14.02 Principles of Macroeconomics
Solutions to Problem Set # 2

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1 True/False/Uncertain [20 points]

Please state whether each of the following claims are True, False or Uncertain, and provide a brief justification for your answer.

1. "An increase in the real interest rate makes current consumption relatively cheaper than future consumption" [6 points]

   ANSWER. FALSE. It is the other way around. By increasing the return on savings, an increase in the real interest rate makes future consumption relatively more attractive.

2. "The Keynesian consumption function, which implies that agents consume a constant fraction of their income, does not match short run household data" [7 points]

   ANSWER.FALSE. Agents consume a constant fraction of an extra dollar of income. However, the average propensity to consume (C/Y_d) falls with income (rich people save more). The part about not matching short run data is correct.

3. "If an agent is liquidity constrained, then an unexpected increase in transitory income of $100 can result in the agent's current consumption increasing by $100" [7 points]

   ANSWER. TRUE (or Uncertain). Without liquidity constraints we know that agents will only consume a fraction of the $100 and save the rest. But if agents are liquidity constrained, and are consuming less than what they would want to smooth consumption, then they will consume the extra income 1 to 1"

2 Labor market [40 points]

Consider an economy in which the labor market is competitive. There is a large number of firms each with technology given by the following production function

$$Y = AK^{\alpha}N^{1-\alpha}$$

where Y denotes the (unique) consumption good, K denotes capital stock, and N denotes labor input (in hours) in production.

There is also a large number of agents who supply labor, with preferences given by

$$u(c, l) = \log(c) + \eta \log(l)$$

where l denotes leisure and c denotes consumption. Each of these agents has T hours of time (monthly) to allocate between leisure and labor. The only source of income for the agents is labor income.

Assume that the price of the consumption good (denote it by P) is exogenously given and equal to 2. Use W to denote the hourly nominal wage in the economy

1. What is the marginal product of labor (MPN)? How does MPN change as N increases? Explain briefly why. [4 points]

   ANSWER.

   $$MPN = (1-\alpha) A \frac{K^\alpha}{N^\alpha}$$

   MPN is decreasing in N. The reason is that there is a fixed factor of production, capital (K). As we add units of labor output increases but each time by less, since the additional units of labor need to share the fixed amount of capital available.

2. What is the optimal amount of labor that each firm will demand? Sketch the labor demand as a function of the real wage (using the y-axis for the real wage, and x-axis for labor) [4 points]

   ANSWER.

   $$\begin{aligned} MPN &= W/P \\ (1-\alpha) A \frac{K^\alpha}{N^\alpha} &= W/2 \\ N &= \left(\frac{(1-\alpha) A2}{W}\right)^{\frac{1}{\alpha}} K \end{aligned}$$

3. Now suppose that the Government introduces a labor tax: for each hour of labor hired, the firm needs to pay τ to the Government. Find the new optimal demand for labor. Is it higher or lower than in the absence of the tax? [4 points]

   ANSWER.

   $$\begin{aligned} MPN &= (1+\tau) W/P \\ (1-\alpha) A \frac{K^\alpha}{N^\alpha} &= (1+\tau) W/P \\ N &= \left(\frac{(1-\alpha) A2}{(1+\tau) W}\right)^{\frac{1}{\alpha}} K \end{aligned}$$

   For each W/P, the amount of labor demanded by the firm is **lower**.

4. Consider now the decision problem of the agents. Let n denote the number of hours per month that an agent works. Write an equation relating n and l. Write the agent's budget constraint. Express the budget constraint in terms of c and l. State the agent's maximization problem. [4 points]

   ANSWER

$$\begin{aligned} n + l &= T \\ nW &= Pc - > TW = Pc + Wl \end{aligned}$$

$$\begin{aligned} & \max_{c,l} \log(c) + \eta \log(l) \\ & s.t. \\ TW &= Pc + Wl \end{aligned}$$

