Principi mrežnog programiranja

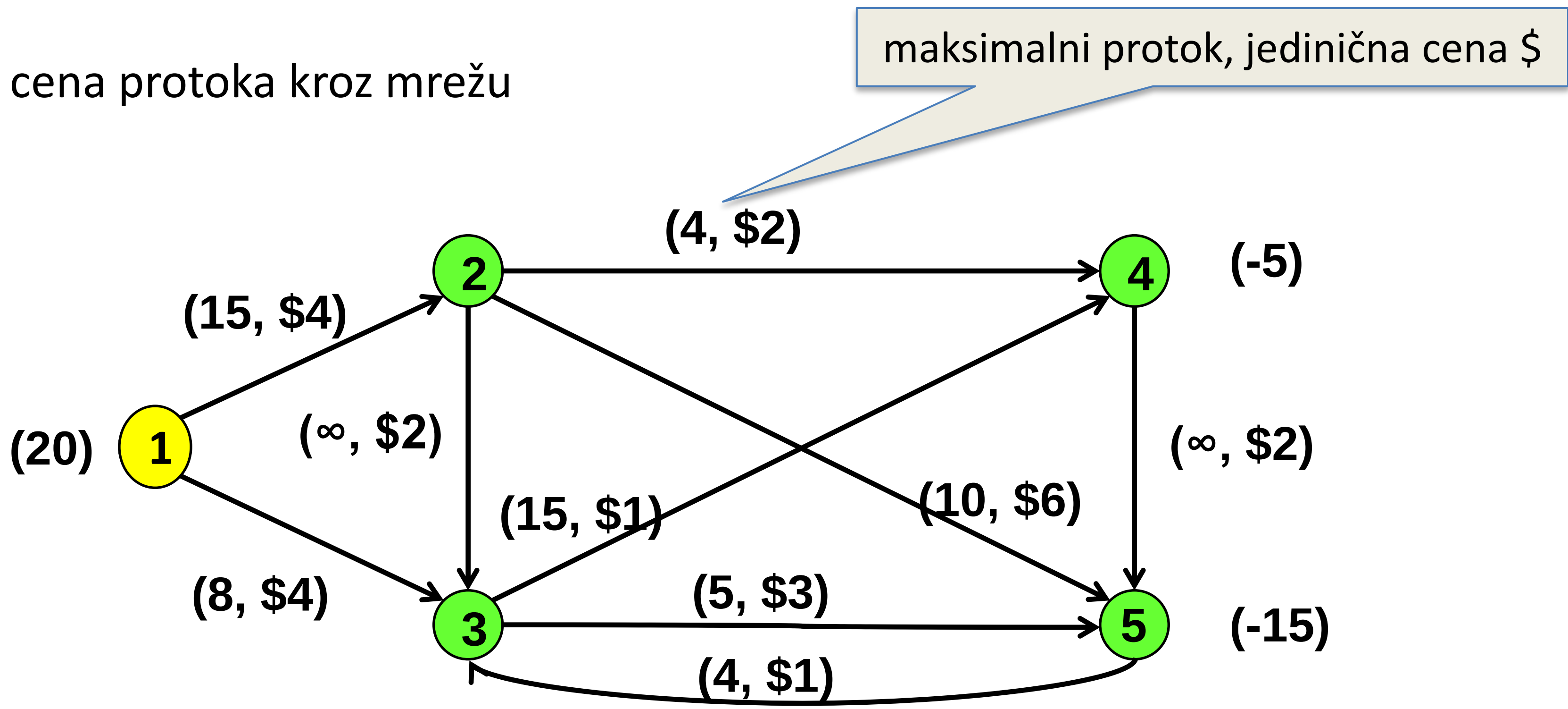
Novembar 2014

automation & control systems group

Problemi koji se mogu predstaviti mrežom

	Gradski transport	**Kominukicaioni sistemi**	**Vodovodi**
Proizvod	Autobusi, automobili	Poruke	Voda
Čvorovi	Stajališta, raskrsnice	Komunikacioni centri, relejne stanice	Pumpne stanice, rezervoari, korisnici
Grane	Ulice	Komunikacioni kanali	Cevovodi

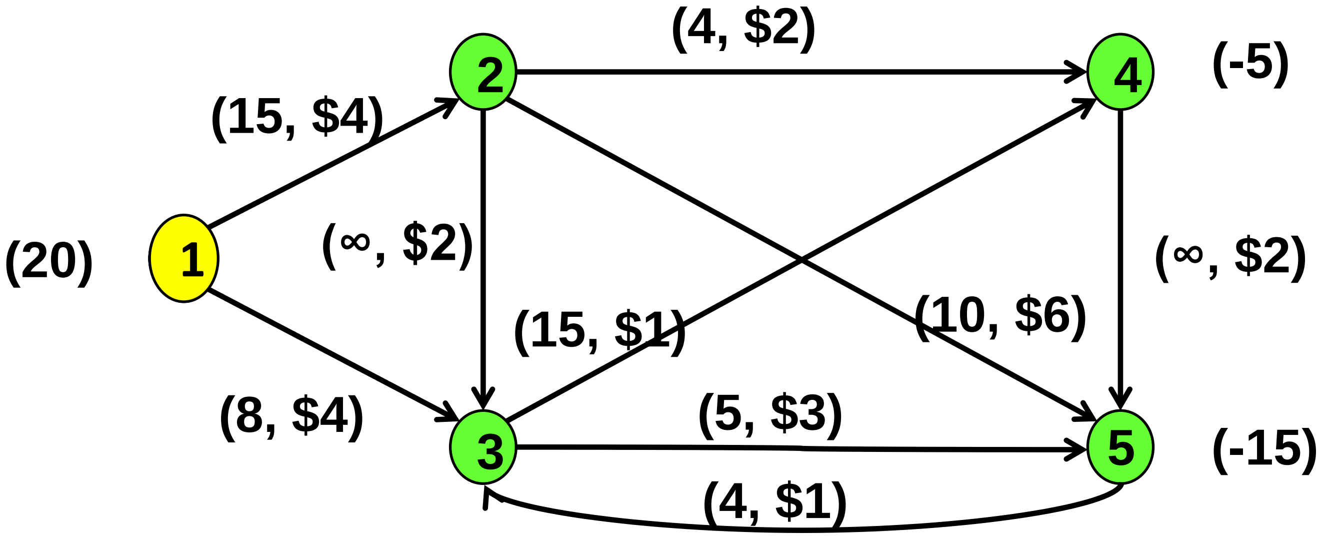
Minimalna cena protoka kroz mrežu



$$x_{ij} = \quad protok\ kroz\ granu\ i - j$$

$$\binom{Protok\ iz}{\check{c}vora} - \binom{Protok\ u}{\check{c}vor} = \binom{Snadbevenost}{\check{c}vora}$$

$$\begin{array}{ccccccccccc} x_{12} & + & x_{13} & & & & & & & = & 20 \\ x_{23} & + & x_{24} & + & x_{25} & - & x_{12} & & & = & 0 \end{array}$$



	x12	x13	x23	x24	x25	x34	x35	x45	x53	balans
Č1	1	1								20
Č2	-1		1	1	1					0
Č3		-1	-1			1	1		-1	0
Č4				-1		-1		1		-5
Č5					-1		-1	-1	1	-15
Kap	15	8	∞	4	10	15	5	∞	4	
KO	4	4	2	2	6	1	3	2	1	

	x12	x13	x23	x24	x25	x34	x35	x45	x53	balans
Č1	1	1								20
Č2	-1		1	1	1					0
Č3		-1	-1			1	1		-1	0
Č4				-1		-1		1		-5
Č5					-1		-1	-1	1	-15
Kap	15	8	∞	4	10	15	5	∞	4	
KO	4	4	2	2	6	1	3	2	1	

$$\text{Min } z = \sum_i \sum_j c_{ij} x_{ij}$$

$$\sum_j x_{ij} - \sum_k x_{ki} = b_i$$

$$l_{ij} \le x_{ij} \le u_{ij}$$

Transportni problem

$a_i =$ broj jedinica raspoloživih na izvoru i
$(i = 1 \cdots m)$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$b_j =$ broj jedinica potrebnih na cilju j $(j = 1 \cdots n)$

$c_{ij} =$ troškovi transporta od izvora i do cilja j
$(i = 1 \cdots m, j = 1 \cdots n)$

$x_{ij} =$ broj jedinica distribuiranih od izvora i do cilja j

$$\text{Min } z = \sum_i \sum_j c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad (i = 1,2, \dots m)$$

$$\sum_{i=1}^{m} (-x_{ij}) = -b_j \qquad (j = 1,2, \dots m)$$

$$x_{ij} \geq 0 \;\; (i = 1 \cdots m, j = 1 \cdots n)$$

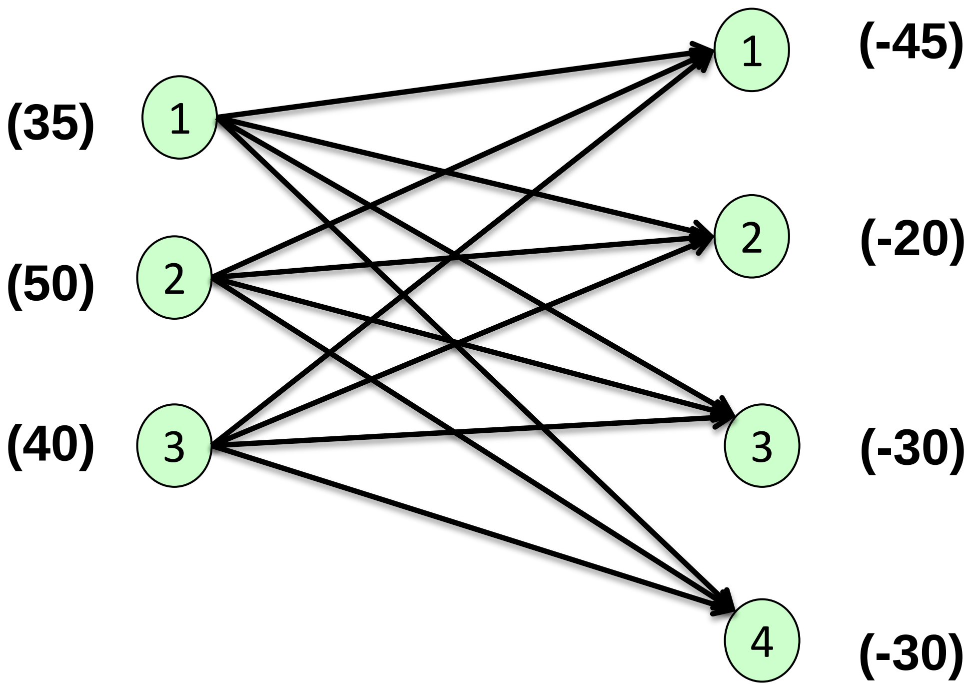
Transportni problem

$$\text{Min } z = \sum_{i}\sum_{j} c_{ij}x_{ij}$$

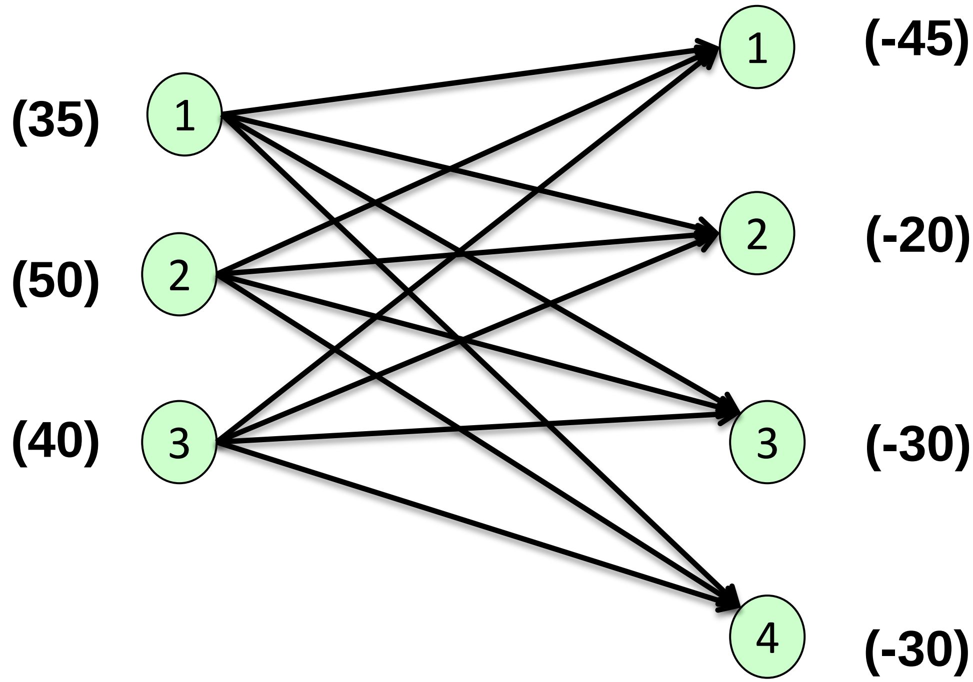
$$\sum_{j=1}^{n} x_{ij} = a_i \qquad (i = 1,2, \dots m)$$

$$\sum_{i=1}^{m} (-x_{ij}) = -b_j \qquad (j = 1,2, \dots m)$$

$$x_{ij} \geq 0 \;\; (i = 1 \cdots m, j = 1 \cdots n)$$



Transportni problem



Fabrike	**Distributivni centri**				**Raspoloživo**
	Dallas	Atlanta	San F	Phily	
Cleveland	8	6	10	9	35
Chicago	9	12	13	7	50
Boston	14	9	16	5	40
Potrebe	45	20	30	30	(125)

