1 Multiple Choice (30 points)

Answer the following questions. You DO NOT need to justify your answer.

1. (6 Points) Consider an economy with two goods and two periods. Data are

   Good 1

   $p_t^1 = 1$

   $p_{t+1}^1 = 1$

   $q_t^1 = 1$

   $q_{t+1}^1 = 1.1$

   Good 2

   $p_t^2 = 1$

   $p_{t+1}^2 = 1.4$

   $q_t^2 = 1$

   $q_{t+1}^2 = 1.3$

   where q_t^1 stands for the quantity of good 1 produced in period t, q_t^2 stands for the quantity of good 2 produced in period t, p_t^1 stands for the price of good 1 in period t and p_t^2 stands for the price of good 2 in period t.

   The office of national accounts wants to calculate the real GDP growth rate in this economy using the chain index and they hire you to do so. What is the correct growth rate you should report to the office of national accounts?

   (a) 19.7%

   (b) 20.8%

   (c) 20.1%

   Remember that we can calculate growth in real GDP using the chain index with the following formula

$$
\begin{aligned}
g_{t+1,t} &= \frac{1}{2}\left(\frac{p_{t+1}^1 q_{t+1}^1 + p_{t+1}^2 q_{t+1}^2}{p_{t+1}^1 q_t^1 + p_{t+1}^2 q_t^2} + \frac{p_t^1 q_{t+1}^1 + p_t^2 q_{t+1}^2}{p_t^1 q_t^1 + p_t^2 q_t^2}\right) - 1 \\
&= \frac{1}{2}\left(\frac{1.1 + 1.82}{1 + 1.4} + \frac{1.1 + 1.3}{1 + 1}\right) - 1 \\
&= \frac{1}{2}\left(\frac{2.92}{2.4} + \frac{2.4}{2}\right) - 1 = 0.2083
\end{aligned}
$$

   which implies that the correct answer is b.

2. (6 Points) When we characterize labor supply as an upward sloping relationship between hours worked and wages, we assume that

   (a) the substitution effect dominates the income effect.

   (b) the income effect dominates the substitution effect.

   (c) the income and the substitution effect cancel out making the relationship between hours worked and wages unambiguous.

   Remember that the budget constraint of an worker is $wT = pc + wl$ where T is the total number of hours available to distribute between leisure and work, w is the hourly wage rate, p is the price of the consumption good, c id the consumption of the consumption good, and l is hours of leisure consumed by the agent.

   Notice that an increase in the wage, w has two effects: on the one hand it increases the left-hand side of the equation, which makes the agent consume more c and enjoy more leisure, hence work less. On the other hand, an increase in w makes the price of leisure more expensive, hence making leisure decrease.

   When we assume that hours worked increase with the wage we assume that the second effect is stronger than the first one, making a the correct answer.

3. (6 Points) According to the Solow growth model, which of the following statements is FALSE?

   (a) A country which experiences higher population growth than another will have a lower output per worker in steady state.

   (b) Steady state consumption in a country which saves more will always be higher than steady state consumption of a country with a lower savings rate.

   (c) Capital accumulation alone can not sustain long run growth in capital per worker.

   We know from the Solow model that population growth depreciates capital per worker which causes countries with high population rate to have low levels of capital per worker. This makes the statement in a correct.

   We also know that if there is no technological growth, in the long run (in stead state) capital per worker will stop growing, which makes the statement in c correct.

   Finally notice that the statement in b is false. To see this, compare the steady state consumption of a country with the maximum level of savings, $s = 1$. This country will have a steady state level of consumption c^* equal to

$$c^*(s = 1) = (1 - s)AF(k^*, 1) = 0$$

However a country with a smaller savings rate, $s < 1$ will have a consumption level of

$$c^* = (1 - s)AF(k^*, 1) > 0$$

as long as $k^* > 0$.

4. (6 Points) Consider a consumer who follows a PIH consumption rule and who is a saver in the first period (i.e., in the first period he consumes less than his current income). Which of the following statements is TRUE?

   (a) An increase in the interest rate in the first period will have no effect in the first period's consumption since current consumption only depends on current income.

