domaći

REKURENTNE RELACIJE

2. d) $f_{n+3} = 4f_{n+2} - f_{n+1} - 6f_n$

$f_0 = 1 \quad f_1 = 2 \quad f_2 = 4$

$$f_n = t^n \Rightarrow \quad t^3 = 4t^2 - t - 6$$

$$t^3 - 4t^2 + t + 6 = 0$$

$$t^3 + t^2 + t^2 + t - 6t^2 + 6 = 0$$

$$t^2(t+1) + t(t+1) - 6(t-1)(t+1) = 0$$

$$(t+1)(t^2 + t - 6t + 6) = 0$$

$$(t+1)(t^2 - 5t + 6) = 0$$

$$(t+1)(t-2)(t-3) = 0$$

$$t_1 = -1 \quad t_2 = 2 \quad t_3 = 3$$

$$f_n = A \cdot (-1)^n + B \cdot 2^n + C \cdot 3^n$$

$$\left.\begin{array}{l} 1 = f_0 = A + B + C \\ 2 = f_1 = -A + 2B + 3C \\ 4 = f_2 = A + 4B + 9C \end{array}\right\} \begin{array}{l} \Rightarrow 3 = 3B + 4C \\ \Rightarrow 6 = 6B + 12C \Rightarrow 2 = 2B + 4C \end{array}$$

$$\Rightarrow B = 1 \;,\; C = 0 \;,\; A = 0$$

$$\Rightarrow f_n = 2^n$$

5. b) $a_n = a_{n-1} a_{n-2}^2 \quad , \quad a_0 = a_1 = 2$

$$\log_2 a_n = \log_2 a_{n-1} + 2\log_2 a_{n-2}$$

$b_n = \log_2 a_n \quad \Rightarrow \quad b_n = b_{n-1} + 2b_{n-2}$

$b_n = t^n \quad \Rightarrow \quad t^n = t^{n-1} + 2t^{n-2} \quad / : t^{n-2}$

$t^2 = t + 2 \Rightarrow t^2 - t - 2 = 0$

$(t-2)(t+1) = 0$

$\Rightarrow b_n = A(-1)^n + B \cdot 2^n \qquad t_1 = -1 \quad t_2 = 2$

$b_0 = \log_2 a_0 = \log_2 2 = 1 \quad \Rightarrow \quad 1 = A + B$

$b_1 = \log_2 a_1 = \log_2 2 = 1 \quad \Rightarrow \quad 1 = -A + 2B$

$\Rightarrow 3B = 2$

$A = \frac{1}{3} \quad B = \frac{2}{3}$

$$b_n = \frac{1}{3} \cdot (-1)^n + \frac{2}{3} \cdot 2^n = \frac{(-1)^n + 2^{n+1}}{3}$$

$$b_n = \log_2 a_n \Rightarrow a_n = 2^{b_n} = 2^{\frac{(-1)^n + 2^{n+1}}{3}}$$

9.

$$\underline{\ \ } \ \underbrace{\underline{\ \ }\ \underline{\ \ }\ \ldots\ \underline{\ \ }}_{n-1}$$

- riječi koje počinju sa 1 ili 2: $1/2 \ \underbrace{\underline{\ \ }\ \underline{\ \ }\ \ldots\ \underline{\ \ }}_{n-1} \Rightarrow 2 \cdot \underbrace{f_{n-1}}_{\text{paran br. nula}}$

- riječi koje počinju sa 0: treba se osigurati da u ostatku riječi ima neparan broj nula
  (neparan br. nula =
  ukupan br. riječi dužine $n-1 \qquad \Rightarrow 3^{n-1}$
  $-$ br. riječi sa parnim br. nula $\qquad \Rightarrow -f_{n-1}$
  $\Rightarrow 3^{n-1} - f_{n-1}$

$$f_n = 2f_{n-1} + 3^{n-1} - f_{n-1}$$
$$= f_{n-1} + 3^{n-1}$$

$f_1 = 2 \quad f_2 = 5$

homogeno rješenje: $\quad f_n - f_{n-1} = 0 \quad \Rightarrow t - 1 = 0 \quad \Rightarrow t = 1$

$\cancel{f_n^{(h)} = A \cdot 1^n}$

$f_n^{(h)} = (An + B) \cdot 1^n = An + B$

partikularno rješenje: $f_n^{(p)} = C \cdot 3^n$

$$C \cdot 3^n = C \cdot 3^{n-1} + 3^{n-1} = (C+3) \cdot 3^{n-1} \quad / : 3^{n-1}$$

$$3C = C+1 \Rightarrow C = \frac{1}{2}$$

$$\Rightarrow f_n^{(p)} = \frac{1}{2} 3^n$$

opšte rješenje: $f_n = f_n^{(h)} + f_n^{(p)} =$

$$f_n = An + B + \frac{1}{2} 3^n$$

$$\left.\begin{aligned} f_1 = 2 &= A + B + \frac{3}{2} \\ f_2 = 5 &= 2A + B + \frac{9}{2} \end{aligned}\right\} \Rightarrow 3 = A + \frac{6}{2} \Rightarrow A = 0$$

$$B = \frac{1}{2}$$

$$f_n = \frac{1}{2} + \frac{1}{2} 3^n = \frac{3^n + 1}{2}$$