JACK JAMES

Friday, September 25, 2020 5:50 AM

# 3.3

a) Korteweg-de Vries (KdV)

$$\frac{\partial u}{\partial t} + 6u(x,t)\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$

$u(x,t) = V(x-ct)$ from d'Alembert

let $x-ct=y$

Need $\frac{\partial V(y)}{\partial y}$ from $\frac{\partial u}{\partial t}$

$x - ct = y$

$dx - c\,dt = dy$

$dt = \frac{-dy + dx}{c}$

$dx = c\,dt + dy$

+ $\frac{\partial V(y)}{\partial y}$ from $\frac{\partial u}{\partial x}$

$u(x,t) = V(x-ct)$

as well as $\frac{\partial^3 V(y)}{\partial y}$ from $\frac{\partial^3 u}{\partial x^3}$

Need Math Methods notes

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D'alembert

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial \xi}{} \frac{\partial}{} + \frac{\partial \eta}{} \frac{\partial}{\partial \eta} = \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta} = \frac{\partial}{\partial \xi} + \frac{\partial}{\partial \eta}$$

$$\frac{\partial}{\partial t} = c\frac{\partial}{\partial \xi} - c\frac{\partial}{\partial \eta}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial \xi^2} + \cancel{2\frac{\partial^2 y}{\partial \xi \partial \eta}} + \frac{\partial y^2}{\partial \eta^2}$$

$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial \xi^2} - \cancel{2c^2\frac{\partial^2 y}{\partial \xi \partial \eta}} + c^2\frac{\partial y^2}{\partial \eta^2}$$

middle terms $\to 0$

$$y(x,t) = f(\xi) + g(\eta) = f(x-ct) + g(x+ct)$$

going forward in time so use $f(x-ct)$ w/ $g=0$

$$\therefore \frac{\partial u}{\partial t} = -c\frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

from substituting given variables back in for traditional $\xi$ + $\eta$

$$\frac{\partial^3 u}{\partial x^3} = \frac{\partial^3 v}{\partial y^3}$$

Equation becomes: $-c\frac{\partial v}{\partial y} + 6v\frac{\partial v}{\partial y} + \frac{\partial^3 v}{\partial y^3} = 0$

Equation $\partial y$ $\partial y$ $\partial y$

✓

b) $z = y\sqrt{c}$ $\quad V(y) = c\,w(z)$

$dz = \sqrt{c}\,dy$ $\quad dV = c\,dw$

$$\frac{-c}{\sqrt{c}}\frac{c\,dw}{\partial z} + 6\frac{c\,w}{\sqrt{c}}\frac{c\,dw}{\partial z} + \frac{\partial^3 (c\,w)}{\frac{1}{c^{3/2}}\partial z^3} \stackrel{?}{=} 0$$

$$\frac{c^2}{\sqrt{c}}\left[\qquad\qquad\right] = 0$$

coefficients can now be cancelled. ✓