Analitička geometrija

Vektorski prostor slobodnih vektora nad poljem realnih brojeva u odnosu na sabiranje vektora i množenje broja i slobodnog vektora, izomorfan je sa vektorskim prostorom uređenih trojki realnih brojeva u odnosu na sabiranje uređenih trojki i množenje broja i uređene trojke.

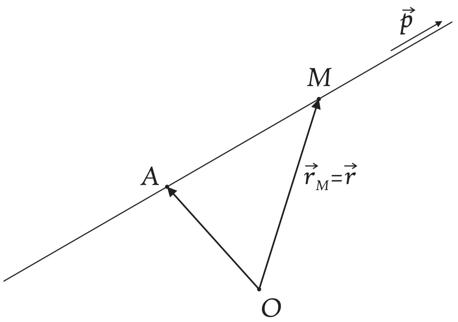
$$(V,\mathbb{R},+,\cdot) \cong (\mathbb{R}^3,\mathbb{R},+,\cdot)$$

Proizvoljna tačka $A(x_A,y_A,z_A)$

$\vec{r}_A = \overrightarrow{OA} = (x_A,y_A,z_A) = x_A\vec{i}+y_A\vec{j}+z_A\vec{k}$

Vektor $\overrightarrow{AB} = \overrightarrow{AO}+\overrightarrow{OB} = \vec{r}_B - \vec{r}_A$

Jednačina prave



$A(x_A,y_A,z_A) \in p, \quad \vec{p} = (p_1,p_2,p_3) \| p$

$M(x,y,z)$ promenljiva tačka
$\vec{r} = x\vec{i}+y\vec{j}+z\vec{k}$ promenljivi vektor

$M \in p \Leftrightarrow \overrightarrow{AM} \| \vec{p} \Leftrightarrow \overrightarrow{AM} = t\cdot\vec{p} \Leftrightarrow$
$\vec{r}_M - \vec{r}_A = t\cdot\vec{p} \Leftrightarrow$

$\boxed{\vec{r} = \vec{r}_A + t\cdot\vec{p}}$ vektorski oblik jednačine prave

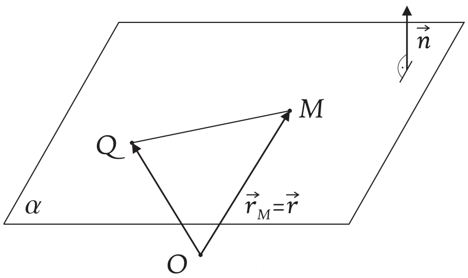
parametarski oblik

$$\boxed{\begin{array}{c} x = x_A + tp_1 \\ y = y_A + tp_2 \\ z = z_A + tp_3 \end{array}} \quad \begin{array}{l} \Rightarrow t = \dfrac{x-x_A}{p_1} \\ \Rightarrow t = \dfrac{y-y_A}{p_2} \\ \Rightarrow t = \dfrac{z-z_A}{p_3} \end{array}$$

$$\boxed{\frac{x-x_A}{p_1} = \frac{y-y_A}{p_2} = \frac{z-z_A}{p_3}}$$

kanonički oblik

Jednačina ravni



$Q \in \alpha, \quad \alpha \perp \vec{n}$

$M(x,y,z)$ promenljiva tačka

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ promenljivi vektor

$M \in \alpha \iff \overrightarrow{QM} \perp \vec{n} \iff \overrightarrow{QM} \cdot \vec{n} = 0 \iff$

$(\vec{r}_M - \vec{r}_Q) \cdot \vec{n} = 0 \iff$

$\boxed{\vec{r} \cdot \vec{n} = \vec{r}_Q \cdot \vec{n}}$ vektorski oblik jednačine ravni

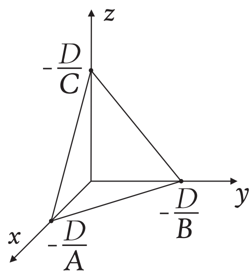
$\vec{n} = (A,B,C), \quad \vec{r} = (x,y,z)$

$(x,y,z) \cdot (A,B,C) = \vec{r}_Q \cdot \vec{n} = -D \iff$

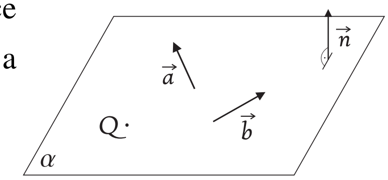
$\boxed{Ax + By + Cz + D = 0}$ opšti oblik jednačine ravni

$$Ax + By + Cz + D = 0 \iff \frac{x}{-\frac{D}{A}} + \frac{y}{-\frac{D}{B}} + \frac{z}{-\frac{D}{C}} = 1, \quad (A,B,C,D) \neq (0,0,0,0)$$

Brojevi u imeniocu predstavljaju redom odsečke koje ta ravan odseca na osama x, y, z.



