

domaći

REKURENTNE RELACIJE

$$2. d) f_{n+3} = 4f_{n+2} - f_{n+1} - 6f_n$$

$$f_0 = 1 \quad f_1 = 2 \quad f_2 = 4$$

$$f_n = t^n \Rightarrow$$

$$t^3 = 4t^2 - t - 6$$

$$t^3 - 4t^2 + t + 6 = 0$$

$$t^3 + t^2 + t^2 + t - 6t^2 + 6 = 0$$

$$t^2(t+1) + t(t+1) - 6(t-1)(t+1) = 0$$

$$(t+1)(t^2 + t - 6t + 6) = 0$$

$$(t+1)(t^2 - 5t + 6) = 0$$

$$(t+1)(t-2)(t-3) = 0$$

$$t_1 = -1 \quad t_2 = 2 \quad t_3 = 3$$

$$f_n = A \cdot (-1)^n + B \cdot 2^n + C \cdot 3^n$$

$$1 = f_0 = A + B + C$$

$$2 = f_1 = -A + 2B + 3C$$

$$4 = f_2 = A + 4B + 9C$$

$$\left. \begin{array}{l} 1 = f_0 = A + B + C \\ 2 = f_1 = -A + 2B + 3C \end{array} \right\} \Rightarrow 3 = 3B + 4C$$

$$\left. \begin{array}{l} 2 = f_1 = -A + 2B + 3C \\ 4 = f_2 = A + 4B + 9C \end{array} \right\} \Rightarrow 6 = 6B + 12C \Rightarrow 2 = 2B + 4C$$

$$\Rightarrow B = 1, C = 0, A = 0$$

$$\Rightarrow f_n = 2^n$$

5. b) $a_n = a_{n-1} a_{n-2}^2$, $a_0 = a_1 = 2$

$$\log_2 a_n = \log_2 a_{n-1} + 2 \log_2 a_{n-2}$$

$$b_n = \log_2 a_n \Rightarrow b_n = b_{n-1} + 2b_{n-2}$$

$$b_n = t^n \Rightarrow t^n = t^{n-1} + 2t^{n-2} \quad | : t^{n-2}$$

$$t^2 = t + 2 \Rightarrow t^2 - t - 2 = 0$$

$$(t-2)(t+1)=0$$

$$t_1 = -1 \quad t_2 = 2$$

$$\Rightarrow b_n = A(-1)^n + B \cdot 2^n$$

$$b_0 = \log_2 a_0 = \log_2 2 = 1$$

$$\Rightarrow I = A + B$$

$$b_1 = \log_2 a_1 = \log_2 2 = 1$$

$$\Rightarrow 1 = -A + 2B$$

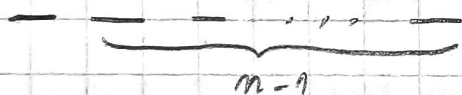
$$\begin{aligned} 2 & \Rightarrow 3B = 2 \\ A & = \frac{1}{3} \quad B = \frac{2}{3} \end{aligned}$$

$$A = \frac{1}{3} \quad B = \frac{2}{3}$$

$$b_n = \frac{1}{3} \cdot (-1)^n + \frac{2}{3} \cdot 2^n = \frac{(-1)^n + 2^{n+1}}{3}$$

$$b_n = \log_2 a_n \Rightarrow a_n = 2^{b_n} = 2$$

9.



- riječi koje počinju sa 1 ili 2: $\frac{1}{2} \underbrace{\quad \dots \quad}_{n-1} \Rightarrow 2 \cdot \underbrace{f_{n-1}}_{\text{paran br. nula}}$

paran
br. mula

- riječi koje počinju sa 0: treba se osigurati da u ostatku riječi ima neparan broj nula (neparan br. nula = 1, 3, 5, ...)

ukupan br. riječi dužine $n-1 \Rightarrow 3^{n-1}$

- br. riječi sa parnim br. nula $\Rightarrow -f_{n-1}$

$$\Rightarrow 3^{n-1} - f_{n-1}$$

$$f_n = 2f_{n-1} + 3^{n-1} - f_{n-1}$$

$$= f_{n-1} + 3^{n-1}$$

$$= f_{n-1} + 3^{n-1}$$

$$f_1 = 24 \quad f_2 = 5$$

homogeneous equation: $f_n - f_{n-1} = 0 \Rightarrow t-1=0 \Rightarrow t=1$
 $1 \cdot 1^n - 4 \cdot 1^n \neq 0$

$$f(h) = (A, 1^m) \times$$

$$f_n^{(h)} = (An + B) \cdot 1^n = An + B$$

partikularno rješenje: $f_n^{(p)} = C \cdot 3^n$

$$C \cdot 3^n = C \cdot 3^{n-1} + 3^{n-1} = (C+3) \cdot 3^{n-1} \quad / : 3^{n-1}$$

$$3C = C+1 \Rightarrow C = \frac{1}{2} \Rightarrow f_n^{(p)} = \frac{1}{2} \cdot 3^n$$

opšte rješenje: $f_n = f_n^{(h)} + f_n^{(p)} =$

$$f_n = An + B + \frac{1}{2} 3^n$$

$$\left. \begin{array}{l} f_1 = 2 = A + B + \frac{3}{2} \\ f_2 = 5 = 2A + B + \frac{9}{2} \end{array} \right\} \Rightarrow \begin{array}{l} 3 = A + \frac{6}{2} \Rightarrow A = 0 \\ B = \frac{1}{2} \end{array}$$

$$f_n = \frac{1}{2} + \frac{1}{2} 3^n = \frac{3^n + 1}{2}$$