Satellite-to-Satellite Relative Navigation Estimation

Status Update 1

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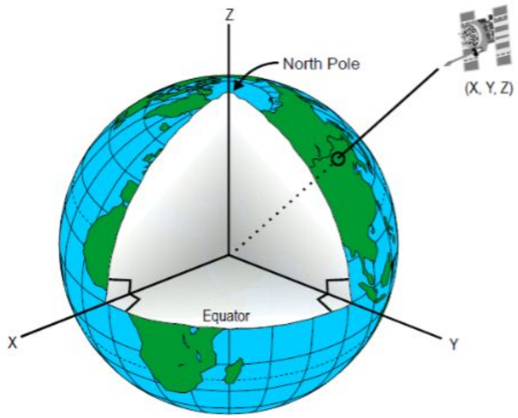
Problem Motivation

- Estimate the state of a secondary satellite using satellite-to-satellite measurements
  - Reduces need for ground station resources
  - Capable of estimating space debris
  - Applicable to relative navigation (i.e. rendezvous, proximity operations)
- Estimate the state of both the primary and secondary satellites without prior knowledge of either
  - Removes the significant constraint of exactly knowing the chief satellite’s orbit
  - Allows this approach to be applied in many more applications

Problem Definition



(a) Earth Centered Inertial Coordinates

$$\ddot{X} = -\frac{\mu}{r^3}X + u_1 + \tilde{w}_1$$
$$\ddot{Y} = -\frac{\mu}{r^3}Y + u_2 + \tilde{w}_2$$
$$\ddot{Z} = -\frac{\mu}{r^3}Z + u_3 + \tilde{w}_3$$

(b) Equations of Motion

Figure 1. Coordinate System and Equations of Motion

$$x_s(t) = [X, \dot{X}, Y, \dot{Y}, Z, \dot{Z}]^T$$
$$u_s(t) = [u_1, u_2, u_3]^T$$
$$\tilde{w}_s(t) = [\tilde{w}_1, \tilde{w}_2, \tilde{w}_3]^T$$

- Two satellites orbiting the Earth in unique orbits (chief satellite and deputy satellite)
- Chief satellite's orbit is initially known
- Chief satellite records range, range rate, azimuth, and elevation measurements of the deputy satellite
- Chief satellite estimates the position and velocity of the deputy satellite

Technical Objectives

Level 1:

- Demonstrate ability to implement UKF for orbit estimation of deputy satellite assuming the states of the chief satellite are known
- Develop baseline for magnitude of estimation error and measurement residuals
- Explore state space and initial condition options. Select example case for use in further levels.

Level 2:

- Demonstrate ability to implement SIR particle filter for orbit estimation of deputy satellite assuming the states of the chief satellite are known
- Develop MAP and MMSE estimators and transition functions for satellite particles
- Compare estimator performance between UKF and SIR particle filter through state estimation error

Level 3:

- Use a particle filter to estimate the state of the chief satellite and a UKF to estimate the state of the deputy satellite. UKF will be conditioned on the particles from the particle filter
- Demonstrate ability of a Rao-Blackwell Particle Filter to handle computationally expensive task through conditioning UKF on the state of the chief satellite.
- Analyze how long the estimation error will remain bounded and how reasonable of an estimate we can attain.
- Determine impact of orbital drift due to rank deficiency

Levels of Success and Milestones

Levels of Success:

Level 1:

- Implementation of UKF to estimate deputy satellite orbit given known chief satellite orbit

Level 2:

- Implementation of bootstrap particle filter to estimate deputy satellite orbit
- Comparison to UKF and error quantification

Level 3:

- Adjust UKF to operate on output of particle filter.
- Implementation of RBPF
- Further error quantification and observability discussion

Milestones:

Level 1:

- Ground truth model implementation
- UKF implementation

Level 2:

- Bootstrap particle filter implementation
- Comparison to UKF

Level 3:

- RBPF implementation
- Expansion to multiple deputy satellites

Completed and Pending Tasks

Completed:

Level 1

- Truth model created
- Plots generated
- UKF implemented and tuned

Level 2

- Implement PF
- Estimation error plots*

Pending:

Level 1

- Vary initial conditions to select a nominal case
- Prepare final iterations of plots for report

Level 2

- Update truth model with realistic noise statistics
- Update truth model with line of sight restrictions
- Tune PF

*See appendix for preliminary results

Level 1: Assumptions and Technical Approach

- The exact state of the chief satellite is known for all time
- Sensors are capable of making measurements at any range and any combination of azimuth and elevation
- Measurements are not impeded by the Earth or other physical constraints
  - This assumption will be resolved during level 2 objectives
- Measurement and process noise covariances are constant for all time

