14.02 Principles of Macroeconomics
Problem Set # 3
Due: October 23, 2009

October 16, 2009

1 True/False/Uncertain [30 points]

1. "In the Solow model, an economy that starts with a higher stock of capital per capita will reach a higher steady state level of capital per capita" [7 points]

2. " In the money market, a decrease in real income will tend to decrease the equilibrium interest rate (abstracting from the goods market)" [7 points]

3. "If prices adjust inmediately, then monetary policy will have no effect whatsover" [8 points]

4. "Fiscal policy cannot affect output in the long run" [8 points]

2 Solow Model [40 points]

Consider the basic solow model. Assume total labor is fixed at $L = 1$. Time is discrete, and indexed by $t = 0, 1, 2, ...$ The production function is Cobb Douglas:

$$y_t = f(k_t) = Ak_t^{\alpha}$$

where y_t is per capita income at time t and k_t is capital per capita at time t. The law of motion for the capital stock is

$$k_{t+1} = (1 - \delta) k_t + i_t$$

where i_t is per capita investment. The economy consumes a constant fraction of output:

$$c_t = (1 - s) y_t \tag{1}$$

where $s \in (0, 1)$. Assume that the economy starts with initial capital $k_0 > 0$.

1. Show that equation (1) implies that per capita investment is also a constant fraction of per capita income. Which equation, which was not stated in the set up of this question, do you need to show this? [4 points]

2. Using the previous result, state a law of motion for capital per capita. This is, an equation relating k_{t+1} to k_t. [4 points]

3. Compute the steady state capital per capita in this economy. Call it k. Also, compute the level of investment per capita (i) and income per capita (y) in the steady state. [6 points]

4. We will now derive an *analytical expression for the evolution of the per capita capital stock in this economy.* Assume from now on that $\boldsymbol{\delta = 1}$, that is, that the capital stock fully depreciates in one period.

   (a) Write k_1 as a function of k_0. [2 points]

   (b) Write k_2 as a function of k_1, and, using (a), as a function of k_0. [2 points]

   (c) Write k_3 as a function of k_0. [2 points]

   (d) Now, in light of the 3 previous answers, write an equation that generalizes to any t. That is, write k_t as a function of k_0 for any $t \geq 0$[1]. [7 points]

   (e) Show that the limit (as $t \to \infty$) of the expression you just got is exactly k (the steady state level of capital derived in point 4). That is, show $lim_{t\to\infty} k_t = k$. [Hint: Recall that for any constant $x \in (0,1)$: $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$] [3 points]

5. Using the analytical expression derived in point 5 (d), plot the evolution of the per capita capital stock in this economy. Take $s = 0.2$, $A = 1$, $\alpha = 0.5$ (and of course, $\delta = 1$). That is, plot k_t as a function of t. Do this graph for both the case in which $k_0 = 1$ and $k_0 = 0.01$.[10 points]

3 IS-LM [30 points]

Consider the following version of the IS-LM model, where expected inflation is zero ($\pi^e = 0$). Assume net exports (NX) are zero.

$$\begin{aligned} C &= c_0 + c_1 (Y - T) \\ I &= b_0 + b_1 Y - b_2 i \\ M^d/P &= d_1 Y - d_2 i \\ M^s &= M \end{aligned}$$

where all the variables are as defined in class, and T represents taxes.

[1] This procedure is called recursive substitution.

1. Find the IS relation. What assumption on the parameters do we have to impose to ensure that the goods market reaches an equilibrium? Show graphically happens to this relation when taxes increase. [6 points]

2. Derive the LM relation. What happens when M increases? Explain the intuition. [6 points]

3. Find the short-run equilibrium (i.e. find the pair (Y, i) that makes the goods and money markets both be in equilibrium at the same time) [6 points]

4. What happens to equilibrium output and the interest rate as taxes increase? Use your results from the previous part, as well as the IS-LM graph to illustrate your answer. [6 points]

5. What happens to equilibrium output and the interest rate as money supply increases? [6 points]

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