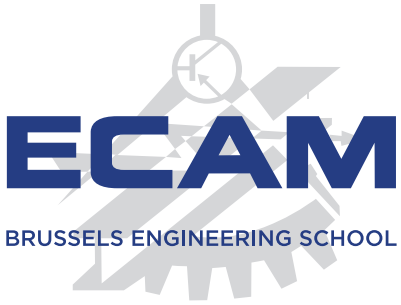


E301B C Programming

Session 2

Data Memory Representation

Sébastien Combéfis

Fall 2019

Objectives

- Analysing how data are represented in the computer
  - Number representation, positional system and binary system
  - `int` (two's complement), `float` (IEEE 754) and `char` (ASCII)
  - Arithmetic operations with integer and floating-point numbers
- Data conversion between different data types

  Rules of implicit and explicit conversions (cast)
- Specific operators to manipulate bits of data

  Logical operators, bit shift operators and ones' complement

Number Representation

Information

- Information can take several different forms

  Number, instruction, image, sound, video, etc.

- The bit is the elementary information
  - 0 or 1 (binary digit)
  - Encoding to map external/internal representation ($A \leftrightarrow 65...$)
- Two kinds of information are dealt with by the computer
  - **Instructions** are executed by the computer
  - **Data** are manipulated by these latter to perform computation

Bit Grouping

- Byte (*B*) sometimes referred to as *char*
  - Set of adjacent bits, often 8, but 6, 7, 9 also exist
  - Smallest data unit accessible through a data bus
- *Eight-bit byte* (*o*) sometimes referred to as octet

  A byte with eight bits encoding a given information
- Word groups bytes or octets together

  We can obtain 32-bit words with four octets/word

char Size

- Size of a `char` can be found in the `limits.h` file

  Number of bits for a `char` stored in the `CHAR_BIT` constant

- Defines the length of a byte for a specific machine

  The value will typically be 8 on most modern system

```c
#include <limits.h>
#include <stdio.h>

int main()
{
    printf("A char has %d bits.\n", CHAR_BIT);

    return 0;
}
```

Amount of Information

- Two kinds of amounts of information used in computer science
  - Memory capacity, disk space, file size, etc. (GB)
  - Speed and transfer speed (Gbps, Gb/s)
- Two kinds of prefix can be used with these units
  - **International System of Units** (SI): powers of 10
  - **International Electrotechnical Commission** (IEC): binary

Binary Prefix

- First definition IEC 60027-2 or more recent IEEE 1541-2002

  Used by Ubuntu >10.10 and macOS >10.6

- Using binary prefixes instead of powers of 10

  Greater difference with large factors

SI Prefix			Binary Prefix			Error
kilo	kB	10^3 B	kibi	KiB	2^{10} B	2%
mega	MB	10^6 B	mébi	MiB	2^{20} B	5%
giga	GB	10^9 B	gibi	GiB	2^{30} B	7%
tera	TB	10^{12} B	tébi	TiB	2^{40} B	10%
peta	PB	10^{15} B	pébi	PiB	2^{50} B	13%
exa	EB	10^{18} B	exbi	EiB	2^{60} B	15%
zetta	ZB	10^{21} B	zébi	ZiB	2^{70} B	18%
yotta	YB	10^{24} B	yobi	YiB	2^{80} B	21%

Legal Disputes

- Hard drives are sometimes sold following decimal meaning...

  A "160 GB" hard drives have 160 billions bytes

- But some operating systems just show "149.01 GB"

  They are counting using binary sense but not showing it so...

- A 100 GB hard drive "only" has 93.13 GiB
  - Class action lawsuits against digital storage manufacturers
  - e.g. Safier v. Western Digital Corporation, July 7, 2005...
  - Added disclaimer on packaging, compensation for customers

1024 or 1000?

THERE'S BEEN A LOT OF CONFUSION OVER 1024 vs 1000, KBYTE vs KBIT, AND THE CAPITALIZATION FOR EACH.