Level 1 Results

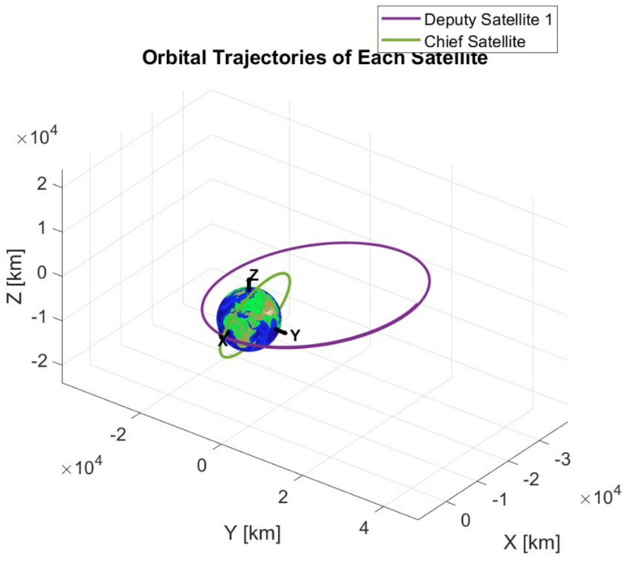
Truth Model:

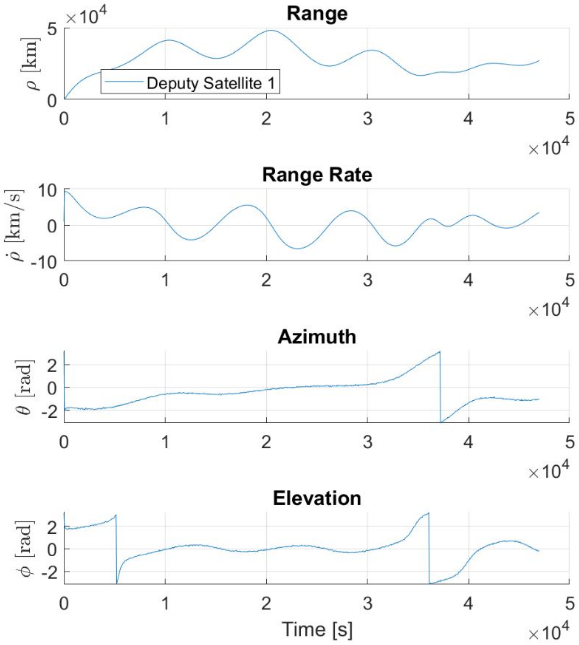
- Simulates the orbits of both satellites for ~1 orbit
  - Uses a process noise covariance of:

$$Q = 1 \times 10^{-10} I_{3\times3}$$

- Records noisy measurements of deputy from chief
  - Uses a measurement noise covariance of:

$$R = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 5\times10^{-6} & 0 & 0 \\ 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0.001 \end{bmatrix}$$

