

$$1. (1, 1, 1, 1), (2, 1, 1, 1), (2, 2, 1, 1), \dots, (n, n, n, n)$$

↳ broj multistupova sastavljenih od elemenata
 $1, 2, \dots, n \rightarrow (x, y, z, w) \quad x \leq y \leq z \leq w \leq n$

$$\binom{4+n-1}{4} = \binom{n+3}{4}$$

$$2. \binom{12}{3} \cdot \binom{9}{3} \cdot \binom{6}{3} \cdot \binom{3}{3}$$

$$3. 3 \cdot \binom{4}{0,0,4} \cdot 2^4 = \binom{4}{4,0,0} \cdot 2^4 + \binom{4}{0,4,0} \cdot 2^4 + \binom{4}{0,0,4} \cdot 2^4$$

$$\begin{aligned} 6 \cdot \binom{4}{0,1,3} \cdot 2^4 &= \binom{4}{0,1,3} 2^1 2^3 + \binom{4}{1,0,3} 2^1 2^3 + \\ &+ \binom{4}{1,3,0} 2^1 2^3 + \binom{4}{0,3,1} 2^1 2^3 + \\ &+ \binom{4}{3,0,1} 2^3 2^1 + \binom{4}{3,1,0} 2^3 2^1 \end{aligned}$$

$$= \sum_{\substack{i+j+k=4 \\ 0 \leq i,j,k \leq 4}} \binom{4}{i,j,k} 2^i 2^j 2^k = (2+2+2)^4 = 8^4$$

$$4. a_n = 5a_{n-1} + 2 \quad a_0 = 2 \quad n \geq 1$$

$$a_n z^n = 5a_{n-1} z^n + 2z^n$$

$$\sum_{n \geq 1} a_n z^n = 5 \sum_{n \geq 1} a_{n-1} z \cdot z^{n-1} + 2 \sum_{n \geq 1} z^n$$

$$\sum_{n \geq 0} a_n z^n - a_0 z^0 = 5z \sum_{n \geq 0} a_n z^n + 2 \cdot \frac{1}{1-z} - 2$$

$$\underbrace{\sum_{n \geq 0} a_n z^n}_{A(z)} - \cancel{2} = 5z \underbrace{\sum_{n \geq 0} a_n z^n}_{A(z)} + \frac{2}{1-z} - \cancel{2}$$

$$(1-5z) A(z) = \frac{2}{1-z} + A(z) \frac{2}{(1-5z)(1-z)} =$$

$$= 2(1-5z)^{-1} (1-z)^{-1}$$

$$\binom{-1}{k} = \frac{(-1)(-2)\dots(-1-k+1)}{k!} = \frac{(-1)^k 1 \cdot 2 \cdot \dots \cdot k}{k!} = (-1)^k$$

$$A(z) = 2 \sum_{n \geq 0} (-1)^n (-5z)^n \sum_{n \geq 0} (-1)^n (-z)^n =$$

$$= 2 \sum_{n \geq 0} 5^n z^n \cdot \sum_{n \geq 0} z^n =$$

$$= 2 \sum_{n \geq 0} z^n \left(\sum_{k=0}^n 5^k \cdot 1 \right) =$$

$$= 2 \sum_{n \geq 0} z^n \cdot \left(\sum_{i=1}^n 5^i + 5^0 \right) =$$

$$= 2 \sum_{n \geq 0} z^n \cdot \left(5 \cdot \frac{5^n - 1}{5 - 1} + \frac{1}{4} \right) =$$

$$= \frac{1}{2} \sum_{n \geq 0} z^n \cdot (5^{n+1} - 1) =$$

$$\Rightarrow a_n = \frac{1}{2} (5^{n+1} - 1)$$

$$\begin{array}{lcl}
 5. & 1 \quad \overbrace{\quad \quad \quad}^{n-1} \quad \dots \quad \quad \quad & \Rightarrow a_{n-1} \\
 & 2 \quad \overbrace{\quad \quad \quad}^{n-2} \quad \dots \quad \quad \quad & \Rightarrow a_{n-2} \\
 & 3 \quad \overbrace{\quad \quad \quad}^{n-3} \quad \dots \quad \quad \quad & \Rightarrow a_{n-3}
 \end{array}
 \left\{
 \begin{array}{l}
 a_n = a_{n-1} + a_{n-2} + a_{n-3} \\
 a_1 = 1 \\
 a_2 = 3 \\
 a_3 = 4
 \end{array}
 \right.$$

6. n zeka m korpica, $n \geq m$

$$\Rightarrow \text{UDP} \quad m = k \left\lfloor \frac{n}{k} \right\rfloor + r \quad \hookrightarrow r \geq 0$$

makar $k = \left\lceil \frac{n}{m} \right\rceil + 1$ u jednog korpici