DISKRETNA MATEMATIKA

- PREDAVANJE -

Jovanka Pantović

1. Binomni koeficijenti
2. Binomna formula
3. Polinomni koeficijenti
4. Polinomna formula

Tema 1

Binomni koeficijenti

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Binomni koeficijenti - osobina1

Lema

Neka su m i n celi brojevi sa osobinom $0 \leq m \leq n$. Tada važi

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

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Ako je $m = 0$ imamo

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Ako je $m = 0$ imamo

$$\binom{n}{0} = 1 \qquad \frac{n!}{0! \cdot n!} = 1$$

Ako je $m > 0$

$$\binom{n}{m} = \frac{n(n-1)\dots(n-m+1)}{m(m-1)\dots 2 \cdot 1} \cdot \frac{(n-m)!}{(n-m)!} =$$

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Neka su m *i* n *celi brojevi sa osobinom* $0 \leq m \leq n$. *Tada važi*

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$$\binom{n}{m} = \frac{n(n-1)\dots(n-m+1)}{m(m-1)\dots 2 \cdot 1} \cdot \frac{(n-m)!}{(n-m)!} = \frac{n!}{m!(n-m)!}.$$

Binomni koeficijenti - osobina2

Lema

Neka su n i m celi brojevi sa osobinom $0 \leq m \leq n$.

$$\binom{n}{m} = \binom{n}{n-m}$$

Dokaz. (algebarski)
Na osnovu osobine1, možemo izvesti sledeće:

$$\binom{n}{m} = \frac{n!}{m!(n-m)!} = \frac{n!}{(n-(n-m))!(n-m)!} = \binom{n}{n-m}.$$

Binomni koeficijenti - osobina2

Lema

Neka su n *i* m *celi brojvei sa osobinom* $0 \leq m \leq n$.

$$\binom{n}{m} = \binom{n}{n-m}$$

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Dokaz. (kombinatorno)

Binomni koeficijenti - osobina2

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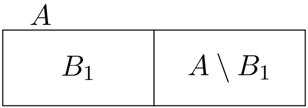
Neka su n *i* m *celi brojvei sa osobinom* $0 \leq m \leq n$.

$$\binom{n}{m} = \binom{n}{n-m}$$

Dokaz. (kombinatorno)
Broj m-točlanih podskupova skupa od n elemenata jednak je broju $(n-m)$-točlanih podskupova skupa od n elemenata.



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Pokazati kombinatorno da je $\binom{5}{2} = \binom{5}{3}$, *koristeći kombinacije bez ponavljanja.*

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Neka je $A = \{a, b, c, d, e\}$ i $B \in \binom{A}{2}$.

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Neka je $A = \{a, b, c, d, e\}$ i $B \in \binom{A}{2}$.

B	$A \setminus B$
$\{a, b\}$	$\{c, d, e\}$
$\{a, c\}$	$\{b, d, e\}$
$\{a, d\}$	$\{b, c, e\}$
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B	$A \setminus B$
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$\{c, e\}$	$\{a, b, d\}$
$\{d, e\}$	$\{a, b, c\}$

Paskalov trougao (osobina 3)

Lemma

Neka su n *i* m *celi brojevi sa osobinom* $0 \leq m \leq n$. *Tada važi*

1. $\binom{n}{n} = \binom{n}{0} = 1;$
2. $\binom{n}{m} = \binom{n-1}{m} + \binom{n-1}{m-1}, 0 < m < n.$ *(Paskalov identitet)*

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Dokaz. (algebarski)

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&= \frac{\color{red}{m}\cdot(n-1)! + \color{red}{(n-m)}\cdot(n-1)!}{m\cdot(m-1)!\cdot(n-m)\cdot(n-m-1)!} \\
&= \frac{(m+n-m)\cdot(n-1)!}{m!(n-m)!} = \frac{n!}{m!(n-m)!} = \binom{n}{m}
\end{aligned}$$

Paskalov identitet

Dokaz. (kombinatorno)
Neka je $A = \{a_1, a_2, \ldots, a_n\}$. Skup svih podskupova jednak je uniji skupa podskupova koji sadrže a_1 i skupa podskupova koji ne sadrže a_1.

