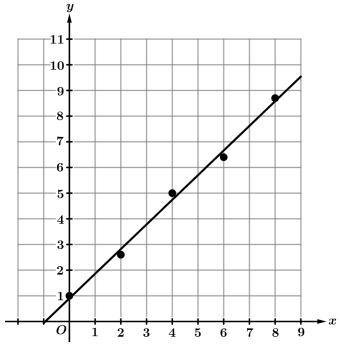
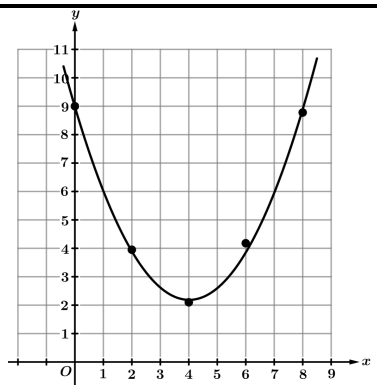
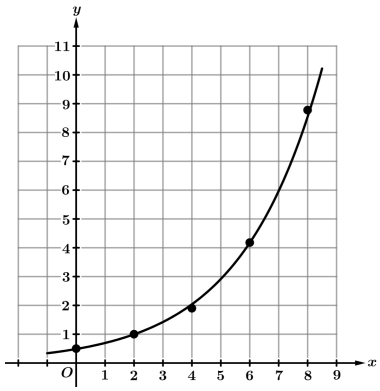
Throughout this course, we will have many opportunities to work with and create models based on a given data set. Three particularly important examples of this are **linear**, **quadratic**, and **exponential** models. Because these three types of models occur frequently in the real world, there is a heightened focus on them in AP Precalculus.

On the AP Exam, students will be expected to determine which model is most appropriate given a set of data. There are a few strategies and methods that we can use to determine whether a linear, quadratic, or exponential model is most appropriate for a given data set.

Comparing Linear, Quadratic, and Exponential Models

Type of Model	Example Graph/Data	Example Data Table	Important Features
Linear		x | y 0 | 1 2 | 2.6 4 | 5 6 | 6.4 8 | 8.7	Use a **linear model** when the data reveals a relatively **constant rate of change**.
Quadratic		x | y 0 | 9 2 | 3.95 4 | 2.1 6 | 4.2 8 | 8.8	Use a **quadratic model** when the **rates of change** are **increasing/decreasing** at a relatively **constant rate**. Data generally follows a "**u**"**-shaped** pattern (or upside-down).
Exponential		x | y 0 | 0.5 2 | 1 4 | 1.9 6 | 4.2 8 | 8.8	Use an **exponential model** when the **output values** are roughly **proportional**. **Each successive output** is approximately the result of **repeated multiplication**.

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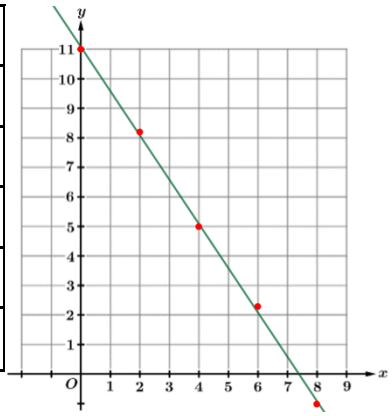
Example 1: Selected values from several functions are given in the tables below. Sketch the scatterplot for each table. Then determine if a linear, quadratic, or exponential model is most appropriate.

a)

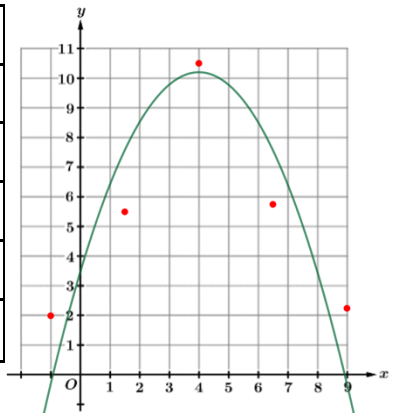
x	$f(x)$
0	11
2	8.2
4	5
6	2.3
8	−1



Linear

b)

