1. NAĆI PRVI IZVOD SLEDEĆIH FUNKCIJA:

1. a) $y = x^3 - 3x^2 + x^6 - 2$

$y' = 3x^2 - 3\cdot 2x + 6x^5 - 0$

b) $y = \sqrt[3]{x} - \frac{2}{x^4} + 5\frac{1}{\sqrt[4]{x^7}} + \frac{1}{x}$

$y = x^{\frac{1}{3}} - 2\cdot x^{-4} + 5\cdot x^{-\frac{7}{4}} + x^{-1}$

$y' = \frac{1}{3}x^{-\frac{2}{3}} - 2\cdot(-4)x^{-5} + 5\left(-\frac{7}{4}\right)x^{-\frac{11}{4}} - 1\cdot x^{-2}$

c) $y = 5^x + \sin x - 5^{-2} + e^x - e^3$

$y' = 5^x \ln 5 + \cos x - 0 + e^x - 0$

d) $y = \frac{\ln x}{\cos x} + 3\cdot \operatorname{tg} x \cdot e^x$

$y' = \frac{\frac{1}{x}\cos x - \ln x\cdot(-\sin x)}{\cos^2 x} + 3\left(\frac{1}{\cos^2 x}\cdot e^x + \operatorname{tg} x\cdot e^x\right)$

e) $y = 2\cdot x^3\cdot \sin x - e^3\cdot e^x + 2\cdot\frac{\ln x}{x^4}$

$y' = 2\left(3x^2\cdot \sin x + x^3\cdot\cos x\right) - e^3\cdot e^x + 2\,\frac{\frac{1}{x}\cdot x^4 - \ln x\cdot 4x^3}{x^8}$

f) $y = \left(\frac{2}{3}\right)^x + \frac{2^x}{3} + \frac{2}{3^x}$

$y = \left(\frac{2}{3}\right)^x + \frac{1}{3}\cdot 2^x + 2\cdot\left(\frac{1}{3}\right)^x$

$y' = \left(\frac{2}{3}\right)^x \ln\frac{2}{3} + \frac{1}{3}2^x \ln 2 + 2\left(\frac{1}{3}\right)^x \ln\frac{1}{3}$

2. a) $y = e^{\sin x} + e^{x^3} + e^{e^x} - e^{\operatorname{arctg} x} + e^{e^e}$

$y' = e^{\sin x} \cdot \cos x + e^{x^3} \cdot 3x^2 + e^{e^x} \cdot e^x - e^{\operatorname{arctg} x} \cdot \frac{1}{1+x^2} + 0$

b) $y = \ln^5 x + \sin^2 x - \cos^3 x$

$y' = 5 \ln^4 x \cdot \frac{1}{x} + 2 \cdot \sin x \cdot \cos x - 3\cos^2 x \cdot (-\sin x)$

c) $y = \frac{1}{\ln x} + \frac{1}{\cos^2 x} + \frac{1}{\operatorname{tg}^3 x}$

$y = (\ln x)^{-1} + (\cos x)^{-2} + (\operatorname{tg} x)^{-3}$

$y' = -1(\ln x)^{-2} \cdot \frac{1}{x} - 2(\cos x)^{-3}(-\sin x) - 3(\operatorname{tg} x)^{-4} \cdot \frac{1}{\cos^2 x}$

d) $y = \operatorname{arctg}^3 x - \frac{1}{\sqrt{\ln x}} + \sqrt[3]{\sin^5 x}$

$y = \operatorname{arctg}^3 x - (\ln x)^{-\frac{1}{2}} + (\sin x)^{\frac{5}{3}}$

e) $y = \ln \sin x - \ln \ln x + \ln \operatorname{arctg} x + \ln 5 - \ln x^8$

$y' = \frac{1}{\sin x} \cdot \cos x - \frac{1}{\ln x} \cdot \frac{1}{x} + \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{1+x^2} + 0 - \frac{1}{x^8} \cdot 8x^7$

f) $y = \sin e^x - \sin \ln x + \sin \sin x - \sin \cos x + \sin x^5$

$y = \cos e^x \cdot e^x - \cos \ln x \cdot \frac{1}{x} + \cos \sin x \cdot \cos x - \cos \cos x(-\sin x) + \cos x^5 \cdot 5x^4$

g) $y = 5^{\sin x} - 3^{\operatorname{tg} x} + 2^{e^x} + 2^{e^2}$

$y' = 5^{\sin x} \ln 5 \cdot \cos x - 3^{\operatorname{tg} x} \ln 3 \cdot \frac{1}{\cos^2 x} + 2^{e^x} \ln 2 \cdot e^x + 0$

h) $y = \operatorname{arctg} e^x - \operatorname{arctg} x^3 + \operatorname{arctg} \ln x$