$\mathcal{P}(A)$

$\{B : B \subseteq A, a_1 \in B\}$	$\{B : B \subseteq A, a_1 \notin B\}$

Prema principu zbira, dobijamo traženi identitet.

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Paskalov trougao

$$
\begin{array}{lccccccccccc}
n=0 & & & & & & \binom{0}{0} & & & & & \\
n=1 & & & & & \binom{1}{0} & & \binom{1}{1} & & & & \\
n=2 & & & & \binom{2}{0} & & \binom{2}{1} & + & \binom{2}{2} & & & \\
n=3 & & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & & \\
n=4 & & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} & \\
n=5 & \binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5} \\
 & & & & & & \cdots & & & & &
\end{array}
$$

Paskalov identitet (osobina 3)

$$
\begin{array}{lccccccccccc}
n=0 & & & & & & 1 & & & & & \\
n=1 & & & & & 1 & & 1 & & & & \\
n=2 & & & & 1 & & 2 & + & 1 & & & \\
n=3 & & & 1 & & 3 & & 3 & & 1 & & \\
n=4 & & 1 & & 4 & & 6 & & 4 & & 1 & \\
n=5 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
 & & & & & & \dots & & & & &
\end{array}
$$

Paskalov trougao u tabelarnom prikazu

(n, m)	0	1	2	3	4	5	6	7	...
0	1								
1	1	1							
2	1	2	1						
3	1	3	3	1					
4	1	4	6	4	1				
5	1	5	10	10	5	1			
6	1	6	15	20	15	6	1		
7	1	7	21	35	35	21	7	1	

Tema 2

Binomna formula

Binomna formula

Teorema (Binomna formula)

Neka je $n \geq 1$. *Tada je*

$$(x+y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y + \ldots + \binom{n}{k} x^{n-k} y^k + \ldots + \binom{n}{n} x^0 y^n$$

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$$(x+y)^1 \quad = \quad \tbinom{1}{0}x + \tbinom{1}{0}y$$

Binomna formula

Teorema (Binomna formula)

Neka je $n \geq 1$. *Tada je*

$$(x+y)^n = \binom{n}{0}x^n y^0 + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{k}x^{n-k}y^k + \ldots + \binom{n}{n}x^0 y^n$$

$$\begin{aligned}(x+y)^1 &= \tbinom{1}{0}x + \tbinom{1}{0}y \\ (x+y)^2 &= \tbinom{2}{0}x^2y^0 + \tbinom{2}{1}x^1y^1 + \tbinom{2}{2}x^0y^2 =\end{aligned}$$

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$$\begin{aligned}
(x+y)^1 &= \tbinom{1}{0} x + \tbinom{1}{0} y \\
(x+y)^2 &= \tbinom{2}{0} x^2 y^0 + \tbinom{2}{1} x^1 y^1 + \tbinom{2}{2} x^0 y^2 = x^2 + 2xy + y^2
\end{aligned}$$

Binomna formula

Teorema (Binomna formula)

Neka je $n \geq 1$. *Tada je*

$$(x+y)^n = \binom{n}{0} x^n y^0 + \binom{n}{1} x^{n-1} y + \ldots + \binom{n}{k} x^{n-k} y^k + \ldots + \binom{n}{n} x^0 y^n$$

$$\begin{aligned}
(x+y)^1 &= \tbinom{1}{0} x + \tbinom{1}{0} y \\
(x+y)^2 &= \tbinom{2}{0} x^2 y^0 + \tbinom{2}{1} x^1 y^1 + \tbinom{2}{2} x^0 y^2 = x^2 + 2xy + y^2 \\
(x+y)^3 &= \tbinom{3}{0} x^3 y^0 + \tbinom{3}{1} x^2 y^1 + \tbinom{3}{2} x^1 y^2 + \tbinom{3}{3} x^0 y^3
\end{aligned}$$

