domaći

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3. $$\sum_{k=0}^{n}(k+1)\binom{n}{k} = \sum_{k=0}^{n}\left(k\binom{n}{k}+\binom{n}{k}\right) = \sum_{k=0}^{n}k\binom{n}{k}+\sum_{k=0}^{n}\binom{n}{k} =$$

$$= \underbrace{0\cdot\binom{n}{0}}_{k=0} + \sum_{k=1}^{n}k\binom{n}{k} + 2^n = \sum_{k=1}^{n}k\frac{n!}{k!(n-k)!} + 2^n =$$

$$= \sum_{k=1}^{n}\frac{n(n-1)!}{(k-1)!(n-k)!} + 2^n = \sum_{k=1}^{n}n\binom{n-1}{k-1} + 2^n =$$

$$= n\sum_{i=k-1=0}^{n-1}\binom{n-1}{i} + 2^n = n\cdot 2^{n-1} + 2^n =$$

$$= n\cdot 2^{n-1} + 2\cdot 2^{n-1} = (n+2)\cdot 2^{n-1}$$

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$\star$ $$\sum_{\substack{i+j+k=n\\ 0\le i,j,k\le n-1}}\binom{n}{i,j,k}2^j = \sum_{\substack{i+j+k=n\\ 0\le i,j,k\le n}}\binom{n}{i,j,k}2^j - \binom{n}{n,0,0} - \binom{n}{0,n,0}2^n - \binom{n}{0,0,n}$$

$$= (1+2+1)^n - \frac{n!}{n!0!0!} - \frac{n!}{0!n!0!}2^n - \frac{n!}{0!n!0!} =$$

$$= 4^n - 2 - 2^n$$