5. Use the fact that the marginal utility of consumption and leisure is given by

$$\begin{aligned} MU(C) &= 1/c \\ MU(L) &= \eta/l \end{aligned}$$

   to find an equation that characterizes the optimal behavior of the agent regarding consumption and leisure. This should be an equation on c and l as a function of the real wage. Use this equation, and the budget constraint of the agent to find the optimal amount of leisure. Use the equation relating n and l from the previous question to find the optimal number of hours n. [4 points]

   ANSWER. From p. 21 in the lecture notes:

$$\begin{aligned} MU(L)/MU(C) &= W/P \\ \frac{\eta}{l} c &= \frac{W}{P} \\ Pc &= Wl \frac{1}{\eta} \end{aligned}$$

   Plugging into the budget constraint

$$TW = Wl\frac{1}{\eta} + Wl$$

$$\begin{aligned} l &= \frac{\eta}{1+\eta} T \\ n &= T - \frac{\eta}{1+\eta} T \\ n &= T \frac{1}{1+\eta} \end{aligned}$$

6. Suppose that the real wage increases. Does the agent work more, less or the same? Explain why. [2 points]

   ANSWER. The last equation shows that the agent works the same. Labor supply is independent of the real wage. This is just a very particular example in which income and substitution effects just cancel out.

7. Suppose that η increases. What is the effect on the agents labor supply? [2 points]

   ANSWER. n decreases. η measures how much the agent cares of leisure.

8. Suppose that there are M_a agents in the economy (where M_a is a very large number). What is the total labor supply? (denote it by N^s) Suppose that there are M_f firms in the economy (where M_f is a very large number). What is the total labor demand in the economy? (denote it by N^d) (we continue to assume that the Government is taxing labor). [4 points]

   ANSWER.

$$\begin{aligned} N^d &= M_f \left(\frac{(1-\alpha) A2}{(1+\tau) W} \right)^{\frac{1}{\alpha}} K \\ N^s &= M_a T \frac{1}{1+\eta} \end{aligned}$$

9. Assume that $M_f = M_a$. Find the real wage that makes total labor supply and total labor demand exactly equal to each other (this is called, the equilibrium real wage). Find the equilibrium number of hours worked in this economy.[4 points]

   ANSWER.

$$\begin{aligned} M_f \left(\frac{(1-\alpha) A2}{(1+\tau) W} \right)^{\frac{1}{\alpha}} K &= M_a T \frac{1}{1+\eta} \\ \left(\frac{W}{2} \right)^* &= \frac{(1-\alpha) A}{1+\tau} \left(\frac{K}{T} (1+\eta) \right)^{\alpha} \end{aligned}$$

10. For each of the following cases, explain what is the effect on the equilibrium wage and number of hours worked. Provide a brief sketch showing how labor supply and demand shift. Also provide intuition. [8 points]

    a. Increase in T

    ANSWER. The equilibrium wage decreases and the equilibrium number of hours worked increases, when T goes up. The reason is that labor supply goes up.

    b. Increase in K

    ANSWER. An increase in K increases labor demand (due to complementarities between inputs, K increases MPN). Thus the equilibrium wage goes up, and the equilibrium number of hours stays constant (since supply is vertical).

c. Increase in τ.

ANSWER. An increase in τ decreases the real wage (and has no effect on the number of hours worked) since it decreases labor demand (τ increases the effective cost of workers faced by firms).

3 Consumption [40 points]

Consider an economy with two periods (current and future). There are 2 agents (call them agent A and agent B). There is only one good in the economy[1]. Suppose that agent $i \in \{A, B\}$ has an income of y^i in the current periods and y^i_f in the future period. Note that we are allowing income endowments to be different for different agents and periods. Both agents are foward looking (which just means they care about the present and the future). In particular, they consume according to the following rule

$$c^i_1 = \alpha^i PVLR^i$$

where c^i is consumption in the current by agent i, PVLR is the present value of lifetime resources, and α^A, α^B are numbers such that $\alpha^i \in (0, 1)$ for $i = A$ and $i = B$. Note that we are allowing the consumption rule to vary across agents. Denote consumption in the future period for agent i by c^i_f and the real interest rate by r.