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(x+y)^2 &= \binom{2}{0} x^2 y^0 + \binom{2}{1} x^1 y^1 + \binom{2}{2} x^0 y^2 = x^2 + 2xy + y^2 \\
(x+y)^3 &= \binom{3}{0} x^3 y^0 + \binom{3}{1} x^2 y^1 + \binom{3}{2} x^1 y^2 + \binom{3}{3} x^0 y^3 \\
&= x^3 + 3x^2 y + 3xy^2 + 3y^3
\end{aligned}$$

Leva strana jednakosti

$$(x+y)^2 = (x+y)(x+y)$$

Leva strana jednakosti

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Binomna formula

Dokaz. (kombinatorna interpretacija)

$$(x+y)^n = \underbrace{(x+y)(x+y)\dots(x+y)}_{n \text{ puta}}$$

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&= (xxx+xxy+xyx+xyy+yxx+yxy+yyx+yyy)(x+y)\dots(x+y) \\
&= \ x^n \\
&\quad + x^{n-1}y + \dots + x^{n-1}y \\
&\quad + x^{n-2}y^2 + \dots + x^{n-2}y^2 \\
&\quad + \dots \\
&\quad + xy^{n-1} + \dots + xy^{n-1} \\
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&= \quad x^n \\
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$$

Ako iz m zagrada izaberemo y, a iz $n-m$ zagrada izaberemo x:

$$\boxed{x^{n-m}y^m}$$

Binomna formula

Dokaz. (kombinatorna interpretacija)

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$$= (xxx+xxy+xyx+xyy+yxx+yxy+yyx+yyy)(x+y)\dots(x+y)$$
$$= x^n + x^{n-1}y+\dots+x^{n-1}y + x^{n-2}y^2+\dots+x^{n-2}y^2 + \dots + xy^{n-1}+\dots+xy^{n-1} + y^n$$

Ako iz m zagrada izaberemo y, a iz $n-m$ zagrada izaberemo x:

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Broj načina da izaberemo m zagrada iz kojih ćemo izabrati y jednak je

$$\boxed{\binom{n}{m}}$$

Binomna formula

Dokaz. (algebarski) indukcijom po n
$n = 1 : (x + y)^1 = x + y$

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$n = 1 : (x+y)^1 = x + y$
$T_n \Rightarrow T_{n+1}$:

$$(x+y)^n(x+y) \quad = \quad (x^n + nx^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \ldots + nxy^{n-1} + y^n)(x+y)$$

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&= \begin{cases} x^{n+1} \quad +nx^n y \quad +\binom{n}{2}x^{n-1}y^2 \ +\ldots \quad +\binom{n}{n-1}x^2y^{n-1} + \quad xy^n \\ \end{cases}
\end{aligned}$$

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$n=1:(x+y)^1=x+y$
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&= \left\{\begin{array}{llllll} x^{n+1} & +nx^ny & +\binom{n}{2}x^{n-1}y^2 & +\ldots & +\binom{n}{n-1}x^2y^{n-1}+ & xy^n \\ & +\ x^ny & +\binom{n}{1}x^{n-1}y^2 & +\ldots & +\binom{n}{n-2}x^2y^{n-1}+ & nxy^n+y^n \end{array}\right.\\
&= x^{n+1}+(n+1)x^ny+\left(\binom{n}{1}+\binom{n}{2}\right)x^{n-1}y^2+\ldots\\
&\quad +\left(\binom{n}{n-1}+\binom{n}{n-2}\right)x^2y^{n-1}+(n+1)xy^n+y^{n+1}
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&= x^{n+1} + (n+1)x^n y + \left(\binom{n}{1} + \binom{n}{2}\right)x^{n-1}y^2 + \ldots \\
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&= x^{n+1} + (n+1)x^n y + \binom{n+1}{2}x^{n-1}y^2 + \ldots \\
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&= x^{n+1} + (n+1)x^n y + \binom{n+1}{2}x^{n-1}y^2 + \ldots \\
&\quad + \binom{n+1}{n-1}x^2y^{n-1} + (n+1)xy^n + y^{n+1} \\
&= \sum_{m=0}^{n+1} \binom{n+1}{m} x^{n+1-m}y^m.
\end{aligned}
$$