Transportni problem

Fabrike	**Distributivni centri**				**Raspoloživo**
	Dallas	Atlanta	San F	Phily	
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Potrebe	45	20	30	30	(125)

$$\begin{aligned}\text{Min } z = {} & 8x_{11} + 6x_{12} + 10x_{13} + 9x_{14} + 9x_{21} + 12x_{22} \\ & + 13x_{23} + 7x_{24} + 14x_{31} + 9x_{32} + 16x_{33} \\ & + 5x_{34}\end{aligned}$$

$$\begin{array}{cccccccccccccccccccccccc}
x_{11} & + & x_{12} & + & x_{13} & + & x_{14} & & & & & & & & & & & & & & & = & 35 \\
& & & & & & & & x_{21} & + & x_{22} & + & x_{23} & + & x_{24} & & & & & & & = & 50 \\
& & & & & & & & & & & & & & & & x_{31} & + & x_{32} & + & x_{33} & + & x_{34} & = & 40 \\
-x_{11} & & & & & & & & -x_{21} & & & & & & & & -x_{31} & & & & & & & = & -45 \\
& & -x_{12} & & & & & & & & -x_{22} & & & & & & & & -x_{32} & & & & & = & -20 \\
& & & & -x_{13} & & & & & & & & -x_{23} & & & & & & & & -x_{33} & & & = & -30 \\
& & & & & & -x_{14} & & & & & & & & -x_{24} & & & & & & & & -x_{34} & = & -30
\end{array}$$

Problem dodele zadataka

$$x_{ij} = \begin{cases} 1 & \text{ako je osobi } i \text{ dodeljen posao } j \\ 0 & \text{svi ostali slučajevi} \end{cases}$$

$$\text{Min } z = \sum_i \sum_j c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad (i = 1,2, \dots n)$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad (j = 1,2, \dots n)$$

$$x_{ij} = 0 \ \text{ ili } \ 1 \ \ (i = 1 \cdots n, j = 1 \cdots n)$$

Problem maksimalnog protoka

$$\max v$$

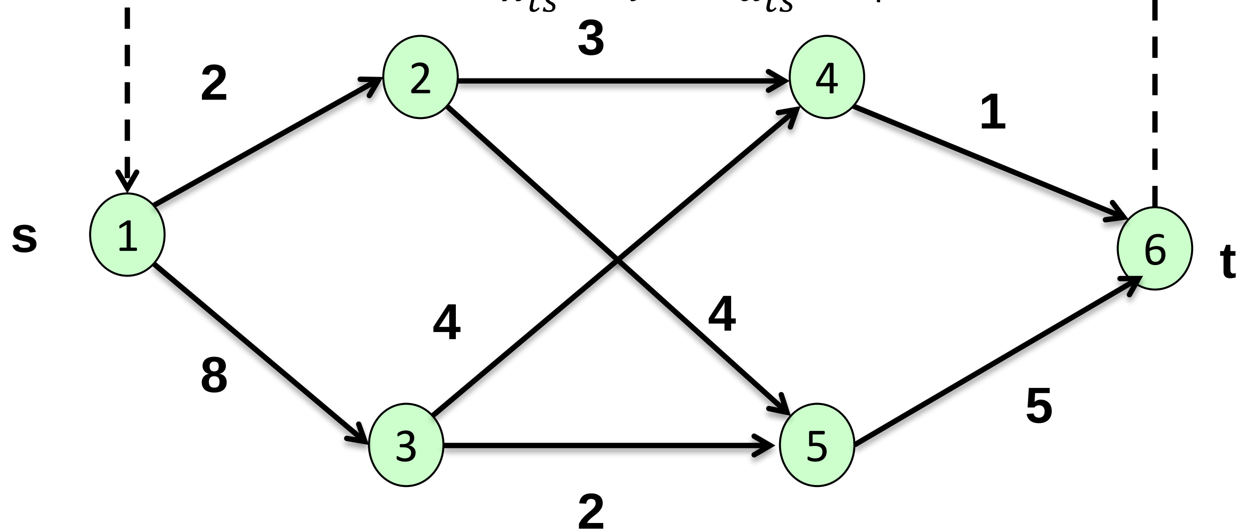
$$\sum_j x_{ij} - \sum_k x_{ki} = \begin{cases} v & ako\ je\ i = s \\ -v & ako\ je\ i = t \\ 0 & ostali\ slučajevi \end{cases}$$

$$0 \leq x_{ij} \leq u_{ij} \ (i = 1 \cdots n, j = 1 \cdots n)$$

$$\max x_{ts}$$

$$\sum_j x_{ij} - \sum_k x_{ki} = 0 \quad (i = 1 \ldots n)$$

$$x_{ts} = v \qquad u_{ts} = +\infty$$

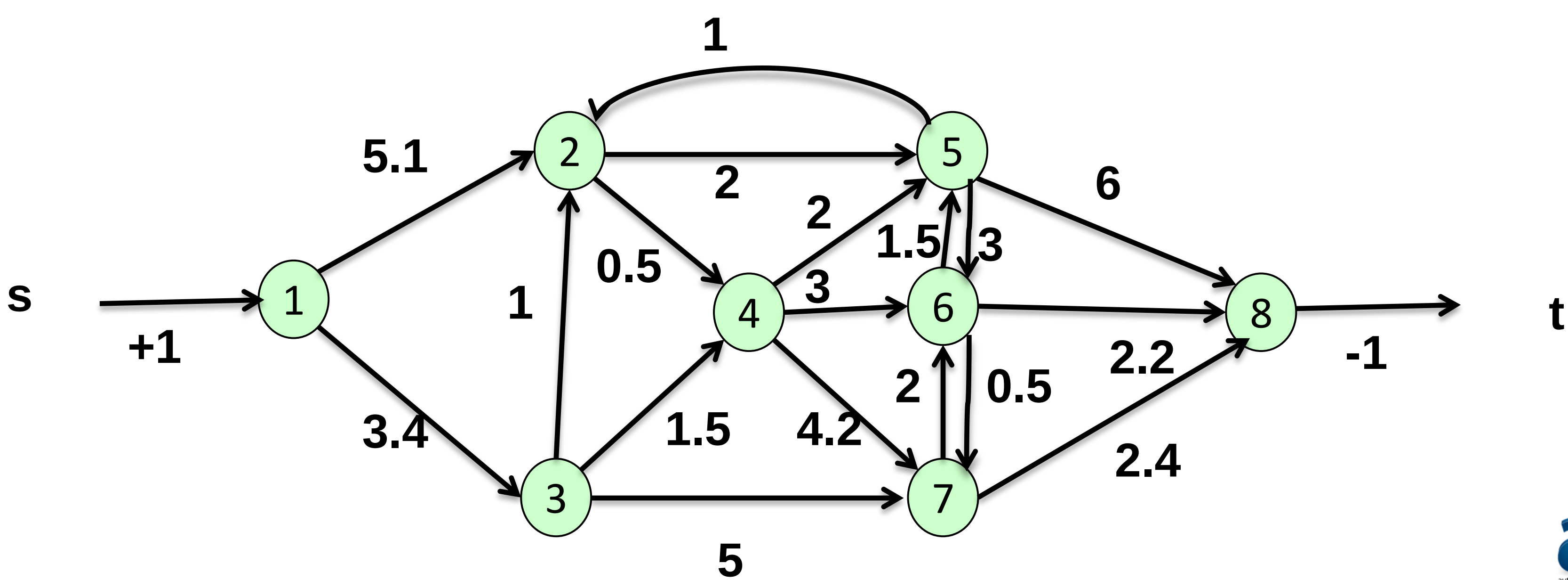


Problem najkraće putanje

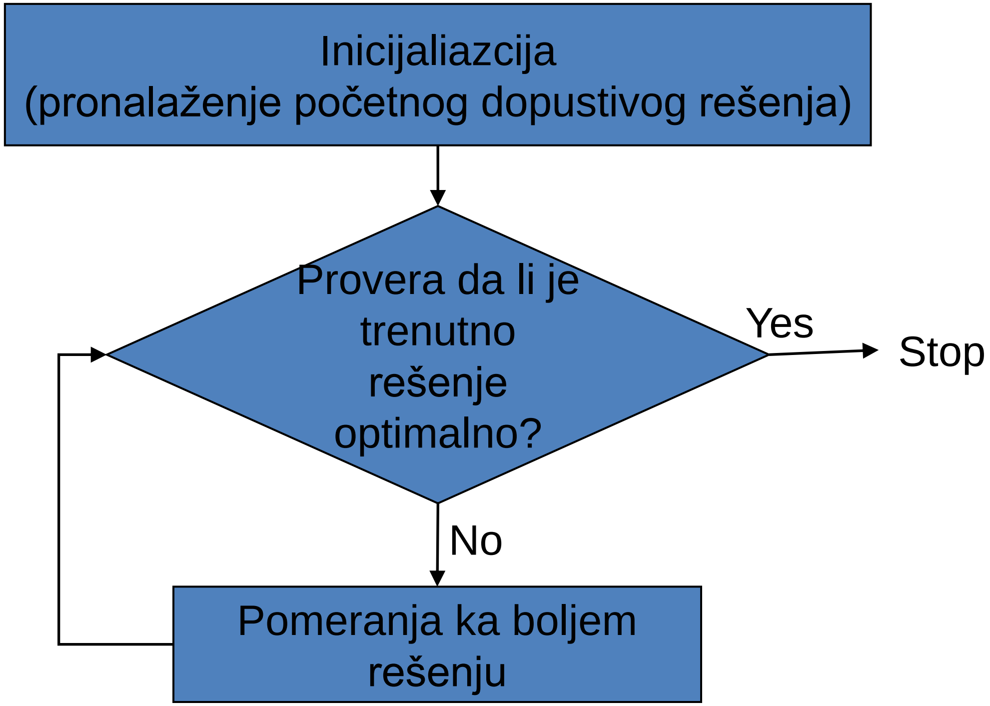
$$\text{Min } z = \sum_i \sum_j c_{ij} x_{ij}$$

$$\sum_j x_{ij} - \sum_k x_{ki} = \begin{cases} 1 & ako\ je\ i = s \\ -1 & ako\ je\ i = t \\ 0 & ostali\ slučajevi \end{cases}$$

$$x_{ij} \geq 0$$



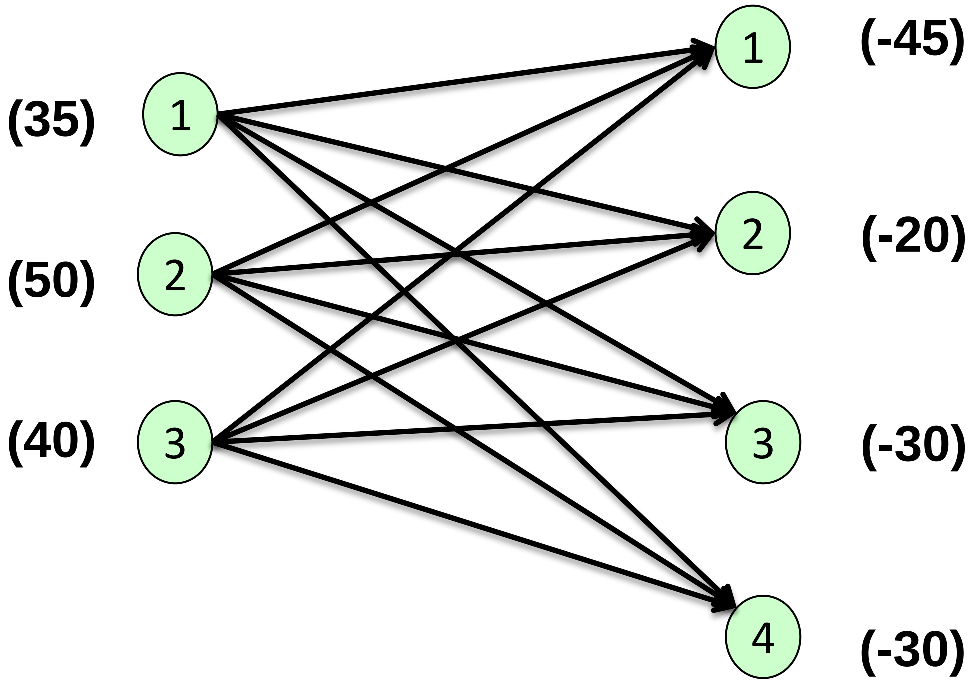
Rešavanje transportnog problema



Rešavanje transportnog problema

	Cilj					
Izvor	**1**	**2**	**…**	**n**	**Ponuda**	u_i
1	c_{11} x_{11}	c_{12} x_{12}	…	c_{1n} x_{1n}	a_1	
2	c_{21} x_{21}	c_{22} x_{21}	…	c_{2n} x_{2n}	a_2	
…	…	…	…	…	…	
m	c_{m1} x_{m1}	c_{m2} x_{m2}	…	c_{mn} x_{mn}	a_m	
Potreba	b_1	b_2	…	b_n	$Z =$	
v_j						