Napomena: Do jednačine ravni α (ili vektora normale $\vec{n}$) najčešće se dolazi tako što se pronađu dva nekolinearna vektora $\vec{a}$ i $\vec{b}$ koja su paralelna sa ravni α.

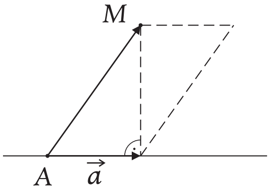


$\vec{a} \times \vec{b} = \vec{n} \ \wedge \ \vec{n} \perp \alpha$

$(\vec{r} - \vec{r}_Q) \cdot \vec{n} = 0 \iff (\vec{r} - \vec{r}_Q)(\vec{a} \times \vec{b}) = 0 \iff (\vec{r} - \vec{r}_Q)(A,B,C) = 0 \iff$

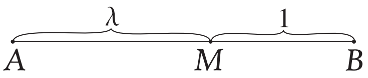
$\boxed{A(x - x_1) + B(y - y_1) + C(z - z_1) = 0}$ jednačina ravni kroz tačku, gde $Q(x_1, y_1, z_1) \in \alpha, \quad \vec{n} = (A,B,C), \quad \vec{n} \perp \alpha$

Rastojanje tačke M od prave



$|\overrightarrow{AM} \times \vec{a}| = d \cdot |\vec{a}| \ \Rightarrow \ \boxed{d = \dfrac{|\overrightarrow{AM} \times \vec{a}|}{|\vec{a}|}}$

Deoba duži u datoj razmeri



$$\frac{|\overrightarrow{AM}|}{|\overrightarrow{MB}|} = \frac{\lambda}{1} \Leftrightarrow |\overrightarrow{AM}| = \lambda|\overrightarrow{MB}| \Leftrightarrow \vec{r}_M - \vec{r}_A = \lambda(\vec{r}_B - \vec{r}_M) \Leftrightarrow$$

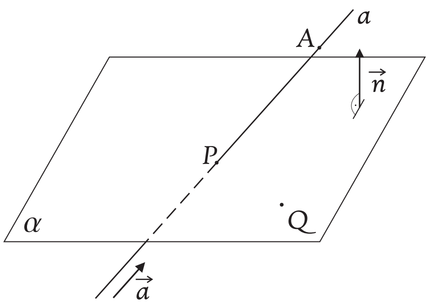
$$\vec{r}_M(1+\lambda) = \lambda\vec{r}_B + \vec{r}_A \Leftrightarrow \boxed{\vec{r}_M = \frac{\vec{r}_A + \lambda\vec{r}_B}{1+\lambda}}$$

Napomena:

Kada je $\dfrac{|\overrightarrow{AM}|}{|\overrightarrow{MB}|} = \dfrac{m}{n}$, zbog praktičnosti se uzima $\dfrac{m}{n} = \lambda$,

pa dobijamo $\dfrac{|\overrightarrow{AM}|}{|\overrightarrow{MB}|} = \dfrac{\lambda}{1}$

Prodor prave kroz ravan



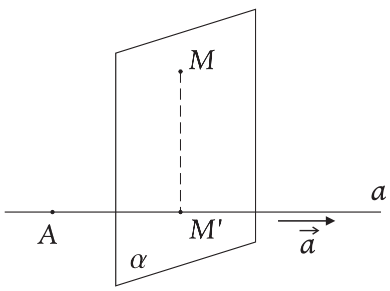
$a : \vec{r} = \vec{r}_A + t \cdot \vec{a}$ sve tačke prave

$\alpha : (\vec{r} - \vec{r}_Q) \cdot \vec{n} = 0$ sve tačke ravni

$$(\vec{r}_A + t\vec{a} - \vec{r}_Q) \cdot \vec{n} = 0 \Rightarrow t = \frac{\vec{n} \cdot (\vec{r}_Q - \vec{r}_A)}{\vec{a} \cdot \vec{n}}$$

$$\boxed{\vec{r}_P = \vec{r}_A + \frac{(\vec{r}_Q - \vec{r}_A) \cdot \vec{n}}{\vec{a} \cdot \vec{n}} \cdot \vec{a}}$$

