

03.07.2021.

1. 1 шара со p шарика
 $\binom{n-p}{2}$ шаров без p шарика

$\frac{p(n-p)}{2}$ шаров p шарика, шаров не p шарика

$$\Rightarrow 1 + \binom{n-p}{2} + \frac{p(n-p)}{2}$$

$$2. \binom{n+1}{n-1}^2 - \binom{n}{n-2}^2 = \left(\binom{n+1}{n-1} - \binom{n}{n-2} \right) \left(\binom{n+1}{n-1} + \binom{n}{n-2} \right)$$

$$= \left(\frac{(n+1)!}{(n-1)!2!} - \frac{n!}{(n-2)!2!} \right) \left(\frac{(n+1)!}{(n-1)!2!} + \frac{n!}{(n-2)!2!} \right) =$$

$$= \left(\frac{n(n+1)}{2} - \frac{n(n-1)}{2} \right) \left(\frac{n(n+1)}{2} + \frac{n(n-1)}{2} \right) =$$

$$= \frac{n(n+1-n+1)}{2} \cdot \frac{n(n+1+n-1)}{2} = n \cdot n^2 = n^3$$

3. A, B, C, 0, 1

$$x_1 + x_2 + x_3 + x_4 + x_5 = 15$$

$$\binom{15+5-1}{5-1} = \binom{19}{4}$$

$$4. 10a_n = 9a_{n-1} - 2b_{n-1} \Rightarrow 2b_{n-1} = 9a_{n-1} - 10a_n$$

$$5b_n = -a_{n-1} + 3b_{n-1} \quad b_{n-1} = \frac{2}{5}a_{n-1} - 5a_n$$

$$5\left(\frac{9}{2}a_n - 5a_{n+1}\right) = -a_{n-1} + \frac{99}{2}a_{n-1} - 15a_n \quad | \cdot 2$$

$$45a_n - 50a_{n+1} = -2a_{n-1} + 99a_{n-1} - 30a_n$$

$$75a_n - 50a_{n+1} - 25a_{n-1} = 0 \quad | : (-25)$$

$$2a_{n+1} - 3a_n + a_{n-1} = 0 \Rightarrow 2x^2 - 3x + 1 = 0$$

$$x_{1,2} = \frac{3 \pm \sqrt{9-8}}{4} = \frac{3 \pm 1}{4} \quad t_1 = \frac{1}{2} \quad t_2 = 1$$

$$a_n = A\left(\frac{1}{2}\right)^n + B \cdot 1^n = A\left(\frac{1}{2}\right)^n + B$$

$$a_0 = 4 = A + B$$

$$10a_1 = 9a_0 - 2b_0 = 9 \cdot 4 - 2 \cdot 3 = 36 - 6 = 30 \Rightarrow a_1 = 3$$

$$a_1 = \frac{1}{2}A + B = 3 \rightarrow \ominus \Rightarrow 4 - 3 = \left(1 - \frac{1}{2}\right)A \Rightarrow A = \frac{2}{1}A$$

$$a_n = 2 \cdot \left(\frac{1}{2}\right)^n + 2 = 2(2^{-n} + 1)$$

$$A = 2 \\ B = 2$$

$$b_n = \frac{9}{2}a_n - 5a_{n+1} = \frac{9}{2}2(2^{-n} + 1) - 5 \cdot 2(2^{-n-1} + 1) =$$

$$= 9(2^{-n} + 1) - 10(2^{-n-1} + 1) = (9-5)2^{-n} - 1 = 2^{-n+2} - 1$$