Worksheet A: (Topic 1.8) Rational Functions and Zeros Name: Solutions

Directions: Find the zeros for the following rational functions.

1. $y=\dfrac{(x+3)(x-2)}{(x-1)(x+5)}$

1. Zeros: $x=-3,2$

2. $y=\dfrac{x^2}{(x-1)(x+2)}$

2. Zeros: $x=0$

3. $y=\dfrac{(x-3)(x-6)}{(x-6)(x-2)}$

There is a hole at $x=6$

3. Zeros: $x=3$

4. $h(x)=\dfrac{x^2-x-20}{x^2+x-20}$

$=\dfrac{(x+4)(x-5)}{(x+5)(x-4)}$

4. Zeros: $x=-4,5$

5. $f(x)=\dfrac{x^2-9}{x^2-2x-15}$

$=\dfrac{(x+3)(x-3)}{(x+3)(x-5)}$

There is a hole at $x=-3$

5. Zeros: $x=3$

6. $g(x)=\dfrac{x^3-4x^2-32x}{2x^2+7x-4}$

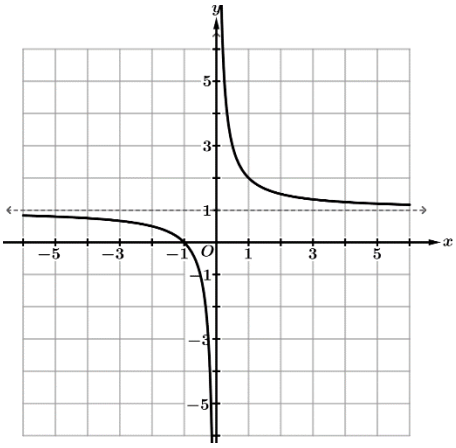
$=\dfrac{x(x^2-4x-32)}{(2x-1)(x+4)}$

$=\dfrac{x(x+4)(x-8)}{(2x-1)(x+4)}$

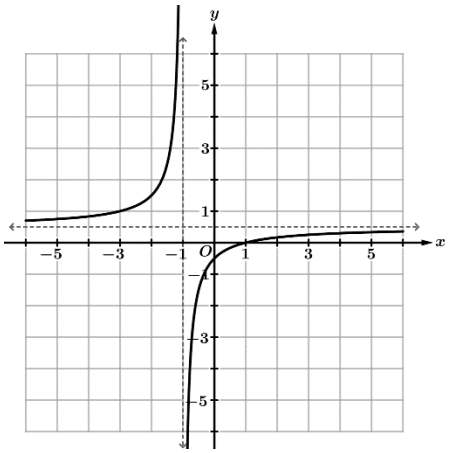
There is a hole at $x=-4$

6. Zeros: $x=0,8$

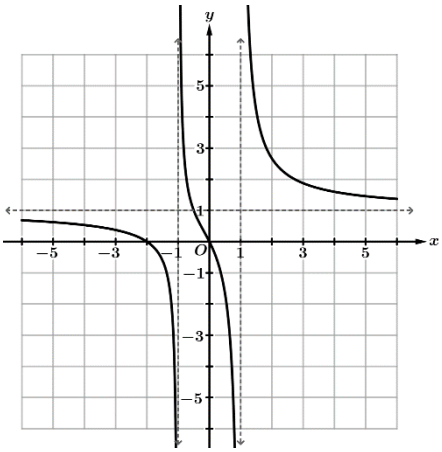
Directions: The graphs of several rational functions are given below. Use the graphs to solve the following inequalities.



Graph of f



Graph of g



Graph of h

7. $f(x)>0$

$(-\infty,-1)\cup(0,\infty)$

8. $g(x)\le 0$

$(-1,1]$

9. $h(x)\ge 0$

$(-\infty,-2]\cup(-1,0]\cup(1,\infty)$

Directions: Solve the following inequalities. Write your answers using interval notation.

10. $\dfrac{x+2}{x-1}<0$ $(-2,1)$

$\overset{\frac{(-)}{(-)}}{+}\ <\ \overbrace{---}\ \underset{-2}{+}\ \overbrace{---}^{\frac{(+)}{(-)}}\ \underset{1}{+}\ \overbrace{---}^{\frac{(+)}{(+)}}\ >$

11. $\dfrac{x-3}{x+5}>0$ $(-\infty,-5)\cup(3,\infty)$

$\overset{\frac{(-)}{(-)}}{+}\ <\ \overbrace{---}\ \underset{-5}{+}\ \overbrace{---}^{\frac{(-)}{(+)}}\ \underset{3}{+}\ \overbrace{---}^{\frac{(+)}{(+)}}\ >$

Directions: Solve the following inequalities. Write your answers using interal notation.