Projekcija tačke na pravu



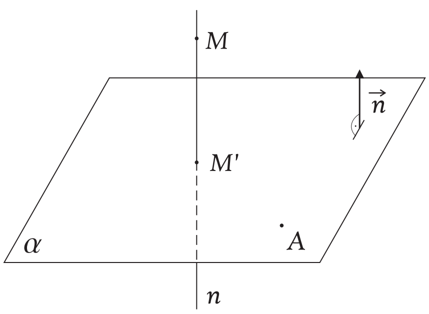
$a : \vec{r} = \vec{r}_A + t \cdot \vec{a}$

$M \in \alpha \wedge \vec{n}_\alpha = \vec{a} \Rightarrow \alpha : (\vec{r} - \vec{r}_M) \cdot \vec{a} = 0$

$a \cap \alpha = \{M'\}$

$$\boxed{\vec{r}_{M'} = \vec{r}_A + \frac{(\vec{r}_M - \vec{r}_A) \cdot \vec{a}}{\vec{a} \cdot \vec{a}} \cdot \vec{a}}$$

Projekcija tačke na ravan



$\alpha : (\vec{r} - \vec{r}_A) \cdot \vec{n}_\alpha = 0$

$M \in n \wedge \vec{n}_\alpha = \vec{n} \Rightarrow n : \vec{r} = \vec{r}_M + t \cdot \vec{n}_\alpha$

$n \cap \alpha = \{M'\}$

$$\boxed{\vec{r}_{M'} = \vec{r}_M + \frac{(\vec{r}_A - \vec{r}_M) \cdot \vec{n}_\alpha}{\vec{n}_\alpha \cdot \vec{n}_\alpha} \cdot \vec{n}_\alpha}$$

Odnos između pravih a i b

$$a : \frac{x-x_A}{a_1} = \frac{y-y_A}{a_2} = \frac{z-z_A}{a_3} = t_1 \qquad \vec{a} = (a_1, a_2, a_3)$$

$$b : \frac{x-x_B}{b_1} = \frac{y-y_B}{b_2} = \frac{z-z_B}{b_3} = t_2 \qquad \vec{b} = (b_1, b_2, b_3)$$

1. $\vec{a} = \lambda \cdot \vec{b} \;\;\Rightarrow\;\; a \| b$ ili se a i b poklapaju

   - Ako $A \in b \;\;\Rightarrow\;\; a \equiv b$, a inače $a \| b$.

2. $\vec{a} \neq \lambda \cdot \vec{b} \;\;\Rightarrow\;\; a$ i b se seku ili se mimoilaze

   - Ako je $\overrightarrow{AB} \cdot (\vec{a} \times \vec{b}) = 0 \;\;\Rightarrow\;\; a$ i b se seku.
   - Ako je $\overrightarrow{AB} \cdot (\vec{a} \times \vec{b}) \neq 0 \;\;\Rightarrow\;\; a$ i b se mimoilaze.

Međusobni položaj prave i ravni

$$\alpha : \vec{r} \cdot \vec{n}_\alpha = \vec{r}_Q \cdot \vec{n}_\alpha \qquad Q(x_Q, y_Q, z_Q), \quad \vec{n}_\alpha = (A, B, C)$$

$$p : \vec{r} = \vec{r}_P + t \cdot \vec{p} \qquad P(x_P, y_P, z_P), \quad \vec{p} = (p_1, p_2, p_3)$$

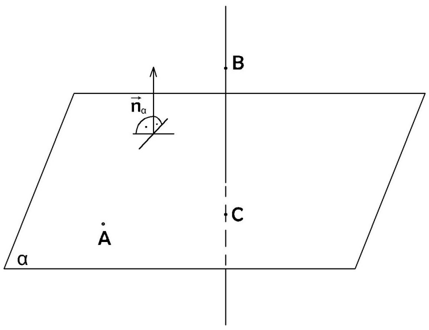
1. $\vec{n}_\alpha \cdot \vec{p} = 0 \;\;\Rightarrow\;\; p \| \alpha$ ili $p \subset \alpha$

   - Tačku P uvrstimo u α i ako jednakost važi, tada $p \subset \alpha$.
   - Ako je jednakost netačna onda je $p \| \alpha$ i $p \not\subset \alpha$.

