IEEE Standard for Digitizing Waveform Recorders

Sponsor
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of the
IEEE Instrumentation and Measurement Society**

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Abstract: Terminology and test methods for describing the performance of waveform recorders are provided.
Keywords: digitizers, digitizing oscilloscopes, error sources, performance, terminology, test methods, waveform recorders

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Introduction

(This introduction is not a part of IEEE Std 1057-1994, IEEE Standard for Digitizing Waveform Recorders.)

Work on this standard unofficially began with the "Seminar on Waveform Recorder Measurement Needs and Techniques for Evaluation/Calibration," hosted by the National Bureau of Standards (now National Institute of Standards and Technology [NIST]) in 1981. Dissatisfied with the state of measurement standards and test techniques for waveform recorders, a group of concerned users, manufacturers, and researchers who attended the seminar formed a committee to crystallize waveform recorder specifications and achieve consensus on them. This effort gathered additional members and became affiliated with the IEEE Instrumentation and Measurement Society in 1983. The initial work was completed in 1988, and a Trial-Use Standard was issued in July of 1989. Since that time, the standard has been substantially revised, resulting in the current standard, which now has full-use status.

The purpose of this standard is to provide common terminology and test methods for describing the performance of waveform recorders. Since these devices give digital answers that are intended to be sent to a computer for analysis, this committee found it necessary to adopt an approach that is perhaps more highly analytical than approaches used by other instrumentation standards committees. A report on the user's reaction to this approach and any additional feedback will be welcomed.

At the time of the approval of this standard, the working group had the following membership:

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IEEE Standard for Digitizing Waveform Recorders

1. Overview

1.1 Scope

Instruments covered by this standard are electronic digitizing waveform recorders, waveform analyzers, and digitizing oscilloscopes with digital outputs. This standard is directed toward, but not restricted to, general-purpose waveform recorders and analyzers. Special applications may require additional manufacturer information and verification tests.

1.2 Purpose

The purpose of this standard is to provide common terminology and test methods for describing the performance of waveform recorders. It is meant for users and manufacturers of these devices. The main body presents many performance features, sources of error, and test methods. The tests are meant to be used by manufacturers to locate and correct sources of error in the hardware.

The information in this standard can also be applied by users to correct some of the errors by software processing after the signal has been recorded. Another planned use for this standard is to aid users in writing specifications for purchasing new waveform recorders.

1.3 Guidance to the user

This standard presents a large number of possible waveform recorder error sources and test methods for quantifying them. Some of these methods are cumbersome in that they require sophisticated test equipment and signal processing, or require a considerable amount of time. It is generally not necessary to perform all of these tests to adequately characterize a particular waveform recorder for a particular application. Also, the existence of some errors can preclude measuring others. For many applications, the two relatively easy tests for determining effective bits and step response may suffice or at least point the user in a direction to pursue other tests. These tests require a high purity sine-wave source and a clean, fast leading-transition step generator. Recommended algorithms for performing the sine fit used in determining effective bits are described in 4.1.3.

The sine-fit test will detect such errors as random noise, nonlinearities, and aperture uncertainty. Some insight into these errors may be gained by examining a plot of the residuals as defined in equations (23) or (53), as appropriate. The direction of further testing may be suggested by whether the residuals appear random or systematic.

The sine-fit test does not measure amplitude flatness or phase linearity. Deviations from the ideal in these parameters may appear as aberrations in the recorder's step response. The step response parameters should be determined over a range of levels to detect amplitude nonlinearity and slew limit effects. When the recorder is not too noisy, the impulse response may be approximated by differentiating the step response. For more information, see [B14], [B17], and [B16].[1]

Waveform recorders often have various controllable settings such as gain, offset, filtering, sampling rate, trigger level, and trigger slope. The performance parameters and test methods described in this standard apply to a single set of settings. Changing a setting can affect the recorder's performance. This is particularly true of gain, offset, and sampling rate. Thus, a waveform recorder should be characterized over the range of settings at which the instrument will be used. Also, some waveform recorders have the capability to automatically self-calibrate, and the data taken before the calibration may differ from the data taken after the calibration. If a test procedure is affected by a self-calibration, the user may have to defeat the self-calibration feature before the test procedure has begun.

Drift in performance over time and temperature are not specifically addressed for most parameters in this standard. An exception is long-term sample time drift. The manufacturer should specify the operating temperature range and recalibration intervals.

Many test methods call for test equipment whose performance is better than that expected from the waveform recorder being measured. The required test equipment may not be available. This is especially the case for recorders at the state of the art in sampling speed or accuracy, or both. A paradox may exist in that high performance waveform recorders are required to develop and evaluate new test equipment. When this is the case, clever engineering is required to develop techniques to measure the parameters of interest using available equipment.

Waveform recorders and digital oscilloscopes may employ either real-time or equivalent-time sampling, or both, at different times. With real-time sampling, samples of the input signal are acquired sequentially, at the stated sampling rate. However, with equivalent-time sampling, the samples may be acquired sequentially or otherwise, at a rate usually much lower than the stated equivalent-time sampling rate. Equivalent-time sampling requires that input waveforms be repetitive, since a waveform sampled in equivalent-time is reconstructed from samples taken over many repetitions. This restriction does not apply for recorders employing real-time sampling.

In this standard, the recommended test methods were designed with real-time sampling instruments specifically in mind. Nevertheless, the test methods will generally still be valid when applied to recorders that use equivalent-time sampling. A few additional points should be kept in mind in such cases.

A common feature of equivalent-time sampling instruments is the ability to perform waveform averaging. This is useful in reducing noise components, both originating in the waveform source and also in the recorder. Therefore, tests that in some way measure the recorder's noise processes will be affected by the amount of averaging applied in the recorder. Examples of such tests are effective bits, signal-to-noise ratio, peak error, random noise, and aperture uncertainty. The results of any such tests performed on recorders employing equivalent-time sampling should include the number of samples or records averaged.

As noted in 4.9.2, the use of equivalent-time sampling also affects the time over which the aperture uncertainty is measured. Therefore, the results of this measurement should include the time required to capture a record, or equivalently, the real sampling rate and the record length.

Finally, some tests of equivalent-time sampling recorders will actually be easier to perform, or will have less uncertainty because the sampling is often phase-locked to the trigger signal. As an example, the measurement of trigger delay and jitter may be more accurate (see 4.10.1.3).

[1]The numbers in brackets correspond to those bibliographic items listed in annex E.

1.4 Manufacturer supplied information

1.4.1 General information

Manufacturers shall supply the following general information:

a) Model number
b) Dimensions and weight
c) Power requirements
d) Environmental conditions: Safe operating, non-operating, and specified performance temperature range; altitude limitations; humidity limits, operating and storage; vibration tolerance; and compliance with applicable electromagnetic interference specifications
e) Any special or peculiar characteristics
f) Compliance with military and other specifications
g) Available options and accessories, including their impact on equipment performance
h) Exceptions to the above parameters where applicable
i) Calibration interval, if required

1.4.2 Performance specifications

The manufacturer shall provide the minimum specifications (see clause 1.4.2.1). See clause 3 for definitions.

1.4.2.1 Minimum specifications

a) Number of digitized bits
b) Sample rates
c) Memory length
d) Input impedance
e) Analog bandwidth
f) Input signal ranges

1.4.2.2 Additional specifications

a) Gain
b) Offset
c) Differential nonlinearity
d) Harmonic distortion and spurious response
e) Integral nonlinearity
f) Maximum static error
g) Signal-to-noise ratio
h) Effective bits
i) Peak error
j) Random noise
k) Frequency response
l) Settling time
m) Transition duration of step response (rise time)
n) Slew limit
o) Overshoot and precursors
p) Aperture uncertainty (short-term time- base stability)
q) Long-term time-base stability
r) Fixed error in sample time
s) Trigger delay and jitter
t) Trigger sensitivity
u) Trigger hysteresis band
v) Trigger minimum rate of change
w) Trigger coupling to signal
x) Crosstalk
y) Monotonicity
z) Hysteresis
aa) Overvoltage recovery

ab) Word error rate
ac) Cycle time
ad) Common mode rejection ratio
ae) Maximum common mode signal level
af) Differential input impedance
ag) Maximum operating common mode signal level

2. References

IEEE Std 181-1977, IEEE Standard on Pulse Measurement and Analysis by Objective Techniques.[2]

3. Definitions and symbols

3.1 Definitions

3.1.1 amplitude flatness: The variation in output amplitude as a function of frequency in response to a constant amplitude sine wave input.

3.1.2 aperture uncertainty: The standard deviation of the sample instant in time. *Syn*: short-term timing instability; timing jitter; timing phase noise.

3.1.3 CMRR: See 3.1.11, common-mode rejection ratio.

3.1.4 code bin k: A digital output that corresponds to a particular set of input values.

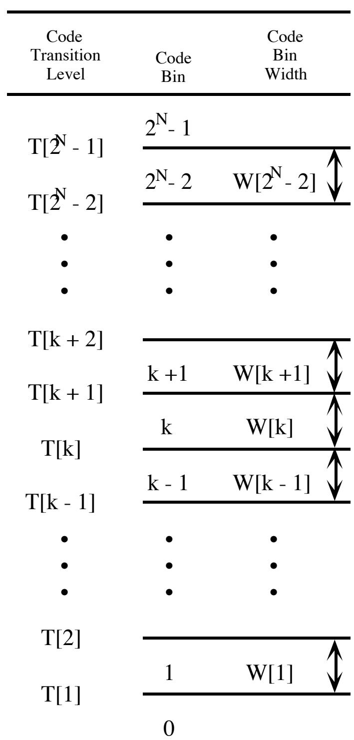

Figure 1—Definitions pertaining to input quantization

[2]IEEE Std 181-1977 has been withdrawn; however, copies can be obtained from the IEEE Standards Department, 445 Hoes Lane, P.O. Box 1331, Piscataway, NJ 08855-1331, USA.

3.1.5 code bin width ***W[k]:*** The difference of the code transition levels that delimit the bin.

$$W[k] = T[k+1] - T[k] \tag{1}$$

3.1.6 code transition level: The boundary between two adjacent code bins.

3.1.7 code transition level ***T [k]:*** The value of the recorder input parameter at the transition point between two given, adjacent code bins. The transition point is defined as the input value that causes 50% of the output codes to be greater than or equal to the upper code of the transition, and 50% to be less than the upper code of the transition. The transition level *T[k]* lies between code bin k–1 and code bin k. *See:* 3.1.61, transition point.

3.1.8 coherent sampling: Sampling of a periodic waveform in which there is an integer number of cycles in the data record. In other words, coherent sampling occurs when the following relationship exists:

$$f_s \cdot M_c = f_0 \cdot M \tag{2}$$

where

f_s is the sampling frequency
M_c is the integer number of cycles in the data record
f_0 is the frequency of the input
M is the number of samples in the record

3.1.9 common-mode overvoltage: A signal level whose magnitude is less than the specified maximum safe common-mode signal but greater than the maximum operating common-mode signal.

3.1.10 common-mode overvoltage recovery time: The time required for the recorder to return to its specified characteristics after the end of a common-mode overvoltage pulse.

3.1.11 common-mode rejection ratio (CMRR): The ratio of the input common-mode signal to the effect produced at the output of the recorder in units of the input.

3.1.12 common mode signal: The average value of the signals at the positive and negative inputs of a differential input waveform recorder. If the signal at the positive input is designated V_+, and the signal at the negative input is designated V_-, then the common mode signal (V_{cm}) is

$$V_{cm} = \frac{V_+ + V_-}{2} \tag{3}$$

3.1.13 crosstalk (multichannel): The ratio of the signal induced in one channel to a common signal applied to all other channels.

3.1.14 cycle time: The real time elapsed (with a recorder continually taking records of data) between the beginning of two records taken in succession.

3.1.15 data window: A set of coefficients by which corresponding samples in the data record are multiplied to more accurately estimate certain properties of the signal, particularly frequency domain properties. Generally, the coefficient values increase smoothly toward the center of the record.

3.1.16 differential input impedance to ground: The impedance between either the positive input and ground or the negative input and ground.

3.1.17 differential nonlinearity: The difference between a specified code bin width and the average code bin width, divided by the average code bin width.

3.1.18 fixed error in sample time: A nonrandom error in the instant of sampling. A fixed error in sample time may be fixed with respect to the data samples acquired or correlated with an event that is detected by the sampling process. Unless otherwise specified, usually taken to mean the maximum fixed error that may be observed.

3.1.19 full-scale range: The difference between the maximum and the minimum recordable input values as specified by the manufacturer.

3.1.20 full-scale signal: A signal that spans the entire manufacturer's specified amplitude range of the instrument.

3.1.21 gain and offset: (A) (independently based). Gain and offset are the values by which the input values are multiplied and then to which the input values are added, respectively, to minimize the mean squared deviation from the output values. (B) (terminal-based). Gain and offset are the values by which the input values are multiplied and then to which the input values are added, respectively, to cause the deviations from the output values to be zero at the terminal points, that is, at the first and last codes.

3.1.22 harmonic distortion: For a pure sine wave input, output components at frequencies that are an integer multiple of the applied sine wave frequency.

3.1.23 hysteresis: The maximum difference in values for a digitizer code transition level when the transition level is approached from either side of the transition.

3.1.24 ideal code bin width *Q*: The full-scale range divided by the total number of code states.

3.1.25 incoherent sampling: Sampling of a waveform such that the relationship between the input frequency, sampling frequency, number of cycles in the data record, and the number of samples in the data record does not meet the definition of coherent sampling.

3.1.26 input impedance: The impedance between the signal input of the waveform recorder and ground.

3.1.27 integral nonlinearity: The maximum difference between the ideal and actual code transition levels after correcting for gain and offset.

3.1.28 kth code transition level *T[k]*: The input value corresponding to the transition between codes k–1 and k.

3.1.29 large signal: A signal whose peak-to-peak amplitude is as large as practical but is recorded by the instrument within, but not including, the maximum and minimum amplitude data codes. As a minimum, the signal must span at least 90% of the full-scale range of the waveform recorder.

3.1.30 long-term settling error: The absolute difference between the final value specified for short-term settling time, and the value 1 s after the beginning of the step, expressed as a percentage of the step amplitude.

3.1.31 long-term timebase stability: The change in time base frequency (usually given in parts per million) over a specified period of time at a specified sampling rate.

3.1.32 maximum common-mode signal level: The maximum level of the common-mode signal at which the common mode rejection ratio is still valid.

3.1.33 maximum operating common-mode signal: The largest common-mode signal for which the waveform recorder will meet its effective bits specifications in recording a simultaneously-applied, normal-mode signal.

3.1.34 maximum static error (MSE): The maximum difference between any code transition level and its ideal value.

3.1.35 monotonic recorder: A recorder that has output codes that do not decrease (increase) for a uniformly increasing (decreasing) input signal, disregarding random noise.

3.1.36 noise: Any deviation between the output signal (converted to input units) and the input signal, except deviations caused by linear time invariant system response (gain and phase shift), a dc level shift, or an error in the sample rate. For example, noise includes the effects of random errors, fixed pattern errors, nonlinearities and time base errors (fixed error in sample time and aperture uncertainty).

3.1.37 normal mode signal (or differential signal): The difference between the signal at the positive input and the negative input of a differential input waveform recorder. If the signal at the positive input is designated V_+, and the signal at the negative input is designated V_-, then the normal mode (or differential) signal (V_{dm}) is

$$V_{dm} = V_+ - V_- \tag{4}$$

3.1.38 normalized peak error: Peak error divided by three times the standard deviation of the differences discussed in 4.5.1.1. See 3.1.42, peak error.

3.1.39 offset: See 3.1.21, gain and offset.

3.1.40 overshoot: The maximum amount by which the step response exceeds the topline, specified as a percentage of (recorded) pulse amplitude.

3.1.41 overvoltage: Any voltage whose magnitude is less than the maximum safe input voltage of the recorder but greater than the full-scale value for the selected range.

3.1.42 peak error: The residual with the largest absolute value.

3.1.43 precursor: A deviation from the baseline prior to the pulse transition.

3.1.44 random noise: A nondeterministic fluctuation in the output of a waveform recorder, described by its frequency spectrum and its amplitude statistical properties.

3.1.45 record of data: A sequential collection of samples acquired by the waveform recorder.

3.1.46 relatively prime: Describes integers whose greatest common divisor is 1.

3.1.47 residuals: The differences between the recorded data and the fitted sine wave for sine-wave curve fitting.

3.1.48 reverse coupling: The ratio of the spurious signal generated by a signal at some other input to the recorder and the signal recorded at the specified input of the recorder.

3.1.49 settling time: Measured from the mesial point (50%) of the output, the time at which the step response enters and subsequently remains within a specified error band around the final value. The final value is defined to occur 1 s after the beginning of the step.

3.1.50 short-term settling time: Measured from the mesial point (50%) of the output, the time at which the step response (see 4.1.4) enters and subsequently remains within a specified error band around the final value. The final value is defined to occur at a specified time less than one second after the beginning of the step.

3.1.51 short-term timing instability: See 3.1.2, aperture uncertainty.

3.1.52 signal-to-noise ratio: The ratio of a signal to the noise.

3.1.53 single-ended recorder: A non-differential waveform recorder, i.e., one that does not subtract the signals at two input terminals.

3.1.54 slew limit: The value of output transition rate of change for which an increased amplitude input step causes no change.

3.1.55 spurious components: Persistent sine waves at frequencies other than the harmonic frequencies. See 3.1.22, harmonic distortion.

3.1.56 step response: The recorded output response for an ideal input step with designated baseline and topline.

3.1.57 timing jitter: See 3.1.2, aperture uncertainty.

3.1.58 timing phase noise: See 3.1.2, aperture uncertainty.

3.1.59 total harmonic distortion: The root sum square of all harmonic distortion components including their aliases.

3.1.60 transition duration of a step response: The duration between the proximal point (10%) and the distal point (90%) on the recorded output response transition, for an ideal input step with designated baseline and topline.

3.1.61 transition point: The input value that causes 50% of the output codes to be greater than or equal to the upper code of the transition, and 50% to be less than the upper code of the transition.

3.1.62 trigger delay: The elapsed time from the occurrence of a trigger pulse at the trigger input connector to the time at which the first or a specified data sample is recorded.

3.1.63 trigger jitter: The standard deviation in the trigger delay time over multiple records.

3.1.64 trigger minimum rate of change: The slowest rate of change of the leading edge of a pulse of a specified level that will trigger the recorder.

3.1.65 trigger signal coupling: The ratio of the spurious signal level (that is recorded by an input to the recorder) to the trigger signal level.