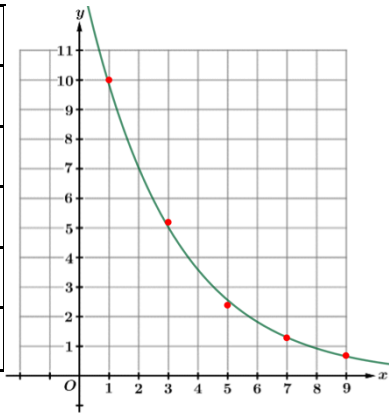
x	$g(x)$
−1	2
1.5	5.5
4	10.5
6.5	5.75
9	2.25



Quadratic

c)

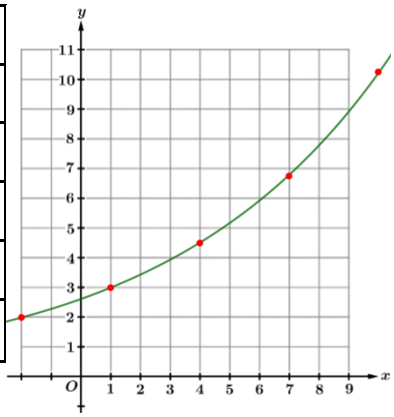
x	$h(x)$
1	10
3	5.2
5	2.4
7	1.3
9	0.7



Exponential

d)

x	$k(x)$
−2	2
1	3
4	4.5
7	6.75
10	10.25



Exponential

Residuals

When we create a model, we can use our model to **predict** values for the **dependent** variable (output) given an independent variable (input).

When using a model, it is important to remember this is only a **PREDICTED** value.

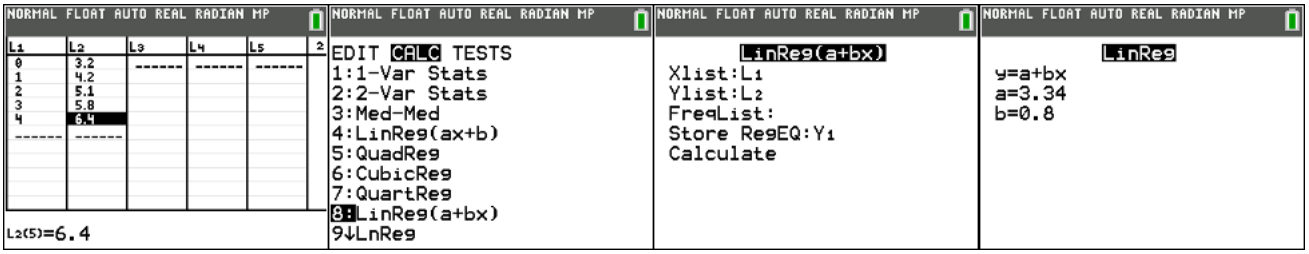
If we use the input value from a known data point to find a predicted output using our model, the difference between the actual output and our predicted output is called a **residual**.

Residual = Actual Output Value – Predicted Output Value

t	$W(t)$
0	3.2
1	4.2
2	5.1
3	5.8
4	6.4

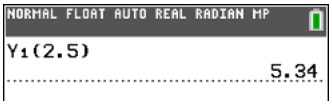
Example 2: The weight of newborn babies can be modeled by a linear function for the first four months after birth. Selected values for the weight $W(t)$, in kilograms, of a particular newborn baby are given in the table above, where t represents the number of months since birth.

a) Use the regression capabilities on your calculator to data find a linear model of the form $y = a + bx$ for the weight (in kg) of this particular baby x months after birth. $w(x) = 3.34 + 0.8x$



b) Use the model found in part a to predict the weight (in kg) of this baby 2.5 months after birth.

