Reluctance coil-gun modeling and energy transfer optimization

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Abstract

We focus on a simple coil-gun model, free of any constraints regarding the projectile. Instead of taking into account the mutual inductance we propose an original model for the system mutual inductance by directly taking it into account in a generalized inductance as a function of the projectile position, under the assumption that no Eddy currents appear in the projectile: the projectile is an ideal magnetic material. This allows for a steady state analysis and fast computer simulations. Eventually we propose a method to optimize a multi-stage reluctance accelerator.

I. Introduction

Coil-gun gained some scientific interest in the 1990's and were used to settle ground breaking super velocity launchers [5, 3, 4]. Most studies focus on an annular projectile in a multi-stage coil gun. Several coils geometry were studied, coil-in-coil or even E-shape systems [8]. Studying the coilgun is a complex task as it is always in a transient state. Some studies have shown excellent comparison vs. theory with transient models and solvers [2]. We will here focus on a reluctance coil-gun [6, 10]. We develop a simple model in between steady-state and transient. This model tries to be as generalist as possible and can still be easily simulated with modern computation methods. Main difference between past researches is to solve the dynamic of the system only by studying the RLC circuit thanks to a generalized definition of inductance.

II. Problem definition

At first we focus on a single-stage, capacitively driven coilgun. We make the following assumptions:

- Quasi-steady state approximation is valid
- No Eddy currents appear in the projectile (laminated projectile)
- The projectile's magnetic susceptibility is constant over time

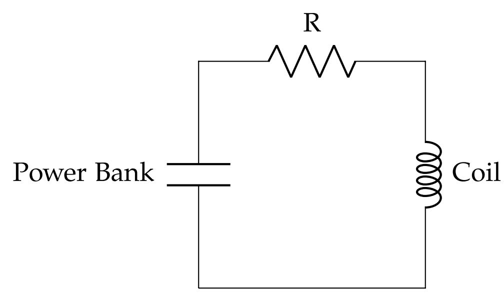

Figure 1: *Single stage electrical circuit, the resistor contains all parasitic resistances of the circuit.*

The circuit in its most simple form is represented in Figure 1. For the sake of simplicity all switching logic has been hidden as well as protections.

We do not make any assumptions regarding the projectile's shape. The projectile is only able to move along the axis of the coil, denoted z. We assume that the power bank is originally loaded at a voltage E, its total capacitance is noted C. The coil inductance is denoted $L(z)$, z being the position of the projectile (ie. it takes into account all side effects of the projectile magnetization). One should note that z is a function of time. The resistor represents all resistances in the circuit and is noted R.

III. Dynamic of the system

We establish the dynamic of the whole system by studying only the electrical equations that occurs in the circuit. Using Lenz-Faraday law:

$$\begin{aligned}
0 &= u + Ri + \frac{d\left[i(t)L(z)\right]}{dt} \\
&= Cu + \left(R + \dot{z}\frac{dL}{dz}(z)\right) Ci + L(z)C\frac{di}{dt} \\
&= L(z)C\frac{d^2i}{dt^2} + \left(R + 2\dot{z}\frac{dL}{dz}(z)\right) C\frac{di}{dt} \\
&+ \left(\left(\ddot{z}\frac{dL}{dz}(z) + \dot{z}^2\frac{d^2L}{dz^2}(z)\right) C + 1\right) i
\end{aligned}$$

One can rewrite this equation using standard notation:

$$0 = \frac{d^2i}{dt^2} + \frac{\omega_0}{Q}(z)\frac{di}{dt} + \omega_0^2(z)i \qquad (1)$$

With :

$$\omega_0(z) = \sqrt{\frac{\left(\ddot{z}\frac{dL}{dz}(z) + \dot{z}^2\frac{d^2L}{dz^2}(z)\right) C + 1}{L(z)C}}$$

And:

$$\frac{\omega_0}{Q}(z) = \frac{R + 2\dot{z}\frac{dL}{dz}(z)}{L(z)}$$

This formulation of the problem with a variable inductance, although original, has already been introduced [9]. However, we do not make any assumptions on the expression of $L(z)$ here.