3.2 Symbols

$\alpha_n = \cos(\omega t_n + \theta)$

$\bar{\alpha}$ = average of α_n over M samples

$\beta_n = \sin(\omega t_n + \theta)$

$\bar{\beta}$ = average of β_n over M samples

ε = error, used for total error and error band

ε_{rms} = root-mean-square value of ε

$\varepsilon[f_s]$ = error in sampling rate

$\varepsilon[f_r]$ = measured error in input reference frequency

$\varepsilon[k]$ = difference between $T[k]$ and ideal value of $T[k]$ computed from G and V_{os}

θ = phase, expressed as radians

π = constant, ratio of the circumference of a circle to the diameter

ρ = reflection coefficient

σ = standard deviation; sometimes used as noise amplitude, which is the standard deviation of the random component of a signal

σ_{σ} = standard deviation of the standard deviation (for example, standard deviation of the noise amplitude)

σ_t = aperture uncertainty

τ = sampling period, the inverse of f_s

$\Phi[X]$ = cumulative density function for a Gaussian distribution (the fraction of the total distribution covered from negative infinity to X)

ϕ = phase estimate, expressed in radians

ψ = frequency estimate, expressed in radians per second

ω = frequency, expressed in radians per second

A = sinusoidal amplitude

A_1 = amplitude estimate of a sinusoid

A_D = amplitude denominator, when A (or A_1) is expressed as a fraction

A_N = amplitude numerator, when A (or A_1) is expressed as a fraction

$A[i]$ = average of a record

a_{11}, a_{12}, a_{21}, a_{22} = linear equation matrix elements for four parameter fit iteration

B = test tolerance in fractions of an ideal code-bin width (Q). Also used as an amplitude

B_1 = amplitude estimate of a cosine wave

B_D = amplitude denominator, when B (or B_1) is expressed as a fraction

B_N = amplitude numerator, when B (or B_1) is expressed as a fraction

C = offset

DNL = maximum differential nonlinearity over all k

$DNL[k]$ = differential nonlinearity of code k

E = number of effective bits

$ENBW$ = equivalent noise bandwidth

$e_m[f]$ = aliasing and first differencing magnitude errors

$e_p[f]$ = aliasing and first differencing phase errors

f = frequency, Hz

f_0 = frequency of the input signal

f_{co} = upper frequency for which the amplitude response is –3 dB

f_{eq} = equivalent sampling rate

f^* = Approximate desired input frequency

$f[k]$ = correction function converting sine wave probability distribution function to uniform distribution function

f_{opt} = optimal input frequencies for the sine wave curve fit

f_r = input signal reference frequency or input signal repetition rate

f_s = sampling frequency

G = gain of waveform recorder, ideally = 1

$hn(i)$ = Hann window time coefficients

INL = integral nonlinearity

K = number of data records

k = code bin

L = equivalent time sampling parameter

M = number of sequential samples

M_c = number of input sine wave cycles

MSE = maximum static error

m = root-mean-square of rising and falling slopes of selected portions of a waveform

mse = mean square error

N = number of digitized bits

n = sample index within a record

$P+$, $P-$ = maximum and minimum peak regions of a sine wave

p = probability

Q = ideal code-bin width, expressed in input units

Q' = average code bin width

R = error parameter, or number of records

$R+$, $R-$ = rising and falling regions of a sine wave

S = set of samples collected over more than one record, also used as an error parameter

snr = signal-to-noise ratio

Δt = interval between equivalent time samples

$T[k]$ = code transition level between code k and code $k-1$

T_k = ideal value of $T[k]$ or $[Q \cdot (k - 1) + T_1]$

T_1 = ideal value of $T[1]$

T_{MIN} = minimum trigger delay required for a large pulse

T_{OD} = trigger delay for a pulse value V_{OD}

t_n = discrete sample times

u = confidence expressed as a fraction

V = full-scale input range of waveform recorder

V_{HYST} = the trigger hysteresis level

$V(t)$ = the settled value after a specified time

V_O = overdrive voltage

V_{OD} = a small trigger pulse level

V_{TH} = minimum level that will trigger the recorder

V_{os} = input offset of waveform recorder, ideally = 0

$W[k]$ = code bin width of code bin k

w = estimated word error rate

w' = worst-case word error rate

$w(i)$ = window function (for a Discrete Fourier Transform [DFT])

X = number of standard deviations of a Gaussian distribution

x = number of errors detected

$\bar{y}$ = average of y_n over M samples

y_n = data samples within a record

y'_n = best fit points to a data record

Z_0 = transmission line impedance

Z_τ = waveform recorder input impedance

$Z_{u/2}$ = number of standard deviations that encompass 100 (1–u)% of a Gaussian distribution about the center

4. Test methods

4.1 General methods

4.1.1 Taking a record of data

A record of data is a sequential collection of samples acquired by the waveform recorder. The action of taking a record of data is defined as creating the record of data in the recorder by making a measurement, and then transferring the data record to a computer for analysis.

4.1.2 Locating code transition levels

Since a waveform recorder digitizes the signal for storage, many of the specifications describe the fidelity of the digitization process. The fidelity of digitization depends on how well the code bins (into which the samples fall) match the ideal. The only way of directly measuring the code bins is to identify the end points of the bins. The boundary between two adjacent code bins is called the code transition level. Measurement of the code transition levels is used to determine conformance to several specifications.

The integer k is the recorder's output code, which ranges from 0 to $2^N–1$, where N is the number of digitized bits. The kth code transition level $T[k]$ is defined to be the input value corresponding to the transition between codes k–1 and k. Therefore $T[k]$ is the input value for which 50% of the output codes will be less than k, and 50% will be greater than or equal to k.

4.1.2.1 Static test method

A programmable source (e.g., a digital to analog converter) is required whose range and output parameter (voltage or current) is compatible with the waveform recorder, and whose resolution is at least four times that of the waveform recorder. The static transfer characteristic of the source should be known. The code transition levels are then determined as follows:

The output of the source is connected to the input of the waveform recorder. The following steps should be performed:

a) Begin with $k = 1$.
b) Apply an input level slightly lower than the expected code transition level. For $k = 1$, begin with a value slightly lower (for example 2%) than the minimum level recordable by the waveform recorder.
c) Take a record of data. Evaluate the record to determine the percentage of codes in the record that are less than k. Temporarily store the percentage along with the corresponding input value.
d) If the percentage from step c) is greater than 50%, then raise the input value by 1/4 Q (of the waveform recorder) or less, and repeat c), updating the stored input value and percentage.
e) The first time the percentage drops to 50% or less, the transition has been crossed. The code transition level is computed by linear interpolation based on the recorded percentages at this level and the previous level. Record the interpolated value as the code transition level $T[k]$.
f) After finding $T[k]$, repeat b) to find $T[k + 1]$ (the final level of step e) for the kth code transition level will generally be a satisfactory starting point for code transition level $k + 1$, step b).
g) Repeat step f) until $T[k]$ has been found for all k.

Care should be taken to ensure the impedance of the source and the input impedance of the recorder do not affect this measurement.

4.1.2.1.1 Comment on significance of record length and the presence of noise

The location of code transition levels is a probabilistic process because of the inevitable presence of noise. As a consequence, the percentages estimated by the measurements of 4.1.2.1 have an associated standard deviation. Corresponding uncertainties in the estimates of the code transition levels result. The uncertainties can be reduced by choosing larger record lengths.

Assuming the noise to have zero mean and a normal distribution, the size of the record required for a given precision in the estimates of code transition levels can be computed. Table 1 gives the precision with a 99.87% (3 σ) confidence level, expressed as a percentage of the root-mean-square (rms) noise value, computed for several record lengths.

If the precision needs to be known absolutely (that is, in terms of input units) rather than as a percentage of the noise, then the noise level must be determined. Subclause 4.5.5 provides procedures for measuring the noise level if the dominant source of noise is the waveform recorder rather than the programmable source. Alternatively, the noise can be estimated as the code transitions are located by examining the probability distribution vs. input level on either side of a code transition (see [B4]). Note that there is limited value in determining the code transition levels with a precision very much smaller than the noise level, unless the results of an intended application can be used in conjunction with noise-reducing signal processing.

Table 1—Precision of estimates of code transition level

Record length (samples)	Precision of estimates of code transition level (% of rms noise)
64	45%
256	23%
1024	12%
4096	6%

4.1.2.2 Dynamic test method

The following method is often easier to implement, especially if one is only interested in determining nonlinearities. A sine wave of amplitude sufficient to slightly overdrive the recorder is recorded many times, and a histogram is constructed. If the input range of the recorder is not symmetrical around 0, then a constant must be added to the sine wave so that the peaks of the combined signal are equidistant from the center of the range. The triggering of the recorder should be asynchronous with the sine wave, and the frequency of the sine wave must be specified. The amount of overdrive required depends on the noise level of the recorder and on the accuracy required; this will be covered in 4.1.2.2.2. If the amplitude and offset of the sine wave are precisely known, this method gives the transition levels to the same precision. If the amplitude of the sine wave and the offset are unknown, this method gives the transition levels to within a gain and offset error. That is, the calculated transition levels, $T'[k]$, will be related to the true transition levels, $T[k]$, by the relation

$$T'[k] = aT[k] + b \tag{5}$$

where a and b are constants.

The sine wave frequency must be chosen as described in 4.1.2.2.1. Take many records of data (the required amount will be covered in 4.1.2.2.2) and keep track of the total number of samples received in each code bin. The transition levels are then given by

$$T[k] = C - A\cos\left[\frac{\pi \cdot H_c[k-1]}{S}\right] \tag{6}$$

where

A	is the amplitude of the sine wave
C	is the offset (dc level) of the applied signal
$H_c[j]$	is $\sum_{i=0}^{j} H[i]$
$H[i]$	is the total number of samples received in code bin i
S	is the total number of samples

If A and C are unknown, they can be determined from the data and an independent estimate of any two of the transition levels. Errors in the values of A and/or C do not induce any errors in the determination of differential or integral nonlinearity from the calculated transition levels because they only induce gain and offset errors in the transition levels as shown in equation (5). These results are derived in [B21].

4.1.2.2.1 Comment on the selection of the frequency and record length

The frequency of the sine wave and the record length of the data collected should be carefully selected in order for the error estimates of the following clause to apply. There should be an exact integer number of cycles in a record, and the number of cycles in a record should be relatively prime to the number of samples in the record. This guarantees that the samples in each record are uniformly distributed in phase from 0 to 2π. If the frequency is low enough that dynamic errors do not arise, this method will give the same results as the static test method. If the frequency is chosen large enough that the dynamic errors are significant, the user should be warned that some dynamic errors will appear in the results, while others will be averaged out by the histogram calculations.

A frequency that meets the above requirements can be selected as follows. Choose the number of cycles per record, M_c, and a record length, M, such that M_c and M have no common factors. Choose the frequency by the following formula:

$$f = \frac{M_c}{M} f_s \tag{7}$$

where

f	is the signal frequency
f_s	is the sampling frequency

In order for the test tolerances in the derivations in 4.1.2.2.2 to be valid, the accuracy of the signal frequency is given by

$$\frac{\Delta f}{f} \le \frac{1}{4(M-1)} \quad \text{if } M_c = 1$$

$$\frac{\Delta f}{f} \le \frac{1}{4(M_c - 1)M} \quad \text{if } M_c > 1 \tag{8}$$

where

Δf is the allowable error in the signal frequency

With larger values of *M,* fewer total samples (the number of records times the number of samples per record) will be required to obtain any given accuracy, but greater accuracy will be required of the signal frequency. The best approach is to use the largest value of *M* compatible with the frequency accuracy obtainable. The frequency accuracy specified by equation (8) guarantees that the phase separation between samples is within ± 25% of the ideal. This tolerance was assumed in the derivation of equation (11).

4.1.2.2.2 Comment on the amount of overdrive and the number of records required

The minimum amount of overdrive required depends on the combined noise level of the signal source and the recorder. In the absence of noise, the overdrive need only be sufficient to receive at least one count in each of the first and last code bins. If noise is present, it will modify the probabilities of samples falling in various code bins, and the effect will be largest near the peaks where the curvature of the probability density is greatest. This effect can be made as small as desired by making the overdrive large enough. The amount of overdrive required to obtain a specified accuracy also depends on whether the specified accuracy is for the code bin widths (i.e., differential nonlinearity) or for the transition levels (i.e., integral nonlinearity).

The overdrive required to obtain a specified tolerance in code bin widths is given by

$$V_O \ge \text{maximum of } (3\ \sigma) \text{ or } \sigma\sqrt{\frac{3}{2B}} \tag{9}$$

where

σ is the rms value of the random noise in input units
B is the desired tolerance as a fraction of the code bin width
V_O is the overdrive voltage, the difference between the positive (negative) peaks of the sine wave, and the most positive (negative) transition level of the recorder

This amount of overdrive guarantees that the error caused by the noise is ≤ 1/3 of the desired tolerance.

The overdrive required to obtain a specified tolerance in transition levels is given by

$$V_O \ge \text{maximum of } (2\sigma) \text{ or } \frac{\sigma^2 2^N}{VB} \tag{10}$$

where

V is the full scale voltage of the instrument in input units
σ is the rms value of the random noise in input units
B is desired tolerance as a fraction of the code bin width
N is the number of bits of the waveform recorder
V_O is the overdrive voltage

The values of overdrive in equations (9) and (10) are adequate to keep the errors due to noise to equal to or less than *B*/3 code bin widths so that these errors are negligible when added to the statistical errors due to taking a finite number of samples.

The number of records required depends on several factors. It depends on the combined noise level of the recorder and the signal source, on the desired test tolerance and confidence level, on whether the tolerance and confidence level is for INL (transition levels) or DNL (code bin widths), and on whether one wants to obtain the desired confidence for a particular width or transition level or for the worst case for all widths or transition levels. The number of records required for a given test tolerance and a given confidence in code bin widths is given by

$$R = J\left[\frac{2^{N-1}K_u}{B}\right]^2\left[\frac{c\pi}{M}\right]\left\{1.1\left[\frac{\sigma^*}{V}\right] + 0.2\left[\frac{c\pi}{M}\right]\right\} \tag{11}$$

where

R	is minimum required number of records
J	is 1 for INL and J = 2 for DNL
M	is the number of samples per record
c	is $1 + 2(V_O/V)$
V	is the full scale voltage of the instrument in volts
V_O	is overdrive voltage
K_u	is $Z_{u/2}$ for obtaining the specified confidence in an individual transition level or code bin width
K_u	is $Z_{N,u/2}$ for obtaining the specified confidence in the worst case transition level or code bin width
u	is $1–v$, with v the desired confidence level expressed as a fraction
σ^*	is σ, the rms random noise level in volts, for INL
σ^*	is the minimum of σ or $Q/1.1$ for DNL (Q = code bin width)
N	is number of bits of the waveform recorder
B	is desired test tolerance as a fraction of the code bin width

The values for $Z_{u/2}$ and $Z_{N,u/2}$ can be obtained from table 2. For values of N between those in the table, use linear interpolation. Expression $Z_{u/2}$ is defined such that the probability is $1–u$ that the absolute value of a Gaussian distributed random variable with a mean of zero and a standard deviation of one is $\leq Z_{u/2}$. Expression $Z_{N,u/2}$ is defined such that the probability is $1–u$ that the maximum of the absolute values of 2^N Gaussian distributed random variables with a mean of zero and a standard deviation of one will be $\leq Z_{N,u/2}$.

Table 2—Values of $Z_{u/2}$ and $Z_{N, u/2}$

u	$Z_{u/2}$	$Z_{4, u/2}$	$Z_{8, u/2}$	$Z_{12, u/2}$	$Z_{16, u/2}$	$Z_{20, u/2}$	$Z_{24, u/2}$
0.2	1.28	2.46	3.33	4.04	4.64	5.19	5.68
0.1	1.64	2.72	3.53	4.21	4.80	5.33	5.81
0.05	1.96	2.95	3.72	4.37	4.94	5.46	5.93
0.02	2.33	3.22	3.95	4.57	5.12	5.62	6.08
0.01	2.58	3.42	4.11	4.71	5.25	5.74	6.19
0.005	2.81	3.60	4.27	4.85	5.38	5.85	6.30
0.002	3.09	3.84	4.47	5.03	5.54	6.01	6.44
0.001	3.29	4.00	4.62	5.16	5.66	6.12	6.54

Equations (9) through (11) and the values in table 2 are derived in [B2]. For further information, see [B15].

4.1.3 Fitting sine waves to recorded sine wave data

Apply a sine wave with specified parameters to the input of the recorder. Take a record of data. Fit a sine wave function to the record by varying the phase, amplitude, DC value, and (if needed) frequency of the fit function to minimize the sum of the squared difference between the data and the function. Two suggested approaches for performing the least squares fit are given below—one for known frequency solutions, which can be used when the sample clock and input frequencies are both known and stable, and one for unknown frequency solutions for all other cases. Two algorithms are included for each approach—one that requires matrix operations, and another that does not. For the known frequency approach, the two algorithms give identical results. For the approach used when the frequency is unknown, the two algorithms give the same results when the initial conditions are the same, although the convergence properties of the two differ. The matrix algorithm converges faster than the non-matrix algorithm, especially for records that contain five or fewer periods of the sine wave. For this reason, the matrix algorithm is recommended for use when the host computer supports matrix operations.

NOTE—The three parameter algorithms of 4.1.3.1 (for known frequency) are closed form solutions that will always produce an answer, but the answer will be a poorer fit than the four parameter algorithms of 4.1.3.2 if the actual frequency applied to the recorder differs from the frequency used by the algorithm. The algorithms of 4.1.3.2, on the other hand, employ an iterative solution that may diverge for bad initial estimates, or especially corrupted data. The method used should be stated.

