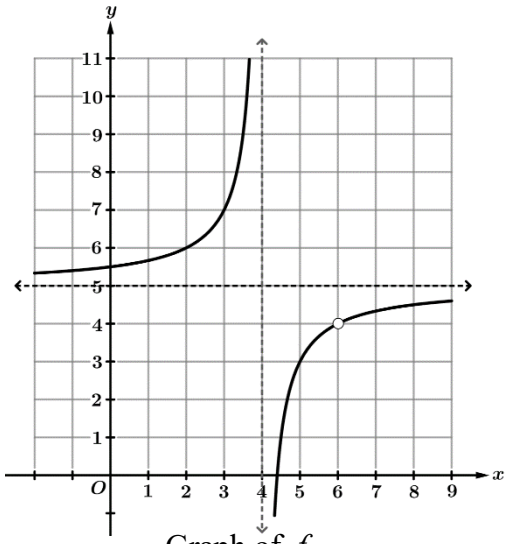


Graph of f

Directions: The rational function f is shown above and has a vertical asymptote at $x=4$, a hole at $x=6$, and a horizontal asymptote of $y=5$. Use the graph of f to answer the following questions.

1. Which of the following limit statements about f is correct?

(A) $\lim\limits_{x\to-\infty} f(x)=4$ (B) $\lim\limits_{x\to\infty} f(x)=4$ (C) $\lim\limits_{x\to4^-} f(x)=\infty$ (D) $\lim\limits_{x\to4^+} f(x)=\infty$

2. Which of the following limit statements about f is correct?

(A) $\lim\limits_{x\to4^-} f(x)=6$ (B) $\lim\limits_{x\to4^+} f(x)=6$ (C) $\lim\limits_{x\to6^-} f(x)=5$ (D) $\lim\limits_{x\to6^+} f(x)=4$

3. Which of the following limit statements about f is correct?

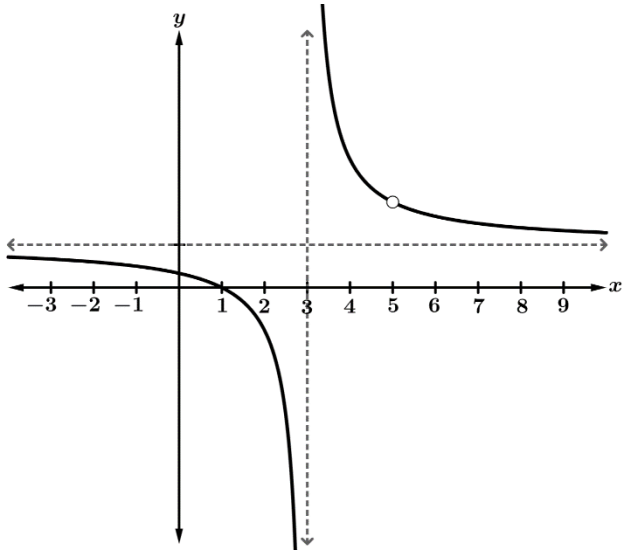
(A) $\lim\limits_{x\to5} f(x)=-\infty$ (B) $\lim\limits_{x\to5} f(x)=\infty$ (C) $\lim\limits_{x\to-\infty} f(x)=5$ (D) $\lim\limits_{x\to\infty} f(x)=4$

4. Which of the following could be the equation for $f(x)$?

(A) $f(x)=-\dfrac{(5x-22)(x-6)}{(x-6)(x-4)}$ (B) $f(x)=\dfrac{(5x-22)(x-6)}{(x-6)(x-4)}$

(C) $f(x)=-\dfrac{(5x-22)(x-4)}{(x-6)(x-4)}$ (D) $f(x)=-\dfrac{(x-6)(x-4)}{(5x-22)(x-6)}$

Denominator factor $(x-4)\Rightarrow$ VA @ $x=4$
Common factor $(x-6)\Rightarrow$ hole @ $x=6$
Quotient of leading terms $\lim\limits_{x\to\infty}\frac{5x^2}{x^2}=5\Rightarrow$ HA of $y=5$



Graph of g

5. The graph of the rational function g is shown above and has vertical asymptote at $x=3$, a hole at $x=5$, a zero at $x=1$, and a horizontal asymptote of $y=1$. Which of the following could be an expression for $g(x)$?

(A) $\dfrac{(x-3)(x-5)}{(x-5)(x-1)}$

(B) $\dfrac{(x-1)(x-5)}{(x-5)(x-3)}$

(C) $\dfrac{(x-3)(x-1)}{(x-5)(x-3)}$

(D) $\dfrac{(x-3)(x-5)}{(x-3)(x-1)}$

Denominator factor $(x-3) \Rightarrow$ VA @ $x=3$
Common factor $(x-5) \Rightarrow$ hole @ $x=5$
Numerator factor $(x-1) \Rightarrow$ zero @ $x=1$
Quotient of leading terms $\lim\limits_{x\to\infty} \frac{x^2}{x^2} = 1 \Rightarrow$ HA of $y=1$

6. Let h be a rational function whose graph has a hole at $x=-4$ and a vertical asymptote at $x=-5$. Which of the following could be an equation for $h(x)$?

