Previously (in Topics 3.4 – 3.5), we learned how to identify the midline, amplitude, and period from the graph of a sinusoidal function. Now, we will learn how those properties affect the equations of sinusoidal functions.

To do this, let's consider how the midline, amplitude, and period affect the graph of a sinusoidal function through the lens of transformations that we learned in Unit 1.

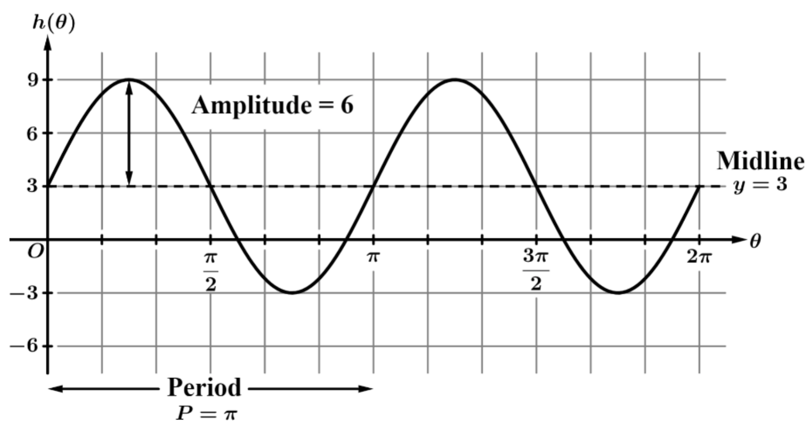
The **midline** of a sinusoidal function is simply a **vertical translation**.

The **amplitude** of a sinusoidal function is the **vertical dilation**.

The **period** of a sinusoidal function is the result of a **horizontal dilation**.

Given a function written as $f(\theta)=a\sin(b\theta)+d$ or $k(\theta)=a\cos(b\theta)+d$, the graphs of f and k will have the following transformations:

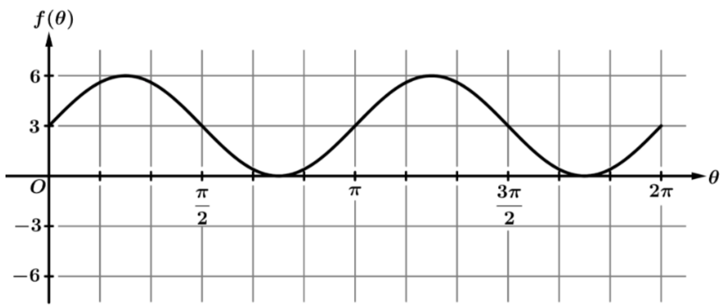
1. A vertical dilation by a factor of a. (a represents the amplitude of the graph)
2. A vertical translation of d units. (d represents the midline of the graph)
3. A horizontal dilation by a factor of $\left|\frac{1}{b}\right|$. (The period is given by $P=\left|\frac{2\pi}{b}\right|$)



Graph of *h*

Example 1: The graph of the sinusoidal function h is shown in the figure above, along with information about the amplitude, midline, and period of the graph. The function h can be written as $h(\theta)=a\sin(b\theta)+d$. Find the values of the constants a, b, and d.

$a = 6$ the amplitude period $= \pi = \frac{2\pi}{b} \Rightarrow b = 2$ $d = 3$ the veritical translation

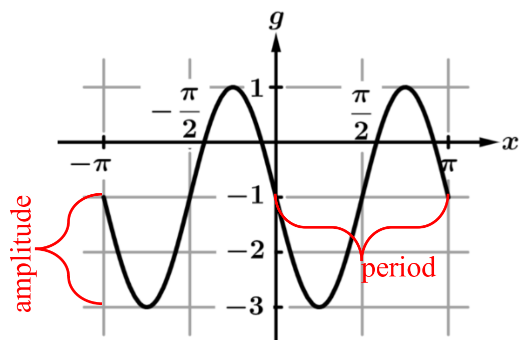


Graph of f

Example 2: The figure shows the graph of a trigonometric function f. Write an expression for $f(\theta)$.

The midline is $y = 3$, so there is a vertical translation of $d = 3$. The period is π, so $\pi = \frac{2\pi}{b} \Rightarrow b = 2$.

The amplitude is 3 so $a = 3$ $\qquad f(\theta) = 3\sin(2\theta) + 3$



Graph of g

Example 3: The figure shows the graph of a sinusoidal function g. What are the values of the period and amplitude of g?

(A) The period is π, and the amplitude is 2.

(B) The period is π, and the amplitude is 4.

(C) The period is 2π, and the amplitude is 2.

(D) The period is 2π, and the amplitude is 4.