Zadaci

1. $$\sum_{k=0}^{n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} =$$

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4. $$\sum_{k=0}^{n} 2^k \binom{n}{k} = \binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} \ldots + 2^n \binom{n}{n} =$$

Zadaci

1. $$\sum_{k=0}^{n}\binom{n}{k}=\binom{n}{0}+\binom{n}{1}+\ldots+\binom{n}{n}=(1+1)^n=2^n$$
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4. $$\sum_{k=0}^{n}2^k\binom{n}{k}=\binom{n}{0}+2\binom{n}{1}+2^2\binom{n}{2}\ldots+2^n\binom{n}{n}=(1+2)^n=3^n$$

Tema 3

Polinomni koeficijenti

Polinomni koeficijenti

Neka je $l \geq 1, m_1, \ldots, m_l \geq 0$ i $n = m_1 + \ldots + m_l$.

Polinomni koeficijenti

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Definicija

Polinomni koeficijent:

$$\binom{n}{m_1, m_2, \ldots, m_l} = \frac{n!}{m_1! \cdot \ldots \cdot m_l!}$$

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Definicija

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$$\binom{n}{m_1, m_2, \ldots, m_l} = \frac{n!}{m_1! \cdot \ldots \cdot m_l!}$$

Primer

$$\binom{5}{1, 3, 1} = \frac{5!}{1!3!1!} = 20$$

Polinomni koeficijenti-osobina1

Lema

$$\binom{n}{m_1, m_2, \ldots, m_l} = \binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\cdots\binom{n-(m_1+\ldots+m_{l-1})}{m_l}$$

Polinomni koeficijenti-osobina1

Lema

$$\binom{n}{m_1,m_2,\ldots,m_l} = \binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\ldots\binom{n-(m_1+\ldots+m_{l-1})}{m_l}$$

$$\binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\ldots\binom{m_l}{m_l}$$

Polinomni koeficijenti-osobina1

Lema

$$\binom{n}{m_1, m_2, \ldots, m_l} = \binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\cdots\binom{n-(m_1+\ldots+m_{l-1})}{m_l}$$

$$\binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\cdots\binom{m_l}{m_l}$$

$$= \frac{n!}{m_1!(n-m_1)!}\,\frac{(n-m_1)!}{m_2!(n-m_1-m_2)!}\,\frac{(n-m_1-m_2)!}{m_3!(n-m_1-m_2-m_3)!}\cdots\frac{m_l!}{m_l!0!}$$

Polinomni koeficijenti-osobina1

Lema

$$\binom{n}{m_1, m_2, \ldots, m_l} = \binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\cdots\binom{n-(m_1+\ldots+m_{l-1})}{m_l}$$

$$\binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\cdots\binom{m_l}{m_l}$$

$$= \frac{n!}{m_1!(n-m_1)!}\frac{(n-m_1)!}{m_2!(n-m_1-m_2)!}\frac{(n-m_1-m_2)!}{m_3!(n-m_1-m_2-m_3)!}\cdots\frac{m_l!}{m_l!0!}$$

$$= \frac{n!}{m_1!m_2!\ldots m_l!} =$$

Polinomni koeficijenti-osobina1

Lema

$$\binom{n}{m_1, m_2, \ldots, m_l} = \binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\cdots\binom{n-(m_1+\ldots+m_{l-1})}{m_l}$$

$$\begin{aligned}
&\binom{n}{m_1}\binom{n-m_1}{m_2}\binom{n-(m_1+m_2)}{m_3}\cdots\binom{m_l}{m_l} \\
&\qquad = \frac{n!}{m_1!(n-m_1)!}\frac{(n-m_1)!}{m_2!(n-m_1-m_2)!}\frac{(n-m_1-m_2)!}{m_3!(n-m_1-m_2-m_3)!}\cdots\frac{m_l!}{m_l!0!} \\
&\qquad = \frac{n!}{m_1!m_2!\ldots m_l!} = \binom{n}{m_1, m_2, \ldots, m_l}
\end{aligned}$$

