Reminder: A **geometric sequence** has successive terms that have a **common ratio (r)**. This means that each term is multiplied by the same number to generate the next term.

The general form of a geometric sequence is usually the best form to use: $g_n = g_k(r)^{n-k}$

Example 1: Find an equation for the geometric sequence with $g_3 = -3$ and $r = 10$.

$$g_n = g_3(10)^{n-3} = -3(10)^{n-3}$$

Example 2: Let g_n be a geometric sequence with $g_2 = 24$ and $r = \frac{1}{2}$. Find an expression for g_n. Use the expression for g_n to find g_5.

$$g_n = g_2\left(\frac{1}{2}\right)^{n-2} = 24\left(\frac{1}{2}\right)^{n-2} \qquad g_5 = 24\left(\frac{1}{2}\right)^{5-2} = 24\left(\frac{1}{2}\right)^{3} = 24\left(\frac{1}{8}\right) = 3$$

The examples above were relatively quick and easy because we were given one term of the sequence and the value of r. However, how can we deal with problems that give us two terms but not the value of r?

Geometric Sequences: Given 2 Terms

1. Use both terms to create one equation in general form – Use the larger k value term as the g_n term.
2. Solve for r using the equations created from Step 1.
3. Use the value of r found in Step 2 along with either term given in the problem to write the general form of the geometric sequence.

Example 3: Let g_n be a geometric sequence with $g_3 = -2$ and $g_6 = 128$. Find an expression for g_n. Use the expression for g_n to find g_{11}.

$$g_6 = g_3(r)^{6-3} \Rightarrow 128 = -2(r)^3 \Rightarrow -64 = (r)^3 \Rightarrow r = -4$$
$$g_n = g_3(-4)^{n-3} = -2(-4)^{n-3}$$
$$g_{11} = -2(-4)^{11-3} = -2(-4)^8 = -2(65536) = -131072$$

Example 4: Let g_n be a geometric sequence with $g_2 = 48$ and $g_7 = 1.5$. Find an expression for g_n. Use the expression for g_n to find g_{11}.

$$g_7 = g_2(r)^{7-2} \Rightarrow 1.5 = 48(r)^5 \Rightarrow \frac{3}{2}\cdot\frac{1}{48} = \frac{1}{32} = (r)^5 \Rightarrow r = \frac{1}{2}$$
$$g_n = 48\left(\frac{1}{2}\right)^{n-2} \qquad g_{11} = 48\left(\frac{1}{2}\right)^{11-2} = 48\left(\frac{1}{2}\right)^{9} = 48\left(\frac{1}{512}\right) = \frac{48}{512} = \frac{24}{256} = \frac{12}{128} = \frac{3}{32}$$

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