2. $\vec{n}_\alpha \cdot \vec{p} \neq 0 \;\;\Rightarrow\;\; p$ i α se seku

Zadatak 1 *Napisati jednačinu ravni α koja prolazi kroz tačku $A(2,3,0)$ i normalna je na pravu određenu tačkama $B(1,1,-1)$ i $C(0,0,3)$. Da li sredina duži BC pripada ravni α?*

Rešenje:



$$\vec{n}_\alpha = \overrightarrow{BC} = \vec{r}_C - \vec{r}_B = (-1,-1,4)$$

$$\alpha : (\vec{r} - \vec{r}_A) \cdot \vec{n}_\alpha = 0$$

$$((x,y,z) - (2,3,0)) \cdot (-1,-1,4) = 0$$

$$-(x-2) - (y-3) + 4z = 0 \;\Rightarrow\; \alpha : -x - y + 4z + 5 = 0$$

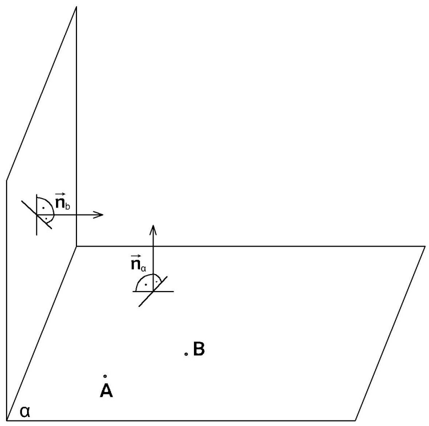
$$\vec{r}_S = \frac{\vec{r}_B + \vec{r}_C}{2} = (\tfrac{1}{2}, \tfrac{1}{2}, 1)$$

Uvrštavanjem koordinata tačke S u jednačinu ravni α dobijamo $-\frac{1}{2} - \frac{1}{2} + 4 \cdot 1 + 5 \neq 0 \;\Rightarrow\; S \notin \alpha$.

□

Zadatak 2 *Naći jednačinu ravni α koja sadrži tačke $A(1,2,3)$ i $B(3,2,1)$ i koja je normalna na ravan $\beta : 4x - y + 2z = 7$.*

Rešenje:



$$\vec{n}_\beta = (4,-1,2)$$

$$\overrightarrow{AB} = \vec{r}_B - \vec{r}_A = (2,0,-2)$$

$$\left.\begin{array}{ccc} \vec{n}_\alpha & \perp & \overrightarrow{AB} \\ \vec{n}_\alpha & \perp & \vec{n}_\beta \end{array}\right\} \Rightarrow \vec{n}_\alpha \parallel \overrightarrow{AB} \times \vec{n}_\beta \;\Rightarrow\; \vec{n}_\alpha = k \cdot (\overrightarrow{AB} \times \vec{n}_\beta)$$

$$\overrightarrow{AB} \times \vec{n}_\beta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -2 \\ 4 & -1 & 2 \end{vmatrix} = (2,12,2) = 2(1,6,1)$$

$$\Rightarrow \vec{n}_\alpha = (1,6,1)$$

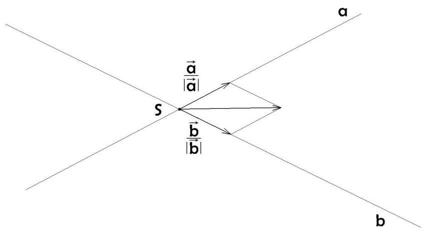
$$\alpha : (\vec{r} - \vec{r}_A) \cdot \vec{n}_\alpha = 0 \;\Leftrightarrow\; ((x,y,z) - (1,2,3)) \cdot (1,6,1) = 0$$

$$\alpha : x + 6y + z - 16 = 0$$

□

Zadatak 3 *Odrediti simetralu oštrog ugla između pravih a i b koje se seku u tački S.*

Rešenje:



$$a \cap b = \{S\}$$

$$s_{1,2}: \ \vec{r} = \vec{r}_S + t \cdot \vec{s}_{1,2} = \vec{r}_S + t \cdot (\frac{\vec{a}}{|\vec{a}|} \pm \frac{\vec{b}}{|\vec{b}|})$$

$$\text{Ako } \vec{a} \cdot \vec{b} > 0 \ \Rightarrow \ \cos(\vec{a}, \vec{b}) > 0 \ \Rightarrow \ \measuredangle(\vec{a}, \vec{b}) \in (0, \tfrac{\pi}{2})$$

$$\vec{s}_1 = \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|} \ \| \ |\vec{b}| \cdot \vec{a} + |\vec{a}| \cdot \vec{b}$$

$$\text{Ako } \vec{a} \cdot \vec{b} < 0 \ \Rightarrow \ \cos(\vec{a}, \vec{b}) < 0 \ \Rightarrow \ \measuredangle(\vec{a}, \vec{b}) \in (\tfrac{\pi}{2}, \pi)$$

$$\vec{s}_2 = \frac{\vec{a}}{|\vec{a}|} - \frac{\vec{b}}{|\vec{b}|} \ \| \ |\vec{b}| \cdot \vec{a} - |\vec{a}| \cdot \vec{b}$$

$$\vec{s}_1 \cdot \vec{s}_2 = 0 \ \Leftrightarrow \ \vec{s}_1 \perp \vec{s}_2$$

□

Zadatak 4 *Naći jednačinu presečne prave p ravni* $\alpha: 2x+3y-3z+1=0$ *i* $\beta: 4x+7y-z-2=0$.