Transportni problem



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	Dallas	Atlanta	San F	Phily	
Cleveland	8	6	10	9	35
Chicago	9	12	13	7	50
Boston	14	9	16	5	40
Potrebe	45	20	30	30	(125)

Transportni problem

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}$$

Fabrike	**Distributivni centri**				**Raspoloživo**
	Dallas	Atlanta	San F	Phily	
Cleveland	8	6	10	9	35
Chicago	9	12	13	7	50
Boston	14	9	16	5	40
Potrebe	45	20	30	30	(125)

$$\text{Min } z = 8x_{11} + 6x_{12} + 10x_{13} + 9x_{14} + 9x_{21} + 12x_{22} + 13x_{23} + 7x_{24} + 14x_{31} + 9x_{32} + 16x_{33} + 5x_{34}$$

$$x_{11} + x_{12} + x_{13} + x_{14} = 35$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 50$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 40$$

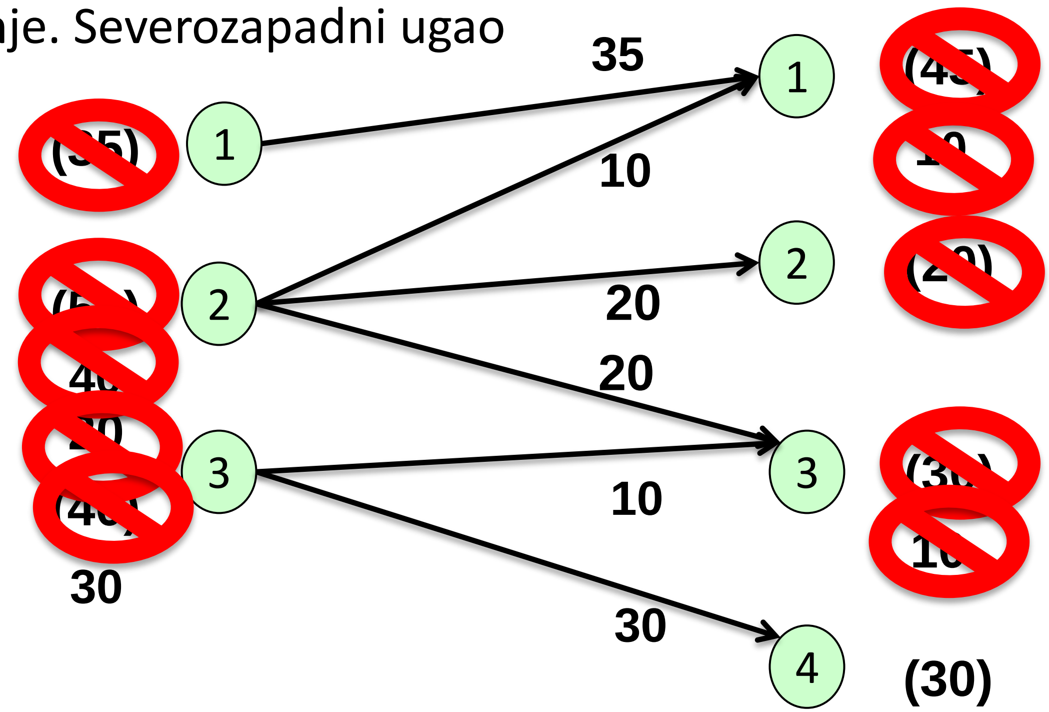
$$-x_{11} - x_{21} - x_{31} = -45$$

$$-x_{12} - x_{22} - x_{32} = -20$$

$$-x_{13} - x_{23} - x_{33} = -30$$

$$-x_{14} - x_{24} - x_{34} = -30$$

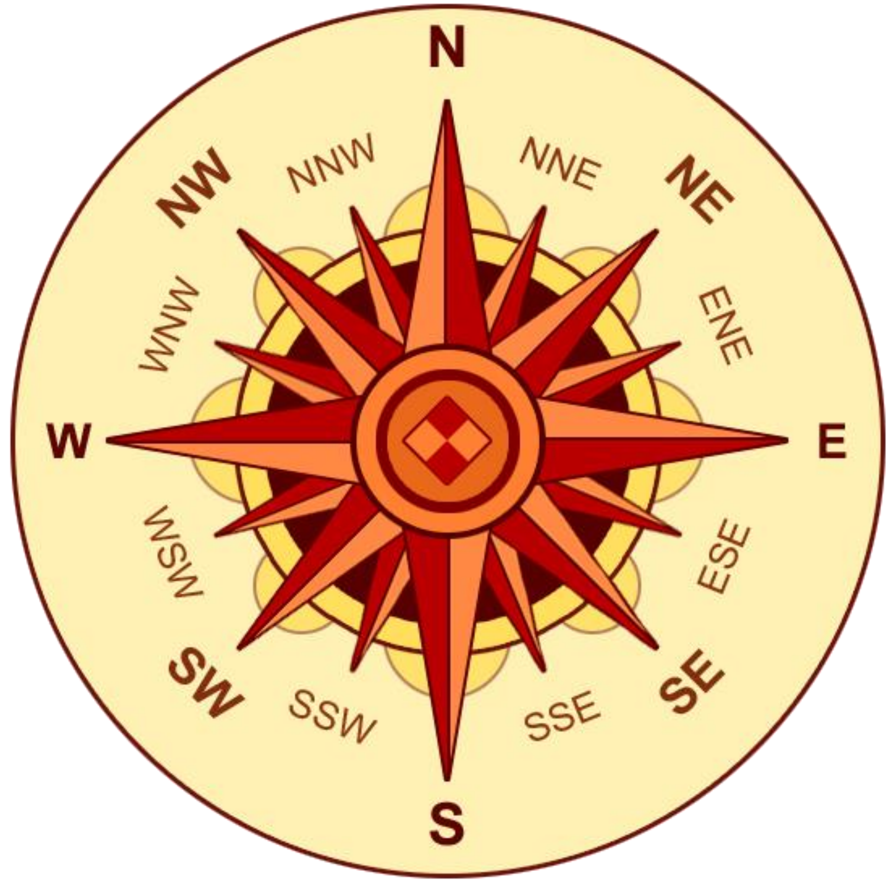
Transportni problem, početno rešenje. Severozapadni ugao



Fabrike	Distributivni centri				Raspoloživo
	Dallas	Atlanta	San F	Phily	
Cleveland	35				35
Chicago	10	20	20		50
Boston			10	30	40
Potrebe	45	20	30	30	(125)

Transportni problem, početno rešenje. Severozapadni ugao

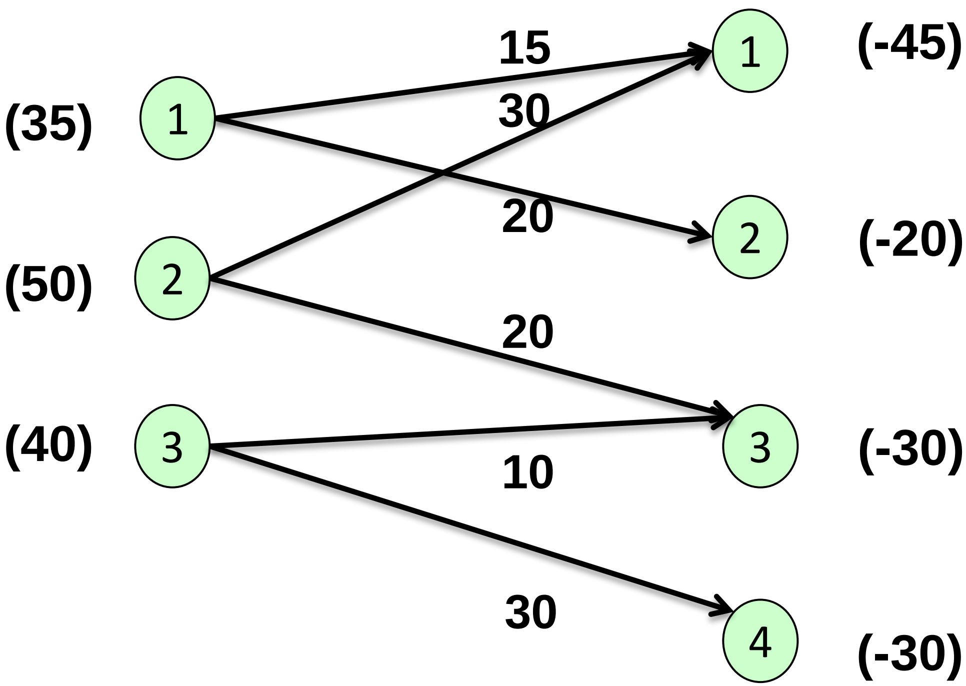
NIJE North West, već Northwest



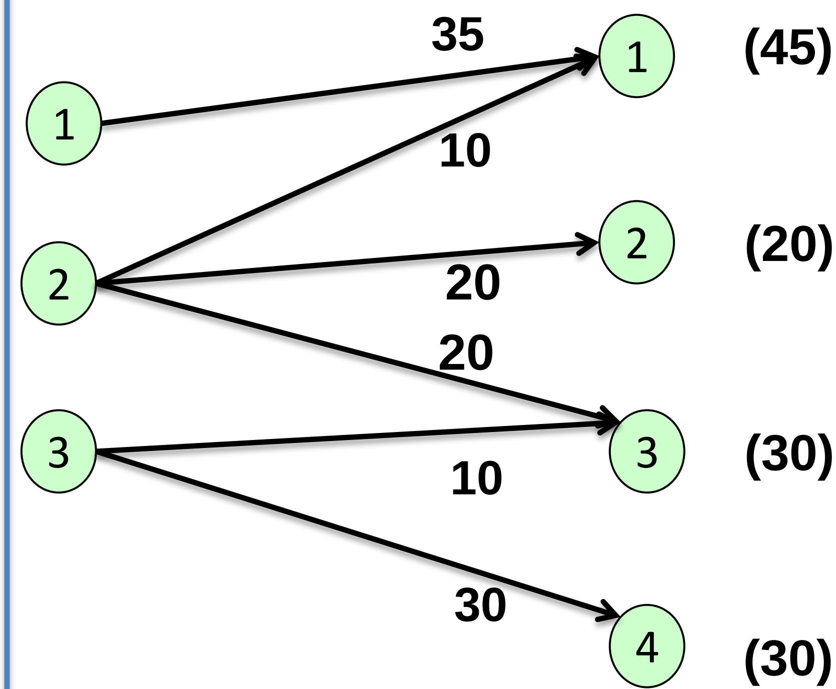
Transportni problem, početno rešenje. Minimalna matrica

	Cilj				
Izvor	**1**	**2**	**3**	**4**	**Ponuda**
1	8 / **15**	6 / **20**	10	9	35
2	9 / **30**	12	13 / **20**	7	50
3	14	9	16 / **10**	5 / **30**	40
Potreba	45	20	30	30	

Transportni problem, početno rešenje.



MINIMALNA MATRICA



SEVEROZAPADNI UGAO

RAZAPINJUĆE ili OBUHVATNO STABLO

1.Svi čvorovi su međusobno povezani, preko međusobno povezanih grane, zanemarujući orijentaciju grana

2. Nema petlji, čvor nije povezan sa samim sobom – preko međusobno povezanih grana, zanemarujući orijentaciju grana

Transportni problem

Provera da li je trenutno rešenje optimalno?