1. Let s^i denote agent i's savings in the current [In the lecture notes, this is just the leftover after consumption]. Write the budget constraint for each period for agent i. These are called "flow" budget constraints. Show how you can obtain an intertemporal budget constraint for agent i out of the two flow budget constraints. [Basically just substitute out "s^i" to obtain an equation that relates the PVLR with c^i and c^i_f.] [5 points]

   ANSWER. For $i \in \{A, B\}$

   $$\begin{aligned} c^i + s^i &= y^i \\ c^i_f &= (1 + r)\, s^i + y^i_f \end{aligned}$$

   Then substitute s^i to get

   $$c^i + \frac{c^i_f}{1 + r} = y^i + \frac{y^i_f}{1 + r} = PVLR^i \qquad \text{(IBC)}$$

2. State the $PLVR^i$ in terms of income in the two periods. [5 points]

   ANSWER.

   $$PVLR^i = y^i + \frac{y^i_f}{1 + r}$$

[1] We will assume that its price is equal to 1.

3. For a given interest rate r, plot the combinations of c^i and c^i_f that the agent can choose. The set you just plotted is called the budget set. [5 points]

   ANSWER. See the graph on page 12 in Lecture notes.

4. Show graphically the effect of an increase in the interest rate on the budget set of agent i. [5 points]

   ANSWER. See page 33 in lecture notes.

5. Express c^i, c^i_f as functions of α^i, y^i, y^i_f and r. [6 points]

   ANSWER.

$$c^i = \alpha^i y^i_1 + \alpha^i \frac{y^i_2}{1+r}$$

   To get c^i_f, express (**??**) as

$$\begin{aligned}
(1+r)\,c^i + c^i_f &= (1+r)\,y^i_1 + y^i_f \\
c^i_f &= (1+r)\,y^i_1 + y^i_f - (1+r)\,c^i \\
c^i_f &= (1+r)\,y^i + y^i_f - (1+r)\left[\alpha^i y^i + \alpha^i \frac{y^i_f}{1+r}\right] \\
c^i_f &= \left(1-\alpha^i\right)\left[(1+r)\,y^i + y^i_f\right]
\end{aligned}$$

6. Write an equation that states that the total demand for the good in the current period equals total supply [total demand is the sum of consumption by each agent, and total supply is the sum of each agent's income]. We call this equation the goods market clearing condition. Find the equilibrium interest rate in this economy, and denote it by r* [this is just the interest rate that makes supply and demand coincide] [7 points]

   ANSWER.

$$\begin{aligned}
c^A + c^B &= y^A + y^B \\
\alpha^A y^A + \alpha^A \frac{y^A_f}{1+r} + \alpha^B y^B + \alpha^B \frac{y^B_f}{1+r} &= y^A + y^B \\
\alpha^A \frac{y^A_f}{1+r} + \alpha^B \frac{y^B_f}{1+r} &= \left(1-\alpha^A\right) y^A + \left(1-\alpha^B\right) y^B \\
\alpha^A y^A_f + \alpha^B y^B_f &= (1+r)\left[\left(1-\alpha^A\right) y^A + \left(1-\alpha^B\right) y^B\right]
\end{aligned}$$

$$1+r = \frac{\alpha^A y^A_f + \alpha^B y^B_f}{(1-\alpha^A)\,y^A + (1-\alpha^B)\,y^B}$$

7. Suppose $y^A = y^B_f = 2$, $y^A_f = y^B = 4$ and $\alpha^A = \alpha^B = 1/2$. Find the equilibrim interest rate, and the levels of consumption for both periods for each agent. Relate your numerical answers to the concept of consumption

smoothing. Who is borrowing and who is lending in the current period in this economy? [7 points]

ANSWER.

$$1 + r = \frac{3}{3} = 1 \rightarrow r = 0$$

$$\begin{aligned} c^A &= \frac{1}{2}2 + \frac{1}{2}4 = 3 = c^B \\ c_f^A &= 3 = c_f^B \end{aligned}$$

Both agents are able to perfectly smooth consumption over time. In the first period A is borrowing 1 unit of the good from B. In period 2, A returns that unit of the good to B so that both agents consume 3 in each period.

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