4.1.3.1 An algorithm for three parameter (known frequency) least squares fit to sine wave data using matrix operations

Assuming the data record contains the sequence of samples y_1, y_2,...y_m, taken at times t_1, t_2,... t_m, this algorithm finds the values of A_0, B_0, and C_0 that minimize the following sum of squared differences:

$$\sum_{n=1}^{M}[y_n - A_0\cos(\omega_0 t_n) - B_0\sin(\omega_0 t_n) - C_0]^2 \tag{12}$$

where

ω_0 is the frequency applied to the waveform recorder input

To find the values for A_0, B_0, and C_0, first create the following matrices:

$$D_0 = \begin{bmatrix} \cos(\omega_0 t_1) & \sin(\omega_0 t_1) & 1 \\ \cos(\omega_0 t_2) & \sin(\omega_0 t_2) & 1 \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \cos(\omega_0 t_M) & \sin(\omega_0 t_M) & 1 \end{bmatrix} \tag{13}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ \bullet \\ y_M \end{bmatrix} \tag{14}$$

$$x_0 = \begin{bmatrix} A_0 \\ B_0 \\ C_0 \end{bmatrix} \tag{15}$$

In matrix notation, the sum of squared differences in equation (12) is given by

$$(y - D_0 x_0)^T (y - D_0 x_0) \tag{16}$$

where

$(*)^T$ designates the transpose of (*)

Compute the least-squares solution, $\hat{x}_0$, that minimizes equation (16) using

$$\hat{x}_0 = (D_0^T D_0)^{-1} (D_0^T y) \tag{17}$$

The fitted function is then given by

$$y_n' = A_0 \cos(\omega_0 t_n) + B_0 \sin(\omega_0 t_n) + C_0 \tag{18}$$

To convert the amplitude and phase to the form

$$y_n' = A \cos(\omega_0 t_n + \theta) + C \tag{19}$$

use

$$A = \sqrt{A_0^2 + B_0^2} \tag{20}$$

$$\theta = \tan^{-1}\left[\frac{-B_0}{A_0}\right] \qquad \text{if } A_0 \geq 0 \tag{21}$$

$$\theta = \tan^{-1}\left[\frac{-B^0}{A^0}\right] + \pi \qquad \text{if } A_0 < 0 \tag{22}$$

The residuals, r_n, of the fit are given by

$$r_n = y_n - A_0 \cos(\omega_0 t_n) - B_0 \sin(\omega_0 t_n) - C_0 \tag{23}$$

and the rms error is given by

$$\varepsilon_{\text{rms}} = \sqrt{\frac{1}{M} \sum_{n=1}^{M} r_n^2} \tag{24}$$

4.1.3.2 A non-matrix algorithm for three parameter (known frequency) least squared fit to sine wave data

Assuming the data record contains the sequence of samples y_n taken at times t_n, the following nine sums are computed using the angular frequency applied to the waveform recorder input as ω:

Define

$$\alpha_n = \cos(\omega t_n) \tag{25}$$

$$\beta_n = \sin(\omega t_n) \tag{26}$$

Then calculate

$$\sum_{n=1}^{M} y_n \qquad \sum_{n=1}^{M} \alpha_n \qquad \sum_{n=1}^{M} \beta_n$$

$$\sum_{n=1}^{M} \alpha_n \beta_n \qquad \sum_{n=1}^{M} \alpha_n^2 \qquad \sum_{n=1}^{M} \beta_n^2$$

$$\sum_{n=1}^{M} y_n \alpha_n \qquad \sum_{n=1}^{M} y_n \beta_n \qquad \sum_{n=1}^{M} y_n^2$$

Using these sums, compute

$$A_1 = \frac{A_N}{A_D} \tag{27}$$

where

$$A_N = \frac{\sum_{n=1}^{M} y_n \alpha_n - \bar{y} \sum_{n=1}^{M} \alpha_n}{\sum_{n=1}^{M} \alpha_n \beta_n - \bar{\beta} \sum_{n=1}^{M} \alpha_n} - \frac{\sum_{n=1}^{M} y_n \beta_n - \bar{y} \sum_{n=1}^{M} \beta_n}{\sum_{n=1}^{M} \beta_n^2 - \bar{\beta} \sum_{n=1}^{M} \beta_n} \tag{28}$$

$$A_D = \frac{\sum_{n=1}^{M} \alpha_n^2 - \bar{\alpha} \sum_{n=1}^{M} \alpha_n}{\sum_{n=1}^{M} \alpha_n \beta_n - \bar{\beta} \sum_{n=1}^{M} \alpha_n} - \frac{\sum_{n=1}^{M} \alpha_n \beta_n - \bar{\alpha} \sum_{n=1}^{M} \beta_n}{\sum_{n=1}^{M} \beta_n^2 - \bar{\beta} \sum_{n=1}^{M} \beta_n} \tag{29}$$

$$B_1 = \frac{B_N}{B_D} \tag{30}$$

where

$$B_N = \frac{\sum_{n=1}^{M} y_n\alpha_n - \bar{y}\sum_{n=1}^{M}\alpha_n}{\sum_{n=1}^{M}\alpha_n^2 - \bar{\alpha}\sum_{n=1}^{M}\alpha_n} - \frac{\sum_{n=1}^{M} y_n\beta_n - \bar{y}\sum_{n=1}^{M}\beta_n}{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\alpha}\sum_{n=1}^{M}\beta_n} \tag{31}$$

$$B_D = \frac{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\beta}\sum_{n=1}^{M}\alpha_n}{\sum_{n=1}^{M}\alpha_n^2 - \bar{\alpha}\sum_{n=1}^{M}\alpha_n} - \frac{\sum_{n=1}^{M}\beta_n^2 - \bar{\beta}\sum_{n=1}^{M}\beta_n}{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\alpha}\sum_{n=1}^{M}\beta_n} \tag{32}$$

$$C = \bar{y} - A_1\bar{\alpha} - B_1\bar{\beta} \tag{33}$$

where

$$\bar{y} = \frac{1}{M}\sum_{n=1}^{M} y_n \tag{34}$$

$$\bar{\alpha} = \frac{1}{M}\sum_{n=1}^{M} \alpha_n \tag{35}$$

$$\bar{\beta} = \frac{1}{M}\sum_{n=1}^{M} \beta_n \tag{36}$$

This produces a fitted function of the form

$$y_n' = A_1\cos(\omega t_n) + B_1\sin(\omega t_n) + C \tag{37}$$

where

y_n' is fitted points

Now the rms error can be computed as follows:

$$\varepsilon_{\mathrm{rms}} = \sqrt{\frac{\varepsilon}{M}} \tag{38}$$

where

$$\varepsilon = \sum_{n=1}^{M} y_n^2 + A_1^2 \sum_{n=1}^{M} \alpha_n^2 + B_1^2 \sum_{n=1}^{M} \beta_n^2 + MC^2$$

$$- 2A_1 \sum_{n=1}^{M} \alpha_n y_n - 2B_1 \sum_{n=1}^{M} \beta_n y_n - 2C \sum_{n=1}^{M} y_n$$

$$+ 2A_1 B_1 \sum_{n=1}^{M} \alpha_n \beta_n + 2A_1 C \sum_{n=1}^{M} \alpha_n + 2B_1 C \sum_{n=1}^{M} \beta_n \tag{39}$$

To convert the amplitude and phase to the form

$$y'_n = A\cos(\omega t_n + \theta) + C \tag{40}$$

use

$$A = \sqrt{(A_1^2 + B_1^2)} \tag{41}$$

$$\theta = \tan^{-1}\left(-\frac{B_1}{A_1}\right) \qquad \text{if } A_1 \geq 0$$

$$\theta = \tan^{-1}\left(-\frac{B_1}{A_1}\right) + \pi \qquad \text{if } A_1 < 0 \tag{42}$$

4.1.3.3 An algorithm for four parameter (general use) least squares fit to sine wave data using matrix operations

Assuming the data record contains the sequence of samples y_1, y_2,...y_M taken at times t_1, t_2,...t_M, this algorithm uses an iterative process to find the values of A_i, B_i, C_i and ω_i, that minimize the following sum of squared differences:

$$\sum_{n=1}^{M} \left[y_n - A_i\cos(\omega_i t_n) - B_i\sin(\omega_i t_n) - C_i\right]^2 \tag{43}$$

where

ω_i is the frequency applied to the waveform recorder input

Perform the following steps:

a) Set index $i = 0$. Make an initial estimate of the angular frequency ω_0 of the recorded data. The frequency may be estimated by using a Discrete Fourier Transform (DFT) (either on the full record or a portion of it), or by counting zero crossings, or simply by using the applied input frequency. Perform a prefit using the three-parameter matrix algorithm given in 4.1.3.1 to determine A_0, B_0, and C_0.

b) Set $i = i + 1$ for the next iteration.

c) Update the angular frequency estimate using:

$$\omega_i = \omega_{i-1} + \Delta\omega_{i-1} \qquad (\Delta\omega_{i-1} = 0 \text{ for } i = 1) \tag{44}$$

d) Create the following matrices:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \bullet \\ \bullet \\ y_M \end{bmatrix} \tag{45}$$

$$D_\mathrm{i} = \begin{bmatrix} \cos(\omega_i t_1) & \sin(\omega_i t_1) & 1 & -A_{i-1} t_1 \sin(\omega_\mathrm{i} t_1) + B_{i-1} t_1 \cos(\omega_i t_1) \\ \cos(\omega_i t_2) & \sin(\omega_i t_2) & 1 & -A_{i-1} t_2 \sin(\omega_\mathrm{i} t_2) + B_{i-1} t_2 \cos(\omega_i t_2) \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \cos(\omega_i t_M) & \sin(\omega_i t_M) & 1 & -A_{i-1} t_M \sin(\omega_i t_M) + B_{i-1} t_1 \cos(\omega_i t_M) \end{bmatrix} \tag{46}$$

$$x_\mathrm{i} = \begin{bmatrix} A_i \\ B_i \\ C_i \\ \Delta\omega_i \end{bmatrix} \tag{47}$$

e) Compute the least-squares solution, $\hat{x}$, that minimizes equation (43) using

$$\hat{x}_i = (D_i^T D_i)^{-1} (D_i^T y) \tag{48}$$

f) Compute the amplitude, A, and phase, θ, for the form

$$y_n' = A\cos(\omega t_n + \theta) + C \tag{49}$$

using

$$A = \sqrt{A_i^2 + B_i^2} \tag{50}$$

and

$$\theta = \tan^{-1}\left[-\frac{B_\mathrm{i}}{A_\mathrm{i}}\right] \text{ if } A_i \geq 0 \tag{51}$$

$$\theta = \tan^{-1}\left[-\frac{B_i}{A_i}\right] + \pi \quad \text{if } A_i < 0 \tag{52}$$

g) Repeat steps b) through f), recomputing the model based on the new values of A_i, B_i, and ω_i, calculated from the previous iteration. Continue to iterate until the changes in A, ω, θ, and C are suitably small.

The residuals, r_n, of the fit are given by

$$r_n = y_n - A_i\cos(\omega_i t_n) - B_i\sin(\omega_i t_n) - C_i \tag{53}$$

and the rms error is given by

$$\varepsilon_{\mathrm{rms}} = \sqrt{\frac{1}{M}\sum_{n=1}^{M} r_n^2} \tag{54}$$

4.1.3.4 A non-matrix algorithm for four parameter (general use) least squared fit to sine wave data

Make estimates of frequency ω and phase θ of the recorded data, where phase is referred to the first point in the record.

The frequency expressed in radians per second, may be estimated by using a DFT (either on the full record or a portion of it), or by counting zero crossings. Alternatively, the frequency can be estimated by using the applied input frequency.

The phase expressed in radians, may be estimated by closed form calculation using the algorithm of 4.1.3.1 given a frequency estimate. Another method is to use a linear least squares fit to determine the phase as the intercept of the relation between the record's zero-crossing times and consecutive integer multiples of π. A two-point estimate of phase can be arrived at by evaluating

$$\left[\mathrm{sign}(y_2 - y_1)\right] \cos^{-1}\left(\frac{y_1 - C}{A}\right) \tag{55}$$

where

sign $(y_2\text{-}y_1) = 1$ for $y_2 \geq y_1$, or -1 for $y_2 < y_1$

y_1 is the first sample in record where $t = 0$

y_2 is the sample immediately following y_1

C, A is sine wave offset and amplitude, respectively

To make amplitude estimates for this purpose, half of the algebraic difference between the maximum and minimum values in the data record can be used if the recorder is free from large random errors. Alternatively, a histogram representing the probability density function of the digital codes can be employed. To make offset estimates for this purpose, half the sum of the maximum and minimum value of the record should be taken. Another means for offset estimation is to take the average value of the data points over an integer number of cycles (or over the entire record if it contains many cycles). Note that sign $(y_2 - y_1)$ can give an erroneous sign if there are too few points per cycle, particularly for values of $\cos^{-1}$ close to 0 or π.

For more information on making initial estimates, see [B10].

Assuming the data record contains the sequence of samples y_n taken at times t_n, and using the estimates for w and θ, compute the following 16 sums:

$$\sum_{n=1}^{M} \alpha_n \qquad \sum_{n=1}^{M} \beta_n \qquad \sum_{n=1}^{M} y_n \qquad \sum_{n=1}^{M} \alpha_n y_n$$

$$\sum_{n=1}^{M} \beta_n y_n \qquad \sum_{n=1}^{M} \alpha_n \beta_n \qquad \sum_{n=1}^{M} \beta_n^2 \qquad \sum_{n=1}^{M} \alpha_n^2$$

$$\sum_{n=1}^{M} y_n^2 \qquad \sum_{n=1}^{M} \beta_n t_n y_n \qquad \sum_{n=1}^{M} \alpha_n \beta_n t_n \qquad \sum_{n=1}^{M} \beta_n^2 t_n$$

$$\sum_{n=1}^{M} \beta_n^2 t_n^2 \qquad \sum_{n=1}^{M} \beta_n t_n \qquad \sum_{n=1}^{M} \alpha_n t_n \qquad \sum_{n=1}^{M} \beta_n t_n^2$$

where

$$\alpha_n = \cos(\omega t_n + \theta) \tag{56}$$

$$\beta_n = \sin(\omega t_n + \theta) \tag{57}$$

Now using the estimates for ω and θ, compute

$$\psi = \omega + \frac{a_{22}R - a_{12}S}{a_{11}a_{22} - a_{12}a_{21}} \tag{58}$$

$$\phi = \theta + \frac{a_{11}S - a_{21}R}{a_{11}a_{22} - a_{12}a_{21}} \tag{59}$$

where

$$a_{11} = \frac{\sum_{n=1}^{M} \beta_n t_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \alpha_n t_n(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2}$$

$$-\frac{\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \beta_n t_n^2(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2} \tag{60}$$

$$a_{12} = \frac{\sum_{n=1}^{M} \beta_n t_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \alpha_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha})\right]^2}$$

$$- \frac{\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \beta_n t_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha})\right]^2} \tag{61}$$

$$a_{21} = \frac{\sum_{n=1}^{M} \beta_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \alpha_n t_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha})\right]^2}$$

$$- \frac{\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \beta_n t_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha})\right]^2} \tag{62}$$

$$a_{22} = \frac{\sum_{n=1}^{M} \beta_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \alpha_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha})\right]^2}$$

$$- \frac{\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \beta_n (\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n (\alpha_n - \bar{\alpha})\right]^2} \tag{63}$$

$$R = \frac{\sum_{n=1}^{M} (y_n - \bar{y}) t_n \beta_n}{\sum_{n=1}^{M} (y_n - \bar{y}) \alpha_n} - \frac{\sum_{n=1}^{M} (\alpha_n - \bar{\alpha}) t_n \beta_n}{\sum_{n=1}^{M} (\alpha_n - \bar{\alpha}) \alpha_n} \tag{64}$$

$$S = \frac{\sum_{n=1}^{M}(y_n - \bar{y})\beta_n}{\sum_{n=1}^{M}(y_n - \bar{y})\alpha_n} - \frac{\sum_{n=1}^{M}(\alpha_n - \bar{\alpha})\beta_n}{\sum_{n=1}^{M}(\alpha_n - \bar{\alpha})\alpha_n} \tag{65}$$

$$\bar{y} = \frac{1}{M}\sum_{n=1}^{M} y_n \tag{66}$$

$$\bar{\alpha} = \frac{1}{M}\sum_{n=1}^{M} \alpha_n \tag{67}$$

$$\bar{\beta} = \frac{1}{M}\sum_{n=1}^{M} \beta_n \tag{68}$$

$$\alpha_n = \cos(\omega t_n + \theta) \tag{69}$$

$$\beta_n = \sin(\omega t_n + \theta) \tag{70}$$

Repeat this procedure, using ψ and ϕ as new estimates for ω and θ, until the difference between the two is suitably small. This produces a fitted function of the form

$$y_n' = A\cos(\psi t_n + \phi) + C \tag{71}$$

where

A is the fitted amplitude
C is the fitted offset

and may be computed from

$$A = \frac{\sum_{n=1}^{M}(y_n - \bar{y})(\alpha_n + \beta_n + \beta_n t_n)}{\sum_{n=1}^{M}(\alpha_n - \bar{\alpha})(\alpha_n + \beta_n + \beta_n t_n)} \tag{72}$$

$$C = \bar{y} - A\bar{\alpha} \tag{73}$$

The actual rms error is $\left(\frac{\varepsilon}{M}\right)^{\frac{1}{2}}$

where

$$\frac{\varepsilon}{M} = \frac{1}{M}\sum_{n=1}^{M} y_n^2 + \frac{A^2}{M}\sum_{n=1}^{M} \alpha_n^2 - 2\frac{A}{M}\sum_{n=1}^{M} \alpha_n y_n + C^2 - 2C\bar{y} + 2AC\bar{\alpha} \tag{74}$$

Since this is an iterative process, it can diverge for initial estimates of ω and θ that are grossly incorrect.

4.1.3.5 Comment on choice of input frequency, significance of record size and number of cycles per record

The precise selection of the frequency used is very important. There are frequencies for which there is a potential for hiding errors, and there are frequencies that provide particularly good error coverage. The best and worst frequency can differ by only a fraction of a percent. Input frequency selection becomes more important as the input frequency becomes higher with respect to the sampling frequency.

An optimum frequency is one for which there are *M* distinct phases sampled which are uniformly distributed between 0 and 2π radians, where *M* is the number of samples in a record. It can be shown that the optimum frequencies are given by

$$f_{opt} = \left(\frac{J}{M}\right) f_s \tag{75}$$

where

J is an integer which is relatively prime to *M*
f_S is the sampling frequency

Note that the condition of being relatively prime means that *M* and *J* have no common factors and that for the optimum frequency there are exactly *J* cycles in a record. If *M* is a power of two, then any odd value for *J* meets this condition.

As an example, a frequency, *f,* which gives poor error coverage, is

$$f = \frac{f_s}{L} \tag{76}$$

where

L is a small integer

In this case there are only *L* distinct phases sampled.

Sometimes the optimum frequencies are too far apart to adequately approximate the desired frequency. The following is a simple approach for calculating a near-optimum frequency that is close to any desired frequency.

a) Find an integer, *n,* such that the desired frequency is approximately f_s/n
b) Let $D = \text{int}(M/n)$ = the number of full cycles that can be recorded at this frequency
c) Let $f^* = Df_s/(nD–1)$ = the near-optimum frequency

Using this frequency guarantees $nD–1$ distinct samples with phases uniformly distributed between 0 and 2π radians.

For example, if the sampling frequency is 1G sample/s, the desired input frequency is 10 MHz and the record length is 1024, then $n = 100$, $D = 10$ and $f^* = [10/((100)(10) - 1)][1000] = 10.01001$ MHz. This frequency guarantees 999 distinct uniformly distributed phases and differs from the desired frequency by 0.1%. The nearest optimum frequency is (11/1024)1000 = 10.742MHz. This differs from the desired frequency by 7.4% but guarantees 1024 distinct uniformly distributed phases. The desired frequency yields only 100 distinct phases because its period is exactly 100 samples.

For an ideal transfer characteristic in the absence of random noise, the minimum record size that will ensure a representative sample of every code bin is $2\pi 2^N$ when the input frequency is chosen as above.

The number of cycles of the sine wave contained in the record also affects the estimates obtained in the curve fit. This is especially true for the four parameter sine fit algorithms, in which frequency is not fixed. As the number of cycles in the record increases, the estimated frequency becomes more and more tightly constrained by the data since the number of zero crossings increases. Therefore, for records containing large numbers of cycles, the sine fit will return frequency estimates that are very close to the fundamental frequency of the input signal. However, for records containing only a few cycles, e.g., < 5, the estimated frequency is not strongly constrained by the fundamental frequency and therefore can also change to accommodate error components due to noise, harmonics, or jitter. The result is a fit that returns incorrect estimates, i.e., that do not exactly correspond to the parameters of the fundamental component of the signal, but nevertheless gives smaller mean squared error, and thus smaller residuals. Estimates of all four parameters—frequency, phase, amplitude and offset—are affected. The differences in the estimates depend on the amount of noise, harmonics, and jitter present, as well as on the accuracy of the initial frequency estimate; if the signal is essentially free of contamination, the estimates will be quite good even for records containing only one cycle. A good rule of thumb is to use records containing at least five cycles. If the harmonics and noise are each less than $5Q$ (where Q is the ideal code bin width of the recorder), then the estimates will have sufficient accuracy for most applications.

4.1.4 Step response

The recorded output response for an ideal input step with designated baseline and topline. If unspecified, the baseline of the input step is 10% and the topline is 90% of full-scale.

4.1.4.1 Test method

Apply a suitable input step and take a record sufficiently long to include all features of interest, e.g., precursors, and electrical and thermal settling. A suitable step is one that has transition duration, overshoot, and settling time no greater than one fourth of those expected from the recorder under test.

4.1.5 Equivalent time sampling

The sampling rate of a real time recorder limits the measurement bandwidth. If the sampling rate is not at least twice the frequency of the highest frequency component of the input signal, then aliasing errors can result. If the input signal is repetitive, these limitations can be reduced by equivalent time sampling. Several methods of equivalent time sampling exist. The method described here consists of extracting equivalent time samples from a single record using the recorder's internal time base, provided that the input signal's repetition rate is selected appropriately.