$w(2.5) = 3.34 + 0.8(2.5) = 5.34$ kg 2.5 months after birth.



c) The actual weight of this baby 2.5 months after birth was 5.5 kilograms. What is the residual for this weight? Did our model underestimate or overestimate the weight of this baby 2.5 months after birth?

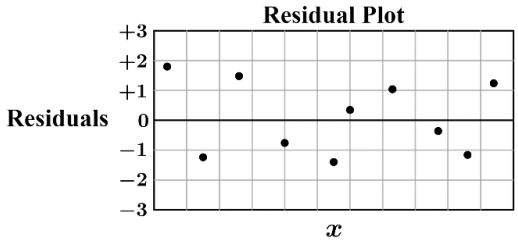
Residual $= 5.5 - 5.34 = 0.16$ The model underestimated the weight of the baby 2.5 months after birth.

Using a Residual Plot to Check the Appropriateness of a Model

A residual plot shows all the residuals for a set of data and can be used to help determine if a model is appropriate for a given set of data.

If a model for a given set of data is appropriate, the residual plot should appear without pattern.

If we can see a clear pattern in a residual plot, then the model used for the regression was not appropriate!



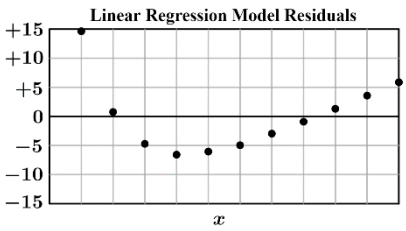
Example 3: An exponential regression was used to model a data set. The residual plot for the exponential regression model is shown above. Which of the following is the best conclusion about the appropriateness of the exponential regression model based on the corresponding residual plot?

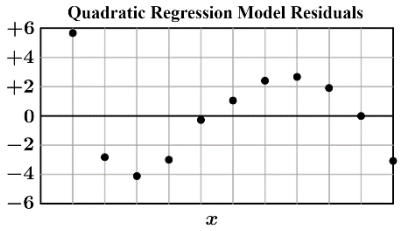
(A) The exponential model is not appropriate because the residuals show no pattern.

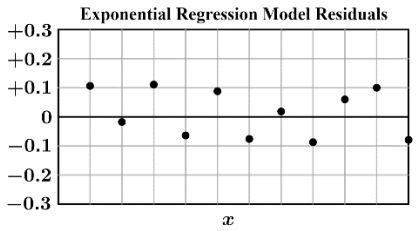
(B) The exponential model is not appropriate because the residuals show a pattern.

(C) The exponential model is appropriate because the residuals show no pattern.

(D) The exponential model is appropriate because the residuals show a pattern.







Example 4: A group of AP Precalculus students used a set of data to create linear, quadratic, and exponential regression models. After creating the three models, the students created a residual plot for each model type (see above). Based on the three residual plots above, which model was most appropriate for the data? Give a reason for your answer based on the residual plots above.

The exponential model was most appropriate for the data because the residual shows no pattern.

Example 5: Mr. Passwater hopes to make it into the Guiness Book of World Records by painting the world's biggest mural on the sides of several downtown buildings. His mural will consist of many painted circles of various sizes, where each circle is painted a different color. He wants to create a model to determine how much paint is needed (in quarts) for a circle of radius r (in feet).

a) Should Mr. Passwater use a linear, quadratic, or exponential model in this situation? Explain your reasoning.

Mr. Passwater should use a quadratic model because the amount of paint depends on the area of each circle given by the model $A(r) = \pi r^2$.

b) After creating his model, Mr. Passwater uses it to purchase different special paint colors to use on his mural. In this situation do you think it is more appropriate for the model to underestimate or overestimate the actual amount of paint needed? Give a reason for your answer.

It is more appropriate for the model to overestimate because Mr. Passwater does not want to run out of paint.