   (b) An increase in the interest rate will make the price of the first period's consumption compared to the second period's consumption more expensive, which means that the first period's consumption will decrease unambiguously.

   (c) An increase in the interest rate makes the price of the first period's consumption compared to the second period's consumption more expensive but makes the consumer richer, hence the effect of the first period's consumption after an increase in the interest rate is ambiguous.

   Statement a is false because the consumption of a PIH consumer is a function of current an future income.
   Statement b is false. Notice that the budget constraint of a PIH consumer is

   $$(1 + r)y_1 + y_2 = c_1(1 + r) + c_2$$

   This means that an increase in the interest rate will have two effects. The first effect is that it makes future consumption cheaper compared to current consumption (substitution effect) which makes the current consumption decrease. However, there is a second effect, the increase in the interest rate makes the agent richer (since he is a saver) making him want to consume more in both periods. Hence the effect of an increase in the interest rate on consumption is ambiguous.

5. (6 Points) The relationship between investment and the interest rate will be negative:

   (a) Only for firms who finance investment through borrowing.

   (b) Because the interest rate determines the opportunity cost of investing, and is a component of the user cost of capital.

(c) Because the interest rate decreases the marginal product of capital making firms want to install less capital.

Remember that the user cost of capital is $p_k(r+\delta)$, where p_k is the price of capital goods, r is the real interest rate, and δ is the depreciation. The optimal capital choice of the firm comes from the optimality condition

$$MPK = \text{user cost of capital}$$

where MPK is $F_k(K, N)$. This implies that the effect of the interest rate on investment works through the user cost of capital.

2 Consumption with Borrowing Constraints (35 points)

Consider a PIH consumer, Anna, who receives an income of \$4 when she is young and an income of \$10 when old. Anna is born with no assets, so $a = 0$. The real interest rate that Anna faces for borrowing and saving is equal to r.

1. (7 points) Write down Anna's intertemporal budget constraint, *i.e.* the budget constraint that relates Anna's lifetime income with Anna's lifetime consumption. Explain briefly what this budget constraint tells us.

   Anna's intertemporal budget constraint is

   $$4 + \frac{10}{1+r} \geq c^y + \frac{c^o}{1+r}$$

   This intertemporal budget constraint tells us that the lifetime present value of Anna's income has to be greater or equal than the present value of Anna's consumption.

2. (7 points) Assume that preferences are logarithmic, that is $U(c^y, c^o) = \ln c^y + \beta \ln c^o$, where $\beta \in [0,1]$. Use Anna's optimality condition, $c^o = \beta(1+r)c^y$, and the budget constraint you found in part 1 to find Anna's consumption when young and old as a function of the discount factor, β, the real interest rate, r, the income in period 1 and the income in period 2. Calculate Anna's savings (or borrowing) when young. Under what condition will Anna be a borrower in the first period? Explain briefly how Anna's consumption when young differs from an agent who follows a Keynesian consumption rule, where consumption at time t is equal to $c_t = 0.9Y_t$, where Y_t is income at time t.

Using the optimality condition and the budget constraint we can write the Anna's consumption when young as

$$c^y = \frac{1}{1+\beta}\left[4 + \frac{10}{1+r}\right]$$

which implies that young Anna consumes a fraction $\frac{1}{1+\beta}$ of her present value life-time income. This consumption is higher the the lower β and the lower r.

Anna's consumption when old will be

$$c^o = \frac{\beta(1+r)}{1+\beta}\left[4 + \frac{10}{1+r}\right]$$

Notice that Anna's savings will be

$$\begin{aligned} s &= 4 - c^y \\ &= \frac{1}{(1+\beta)(1+r)}\left(4\beta(1+r) - 10\right) \end{aligned}$$

This implies that Anna will be a borrower in the first period if

$$s < 0$$

which holds if

$$\beta(1+r) < \frac{5}{2}$$

3. (7 points) Suppose that Anna's income when young increases by ε but her income when old remains constant. Calculate the increase in Anna's consumption when young, Δc^y, and calculate Anna's marginal propensity to consume when young ($\Delta c^y/\Delta y^y$). Compare this to the marginal propensity to consume of a Keynesian consumer and briefly comment on the differential response of the two consumers to transitory shocks.