Rešenje: Do jednačine prave p dolazimo rešavanjem sistema jednačina ravni α i β:

$$\begin{array}{rcr} 2x+3y-3z &=& -1 \\ 4x+7y-\ z &=& 2 \\ \hline 2x+3y-3z &=& -1 \\ y+5z &=& 4 \\ \hline \end{array}$$

$$(x,y,z) = \{(\frac{-13}{2} + 9z, 4-5z, z) | z \in \mathbb{R}\}$$

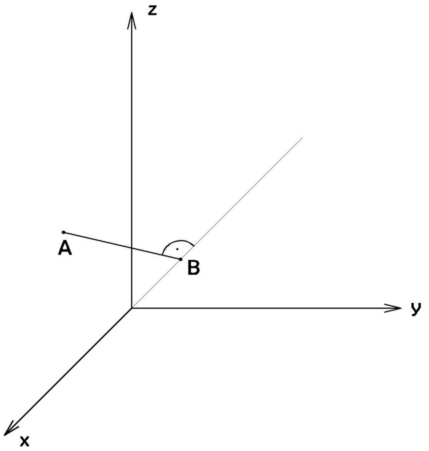
Sistem jednačina je jednostruko neodređen i rešenje sistema upravo određuje jednačinu prave p sa vektorom pravca $\vec{p} = (9, -5, 1)$ i tačkom $P(-\frac{13}{2}, 4, 0)$:

$$p: \frac{x + \frac{13}{2}}{9} = \frac{y-4}{-5} = \frac{z-0}{1}.$$

□

Zadatak 5 *Neka je tačka A određena vektorom položaja $\vec{r}_A$. Izračunati vektor položaja tačke B koja je jednako udaljena od sve tri koordinatne ose, ali tako da duž AB bude minimalne dužine. Posmatrati samo prvi oktant.*

Rešenje:



Jednačina prave koja je jednako udaljena od sve tri koordinatne ose, a prolazi kroz prvi oktant je

$p : x = y = z.$

Tačka B je projekcija tačke A na pravu p.

$$\vec{r}_B = \vec{r}_O + \frac{(\vec{r}_A - \vec{r}_O)\cdot\vec{p}}{\vec{p}\cdot\vec{p}}\cdot\vec{p} = \frac{\vec{r}_A\cdot(1,1,1)}{(1,1,1)\cdot(1,1,1)}\cdot(1,1,1)$$

□

Zadatak 6 *Data je tačka $A(1,1,1)$, ravan $\alpha : 2x - y - z + 1 = 0$ i prava $p : \frac{x+1}{2} = y = \frac{z-1}{2}$.*

a) Napisati jednačinu ravni β koja je paralelna ravni α i sadrži tačku A.

b) Izračunati koordinate tačke T koja pripada pravoj p i ravni β.

c) Napisati jednačinu prave q koja je paralelna ravni α, seče pravu p i sadrži tačku A.

Rešenje:

$\vec{r}_A = (1,1,1)$

$\alpha : \vec{n}\cdot\vec{r} = \vec{n}\cdot\vec{r}_Q \quad \vec{n} = (2,-1,-1) \quad \vec{r}_Q = (0,0,1)$

$p : \vec{r} = \vec{r}_P + t\cdot\vec{p} \quad \vec{r}_P = (-1,0,1) \quad \vec{p} = (2,1,2)$

a) $\beta : \vec{n}\cdot\vec{r} = \vec{n}\cdot\vec{r}_A$

$(2,-1,-1)\cdot(x,y,z) = (2,-1,-1)\cdot(1,1,1)$

$\beta : 2x - y - z = 0$

b) $p \cap \beta = \{T\}$

$$\vec{r}_T = \vec{r}_P + \frac{(\vec{r}_A - \vec{r}_P)\cdot\vec{n}}{\vec{p}\cdot\vec{n}}\cdot\vec{p} = (-1,0,1) + \frac{(2,1,0)(2,-1,-1)}{(2,1,2)(2,-1,-1)}\cdot(2,1,2) = (-1,0,1) + \frac{3}{1}(2,1,2) = (5,3,7)$$

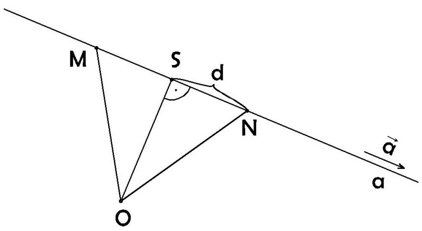
c) $q \parallel \alpha \wedge A \in \alpha$

$q : \vec{r} = \vec{r}_A + t\cdot\overrightarrow{AT} = (1,1,1) + t\cdot(4,2,6)$

□

Zadatak 7 *Odrediti* $\vec{r}_M$ *i* $\vec{r}_N$ *tako da* OMN *bude jednakostraničan trougao i da tačke* M *i* N *pripadaju pravoj* $a : \vec{r} = \vec{r}_A + t\vec{a}$, O *je* $(0,0,0)$.