To derive the mechanical dynamic of the system we perform an energy analysis. The magnetic energy can be computed as:

$$\begin{aligned}
E_{coil}(t) &= \int_0^t \frac{d\Phi}{dt} i(s) ds \\
&= \int_0^t \dot{z}(s)\frac{dL}{dz}(z(s))i(s)^2 ds \\
&+ \int_0^t L(z(s))\frac{di}{dt}(s)i(s)ds \\
&= \int_0^t \dot{z}(s)\frac{dL}{dz}(z(s))i(s)^2 ds \\
&+ \int_0^t L(z(s))\frac{d}{dt}\left[\frac{1}{2}i(t)^2\right](s)ds
\end{aligned}$$

An integration by parts on the second term of the last equation gives:

$$\begin{aligned}
E_{coil}(t) &= \left[\frac{1}{2}L(z(s))i(s)^2\right]_0^t \\
&+ \left(1 - \frac{1}{2}\right)\int_0^t \dot{z}(s)\frac{dL}{dz}(z(s))i(s)^2 ds \\
&= \frac{1}{2}L(z(t))i(t)^2 \\
&+ \frac{1}{2}\int_0^t \dot{z}(s)\frac{dL}{dz}(z(s))i(s)^2 ds
\end{aligned}$$

Here we recognize the first term as being to total magnetic energy stored in space:

$$\frac{1}{2}L(z(t))i(t)^2 = \frac{1}{2}\iiint_{\mathbb{R}^3} \frac{||B(\tau,t)||^2}{\mu} d\tau$$

As we neglected all losses otherwise, the law of conservation of energy implies that:

$$\frac{1}{2}\int_0^t \dot{z}(s)\frac{dL}{dz}(z(s))i(s)^2 ds = \frac{1}{2}m\dot{z}^2 = E_p \qquad (2)$$

Where E_p is the projectile's energy. Let's derive this equation:

$$\begin{aligned}
\frac{1}{2}\dot{z}\frac{dL}{dz}(z)i^2 &= m\dot{z}\ddot{z} \\
\frac{1}{2}\frac{dL}{dz}(z)i^2 &= m\ddot{z}
\end{aligned}$$

Therefore thanks to Newton's first law, we proved that the force exerted on the projectile is:

$$F(t) = \frac{1}{2}\frac{dL}{dz}(z(t))i(t)^2 \qquad (3)$$

We now have all the equations that describe the dynamic of the coupled system. Furthermore, the last part of this study to derive the force on the projectile should be true for any coil in any system, as it doesn't use any information about the whole electrical circuit.

IV. Simulation

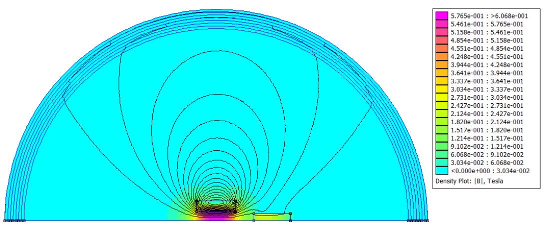

Figure 2: *Modeling of a coil/projectile setup in FEMM. Longitudinal section through the axis of revolution.*

In order to simulate the system we only need to compute $L(z)$, or in practice $\dfrac{dL}{dz}$. To solve this problem we propose to use finite element methods. We will use FEMM [1] as it provides functions to compute the force applied on an element, and therefore using (3) we can compute the inductance. The computations are made in steady-state with an arbitrary amperage. We study a simple projectile shape[1]:

- The projectile is a cylindrical piece of iron
- It's outer radius is equal to the inner radius of the coil minus 1mm

We will estimate the force for different position of the projectile, as well as the inductance without projectile. An example of the modeling is provided in Figure 2.

Using finite elements to compute the inductance raises some difficulties. As shown in equation (1), we also need the second order derivative of the inductance. Therefore any noise due to the finite elements lack of convergence will result in important noise on the final solution. We will now introduce a way to solve this problem as standard filtering techniques will not provide good results (essentially because of Biggs phenomenom).

[1]These assumptions are arbitrary and any projectile shape could be studied.

V. Inductance denoising

First of all let's define some notation. We name R_{bo} the coil outside radius, L_b the coil length and L_p the projectile length. Thanks to the symmetries of the problem we can limit our study to $z \leq 0$. An example of inductance (raw output from the finite elements) and its four first derivatives are shown in Figure 3.

The aim of this section is to get a perfectly smooth second order derivative of the inductance.