NO

$$\bar{c}_{ij} = c_{ij} - u_i - v_j \geq 0 \qquad \text{ako je } x_{ij} = 0$$

$$\bar{c}_{ij} = c_{ij} - u_i - v_j = 0 \qquad \text{ako je } x_{ij} > 0$$

$$\text{Min } z = \sum_i \sum_j c_{ij} x_{ij}$$

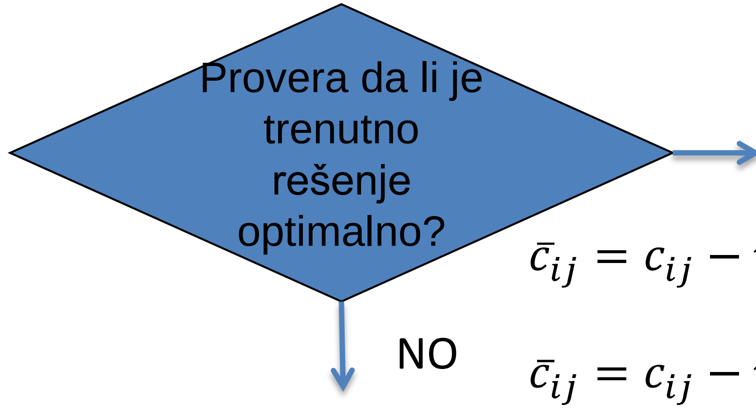
Skiveni troškovi

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad (i = 1,2, \ldots m) \qquad u_i$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad (i = 1,2, \ldots n) \qquad v_j$$

$$x_{ij} \geq 0 \;\; (i = 1 \cdots m, j = 1 \cdots n)$$

Transportni problem



$$\bar{c}_{ij} = c_{ij} - u_i - v_j \geq 0 \qquad \text{ako je } x_{ij} = 0$$

$$\bar{c}_{ij} = c_{ij} - u_i - v_j = 0 \qquad \text{ako je } x_{ij} > 0$$

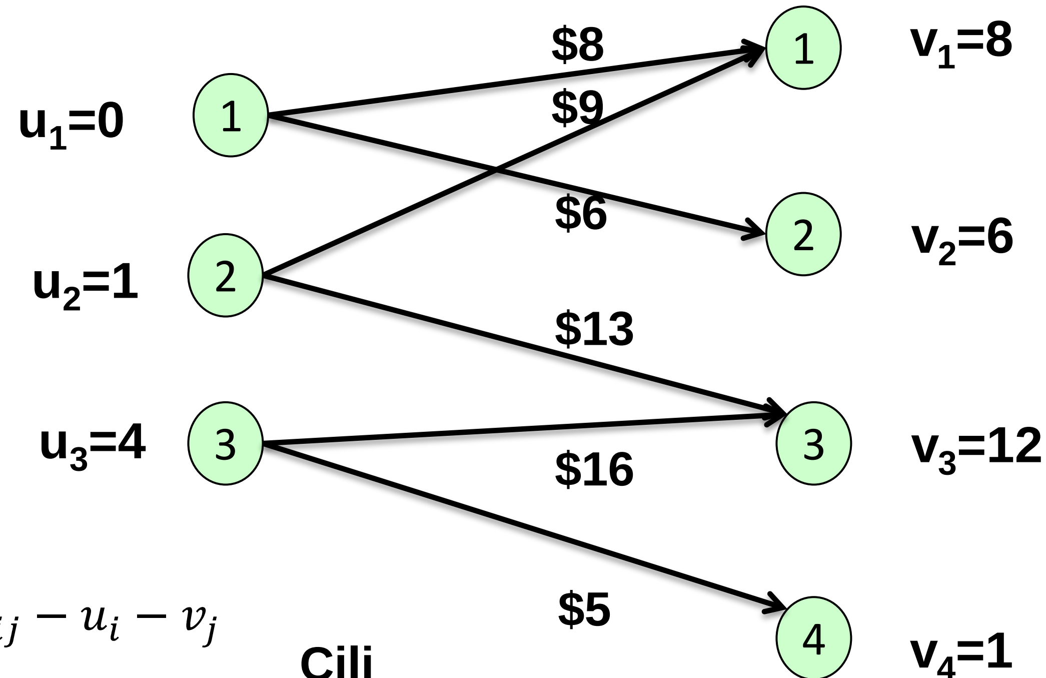
Za dopustivo početno rešenje izrčunamo u_i i v_j

$$u_i + v_j = c_{ij}$$

Za ove simplex množitelje, izračunvamo umanjene troškove, za ne bazna rešenja

$$\bar{c}_{ij} = c_{ij} - u_i - v_j$$

Transportni problem



$$\bar{c}_{ij} = c_{ij} - u_i - v_j$$

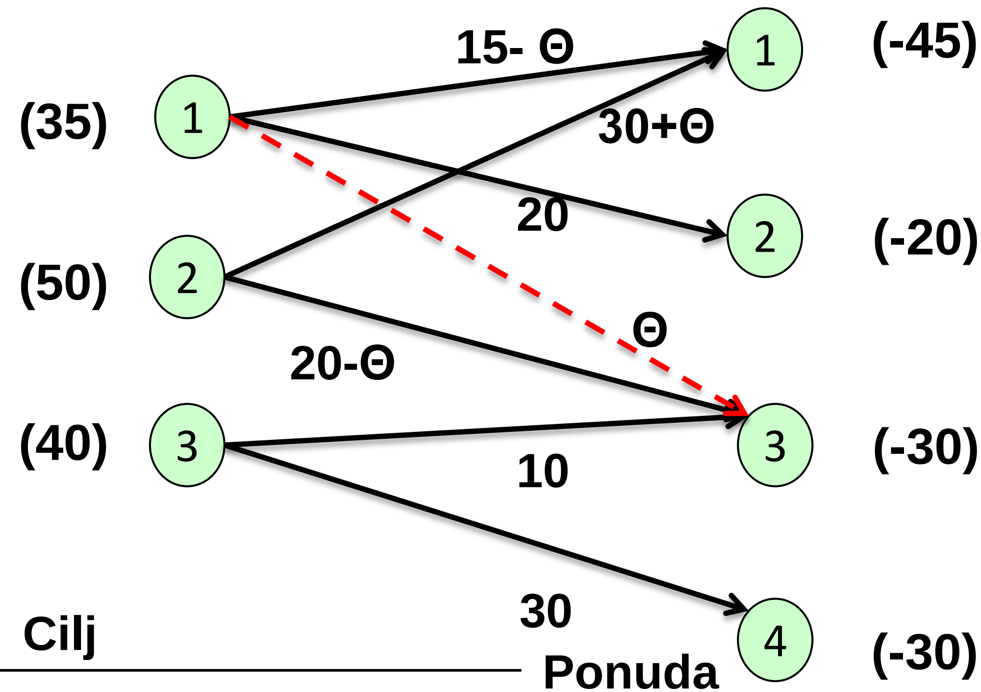
Izvor	Cilj 1	Cilj 2	Cilj 3	Cilj 4	Ponuda
1	8 / -	6 / -	10 / -2	9 / 8	0
2	9 / -	12 / 5	13 / -	7 / 5	1
3	14 / 2	9 / -1	16 / -	5 / -	4
Potreba	8	6	12	1	

Transportni problem

Izvor	Cilj 1	Cilj 2	Cilj 3	Cilj 4	Ponuda
1	8 -	6 -	10 -2	9 8	0
2	9 -	12 5	13 -	7 5	1
3	14 2	9 -1	16 -	5 -	4
Potreba	8	6	12	1	

Transportni problem

Pomeranja ka boljem rešenju

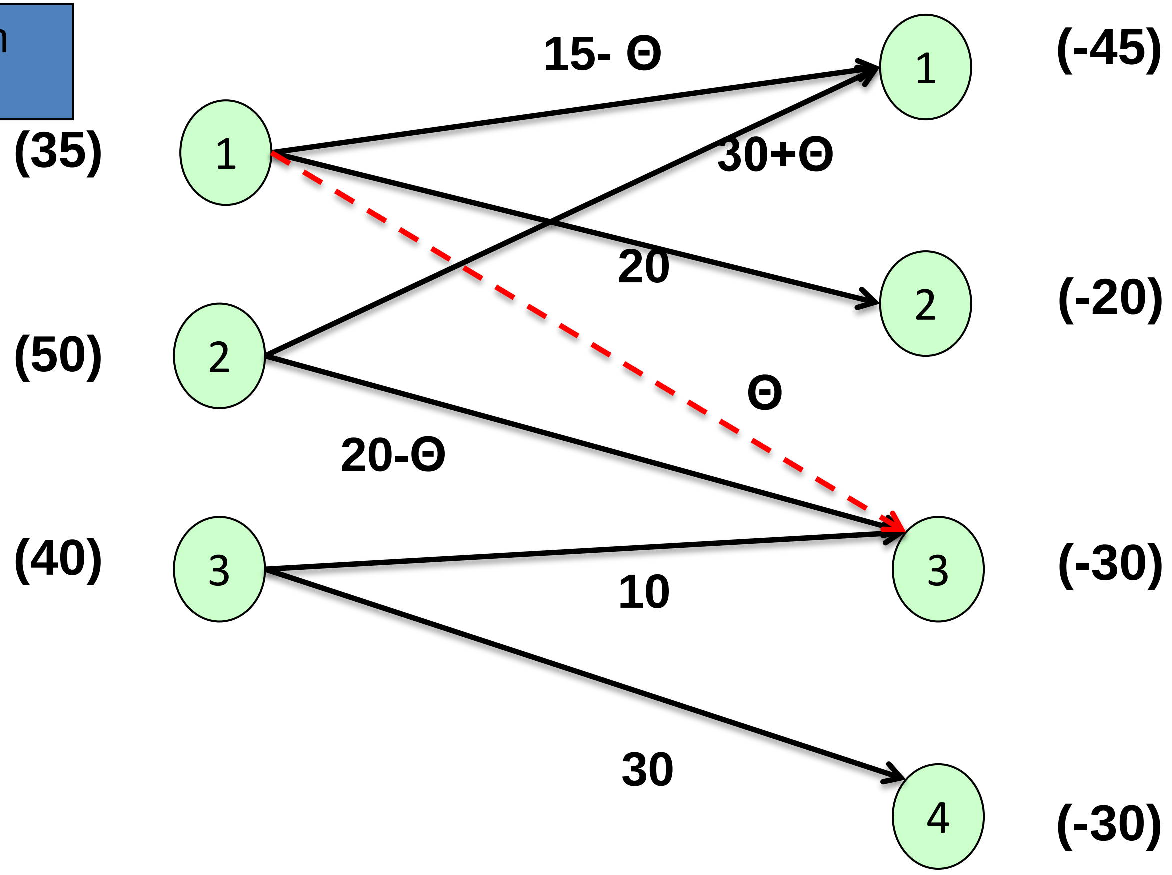


$$u_i + v_j = c_{ij}$$

	Cilj				
Izvor	**1**	**2**	**3**	**4**	**Ponuda**
1	8 / -	6 / -	10 / -2	9 / 8	0
2	9 / -	12 / 5	13 / -	7 / 5	1
3	14 / 2	9 / -1	16 / -	5 / -	4
Potreba	8	6	12	1	