Primer

$$\binom{5}{1,3,1} = \binom{5}{1}\cdot\binom{4}{3}\cdot\binom{1}{1} = 5\cdot 4 = 20$$

Polinomni koeficijenti-osobina2

Lema

$\binom{n}{m_1, m_2, \ldots, m_l} = \binom{n}{k_1, k_2, \ldots, k_l}, \qquad \{\{m_1, \ldots, m_l\}\} = \{\{k_1, \ldots, k_l\}\}$

Polinomni koeficijenti-osobina2

Lema

$$\binom{n}{m_1, m_2, \ldots, m_l} = \binom{n}{k_1, k_2, \ldots, k_l}, \qquad \{\{m_1, \ldots, m_l\}\} = \{\{k_1, \ldots, k_l\}\}$$

Primer

$$\binom{4}{0,1,3} = \binom{4}{0,3,1} = \binom{4}{1,0,3} = \binom{4}{1,3,0} = \binom{4}{3,1,0} = \binom{4}{3,0,1} = \frac{4!}{0!1!3!} = 4$$

Polinomni koeficijenti-osobina3

Neka je $1 \leq m_1, \ldots, m_l \leq n-1$.

Lema

$$\binom{n}{m_1, m_2, \ldots, m_l} = \binom{n-1}{m_1-1, m_2, \ldots, m_l} + \binom{n-1}{m_1, m_2-1, \ldots, m_l} + \ldots + \binom{n-1}{m_1, m_2, \ldots, m_l-1}$$

Polinomni koeficijenti-osobina3

Neka je $1 \leq m_1, \ldots, m_l \leq n-1$.

Lema

$$\binom{n}{m_1, m_2, \ldots, m_l} = \binom{n-1}{m_1-1, m_2, \ldots, m_l} + \binom{n-1}{m_1, m_2-1, \ldots, m_l} + \ldots + \binom{n-1}{m_1, m_2, \ldots, m_l-1}$$

$$P(m_1, m_2, \ldots, m_l) = P(m_1-1, m_2, \ldots, m_l) + P(m_1, m_2-1, \ldots, m_l) + \ldots + P(m_1, m_2, \ldots, m_l-1)$$

Kako je $m_1 \geq 1$, svaka permutacija kao prvu koordinatu ima a_1 ili a_2 ili...ili a_l:

$$\begin{aligned} |P(M)| &= |P(M_{a_1}) \cup P(M_{a_2}) \ldots \cup P(M_{a_l})| \\ &= |P(M \setminus \{a_1\})| + |P(M \setminus \{a_2\})| + \ldots + |P(M \setminus \{a_l\})| \end{aligned}$$

$P(M) =$ skup permutacija multiskupa M

$P(M_{a_i}) =$ skup permutacija multiskupa M sa prvom koordinatom a_i

Polinomni koeficijenti-osobina4

Lema

$$\binom{n}{m_1, m_2, \ldots, m_{l-1}, 0} = \binom{n}{m_1, m_2, \ldots, m_{l-1}}$$

Tema 4

Polinomna formula

Polinomna formula

Teorema (Polinomna formula)

Neka je $l \geq 2$ *i* $n \geq 0$.

$$(x_1 + \ldots + x_l)^n = \sum_{\substack{m_1 + \ldots + m_l = n \\ m_1 \geq 0 \ldots m_l \geq 0}} \binom{n}{m_1, \ldots, m_l} x_1^{m_1} x_2^{m_2} \ldots x_l^{m_l}$$