$y' = \frac{1}{1+(e^x)^2} \cdot e^x - \frac{1}{1+(x^3)^2} \cdot 3x^2 + \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x}$

3. a) $y = e^{\sin x} + \sin e^x$

$y' = e^{\sin x} \cdot \cos x + \cos e^x \cdot e^x$

b) $y = \ln \cos x + \cos \ln x$

$y' = \frac{1}{\cos x} \cdot (-\sin x) + (-\sin \ln x) \cdot \frac{1}{x}$

c) $y = \operatorname{arctg} e^x - e^{\operatorname{arctg} x}$

$y' = \frac{1}{1+(e^x)^2} \cdot e^x - e^{\operatorname{arctg} x} \cdot \frac{1}{1+x^2}$

d) $y = \sin^3 x + \sin x^3$

$y' = 3\sin^2 x \cdot \cos x + \cos x^3 \cdot 3x^2$

e) $y = \sqrt{\cos x} + \cos \sqrt{x}$

$y = (\cos x)^{\frac{1}{2}} + \cos x^{\frac{1}{2}}$

$y' = \frac{1}{2} (\cos x)^{-\frac{1}{2}} \cdot (-\sin x) + (-\sin x^{\frac{1}{2}}) \cdot \frac{1}{2} x^{-\frac{1}{2}}$

4. a) $y = e^{x^4} - \sin \ln x + \underset{\substack{\| \\ (\cos x)^{-1}}}{\frac{1}{\cos x}}$

$y' = e^{x^4} \cdot 4x^3 - \cos \ln x \cdot \frac{1}{x} + (-1)(\cos x)^{-2} \cdot (-\sin x)$

b) $y = 3^{\operatorname{arctg} x} + \operatorname{ctg} e^x - \arcsin x^2$

$y' = 3^{\operatorname{arctg} x} \ln 3 \cdot \frac{1}{1+x^2} + \frac{-1}{\sin^2 e^x} \cdot e^x - \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$

c) $y = e^{\arccos x} + \ln \operatorname{tg} x - \arccos^3 x$

$y' = e^{\arccos x} \cdot \frac{-1}{\sqrt{1-x^2}} + \frac{1}{\operatorname{tg} x} \cdot \frac{1}{\cos^2 x} - 3\arccos^2 x \frac{-1}{\sqrt{1-x^2}}$

5. a) $y = x^{x^2}$

$\ln y = \ln x^{x^2}$

$\ln y = x^2 \cdot \ln x \quad /'$

$\frac{1}{y} \cdot y' = 2x \cdot \ln x + \cancel{x^2} \cdot \frac{1}{\cancel{x}}$

$y' = x^{x^2}(2x \ln x + x)$

b) $y = x^{\sin x}$

$\ln y = \ln x^{\sin x}$

$\ln y = \sin x \cdot \ln x \quad /'$

$\frac{1}{y} \cdot y' = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x}$

$y' = x^{\sin}\left(\cos x \ln x + \frac{\sin x}{x}\right)$

c) $y = x^{\operatorname{tg} x}$

$\ln y = \ln x^{\operatorname{tg} x}$

$\ln y = \operatorname{tg} x \cdot \ln x$

$\frac{1}{y} \cdot y' = \frac{1}{\cos^2 x} \cdot \ln x + \operatorname{tg} x \cdot \frac{1}{x}$