HERE, AT LAST, IS A SINGLE, DEFINITIVE STANDARD:

SYMBOL	NAME	SIZE	NOTES
kB	KILOBYTE	1024 BYTES OR 1000 BYTES	1000 BYTES DURING LEAP YEARS, 1024 OTHERWISE
KB	KELLY-BOOTLE STANDARD UNIT	1012 BYTES	COMPROMISE BETWEEN 1000 AND 1024 BYTES
KiB	IMAGINARY KILOBYTE	1024 √-1 BYTES	USED IN QUANTUM COMPUTING
kb	INTEL KILOBYTE	1023.937528 BYTES	CALCULATED ON PENTIUM F.P.U.
Kb	DRIVEMAKER'S KILOBYTE	CURRENTLY 908 BYTES	SHRINKS BY 4 BYTES EACH YEAR FOR MARKETING REASONS
KBa	BAKER'S KILOBYTE	1152 BYTES	9 BITS TO THE BYTE SINCE YOU'RE SUCH A GOOD CUSTOMER

Bit Sequence

- Information represented by bit sequences

  Typically words or sequences of bytes/octets

- Interpretation according to type of encoded information

  Distinction to make between binary number and bit sequence

- Hexadecimal notation to shorten long sequences

  A nibble (4 bits) represents a hexadecimal digit

$$\underbrace{0010}_{2}\ \underbrace{1101}_{D}\ \underbrace{0100}_{4}$$

Number

- Concept to evaluate and compare quantities

  Can also be used to order items by numbering

- Number relation studied by arithmetic and number theory

  Interaction through operations summarised by calculation rules

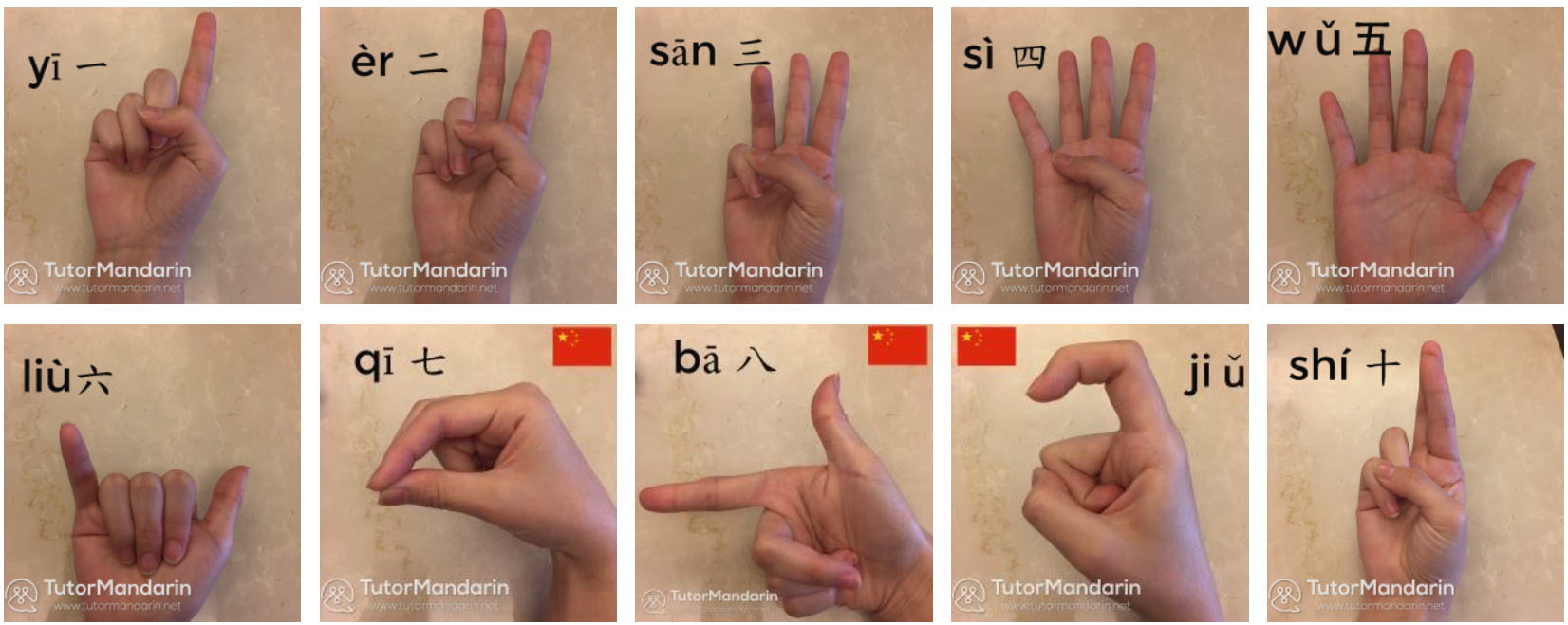
- Numbering system to represent numbers

  Set of signs, words, gestures to write/state numbers

Chinese Number Gesture

- Chinese people can count from one to nine with only one hand

  Ordering food at noisy market, tell the waiter number of people...