(A) $h(x)=\dfrac{(x+4)(x+5)}{(x+4)(x-2)}$

(B) $h(x)=\dfrac{(x-2)(x+5)}{(x+4)(x+5)}$

(C) $h(x)=\dfrac{(x+4)^2(x+5)}{(x+4)(x+5)^2}$

(D) $h(x)=\dfrac{(x+4)(x+5)^2}{(x+4)^2(x+5)}$

Extra denominator factor $(x+5) \Rightarrow$ VA @ $x=-5$
Common factor $(x+4) \Rightarrow$ hole @ $x=-4$

7. The graph of $k(x)=\dfrac{(x-1)(x+3)}{(x+3)(x-2)}$ has

(A) a vertical asymptote at $x=2$ and a hole at $x=-3$.
(B) a vertical asymptote at $x=-3$ and a hole at $x=2$.
(C) a vertical asymptote at $x=1$ and a hole at $x=-3$.
(D) a vertical asymptote at $x=2$ and a hole at $x=1$.

Denominator factor $(x-2) \Rightarrow$ VA @ $x=2$
Common factor $(x+3) \Rightarrow$ hole @ $x=-3$

8. The polynomial function $f(x)$ has a zero at $x=1$ with a multiplicity of two. The polynomial function $g(x)$ has a zero at $x=1$ with a multiplicity of three, and the polynomial function $h(x)$ has a zero at $x=1$ with a multiplicity of one. Which of the following rational functions has a hole at $x=1$?

(A) $y=\dfrac{f(x)}{g(x)}$ (B) $y=\dfrac{h(x)}{g(x)}$ (C) $y=\dfrac{h(x)}{f(x)}$ (D) $y=\dfrac{g(x)}{f(x)}$

$f(x)=(x-1)^2\ldots\quad g(x)=(x-1)^3\ldots\quad h(x)=(x-1)$

$\dfrac{f(x)}{g(x)}=\dfrac{1\ldots}{(x-1)}\Rightarrow \text{VA @ } x=1\qquad \dfrac{h(x)}{g(x)}=\dfrac{1\ldots}{(x-1)^2}\Rightarrow \text{VA @ } x=1\qquad \dfrac{h(x)}{f(x)}=\dfrac{1\ldots}{(x-1)}\Rightarrow \text{VA @ } x=1$

$\dfrac{g(x)}{f(x)}=(x-1)\Rightarrow \text{hole @ } x=1$

9. Let r be a rational function with the following properties

$$\lim_{x\to 4^-} r(x)=-5 \qquad \lim_{x\to 2^+} r(x)=-\infty \qquad \lim_{x\to\infty} r(x)=-1$$

Which of the following could be an expression for $r(x)$?

(A) $\dfrac{(x+1)(x-2)}{(x-4)(x-2)}$ $\lim\limits_{x\to 4^-}\dfrac{(x+1)(x-2)}{(x-4)(x-2)}\to\dfrac{(5)(2)}{(0^-)(2)}\to-\infty$

(B) $\dfrac{(x+1)(x-4)}{(x-4)(x-2)}$ $\lim\limits_{x\to 4^-}\dfrac{(x+1)(x-4)}{(x-4)(x-2)}=\dfrac{(5)}{(2)}$

(C) $-\dfrac{(x+6)(x-4)}{(x-4)(x-2)}$ $\lim\limits_{x\to 4^-}\dfrac{(x+6)(x-4)}{(x-4)(x-2)}=-\dfrac{(10)}{(2)}=-5$ $\lim\limits_{x\to 2^+}-\dfrac{(x+6)}{(x-2)}\to\dfrac{-8}{0}\to-\infty$ $\lim\limits_{x\to\infty}-\dfrac{x^2}{x^2}=-1$

(D) $-\dfrac{5(x+1)(x-4)}{(x-4)(x-2)}$ $\lim\limits_{x\to 4^-}-\dfrac{5(x+1)(x-4)}{(x-4)(x-2)}=-\dfrac{(25)}{(2)}$

10. Which of the following statements about the graph of the rational function $y=\dfrac{(x-2)^2(x+3)(x-5)^6}{(x-2)^3(x+3)(x-5)^2}$ is correct?

(A) The graph has three vertical asymptotes and no holes.

(B) The graph has two vertical asymptotes and one hole.

(C) The graph has one vertical asymptote and two holes.

(D) The graph has no vertical asymptotes and three holes.

$y=\underbrace{\dfrac{(x-5)^4}{(x-2)}}_{\text{VA } x=2}\cdot\underbrace{\dfrac{x+3}{x+3}}_{\text{hole } x=-3}\cdot\underbrace{\dfrac{(x-5)^2}{(x-5)^2}}_{\text{hole } x=5}$