**Inverse Functions**

If $f$ and $g$ are inverse functions, then…

1. $g(x) = f^{-1}(x)$
2. If $(x, y)$ is a point on the graph of $f(x)$, then $(y, x)$ is a point on the graph of $g(x)$.
3. With inverse functions, all of the $x$ and $y$ values are "switched", so the graphical behaviors in terms of $x$ and $y$ will also be switched. For example, the **domain of $f$** is the **range of $f^{-1}$**.
4. A continuous function will only have an inverse function if it is strictly increasing or strictly decreasing. If a function changes from increasing to decreasing (or vice versa), it will not pass the horizontal line test and its inverse relation will not pass the vertical line test as a result.

| $x$ | $-3$ | $-2$ | $0$ | $1$ | $4$ | $6$ |
|---|---|---|---|---|---|---|
| $f(x)$ | $6$ | $3$ | $1$ | $-1$ | $-3$ | $-7$ |
| $x$ | $6$ | $3$ | $1$ | $-1$ | $-3$ | $-7$ |
| $g(x)$ | $-3$ | $-2$ | $0$ | $1$ | $4$ | $6$ |

**Example 1:** Let $f$ be a continuous function with selected values in the table below. Let $g$ be the inverse of $f$, such that $g(x) = f^{-1}(x)$. Find the following values if possible.

a) $f(f(0)) = f(1) = -1$

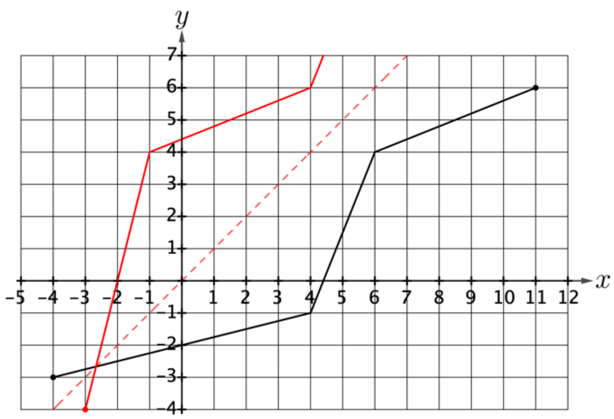
b) $g(-3) = 4$

c) $g(6) = -3$

d) $g(g(-1)) = g(1) = 0$

e) $(f^{-1} \circ f)(-2) = f^{-1}(3)$
$= g(3) = -2$

f) $f^{-1}(-3) = g(-3) = 4$

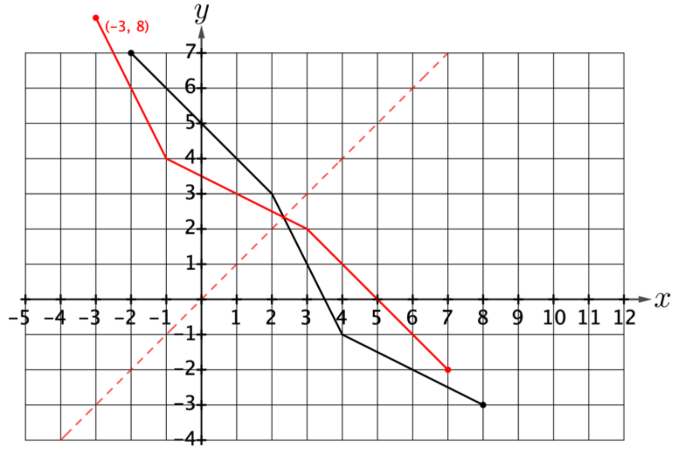



**Graph of $k$**

**Example 2:** The function $k$ is defined over the interval $-4 \le x \le 11$ as shown above. Let $k^{-1}$ represent the inverse of $k$.

a) What is the minimum value of $k(x)$? $-3$ What is the minimum value of $k^{-1}(x)$? $-4$ graphically but this is the least $x$–value of $k$

b) Find $k^{-1}(6)$ and $k^{-1}(4)$. $k^{-1}(6) = 11$ because this is the $x$–value where $k = 6$
$k^{-1}(4) = 6$ because this is the $x$–value where $k = 4$



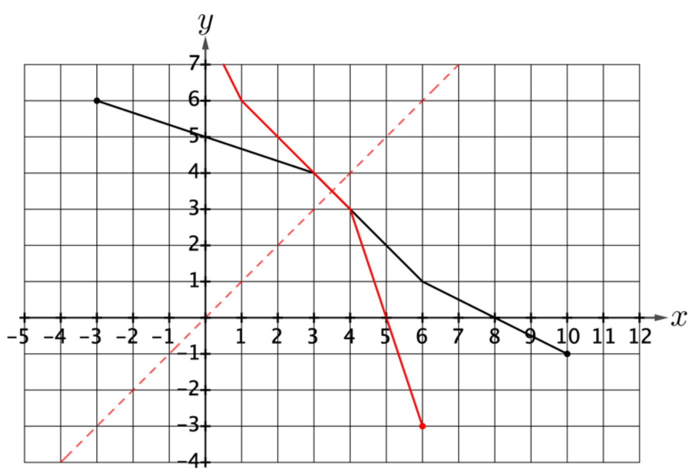

**Graph of $f$**

**Example 3:** The function $f$ is defined over the interval $-2 \le x \le 8$ as shown above. Let $f^{-1}$ represent the inverse of $f$.

a) What is the maximum value of $f^{-1}(x)$? 8 graphically but this is the maximum *x*–value of *f*.

b) Find $f^{-1}(3)$ and $f^{-1}(1)$. $f^{-1}(3) = 2$ because this is the *x*–value where $f = 3$
$f^{-1}(1) = 3$ because this is the *x*–value where $f = 1$

c) What is the domain of $f^{-1}$? $[-3,7]$ because the domain of $f^{-1}$ is the range of *f*.



**Graph of $g$**

| $x$ | $-5$ | $-1$ | $0$ | $2$ | $5$ | $6$ |
|---|---|---|---|---|---|---|
| $h(x)$ | $-3$ | $0$ | $3$ | $5$ | $8$ | $10$ |
| $x$ | $-3$ | $0$ | $3$ | $5$ | $8$ | $10$ |
| $h^{-1}(x)$ | $-5$ | $-1$ | $0$ | $2$ | $5$ | $6$ |

**Example 4:** The function $g$ is defined over the interval $-3 \le x \le 10$ as shown above. Let $g^{-1}$ represent the inverse of $g$. Values of the increasing function $h$ are given in the table above for selected values of $x$. Find the following, if possible.

a) $g(h(6)) = g(10) = -1$

b) $g^{-1}(h(0)) = g^{-1}(3) = 4$

c) $h^{-1}(g(8)) = h^{-1}(0) = -1$

d) $h^{-1}(g^{-1}(-1)) = h^{-1}(10) = 6$
$g^{-1}(-1) = 10$ because this is the *x*–value where $g = -1$