   The change in Anna's consumption will be

$$\Delta c^y = \frac{1}{1+\beta}\varepsilon$$

   which implies that her marginal propensity to consume will be

$$\frac{\Delta c^y}{\Delta y^y} = \frac{1}{1+\beta}$$

   Notice that the Anna's marginal propensity to consume will be smaller than that of a Keynesian consumer if

$$\beta > \frac{1}{9}$$

   Notice that the more weight Anna puts on future consumption the less she will respond to transitory shocks when young in order to smooth consumption between periods.

4. (7 points) Now suppose that Anna faces liquidity constraints, which means Anna can save but is unable to borrow when she is young. Draw the budget constraint in the c^y, c^o axis. Using the same income stream as above and setting $r = 0, \beta = 1$, calculate Anna's optimal consumption decision when young. Explain briefly how this compares to the consumption when Anna is young and faces no liquidity constraints.

   Anna's budget constraint will be the intersection of the following two restrictions

$$\begin{aligned} c^o &\leq 10 + (4 - c^y) \\ c^y &\leq 4 \end{aligned}$$

   Notice that Anna's decision if she could borrow would imply

$$c^y = 7$$

   This however can't be attained because of Anna's inability to borrow. This will imply that Anna's consumption rule will be

$$c^y = \begin{cases} y^y \text{ if } y^y \leq y^o \\ \frac{y^y + y^o}{2} \text{ if } y^y > y^o \end{cases}$$

   Anna's problem is that expecting to be richer in the future, Anna would like to consume more today than her current income. That is, Anna would like to be a borrower, but she is unable to do so.

5. (7 points) Suppose as above that Anna faces liquidity constraints when young and her income when young increases by ε but her income when old remains constant. Calculate the increase in Anna's consumption when young, Δc^y and calculate Anna's marginal propensity to consume when young $(\Delta c^y/\Delta y^y)$. How does Anna's marginal propensity to consume compares to the marginal propensity to consume of a Keynesian consumer? Comment.

   Notice now that as long as $\varepsilon < 6$ Anna's liquidity constraint will still bind. This implies that Anna's consumption when young will vary one to one with y^y. This implies that Anna's marginal propensity to consume of both permanent and transitory shocks will be 1 when $\varepsilon < 6$. This highlights that under liquidity constraints PIH agents behavior becomes closer to that of a Keynesian consumer.

3 Technological Change and the Labor Market (35 points)

In country B there is a firm that produces the unique good of the economy using the following production function:

$$Y = F(L_s, L_u) = \left(AL_s^{1/2} + L_u^{1/2}\right)^2$$

where A is a technological parameter, L_s is the number of hours of skilled workers the firm hires monthly, and L_u is the number of hours of unskilled workers the firm hires monthly.

There are two groups of agents in the economy, one group is composed of skilled agents and the other group is composed of unskilled agents. Both groups are equally sized ($N^S = N^U = N$) and have the same preferences over consumption and leisure which are represented by the following utility function

$$U(c, l) = \ln c + \eta \ln l$$

where η is a positive constant. The only source of income of the consumers is their wage income, and the two groups of consumers only differ in the wage they receive, which is w^S for skilled workers and w^U for unskilled workers. The monthly time endowment of a worker is T. Both the firm and the consumers are price takers, meaning they take the wages, w^S, w^U, and the price of the final good, P, as given.

1. (3 points) Show that the production function satisfies constant returns to scale in the two labor types.

   Notice that if we multiply the two labor types by a constant k we have that

   $$\mathbf{F}(\mathbf{kL}_s, \mathbf{kL}_u) = \mathbf{k}\left(AL_s^{1/2} + L_u^{1/2}\right)^2 = \mathbf{kF}(\mathbf{L}_s, \mathbf{L}_u)$$

   hence, the production function exhibits constant returns to scale.