Unary System

- Juxtaposition of the corresponding amount of a symbol

  One unique symbol available that represents the unit

- Definition of the four basic operations
  - **Addition** by concatenation: $|||+||=|||||$
  - **Subtraction** by withdrawal: $|||-||=|$
  - **Multiplication** by substitution: $|||\times||=||||||$
  - **Division** by substitution: $|||\div||=|$, with remainder $|$

Positional Notation

- Other numbering systems: binary, quinary, vigesimal...

  Grouping units by packets following a numbering base

- Positional system to write down a number with digits
  - Position of the digit in the writing indicates weight assigned
  - System in base b requires b digits (from 0 to $b-1$)

- Value in base 10 of the number n written $d_{k-1}...d_1 d_0$ in base b

$$n = \left(\sum_{i=0}^{k-1} d_i b^i \right)_{10} = \left(d_0 + d_1 \cdot b + ... + d_{k-1} \cdot b^{k-1} \right)_{10}$$

with $d_i \in \{0, ..., b-1\}$

Common Base

- Several common bases often used in computer science
  - Binary (base 2): bit vectors, flags, etc.
  - Octal (base 8): not much used anymore
  - Hexadecimal (base 16): colour codes, MD5 checksum, etc.
- For example, let's take the $n = (42)_{10}$ number
  - $n = (101010)_2 = 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^1 = (42)_{10}$
  - $n = (52)_8 = 5 \times 8^1 + 2 \times 8^0 = (42)_{10}$
  - $n = (2A)_{16} = 2 \times 16^1 + 10 \times 16^0 = (42)_{10}$ $\qquad (A \leftrightarrow 10\ldots)$

Literal Form of `int`

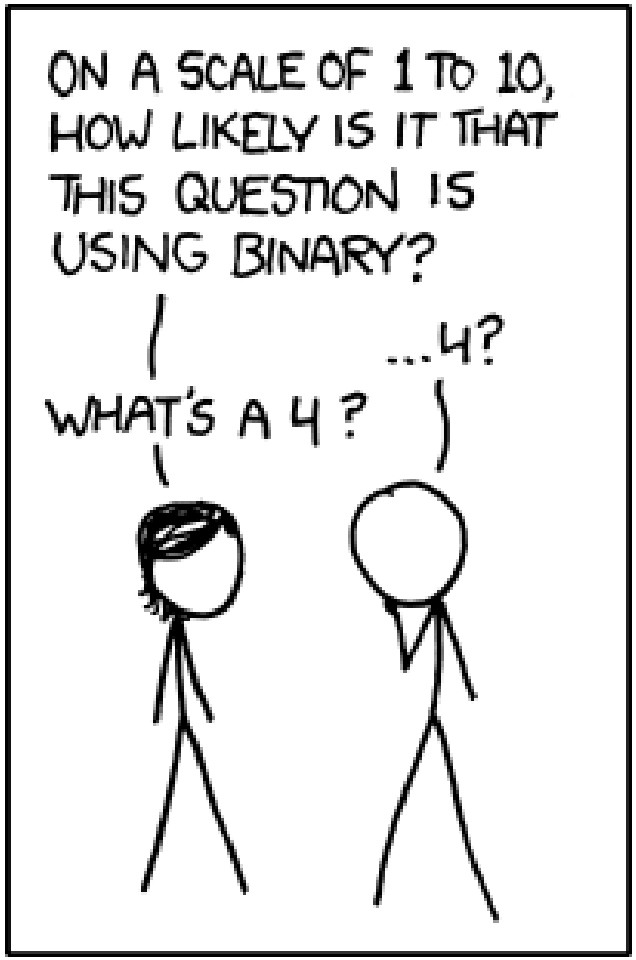
- Writing a constant integer number with its literal form

  Just write its digits one after the other, with a possible sign

- Using a prefix to determine the base
  - By default, integer literal written in base 10
  - Adding `0b` for base 2, `0` for base 8 and `0x` for base 16

```
1  int a = 0b101010;
2  int b = 0123;
3  int c = 0xABC;
4
5  printf("%d %d %d\n", a, b, c);       // Prints "42 83 2748"
```

Binary...


