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3.2a)

$$\ddot{x} = -\omega^2 x$$

$$\hookrightarrow x(t) = x_0 \cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)$$

Let $\tau = \omega t$, Find $\dot{x}(\tau)$

$$\ddot{x} + \omega^2 x = 0$$

guess $x(t) = e^{\pm i\omega t}$

$$\frac{d}{dt}\left(e^{i\omega t}\right) = \pm(i\omega)e^{\pm i\omega t}$$

$$\frac{d^2}{dt^2}\left(e^{i\omega t}\right) = \pm(i\omega)^2 e^{\pm i\omega t}$$

$$= \mp\omega^2 e^{\pm i\omega t}$$ ✓

$$x = x_0 e^{\pm i\omega t} = e^{\pm i\tau} = x(t) = x\left(\frac{\tau}{\omega}\right)$$

$$v = \pm i\omega e^{\pm i\omega t} = \pm i\frac{\tau}{t} e^{\pm i\tau}$$

$$vt = \pm i\tau e^{\pm i\tau} = [\text{length}]$$

$$x_0 e^{i\tau} = x_0(\cos\tau + i\sin\tau)$$

$$x_0 e^{-i\tau} = x_0(\cos\tau - i\sin\tau)$$

from Euler $\frac{e^{i\tau} + e^{-i\tau}}{2} = \cos\tau$ $\frac{e^{i\tau} - e^{-i\tau}}{i2} = \sin\tau$

add/sub formulas gives $2e^{i\tau}$

$$\therefore \quad \begin{aligned} \cos\tau + i\sin\tau &= 2e \\ \cos\tau - i\sin\tau &= 2e^{-i\tau} \end{aligned}$$

@$\tau = 0$, $\sin\tau = 0$ so coefficient for cos is X_0

@$\tau = \frac{\pi}{2}$, $\cos\tau = 0$ so " " sin is

$$Ae^{i\tau} + Be^{-i\tau} = X\left(\frac{\tau}{t}\right)$$

$$X(t) = \frac{\tilde{A}}{2}(\cos\tau + i\sin\tau) + \frac{\tilde{B}}{2}(\cos\tau - i\sin\tau)$$ ← combination

$$X = (A + B)\cos\tau + i(A - B)\sin\tau$$

$$X(0) = A + B = X_0 \qquad \therefore \quad (A - B) = V_0$$

$$\dot{X}(0) = i\frac{\omega}{\omega}(A - B)\sin\tau$$

↑ w/respect to τ since

$$\tau = \omega t \rightarrow d\tau = \omega\, dt$$

$$\frac{\partial X}{\partial \tau} = \frac{dx}{\partial t}\frac{\partial t}{\partial \tau} = \frac{1}{\omega}\frac{dx}{dt}$$

Let's try different way.

$$\therefore X\left(\frac{\tau}{\omega}\right) = X_0\cos\tau + V_0\sin\tau$$

$$X(t) = X_0\cos\omega t + \frac{V_0}{\omega}\sin\omega t$$

@$\tau = 0$,

run into problem.

$$\ldots\cos\omega t + V_0\sin\omega t$$

$$\omega \cdot x(t) = x_0 \omega \cos \omega t + v_0 \sin \omega t$$

$$v_0 = \omega \left(x(t) - x_0 \cos \omega t \right) / \sin(\omega t)$$

$$= \omega \left(x\left(\frac{\tau}{t}\right) - x_0 \cos(\tau) \right) / \sin \tau$$

$$\boxed{v_0 = \frac{\omega}{\sin \tau} x\left(\frac{\tau}{t}\right) - \omega x_0 \cot(\tau)}$$

run into problem. L'hopital rule forgotten, came back.

$$x(\tau) = x_0 \cos \tau + \frac{v_0 t}{\tau} \sin \tau$$

$$\frac{\tau}{t \sin \tau} \left(x - x_0 \cos \tau \right) = v_0$$

$$\frac{\omega t}{t \sin \omega t} \xrightarrow{L'H} \lim_{t \to 0} \frac{\omega}{\omega t \cos \omega t + \cancel{\sin \omega t}} = \frac{\omega}{\omega t} = \frac{1}{0} \text{ still}$$

$$\sim \frac{\omega}{\sin \tau}$$