Notes: (Topic 2.12) Logarithmic Function Manipulation Solutions

Since logarithmic expressions can also be written in exponential form, the same properties that apply to exponents can also be applied to expressions involving logarithms. These properties allow us to manipulate logarithmic expressions to form equivalent expressions that may be useful when solving equations or graphing functions involving logarithms.

Properties of Logarithms		
Product Property	**Quotient Property**	**Power Property**
$\log_b(xy) = \log_b x + \log_b y$	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log_b x^n = n\log_b x$
Example: $\log_2(5x) = \log_2 5 + \log_2 x$	**Example:** $\log_3\left(\frac{y}{4}\right) = \log_3 y - \log_3 4$	**Example:** $\log_7 3^x = x\log_7 3$

These properties can be used to show that various transformations of logarithmic functions can be written in equivalent ways.

Example 1: For each of the following, write an equivalent expression by condensing each expression to a single logarithm.

a) $\log_4 x + \log_4 y$

$\log_4(xy)$

b) $\log_3 5 - \log_3 z$

$\log_3\left(\frac{5}{z}\right)$

c) $\log_{10} x - \log_{10} 5 - \log_{10} z$

$\log_{10}\left(\frac{x}{5z}\right)$

d) $3\log_2 x - \log_2 y$

$\log_2\left(\frac{x^3}{y}\right)$

e) $2\log_7 a - 5\log_7 b + \log_7 4$

$\log_7\left(\frac{4a^2}{b^5}\right)$

f) $2 + \log_6 x$

$\log_6(36x)$

Example 2: Which of the following expressions is equivalent to $\log_3\left(\frac{x^2}{y}\right)$?

(A) $\log_3(2x) - \log_3 y$

(B) $2(\log_3 x - \log_3 y)$

(C) $2\log_3 x - \log_3 y$

(D) $\log_3 x + \log_3 2 - \log_3 y$

Example 3: The function $f(x) = \log(6x)$ is a horizontal dilation of the function $g(x) = \log(x)$. Show that f can also be written as a vertical translation of g, with $f(x) = g(x) + k$, where k is a constant.

$f(x) = \log x + \log 6 = g(x) + k$ where $k = \log 6$

Change of Base Property
$\log_b x = \dfrac{\log_a x}{\log_a b}$ where $a > 0$ and $a \neq 1$
Note: This property illustrates that all logarithmic functions are vertical dilations of each other!

Example 4: Let $f(x) = \log_4 x$ and $g(x) = \log_9 x$. Show that f is a vertical dilation of g by finding the value of k such that $f(x) = k \cdot g(x)$.

$f(x) = \log_4 x = \dfrac{\log_9 x}{\log_9 4} = \dfrac{1}{\log_9 4} g(x) \quad k = \dfrac{1}{\log_9 4}$

The Natural Logarithm Function
The logarithm with base e is called the natural logarithm: $\log_e x = \ln x$.

Example 5: Which of the following expressions is equivalent to $3\ln x - 4\ln y$?

(A) $\ln\left(\dfrac{x^3}{y^4}\right)$ (B) $\ln\left(\dfrac{3x}{4y}\right)$ (C) $\ln(x^3 - y^4)$ (D) $\ln(3x - 4y)$