31.08.2021.

1. 25 кутија 

3 врсте

$25 = 3\cdot 8 + 1$ => ако се равноправно распореди сигурно је 9 у истој кутији

2. 5-циф. 2 парне

1° прва парна

2° прва непарна

----- $4 * \binom{4}{1} * 5^4$

----- $\binom{4}{2} * 5^5$

} +

3. 142 студента , $\underbrace{56+52+30+30}_{\text{амфитеатри}} = 168$

→ небитан редослед студената

$$x_1 + x_2 + x_3 + x_4 = 142 \qquad y_i = x_i - \text{капацит.}$$

$$0 \le x_1 \le 56 \quad 0 \le x_2 \le 52 \quad 0 \le x_3 \le 30 \quad 0 \le x_4 \le 30$$

$S_1 : x_1 \ge 57 \Rightarrow y_1 + x_2 + x_3 + x_4 = 85 \quad N(S_1) = \binom{88}{3}$

$S_2 : x_2 \ge 53 \Rightarrow x_1 + y_2 + x_3 + x_4 = 89 \quad N(S_2) = \binom{92}{3}$

$S_3 : x_3 \ge 31 \Rightarrow x_1 + x_2 + y_3 + x_4 = 111 \quad N(S_3) = S_4 = \binom{114}{3}$

$N(S_1 S_2) = \binom{35}{3} \quad N(S_1 S_3) = N(S_1 S_4) = \binom{57}{3} \quad N(S_2 S_3) = N(S_2 S_4) = \binom{61}{3}$

$N(S_3 S_4) = \binom{83}{3} \quad N(S_1 S_2 S_3) = N(S_1 S_2 S_4) = \binom{4}{3} \quad N(S_1 S_3 S_4) = \binom{26}{3}$

$N(S_2 S_3 S_4) = \binom{31}{3} \quad S_1 S_2 S_3 S_4 = 0$

$N = \binom{145}{3}$

$$\Rightarrow N(S_1' S_2' S_3' S_4') = \binom{145}{3} - \binom{88}{3} - \binom{92}{3} - 2\cdot\binom{114}{3} + \binom{35}{3} + 2\binom{57}{3} + 2\binom{61}{3} + \binom{83}{3} - 2\binom{4}{3} - \binom{26}{3} - \binom{31}{3}$$

4. $a_0 = 1 \qquad a_1 = 4$

$a_{n+2} = a_{n+1}\, a_n^3$

$\log_2 a_{n+2} = 3\log_2 a_{n+1} + 4\log_2 a_n$ $\qquad b_n = \log_2 a_n$

$b_{n+2} = 3b_{n+1} + 4b_n$ $\qquad b_n = t^n$

$t^{n+2} = 3t^{n+1} + 4t^n$

$t^2 - 3t - 4 = (t-4)(t+1) = 0$

$b_n = A\cdot(-1)^n + B\cdot 4^n$

$b_0 = \log_2 1 = 0 = A + B$

$b_1 = \log_2 4 = 2 = -A + 4B$

$\Leftrightarrow 5B = 2 \Rightarrow B = \frac{2}{5}$

$A = -\frac{2}{5}$

$\Rightarrow b_n = -\frac{2}{5}\left((-1)^n - 4^n\right)$

$a_n = 2^{-\frac{2}{5}\left((-1)^n - 4^n\right)}$