12. $\dfrac{-3}{x-1} \geq 0$ $(-\infty, 1)$

$\overset{\frac{(-)}{(-)}}{+}$ $\overset{\frac{(-)}{(+)}}{-}$

$< \underbrace{---}_{+} \underset{1}{\underbrace{+}} \underbrace{---}_{-} >$

13. $\dfrac{x}{(x-4)^2} < 0$ $(-\infty, 0)$

$\overset{\frac{(-)}{(+)}}{-}$ $\overset{\frac{(+)}{(+)}}{+}$ $\overset{\frac{(+)}{(+)}}{+}$

$< \underbrace{---}_{-} \underset{0}{\underbrace{+}} \underbrace{---}_{+} \underset{4}{\underbrace{+}} \underbrace{---}_{+} >$

14. $\dfrac{(x-1)(x+6)}{x+2} \leq 0$ $(-\infty, -6] \cup (-2, 1]$

$\overset{\frac{(-)(-)}{(-)}}{-}$ $\overset{\frac{(+)(-)}{(-)}}{+}$ $\overset{\frac{(+)(-)}{(+)}}{-}$ $\overset{\frac{(+)(+)}{(+)}}{+}$

$< \underbrace{---}_{-} \underset{-6}{\underbrace{+}} \underbrace{---}_{+} \underset{-2}{\underbrace{+}} \underbrace{---}_{-} \underset{1}{\underbrace{+}} \underbrace{---}_{+} >$

15. $\dfrac{(2x-3)(x+4)}{x^2} \geq 0$ $(-\infty, -4] \cup [3/2, \infty)$

$\overset{\frac{(-)(-)}{(+)}}{+}$ $\overset{\frac{(-)(+)}{(+)}}{-}$ $\overset{\frac{(-)(+)}{(+)}}{-}$ $\overset{\frac{(+)(+)}{(+)}}{+}$

$< \underbrace{---}_{+} \underset{-4}{\underbrace{+}} \underbrace{---}_{-} \underset{0}{\underbrace{+}} \underbrace{---}_{-} \underset{3/2}{\underbrace{+}} \underbrace{---}_{+} >$

16. $\dfrac{x^2-9}{x+2} > 0$ $(-3, -2) \cup (3, \infty)$

$\dfrac{(x+3)(x-3)}{x+2} > 0$

$\overset{\frac{(-)(-)}{(-)}}{-}$ $\overset{\frac{(+)(-)}{(-)}}{+}$ $\overset{\frac{(+)(-)}{(+)}}{-}$ $\overset{\frac{(+)(+)}{(+)}}{+}$

$< \underbrace{---}_{-} \underset{-3}{\underbrace{+}} \underbrace{---}_{+} \underset{-2}{\underbrace{+}} \underbrace{---}_{-} \underset{3}{\underbrace{+}} \underbrace{---}_{+} >$

17. $\dfrac{x^2-3x-10}{x+8} < 0$ $(-\infty, -8) \cup (-2, 5)$

$\dfrac{(x+2)(x-5)}{x+8} < 0$

$\overset{\frac{(-)(-)}{(-)}}{-}$ $\overset{\frac{(-)(-)}{(+)}}{+}$ $\overset{\frac{(+)(-)}{(+)}}{-}$ $\overset{\frac{(+)(+)}{(+)}}{+}$

$< \underbrace{---}_{-} \underset{-8}{\underbrace{+}} \underbrace{---}_{+} \underset{-2}{\underbrace{+}} \underbrace{---}_{-} \underset{5}{\underbrace{+}} \underbrace{---}_{+} >$

18. $\dfrac{x^2-6x+9}{x^2+1} > 0$ $(-\infty, 3) \cup (3, \infty)$

$\dfrac{(x-3)^2}{x^2+1} > 0$

$\overset{\frac{(+)}{(+)}}{+}$ $\overset{\frac{(+)}{(+)}}{+}$

$< \underbrace{---}_{+} \underset{3}{\underbrace{+}} \underbrace{---}_{+} >$

19. $\dfrac{3x^2-2x-8}{x+1} \leq 0$ $(-\infty, -4/3] \cup (-1, 2]$

$\dfrac{(3x+4)(x-2)}{x+1} \leq 0$

$\overset{\frac{(-)(-)}{(-)}}{-}$ $\overset{\frac{(+)(-)}{(-)}}{+}$ $\overset{\frac{(+)(-)}{(+)}}{-}$ $\overset{\frac{(+)(+)}{(+)}}{+}$

$< \underbrace{---}_{-} \underset{-4/3}{\underbrace{+}} \underbrace{---}_{+} \underset{-1}{\underbrace{+}} \underbrace{---}_{-} \underset{2}{\underbrace{+}} \underbrace{---}_{+} >$