ON A SCALE OF 1 TO 10, HOW LIKELY IS IT THAT THIS QUESTION IS USING BINARY?
WHAT'S A 4?
...4?

Binary System

- Number n written $b_{k-1}...b_1 b_0$ as a sum of powers of two

$$n = \left(\sum_{i=0}^{k-1} b_i 2^i \right)_{10} = \left(b_0 + b_1 \cdot 2 + ... + b_{k-1} \cdot 2^{k-1} \right)_{10}$$

with $b_i \in \{0, 1\}$

- In computer science, limited length binary numbers

  Due to memory limitation and computation performances

MSB ↓ *LSB* ↓

$$1011010100 = (\mathbf{1} \cdot 2^9 + \mathbf{0} \cdot 2^8 + ... + \mathbf{0} \cdot 2^0)_{10}$$

Integer Number

Integer Number Representation

- Representing integers on a finite number of bits k
  - Limits on the number of representable numbers
  - With k bits, there are only 2^k possibilities
- Several constraints influence the representation
  - Should it be possible to represent negative numbers?
  - Should all the available space be used or not?
  - Should some kinds of operations be easier to compute?

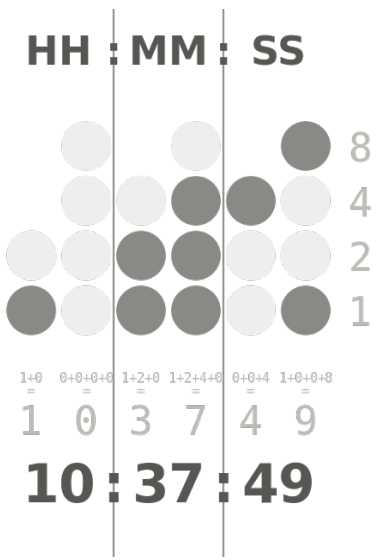
Binary-Coded Decimal (BCD)

- An integer is represented by a sequence of nibbles

  Any decimal digit can be represented with 4 bits

- Encoding in compacted or expanded form in octets

  This is using or not the two nibbles of an octet



Sign Bit

- Most significant bit used to indicate the sign

  Positive integer with 0 *and negative integer with* 1

- Other bits represent the integer as a binary number

  Bit usage: 1 *sign bit and* $k-1$ *information bits*

- Space of representable integers of size $2 \cdot 2^{k-1} - 1$
  - Integers between $-(2^{k-1}-1)$ and $2^{k-1}-1$
  - Be careful, two possible representations for zero (-0 and $+0$)

$$00101010 : (42)_{10} \qquad 10101010 : (-42)_{10}$$

Ones' Complement

- Ones' complement of a binary number by inverting all its bits
  - Negative of original number for some operations
  - Ones' complement of n can be computed as $(2^k - 1) - n$
- Space of representable integers of size $2^k - 1$
  - Integers between $-(2^{k-1} - 1)$ and $2^{k-1} - 1$
  - Be careful, two possible representations for zero (-0 and $+0$)
  - First bit used to identify the sign of the integer

$$00000010 : (2)_{10} \qquad 11111101 : (-2)_{10}$$

Two's Complement (1)

- Two's complement of k-bit number with respect to 2^k
  - Ones' complement of n can be computed as $2^k - n$
  - Two's complement by adding 1 to the ones' complement
- Space of representable integers of size 2^k
  - Integers between -2^{k-1} and $2^{k-1} - 1$
  - First bit used to identify the sign of the integer

$$00000010 : (2)_{10} \qquad 11111110 : (-2)_{10}$$

Two's Complement (2)

- Two's complement eases fundamental arithmetic operations
  - Used to represent fixed point binary values
  - Simple operations on the underlying bits
- Automatic detection of overflow
  - Two operands with the same sign and result with opposite sign
  - No overflow with two opposite signs operands
- Sign extension to increase the number of bits