Rešenje:



Tačka S je projekcija koordinatnog početka na pravu a:

$$\vec{r}_S = \vec{r}_A + \frac{(\vec{r}_O - \vec{r}_A)\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\cdot a = \vec{r}_A - \frac{\vec{r}_A\cdot\vec{a}}{\vec{a}\cdot\vec{a}}\cdot a$$

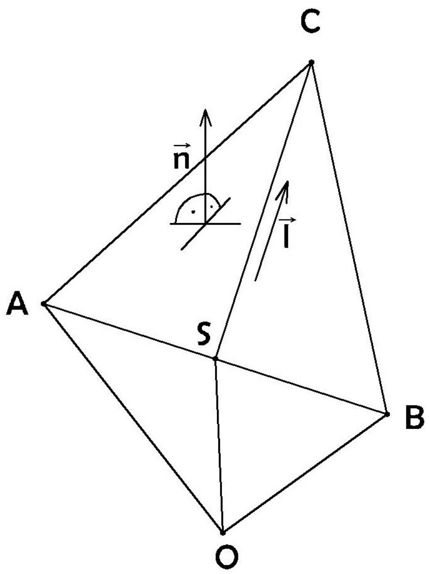
$$|\overrightarrow{OS}| = 2\cdot d\cdot\frac{\sqrt{3}}{2} \quad\Rightarrow\quad d = \frac{|\overrightarrow{OS}|}{\sqrt{3}} = \frac{|\vec{r}_S|}{\sqrt{3}}$$

$$\vec{r}_{M,N} = \vec{r}_S \pm \frac{|\vec{r}_S|}{\sqrt{3}}\cdot\frac{\vec{a}}{|\vec{a}|}$$

□

Zadatak 8 *Dati su vektori* $\vec{r}_A$ *i* $\vec{r}_B$. *Naći* $\vec{r}_C$ *tako da* ABC *bude jednakostraničan trougao i da su* $OABC$ *komplanarne tačke.*

Rešenje:



$$\vec{r}_S = \frac{\vec{r}_A + \vec{r}_B}{2}$$

$$\overrightarrow{AB} = \vec{r}_B - \vec{r}_A$$

$$\vec{r}_{C_1,C_2} = \vec{r}_S \pm |\overrightarrow{AB}|\cdot\frac{\sqrt{3}}{2}\cdot\frac{\vec{l}}{|\vec{l}|}$$

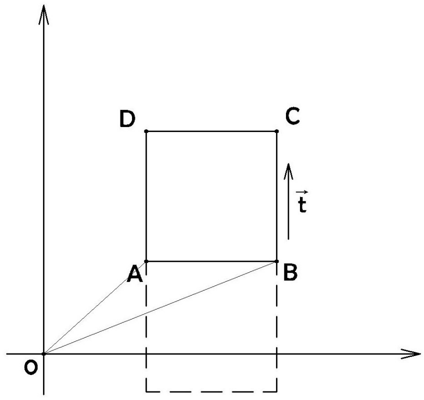
$$\left.\begin{array}{ccc}\vec{l} & \perp & \overrightarrow{AB}\\ \vec{l} & \perp & \vec{n}\end{array}\right\} \quad\Rightarrow\quad \vec{l} = \overrightarrow{AB}\times(\overrightarrow{AB}\times\vec{r}_A)$$

$$\left.\begin{array}{ccc}\vec{n} & \perp & \overrightarrow{AB}\\ \vec{n} & \perp & \vec{r}_A\end{array}\right\} \quad\Rightarrow\quad \vec{n} = \overrightarrow{AB}\times\vec{r}_A$$

□

Zadatak 9 *Dati su vektori $\vec{r}_A$ i $\vec{r}_B$. Naći $\vec{r}_C$ i $\vec{r}_D$ tako da ABCD bude kvadrat i da su OABCD komplanarne tačke.*