Example 4: The trigonometric function k has a maximum at the point $(0, 6)$. After this maximum, the next minimum occurs at the point $\left(\frac{\pi}{2}, -4\right)$. Which of the following could be an expression for $k(x)$?

(A) $10\cos(\pi x) + 1$

(B) $10\cos(2x) + 1$

(C) $5\cos(\pi x) + 1$

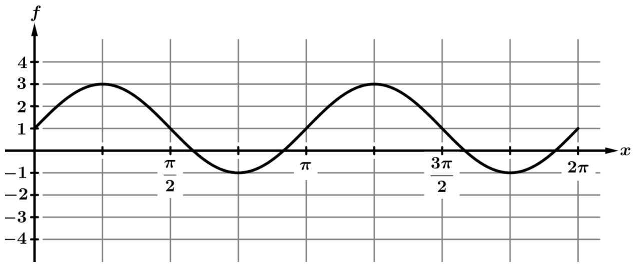
(D) $5\cos(2x) + 1$

Half the period is from $x = 0$ to $x = \frac{\pi}{2}$ so the period is π. $\quad \pi = \frac{2\pi}{b} \Rightarrow b = 2$.

The midline is $\frac{6 + (-4)}{2} = 1$ and the amplitude is $6 - 1 = 5$.

Example 5: The trigonometric function f is given by $f(x)=3\sin(2x)-1$. Which of the following could be the graph of $f(x)$? amplitude is 3 and midline is $y=-1$ with a period of π $b=2 \Rightarrow \frac{2\pi}{b}=\pi$

(A)



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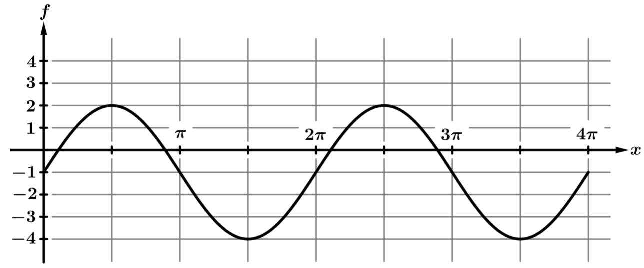

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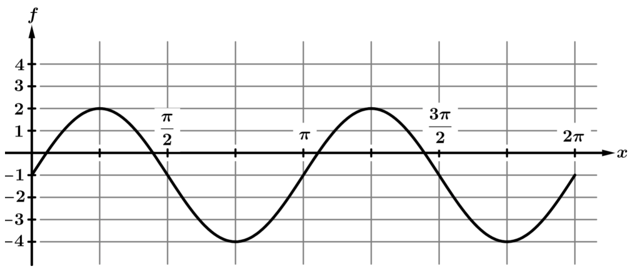
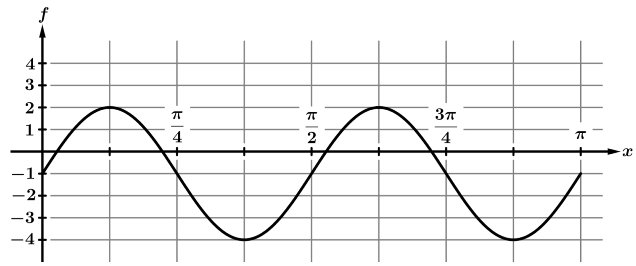


(D)

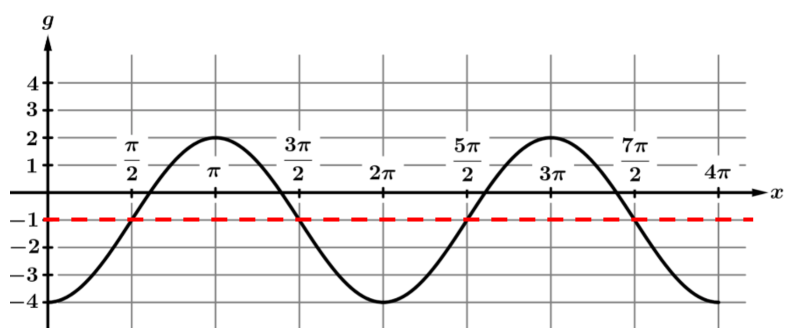


Phase Shift of a Sinusoidal Function

A horizontal translation of a sinusoidal function is called a **phase shift.**

The graph of $g(x)=\sin(x+c)$ is a **phase shift** of the graph of $f(x)=\sin(x)$ by $-c$ units.

The same results can be applied to the cosine function.



Graph of g

Example 6: The figure shows the graph of a trigonometric function g. Which of the following could be an expression for $g(x)$? All choices show a midline $y=-1$ and an amplitude of 3 and period of 2π with no horizontal dilation.