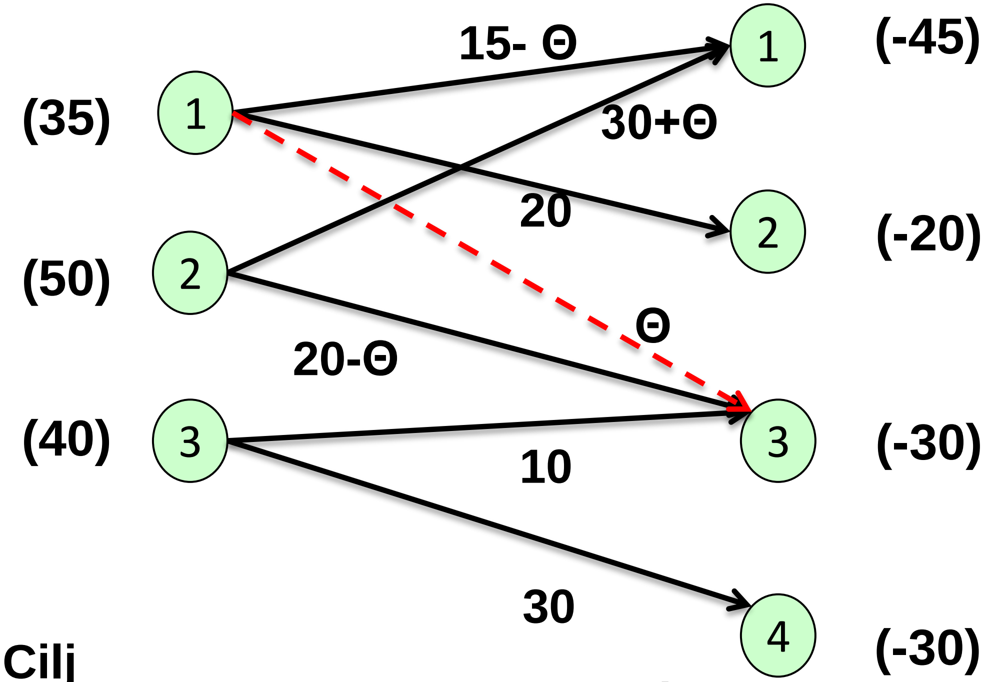
Transportni problem

Pomeranja ka boljem rešenju



Transportni problem

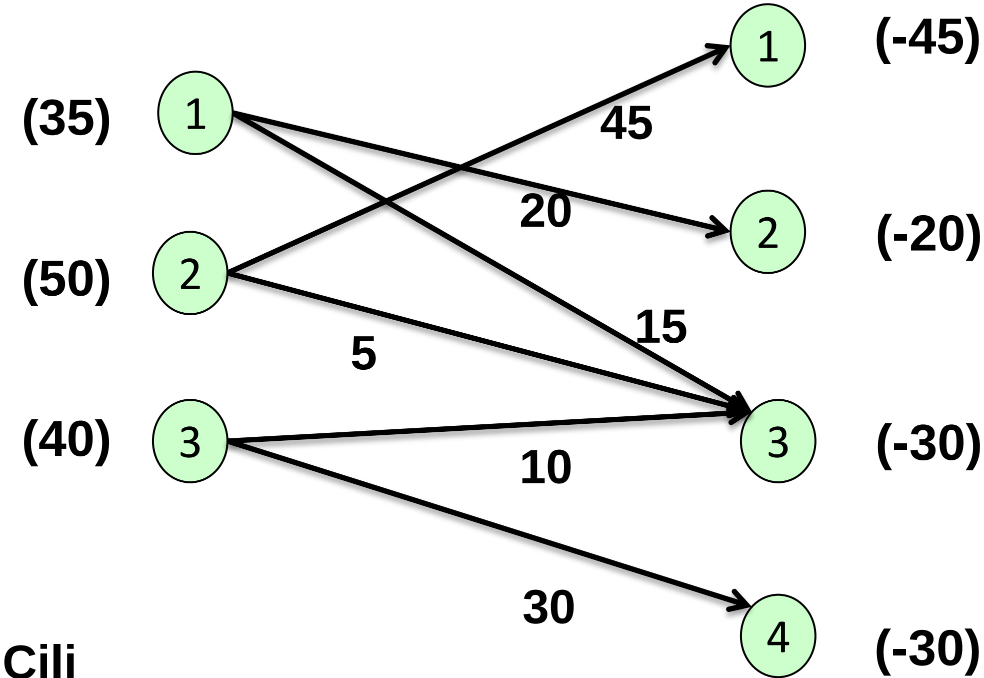
Pomeranja ka boljem rešenju



Izvor	Cilj 1	2	3	4	Ponuda
1	8 / 15-Θ	6 / 20	10 / Θ	9	35
2	9 / 30+Θ	12	13 / 20-Θ	7	50
3	14	9	16 / 10	5 / 30	40
Potreba	45	20	30	30	

Transportni problem

Pomeranja ka boljem rešenju



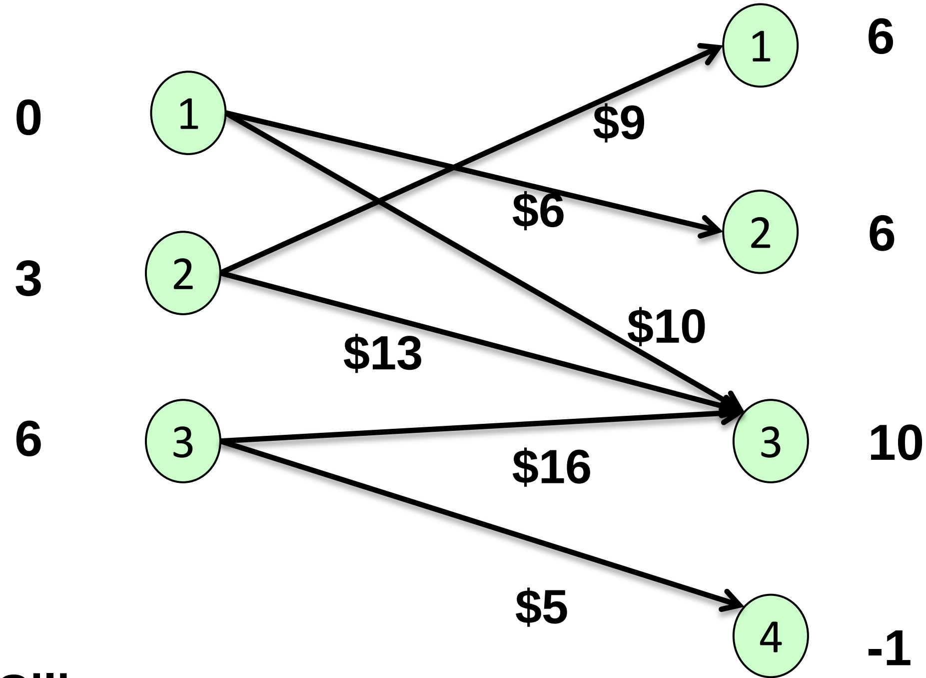
Izvor	Cilj 1	Cilj 2	Cilj 3	Cilj 4	Ponuda
1	8	6 / **20**	10 / **15**	9	35
2	9 / **45**	12	13 / **5**	7	50
3	14	9	16 / **10**	5 / **30**	40
Potreba	45	20	30	30	

Transportni problem

Pomeranja ka boljem rešenju

$$u_i + v_j = c_{ij}$$

$$\bar{c}_{ij} = c_{ij} - u_i - v_j$$



Izvor	Cilj 1	Cilj 2	Cilj 3	Cilj 4	Ponuda
1	8 / 2	6 / -	10 / -	9 / 10	0
2	9 / -	12 / 3	13 / -	7 / 5	3
3	14 / 2	9 / -3	16 / -	5 / -	6
Potreba	6	6	10	-1	

Transportni problem

Pomeranja ka boljem rešenju

Izvor	Cilj 1	Cilj 2	Cilj 3	Cilj 4	Ponuda
1	8	6 / **10**	10 / **25**	9	35
2	9 / **45**	12	13 / **5**	7	50
3	14	9 / **10**	16	5 / **30**	40
Potreba	45	20	30	30	

Transportni problem

Pomeranja ka boljem rešenju

Provera da li je trenutno rešenje optimalno?

YES

	Cilj				
Izvor	**1**	**2**	**3**	**4**	**Ponuda**
1	8 / 2	6 / -	10 / -	9 / 7	0
2	9 / -	12 / 3	13 / -	7 / 2	3
3	14 / 5	9 / -	16 / 3	5 / -	3
Potreba	6	6	10	2	

Transportni problem, dodatak

Ponuda različita od potražnje

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j = d > 0$$

$$\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i = d > 0$$

$$b_{n+1} = d \quad c_{i,n+1} = 0$$

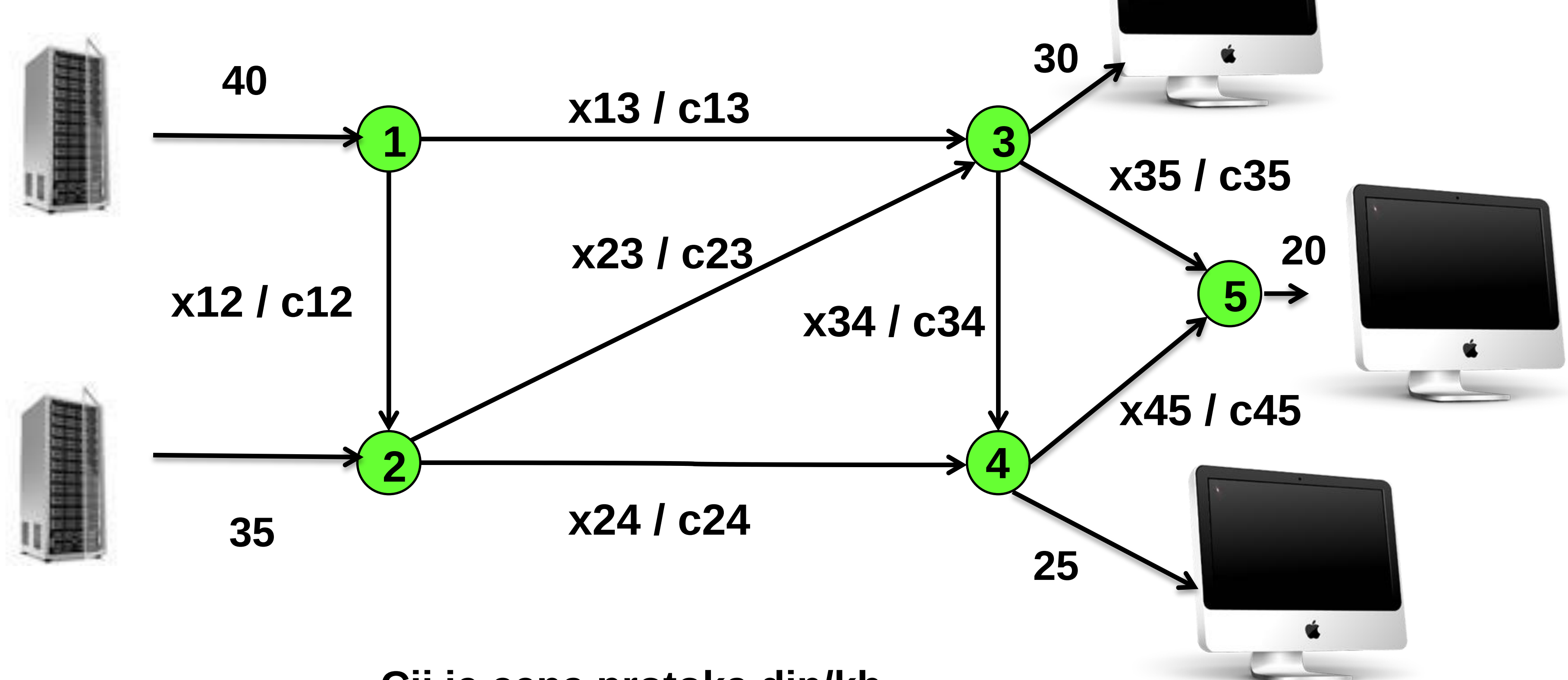
$$a_{m+1} = d \quad c_{m+1,j} = 0$$

Zabranjene putanje

$C_{i,j}$=M gde je M veoma veliki broj

Degenracija

Minimalna cena protoka kroz mrežu

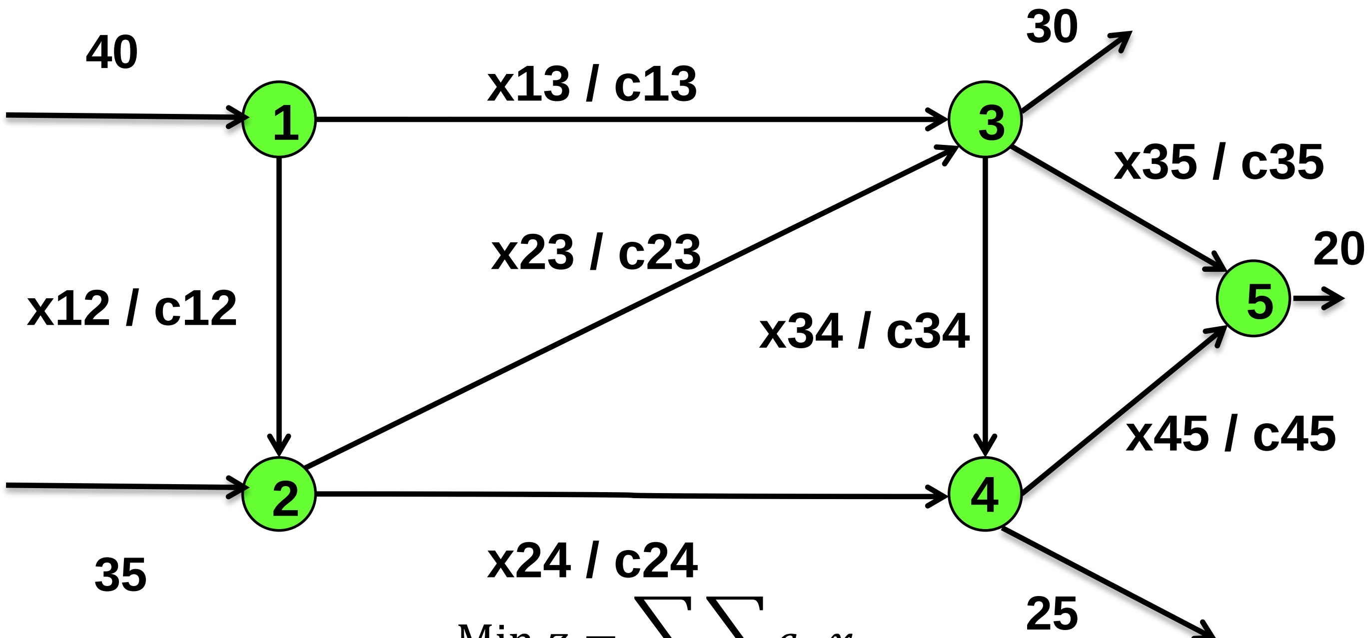


Cij je cena protoka din/kb

$$\binom{Protok\ iz}{\check{c}vora} - \binom{Protok\ u}{\check{c}vor} = \binom{Snadbevenost}{\check{c}vora}$$

