

14.02 Principles of Macroeconomics  
Problem Set # 2  
Due: October 2nd, 2009

September 25, 2009

**1 True/False/Uncertain [20 points]**

*Please state whether each of the following claims are True, False or Uncertain, and provide a brief justification for your answer.*

1. "An increase in the real interest rate makes current consumption relatively cheaper than future consumption" [6 points]
2. "The Keynesian consumption function, which implies that agents consume a constant fraction of their income, does not match short run household data" [7 points]
3. "If an agent is liquidity constrained, then an unexpected increase in transitory income of \$100 can result in the agent's current consumption increasing by \$100" [7 points]

**2 Labor market [40 points]**

Consider an economy in which the labor market is competitive. There is a large number of firms each with technology given by the following production function

$$Y = AK^\alpha N^{1-\alpha}$$

where  $Y$  denotes the (unique) consumption good,  $K$  denotes capital stock, and  $N$  denotes labor input (in hours) in production.

There is also a large number of agents who supply labor, with preferences given by

$$u(c, l) = \log(c) + \eta \log(l)$$

where  $l$  denotes leisure and  $c$  denotes consumption. Each of these agents has  $T$  hours of time (monthly) to allocate between leisure and labor. The only source of income for the agents is labor income.

Assume that the price of the consumption good (denote it by  $P$ ) is exogenously given and equal to 2. Use  $W$  to denote the hourly nominal wage in the economy

1. What is the marginal product of labor (MPN)? How does MPN change as  $N$  increases? Explain briefly why. [4 points]
2. What is the optimal amount of labor that each firm will demand? Sketch the labor demand as a function of the real wage (using the y-axis for the real wage, and x-axis for labor) [4 points]
3. Now suppose that the Government introduces a labor tax: for each hour of labor hired, the firm needs to pay  $\tau$  to the Government. Find the new optimal demand for labor. Is it higher or lower than in the absence of the tax? [4 points]
4. Consider now the decision problem of the agents. Let  $n$  denote the number of hours per month that an agent works. Write an equation relating  $n$  and  $l$ . Write the agent's budget constraint. Express the budget constraint in terms of  $c$  and  $l$ . State the agent's maximization problem. [4 points]
5. Use the fact that the marginal utility of consumption and leisure is given by

$$\begin{aligned} MU(C) &= 1/c \\ MU(L) &= \eta/l \end{aligned}$$

to find an equation that characterizes the optimal behavior of the agent regarding consumption and leisure. This should be an equation on  $c$  and  $l$  as a function of the real wage. Use this equation, and the budget constraint of the agent to find the optimal amount of leisure. Use the equation relating  $n$  and  $l$  from the previous question to find the optimal number of hours  $n$ . [4 points]

6. Suppose that the real wage increases. Does the agent work more, less or the same? Explain why. [2 points]
7. Suppose that  $\eta$  increases. What is the effect on the agents labor supply? [2 points]
8. Suppose that there are  $M_a$  agents in the economy (where  $M_a$  is a very large number). What is the total labor supply? (denote it by  $N^s$ ) Suppose that there are  $M_f$  firms in the economy (where  $M_f$  is a very large number). What is the total labor demand in the economy? (denote it by  $N^d$ ) (we continue to assume that the Government is taxing labor). [4 points]
9. Assume that  $M_f = M_a$ . Find the real wage that makes total labor supply and total labor demand exactly equal to each other (this is called, the equilibrium real wage). Find the equilibrium number of hours worked in this economy. [4 points]

10. For each of the following cases, explain what is the effect on the equilibrium wage and number of hours worked. Provide a brief sketch showing how labor supply and demand shift. Also provide intuition. [8 points]
- a. Increase in  $T$
  - b. Increase in  $K$
  - c. Increase in  $\tau$ .

### 3 Consumption [40 points]

Consider an economy with two periods (current and future). There are 2 agents (call them agent A and agent B). There is only one good in the economy<sup>1</sup>. Suppose that agent  $i \in \{A, B\}$  has an income of  $y^i$  in the current periods and  $y_f^i$  in the future period. Note that we are allowing income endowments to be different for different agents and periods. Both agents are forward looking (which just means they care about the present and the future). In particular, they consume according to the following rule

$$c^i = \alpha^i PVL R^i$$

where  $c^i$  is consumption in the current by agent i, PVL R is the present value of lifetime resources, and  $\alpha^A, \alpha^B$  are numbers such that  $\alpha^i \in (0, 1)$  for  $i = A$  and  $i = B$ . Note that we are allowing the consumption rule to vary across agents. Denote consumption in the future period for agent i by  $c_f^i$  and the real interest rate by  $r$ .

1. Let  $s^i$  denote agent i's savings in the current [In the lecture notes, this is just the leftover after consumption]. Write the budget constraint for each period for agent  $i$ . These are called "flow" budget constraints. Show how you can obtain an intertemporal budget constraint for agent i out of the two flow budget constraints. [Basically just substitute out " $s^i$ " to obtain an equation that relates the PVL R with  $c^i$  and  $c_f^i$ .] [5 points]
2. State the  $PVL R^i$  in terms of income in the two periods. [5 points]
3. For a given interest rate  $r$ , plot the combinations of  $c^i$  and  $c_f^i$  that the agent can choose. The set you just plotted is called the budget set. [5 points]
4. Show graphically the effect of an increase in the interest rate on the budget set of agent i. [5 points]
5. Express  $c^i, c_f^i$  as functions of  $\alpha^i, y^i, y_f^i$  and  $r$ . [6 points]

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<sup>1</sup>We will assume that its price is equal to 1.

6. Write an equation that states that the total demand for the good in the current period equals total supply [total demand is the sum of consumption by each agent, and total supply is the sum of each agent's income]. We call this equation the goods market clearing condition. Find the equilibrium interest rate in this economy, and denote it by  $r^*$  [this is just the interest rate that makes supply and demand coincide] [7 points]
7. Suppose  $y^A = y_f^B = 2$ ,  $y_f^A = y^B = 4$  and  $\alpha^A = \alpha^B = 1/2$ . Find the equilibrium interest rate, and the levels of consumption for both periods for each agent. Relate your numerical answers to the concept of consumption smoothing. Who is borrowing and who is lending in the current period in this economy? [7 points]

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