  Just by repeating the sign bit (MSB)

Sum of Integers

- Sum of two integers with two's complement

  Both encoded with the same number k of bits

- Non representable sum results in an overflow

  Overflow indicator as a bit in the processor

- With $k = 6$ bits, possible to represents any $n \in [-32; 31]$

$$\begin{array}{rr} 0\,1\,0\,1\,0\,0 & (20)_{10} \\ +\ 0\,0\,0\,1\,0\,1 & (5)_{10} \\ \hline 0\,1\,1\,0\,0\,1 & (25)_{10} \end{array} \qquad \begin{array}{rr} 0\,1\,0\,1\,0\,0 & (20)_{10} \\ +\ 0\,0\,1\,1\,1\,1 & (15)_{10} \\ \hline 1\,0\,0\,0\,1\,1 & (-29)_{10} \end{array}$$

Subtraction of Integers

- Subtraction of two integers with two's complement

  Both encoded with the same number k of bits

- Just using addition with the negative representation

  Overflow is also possible if the result is too small

- With $k = 6$ bits, possible to represents any $n \in [-32; 31]$

$$\begin{array}{rll} 0\,0\,0\,1\,1\,1 & (7)_{10} \\ +\ 1\,0\,1\,1\,0\,1 & (-19)_{10} \\ \hline 1\,1\,0\,1\,0\,0 & (-12)_{10} \end{array} \qquad \begin{array}{rll} 1\,0\,1\,1\,1\,1 & (-17)_{10} \\ +\ 1\,0\,1\,1\,0\,1 & (-19)_{10} \\ \hline 0\,1\,1\,1\,0\,0 & (28)_{10} \end{array}$$

Multiplication of Integers

- Multiplication of two integers with two's complement
  - Just a succession of staggered additions
  - Be careful that the result can quickly overflow
  - Negate both operands if multiplier is negative
- With $k = 6$ bits, possible to represents any $n \in [-32; 31]$

$$
\begin{array}{rl}
 & 0\,0\,0\,1\,0\,0 \quad (4)_{10} \\
\times & 0\,0\,0\,1\,0\,1 \quad (5)_{10} \\
\hline
 & 0\,0\,0\,1\,0\,0 \\
 & 0\,0\,0\,0\,0\,0 \\
+ & 0\,1\,0\,0\,0\,0 \\
\hline
 & 0\,1\,0\,1\,0\,0 \quad (20)_{10}
\end{array}
\qquad
\begin{array}{rl}
 & 1\,1\,1\,1\,0\,0 \quad (-4)_{10} \\
\times & 0\,0\,0\,1\,0\,1 \quad (5)_{10} \\
\hline
 & 1\,1\,1\,1\,0\,0 \\
 & 0\,0\,0\,0\,0\,0 \\
+ & 1\,1\,0\,0\,0\,0 \\
\hline
 & 1\,0\,1\,1\,0\,0 \quad (-20)_{10}
\end{array}
$$

Division of Integers

- Division of two integers with two's complement
  - Integer division produces a quotient and a remainder
  - Dividing absolute values then computes sign
- With $k = 6$ bits, possible to represents any $n \in [-32; 31]$

$$
\begin{array}{lrl|l|l}
(27)_{10} & 0\,1\,1\,0\,1\,1 & & 1\,0\,1 & (5)_{10} \\
\cline{4-4}
 & 1\,0\,1\,0\,0 & & 0\,1\,0\,1 & (4)_{10} \\
\cline{2-2}
 & 1\,1\,1 & & & \\
 & 1\,0\,1 & & & \\
\cline{2-2}
(2)_{10} & 1\,0 & & &
\end{array}
$$

Integer Overflow

- Multiplication overflows can result in wrong results

  Integer literal are of type `int` by default

- One solution is to use `long` literals constants

  As long as type of variable is big enough for the value to compute

```c
#include <stdio.h>

int main()
{
    long MICROS_DAY = 24 * 60 * 60 * 1000 * 1000;
    long MILLIS_DAY = 24 * 60 * 60 * 1000;

    printf("%d\n", MICROS_DAY / MILLIS_DAY);    // Prints "5"

    return 0;
}
```