$$\text{Min } z = \sum_i \sum_j c_{ij} x_{ij}$$

Minimalna cena protoka kroz mrežu



$$\text{Min } z = \sum_i \sum_j c_{ij} x_{ij}$$

$$x_{12} + x_{13} = 40$$

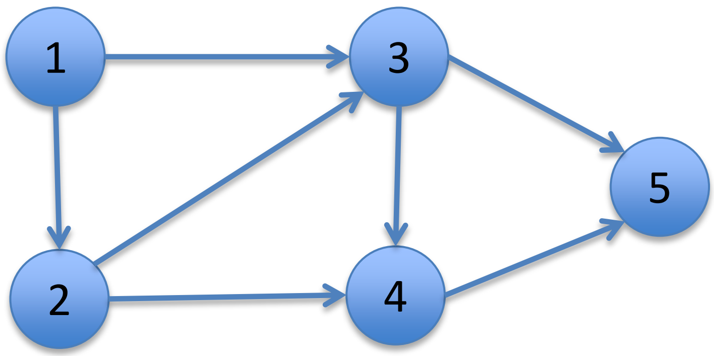
$$-x_{12} + x_{23} + x_{24} = 35$$

$$-x_{13} - x_{23} + x_{34} + x_{35} = -30 \qquad x_{ij} \geq 0$$

$$-x_{24} - x_{34} + x_{45} = -25$$

$$-x_{35} - x_{45} = -20$$

Minimalna cena protoka kroz mrežu, teorija grafova

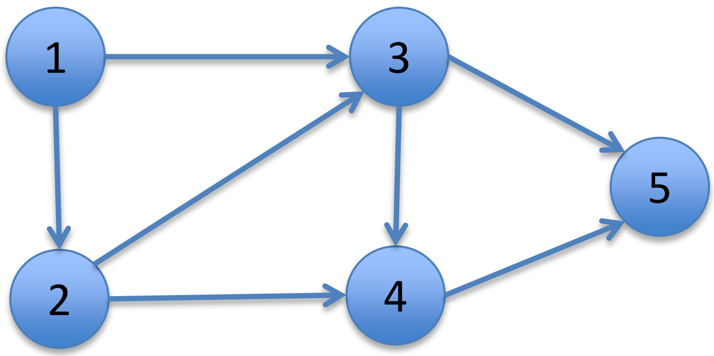


Čvorovi={1, 2, 3, 4, 5}

Grane={(1, 2), (1,3), (2,3), (2,4), (3,4), (3,5), (4,5)}

$$\tilde{A} = \begin{array}{cc} \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} & \begin{matrix} c1 \\ c2 \\ c3 \\ c4 \\ c5 \end{matrix} \\ \begin{matrix} (1,\ 2) & (1,3) & (2,3) & (2,4) & (3,4) & (3,5) & (4,5) \end{matrix} & \end{array}$$

Minimalna cena protoka kroz mrežu, teorija grafova



$$e^T \tilde{A} = 0, \quad \text{gde je} \;\; e^T = [1 \quad \ldots\ldots \quad 1]$$

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

Minimalna cena protoka kroz mrežu, teorija grafova, osnovni pojmovi

- Putanja od čvora *s* do *t* u orijentisanom grafu je povezana sekvenca grana ρ_1 do ρ_N gde važi

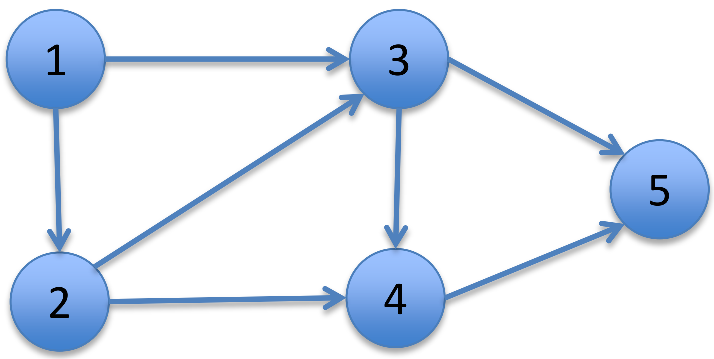
$$\rho_k = (i_{k-1}, i_k) \text{ ili } \rho_k = (i_k, i_{k-1}) \text{ gde je } i_0 = s \text{ i } i_N = t.$$

- Direktna putanja od čvora *s* do *t* u orijentisanom grafu je povezana sekvenca grana ρ_1 do ρ_N gde važi

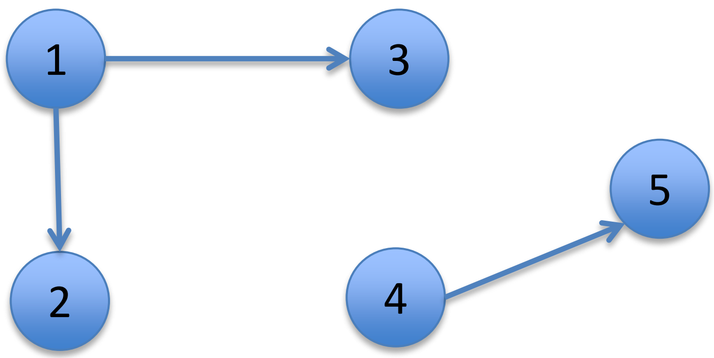
$$\rho_k = (i_{k-1}, i_k) \text{ gde je } i_0 = s \text{ i } i_N = t$$

- (Direktna) petlja je (direktna) putanja koja počinje i završava u istom čvoru
- Graf je povezan ako postoji putanja između bilo koja dva čvora
- Stablo je povezni podskup grafa, koji ne sadrži petlje
- Razapinjuće ili obuhvatno stablo sadrži n čvorova i n-1 grana

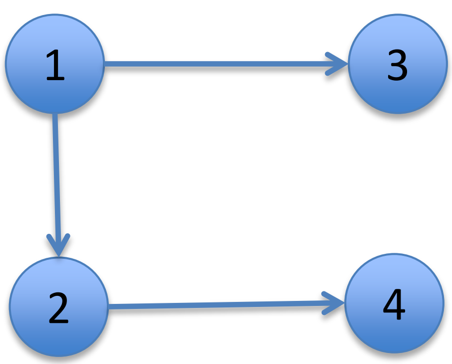
Minimalna cena protoka kroz mrežu, teorija grafova, osnovni pojmovi



Povezan graf, sadrži petlje

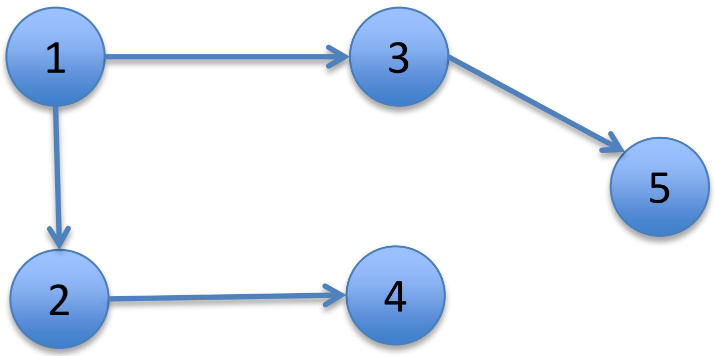


Nepovezan graf, ne sadrži petlje

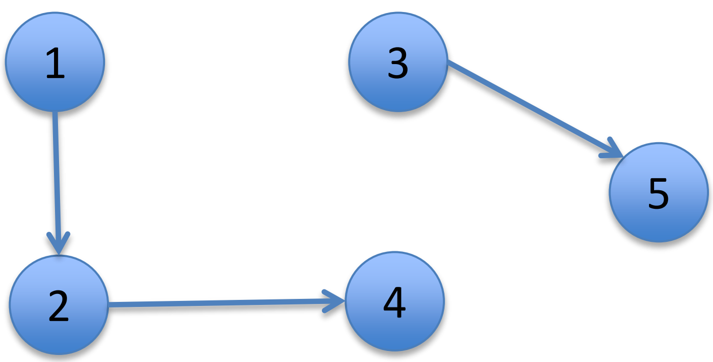


Stablo

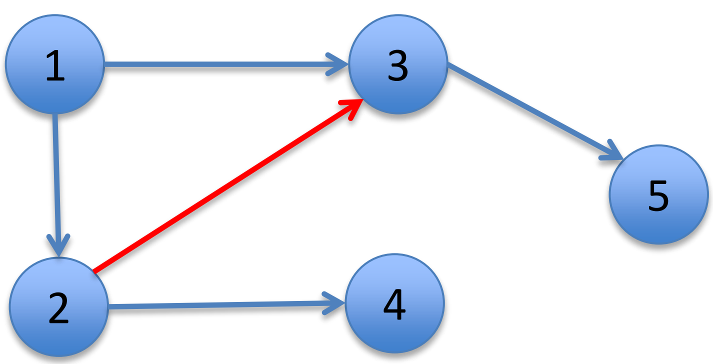
Minimalna cena protoka kroz mrežu, teorija grafova, osnovni pojmovi



