HW 4

Tuesday, October 6, 2020 7:44 PM

$$\frac{dx}{dt} = ax + y - x(x^2+y^2) \qquad \frac{dy}{dt} = -x + ay - y(x^2+y^2)$$

1 a) $x = r\cos\varphi \qquad y = r\sin\varphi \qquad x^2+y^2 = r^2$

$$\sin\varphi = \frac{y}{r} \qquad \cos\varphi = \frac{x}{r} \qquad \tan\varphi = \frac{x}{y}$$

$$\varphi = \sin^{-1}\left(\frac{y}{r}\right) = \cos^{-1}\left(\frac{x}{r}\right) = \tan^{-1}\left(\frac{x}{y}\right)$$

$$r = \frac{x}{\cos\varphi} = \frac{y}{\sin\varphi}$$

$$dx = dr\cos\varphi - r\sin\varphi\, d\varphi$$
$$dy = dr\sin\varphi + r\cos\varphi\, d\varphi$$

$$\frac{dx}{dt} = \cos\varphi\frac{dr}{dt} - r\sin\varphi\frac{d\varphi}{dt} = ar\cos\varphi + r\sin\varphi - r^3\cos\varphi$$

$$\frac{dy}{dt} = \sin\varphi\frac{dr}{dt} + r\cos\varphi\frac{d\varphi}{dt} = -r\cos\varphi + ar\sin\varphi - r^3\sin\varphi$$

from $\frac{dx}{dt}$: $\quad \frac{dr}{dt} = \sec\varphi\left[ar\cos\varphi + r\sin\varphi - r^3\cos\varphi + r\sin\varphi\frac{d\varphi}{dt}\right]$

from $\frac{dy}{dt}$: $\quad = \csc\varphi\left[-r\cos\varphi + ar\sin\varphi - r^3\sin\varphi - r\cos\varphi\frac{d\varphi}{dt}\right]$

distrib $$\frac{dr}{dt} = ar + r\tan\varphi - r^3 + r\tan\varphi \frac{d\varphi}{dt}$$

$$= -r\cot\varphi + ar - r^3 - r\cot\varphi \frac{d\varphi}{dt}$$

subtract: $$0 = r\tan\varphi + r\cot\varphi + \left[r\tan\varphi + r\cot\varphi\right] \frac{d\varphi}{dt}$$

$$-r(\tan\varphi + \cot\varphi)\frac{d\varphi}{dt} = r(\tan\varphi + \cot\varphi)$$

$$\boxed{\therefore \frac{d\varphi}{dt} = -1 \qquad \frac{dr}{dt} = r(a - r^2)}$$

double check $$\frac{dr}{dt} = ar + r\tan\varphi - r^3 - r\tan\varphi = ar - r^3$$

$$= ar - r\cot\varphi - r^3 + r\cot\varphi$$

$$0 = \cancel{r\tan} + \cancel{r\cot} - \cancel{r\tan} - \cancel{r\cot} \quad \checkmark$$

When $a < 0$, $\frac{dr}{dt}$ always negative $\rightarrow$ spirals to zero @ ∞

$a > 0$, $\frac{dr}{dt} = 0$ when $a = r^2$, or $r = \sqrt{a}$.

since $\frac{d\varphi}{dt}$ = constant, @ ∞ becomes circle of radius $\sqrt{a}$.