4.1.5.1 Extraction method

If the input signal is repetitive, sampling rate limitations can be reduced by using the principle of equivalent time sampling to multiply the real-time sampling rate of the waveform recorder by an integer, D. By choosing the repetition rate of the input signal (f_r) appropriately, at least D periods of the input waveform are recorded in a single record; then, upon rearranging the samples with a simple algorithm, a single repetition of the input signal is obtained which is effectively sampled at D times the real time sampling rate. This is illustrated in figure 2 for $D = 4$. To implement this method, choose integer D based on the required equivalent sample rate, f_{eq}, such that $f_{eq} = Df_S$, where f_s is the real-time sampling rate of the waveform recorder. Next, L, the number of real-time samples taken during each repetition of the input waveform, is given by $L = \mathrm{INT}\ (M/D)$, i.e., the integer part of M/D, where M is the number of samples in a record. Finally, the input signal's repetition rate, f_r , is set such that

$$f_r = f_s \frac{D}{LD - 1}, \text{with } LD \leq M \tag{77}$$

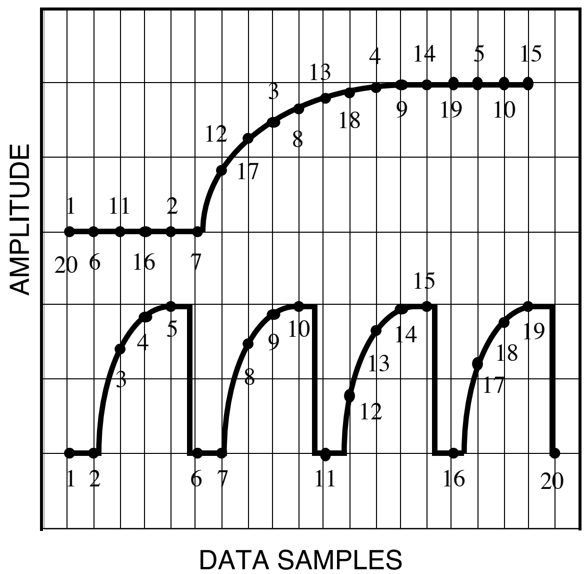

Figure 2—Equivalent time sampling extraction method

For the example of figure 2, $D = 4$, $L = 5$, M could be 20, 21, 22, 23, or 24, and $f_r = 0.2105263f_s$. (Note that data points from LD through M are redundant.)

The following BASIC program implements the algorithm to rearrange the samples in equivalent time:

```
10    F = 0
20    FOR I = 1 TO L
30    FOR J = 1 TO D
40    F = F + 1
50    I2 = I + (J-1) * L
60    E(F) = R(I2)
70    NEXT J
80    NEXT I
```

where

D	is the sample rate multiplier
M	is the record length
L	is INT(M/D) where INT(*) designates the integer part of *
E(*)	is the array containing (L · D) equivalent-time samples
F	is the equivalent-time sampling index
R(*)	is the array containing real-time samples
I2	is the real-time sampling index

4.1.5.2 Comment on the extraction method for equivalent time sampling

This method of achieving higher equivalent sampling rates requires that the repetition rate, f_r , of the input signal be precisely controlled. While the average equivalent-time sample rate is just Df_s, independent of f_r, the relative spacing of the equivalent-time samples becomes non-uniform when f_r deviates from the value given by equation (77). If f_r is too great, D–1 out of D successive samples will occur too late while one sample will be correctly placed; if f_r is too small, D–1 samples will occur too soon. In either case, the maximum sampling time error (Δt_{eq}) is given, to a good approximation, by

$$\Delta t_{eq} \approx \frac{M(D-1)}{Df_{\mathrm{s}}} \frac{\Delta t_r}{t_r} \tag{78}$$

where

t_{eq} is the average equivalent-time sampling, i.e., $1/(D \cdot f_s)$
Δt_{eq} is the maximum sampling time error
$\Delta t_r/t_r$ is the proportional error in the repetition period (or repetition rate)

Note, however, that the errors are not cumulative; the average equivalent-time sampling period is still given by $1/Df_s$.

Of course, the assumption is made in equation (78) that f_s is exactly known; if it is not exactly known, then the additional error given by an expression similar to equation (78) will occur. As an example, if $D = 4$, $M = 1024$, and the equivalent sampling period must be known to 5%, then the repetition rate must be set, and the sampling rate must be known, each with an accuracy of $0.05/(1024 \cdot 3) = 16$ ppm. To achieve such accuracy, it is usually necessary to use a frequency synthesized source. It may sometimes be necessary to measure the frequency of the input signal as well as the frequency of the waveform recorder's internal clock with an accurate frequency counter to assure that they are set with sufficient accuracy. If sufficient accuracy cannot be guaranteed for a specific record length, the accuracy can be improved by decreasing the record length. However, since the lowest frequency component that is represented in a record of length M is given by f_{eq}/M, this limits the range of frequencies that can be represented.

4.1.6 DFT and windowing

Several test methods in this standard call for taking the DFT of a data record and analyzing it. These records are often of sine wave signals, which may have harmonics or other spurious signals. Because of the periodic nature of the DFT, when the data record does not contain an integral number of cycles, the DFT will contain frequency components caused by an abrupt jump between the last sample and first sample in the record. When the record is known to contain a pure sine wave with a non-integral number of cycles, its DFT will not be a single line. Instead it will contain multiple lines in the region of the true line. This effect is often called leakage. See [B7], [B12], [B19], and [B13].

The most common method for reducing leakage is to multiply the data samples in the record by a window function that de-emphasizes the samples towards the ends of the record. A suitable window function is the Hann window shown in the following:

$$hn(i) = \frac{1}{2} - \frac{1}{2}\cos\left[\frac{2\pi(i-1)}{M}\right] \tag{79}$$

where

i varies from 1 to M, and the record contains M samples

A feature of the Hann window is that the function and its first derivative are zero at the end points of the record. This means that it reduces frequency side lobes by at least 18 dB per octave away from the signal. Other window functions may be used (see [B7], [B12], and [B6]).

4.2 Input impedance

The input impedance is the impedance between the signal input of the waveform recorder and ground. The input impedance should be specified at various frequencies. When the frequency is not specified, the input impedance given is the static value (dc resistance). Alternatively, the input impedance can be represented as the parallel combination of a resistance and a capacitance.

4.2.1 Test method

Connect a vector impedance meter of desired accuracy and appropriate output level to the signal input of the waveform recorder and take readings with the input range of the recorder set to values of interest. Sweep the vector impedance meter over the frequency range of interest.

4.2.2 Alternative test method

A time domain reflectometer (TDR) may be used. When the reason for measuring the input impedance is to determine how well the waveform recorder is matched to the rest of the system, use of a TDR may be more appropriate. A TDR gives direct dynamic measurement of a cable system or instrument reflection coefficient ρ. The reflection coefficient can easily be converted to impedance VSWR and power loss factors using simple equations or nomographs. A TDR can be used to measure dc resistance, series inductance, and parallel capacitance. The method is used primarily for 50 Ω systems, but can also be used for other low-impedance systems.

Connect a TDR of appropriate output level to the input of the waveform recorder using a precision 50 Ω cable. The first step is to measure the dc resistance. Adjust the horizontal display sweep rate of the TDR to a window long enough to show where the terminating reflection has settled to its final value. Set the vertical sensitivity high enough to measure any difference in step heights between the test cable and the recorder. Note the position on the screen of the settled reflection. Now repeat the measurement using a precision 50 Ω terminating resistor. The waveform recorder input resistance may be computed from the difference in settled reflection coefficients ρ from equation (80).

$$Z_\tau = Z_0 \frac{1+\rho}{1-\rho} \tag{80}$$

where

Z_τ is the recorder impedance

Z_0 is the input transmission line impedance, nominally 50 Ω

Should the user be interested in small reactive components of the input impedance, a positive reflection pulse corresponds to series inductance, and a negative pulse corresponds to parallel capacitance. For details, see [B20].

4.3 Gain and offset

4.3.1 Gain and offset (independently based)

Gain and offset are the values by which the input values are multiplied and then to which the input values are added, respectively, to minimize the mean squared deviation from the output values. Unless otherwise specified in this standard, gain and offset will be taken to mean independently based gain and offset.

4.3.1.1 Static test method

Locate the code transition levels per 4.1.2. The transfer characteristic can then be represented by

$$G \cdot T[k] + V_{os} + \varepsilon[k] = Q \cdot (k-1) + T_1 \tag{81}$$

where

- $T[k]$ is the input quantity corresponding to the code transition level between codes k and k–1
- T_1 is the ideal code transition level corresponding to $k = 1$
- V_{os} is the output offset in units of the input quantity, nominally = 0
- G is gain, nominally = 1
- Q is the ideal width of a code bin, that is, the full-scale input range divided by the total number of code states
- $\varepsilon[k]$ is the residual error

NOTE—The expression on the right-hand side gives the ideal code transition levels, in input units, as a function of *k,* assumed to be the value of the binary coded output. For waveform recorders that output the data already formatted in input units, the data must be recorded in binary form before using the equation.

Using conventional linear least squares estimation techniques, independently based offset and gain are the values of V_{os} and G that minimize the mean squared value of $\varepsilon[k]$, over all k. The value of G that minimizes ε is given by

$$G = Q\frac{(2^N-1)\sum_{k=1}^{2^N-1} kT[k]}{(2^N-1)\sum_{k=1}^{2^N-1} T^2[k] - \left(\sum_{k=1}^{2^N-1} T[k]\right)^2}$$

$$-Q\frac{(2^N-1)(2^{(N-1)})\sum_{k=1}^{2^N-1} T[k]}{(2^N-1)\sum_{k=1}^{2^N-1} T^2[k] - \left(\sum_{k=1}^{2^N-1} T[k]\right)^2} \tag{82}$$

The value of V_{os} that minimizes ε is

$$V_{os} = T_1 + Q(2^{N-1} - 1) - \frac{G}{2^N - 1} \sum_{k=1}^{2^N - 1} T[k] \tag{83}$$

Given these values for G and V_{os}, $\varepsilon[k]$ is the independently based integral nonlinearity (see 4.4.2).

4.3.1.2 Dynamic test method

Apply a sine wave of specified frequency to the waveform recorder input whose amplitude and dc component are known to the precision desired for the gain and offset. A large signal amplitude is suggested. Take a record of data.

Fit a sine wave to the record per 4.1.3. The ratio of fitted function amplitude to the input signal amplitude is the gain. Multiply the dc value of the input signal by the gain so as to determine the ideal output dc value. Subtract this quantity from the dc value of the fitted function so as to determine the offset.

4.3.1.3 Comment on number of samples required

The record should be long enough to allow the fitting algorithm to find a solution. In particular, records that are significantly shorter than the inverse of the input frequency may fail to converge on a solution.

4.3.2 Gain and offset (terminal based)

Gain and offset are the values by which the input values are multiplied and then to which the input values are added, respectively, to cause the deviations from the output values to be zero at the terminal points, that is, at the first and last codes.

4.3.2.1 Test method

Locate the code transition levels per 4.1.2. The transfer characteristic can be represented by equation (81). Terminal based gain and offset are the values of G and V_{os} that cause $\varepsilon[1] = 0$ and $\varepsilon[2^N-1] = 0$, where N is the number of digitized bits and 2^N-1 is the highest code transition defined.

4.4 Linearity, harmonic distortion, and spurious response

4.4.1 Differential nonlinearity

This is the difference between a specified code bin width and the average code bin width, divided by the average code bin width. When given as one number without a code bin specification, it is the maximum differential nonlinearity of the entire range. In equation form, it is expressed as

$$DNL[k] = \frac{W[k] - Q'}{Q'} \tag{84}$$

$$DNL = \max\left|\frac{W[k] - Q'}{Q'}\right| \tag{85}$$

where

$W[k]$ is the specific code bin width (of code k), and
Q' is the average code bin width = $(T[2^N-1] - T[1])/(2^N - 2)$

Perfect differential linearity is $DNL = 0$.

Note that neither the width of the top bin, $W[2^N-1]$, nor the width of the bottom bin, $W[0]$, is defined.

4.4.1.1 Test method

Locate the code transition levels by any of the methods of clause 4.1.2. A set of code bin widths may then be calculated from

$$W[k] = T[k+1] - T[k] \tag{86}$$

Differential nonlinearity may then be calculated by equation (85).

4.4.2 Integral nonlinearity (ILN)

This is the maximum difference between the ideal and actual code transition levels after correcting for gain and offset. Integral nonlinearity is usually expressed as a percentage of full scale. It will be independently based or terminal based depending on how gain and offset are defined. Integral nonlinearity is defined here to be a static parameter. For dynamic effects, see harmonic distortion in 4.4.4.

4.4.2.1 Test method

Locate the code transition levels by any of the methods in 4.1.2, then determine gain and offset per 4.3.1.1 or 4.3.2.1 as appropriate. The integral nonlinearity is given in percent by

$$\mathrm{INL} = 100\frac{\max|\varepsilon[k]|}{Q \cdot 2^N} \tag{87}$$

Note that if code transitions are determined by a histogram method, and the signal parameters are inaccurately known, then the gain and offset determined here will be in error. However, the error in gain and offset will not affect the calculated INL.

4.4.3 Maximum static error

This is the maximum difference between any code transition level and its ideal value. It is usually expressed as a percentage of full scale.

4.4.3.1 Test method

Locate the code transition levels per 4.1.2. The maximum static error (MSE) is given in percent by

$$\mathrm{MSE} = 100\frac{\max|T[k] - Q \cdot (k-1) - T_1|}{Q \cdot 2^N} \tag{88}$$

where

$T[k]$ is the code transition level for the kth transition (between codes k and $k-1$)
Q is the ideal width of a code bin
T_1 is the ideal input value corresponding to the code transition level $T[1]$
N is the number of digitized bits

4.4.4 Harmonic distortion and spurious components

For a pure sine wave input, the harmonic distortion is the output components at frequencies that are an integer multiple of the applied sine wave frequency. Their amplitudes are generally given as a decibel (dB) ratio with respect to the amplitude of the applied sine wave. Their frequencies are usually expressed as a multiple of the frequency of the applied sine wave. Total harmonic distortion is the root sum of squares of all harmonic distortion components including their aliases. Spurious components are persistent sine waves at frequencies other than those described above as harmonic components. Usually, their amplitudes are expressed as a dB ratio with respect to a full-scale signal.

Harmonic distortion and some spurious components depend on the amplitude and frequency of the applied sine wave. The amplitude and frequency at which the measurements were made shall be specified.

4.4.4.1 Test methods

The coherent sampling test method and the incoherent sampling test method apply a sine wave of specified frequency and amplitude to the input of the waveform recorder. The purity of the sine wave should be consistent with the desired accuracy of the measurement result. If necessary, filter the sine wave to achieve the desired spectral purity. Both methods also require the computation of the DFT (see page 532 in [B9] and page 24 in [B18]) of a record x_n of length M.

$$X_m = \left| \sum_{i=0}^{M-1} x_i e^{-j2\pi i(m/M)} \right| \tag{89}$$

$$m = 0, 1, ..., M-1$$

The indices $m = 0, 1, ..., M-1$ are called DFT frequency bins or bin numbers. Calculate the frequency sequence, f_m, for the DFT using

$$f_m = \begin{cases} \dfrac{mf_s}{M} & m = 0, 1, \ldots, \dfrac{M}{2} \\ -f_s + \dfrac{mf_s}{M} & m = \dfrac{M}{2}+1, \dfrac{M}{2}+2, \ldots, M-1 \end{cases} \tag{90}$$

where

f_s is the sample rate

Note that the frequencies from your DFT or FFT algorithm might be arranged in a different order. The frequencies f_m are sometimes called DFT frequency bins.

4.4.4.1.1 Coherent sampling test method

Theoretically, a sine wave concentrates all of its energy at a single frequency. In practice, the DFT of a sine wave is nonzero in a band of frequencies due to leakage from the fundamental to other frequencies. By carefully choosing the frequency of the sine wave and using a rectangular window, this leakage can be minimized. Sine waves whose frequency exactly equals one of the DFT frequencies f_m described in 4.4.4.1 are called basis or kernel functions. For a rectangular window, the DFT of these basis sine waves contains precisely two nonzero DFT values—one at a positive frequency and one at a negative frequency. To force the test signal to be a basis sine wave, choose its frequency, f_0, as in step c) below. Choosing this relationship between the frequency of the sine wave, the sample rate, and the record length is called coherent sampling, see [B5]. Maintaining this relationship might require that the frequency of the sine wave be phase locked to the sample rate. The advantage of coherent sampling is that rectangular windows can be used without worrying about leakage. The sine wave and each of its harmonics will occupy only one positive and one negative frequency bin. The magnitude of some spurious components may be underestimated when using coherent sampling due to the fact that the spurious components are not harmonically related to the test signal. To estimate the rms value of a harmonic or spurious component, perform the following steps:

a) Select a record length: M
b) Select the sample frequency: f_s
c) Select the frequency, f_0, of the sine wave by selecting M_c

$$f_0 = \frac{M_c \cdot f_s}{M} \tag{91}$$

where

$M_c = 0, 1, \ldots,$ or $\frac{M}{2}$

M_c is the number of sine wave cycles in a data record of length M

d) Take a record of sine wave data: x_n, $n = 0$ to $M-1$.
e) Compute the DFT, X_m, of the record x_n.
f) Compute the rms value of the DFT values at the positive and negative frequencies, f_a and $-f_a$, of the harmonic or spurious component as follows:

$$S_{\text{rms}} = \sqrt{\frac{1}{M^2}\left(\left|X_{fa}\right|^2 + \left|X_{-fa}\right|^2\right)} \tag{92}$$

The S_{rms} is often divided by the rms value of the sine wave input,

$$\frac{A}{\sqrt{2}}$$

and the result is normally converted to either % or dB.

g) To estimate total harmonic distortion, repeat step f) for a set of harmonic components, and calculate the square root of the sum of squares of the S_{rms} values. The choice of harmonic components included in the sum is a trade-off between the desire to include all harmonics with a significant portion of the harmonic distortion energy, but to not include DFT frequency bins whose energy content is dominated by random noise. If not otherwise specified, the set is assumed to be the second through the tenth harmonics, inclusive.

4.4.4.1.2 Incoherent sampling test method

For incoherent sampling, the frequency, f_0, of the test sine wave does not coincide with any DFT basis frequency. This is the normal situation for experimenters and users of digitizing waveform recorders. Determining the frequency bands that correspond to a harmonic of a sine wave or a spurious component requires some thought. The use of a data window is a crucial component in the analysis. Leakage of energy from the fundamental to other frequencies must be controlled, or at least understood so that the positive and negative frequency bands in step h) below can be selected properly. To estimate the rms value of a harmonic or spurious component, perform the following steps:

a) Select a record length: M
b) Select the sample rate: f_s
c) Select the frequency of the sine wave in Hz: f_0
d) Select a data window: w_n, $n = 0$ to $M–1$. In the coherent sampling test method (see 4.4.4.1.1), the data window is the rectangular window.
e) Take a record of sine wave data: y_n, $n = 0$ to $M–1$
f) Multiply the data record point-by-point by the data window: $x_n = w_n y_n$, $n = 0$ to $M–1$
g) Compute the DFT, X_m, of the record x_n
h) When f_0 does not satisfy step c) of the coherent sampling test method (see 4.4.4.1.1), a harmonic of f_0 occupies a band of positive and negative frequencies instead of a single positive and negative frequency. Spurious components can appear at any frequency even though f_0 satisfies step c) of the coherent sampling test method (4.4.4.1.1). The test engineer must select the positive frequency band f_1 to f_2 and the negative frequency band $–f_2$ to $–f_1$ that the harmonic or spurious component occupies. The frequency bands depend on the record length, data window, frequency of the sine wave, and the sample rate.
i) Compute the rms value of the DFT values in the positive and negative frequency bands as follows:

$$S_{\text{rms}} = \sqrt{\frac{1}{M^2 \cdot \text{NNPG}}\left(\sum_{f=f_1}^{f_2} |X_f|^2 + \sum_{f=-f_2}^{-f_1} |X_f|^2\right)} \tag{93}$$

$$\text{NNPG} = \frac{1}{M}\sum_{i=0}^{M-1} w_i^2 \tag{94}$$

The normalized noise power gain, NNPG, is the noise power gain normalized by M, which is the noise power gain of the rectangular window. For the rectangular window, NNPG is unity. The noise power gain is also called the incoherent power gain. It determines how the window modifies the level of the DFT values of noise. The noise power gain also determines how the window modifies the rms value of sine waves.