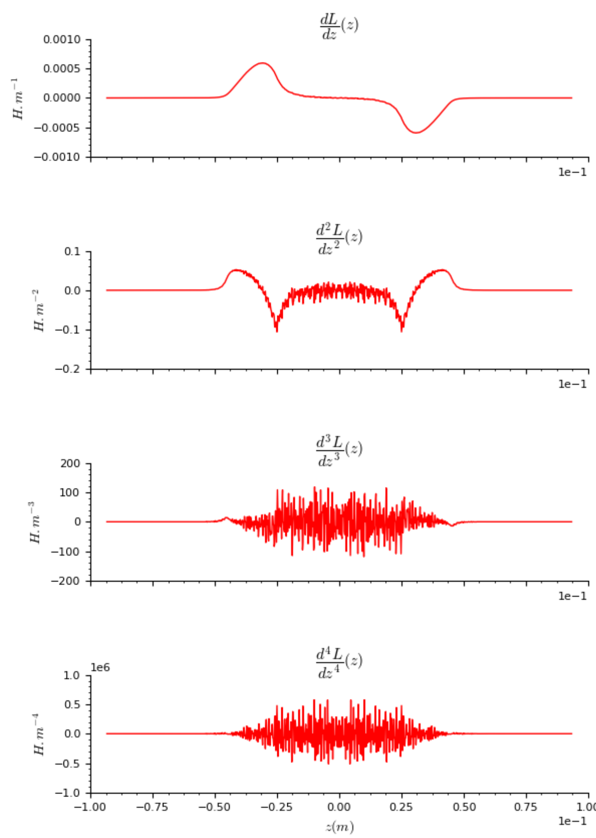

Figure 3: *Raw output of an inductance using finite elements.*

We define empirically a frequency at which the inductance varies[2]:

$$fr_{base} = \frac{4}{\min(L_b, R_{bo}, L_p)} \tag{4}$$

In order to satisfy Shannon's theorem, we need to sample at a frequency at least 2 times higher then fr_{base}. However, to avoid aliasing we oversample the inductance and use a sampling frequency of $16 \times fr_{base}$. This is the frequency used for Figure 3. The idea of this denoising technique is to find the best piece-wise convex/concave approximation of the sampled data.

The denoising algorithm consists of two parts:

1. Guess the position where the fourth order derivative sign changes[3]

2. For each interval of constant sign, find the best convex (or concave) constant piece-wise approximation

1. Finding sign reversals

Algorithm 1: First part of the denoising algorithm.

Result: Position of 3 probable sign reversal

Compute the discrete second order derivative of L(z);
Apply a low-pass FIR filter;
Compute the discrete second order derivative of the output;
Set window size to 1;
while *Number of reversals of sign is above 3* **do**
    Apply moving average on the fourth order derivative;
    Compute number of sign reversals;
    Increase the window size;
end

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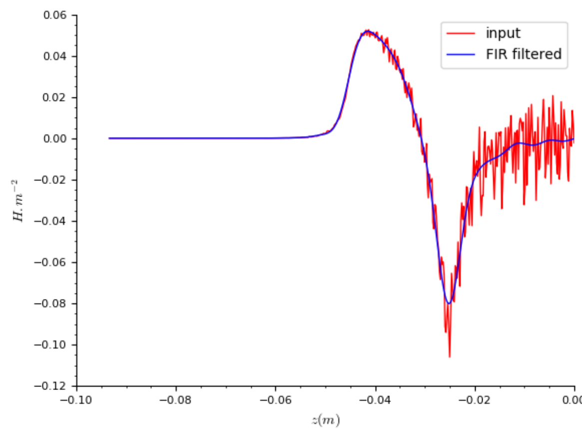
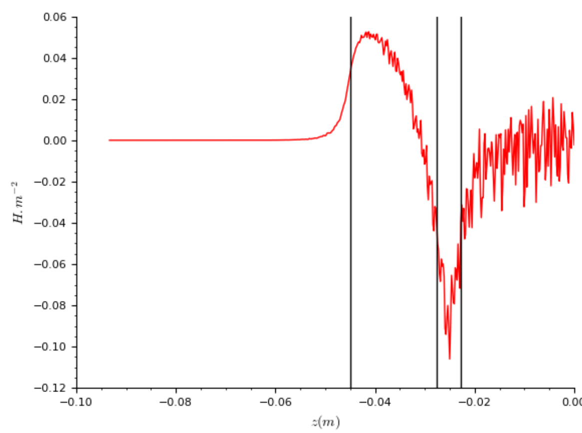
[2]In practice the variation may be higher for some specific coil designs or much smaller. The effect of the projectile's shape has not been studied.

[3]Given the shape of the function we need to find 3 switching of sign.

To detect sign reversal we perform a first low-pass FIR filter to remove most of the high-frequency noise. The cutoff frequency is fr_{base}. The effect of this filter is shown in Figure 4a. The next step is an iterative process. We numerically derive the output of the FIR filter twice, and perform a moving average with a increasing window until we get 3 sign reversals. The entire process is shown in Algorithm 1 and the final output in Figure 4b.



