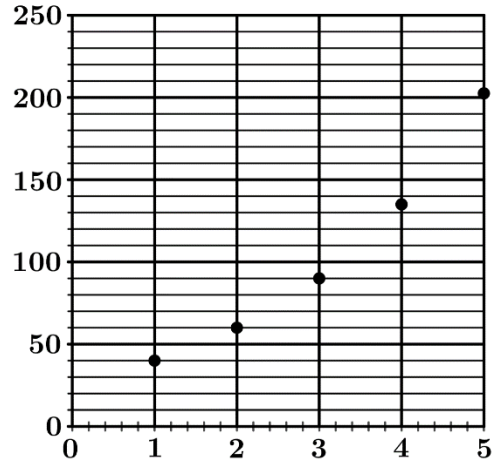
(A)



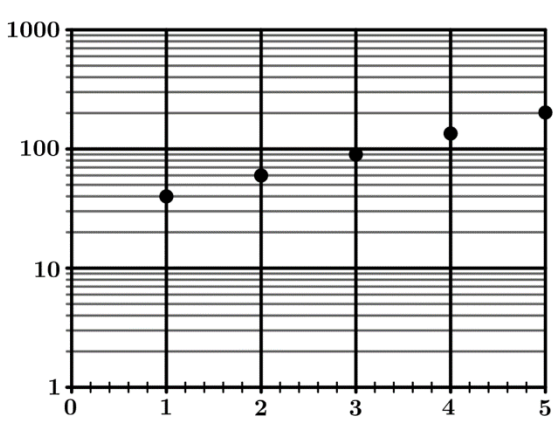
(B)

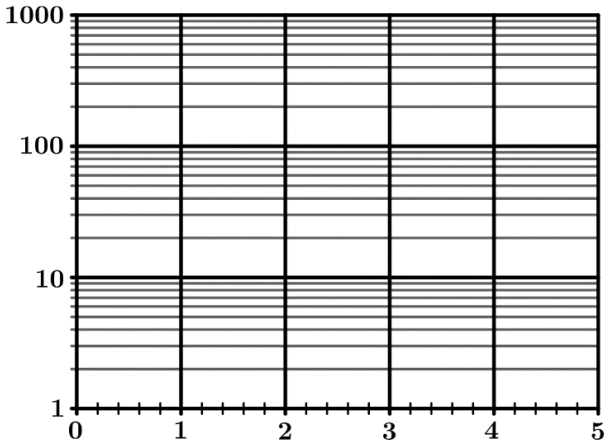


(C)



(D)





Example 4: Plot the following points on the same coordinate plane above.

$\mathbf{A}(0, 200)$ $\mathbf{B}(1, 25)$ $\mathbf{C}(2, 7)$ $\mathbf{D}(3.2,\ 800)$ $\mathbf{E}(4.6,\ 1.5)$

As we have seen, if we graph an exponential function on a semi-log plot, the graph will appear linear. In these cases, we can create a linear model for the graph on the semi-log plot.

Linear Models for a Semi-log Plot

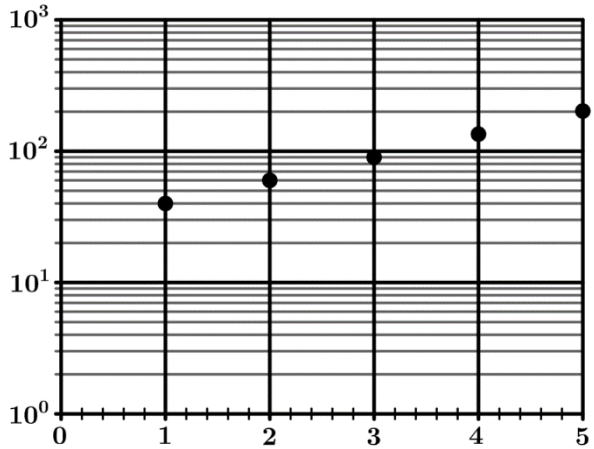
Given the exponential model $y = ab^x$, the corresponding linear model for the semi-log plot is given by

$$y = \left(\log_n b\right)x + \log_n a\,,$$

where $n > 0$ and $n \neq 1$.

Note #1: The slope of our linear function is $\log_n b$ and the y-intercept is $\log_n a$.

Note #2: The base n corresponds to the base used for the scaling of the vertical axis.



Example 5: The semi-log plot above corresponds to the data table for **Example 3**.

a) Write an equation for the linear model for the semi-log plot of the form $y = \left(\log_n b\right)x + \log_n a$.

b) Using the linear model from part a, write the equation of the exponential model $y = ab^x$ for this data.