~~(A)~~ $3\cos(x)-1$

$3\cos(0)-1=3-1=2$
$-3\cos(x)-1$ would be a correct answer, the cosine graph reflected over the midline.

~~(B)~~ $-3\cos(x-\pi)-1$

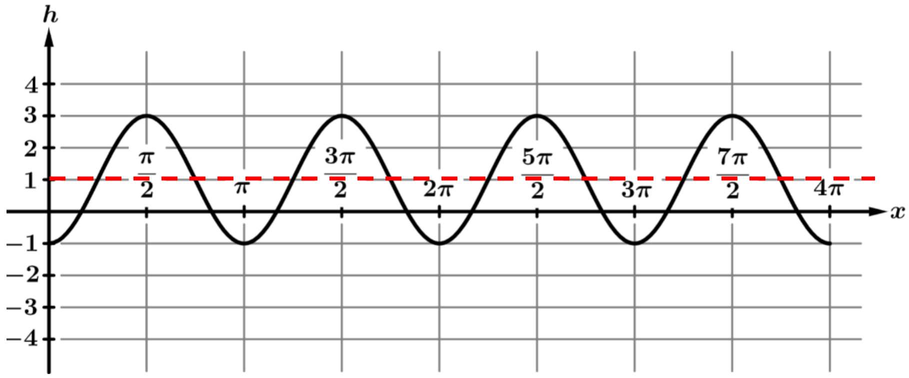
The reflection is in the function but there is horizontal shift to the right of π.

~~(C)~~ $3\sin\left(x+\frac{\pi}{2}\right)-1$

This looks like a sine graph shifted $\frac{\pi}{2}$ to the right but this choice is a shift to the left.

(D) $-3\sin\left(x-\frac{3\pi}{2}\right)-1$

Starting at $x=\frac{3\pi}{2}$, there is a sine curve reflected over the midline.



Graph of h

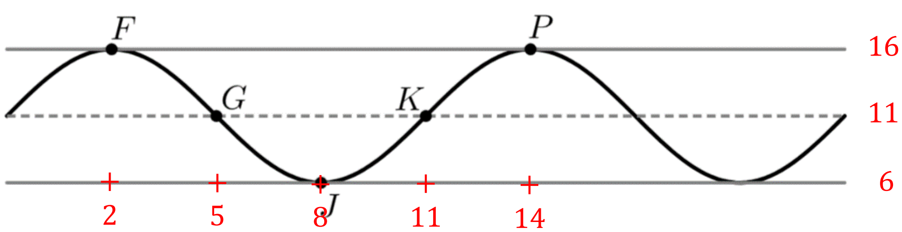
Example 7: The figure shows the graph of a trigonometric function h. Which of the following could be an expression for $h(x)$? All choices show a midline $y = 1$ and an amplitude of 2 with a period of π $\quad b = 2 \Rightarrow \frac{2\pi}{b} = \pi$

(A) $2\cos\left(2\left(x-\frac{\pi}{4}\right)\right)+1$ Starting at $x = \frac{\pi}{4}$ there is a sine curve, not a cosine curve.

(B) $2\cos(2(x-\pi))+1$ Starting at $x = \pi$, there is a reflected cosine curve, no reflection in this choice.

(C) $2\sin\left(2\left(x-\frac{\pi}{4}\right)\right)+1$ Starting at $x = \frac{\pi}{4}$ there is a sine curve, so (C) is the correct choice.

(D) $2\sin(2(x-\pi))+1$ Starting at $x = \pi$, there is not a sine curve.



Example 8: The graph of h and its dashed midline for two full cycles is shown. Five points, $F, G, J, K,$ and P are labeled on the graph. No scale is indicated, and no axes are presented.

The coordinates for the five points: $F, G, J, K,$ and P are: $F(2,16)$, $G(5,11)$, $J(8,6)$, $K(11,11)$, $P(14,16)$.

The function h can be written in the form $h(t) = a\sin(b(t+c))+d$. Find values of constants $a, b, c,$ and d.

Midline $y = 11 \Rightarrow \boxed{d = 11}$ $\quad$ Amplitude is $5 \Rightarrow \boxed{a = 5}$ $\quad$ The period is $12 \Rightarrow \frac{2\pi}{b} = 12 \Rightarrow \boxed{b = \frac{\pi}{6}}$

The sine curve is at its maximum when $b(t+c) = \frac{\pi}{2}$ $\quad$ At $t = 2$ then $\frac{\pi}{6}(2+c) = \frac{\pi}{2}$ $\quad (2+c) = 3$ $\quad \boxed{c = 1}$