Notes: (Topic 2.2) Change in Linear and Exponential Functions Solutions

In the last section, we learned that arithmetic sequences behave like linear functions and that geometric sequences behave like exponential functions.

We will look more closely at these connections today.

Arithmetic Sequences and Linear Functions	
Arithmetic Sequences	**Linear Functions**
$a_n = a_0 + dn$	$f(x) = b + mx$ **Slope-Intercept Form**
$a_n = a_k + d(n - k)$	$f(x) = y_i + m(x - x_i)$ **Point-Slope Form**

Geometric Sequences and Exponential Functions	
Geometric Sequences	**Exponential Functions**
$g_n = g_0 r^n$	$f(x) = ab^x$ or $f(x) = ar^x$
$g_n = g_k r^{(n-k)}$	$f(x) = y_i r^{(x-x_i)}$

It is important to recognize and understand the similarities and differences between **linear** and **exponential** functions.

Linear Functions vs. Exponential Functions	
Linear Functions	**Exponential Functions**
$f(x) = b + mx$	$f(x) = ab^x$
Over equal-length input-value intervals, the output values change at a **constant rate**.	Over equal-length input-value intervals, the output values change **proportionately**.
The change (m) in y is based on **addition**.	The change (b) in y is based on **multiplication**.
If you have **two points**, you can write the equation of a linear function, exponential function, arithmetic sequence, or a geometric sequence.	

Example 1: Selected values of several functions are given in the table below. For each table, determine if the function could be linear, exponential, or neither. Give a reason for your answer.

a)

x	$f(x)$
0	7
3	5
6	3
9	1
12	-1

-2, -2, -2, -2

Linear because over equal-length input-value intervals, the output values of $f(x)$ change at constant rate.

b)

x	$g(x)$
1	0
2	1
3	4
4	9
5	16

1, 3, 5, 7

Neither because over equal-length input-value intervals, the output values of $g(x)$ do not change at a constant rate or proportionally.

c)

x	$h(x)$
0	1
2	2
4	4
6	8
8	16

$\times 2$, $\times 2$, $\times 2$, $\times 2$

exponential because over equal-length input-value intervals, the output values of $h(x)$ change proportionally.

d)

x	$k(x)$
5	80
10	40
15	20
20	10
25	5

$\div 2$, $\div 2$, $\div 2$, $\div 2$

exponential because over equal-length input-value intervals, the output values of $k(x)$ change proportionally.

Example 2: A wild rumor is spreading that Mr. Passwater won 3rd place in the World's Strongest Man Contest (Mr. Passwater ~~definitely~~ probably didn't start the rumor). The number of people that have heard the rumor can be modeled using a geometric sequence, where the 43 people had heard the rumor on day 3 and 140 people have heard the rumor on day 6. According to the model, how many people, to the nearest whole number, have heard the rumor by day 10?

$P(n) =$ the number of people that have heard the rumor on day n $\quad P(3) = 43 \quad P(6) = 140$

$P(n) = P(3)r^{(n-3)} = 43r^{(n-3)} \quad P(6) = 43r^{(6-3)} = 140$

$43r^{(3)} = 140 \Rightarrow r^3 = \dfrac{140}{43} \Rightarrow r = \left(\dfrac{140}{43}\right)^{\frac{1}{3}} \quad P(n) = 43\left(\dfrac{140}{43}\right)^{\frac{1}{3}(n-3)} \quad P(10) = 43\left(\dfrac{140}{43}\right)^{\frac{1}{3}(10-3)} = 675.5758\ldots$

676 people have heard the rumor by day 10.

Example 3: A large theater has rows of seats where the number of seats in each row can be modeled by an arithmetic sequence. If the fifth row has 31 seats and the eleventh row has 49 seats, determine how many seats there are in the twenty-fifth row.

$s(n) =$ the number of seats in row $n \qquad s(n) = s(5) + m(n-5)$

$s(11) = 31 + m(11-5) = 49$

$49 = 31 + m(6) \Rightarrow m = \dfrac{49-31}{6} = \dfrac{18}{6} = 3 \qquad s(25) = 31 + 3(25-5) = 31 + 3(20) = 91$