Worksheet A: (Topic 1.7) Rational Functions and End Behavior Name: Solutions

Directions: For each of the following, determine if the given rational function has a horizontal asymptote. If it does, write the equation of the horizontal asymptote.

1. $f(x)=\dfrac{3x^2-1}{2x^2+5x+7}$

Horizontal Asymptote: Y or N — **Y**

If Yes, Equation: $y=\frac{3}{2}$

$f(x)\to\dfrac{3x^2}{2x^2}=\dfrac{3}{2}\Rightarrow HA: y=\dfrac{3}{2}$

2. $g(x)=\dfrac{x^3+2x^2+x+4}{5x^2+7x+8}$

Horizontal Asymptote: Y or N — **N**

If Yes, Equation: __________

$g(x)\to\dfrac{x^3}{5x^2}=\dfrac{1}{5}x$

3. $h(x)=\dfrac{5x^3-2x^2-1}{x^4-6}$

Horizontal Asymptote: Y or N — **Y**

If Yes, Equation: $y=0$

$h(x)\to\dfrac{5x^3}{x^4}=\dfrac{5}{x}\Rightarrow HA: y=0$

4. $k(x)=\dfrac{6x^3+2x+3}{2x^2-11x+4}$

Horizontal Asymptote: Y or N — **N**

If Yes, Equation: __________

$k(x)\to\dfrac{6x^3}{2x^2}=3x$

5. $r(x)=\dfrac{(2x-1)(5x+6)}{(x+3)(x-6)}$

Horizontal Asymptote: Y or N — **Y**

If Yes, Equation: $y=10$

$r(x)\to\dfrac{(2x)(5x)}{(x)(x)}=\dfrac{10x^2}{x^2}=10$

6. $q(x)=\dfrac{(x^2-3)^2}{3x^3+4x^2+7}$

Horizontal Asymptote: Y or N — **N**

If Yes, Equation: __________

$q(x)\to\dfrac{(x^2)^2}{3x^3}=\dfrac{x^4}{3x^3}=\dfrac{1}{3}x$

7. $p(x)=\dfrac{(3x-1)^2}{2x^2+3x+5}$

Horizontal Asymptote: Y or N — **Y**

If Yes, Equation: $y=\frac{9}{2}$

$p(x)\to\dfrac{(3x)^2}{2x^2}=\dfrac{9x^2}{2x^2}=\dfrac{9}{2}$

8. $y=\dfrac{(x-2)(4-x)}{(x+3)^2}$

Horizontal Asymptote: Y or N — **Y**

If Yes, Equation: $y=-1$

$y\to\dfrac{(x)(-x)}{(x)^2}=-\dfrac{x^2}{x^2}=-1$

9. $s(x)=\dfrac{(2x^2+3)^2(x-4)}{(x^2+5)(x-2)}$

Horizontal Asymptote: Y or N — **N**

If Yes, Equation: __________

$s(x)\to\dfrac{(2x^2)^2(x)}{(x^2)(x)}=\dfrac{4x^5}{x^3}=4x^2$

Directions: Write limit statements for the end behavior of the following rational functions.