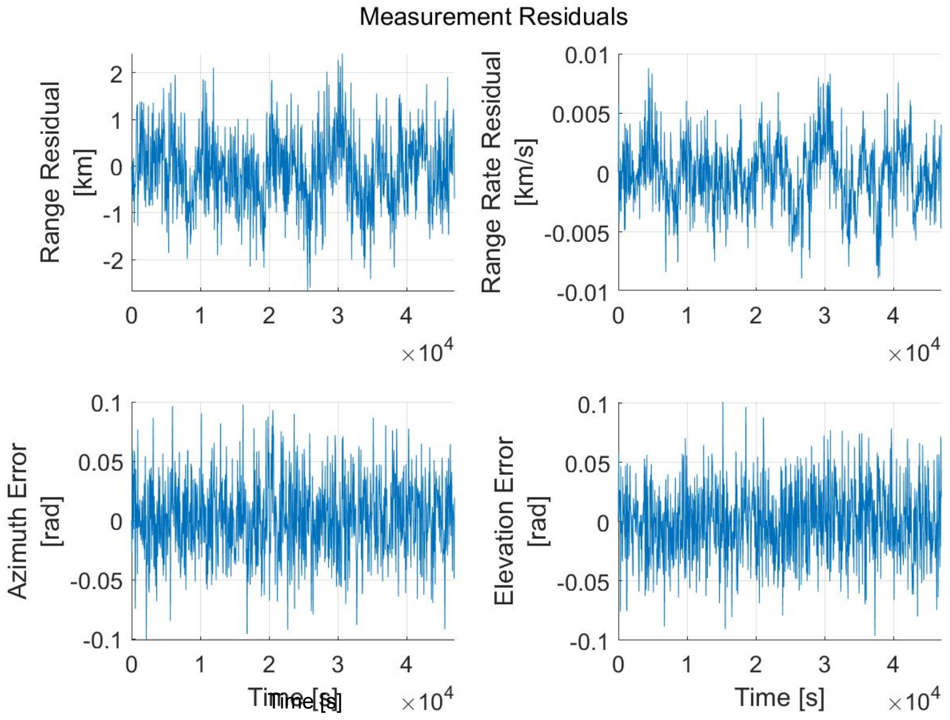




Level 1 Results

State Estimation: Measurement Residuals

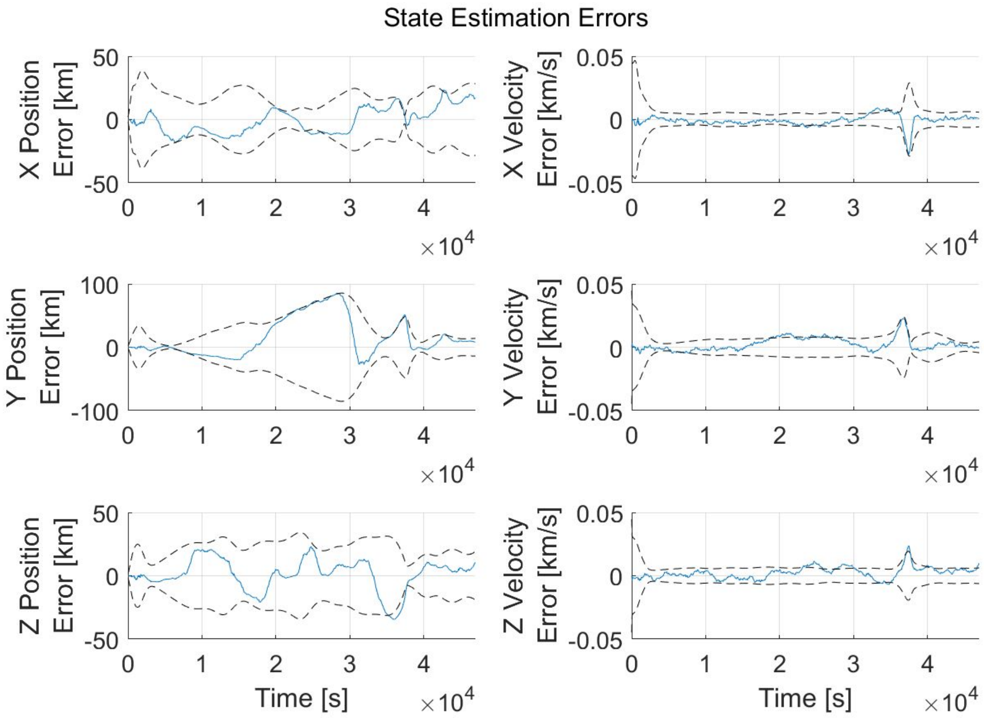
- Compares predicted measurement to actual measurement
- Residuals are as expected
- Appear as random noise
- No apparent systematic error over 1 orbital period



Level 1 Results

State Estimation: Estimation Error

- Error remains bounded for the most part over 1 orbit period
- 2σ bounds closely follow the estimation error, not too large and not too small
  - Indicates that the filter is not over/under confident
  - Points to dynamic consistency



Individual Contributions

Conner Martin:

- Implementation of UKF
- Main debugger for UKF
- Gathered results for progress report
- Basic PF implementation

Jordan Abell:

- Implemented Ground Truth Model
- Assisted in debugging UKF
- UKF Tuning
- Motivation and problem definition

Technical Questions

- What causes the state covariance bounds to “squeeze” in the estimation error plots (specifically the Y-position error on slide 10)?
- It looks like the bulges in covariance align with the bearing angles wrapping. Is there a good solution to make sure the angle wrapping doesn’t negatively impact the filter?
- Looking at satellite laser rangefinders, the accuracies we found are ~500m[2] and ~5mm/s[3], Fig. 13 for range and range rate. Is there a good way to go from sensor accuracy to measurement noise covariance?

References

[1] Liu, Y.-C., and Liu, L., “Orbit Determination Using Satellite-to-Satellite Tracking Data,” 2001.

[2] International Earth Rotation and Reference Systems Services, “Satellite Laser Ranging (SLR)”, 2013

[3] Serrano, L., Kim, D., et al., “A GPS velocity sensor: How accurate can it be? -A first look”, 2004

Appendix: Preliminary PF Results