Rešenje:



$$\vec{r}_{C_1,C_2} = \vec{r}_B \pm |\overrightarrow{AB}| \cdot \frac{\vec{t}}{|\vec{t}|} = \vec{r}_B \pm |\vec{r}_B - \vec{r}_A| \cdot \frac{\vec{t}}{|\vec{t}|}$$

$$\left.\begin{array}{l} \vec{t} \perp \overrightarrow{AB} \\ \vec{t} \perp \vec{n} \end{array}\right\} \Rightarrow \vec{t} = \overrightarrow{AB} \times (\vec{r}_A \times \vec{r}_B)$$

$$\left.\begin{array}{l} \vec{n} \perp \vec{r}_A \\ \vec{n} \perp \vec{r}_B \end{array}\right\} \Rightarrow \vec{n} = \vec{r}_A \times \vec{r}_B$$

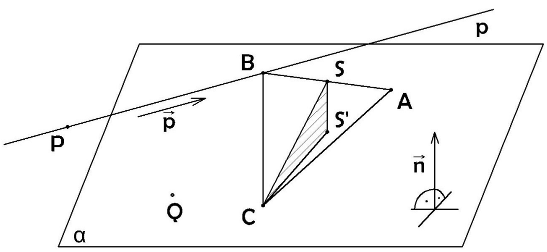
I način: $\vec{r}_{D_1,D_2} = \vec{r}_A \pm |\overrightarrow{AB}| \cdot \frac{\vec{t}}{|\vec{t}|}$

II način: $\vec{r}_D = \vec{r}_A + \overrightarrow{BC} = \vec{r}_A + \vec{r}_C - \vec{r}_B$

□

Zadatak 10 *Data je ravan $\alpha : \vec{n}\vec{r} = \vec{n}\vec{r}_Q$ i prava $p : \vec{r} = \vec{r}_P + t\vec{p}$. Tačka A ne pripada ni ravni α ni pravoj p. U zavisnosti od $\vec{n}$, $\vec{p}$, $\vec{r}_A$, $\vec{r}_Q$ i $\vec{r}_P$ izraziti vektore položaja temena jednakokrakog $\triangle ABC$ čije teme B pripada pravoj p, teme C pripada ravni α i stranica AB je osnovica trougla koja je paralelna ravni α, pri čemu ravan trougla $\triangle ABC$ zaklapa sa ravni α ugao od $\frac{\pi}{4}$.*

Rešenje:



$A \in \beta \ \wedge \ \alpha \parallel \beta$

$\beta : \vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{r}_A$

$p \cap \beta = B$

$$\vec{r}_B = \vec{r}_P + \frac{(\vec{r}_A - \vec{r}_P) \cdot \vec{n}}{\vec{p} \cdot \vec{n}} \cdot \vec{p}$$

$$\vec{r}_S = \frac{\vec{r}_A + \vec{r}_B}{2}$$

$\triangle SS'C$ je jednakokraki, pravougli trougao. Tačka S' je projekcija tačke S na ravan α:

$$\vec{r}_{S'} = \vec{r}_S + \frac{(\vec{r}_Q - \vec{r}_S) \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \cdot \vec{n}$$

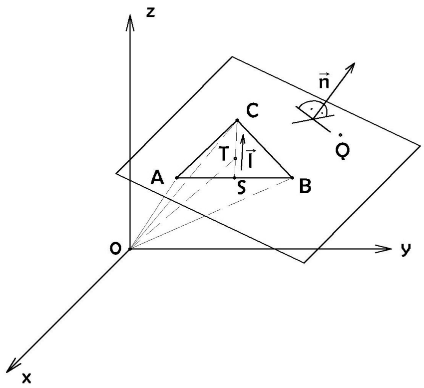
$SS' \perp \alpha \ \wedge \ S' \in \alpha$

$$\left.\begin{array}{l} \overrightarrow{S'C} \perp \vec{n} \\ \overrightarrow{S'C} \perp \overrightarrow{AB} \end{array}\right\} \Rightarrow \overrightarrow{S'C} \parallel \vec{n} \times \overrightarrow{AB} \quad \Rightarrow \quad \vec{r}_C = \vec{r}_{S'} + \overrightarrow{S'C} = \vec{r}_{S'} + |\overrightarrow{SS'}| \cdot \frac{\vec{n} \times \overrightarrow{AB}}{|\vec{n} \times \overrightarrow{AB}|}$$

□

Zadatak 11 *Data je ravan $\alpha : \vec{n}\cdot\vec{r} = \vec{n}\cdot\vec{r}_Q$. Odrediti $\vec{r}_A$, $\vec{r}_B$ i $\vec{r}_C$ tako da OABC bude pravilan tetraedar, gde $\triangle ABC$ pripada ravni α i $AB \parallel xOy$ i $\alpha \nparallel xOy$ ravni.*

Rešenje:

