domaći

MATEMATIČKA INDUKCIJA

5. $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

1° baza indukcije: $n = 1 \Rightarrow 1^2 = \frac{1 \cdot 2 \cdot 3}{6}$

$1 = 1$ ✓

2° induktivna hipoteza: $n = k \Rightarrow 1^2 + 2^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$

3° induktivni korak: $n = k+1 \Rightarrow$

$$1^2 + 2^2 + \ldots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$$

$$= (k+1)\frac{k(2k+1) + 6(k+1)}{6} = (k+1)\frac{2k^2 + 7k + 6}{6} =$$

$$= (k+1)\frac{2k^2 + 4k + 3k + 6}{6} = (k+1)\frac{(k+2)(2k+3)}{6}$$ ✓

7. $9 \mid (13^n - 4^n), \ n \in \mathbb{N}$

1° baza indukcije: $n = 1 \Rightarrow 9 \mid 13 - 4$

$9 \mid 9$ ✓

2° induktivna hipoteza: $n = k \Rightarrow 9 \mid (13^k - 4^k)$

3° induktivni korak: $n = k+1 \Rightarrow$

$$13^n - 4^n = 13^{k+1} - 4^{k+1} = 13 \cdot 13^k - 4 \cdot 4^k =$$

$$= 13 \cdot 13^k - 13 \cdot 4^k + 9 \cdot 4^k = 13 \cdot (13^k - 4^k) + 9 \cdot 4^k$$

$$9 \mid \left(13(13^k - 4^k) + 9 \cdot 4^k\right) \Rightarrow 9 \mid 13^{k+1} - 4^{k+1}$$

12. $4n < 2^n, \ n \geq 5$

1° b. i.: $n = 5 \Rightarrow 4 \cdot 5 < 2^5$

$20 < 32$ ✓

2° i. h.: $n = k \Rightarrow 4k < 2^k$

3° i.k. : $\quad n = k+1 \quad \Rightarrow$

$$4(k+1) = 4k+4 < 2^k + 4 < 2 \cdot 2^k < 2^{k+1}$$

(za $k \geq 5$)

14. $x_n = (n-1)(x_{n-1} + x_{n-2})$, $n \geq 3$, $x_1 = 1$, $x_2 = 2$

$x_n = n!$?

1° b.i. : $\quad n = 3 \Rightarrow x_3 = 2 \cdot (1+2) = 6$

$n! = 3! = 6$ ✓

2° i.h. : Pretpostavka da $\forall k \in [3, n]_{\mathbb{N}}$ važi $k! = (k-1)(x_{k-1} + x_{k-2})$

3° i.k. : dokaz za broj $n+1$

$$x_{n+1} = n \cdot (x_n + x_{n-1}) = n \cdot (n! + (n-1)!) =$$
$$= n \cdot (n \cdot (n-1)! + (n-1)!) =$$
$$= n \cdot (n-1)! \cdot (n+1) = (n+1)!$$

PRINCIP BIJEKCIJE