16.06.2021.

1. a) $x_1+x_2+x_3+x_4+x_5=20$ $\binom{20+4}{4}=\binom{24}{4}$

b) $y_1+y_2+y_3+y_4+y_5=15$ $\binom{15+4}{4}=\binom{19}{4}$

2. $\binom{n}{0}+2\binom{n}{1}+\dots+(n+1)\binom{n}{n}=\sum_{i=0}^{n}(i+1)\binom{n}{i}=$

$=\sum_{i=0}^{n} i\binom{n}{i}+\sum_{i=0}^{n}\binom{n}{i}=\sum_{i=0}^{n} i\binom{n}{i}+\sum_{i=1}^{n} i\binom{n}{i}+2^n=$

$=0+\sum_{i=1}^{n}\frac{i\cdot n!}{i!(n-i)!}+2^n=n\cdot\sum_{i=1}^{n}\frac{(n-1)!}{(i-1)!(n-i)!}+2^n=$

$=n\sum_{i=1}^{n}\binom{n-1}{i-1}+2^n=n\sum_{i-1=0}^{n-1}\binom{n-1}{i-1}+2^n=$

$=n2^{n-1}+2^n$

3. 1 3 5 7 9

S_2 – 2 ma 2, S_4 – 4 ma 4, ...

$N(S_2)=N(S_4)=N(S_6)=N(S_8)=3!$

$N(S_2S_4)=N(S_2S_6)=\dots=2\cdot2\cdot2\cdot1=2!$

$N(S_2S_4S_6)=\dots=1$

$N(S_2S_4S_6S_8)=1$ $N=4!$

$N(S_2'S_4'S_6'S_8')=4!-\binom{4}{1}3!+\binom{4}{2}\cdot2!-\binom{4}{3}1!+\binom{4}{4}$

4. $a_n^{(1)}=An+B$ $a_n^{(p)}=Cn^2$

$0=B$

$Cn^2-2C(n^2-2n+1)+C(n^2-4n+4)$

$1=A+\frac{1}{2}\Rightarrow A=\frac{1}{2}$

$=4C-2C=1\Rightarrow C=\frac{1}{2}$

$a_n=\frac{1}{2}n(n+1)$ $\Rightarrow a_n=An+B+\frac{1}{2}n^2$