The S_{rms} is often divided by the rms value of the sine wave input

$$\frac{A}{\sqrt{2}}$$

and the result can be expressed in either % or dB.

j) To estimate total harmonic distortion, repeat step i) for a set of harmonic components, and calculate the square root of the sum of squares of the S_{rms} values. The choice of harmonic components included in the sum is a trade-off between the desire to include all harmonics with a significant portion of the harmonic distortion energy, but to not include DFT frequency bins whose energy content is dominated by random noise. If not otherwise specified, the set is assumed to be the second through the tenth harmonics, inclusive.

4.4.4.2 Comment on choice of records lengths, sample rate, input frequency, data windows, selection of S, and number of averages

The positive and negative frequency bands occupied by a windowed sine wave depend on the record length, data window, frequency of the sine wave, and sample rate. For most choices of these parameters, these frequency bands will cover several frequencies due to leakage from the fundamental to other frequencies. Coherent sampling eliminates this leakage by forcing the frequency of the test sine wave to coincide with a DFT basis frequency and using a rectangular window. For incoherent sampling, the frequency, f_0, of the test sine wave does not coincide with any DFT basis frequency. Proper analysis in this case requires one to understand the control of leakage through the use of data windows. For more information on windows, refer to 4.1.6, [B7], [B12], [B1], and [B9]. For an alternate technique for determining harmonic distortion and spurious components refer to [B3].

4.5 Noise

Noise is any deviation between the output signal (converted to input units) and the input signal except deviations caused by linear time invariant system response (gain and phase shift), a dc level shift, or an error in the sample rate. For example, noise includes the effects of random errors, fixed pattern errors, nonlinearities, and time base errors (fixed error in sample time and aperture uncertainty).

4.5.1 Signal-to-noise ratio (snr)

The snr is the ratio of the signal to the noise. Unless otherwise specified, it is assumed to be the ratio of rms signal to rms noise for sine wave input signals.

The snr depends on the amplitude and frequency of the applied sinewave. The amplitude and frequency at which the measurement was made shall be specified.

4.5.1.1 Test method

Apply a sine wave of specified frequency and amplitude to the waveform recorder input. A large signal is preferred. Almost any error source in the sine wave other than frequency accuracy, gain accuracy, and dc offset can affect the test result, so it is recommended that a sine wave source of good short-term stability be used and that it be highly filtered to remove distortion and noise.

Take a record of data. To find the rms noise, fit a sine wave to the record per 4.1.3 and use

$$\text{rms noise} = \left[\frac{1}{M}\sum_{n=1}^{M}(y_n - y'_n)^2\right]^{\frac{1}{2}} \tag{95}$$

The snr is given by

$$\text{snr} = \text{rms signal} / \text{rms noise} \tag{96}$$

where

rms signal is the peak amplitude of the fitted sine wave $/\sqrt{2}$
rms noise is given by equation (95)

4.5.2 Effective bits

For an input sine wave of specified frequency and amplitude, after correction for gain and offset, effective bits is E.

$$E = N - \log_2\left(\frac{\text{rms noise}}{\text{ideal rms quantization error}}\right) = \log_2\left(\frac{\text{full scale range}}{\text{rms noise} \cdot \sqrt{12}}\right) \tag{97}$$

where

N is the number of digitized bits

The second equality in equation (97) comes from

$$\text{ideal rms quantization error} = \frac{Q}{\sqrt{12}} \tag{98}$$

Effective bits depend on the amplitude and frequency of the applied sine wave. The amplitude and frequency at which the measurement was made shall be specified.

4.5.2.1 Test method

Find the actual rms error as described in 4.5.1.1 and apply equation (97).

4.5.2.2 Comment on ideal quantization error

When using quantized data, the input value corresponding to a code bin is assumed to be the center of the bin. A signal falling into a code bin not at the center generates quantization error amounting to the distance of the signal from the center of the bin. To evaluate the size of this error over many samples, the probability distribution of the signal over a code bin must be known.

Approximating the ideal rms quantization error by $Q/(\sqrt{12})$ as used in equation (98) arises in the following way. For large sine waves, quantization error is uniformly distributed over a code bin. The standard deviation of a uniform distribution of width Q can be calculated as $Q/(\sqrt{12})$.

4.5.3 Comment on test conditions and interpreting the results of the signal-to-noise and effective bits tests

4.5.3.1 Comment on the relationship of signal-to-noise ratio and effective bits

Snr and effective bits are related by

$$E = \log_2(\text{snr}) - \frac{1}{2}\log_2(1.5) - \log_2\left(\frac{A}{V}\right) \tag{99}$$

$$\text{snr} = \sqrt{1.5}\left(\frac{A}{V}\right)2^E \tag{100}$$

where

A is the amplitude of the fitted sine wave during the test
V is one-half of the full-scale range of the waveform recorder input

A different formulation of effective bits omits the third term on the right-hand side of equation (99). This formulation gives different results for effective bits than 4.5.2 in all cases other than for full-scale test signals.

4.5.3.2 Comment on significance of record size

See 4.1.3.5.

4.5.3.3 Comment on purity of the input sine wave

Two specifications are of interest—phase noise and harmonic distortion.

For an effective bit or signal-to-noise measurement, the noise of the generator can perturb the measurement. Since the bandwidth of interest approximately ranges from the inverse of the record length, $f_s /M,$ to the upper bandwidth limit of the recorder, f_{co}, the noise contribution of the generator covers a similar range centered on the generator reference frequency f_r. The contribution of this phase noise to the measurement is the integrated noise of the generator from $-f_{co}$ to f_{co}, omitting the portion from $f_r - (f_s/M)$ to $f_r + (f_s/M)$. The integrated phase noise should be suitably small compared to the actual rms error computed in 4.5.1.1. In practice, the phase noise of generators decreases with distance in frequency from the carrier. Therefore, to obtain a quick pessimistic check of the integrated noise, multiply the noise density at $f_r + f_s /M$ by the bandwidth of interest, $2 f_{co}$.

The total harmonic distortion must also contribute a suitably small amount to the actual rms error found computed in 4.5.1.1. For many measurements it will be impossible to obtain a generator that simultaneously meets both phase noise and distortion requirements. A generator that meets the phase-noise requirement can be made to meet the distortion requirement with the use of filters.

4.5.3.4 Comment on errors included in and omitted from signal-to-noise and effective bits

Both snr and effective bits are based on total rms error with respect to a best-fit sine wave. Both include quantization error, differential nonlinearity, harmonic distortion, aperture uncertainty, spurious response, and random noise. Since the sine wave fitting procedure permits frequency, amplitude, phase, and offset to vary to obtain the best fit, errors in these parameters are excluded from the rms error derived and, hence, from snr or effective bits. For example, neither snr nor effective bits are a measure of amplitude flatness, phase linearity versus frequency, or long term variations. For further information, see [B11].

4.5.4 Peak error

Of the set of differences between the fitted sine wave and the recorded data points found in 4.5.1.1, peak error is the difference with the largest absolute value. Normalized peak error is the peak error divided by three times the standard deviation of the differences.

4.5.5 Random noise

Random noise is a nondeterministic fluctuation in the output of a waveform recorder, described by its frequency spectrum and its amplitude statistical properties. For the measurements in this clause, the following noise characteristics are assumed. The power spectrum is flat, (white noise), the amplitude probability density function is stationary, and the noise is additive and signal independent.

4.5.5.1 Test method

A noise record may be obtained by digitizing two records with no input signal (but with the waveform recorder input terminated) at a specified input range and subtracting them. The subtraction eliminates fixed-pattern errors that occur in the same location in successive records. The noise variance may be estimated from

$$\sigma^2 = \frac{1}{2M}\sum_{n=1}^{M}(y_{an} - y_{bn})^2 \qquad (101)$$

where

σ^2	is the noise variance
y_{an} and y_{bn}	are the noise record samples
M	is number of samples

We have assumed zero mean noise, since nonzero mean is the same as offset error. When the noise result is 1 Q or less, use the method in 4.5.5.2.

4.5.5.2 Alternative test method for low noise recorders

Connect the output of a triangle wave generator to the signal input of the digitizer. Adjust the output amplitude to 10 Q *p-p* (see 4.5.5.3). Connect the trigger output of the triangle wave generator to the trigger input of the digitizer. Trigger the digitizer on the beginning of the positive-going portion of the triangle. Adjust the frequency of the triangle wave generator such that one period subtends one record length. Capture two records and use equations (102) and (103).

$$\sigma^2 = \left[\left(\frac{\text{mse}}{2}\right)^{-2} + \left(\frac{0.886\,\text{mse}}{Q}\right)^{-4}\right]^{-\frac{1}{2}} \tag{102}$$

where

$$\text{mse} = \frac{1}{M}\sum_{n=1}^{M}(y_{an} - y_{bn})^2 \tag{103}$$

and where

σ^2	is the noise variance
y_{an} *and* y_{bn}	are the noise record samples
M	is the number of samples

As noise increases, these equations converge to that used in 4.5.5.1. For a derivation of equation (102), see annex D.

4.5.5.3 Comment on amplitude of triangle wave used for test

The triangle wave provides a means of slowly slewing the digitizer over a plurality of code bin thresholds at a relatively constant rate. The subtraction process removes the contribution of the triangle wave to the result to the extent that the two repetitions are identical. Any differences due to noise, jitter, etc., will contribute to the apparent result. Consequently, unless the output of the generator can be independently judged to have a sufficiently low noise level, it is best to keep the amplitude low. This means that only a part of the full-scale range of the digitizer can be explored with each measurement.

4.6 Analog bandwidth

Bandwidth is the difference between the upper and lower frequency at which the amplitude response as seen in the data record is 0.707 (–3 dB) of the response as seen in the data record at the specified reference frequency. Usually, the upper and lower limit frequencies are specified rather than the difference between them. When only one number appears, it is taken as the upper limit. Bandwidth may be measured at any stated signal amplitude and sampling rate. When the sampling rate is not specified, bandwidth is measured at the maximum sampling rate. When amplitude is not specified, bandwidth is measured using a large signal. When the amplitude is specified only as a small signal, it is assumed to be a 0.1 full-scale signal.

4.6.1 Test method

Apply a constant amplitude sine wave to the waveform recorder. When not specified, select a reference frequency well within the passband of the recorder and not a subharmonic of the sampling rate. Acquire a sufficient number of records to establish the maximum peak-to-peak range of the recorded data. Divide the recorded peak-to-peak range by the input amplitude to establish the reference amplitude ratio. Change the frequency to another value that is not harmonically related to the sampling rate. Measure the maximum peak-to-peak range of the recorded data, and calculate the amplitude transfer ratio. Find the upper and lower frequencies closest to the reference frequency at which the amplitude transfer function is 0.707 of the reference amplitude transfer function. The difference between these two frequencies is the bandwidth of the recorder. When the bandwidth of the recorder includes dc, then the upper frequency is the bandwidth.

4.6.2 Alternative method using time domain techniques

See 4.7.1.

4.7 Frequency response

The complex gain (magnitude and phase) as a function of input frequency, or the Fourier transform of the impulse response. The preferred method of presentation is in the form of plots of magnitude and phase versus frequency. See [B17].

4.7.1 Test method

Record the step response of the test recorder (see 4.1.4), using equivalent time sampling (see 4.1.5) if necessary. Select the (equivalent time) sampling rate high enough to give negligible aliasing errors based on the bandwidth of the recorder (see 4.7.2). Take a record of data with an epoch long enough to ensure that the topline of the step has settled to within the desired accuracy. Take the first difference of this record, that is, take the discrete time derivative, to find the impulse response. Obtain a Fourier transform of the impulse response using an unweighted (rectangular) window. The result is the frequency response of the recorder. The linear (delay) term in the phase portion of the transform is arbitrary since the equivalent delay of the recorded signal with respect to the input waveform is indeterminate. The nonlinear phase components can be made more apparent by rotating the phase plot so that the first and last components of the phase shift are equal. The result is a plot of the nonlinear phase contribution with measurement error bounded by e_p given in 4.7.2.

This test method makes use of the natural roll-off of the recorder under test as an anti-aliasing filter, attenuating the frequency components of the step that are beyond the Nyquist limit. Bounds on the residual aliasing errors, together with first differencing errors, are given below.

Note that the first differencing operation accentuates high-frequency noise components, such as that due to quantization, and the equivalent noise increases as the square root of record length. When excessive, the noise may be reduced to acceptable levels by digitally filtering the step response record. This can be performed without significantly affecting the measured frequency response. For more information, see [B16].

This test may not produce meaningful results for nonlinear systems.

4.7.2 Comment on aliasing errors

Upper bounds on aliasing and first differencing errors can be estimated by assuming that the recorder's roll-off is dominated by a single pole [B16]. Under these assumptions, the aliasing and first differencing errors measured in percent of full-scale, in the magnitude spectrum as measured above will be no greater than

$$e_m[f] = 400\frac{f \cdot f_{co}}{f_{eq}^2} \text{ percent of full scale} \tag{104}$$

for $f < \frac{f_{eq}}{2}$ and for $f_{eq} \geq 2f_{co} f_{eq} \geq 2f_{co}$

where

f is the frequency of interest
f_{co} is the cutoff frequency (bandwidth) of the test recorder
f_{eq} is the equivalent sampling rate

For the phase spectrum, the aliasing and first differencing errors $e_p[f]$, will be no greater than

$$e_p[f] = 270\frac{f}{f_{eq}} \text{ in degrees} \tag{105}$$

valid for $f \leq \frac{f_{eq}}{4}$, and $f_{eq} \geq 2f_{co}$.

Example: If the expected bandwidth of the test recorder is 10 MHz and an equivalent sampling rate of 100 MHz is chosen, what is the maximum aliasing and first differencing error that can be expected at half the bandwidth (5 MHz)?

$$e_m = \frac{(400 \cdot 5 \cdot 10^6) \cdot 10^7}{10^{16}} = 2\ \% \tag{106}$$

$$e_p = \frac{270 \cdot 5 \cdot 10^6}{10^8} = 13.5 \text{ degrees} \tag{107}$$

4.8 Step response parameters

4.8.1 Settling time parameters

4.8.1.1 Settling time

Measured from the mesial point (50%) of the output, the settling time is the time at which the step response (see 4.1.4) enters and subsequently remains within a specified error band around the final value. The final value is defined to occur 1 s after the beginning of the step.

4.8.1.2 Short-term settling time

Measured from the mesial point (50%) of the output, the time at which the step response (see 4.1.4) enters and subsequently remains within a specified error band around the final value. The final value is defined to occur at a specified time less than one second after the beginning of the step.

4.8.1.3 Long-term settling error

This is the absolute difference between the final value specified for short-term settling time, and the value 1 s after the beginning of the step, expressed as a percentage of the step amplitude.

4.8.1.4 Test method for settling time and short-term settling time

Record the response (per 4.1.4.1) to an input step using a record length sufficient to represent the step over the duration specified, or for at least 1 s when the duration is not specified. Two or more overlapping records with different sample rates may be required to achieve the necessary time resolution and the required duration. To reduce noise or quantization errors, it may be desirable to digitally filter the step response data before computing settling time parameters. For this purpose, apply a moving average filter of the form

$$y_n = \frac{1}{(2r+1)} \sum_{s=-r}^{r} x_{(n-s)} \tag{108}$$

where

$x_{(n-s)}$ is the value of the *(n–s)* data point of the unfiltered step response
y_n is the value of the nth data point of the filtered step response
r is an integer defining the width of the moving average window

If a filter is used, the width of the window, $(2r + 1)$ must be specified.

Determine the time of occurrence of the mesial point of the recorded waveform. Counting from that time the settling time (or the short-term settling time) is the time at which the output waveform last enters the bound given by $V(t) \pm \varepsilon$, where $V(t)$ is the value at the end of the specified duration, and ε is the specified error. (If the specified duration is less than 1 s, then the time thus determined is the short-term settling time.) When the duration is not specified, $V(t)$ is the value 1 s after the beginning of the step.

To measure the long-term settling error, record the same step used to determine the short-term settling time with a record that spans at least a 1 s interval from the beginning of the step. The long-term settling error is the absolute difference between the value 1 s after the beginning of the step, and the value at the end of the specified duration following the step, expressed as a percentage of the step amplitude.

4.8.1.5 Alternate test method for recorders that do not allow records of 1 s or more

For recorders that do not allow records of 1 s or more to be taken, suitably delay the trigger so that the record includes the time one second after the step, and take another record. The long-term settling error is the absolute difference between the value of this record 1 s after the step, and the value at the end of the specified duration when the step was applied, expressed as a percentage of the step amplitude.

4.8.1.6 Comment on settling time

The term *settling time* refers to the time required to settle to the steady state, dc value, to within the given tolerance. The dc value is assumed to be the value after a constant input has been applied for at least 1 s. Changes that occur after one second are considered drift, and may be due to room temperature fluctuations, component aging, and similar effects.

The term *short-term settling time* refers to the time required to settle to a relative value (perhaps different from the steady-state value), defined as the value at the end of a specified duration, for record lengths less than 1 s. If static offset, gain, and linearity corrections are used to assign true values to short-term settling data, the results will have an uncertainty given by the long-term settling error. The uncertainty results because of longer term settling phenomena, such as thermal imbalances that may occur after the short-term duration is complete, but that affect a steady-state measurement.

Note that only short-term settling time can be specified for ac coupled recorders.

4.8.2 Transition duration of step response

This is the duration between the proximal point (10%) and the distal point (90%) on the recorded output response transition, for an ideal input step with designated baseline and topline. The algorithm used to determine the baseline and topline of the output step must be defined. The methods of IEEE Std 181-1977[3] are preferred.

4.8.2.1 Test method

Record the step response (see 4.1.4.1) and determine the proximal and distal lines of the output transition using the algorithm from 4.3.1.1 in IEEE Std 181-1977. Linear interpolation is used to determine the proximal and distal points when insufficient data points are available on the transition. The transition duration of the step response is the time between the first proximal point and the last distal point on the transition.

4.8.2.2 Comment on pathological test results

On some recorders, typically those that do not employ sharp cutoff anti-aliasing filters, the step response can be nonlinear. The degree of nonlinearity increases with the steepness of the applied step. Because of these nonlinear effects, the step response of such a recorder can be misleading. To eliminate the gross nonlinearities, the transition duration of the applied step should be limited to above a minimum value. The value of this limiting transition duration is dependent upon the recorder under test. When the step response is obtained using an input pulse of limited transition duration, the test can be referred to as pulse response. The statement of results should include the transition duration of the applied pulse.