(a) *Output of the FIR filter. Some ringing is present.*



(b) *The sign reversals, guessed by the algorithm, are shown in black.*

Figure 4: *The two key steps of sign reversal detection algorithm.*

2. Best constant piece-wise approximation

The subject of best convex constant piece-wise approximation has been extensively treated [7, 11]. For our problem we consider each subset of the data (defined by the different sign reversals) and fit one piece-wise approximation to each.

In the convex case, the problem is defined as follows. Let $y_{i\in[\![0,n]\!]}$ be the data points, we search a new set $x_{i\in[\![0,n]\!]}$ that verifies:

$$\begin{cases} \min\limits_{x} \sum_{i=0}^{n} ||y_i - x_i||^2 \\ \forall i \in [\![0, n-2]\!], x_{i+2} - x_{i+1} \geq x_{i+1} - x_i \\ x_0 = y_0 \\ x_n = y_n \end{cases}$$

The concave case only requires to reverse the inequalities. The two equalities are added to ensure continuity of the final solution. The first order derivative and inductance are then computed with numerical integration. The final output of the algorithm is shown in Figure 5.

VI. Optimization

Optimizing an uncoupled multistage coil-gun can be done with the following process:

1. Compute a large number of physically possible coils and their raw inductance with finite element methods.
2. For each coil, apply the denoising filter
3. For each denoised inductance, numerically solve the system of differential equations (1) (3) in order to find the best launch position of the projectile [12]
4. Select the best coil and launching position, use the output speed as an initial condition for the next stage.
5. Repeat steps 3 and 4.

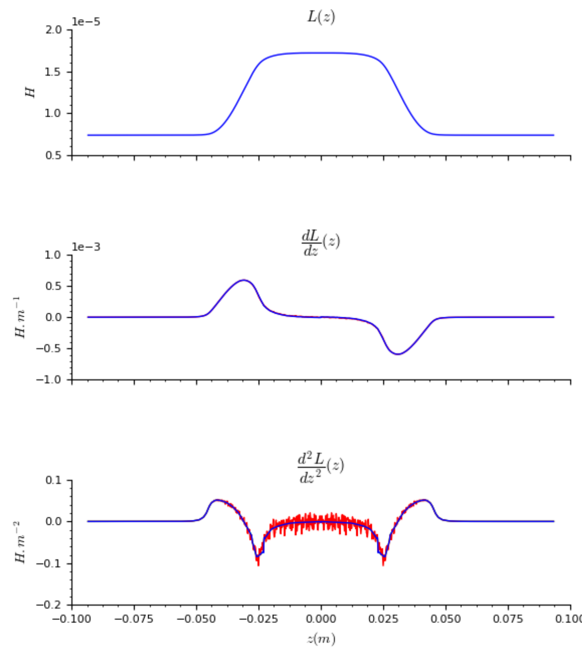


Figure 5: *Output of the complete denoising algorithm.*

VII. Results

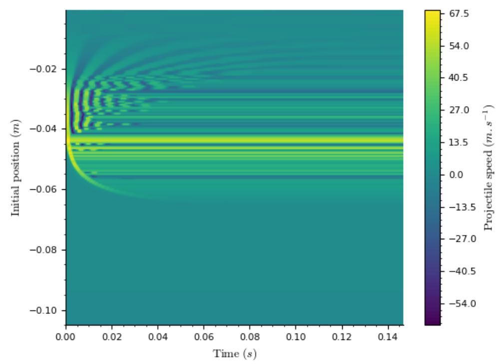
Here we consider a single-stage coil-gun with the given electrical setup: the initial capacitor is charged at $200V$ and has a capcitance of $24mF$. The circuit resistance (without coil) is $70m\Omega$. The projectile length is $20mm$, the inner radius of the coil is $5mm$ and the coil wire diameter is $1mm$. The projectile is made of iron with a magnetic susceptibility of 100.

We define the energy transfer of a single shot as:

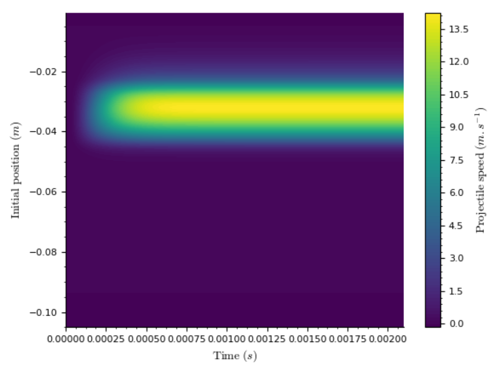
$$\eta = \frac{m \times v_{out}^2}{C \times E^2}$$

where m is the mass of the projectile and v_{out} its output speed.