10. $y=\dfrac{2x^3-5x+6}{6x^3+10x^2-4x-12}$

$y\to\dfrac{2x^3}{6x^3}$

Left: $\lim\limits_{x\to-\infty}\dfrac{2x^3}{6x^3}=\dfrac{2}{6}=\dfrac{1}{3}$

Right: $\lim\limits_{x\to\infty}\dfrac{2x^3}{6x^3}=\dfrac{2}{6}=\dfrac{1}{3}$

11. $y=\dfrac{(4x+3)^2}{(3x-1)(2x+5)}$

$y\to\dfrac{(4x)^2}{(3x)(2x)}=\dfrac{16x^2}{6x^2}$

Left: $\lim\limits_{x\to-\infty}\dfrac{16x^2}{6x^2}=\dfrac{16}{6}=\dfrac{8}{3}$

Right: $\lim\limits_{x\to\infty}\dfrac{16x^2}{6x^2}=\dfrac{16}{6}=\dfrac{8}{3}$

12. $y=\dfrac{x^2}{(x-1)^3}$

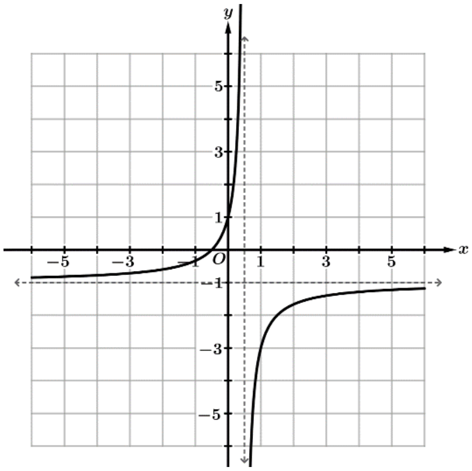
$y\to\dfrac{x^2}{x^3}=\dfrac{1}{x}$

Left: $\lim\limits_{x\to-\infty}\dfrac{1}{x}=0$

Right: $\lim\limits_{x\to\infty}\dfrac{1}{x}=0$

Directions: Write a limit statement describing the output values for the following graphs and verbal descriptions of the input values.

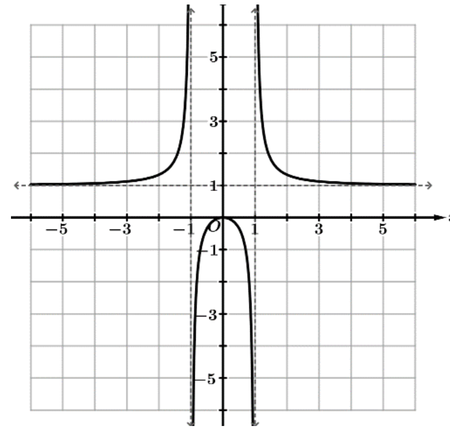
13. The input values decrease without bound



Graph of $f(x)$

13. Limit Statement: $\lim\limits_{x\to-\infty} f(x) = -1$

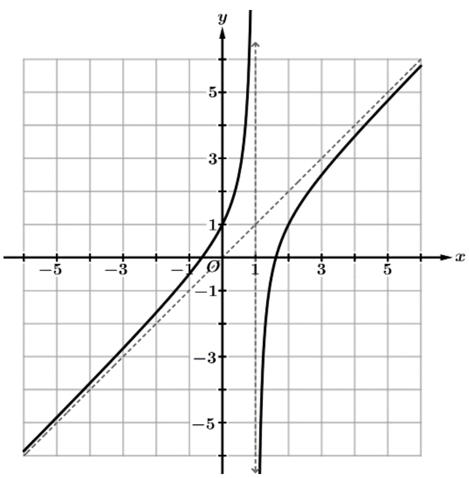
14. The input values increase without bound



Graph of $g(x)$

14. Limit Statement: $\lim\limits_{x\to\infty} g(x) = 1$

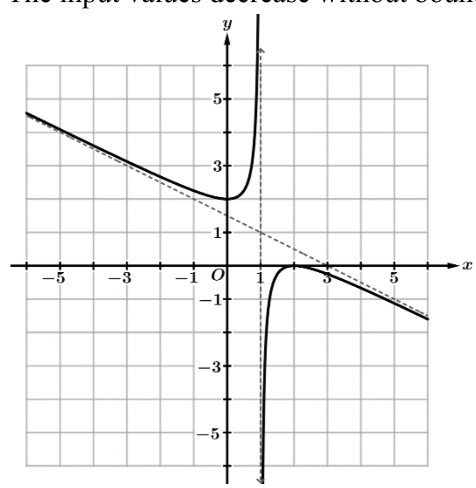
15. The input values increase without bound



Graph of $h(x)$

15. Limit Statement: $\lim\limits_{x\to\infty} h(x) = \infty$

16. The input values decrease without bound



Graph of $k(x)$

16. Limit Statement: $\lim\limits_{x\to-\infty} k(x) = \infty$