4.8.3 Slew limit

This is the value of output transition rate of change for which an increased amplitude input step causes no change.

4.8.3.1 Test method

Record the step response (see 4.1.4.1) for an input step having an amplitude 10% of full-scale. Determine and store the maximum rate of change of the output transition. Repeat this process, increasing the amplitude of the input step each time. When the maximum rate of change ceases to increase with increasing step amplitude, slew limiting is taking place and the slew limit is the largest recorded value for the maximum rate of change.

4.8.4 Overshoot and precursors

Overshoot is the maximum amount by which the step response exceeds the topline, and is specified as a percent of (recorded) step amplitude. Precursors are any deviations from the baseline prior to the step transition. They are specified in terms of their maximum amplitude as a percent of the step amplitude.

4.8.4.1 Test method

Record the step response (see 4.1.4.1). Determine the maximum overshoot and precursors by following the method in IEEE Std 181-1977.

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[3]Information on references can be found in clause 2.

4.9 Time base errors

4.9.1 Fixed error in sample time

Fixed errors in sample time are nonrandom errors in the instant of sampling. They may be fixed with respect to the data samples acquired or correlated with an event that is detected by the sampling process. Examples of correlated events include subharmonics of the fundamental sampling clock itself, pickup from other logic functions within the recorder, nonlinearity in sweep circuits, and interference from external sources. Unless the recorded data accounts for these errors, an apparent amplitude error is generated. For example, if the sampling is performed with multiple interleaved sampling channels, the channel number corresponding to the first sample must be noted in memory to correct fixed errors. The error magnitude is the timing error times the slope of the signal recorded at that instant. Unless otherwise specified, fixed error in sample time is taken to mean the maximum fixed error that may be observed.

4.9.1.1 Test method

Record a large signal sine wave, triggering the recorder independently so that the starting phase of the sine wave is random. The sine wave frequency should be as high as possible without exceeding either the analog bandwidth or one third the sampling rate. Perform a sine fit to the data per 4.1.3, and compute and store the residuals of the fit. Transform the errors into units of time by dividing the residuals by the derivative of the fitted sine wave, sample by sample.

For a fitted function given by

$$y'_n = A \cos(\omega t_n + \theta) + C,$$

the derivative is

$$dy'_n/dt_n = -\omega A \sin(\omega t_n + \theta)$$

To avoid numerical instability and excessive sensitivity to noise and distortion as the derivative approaches zero, omit this step for samples lying within 15 degrees on either side of both (positive and negative) peaks.

Collect K (at least 10) such time records, and average them on a point by point basis. Averaging minimizes unwanted contributions due to amplitude noise, quantization noise, harmonic distortion, and aperture uncertainty. In addition, it provides data where it may have been omitted, by randomizing the occurrence of the sine wave peaks with respect to each record of data. When computing the averages, remember that fewer than K data points may actually be present for any given record location, because of the omitted data. Compute the mean value of all the averages over the record. (The mean may not be zero because of the information lost at the peaks). Fixed errors in sample time are the deviations of the averages from the mean. The maximum fixed error is the deviation that has the largest absolute value.

4.9.2 Aperture uncertainty

Aperture uncertainty is the standard deviation of the sample instant in time. As a measure of short-term stability, the time over which aperture uncertainty is measured is usually no longer than the longest single record that can be taken with the recorder. An exception occurs when the measurement method is equivalent time sampling. In this case, the time over which the aperture uncertainty is measured is the time required to capture a record of data, rather than the equivalent time-duration represented by the record. Aperture uncertainty is also known as timing jitter, timing phase noise, or short-term timing instability. Aperture uncertainty produces a signal amplitude error whose magnitude is the timing error times the slope of the signal recorded at that instant.

4.9.2.1 General test method to determine an upper bound

Perform the test described in 4.9.1.1 for fixed error in sample time. An upper bound for aperture uncertainty for the *n*th sample in the record $\sigma_t(n)$ is the standard deviation (from the mean) of the *m* values of fixed error in sample time for the *n*th sample. An upper bound for over all aperture uncertainty σ_t is the rms value of $\sigma_t(n)$ calculated over all *n*. Note that this measurement also includes error contributions from amplitude noise, quantization noise, and harmonic distortion. The relative contribution from these sources is minimized by selecting the highest test frequency consistent with bandwidth and sampling restrictions.

4.9.2.2 Alternate test method to determine an upper bound

Apply a sine wave at frequency f_1, where f_1 is a low frequency with respect to the bandwidth of the recorder, and determine the rms noise (σ_1) as in 4.5.1.1. Then, apply a sine wave at frequency f_2, where f_2 is a high frequency with respect to the bandwidth of the recorder, and determine the rms noise (σ_2) as in 4.5.1.1. The frequency f_2 should be high enough that the rms noise is at least twice as large as measured for the low frequency (f_1) sine wave of the same amplitude. However, f_2 should not exceed the analog bandwidth of the recorder. An upper bound for the aperture uncertainty is then given by

$$\sigma_t = \frac{\sqrt{\sigma_2^2 - \sigma_1^2}}{\sqrt{2}\pi f A} \tag{109}$$

where

σ_2 is the rms noise level measured at frequency f_2 and measured amplitude A.
σ_1 is the rms noise level measured at frequency f_1 and the same amplitude A.

This technique includes the effects of fixed pattern errors and harmonic distortion; therefore, it can only provide an upper bound for the aperture uncertainty. The error due to the harmonic distortion component can be removed by correcting both σ_2 and σ_1 for harmonic distortion. The mean squared magnitudes of the harmonics can be determined from the data records as described in [B3].

4.9.2.3 Alternative test method for recorders that permit either external sampling clocks or port the internal sampling clock to the user

When the internal sampling clock is available to the user, and when it is compatible with the instrument's input, connect it to the input port. Alternatively, if an external sampling clock is permitted, connect a pulse or sine wave generator, as specified, to both the signal input port and the sampling clock drive port in a manner compatible with each port. The signal generator should have subharmonic and non-harmonic content of less than 1/2 *Q* of the instrument under test. Its transition duration should lie between one and four times the minimum transition duration appropriate for the instrument under test. Vary the time delay between the signal input and sampling clock input to establish that the peak-to-peak range of the generator lies within the center 90% of the amplitude range of the recorder. When it is possible to vary the sampling rate (as with the external generator), do so to ensure that an integral cycle discrepancy does not exist between the signal path and sampler path. This will reduce the effect of signal generator jitter on the measurement.

Adjust the time delay between the signal input and sampler to successively acquire data in the following four regions of interest:

a) Maximum peak (the highest amplitude, minimum slew region) P+
b) Minimum peak (the lowest amplitude, minimum slew region) P–
c) Midpoint of the rising waveform R+
d) Midpoint of falling waveform R–

Record at least 50 points in each region.

For each region, calculate the variance. Find the aperture uncertainty by using equation (110) where *m* is the square root of the average of the squares of the rising and falling slopes.

$$\sigma_t = \frac{1}{m}\left(\frac{\sigma_{R+}^2 + \sigma_{R-}^2}{2} - \frac{\sigma_{P+}^2 + \sigma_{P-}^2}{2}\right)^{\frac{1}{2}} \tag{110}$$

Results from this method must be carefully evaluated. If the internal sampling clock is ported to the signal input, it may be buffered with a noisy amplifier. Further, it is possible in this configuration for the signal and clock paths to have equal time delay, thus inadvertently eliminating internal sampling clock jitter. When using an external signal generator as the sampling clock, the performance may not represent the performance with the internal sampling clock. As a minimum, noise in the sampling clock itself is not included as it is in the method of 4.9.1.1. Additionally, physically different paths may link the internal and external sources to the sampler.

NOTE—A special application of this method is in testing equivalent time sampling systems. With these systems, a variable delay circuit, triggered by the input or trigger signals, usually initiates the sample command. The equivalent aperture uncertainty can be measured for systems that permit a record of data to be taken at fixed delays. The aperture uncertainty will be a function of the delay setting, generally increasing as the delay increases.

4.9.3 Long-term stability

The change in time base frequency (usually given in parts per million) over a specified period of time at a specified sampling rate.

4.9.3.1 General test method

Connect the output of a sine wave generator whose frequency is known to well within the desired accuracy and stability to the input of the digitizer. The frequency should be between 0.5 and 0.25 of the sampling rate, but not simply related to the sampling rate.

Using a sine wave parameter estimation program, such as that in 4.1.3.2, determine the frequency of the sine wave applied to the digitizer. Since the time base of the digitizer is used in the frequency determining process, the sampling rate accuracy is inferred from the measurement of the reference sine wave. Equation (111) relates the standard deviation of the digitizer's measurement of the reference sine wave frequency to the effective bits and the record length of the digitizer. The effective bits measurement of the reference sine wave establishes the bounds for the determination of the confidence interval, in accordance with standard statistical methods. See [B8].

$$\frac{\sigma[f_r]}{f_s} = \frac{0.225}{M^{3/2}2^{E-1}} \tag{111}$$

where

f_r	is the reference sine wave frequency as inferred by the digitizer
$\sigma[f_r]$	is the standard deviation of measurement
f_s	is the sampling rate
M	is the number of equally spaced samples in the record
E	is the number of effective bits

Since the frequency of the reference sine wave is known independently, the error in the sampling rate is then

$$\varepsilon[f_s] = \frac{f_s}{f_r}\varepsilon[f_r] \tag{112}$$

where

$\varepsilon[f_r]$ is error in the measurement of the reference sine wave frequency
$\varepsilon[fs]$ is inferred error in the sampling rate

The above measurement is then made periodically over a specified period to establish the long-term sampling rate stability.

The sample record should be as long as is practical. The number of reference sine wave cycles M_c in the record should be

$$4 \le M_c \le \frac{M}{2} - 4 \tag{113}$$

The effective bits measurement establishes the error bounds that may be due to noise and distortion on the source.

4.9.3.2 Alternative test method for recorders that port the internal sampling clock to the user

Connect the recorder's clock output to a frequency meter having the required resolution and at least four times better stability than that specified for the recorder. Measure the change in sampling rate over the specified period of time at the specified sampling rate.

4.10 Triggering

The trigger function synchronizes the recorded waveform to an external event. The device may be triggered by either the recorded signal or by an external pulse at an independent input. For checkout purposes, the recorder may also be triggered by operator command.

4.10.1 Trigger delay and trigger jitter

Trigger delay is the elapsed time from the occurrence of a trigger pulse at the trigger input connector to the time at which the first or a specified data sample is recorded. The amplitude and duration of the trigger pulse should be specified. The trigger delay may increase with decreasing trigger pulse amplitude or duration. For recorders with pretrigger capability, the trigger delay may be negative. Trigger jitter is the standard deviation in the trigger delay time over multiple records.

4.10.1.1 General test method

A precision delayable pulse generator can be used for measuring delay and jitter. Connect the delayed pulse to the signal input and the undelayed pulse to the trigger input. Adjust the delay so the leading transition duration of the signal pulse is recorded. Adjust the recorder's trigger level to trigger at the midpoint (50% amplitude) of the trigger pulse. The pulse leading transition should be at least three sample periods. Take enough records to get adequate jitter statistics. For each record, find the elapsed time from the start of the record to the occurrence of the interpolated midpoint of the pulse leading transition. If the recorder provides a measure of the time between the trigger pulse and the sample clock, use this time to adjust the elapsed times accordingly. The trigger delay is the indicated delay of the generator minus the average of the elapsed times just measured. The trigger jitter is the standard deviation of the elapsed times. The jitter of the delayable pulse generator should be accounted for in computing the trigger jitter. Since the two are independent, the trigger jitter may be estimated as the square root of the difference of the squares of the measured jitter and the jitter of the delay generator.

4.10.1.2 Alternative test method, only for recorders with pretrigger capability

Delay and jitter measurements on the external trigger of a recorder with pretrigger capability can be made without a delayable pulse generator by splitting a single pulse into two signals and routing one pulse to the signal input and the other to the trigger input. Equal length electrical cables must be used for each leg. The pulse leading transition duration should be at least three sample periods. Adjust the recorder's trigger level to trigger at the midpoint (50% amplitude) of the trigger pulse. Take enough records to get adequate jitter statistics. For each record, find the elapsed time from the start of the record to the occurrence of the interpolated midpoint of the pulse leading transition. If the recorder provides a measure of the time between the trigger pulse and the sample clock, use this time to adjust the elapsed times accordingly. The trigger delay is the nominal pretrigger duration minus the average of the elapsed time just measured. The standard deviation of the elapsed times is the trigger jitter.

4.10.1.3 Comment on the inherent jitter associated with test methods in 4.10.1.1 and 4.10.1.2

Unless the trigger signal is phase locked to the recorder's sampling clock or the phase between the trigger and the sampling clock is measured and provided to the user, measurements made using this test method will include an inherent jitter component due to time quantization with an rms value of

$$\frac{T}{\sqrt{12}} \tag{114}$$

where

T is sampling period

4.10.2 Trigger sensitivity

Trigger sensitivity has the following several components—sensitivity to the input trigger pulse level, to the rate of change of the leading transition, and to pulse duration. Trigger sensitivity is also related to hysteresis, which may be incorporated in a trigger circuit to reduce spurious triggering. A qualitative plot of the envelope of trigger voltage versus trigger delay time is shown in figure 3 for positive going signals. Assume the input level to be initially below V_{HYST}. The minimum level that will trigger the recorder is V_{TH}. The trigger delay associated with a large input pulse is T_{MIN}. A smaller trigger level V_{OD} will cause a longer trigger delay T_{OD}. The minimum pulse duration is the duration of the shortest pulse of specified amplitude that will cause triggering. V_{HYST} is the hysteresis level below which the input must pass before a subsequent input passing V_{TH} will trigger the recorder. The hysteresis band is the absolute value of the difference between V_{TH} and V_{HYST}, and is the minimum pulse amplitude that will repetitively trigger the recorder.

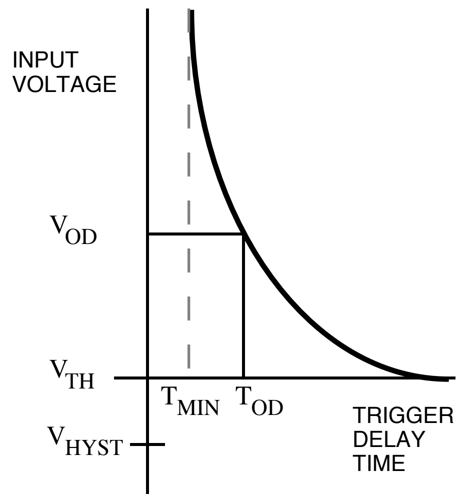

Figure 3—Trigger sensitivity

4.10.2.1 Test methods

To measure the minimum pulse duration, apply a pulse whose duration and amplitude can be varied, and has the proper polarity to ensure leading edge triggering. Set the pulse amplitude to the specified level. Adjust the duration to determine the minimum that will cause triggering. (Check that the pulse amplitude does not decrease as the pulse duration is decreased.)

To measure the hysteresis band triggering on positive-going (negative-going) signals, apply a dc level below (above) V_{HYST} to the trigger input. Arm the recorder. Monotonically increase (decrease) the dc level until it is slightly below (above) V_{TH} and well above (below) V_{HYST}. Add a variable amplitude sine wave of specified frequency to the dc level. Start with a zero amplitude sine wave and increase the level until the recorder triggers. Rearm the recorder. The recorder should not retrigger. Increase the sine wave level until the recorder does trigger. The hysteresis band is the sum of the peak amplitudes of the first and second sine waves.

4.10.3 Trigger minimum rate of change

The minimum rate of change is the slowest rate of change of the leading edge of a pulse of a specified level that will trigger the recorder.

4.10.3.1 Test method

To measure minimum rate of change, apply a ramp or low-frequency sine wave to the trigger input. First set the rate of change to several times faster than specified. Vary the amplitude to determine the minimum trigger level. Again raise the amplitude by a small but specified amount; then vary the rate of change to determine the slowest rate that will trigger the recorder.

4.10.4 Trigger coupling to signal

An external trigger signal to a recorder can cause a spurious signal to appear in the recorded data. Trigger signal coupling is the ratio of the spurious signal level to the trigger signal level.

4.10.4.1 Test method

To measure trigger signal coupling, first terminate and shield the input channel as specified. Apply a trigger pulse of maximum allowable operating amplitude and a leading transition duration as short as is consistent with the trigger channel bandwidth. Take a data record and determine the peak-to-peak signal level. The ratio of this signal level to the peak-to-peak trigger signal level is the trigger signal coupling. Note that this measurement will be limited by the noise and other spurious effects associated with the input channel itself.

4.11 Crosstalk

4.11.1 Multichannel crosstalk

In waveform recorders with multiple independent recording channels, a signal in one channel can cause a spurious signal in another channel. The crosstalk (multichannel) is the ratio of the signal induced in one channel to a common signal applied to all other channels.

4.11.1.1 Test method

To measure multichannel crosstalk, first terminate and shield the channel to be tested as specified. Then apply a maximum allowable amplitude of a specified frequency sine wave to all other channels. To maintain equal sine wave phasing, all inputs should be driven from a common source using cables of equal electrical length. A resistor or transformer coupled signal splitter that maintains proper impedances should be used. (A transformer coupled signal splitter has less insertion loss, but typically has a lower-frequency limit of a few kilohertz.) Take a record of data. The multichannel crosstalk is the ratio of rms level of the spurious signal to the rms level of the test signal. Note that this measurement is limited by the noise and spurious effects of the channel in question.

4.11.2 Multiple input reverse coupling

In waveform recorders with multiple isolated inputs added in a single recording channel, the input to one channel can cause a spurious signal to appear at another input connector. Reverse coupling is the ratio of the spurious signal to the actual signal.

4.11.2.1 Test method

To measure input reverse coupling, apply a sine wave of full scale amplitude and specified frequency to all but one input. Measure the signal level at the input with no applied signal. The reverse coupling is the ratio of the spurious signal rms level to the test signal rms level. A null measurement should also be made with the input cable grounded at the end normally connected to the recorder. This is to ensure that any measured signal is coming from the unit under test and not from stray antenna pickup.

4.12 Monotonicity

A recorder that is monotonic has output codes that do not decrease (increase) for a uniformly increasing (decreasing) input signal, disregarding random noise.

4.12.1 Static test method

A dc signal source, for example, a digital-to-analog converter, is required whose range and output parameter is compatible with the waveform recorder, and whose resolution is at least four times that of the waveform recorder. The source should be monotonic and should have differential nonlinearity better than one fourth of the waveform recorder resolution. Monotonicity is then determined as follows:

a) Connect the source to the waveform recorder input.
b) Apply a voltage slightly lower (for example, 2%) than the minimum voltage recordable by the waveform recorder.
c) Take a record of data. Store the average value of the record as $A[i]$, where i is the step number used to program the source. The $A[i]$ should be calculated and stored with resolution better than 1/4 Q of the waveform recorder.
d) Raise the source level one step (increment i by one).
e) Repeat steps c) and d) until the full range of the waveform recorder is covered.
f) Examine adjacent pairs of $A[i]$ records beginning with the initial average $A[i]$ and the next average $A[i+1]$ for the condition $A[i] \leq A[i+1]$.
g) When the condition stated in step f) is met for all averages that fall within the recorder's input range, then the recorder is monotonic. If any adjacent pairs of averages fail the test, the recorder is not monotonic.

