HW 6 - JACK JAMES

Saturday, December 5, 2020 3:01 PM

HW5, due Wed Nov 4 at 9am

In this assignment, you are free to use any language you wish to answer all computational questions. You *do not* need to use all three languages.

1. **10 points** Modified from Garcia 6.13: In class, we showed both analytically and computationally that neutron emitters, described by $\dot{\rho} = D\rho'' + C\rho$ and $\rho(-L/2, t) = \rho(L/2, t) = 0$, have a critical mass.
   (a) Show analytically that if reflecting boundary conditions are used that the neutron density will *always* diverge for large time, meaning there is no critical mass.
   (b) Use a Crank-Nickelson algorithm to numerically compute the neutron density as a function of time up to $T = 10s$, with $C = 1\text{s}^{-1}$, $L = 1m$, and $D \in \{1, 10, 100\}\text{cm}^2/s$. Comment on similarities or differences you see for the different values of D. *Hint:* The C-N algorithm is implemented in Garcia's `schro.x`, which you are free to modify.

a) $\dot{P} = DP'' + CP$ Reflecting BC

$$\therefore \left.\frac{dP}{dX}\right|_{x=\pm\frac{1}{2}} = 0$$

Let $P(x,t) = X(x)\,T(t)$

So
$$\frac{\cancel{X(x)}\,T'(t)}{\cancel{X(x)}\,T(t)} = \frac{D\,\cancel{T(t)}\,X''(x) + C\,X(x)\,\cancel{T(t)}}{X(x)\,\cancel{T(t)}}$$

$$\frac{T'(t)}{T(t)} = \frac{D\,X''(x)}{X(x)} + C$$

$$\frac{T'(t)}{D\,T(t)} = \frac{X''(x)}{X(x)} + \frac{C}{D} = \lambda$$

$$T'(t) = D\lambda T(t) \longrightarrow T(t) = A\exp(D\lambda t)$$

$$\text{+LW} \quad X''(x) = \left(\lambda - \frac{c}{D}\right) X(x)$$

$$\therefore X(x) = F e^{\sqrt{\lambda - \frac{c}{D}}\,x} + G e^{-\sqrt{\lambda - \frac{c}{D}}\,x}$$

$$P(x,t) = T(t)\,X(x)$$
$$= A e^{D\lambda t}\left(F\exp\left[\left(\lambda - \frac{c}{D}\right)^{1/2} x\right] + G\exp\left[-\left(\lambda - \frac{c}{D}\right)^{1/2} x\right]\right)$$

$$\frac{\partial P}{\partial x} = A e^{D\lambda t}\sqrt{\lambda - \frac{c}{D}}\left[F \underbrace{\exp\left[\left(\lambda - \frac{c}{D}\right)^{1/2} x\right]}_{\equiv \tilde{F}} - G \underbrace{\exp\left[-\left(\lambda - \frac{c}{D}\right)^{1/2} x\right]}_{\equiv \tilde{G}}\right]$$

$$0 = A e^{D\lambda t}\sqrt{\lambda - \frac{c}{D}}\left(F\tilde{F} - G\tilde{G}\right)$$

$$0 = A e^{D\lambda t}\sqrt{\lambda - \frac{c}{D}}\left(F\tilde{F}^{-1} - G\tilde{G}^{-1}\right)$$

$$\therefore F\tilde{F} = G\tilde{G} \quad + \quad F\tilde{F}^{-1} = G\tilde{G}^{-1}$$

I think this means $\lambda = \frac{c}{D}$?

Because $\left[F e^{\pm\sqrt{\lambda - \frac{c}{D}}\,x} = G^{\mp\sqrt{\lambda - \frac{c}{D}}\,x} \right]_{x = \frac{L}{2}}$

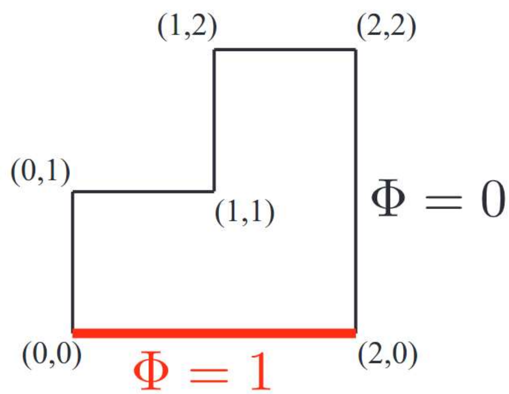
$$\therefore T(t) = A e^{ct}.$$

For neutron emitters, $\cancel{c > 0}$. $c > 0$.

don't think
it can be zero.

Ae^{ct} diverges as $t \to \infty$.

2. **10 points** Modified from Garcia 8.9(a). In the domain sketched below, the black thin walls all have $\Phi = 0$ while the thick red wall has $\Phi = 1$.



   (a) Numerically solve Laplace's equation $\nabla^2\Phi = 0$ with these boundary conditions. You may use any method you would like, but please describe the method you chose, including initial conditions, timestep, grid points, etc for finite difference methods, or number of modes for spectral methods. Show a contour plot for your solution.

   (b) Use the same method to solve Poisson's equation, $\nabla^2\Phi = -\rho(\mathbf{r})/\epsilon_0$, with a point charge located at (0,1/2). Plot a contour plot of the solution.

$\nabla^2 \varphi = 0$