Razapinjuće stablo

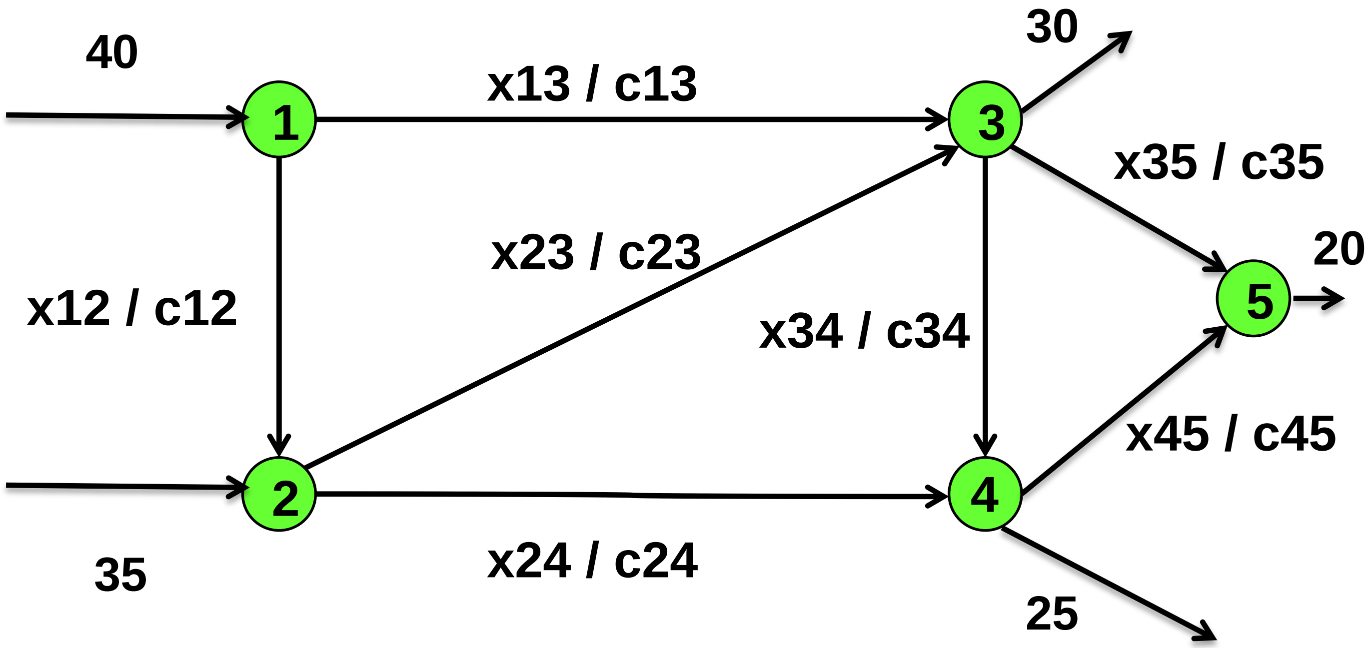


Ako se ukloni grana razapinjućeg stabla, dobijaju se dva stabla



Ako se doda grana na razapinjuće stablo, dobijaju se petalja

Minimalna cena protoka kroz mrežu, balansne jednačine

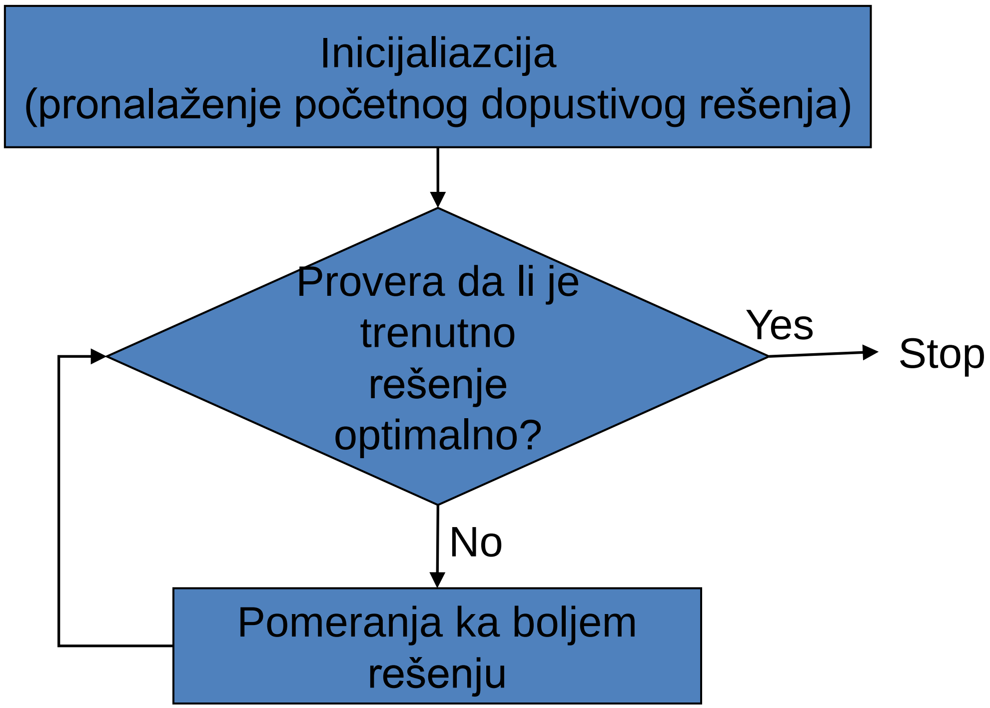


$$\sum_{i=1}^{n} a_{ij}x_{ij} = b_i \quad i = 1, \dots n. \quad \Leftrightarrow \tilde{A}x = \tilde{b}$$

$$e^T\tilde{b} = 0, \quad \text{gde je } \; e^T = [1 \quad \dots\dots \quad 1]$$

$$Ax = b, \;\; \text{gde je} \quad b = [1 \quad 1 \dots\dots \quad 1]^T$$

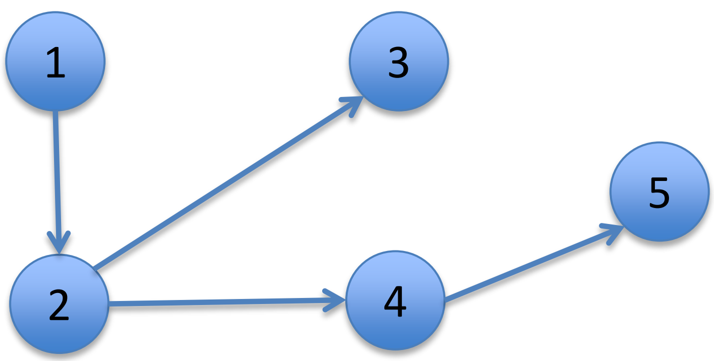
Rešavanje problema



Rešavanje problema

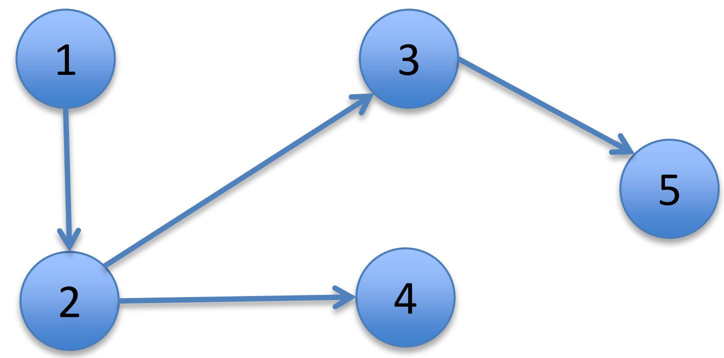
Inicijaliazcija
(pronalaženje početnog dopustivog rešenja)

Teorema m-1 kolona matrice A (m-1)x(n) je je linearno nezavisna akko odgovrajućih m-1 grana formira razapinjuće stablo



β={(1, 2), (2,3), (2,4), (4,5)}

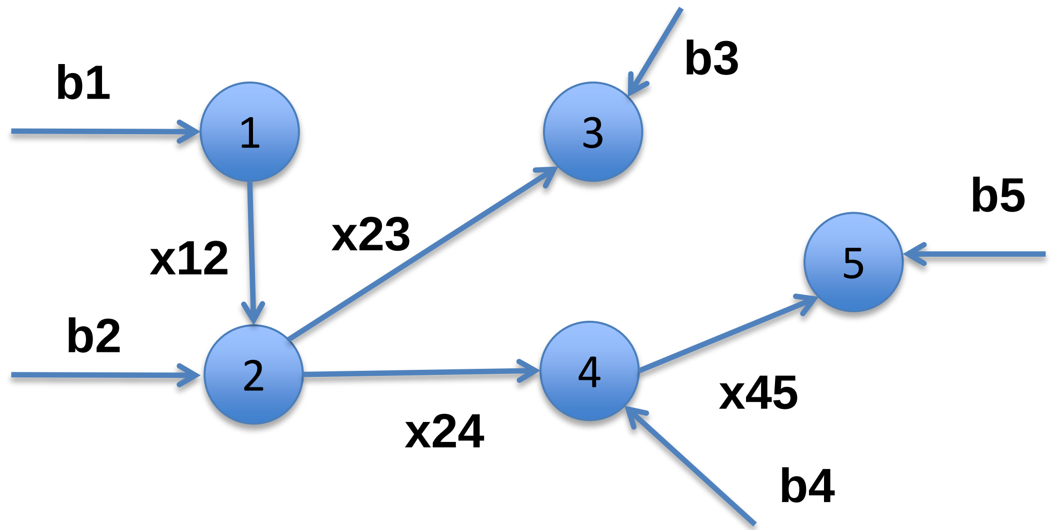
$$A_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$



β={(1, 2), (2,3), (2,4), (3,5)}

$$A_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Rešavanje problema



(b1, b2, b3, b4) = (40, 35, -30, -25) napomena b5=20

$$x_{12} = b_1 = 40$$

$$x_{23} = -b_3 = 30$$

$$x_{24} = b_2 + x_{12} - x_{23} = 45$$

$$x_{45} = x_{24} + b_4 = 20$$

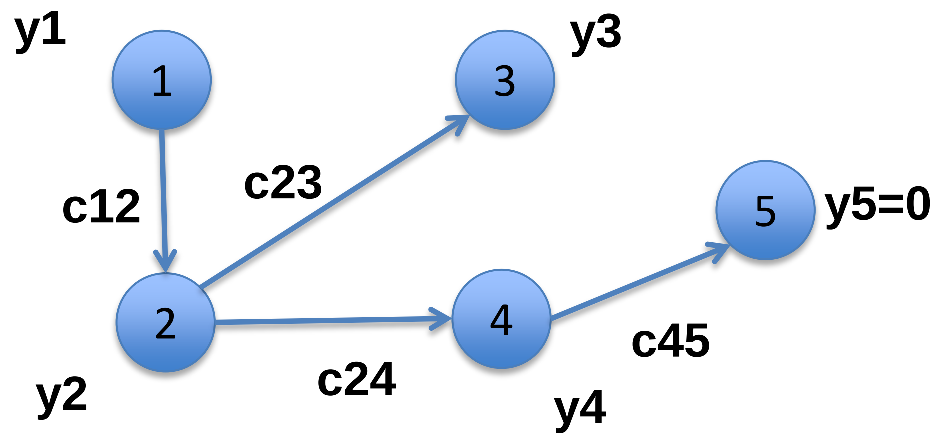
{(1, 2), (2,3), (2,4), (4,5)}

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} x = \begin{bmatrix} b1 \\ b2 \\ b3 \\ b4 \end{bmatrix}$$

x_{ij}>0 dopustivo početo rešenje

Rešavanje problema

Provera da li je trenutno rešenje optimalno?



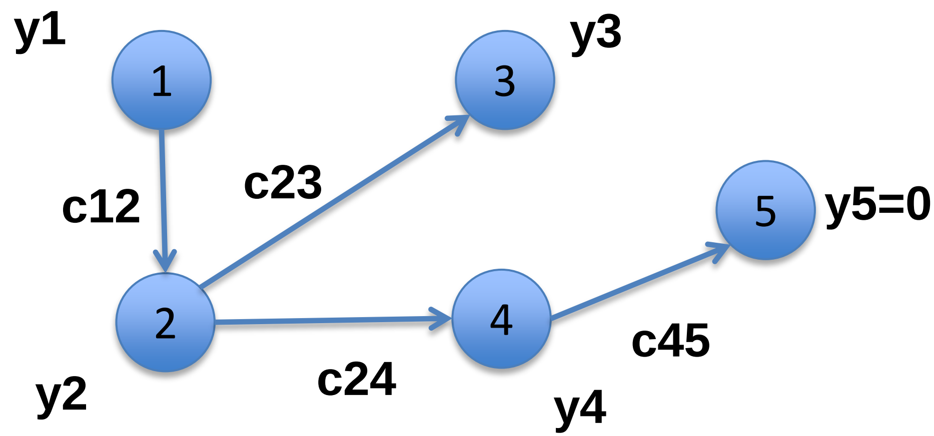
Određvanje simplex multiplikatora $y^T A_\beta = C^T{}_\beta$

$$y_m = 0$$

$y_i - y_j = c_{ij}$ **za sve grane baznog rešenja**

Rešavanje problema

Provera da li je trenutno rešenje optimalno?



$$c^T = (c12, c13, c23, c24, c34, c35, c45) = (2, 5, 2, 2, 1, 1, 2)$$

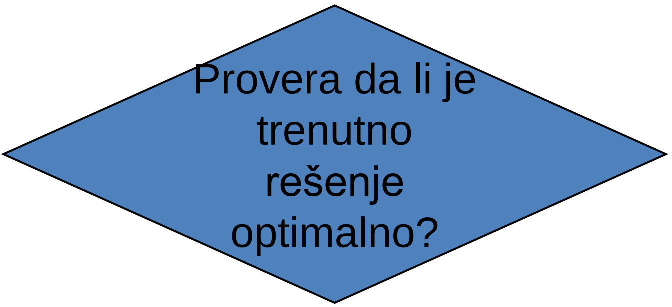
$$y_4 - y_5 = c_{45} \quad \Rightarrow \quad y_4 = c_{45} = 2$$