4.13 Hysteresis

Different values for a digitizer code transition may result when the level is approached by a signal from either side of the transition. Hysteresis is the maximum of such difference seen over the range of the digitizer.

4.13.1 Static test method

Acquire rising waveform data as prescribed in 4.12.1, steps a), b), c), d), and e). Acquire falling waveform data in an analogous manner. Compare each $A[i]$ of the rising waveform with $A[i]$ of the falling waveform. When they are identical, for all $A[i]$, no hysteresis exists. When they are not identical, the hysteresis is the maximum difference between rising and falling $A[i]$.

4.14 Overvoltage recovery

An overvoltage is any voltage whose magnitude is less than the maximum safe input voltage of the recorder but is greater than the full-scale value for the selected range. An overvoltage may produce changes in the characteristics of the input channel, such as saturation of an amplifier or temporary changes in component values caused by thermal effects. The overvoltage recovery time is the time from the end of overvoltage to when the input channel returns to its specified characteristics. Overvoltage recovery occurs according to two different criteria. Relative recovery is achieved when the recorder's normal transfer characteristic is restored in all respects, except for signal propagation time through the recorder. Absolute recovery is achieved when the recorder's normal transfer characteristic is completely regained. Relative recovery is adequate when data before and after the overvoltage need not be related in time. When the data before and after the pulse must be related in time, then the recorder must recover absolutely.

4.14.1 Test method for absolute overvoltage recovery

Arrange a network capable of simultaneously applying both a high-purity sine wave and a specified overvoltage pulse with a flat baseline. The overvoltage pulse should be specified as to its amplitude, duration, polarity, and frequency. Apply a high-purity, large signal sine wave of a convenient, nonharmonically related frequency (e.g., 1/20th the sampling frequency). Take a record of data with the overvoltage pulse occurring near the center of the record. Fit a sine wave to the data prior to the overvoltage pulse. Extrapolate the fitted sine wave to the end of the record. The measure of overvoltage recovery is the deviation of recorded data from the fitted sine wave. Overvoltage recovery time is measured from the last full-scale point associated with the pulse to the first point that deviates less than, and stays within, the desired tolerance of the fitted sine wave.

As a test of the method, record only the sine wave. Fit a sine wave to the portion of the record occurring prior to the point at which the overvoltage pulse will be introduced. Extend the fitted sine wave in the portion of the record where the overvoltage recovery is anticipated to occur. The observed deviation indicates the resolution obtainable when the pulse is applied.

4.14.2 Test method for relative overvoltage recovery

When the occurrence of events before the overvoltage pulse is not relevant to data acquired after the pulse, relative recovery is an appropriate criteria. Relative recovery may also be used when record length precludes the method in 4.14.1. To measure relative recovery, first record several records of the sine wave. Fit each record of data with a sine wave. Find the average amplitude, frequency, and dc offset of the fitted sine waves. Take a record of data in which the overvoltage pulse is removed very early in the record. Align the previously fitted, average sine wave to the latter portion of the record (e.g., the last 1/4 record) by varying the phase only. Extend the aligned sine wave across the entire record. Observe deviations as before.

4.14.3 Comment on test method

In a high-frequency 50 Ω system, the sine wave and the pulse must be added using a resistive adder. An isolating reactive adder generally does not work because the top of the test pulse droops due to the adder's limited low-frequency response. This droop causes an undershoot when the pulse returns to its initial level. The resistive adder feeds some of the pulse back to the sine wave generator, which may degrade the quality of the sine wave. This effect can be checked by applying a sine wave and pulse combination that does not go off scale on the waveform recorder. The degradation can be reduced by placing as large an attenuator as possible at the sine wave input to the resistive adder.

The overvoltage test pulse must return cleanly to its initial level. Any aberrations degrade the sine fit results.

4.15 Word error rate

The word error rate is the probability of receiving an erroneous code for an input after correction is made for gain, offset, and linearity errors, and a specified allowance is made for noise.

4.15.1 Test method

Since the word error rate is small (usually measured in parts per million or parts per billion), a great many samples must be collected to test for it. The number of samples required is discussed in annex C. Before starting, choose a qualified error level. This qualified error level should be the smallest value greater than 2 Q that excludes from this test all other sources of error. Particular attention should be paid to excluding the statistical occurrence of noise on the order of the word error rate.

Apply a large signal to the waveform recorder whose rate of change is significantly less than the equivalent of 1 Q per sample period. Ensure that the peak-to-peak noise of the signal chosen does not exceed a least significant bit. Take the largest possible record of data. Examine the differences between successive samples, and record the number of times this difference exceeds the qualified error level. This number corresponds to twice the number of qualified errors. Take successive records of data and keep a running total of qualified errors until the required number of samples have been examined.

The word error rate is the number of qualified errors found through the test method divided by the number of samples examined.

4.15.2 Comment on the number of samples required for word error rate

While the formulation given in annex C establishes the accuracy of the word-error-rate measurement as a function of samples taken, many users are not interested in knowing an exact word error rate, but are satisfied with an upper limit. To establish only that the error rate is less than some maximum, acquire at least 10 times the number of samples for which it would be expected that a single word error would occur. For example, if the test can tolerate no more than one error per million samples, acquire 10 million samples. After acquiring and examining these samples, there are three possibilities as follows:

a) A few or no errors are found. Then the error rate is certainly less than the maximum.
b) The number of errors is approximately 10. A decision must be made now if the accuracy indicated by the equations in annex C is satisfactory, or if it is necessary to acquire more data.
c) The number of errors is more than 10. If it is closer to 100, then the error rate is known with greater precision. In any case, the maximum chosen is not a good estimate of the upper limit.

4.16 Cycle time

With the recorder continually taking records of data, cycle time is real time elapsed between the beginning of two records taken in succession. The length of data record and sampling rate should be specified. Note that in addition to the internal setup time of the recorder and the experiment duration, cycle time includes the time to transfer data to a computer and the time required for a computer to send instructions to the recorder.

4.16.1 Test method

Apply a trigger signal at a rate much faster than the expected cycle time. Under computer control, take K records of data (see 4.1.1), and measure the elapsed time. Cycle time is the elapsed time divided by K.

4.16.2 Comment

Cycle time may be limited by the computer used rather than the waveform recorder under test.

4.17 Differential input specifications

A waveform recorder with differential inputs produces output codes that are a function of the difference between two input signals. The two input signals are typically called positive and negative. Such devices have a number of performance features in addition to those found in single-ended recorders. These include the impedance of each input (positive and negative) to ground, maximum common mode signal, maximum operating common mode signal, common mode rejection ratio, and common mode overvoltage recovery time.

4.17.1 Differential input impedance to ground

This is the impedance between either the positive input and ground or the negative input and ground. This impedance should be specified at several different frequencies. When the frequency is not specified, the impedance given is the static value. Alternatively, the input impedance can be represented as the parallel combination of a resistance and a capacitance.

4.17.1.1 Test method

For each input, perform the measurement described in 4.2.1 or 4.2.2. When determining the impedance of the positive (negative) input, the negative (positive) input should be appropriately terminated and this termination should be specified.

4.17.2 Common mode rejection ratio (CMRR) and maximum common mode signal level

Common mode rejection ratio (CMRR) is the ratio of the input common mode signal to the effect produced at the output of the recorder in units of the input, T_k. The output codes can be converted to input codes by using

$$V_{out} = Q(k-1) + T_1 \tag{115}$$

CMRR is normally specified as a minimum value in decibels. CMRR shall be specified at various frequencies. The maximum common mode signal level is the maximum level of the common mode signal at which the CMRR is still valid. The maximum common mode signal level must also be specified.

4.17.2.1 Test method

Arrange a network capable of simultaneously applying identical amplitude sine wave signals to both differential inputs. The two common mode signal levels should be identical to within the desired accuracy of the measurement. The common mode signal level (V_{in}) should be large enough to discern an effect in the output data, and it must be equal to or below the specified maximum common mode signal level. Take a record of data. Note the peak-to-peak level of the recorder output [V_{out}, converted into input units using equation (115)] at the common mode sine wave frequency. Compute CMRR in decibels from

$$CMRR = 2\log_{10}\left(\frac{V_{in}}{V_{out}}\right) \tag{116}$$

If no common mode signal is detectable in the output, assign V_{out} the value of $Q/2$. Repeat the measurement at common mode frequencies of interest.

4.17.3 Maximum operating common mode signal

This is the largest common mode signal for which the waveform recorder will meet effective bits specifications in recording a simultaneously-applied normal mode signal.

4.17.3.1 Test method

Arrange a network capable of simultaneously applying identical amplitude sine wave common-mode signals to both differential inputs and a normal-mode large-signal sine wave test signal. Adjust the initial common mode signal level to the specified maximum common mode signal level. Take a record of data. Compute effective bits. Raise or lower the common mode signal amplitude to determine the largest amplitude for which the effective bits specification is met. Repeat the measurement at common mode and normal mode sine wave frequencies of interest.

4.17.4 Common mode overvoltage recovery time

This is the time required for the recorder to return to its specified characteristics after the end of a common mode overvoltage pulse. A common mode overvoltage is a signal level whose magnitude is less than the specified maximum safe common mode signal but greater than the maximum operating common mode signal.

Differential amplifiers often have poor CMRR at high frequencies and performance will be degraded following a high level common mode pulse. The output will typically be driven off scale by a common mode pulse. Comments concerning absolute and relative recovery times for normal mode overvoltage in 4.14 will in general apply for common mode overvoltage.

4.17.4.1 Test method for common mode overvoltage recovery time

Arrange a network capable of simultaneously applying both a high-purity sine wave and a common mode overvoltage pulse of specified amplitude, transition durations, and width. Measure absolute and relative recovery times as described in 4.14.1 and 4.14.2.

Annex A
Derivation of the three parameter (known frequency) sine wave curvefit algorithm

(informative)

A closed form solution for sine wave curvefitting is derived in this annex, for cases in which the frequency of the recorded data is known. Assume a solution in the form of

$$y'_n = A_1 \cos(\omega t_n) + B_1 \sin(\omega t_n) + C \tag{A.1}$$

where

ω is known angular input frequency
t_n is sample times

Given a data record y_n of M samples of the input sinusoid measured at times t_n, the sum of the squares of the error between the measured data and the assumed solution waveform is:

$$\sum_{n=1}^{M} \left[y_n - y'_n \right]^2 = \sum_{n=1}^{M} \left[y_n - A_1 \cos(\omega t_n) - B_1 \sin(w t_n) - C \right]^2 \tag{A.2}$$

Setting the partial derivatives with respect to the parameters being fit to zero gives

$$0 = \frac{\partial \varepsilon}{\partial A_1} = -2 \sum_{n=1}^{M} \left[y_n - A_1 \cos(\omega t_n) - B_1 \sin(\omega t_n) - C \right] \cos(\omega t_n) \tag{A.3}$$

$$0 = \frac{\partial \varepsilon}{\partial B_1} = -2 \sum_{n=1}^{M} \left[y_n - A_1 \cos(\omega t_n) - B_1 \sin(\omega t_n) - C \right] \sin(\omega t_n) \tag{A.4}$$

$$0 = \frac{\partial \varepsilon}{\partial C_1} = -2 \sum_{n=1}^{M} \left[y_n - A_1 \cos(\omega t_n) - B_1 \sin(\omega t_n) - C \right] \tag{A.5}$$

$$\text{Defining } \alpha_n = \cos(\omega t_n) \text{ and } \beta_n = \sin(\omega t_n) \tag{A.6}$$

and rearranging, gives

$$\sum_{n=1}^{M} y_n \alpha_n = A_1 \sum_{n=1}^{M} \alpha_n^2 + B_1 \sum_{n=1}^{M} \alpha_n \beta_n + C \sum_{n=1}^{M} \alpha_n \tag{A.7}$$

$$\sum_{n=1}^{M} y_n \beta_n = A_1 \sum_{n=1}^{M} \alpha_n \beta_n + B_1 \sum_{n=1}^{M} \beta_n^2 + C \sum_{n=1}^{M} \beta_n \tag{A.8}$$

$$\sum_{n=1}^{M} y_n = A_1 \sum_{n=1}^{M} \alpha_n + B_1 \sum_{n=1}^{M} \beta_n + CM \tag{A.9}$$

The three fit parameters are computed from these three linear equations. By linear algebra

$$A_1 = \frac{A_N}{A_D} \tag{A.10}$$

where

$$A_N = \frac{\sum_{n=1}^{M} y_n\alpha_n - \bar{y}\sum_{n=1}^{M}\alpha_n}{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\beta}\sum_{n=1}^{M}\alpha_n} - \frac{\sum_{n=1}^{M} y_n\beta_n - \bar{y}\sum_{n=1}^{M}\beta_n}{\sum_{n=1}^{M}\beta_n^2 - \bar{\beta}\sum_{n=1}^{M}\beta_n} \tag{A.11}$$

$$A_D = \frac{\sum_{n=1}^{M}\alpha_n^2 - \bar{\alpha}\sum_{n=1}^{M}\alpha_n}{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\beta}\sum_{n=1}^{M}\alpha_n} - \frac{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\alpha}\sum_{n=1}^{M}\beta_n}{\sum_{n=1}^{M}\beta_n^2 - \bar{\beta}\sum_{n=1}^{M}\beta_n} \tag{A.12}$$

$$B_1 = \frac{B_N}{B_D} \tag{A.13}$$

where

$$B_N = \frac{\sum_{n=1}^{M} y_n\alpha_n - \bar{y}\sum_{n=1}^{M}\alpha_n}{\sum_{n=1}^{M}\alpha_n^2 - \bar{\alpha}\sum_{n=1}^{M}\alpha_n} - \frac{\sum_{n=1}^{M} \mathrm{y}_n\beta_n - \bar{\mathrm{y}}\sum_{n=1}^{M}\beta_n}{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\alpha}\sum_{n=1}^{M}\beta_n} \tag{A.14}$$

$$B_D = \frac{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\beta}\sum_{n=1}^{M}\alpha_n}{\sum_{n=1}^{M}\alpha_n^2 - \bar{\alpha}\sum_{n=1}^{M}\alpha_n} - \frac{\sum_{n=1}^{M}\beta_n^2 - \bar{\beta}\sum_{n=1}^{M}\beta_n}{\sum_{n=1}^{M}\alpha_n\beta_n - \bar{\alpha}\sum_{n=1}^{M}\beta_n} \tag{A.15}$$

$$C = \bar{y} - A_1\bar{\alpha} - B_1\bar{\beta} \tag{A.16}$$

where

$$\bar{y} = \frac{1}{M}\sum_{n=1}^{M} y_n \tag{A.17}$$

$$\bar{\alpha} = \frac{1}{M}\sum_{n=1}^{M}\alpha_n \tag{A.18}$$

$$\bar{\beta} = \frac{1}{M}\sum_{n=1}^{M}\beta_n \tag{A.19}$$

To make these computations it is necessary to compute the following sums:

$$\sum_{n=1}^{M} y_n \qquad \sum_{n=1}^{M}\alpha_n \qquad \sum_{n=1}^{M}\beta_n \qquad \sum_{n=1}^{M}\alpha_n\beta_n$$

$$\sum_{n=1}^{M}\alpha_n^2 \qquad \sum_{n=1}^{\mathrm{M}}\beta_n^2 \qquad \sum_{n=1}^{M}\mathrm{y}_n\alpha_n \qquad \sum_{n=1}^{M}\mathrm{y}_n\beta_n$$

Now the rms error can be computed as

$$\varepsilon_{rms} = \sqrt{\frac{\varepsilon}{M}} \tag{A.20}$$

Recall from equation (A.2), and using equation (A.6)

$$\varepsilon = \sum_{n=1}^{M}\left[y_n - A_1\alpha_n - B_1\beta_n - C\right]^2$$

$$= \sum_{n=1}^{M} y_n^2 - A_1^2\sum_{n=1}^{M}\alpha_n^2 + B_1^2\sum_{n=1}^{M}\beta_n^2 + MC^2$$

$$-2A_1\sum_{n=1}^{M}\alpha_n y_n - 2B_1\sum_{n=1}^{M}\beta_n y_n - 2C\sum_{n=1}^{M} y_n$$

$$+ 2A_1B_1\sum_{n=1}^{M}\alpha_n\beta_n + 2A_1C\sum_{n=1}^{M}\alpha_n + 2B_1C\sum_{n=1}^{M}\beta_n \tag{A.21}$$

An additional sum is needed beyond the eight already computed

$$\sum_{n=1}^{M} y_n^2 \tag{A.22}$$

To convert the amplitude and phase to the form

$$y'_n = A\cos(\omega t_n + \theta) + C \tag{A.23}$$

use

$$A = \sqrt{A_1^2 + B_1^2} \tag{A.24}$$

$$\theta = \begin{cases} \tan^{-1}\left(\dfrac{-B_1}{A_1}\right) & \text{if } A_1 \geq 0 \\ \tan^{-1}\left(\dfrac{-B_1}{A_1}\right) + \pi & \text{if } A_1 < 0 \end{cases} \tag{A.25}$$

Since the answer is in closed form, convergence is ensured.

Annex B
Derivation of the four parameter (general case) sine wave curvefit algorithm

(informative)

Assume a data record containing M samples of amplitude y_n at times t_n, which consists of a signal that is predominantly a single sine wave. This signal may be contaminated by some form of noise, either *coherent* or *random*. The goal is to obtain estimates, in the least squares sense, of the frequency ω, phase θ, amplitude A, and dc offset C of the sinusoid from these M samples. In addition, we may compute the rms error that may have been introduced into the data by the noise processes.

Assume a solution of the form

$$y_n' = A\cos(\omega t_n + \theta) + C \tag{B.1}$$

The sum of squares of the error between this assumed waveform and the measured data is

$$\varepsilon = \sum_{n=1}^{M}\left[y_n - A\cos(\omega t_n + \theta) - C\right]^2 \tag{B.2}$$

One needs to choose A, ω, θ, and C to minimize the value of ε. This is accomplished by setting four partial derivatives with respect to these parameters to zero, thereby obtaining four simultaneous (but nonlinear) equations in unknowns A, ω, θ, and C. Then eliminate A and C to obtain a pair of transcendental equations in ω and θ. Once these are solved for ω and θ, substitute these into previous equations to obtain A and C.

