

Notes: (Topic 3.9) Inverse Trigonometric Functions

As with all functions, the inverse of a trigonometric function is the result of switching the input (x) and output (y) values of the function. As a result, the output value of an inverse trigonometric function will be an angle measure.

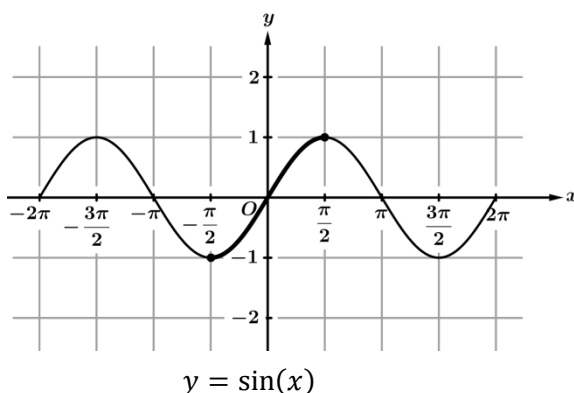
Notation: We can represent inverse trigonometric functions in two different ways: $\sin^{-1}(x)$ or $\arcsin(x)$.

With either notation, we would say “arcsine of x ” when reading it aloud.

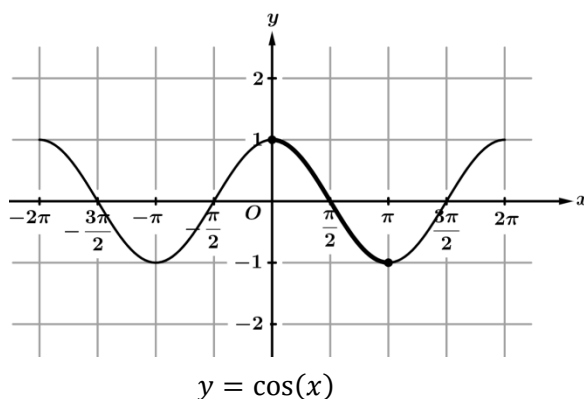
Example 1: Write the statement $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ in an equivalent form using arcsine notation.

Restricted Domains of Inverse Trigonometric Functions

Because trig functions are periodic, we must restrict their domains to create their corresponding inverse functions.



In order to make the inverse a function, we will restrict the function $y = \sin(x)$ to the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, as highlighted in the figure above.

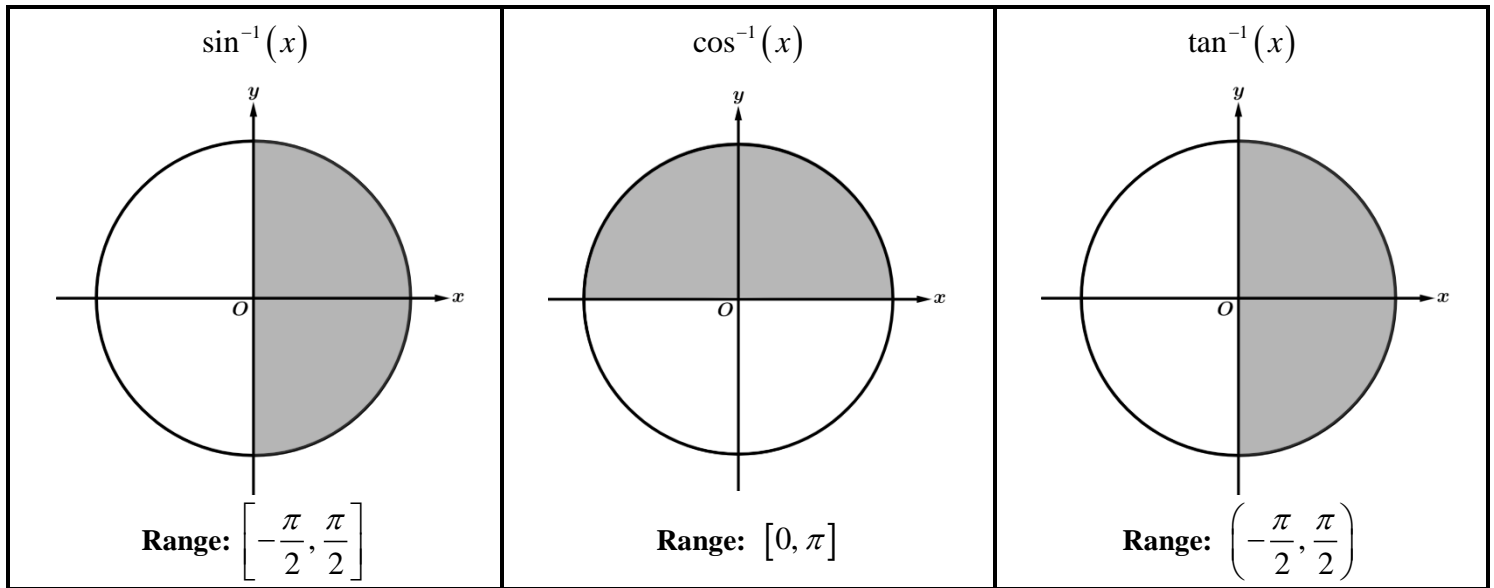


Similarly, we will restrict our cosine function for the same purposes. However, for $y = \cos(x)$, we will restrict our domain to $[0, \pi]$.

For the tangent function, we will restrict the domain of $y = \tan(x)$ to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, due to the vertical asymptotes at

$$x = -\frac{\pi}{2} \text{ and } x = \frac{\pi}{2}.$$

Important Note About Inverse Trig Functions: It is important to always remember and consider the domain restrictions when working with inverse trigonometric functions and values.

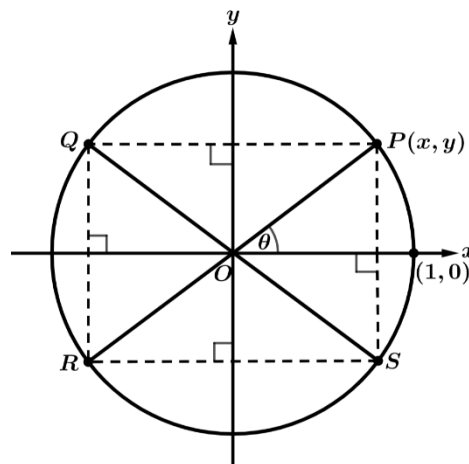


Example 2: Evaluate the following expressions.

a) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$

b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$

c) $\tan^{-1}(\sqrt{3}) =$



Example 3: The angle θ is in standard position. The terminal ray intersects the unit circle at point P , whose coordinates are (x, y) . The points Q , R , and S are the result of the terminal ray being reflected over the y -axis, the origin, and the x -axis respectively. For each of the following expressions, determine which labeled point intersects the terminal ray of the given angles.

a) $\cos^{-1}(x)$

b) $\sin^{-1}(-y)$

c) $\cos^{-1}(-x)$

d) $\tan^{-1}\left(-\frac{y}{x}\right)$