$y' = x^{\operatorname{tg} x}\left(\frac{\ln x}{\cos^2 x} + \frac{\operatorname{tg} x}{x}\right)$

d) $y = (\cos x)^x$

$\ln y = \ln(\cos x)^x$

$\ln y = x \ln \cos x \quad /'$

$\frac{1}{y} \cdot y' = 1 \cdot \ln\cos x + x \frac{1}{\cos x} \cdot (-\sin x)$

$y' = (\cos x)^x (\ln\cos x - x \cdot \operatorname{tg} x)$

6. a) $x = \sin t + t^2$

$y = \ln t - \operatorname{tg} t$

$x'_t = \cos t + 2t$

$y'_t = \frac{1}{t} - \frac{1}{\cos^2 t}$

$y'_x = \frac{y'_t}{x'_t} = \frac{\frac{1}{t} - \frac{1}{\cos^2 t}}{\cos t + 2t}$

b) $x = t^2 \cdot \cos t - e^{2t}$

$y' = \ln \frac{t+1}{t^2-1}$

$x'_t = 2t\cos t + t^2(-\sin t) - e^{2t} \cdot 2$

$y'_t = \frac{1}{\frac{t+1}{t^2-1}} \quad \frac{1 \cdot (t^2-1) - (t+1) \cdot 2t}{(t^2-1)^2}$

$y'_x = \frac{y'_t}{x'_t} = \frac{\frac{-t^2-2t-1}{(t+1)(t^2-1)}}{2(t\cos t - t^2\sin t - 2e^{2t}}$

c) $x = \sqrt[3]{t^4} - \frac{1}{\sqrt[3]{t^4}} = t^{\frac{4}{3}} - t^{-\frac{4}{3}}$

$y = t^4 + \frac{1}{t^4} = t^4 - t^{-4}$

$x'_t = \frac{4}{3} t^{\frac{1}{3}} - \left(-\frac{4}{3}\right) t^{-\frac{7}{3}}$

$y'_t = 4t^3 - (-4) t^{-5}$

$y'_x = \frac{y'_t}{x'_t} = \frac{4t^3 + \frac{4}{t^5}}{\frac{4}{3} t^{\frac{1}{3}} + \frac{4}{3\sqrt[3]{t^7}}}$

7. NAĐI y'':

a) $y = e^{2x} - x^4 + \sin x - \ln x$

$y' = 2e^{2x} - 4x^3 + \cos x - \frac{1}{x}$

$y'' = 4e^{2x} - 12x^2 - \sin x + \frac{1}{x^2}$

b) $y = \sqrt[3]{x} + e^{3x} - \frac{1}{x^2}$

$y = x^{\frac{1}{3}} + e^{3x} - x^{-2}$

$y' = \frac{1}{3}x^{-\frac{2}{3}} + 3e^{3x} + 2x^{-3}$

$y'' = -\frac{2}{9}x^{-\frac{5}{3}} + 9e^{3x} - 6x^{-4}$

8. NAĐI y'''_x

a) $y = 3e^x - x^5 + x - \cos x$

$y' = 3e^x - 5x^4 + 1 + \sin x$

$y'' = 3e^x - 20x^3 + 0 + \cos x$

$y''' = 3e^x - 60x^2 - \sin x$

$x^{\frac{1}{2}}$

b) $y = e^{2x} - x^4 + \sqrt{x}$

$y' = 2e^{2x} - 4x^3 + \frac{1}{2}x^{-\frac{1}{2}}$

$y'' = 4e^{2x} - 12x^2 - \frac{1}{4}x^{-\frac{3}{2}}$

$y''' = 8e^{2x} - 24x + \frac{3}{8}x^{-\frac{5}{2}}$

9. NAĐI $f''(1)$

$f(x) = x^3 - e^{2x} + \ln x$

$f'(x) = 3x^2 - 2e^{2x} + \frac{1}{x}$

$f''(x) = 6x - 4e^{2x} - \frac{1}{x^2}$

$f''(1) = 6\cdot 1 - 4e^{2\cdot 1} - \frac{1}{1^2}$

$f''(1) = 6 - 4e^2 - 1 = 5 - 4e^2$

10. NAĐI $f'''(1)$

$x^{\frac{2}{3}}$

$f(x) = x^4 - e^x + \sqrt[3]{x^2}$

$f'(x) = 4x^3 - e^x + \frac{2}{3}x^{-\frac{1}{3}}$

$f''(x) = 12x^2 - e^x - \frac{2}{9}x^{-\frac{4}{3}}$

$f'''(x) = 24x - e^x + \frac{4}{27}x^{-\frac{7}{3}}$

$f'''(1) = 24 - e + \frac{4}{27}$