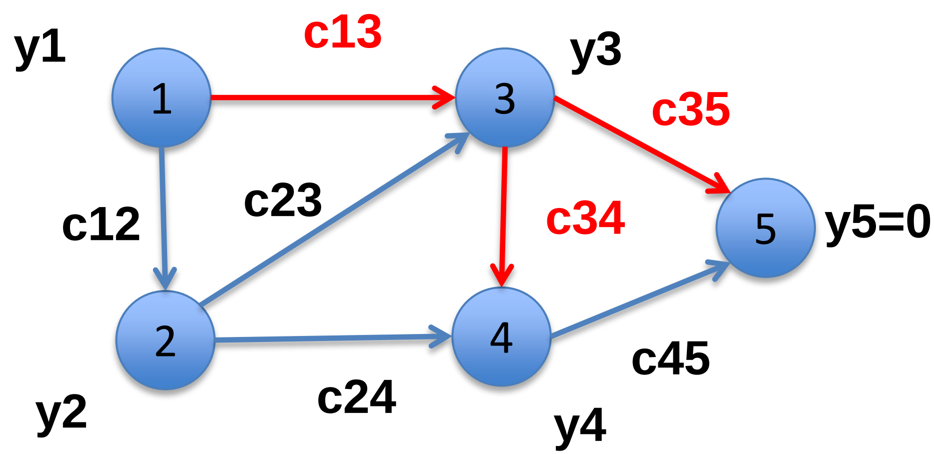
$$y_2 - y_4 = c_{24} \quad \Rightarrow \quad y_2 = y_4 + c_{24} = 2 + 2 = 4$$

$$y_2 - y_3 = c_{23} \quad \Rightarrow \quad y_3 = y_2 - c_{23} = 4 - 2 = 2$$

$$y_1 - y_2 = c_{12} \quad \Rightarrow \quad y_1 = y_2 + c_{12} = 4 + 2 = 6$$

Rešavanje problema





$\mathbf{c}^T$ =(c12, c13, c23, c24, c34, c35, c45) = (2, 5, 2 ,2,1, 1, 2)

$$\bar{c} = c_v^T - y^T A_v$$

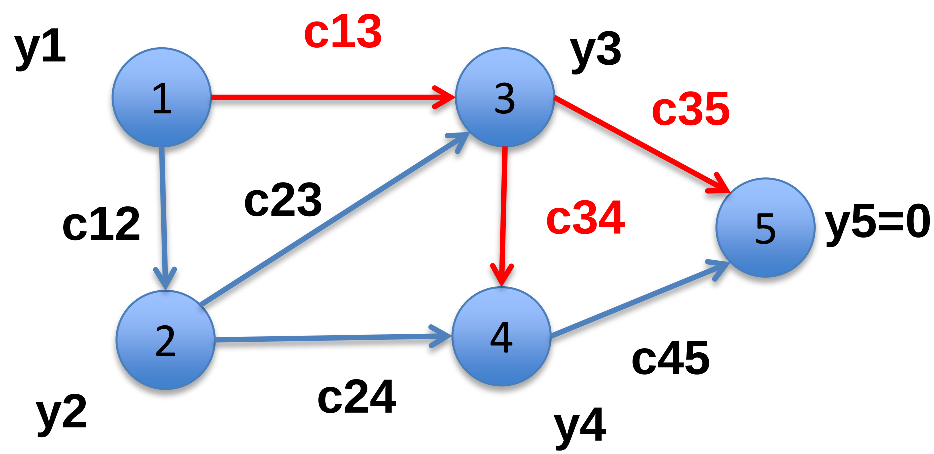
$$\bar{c}_{ij} = c_{ij} - y_i + y_j$$

v = {(1,3), (3,4), (3,5)}

$$A_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Rešavanje problema

Provera da li je trenutno rešenje optimalno?



$\mathbf{c}^T$ =(c12, c13, c23, c24, c34, c35, c45) = (2, 5, 2 ,2,1, 1, 2)

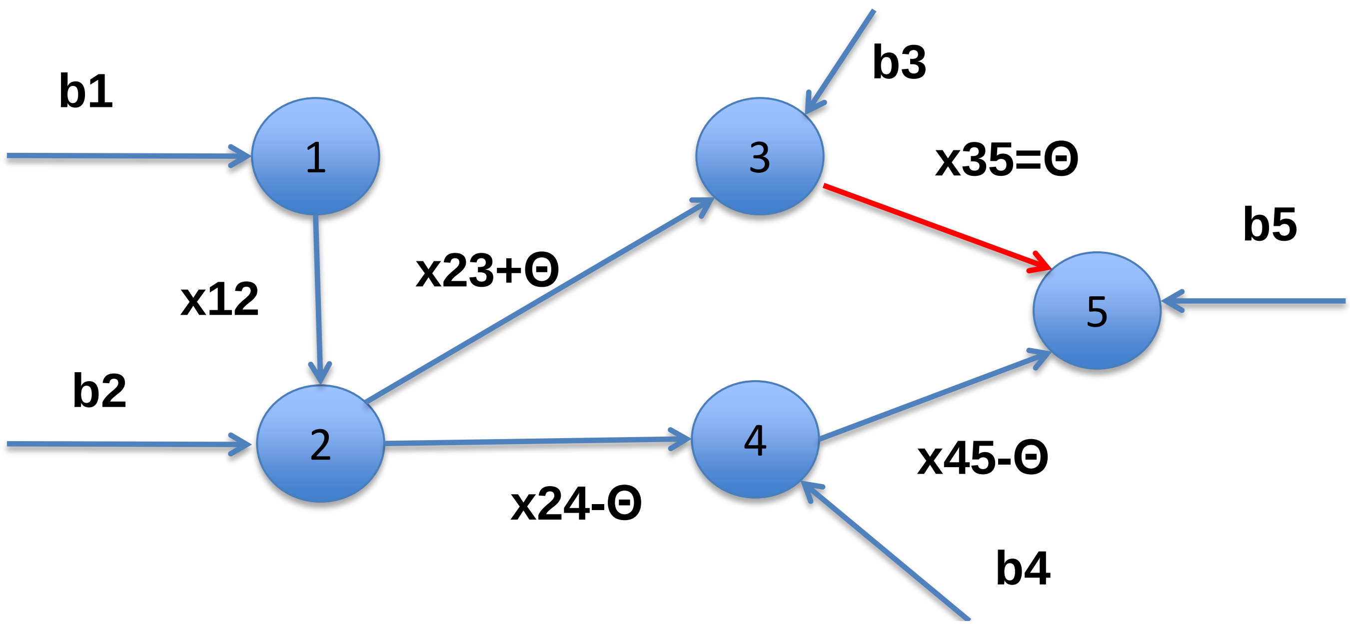
$$\bar{c}_{13} = c_{13} - y_1 + y_3 = 5 - 6 + 2 = 1$$

$$\bar{c}_{34} = c_{34} - y_3 + y_4 = 1 - 2 + 2 = 1$$

$$\bar{c}_{35} = c_{35} - y_3 + y_5 = 1 - 2 + 0 = -1$$

Rešavanje problema

Pomeranja ka boljem rešenju



$$x_{23} = \bar{x}_{23} + \Theta = 30 + \Theta$$

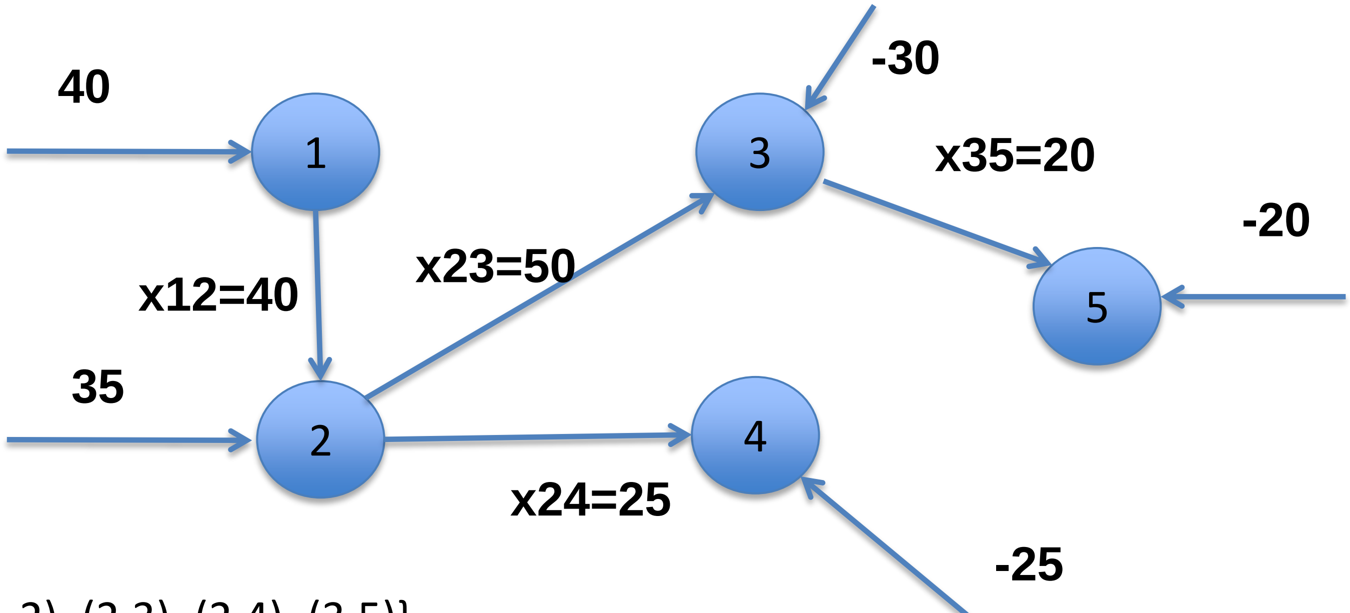
$$x_{24} = \bar{x}_{24} - \Theta = 45 - \Theta$$

$$x_{45} = \bar{x}_{45} - \Theta = 20 - \Theta$$

Θ=20, x_{45}=0

Rešavanje problema

Pomeranja ka boljem rešenju



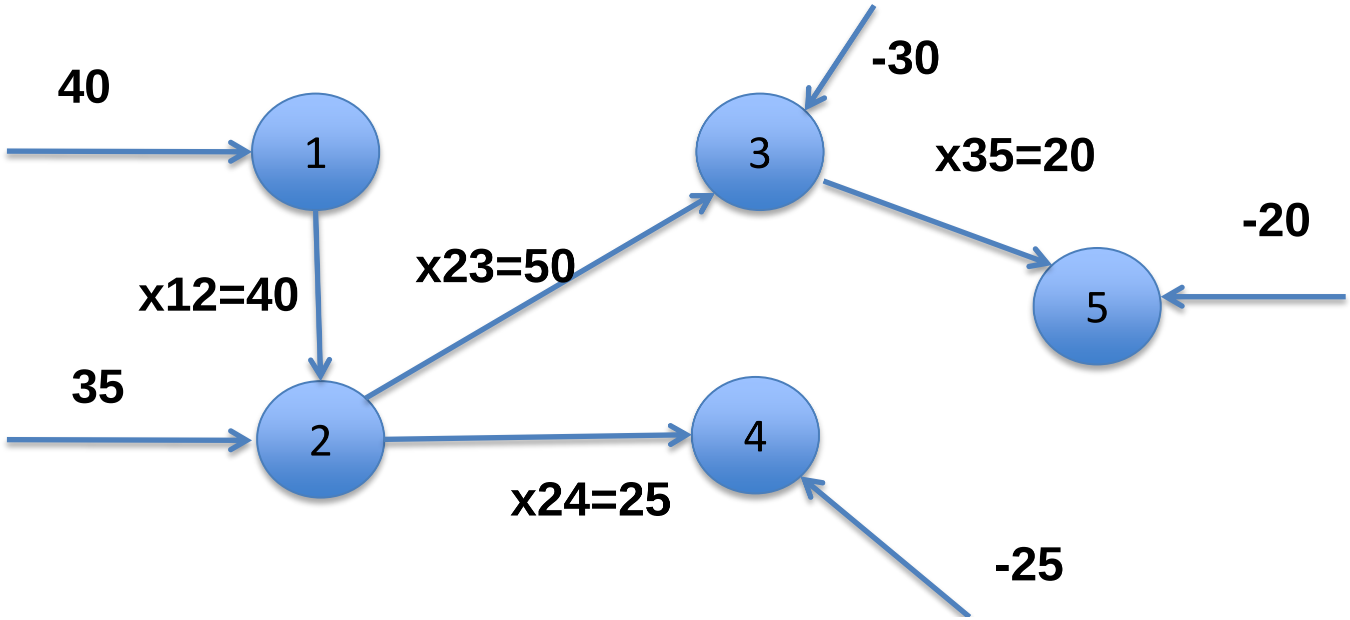
β={(1, 2), (2,3), (2,4), (3,5)}

$$x_\beta = \begin{bmatrix} 40 \\ 50 \\ 25 \\ 20 \end{bmatrix} \geq 0$$

Rešavanje problema

Pomeranja ka boljem rešenju

β={(1, 2), (2,3), (2,4), (3,5)}



$$y = \begin{bmatrix} y1 \\ y2 \\ y3 \\ y4 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 1 \\ 1 \end{bmatrix}$$

$$\bar{c} = \begin{bmatrix} \overline{c_{13}} \\ \overline{c_{34}} \\ \overline{c_{45}} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \geq 0$$

Rezime

i. Početno rešenje odgovara razapinjućem stablu
ii. Iz balansnih jednačina odrede se protoci xij, ako su nenegativni početno rešenje je moguće i dobro izabrano (bazno rešenje)
iii. Odrediti simpleks množitellje iz uslova da je ym=0 i

$$y_i - y_j = c_{ij} \quad \textbf{za sve grane baznog rešenja}$$

iv. Odrediti redukovane troškove za rešenja koja nisu bazna

$$\bar{c}_{ij} = c_{ij} - y_i + y_j$$

v. Ako su redukovani troškovi iz iv ≥ 0, dobili smo optimalno rešenje
vi. Ako je neki od redukovanih troškova negativan, izabrati najveći negativni po apsolutnoj vrednosti i odgovarajuću granu dodati razapinjućem stablu. U dobijenoj petlji naći graničnu vrednost protoka, tako da protok kroz jednu granu pada na 0.
vii. Granu sa protokom 0 izbaciti i grafa, čime se dobija novo razapinjuće stablo.
viii. Ponoviti iii