Polinomna formula

Zadatak

Napisati u razvijenom obliku $(x + y + z)^3$

Polinomna formula

Zadatak

Napisati u razvijenom obliku $(x+y+z)^3$

$$(x+y+z)^3 = \binom{3}{3,0,0}x^3y^0z^0 + \binom{3}{0,3,0}x^0y^3z^0 + \binom{3}{0,0,3}x^0y^0z^3$$

Polinomna formula

Zadatak

Napisati u razvijenom obliku $(x+y+z)^3$

$$\begin{aligned}(x+y+z)^3 \quad = \quad & \binom{3}{3,0,0}x^3y^0z^0 + \binom{3}{0,3,0}x^0y^3z^0 + \binom{3}{0,0,3}x^0y^0z^3 \\ & + \binom{3}{0,1,2}x^0y^1z^2 + \binom{3}{0,2,1}x^0y^2z^1 + \binom{3}{1,0,2}x^1y^0z^2 \\ & + \binom{3}{1,2,0}x^1y^2z^0 + \binom{3}{2,0,1}x^2y^0z^1 + \binom{3}{2,1,0}x^2y^1z^0\end{aligned}$$

Polinomna formula

Zadatak

Napisati u razvijenom obliku $(x+y+z)^3$

$$
\begin{aligned}
(x+y+z)^3 &= \binom{3}{3,0,0}x^3y^0z^0 + \binom{3}{0,3,0}x^0y^3z^0 + \binom{3}{0,0,3}x^0y^0z^3 \\
&\quad + \binom{3}{0,1,2}x^0y^1z^2 + \binom{3}{0,2,1}x^0y^2z^1 + \binom{3}{1,0,2}x^1y^0z^2 \\
&\quad + \binom{3}{1,2,0}x^1y^2z^0 + \binom{3}{2,0,1}x^2y^0z^1 + \binom{3}{2,1,0}x^2y^1z^0 \\
&\quad + \binom{3}{1,1,1}x^1y^1z^1 \\
&= x^3 + y^3 + z^3 + 3yz^2 + 3y^2z + 3xz^2 + 3xy^2 + 3x^2z + 3x^2y + 6xyz
\end{aligned}
$$

Polinomna formula

Zadatak

Odrediti koeficijent uz $x^2y^3z^5$ *u razvoju stepena trinoma* $(x+2y-z)^{10}$

Polinomna formula

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Odrediti koeficijent uz $x^2y^3z^5$ u razvoju stepena trinoma $(x+2y-z)^{10}$

Koeficijent uz $x^2y^3z^5$ je sadržan u sabirku

$$\binom{10}{2,3,5}x^2(2y)^3(-z)^5 = \frac{10!}{2!3!5!}x^2 2^3 y^3 (-1)^5 z^5 = -20160x^2y^3z^5$$

Polinomna formula

Zadatak

Odrediti koeficijent uz x *u razvoju stepena trinoma* $(2x^3 - x + 1)^4$.

Polinomna formula

Zadatak

Odrediti koeficijent uz x *u razvoju stepena trinoma* $(2x^3 - x + 1)^4$.

$$T_{i,j,k} = \binom{4}{i,j,k}(2x^3)^i(-x)^j = \binom{4}{i,j,k}2^i(-1)^j x^{3i+j}$$

$$\begin{array}{rcrcrcl} i & + & j & + & k & = & 4 \\ 3i & + & j & & & = & 1 \end{array}$$

odakle je $(i,j,k) \in \{(0,1,3)\}$ i traženi koeficijent je

$$\binom{4}{0,1,3}2^0(-1)^1 = -4.$$

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2. $$\sum_{\substack{m_1 + \ldots + m_l = n \\ m_1 \geq 0 \ldots m_l \geq 0}} \binom{n}{m_1, \ldots, m_l} = (1 + \ldots + 1)^n = l^n$$