Figure 6 shows two example of solutions. When the electrical circuit is damped the coil-gun is stable. In such case the initial position is not critical but the energy transfer is low. With very low resistance and an oscillating circuit, the coil-gun becomes chaotic and the initial position is critical. A slight error might cause it to shoot backwards.



(a) *Example with a total resistance of* $1m\Omega$*. The electrical circuit is oscillating and the coil-gun is chaotic. There seems to be some noise due to some imperfect inductance denoising or imperfect ODE solver.* $\eta = 24\%$



(b) *Realistic resistance, the circuit is damped. The solution is stable and the initial position is not critical.* $\eta = 1\%$

Figure 6: *The same system is solved with two different resistance values*

Figure 7 shows the effect of the coil dimensions on the energy transfer, therefore providing all the data to build an optimized coil-gun.

VIII. Conclusion

We set a complete reluctance coil-gun model with an original formalism and explained all steps in order to simulate and optimize a coil-gun design. We focused on single-stage coil-gun but the proposed method can be adapted

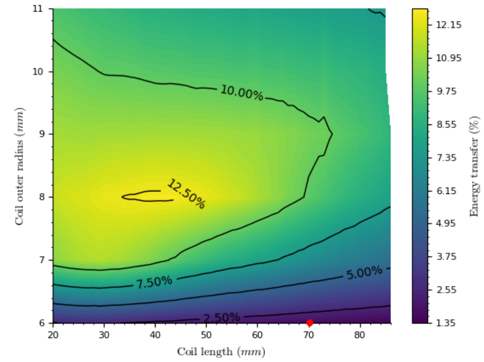


Figure 7: *Final optimization of the considered setup. After computing several hundreds of coils, and their optimal launching position, we can pick the one that gives the best energy transfer in a single shot ($\eta = 13\%$). The red dot is the coil studied in Figure 6.*

for uncoupled multistage coil-gun straightforwardly. The model was not thoroughly tested experimentally but initial tests prove some good correlation between simulation and reality, at least similar to some past researches [10]. This model takes into account the effect of the moving projectile on the electrical circuit and should therefore outperform simpler models.

References

[1] David Meeker. *Finite Element Method Magnetics*. `http://www.femm.info`. 2014-2019.

[2] J. He et al. "Concerning the design of capacitively driven induction coil guns". In: *IEEE Transactions on Plasma Science* 17.3 (June 1989), pp. 429–438. ISSN: 0093-3813. DOI: `10.1109/27.32251`.

[3] E.R. Laithwaite. "Induction coil guns for hypervelocities". In: *Electric Power Applications, IEE Proceedings* - 142 (June 1995), pp. 215–221. DOI: `10.1049/ip-epa:19951839`.

[4] Kim Seo-Whan, Jung Hyun-Kyo, and Song-Yop Hahn. "Optimal design of multistage coilgun". In: *IEEE Transactions on Magnetics* 32.2 (Mar. 1996), pp. 505–508. ISSN: 0018-9464. DOI: 10.1109/20.486539.

[5] Yehudith Naot et al. "DESIGN AND POWER CONDITIONING FOR THE COIL-GUN". In: 2004.

[6] George Slade. "A simple unified physical model for a reluctance accelerator". In: *Magnetics, IEEE Transactions on* 41 (Dec. 2005), pp. 4270–4276. DOI: 10.1109/TMAG.2005.856320.

[7] Alessandro Magnani and Stephen P. Boyd. "Convex Piecewise-Linear Fitting". In: 2006.

[8] L. Gherman et al. "Induction Coilgun Based on "E-Shaped" Design". In: *IEEE Transactions on Plasma Science* 39.2 (Feb. 2011), pp. 725–729. ISSN: 0093-3813. DOI: 10.1109/TPS.2010.2091650.

[9] Ashley Kristin Hallock and Kurt A. Polzin. "Two-dimensional Analysis of Conical Pulsed Inductive Plasma Thruster Performance IEPC-2011-145". In: 2011.

[10] Jeff Holzgrafe et al. "Effect of Projectile Design on Coil Gun Performance". In: (2012).

[11] Alejandro Toriello and Juan Pablo Vielma. "Fitting piecewise linear continuous functions". In: *European Journal of Operational Research* 219 (2012), pp. 86–95.

[12] J. L. Rivas-Camacho, M. Ponce-Silva, and V. H. Olivares-Peregrino. "Experimental results concerning to the effects of the initial position of the projectile on the conversion efficiency of a reluctance accelerator". In: *2016 13th International Conference on Power Electronics (CIEP)*. June 2016, pp. 92–97. DOI: 10.1109/CIEP.2016.7530737.