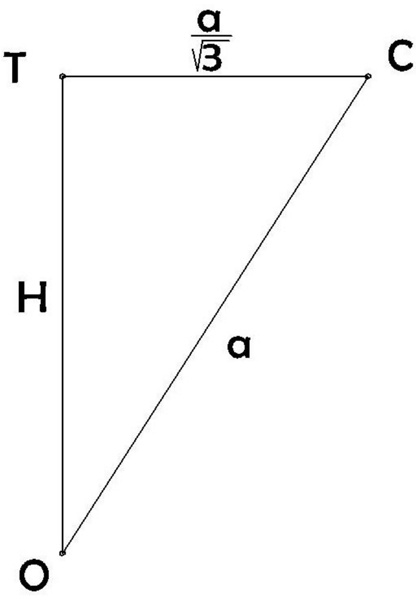


$T \in \alpha \;\wedge\; \overrightarrow{OT} \perp \alpha$

Tačka T je projekcija koordinatnog početka na ravan α:

$$\vec{r}_T = \vec{r}_O + \frac{(\vec{r}_Q - \vec{r}_O)\cdot\vec{n}}{\vec{n}\cdot\vec{n}}\cdot\vec{n}$$

$$|\overrightarrow{TC}| = \tfrac{2}{3}\cdot\tfrac{a\sqrt{3}}{2} = \tfrac{a}{\sqrt{3}}$$



$$H = |\vec{r}_T| = \sqrt{a^2 - \tfrac{a^2}{3}} = \sqrt{\tfrac{2}{3}}a \;\Rightarrow\; a = \tfrac{\sqrt{3}}{\sqrt{2}}H = \tfrac{\sqrt{3}}{\sqrt{2}}\cdot|\vec{r}_T|$$

$$|\overrightarrow{ST}| = \tfrac{1}{3}\cdot\tfrac{a\sqrt{3}}{2} = \tfrac{\sqrt{3}}{6}\cdot\tfrac{\sqrt{3}}{\sqrt{2}}\cdot|\vec{r}_T| = \tfrac{1}{2\sqrt{2}}\cdot|\vec{r}_T|$$

$$\vec{r}_S = \vec{r}_T \pm \tfrac{1}{2\sqrt{2}}|\vec{r}_T|\cdot\tfrac{\vec{l}}{|\vec{l}|}$$

$$\left.\begin{array}{l}\vec{l} \perp \vec{n}\\ \vec{l} \perp \overrightarrow{AB}\end{array}\right\} \Rightarrow \vec{l} = \vec{n}\times(\vec{n}\times\vec{k})$$

$$\left.\begin{array}{l}\overrightarrow{AB} \perp \vec{n}\\ \overrightarrow{AB} \perp \vec{k}\end{array}\right\} \Rightarrow \overrightarrow{AB} \parallel \vec{n}\times\vec{k}$$

$$\vec{r}_{A,B} = \vec{r}_S \pm \tfrac{1}{2}\cdot\tfrac{\sqrt{3}}{\sqrt{2}}\cdot|\vec{r}_T|\cdot\tfrac{\vec{n}\times\vec{k}}{|\vec{n}\times\vec{k}|}$$

$$\vec{r}_C = \vec{r}_T + 2\overrightarrow{ST} = \vec{r}_T + 2(\vec{r}_T - \vec{r}_S) = 3\vec{r}_T - 2\vec{r}_S$$

Postoje dva takva tetraedra. □

Primeri sa testa

• Prave $m: \frac{x-2}{3} = \frac{y-1}{-2} = \frac{z}{5}$ i $n: \frac{x-5}{-6} = \frac{y+1}{4} = \frac{z-5}{-5}$:

a) se mimoilaze b) su paralelne c) se poklapaju d) se seku

• Za koje α i β su vektori $\vec{a} = (1,2,3)$ i $\vec{b} = (1,\alpha,\beta)$:

nekolinearni ________________________ ortogonalni ________________________

• Napisati vektor položaja tačke P simetrične tački $A(1,2,0)$ u odnosu na pravu $p: x = y = z$:

____

• Ako je $Ax+By+C=0$ jednačina prave u ravni xOy i $A \neq B$, tada je vektor normale na tu pravu:

a) (A,B) b) $(A,-B)$ c) $(-A,B)$ d) (B,A) e) $(B,-A)$ f) $(-A,-B)$ g) $(-B,-A)$

• Za ravan $\alpha: 8x = 8$ napisati jedan njen vektor normale $\vec{n}_\alpha = ($__________$)$, koordinate jedne njene tačke $A($__________$)$ i jednačinu prave l normalne na ravan α kroz tačku A: _______________.