2. (5 points) Using the firm's optimality condition we have seen in class, and using the fact that the marginal product of worker of type s is

$$AL_s^{-1/2}\left(AL_s^{1/2}+L_u^{1/2}\right)$$

and the marginal product of labor of a worker of type u is

$$L_u^{-1/2}\left(AL_s^{1/2}+L_u^{1/2}\right),$$

find the relative demand of skilled workers to unskilled workers (L^S/L^U) as a function of the relative wage (w^S/w^U) and the technological parameter.

The maximization problem of the firm is

$$\max_{L_s,L_u}\left\{P\left(AL_s^{1/2}+L_u^{1/2}\right)^2-w_sL_s-w_uL_u\right\}$$

and the optimal demand of the two types of labor satisfy the following two equations:

$$PAL_s^{-1/2}\left(AL_s^{1/2}+L_u^{1/2}\right)=w_s$$

$$PL_u^{-1/2}\left(AL_s^{1/2}+L_u^{1/2}\right)=w_u$$

The wage gap between skilled and unskilled workers will be

$$\frac{w_s}{w_u}=A\left(\frac{L_s}{L_u}\right)^{-1/2}$$

or

$$\frac{L_s}{L_u}=A^2\left(\frac{w^S}{w^U}\right)^{-2}$$

3. (4 points) Using the time constraint $T = n + l$, where n is number of hours worked,write the budget constraint of a consumer of type s and the budget constraint of a consumer of type u as a function of leisure, consumption, the wage rate of the type, the price P, and T.

   The budget constraint is

$$Tw_i = Pc + w_i l$$

4. (5 points) Using the time constraint $T = n+l$, the budget constraint, and the fact the marginal utility of consumption and the marginal utility of leisure are given by

$$\begin{aligned} MU(c) &= 1/c \\ MU(l) &= \eta/l, \end{aligned}$$

find the optimal labor supply of individuals and the optimal demand of the final good for skilled and unskilled consumers.

The optimality condition for the consumer is

$$\begin{aligned} MU(c)/MU(l) &= P/w_i \\ l &= \frac{P}{w_i}c\eta \end{aligned}$$

Using this in the budget constraint we find

$$c = \frac{Tw_i}{P(1+\eta)}$$

The leisure choice of an individual is

$$l = \frac{\eta}{1+\eta}T$$

And using the time constraint this implies that the hours worked are

$$n = \frac{1}{1+\eta}T$$

5. (5 points) Using the labor supplies for the two groups of consumer and the relative labor demand of the firm, find the equilibrium relative wage (w^S/w^U).

   Labor supply for the skilled workers is

$$n^s = N^s \frac{1}{1+\eta} T$$

   and the labor supply for the unskilled is

$$n^u = N^u \frac{1}{1+\eta} T$$

   Using the equilibrium condition $n^i = L^i$ **we have that the wage must solve**

$$w^S/w^U = A$$

6. (5 points) What is the effect of a technological increase (an increase in A) in the wage gap between skilled and unskilled workers? Explain briefly what happens to the labor demand of the two types of workers.

   We can see from the equilibrium conditions that both wages must go up as A increases. However, because the wage rate of the skilled workers goes up by more, the relative wage goes up.

7. (4 points) Now imagine that as a result of a national trend, more and more people go to college so the ratio of skilled to unskilled workers rises (a rise in N^s/N^u). What will happen to the relative wage between skilled and unskilled workers as a result of this national trend?

Notice that the relative supply increases to N^s/N^u. Using the relative demand from part 2, we know that the equilibrium relative wage will be

$$\frac{w_s}{w_u} = A\left(\frac{N^s}{N^u}\right)^{-1/2}$$

which implies that an increase in the ratio of skilled to unskilled workers will decrease the relative wage.

8. (4 points) Finally assume that the current technological level, A, is endogenous and is a function of the ratio between skilled and unskilled workers. In particular, assume $A = (N^s/N^u)^\beta, \beta > 0$. What will happen to the wage ratio as a result of the national trend mentioned before?

Now the wage rate will be

$$\begin{aligned} \frac{w_s}{w_u} &= A\left(\frac{N^s}{N^u}\right)^{-1/2} \\ &= (N^s/N^u)^{\beta-1/2} \end{aligned}$$

which means that the relative wage will increase following the increase in relative supply if $\beta > 1/2$.