08.09.2021.

1. 3 НТ 4 ФР 5 ШП — — — — —

Морају сједати НТ - ФР - НТ - ШП - НТ
или НТ - ШП - НТ - ФР - НТ

$\Rightarrow 2! \cdot 3! \cdot 4! \cdot 5!$

2. $\sum_{j=k}^{n} \binom{n}{j}\binom{j}{k} = \binom{n}{k} 2^{n-k}$

$$\sum_{j=k}^{n} \binom{n}{j}\binom{j}{k} = \sum_{j=k}^{n} \frac{n!}{j!(n-j)!} \cdot \frac{j!}{k!(j-k)!} = \sum_{j=k}^{n} \frac{n!}{(n-j)!\,k!\,(j-k)!}$$

$$= \sum_{j=k}^{n} \frac{n!}{k!\,(n-k)!} \cdot \frac{(n-k)!}{(n-j)!\,(j-k)!} = \frac{n!}{k!(n-k)!} \sum_{j=k}^{n} \frac{(n-k)!}{(n-j)!\,(n-k-n+j)!}$$

$$= \binom{n}{k} \sum_{j=k}^{n} \binom{n-k}{n-j} = \binom{n}{k} \sum_{i=j-k}^{n-k} \binom{n-k}{n-(k+i)} = \binom{n}{k} \sum_{i=0}^{n-k} \binom{n-k}{n-k-i}$$

$$= \binom{n}{k} \sum_{i=0}^{n-k} \binom{n-k}{i} = \binom{n}{k} (1+1)^{n-k} = \binom{n}{k} 2^{n-k}$$

3. $x_1 + x_2 + x_3 + x_4 + x_5 = 9$ $\quad x_1, x_2, x_3, x_4, x_5 \geq 1$ ...

4. $f_n = 5f_{n-2} - 4f_{n-4}, \quad n \geq 4$ $\quad f_0 = -1,\ f_1 = f_2 = 1,\ f_3 = 5$

karakteristična: $t^4 - 5t^2 + 4 = 0$

$(t^2-1)(t^2-4) = 0$ $\quad t_1 = -2,\ t_2 = -1,\ t_3 = 1,\ t_4 = 2$

$(t-1)(t+1)(t-2)(t+2)$

$f_n = A(-2)^n + B(-1)^n + C + D \cdot 2^n$

$f_0 = -1 = A + B + C + D$
$f_1 = 1 = -2A - B + C + 2D$
$f_2 = 1 = 4A + B + C + 4D$
$f_3 = 5 = -8A - B + C + 8D$

$\oplus \Rightarrow 0 = -A + 2C + 3D$
$\oplus \Rightarrow 2 = 2A + 2C + 6D$
$\oplus \Rightarrow 4 = -7A + 2C + 9D$

$\ominus \Rightarrow 2 = 3A + 3D$
$\Rightarrow 2 = -5A + 3D$

$\ominus \Rightarrow A = 0$
$D = \frac{2}{3}$

$0 = 2C + 2 \Rightarrow C = -1$

$-1 = B - 1 + \frac{2}{3} \Rightarrow B = -\frac{2}{3}$

$\Rightarrow \boxed{f_n = \left(-\frac{2}{3}\right)(-1)^n - 1 + \frac{2^{n+1}}{3}}$