The partial derivatives of ε are

$$\frac{\partial\varepsilon}{\partial A} = 0 = -2\sum_{n=1}^{M}\left[y_n - A\cos(\omega t_n + \theta) - C\right]\cos(\omega t_n + \theta) \tag{B.3}$$

$$\frac{\partial\varepsilon}{\partial C} = 0 = -2\sum_{n=1}^{M}\left[y_n - A\cos(\omega t_n + \theta) - C\right] \tag{B.4}$$

$$\frac{\partial\varepsilon}{\partial \omega} = 0 = -2\sum_{n=1}^{M}\left[y_n - A\cos(\omega t_n + \theta) - C\right]At_n\sin(\omega t_n + \theta) \tag{B.5}$$

$$\frac{\partial\varepsilon}{\partial \theta} = 0 = -2\sum_{n=1}^{M}\left[y_n - A\cos(\omega t_n + \theta) - C\right]A\sin(\omega t_n + \theta) \tag{B.6}$$

So the following four simultaneous equations are obtained:

$$\sum_{n=1}^{M} y_n\cos(\omega t_n + \theta) = A\sum_{n=1}^{M}\cos^2(\omega t_n + \theta) + C\sum_{n=1}^{M}\cos(\omega t_n + \theta) \tag{B.7}$$

$$\sum_{n=1}^{M} y_n = A \sum_{n=1}^{M} \cos(\omega t_n + \theta) + MC \tag{B.8}$$

$$\sum_{n=1}^{M} y_n t_n \sin(\omega t_n + \theta) =$$

$$A \sum_{n=1}^{M} \cos(\omega t_n + \theta) \sin(\omega t_n + \theta) + C \sum_{n=1}^{M} t_n \sin(\omega t_n + \theta) \tag{B.9}$$

$$\sum_{n=1}^{M} y_n \sin(\omega t_n + \theta) =$$

$$A \sum_{n=1}^{M} \cos(\omega t_n + \theta) \sin(\omega t_n + \theta) + C \sum_{n=1}^{M} \sin(\omega t_n + \theta) \tag{B.10}$$

C can be disposed of by rearranging equation (B.8) and substituting into the other equations as follows:

$$C = \frac{1}{M}\left[\sum_{n=1}^{M} y_n - A \sum_{n=1}^{M} \cos(\omega t_n + \theta)\right] \tag{B.11}$$

into equation (B.7)

$$\sum_{n=1}^{M} y_n \cos(\omega t_n + \theta) - \frac{1}{M} \sum_{n=1}^{M} y_n \sum_{n=1}^{M} \cos(\omega t_n + \theta) =$$

$$A\left[\sum_{n=1}^{M} \cos^2(\omega t_n + \theta) - \frac{1}{M}\left(\sum_{n=1}^{M} \cos(\omega t_n + \theta)\right)^2\right] \tag{B.12}$$

into equation (B.9)

$$\sum_{n=1}^{M} y_n t_n \sin(\omega t_n + \theta) - \frac{1}{M} \sum_{n=1}^{M} y_n \sum_{n=1}^{M} t_n \sin(\omega t_n + \theta) = \tag{B.13}$$

$$A\left[\sum_{n=1}^{M} t_n \cos(\omega t_n + \theta) \sin(\omega t_n + \theta) - \frac{1}{M} \sum_{n=1}^{M} \cos(\omega t_n + \theta) \sum_{n=1}^{M} t_n \sin(\omega t_n + \theta)\right]$$

into equation (B.10)

$$\sum_{n=1}^{M} y_n \sin(\omega t_n + \theta) - \frac{1}{M} \sum_{n=1}^{M} y_n \sum_{n=1}^{M} \sin(\omega t_n + \theta) = \tag{B.14}$$

$$A\left[\sum_{n=1}^{M}\cos(\omega t_n+\theta)\sin(\omega t_n+\theta)-\frac{1}{M}\sum_{n=1}^{M}\cos(\omega t_n+\theta)\sum_{n=1}^{M}\sin(\omega t_n+\theta)\right]$$

Eliminate A by dividing equation (B.13) and equation (B.14) by equation (B.12), leaving two equations in the two unknowns ω and θ, as follows:

$$\frac{\sum_{n=1}^{M}(y_n-\bar{y})t_n\beta_n}{\sum_{n=1}^{M}(y_n-\bar{y})\alpha_n} = \frac{\sum_{n=1}^{M}(\alpha_n-\bar{\alpha})t_n\beta_n}{\sum_{n=1}^{M}(\alpha_n-\bar{\alpha})\alpha_n} \tag{B.15}$$

$$\frac{\sum_{n=1}^{M}(y_n-\bar{y})\beta_n}{\sum_{n=1}^{M}(y_n-\bar{y})\alpha_n} = \frac{\sum_{n=1}^{M}(\alpha_n-\bar{\alpha})\beta_n}{\sum_{n=1}^{M}(\alpha_n-\bar{\alpha})\alpha_n} \tag{B.16}$$

where

$$\bar{y} = \frac{1}{M}\sum_{n=1}^{M}y_n \tag{B.17}$$

$$\bar{\alpha} = \frac{1}{M}\sum_{n=1}^{M}\cos(\omega t_n+\theta) \tag{B.18}$$

$$\alpha_n = \cos(\omega t_n+\theta) \tag{B.19}$$

$$\beta_n = \sin(\omega t_n+\theta) \tag{B.20}$$

These are nonlinear equations, and must be solved in an iterative fashion, by first assuming values for θ and ω, and then correcting these values in sequence until these two equations are satisfied. For the moment, assume that suitably close starting values for θ and ω are known. Define two error parameters R and S as follows:

$$R = \frac{\sum_{n=1}^{M}(y_n-\bar{y})t_n\beta_n}{\sum_{n=1}^{M}(y_n-\bar{y})\alpha_n} - \frac{\sum_{n=1}^{M}(\alpha_n-\bar{\alpha})t_n\beta_n}{\sum_{n=1}^{M}(\alpha_n-\bar{\alpha})\alpha_n} \tag{B.21}$$

$$S = \frac{\sum_{n=1}^{M}(y_n-\bar{y})\beta_n}{\sum_{n=1}^{M}(y_n-\bar{y})\alpha_n} - \frac{\sum_{n=1}^{M}(\alpha_n-\bar{\alpha})\beta_n}{\sum_{n=1}^{M}(\alpha_n-\bar{\alpha})\alpha_n} \tag{B.22}$$

These two parameters are determined by the measured numbers y_n and t_n, together with the estimates of θ and ω, and will both go to zero when the correct values of θ and ω are found. To handle these nonlinear equations, use an approximation technique to derive an iteration algorithm. Assume that the true sinusoid is given by

$$y = B\cos(\psi t + \phi) + D \tag{B.23}$$

then estimate y by using y_n, (the values of B, ψ, ϕ, or D are not known, but A, ω, θ, and C are the best estimates when $R = S = 0$); therefore,

$$y_n \approx A\ \cos(\omega t_n + \theta) + C \tag{B.24}$$

Assume that ψ and ϕ are reasonably close to ω and θ; then expand R and S in two dimensional Taylor's series about the point (ω, θ). Keeping only the first order terms from each series, write

$$R(\psi,\phi) \approx \frac{\partial R}{\partial \psi}\bigg|\omega,\theta(\psi - \omega) + \frac{\partial R}{\partial \phi}\bigg|\omega,\theta(\phi - \theta) \tag{B.25}$$

$$S(\psi,\phi) \approx \frac{\partial S}{\partial \psi}\bigg|\omega,\theta(\psi - \omega) + \frac{\partial S}{\partial \phi}\bigg|\omega,\theta(\phi - \theta) \tag{B.26}$$

This is a pair of linear equations in ψ and ϕ with all other terms known, so one can solve for ψ and ϕ, and use these values as new estimates for ω and θ. Since y_n is dependent on ψ and ϕ, use equation (B.24) to assist in taking the partials.

$$\bar{y} = \frac{B}{M}\sum_{n=1}^{M}\cos(\psi t_n + \phi) + D \tag{B.27}$$

$$y_n - \bar{y} = B\left[\cos(\psi t_n + \phi) - \frac{1}{M}\sum_{n=1}^{M}\cos(\psi t_n + \phi)\right] \tag{B.28}$$

$$\frac{\partial(y_\mathrm{n} - \bar{y})}{\partial\psi}\bigg|_{\omega,\theta} = -Bt_n(\beta_n - \bar{\beta}) \tag{B.29}$$

$$\frac{\partial(y_\mathrm{n} - \bar{y})}{\partial\phi}\bigg|_{\omega,\theta} = B(\beta_n - \bar{\beta}) \tag{B.30}$$

where

$$\bar{\beta} = \frac{1}{M}\sum_{n=1}^{M}\beta_n \tag{B.31}$$

Using this information, one can now write the partials of R and S as follows:

$$a_{11} = \frac{\partial R}{\partial \psi}\bigg|_{\omega,\theta} \tag{B.32}$$

$$= \frac{\sum_{n=1}^{M} \beta_n t_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \alpha_n t_n(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2} - \frac{\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \beta_n t_n^2(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2}$$

$$a_{12} = \left.\frac{\partial R}{\partial \phi}\right|_{\omega,\theta} \tag{B.33}$$

$$= \frac{\sum_{n=1}^{M} \beta_n t_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \alpha_n(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2} - \frac{\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \beta_n t_n(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2}$$

$$a_{21} = \left.\frac{\partial S}{\partial \psi}\right|_{\omega,\theta} \tag{B.34}$$

$$= \frac{\sum_{n=1}^{M} \beta_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \alpha_n t_n(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2} - \frac{\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \beta_n t_n(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2}$$

$$a_{22} = \left.\frac{\partial S}{\partial \phi}\right|_{\omega,\theta} \tag{B.35}$$

$$= \frac{\sum_{n=1}^{M} \beta_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \alpha_n(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2} - \frac{\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha}) \sum_{n=1}^{M} \beta_n(\beta_n - \bar{\beta})}{\left[\sum_{n=1}^{M} \alpha_n(\alpha_n - \bar{\alpha})\right]^2}$$

Rewrite equations (B.25) and (B.26) as

$$R = a_{11}(\psi - \omega) + a_{12}(\phi - \theta) \tag{B.36}$$

$$S = a_{21}(\psi - \omega) + a_{22}(\phi - \theta) \tag{B.37}$$

from which one obtains

$$\psi = \omega + \frac{a_{22}R - a_{12}S}{a_{11}a_{22} - a_{12}a_{21}} \tag{B.38}$$

$$\phi = \theta + \frac{a_{11}S - a_{21}R}{a_{11}a_{22} - a_{12}a_{21}} \tag{B.39}$$

Replace the old estimates of ω and θ with these newly calculated values of ω and θ respectively, and recalculate the 15 sums involving α_n, β_n, t_n, and y_n, to obtain new values of *R, S,* and the four partial derivatives a_{11}, a_{12}, a_{21}, and a_{22}. This iterative procedure is repeated until *R* and *S* are driven suitably close to zero. The 15 sums needed are as follows:

$$\sum_{n=1}^{M} \alpha_n \qquad \sum_{n=1}^{M} \beta_n \qquad \sum_{n=1}^{M} y_n$$

$$\sum_{n=1}^{M} \alpha_n y_n \qquad \sum_{n=1}^{M} \beta_n y_n \qquad \sum_{n=1}^{M} \alpha_n \beta_n$$

$$\sum_{n=1}^{M} \beta_n^2 \qquad \sum_{n=1}^{M} \alpha_n^2 \qquad \sum_{n=1}^{M} \beta_n t_n y_n$$

$$\sum_{n=1}^{M} \alpha_n \beta_n t_n \qquad \sum_{n=1}^{M} \beta_n^2 t_n \qquad \sum_{n=1}^{M} \beta_n^2 t_n^2$$

$$\sum_{n=1}^{M} \beta_n t_n \qquad \sum_{n=1}^{M} \alpha_n t_n \qquad \sum_{n=1}^{M} \beta_n t_n^2$$

The resulting values of ω and θ are the best least squares estimates of the angular frequency and phase of the measured signal. One can use equations (B.12), (B.13), or (B.14) to determine *A*. Summing all three equations gives

$$A = \frac{\sum_{n=1}^{M} (y_n - \bar{y})(\alpha_n + \beta_n + \beta_n t_n)}{\sum_{n=1}^{M} (\alpha_n - \bar{\alpha})(\alpha_n + \beta_n + \beta_n t_n)} \tag{B.40}$$

and, from equation (B.11), the dc offset is given by

$$C = \bar{y} - A\bar{\alpha} \tag{B.41}$$

This process can diverge for initial estimates of ω and θ that are grossly incorrect.

The rms error is

$$\left(\frac{\varepsilon}{M}\right)^{\frac{1}{2}} \tag{B.42}$$

Recalling that

$$\varepsilon = \sum_{n=1}^{M} \left[y_n - A\cos(\omega t_n + \theta) - C\right]^2$$

$$= \sum_{n=1}^{M} \left[y_n^2 + A^2\cos^2(\omega t_n + \theta) + C^2 - 2Ay_n\cos(\omega t_n + \theta) - 2Cy_n + 2AC\cos(\omega t_n + \theta)\right]$$

$$= \sum_{n=1}^{M} \left[y_n^2 + A^2\alpha_n^2 + C^2 - 2Ay_n\alpha_n - 2Cy_n + 2AC\alpha_n\right]$$

$$\varepsilon = \sum_{n=1}^{M} y_n^2 + A^2 \sum_{n=1}^{M} \alpha_n^2 - 2A \sum_{n=1}^{M} \alpha_n y_n + C^2 \sum_{n=1}^{M} 1$$

$$-2C \sum_{n=1}^{M} y_n + 2AC \sum_{n=1}^{M} \alpha_n \qquad \text{(B.43)}$$

therefore

$$\frac{\varepsilon}{M} = \frac{1}{M}\sum_{n=1}^{M} y_n^2 + \frac{A^2}{M}\sum_{n=1}^{M} \alpha_n^2 - 2\frac{A}{M}\sum_{n=1}^{M} \alpha_n y_n + C^2 - 2C\bar{y} + 2AC\bar{\alpha} \qquad \text{(B.44)}$$

With the addition of the sixteenth sum,

$$\sum_{n=1}^{M} y_n^2$$

the previous iteration's rms error can be calculated from the sums already computed in each iteration.

Annex C
Comments on errors associated with word-error-rate measurement

(informative)

There are statistical errors associated with word error rate measurements. Assuming the source of word errors is purely random and that both the number of observed word errors and the total number of trial samples are both statistically significant numbers (see 4.15.2), then the best estimate of error associated with the error rate is given as follows:

$$\sigma(e) = \frac{\left[x\left(1 - \frac{x}{S}\right)\right]^{\frac{1}{2}}}{S} \tag{C.1}$$

where

x is the number of word errors detected

S is the total number of trial samples

w is $\frac{x}{S}$, the estimated word error rate (uncorrected)

For a given confidence level, Φ *[X],* expressed as a fraction, the worst-case error rate w′ is given by

$$w' = w + X\sigma \tag{C.2}$$

where

X is number of standard deviations covered by confidence Φ *[X]* in a Gaussian distribution, and may be found in standard statistical tables

A few examples of X should cover most applications as follows in table C.1:

Table C.1—Gaussian distribution

Φ [X]	X
0.80	0.84
0.90	1.29
0.95	1.65
0.99	2.33

For some measurements, where word error rates are very low, it may be that x is small or even zero. The formula above is as good as any other estimate provided that x isn't zero. However, even for $x = 0$ there is a way to estimate an upper limit on the actual word error rate.

For the case of zero observed word errors (i.e., $x = 0$), for any assumed worst-case error rate (w') one may calculate the probability that not one word error would be seen for S trials as follows:

$$p = (1 - w')^S \tag{C.3}$$

When a confidence level of $\Phi[X]$ is desired, expressed as a fraction that w′ is the worst-case estimate of the real word error rate, then it follows that

$$w' \le 1 - (1 - \Phi[X])^{1/S} \tag{C.4}$$

For example, when $x = 0$, and $S = 100\ 000$ for a 0.95 confidence level, according to equation (C.4), the worst-case word error rate is $3.0 \cdot 10^{-5}$ or less. For the same confidence level, using the sigma from equation (C.1) (with the approximation that $x = 1$, versus 0), and solving equation (C.2) with $x = 0$ (and therefore $w = 0$), a worst-case word error rate of $1.6 \cdot 10^{-5}$ is determined. This is a reasonable correspondence to the value calculated using equation (C.4).

Annex D
Measurement of random noise below the quantization level

(informative)

For digitizers with random noise (rms) greater than one quantization level, the quantization process contributes relatively little additional error, and can be ignored when measuring and calculating the noise level.

For digitizers with random noise (rms) much smaller than one quantization level, however, the measurement technique and the calculation must take into account the effects of quantization. First, the measurement must be taken with some input signal sufficient to exercise the digitizer over several code bins to avoid understatement or overstatement of the effects of random noise as would occur if the input were centered within a code bin or on a transition level. Second, a different equation must be used to calculate the rms input noise, because the rms output error is not linearly related to the rms input.

If the random noise is small enough that, for any given input level, the noise will cause the output to flip between at most two adjacent codes, the noise behavior can be analyzed around each code transition level independently. Assuming the noise has a Gaussian probability distribution function, the probability of seeing output code *k* is

$$P_k(v) = \Phi\left(\frac{v}{\sigma}\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{v/\sigma} e^{-\frac{1}{2}t^2}\,dt \tag{D.1}$$

where

- v is the input value relative to $T(k)$
- σ is the rms noise of the digitizer in the same units as v

Thus, the probability of two samples taken at the same input level near $T(k)$ having different output codes (differing by ± one count) is

$$P_\Delta(v) = 2 \cdot P_k(v) \cdot \left[1 - Pk(v)\right] \tag{D.2}$$

Since this probability of different output codes repeats for every code transition level, the average probability of two samples taken at the same input level having output codes differing by ± one count (where the average is taken over a range of input levels for the pairs) is

$$\bar{P}_\Delta = \frac{1}{Q}\int_{-Q/2}^{Q/2} P_\Delta(v)dv \tag{D.3}$$

Output codes differing by ± one count contribute a squared error of Q^2 so the mean square error between two samples taken at the same input level (again, where the mean is taken over a range of input levels for the pairs) is

$$\text{mse} = Q\int_{-Q/2}^{Q/2} P_\Delta(v)dv \tag{D.4}$$

Assuming that the noise was small enough to not cause more than two adjacent output codes for a given input level

$$P_{\Delta} v \rightarrow 0 \text{ for } |v| > \frac{Q}{2} \tag{D.5}$$

So we can extend the limits of the integration to ±∞ without any effect on the validity of equation (D.4)

$$\text{mse} = Q \int_{-\infty}^{\infty} P_{\Delta}(v) dv \tag{D.6}$$

Combining equations (D.1), (D.2), and (D.6)

$$\text{mse} = Q \int_{-\infty}^{\infty} 2 \left[\Phi\left(\frac{v}{\sigma}\right)\right] \left[1 - \Phi\left(\frac{v}{\sigma}\right)\right] dv \tag{D.7}$$

Letting $s = v/s$

$$\text{mse} = (Q \cdot \sigma) \int_{-\infty}^{\infty} 2 \left[\Phi(s)\right] \left[1 - \Phi(s)\right] ds \tag{D.8}$$

The value of the integral in equation (D.8) is a constant independent of any parameters of the digitizer being tested and can be evaluated by numerical integration to be approximately 1.1284 (or 1/0.886). Using this value for the integral, and solving for σ

$$\sigma = \frac{0.886}{Q} \text{mse} \tag{D.9}$$

Equation (D.9) provides an accurate estimate of a digitizer's random noise level for levels well below 1 Q. As mentioned earlier, for random noise levels above 1 Q, the quantizing effect can be ignored, and the noise level can be accurately estimated from equation (101), which can be rewritten in terms of the mean square error between pairs of samples as

$$\sigma = \left(\frac{\text{mse}}{2}\right)^{\frac{1}{2}} \tag{D.10}$$

Monte Carlo simulations show equation (D.9) is accurate within a few percent for values of σ less than $Q/4$, and that equation (D.10) is accurate to within a few percent for values of σ greater than Q. Both formulas err by almost 20% for $\sigma = Q/2$. A heuristically derived formula that automatically crosses between equation (D.9) and equation (D.10) based on the value of the mean square error relative to Q is

$$\sigma = \left[\left(\frac{\text{mse}}{2}\right)^{-2} + \left(\frac{0.886\text{mse}}{Q}\right)^{-4}\right]^{-\frac{1}{4}} \tag{D.11}$$

Monte Carlo simulations show equation (D.11) is accurate to within a few percent for any value of σ.

Annex E
Bibliography

(informative)

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