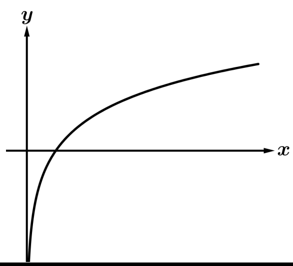
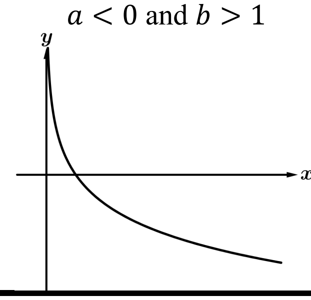
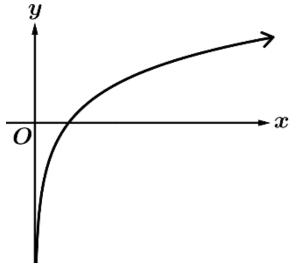
Because logarithmic functions and exponential functions are inverse functions, the characteristics of their graphs will have inverse relationships.

Key Characteristics of Logarithmic Functions	
A logarithmic function has the general form $f(x) = a\log_b x, \quad b > 0$ where a and b are constants with $a \neq 0$ and $b \neq 1$.	**Domain**: $(0, \infty)$ **Range**: $(-\infty, \infty)$
Logarithmic Functions $a > 0$ and $b > 1$ 	**Logarithmic Functions** $a < 0$ and $b > 1$ 
Increasing vs. Decreasing Logarithmic functions are **always increasing** or **always decreasing**! They will never switch from one to the other, so they have **no relative (local) extrema** (unless on a closed interval).	**Concave Up vs. Concave Down** Logarithmic functions are **always concave up** or **always concave down**! They will never switch concavity, so they have **no points of inflection**.
End Behavior For logarithmic functions in general form, as the input values (x) increase without bound, the output values (y) will increase/decrease without bound. Since logarithmic functions have a restricted domain, they are vertically asymptotic to $x = 0$. As a result, the left end behavior will occur as $x \to 0^+$.	**End Behavior Limit Statements** $\lim_{x\to 0^+} a\log_b x = \pm\infty$ and $\lim_{x\to +\infty} a\log_b x = \pm\infty$

Example 1: Write limit statements for the end behavior of the following logarithmic functions.

a)

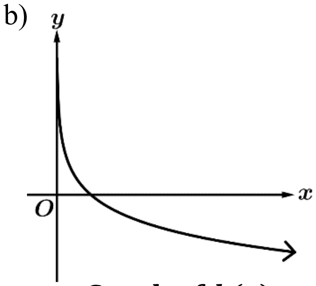


Graph of $f(x)$

Left: $\lim_{x\to 0^+} f(x) =$ _______

Right: ______________

b)



Graph of $h(x)$

Left: ______________

Right: ______________

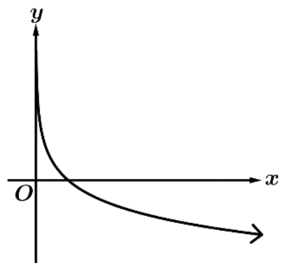
c) $g(x) = 2\log_3 x$

Left: ______________

Right: ______________

Example 2: For each of the following, determine if the logarithmic function is increasing/decreasing and concave up/down.

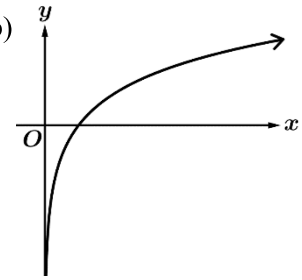
a)



Concave Up or Concave Down

Increasing or Decreasing

b)



Concave Up or Concave Down

Increasing or Decreasing

c) $h(x) = -4\log_6 x$

Concave Up or Concave Down

Increasing or Decreasing

Example 3: Selected values of the several logarithmic functions are shown in the tables below. For each table, find the value of the constant k.

x	$f(x)$
1	1
2	2
k	3
8	4
16	5

x	$g(x)$
k	0
6	5
18	10
54	15
162	20

x	$h(x)$
4	10
5	0
7	-10
k	-20
19	-30

x	$l(x)$
e^{-2}	7
e	14
k	21
e^{7}	28
e^{10}	35

Example 4: Find the domain and range of the following logarithmic functions.

a) $f(x) = 2\log_3 x$

Domain: ______________

Range: ______________

b) $g(x) = -5\log_2(x-3)$

Domain: ______________

Range: ______________

c) $h(x) = 8\log(2x+3)$

Domain: ______________

Range: ______________