Floating-Point Number

Binary Fraction

- Integer and fractional parts separated by a radix point

  Typically the point (.) in the scientific world

- Negative powers of two for the fractional part

  Fundamental arithmetic operations performed as previously

- Value in base 10 of binary fraction n written $l_{k-1}...l_1 l_0.r_1 r_2...r_\ell$

$$
\begin{aligned}
n &= \left(\sum_{i=0}^{k-1} l_i 2^i + \sum_{i=1}^{\ell} r_i 2^{-i} \right)_{10} \\
&= \left(l_0 + l_1 \cdot 2 + ... + l_{k-1} \cdot 2^{k-1} + r_1 \cdot \frac{1}{2} + ... + r_\ell \cdot \frac{1}{2^\ell} \right)_{10}
\end{aligned}
$$

with $l_i, r_i \in \{0, 1\}$

Real Number Representation

- All real numbers cannot be represented

  At least if the number of bits to represent the number is limited

- Fixed point representation with a binary fraction

  Position of the radix point chosen by the programmer

- Floating-point representation similar to scientific notation
  - Based on a significand scaled using an exponent in a given base
  - Trade-off between range and precision

$n = \mathit{significand} \times \mathit{base}^{\mathit{exponent}} \qquad 1.2345 = 12345 \times 10^{-4}$

IEEE 754

- Standardised norm to represent binary floating-point number

  Most common norm in computer science, used by CPU and FPU

- Representation format and special values (infinites and NaN)

  Sign, mantissa, exponent and denormalised number

- IEEE 754 defines five basic formats with extended formats
  - Single precision (32 bits with 24 for significand) `float`
  - Double precision (64 bits with 53 for significand) `double`
  - Double extended ($\geq$ 79 bits, but often 80 bits) `long double`

Single Precision (1)

- Real number n encoded with 32 bits in single precision
  - Sign bit S (1 bit): $s = \pm 1$
  - Exponent E (8 bits): $-126 \leq e \leq 127$
  - Fraction F (23 bits): $1 \leq 1.f < 2$
- Mantissa composed of an implicit leading bit and fraction bits

$$n = (-1)^S \times 2^{(E)_2 - 127} \times (1.F)$$

- $-12.5 = (-1)^1 \times 2^{130-127} \times 1.1001 = -1 \times 2^3 \times \left(1 + \frac{1}{2^1} + \frac{1}{2^4}\right)$

$$\underbrace{1}_{S}\ \underbrace{10000010}_{E}\ \underbrace{10010000000000000000000}_{M}$$

Single Precision (2)

- Denormalised number when zero E and non zero F

  Smaller fractions than with normalised number, closer to 0

- Can represents infinity and special Not A Number (NaN)

  Indeterminate operation (QNaN) or invalid operation (SNaN)

E	**F**	**Type**	**Value**
0	$= 0$	Zero	± 0
0	$\neq 0$	Denormalised number	$(-1)^S \times 2^{-126} \times 0.F$
Between 1 and 254		Normalised number	$(-1)^S \times 2^{(E)_2-127} \times 1.F$
255	$= 0$	Infinite	$\pm\infty$
255	$\neq 0$	NaN	quiet (QNaN) or signalling (SNaN)

IEEE 754 for -17.5

- Transform to $1.F$ by successively dividing 17.5 by 2

$$17.5 = 8.75 \times 2 = 4.375 \times 2^2 = 1.1875 \times 2^3 = 1.09375 \times 2^4$$

- Find the binary fraction F by successively multiplying by 2

$$\begin{aligned} 1.09375 &= \mathbf{1} + 0.09375 \\ 0.1875 &= \mathbf{0} + 0.1875 \\ 0.375 &= \mathbf{0} + 0.375 \\ 0.75 &= \mathbf{0} + 0.75 \\ 1.5 &= \mathbf{1} + 0.5 \\ 1 &= \mathbf{1} + 0 \end{aligned}$$

$$\Rightarrow 1.09375 = 1 + 0.0625 + 0.03125 = 1 + \tfrac{1}{2^4} + \tfrac{1}{2^5} = (1.00011)_2$$

$$\underbrace{1}_{S} \; \underbrace{10000011}_{E = 131} \; \underbrace{00011000000000000000000}_{M}$$

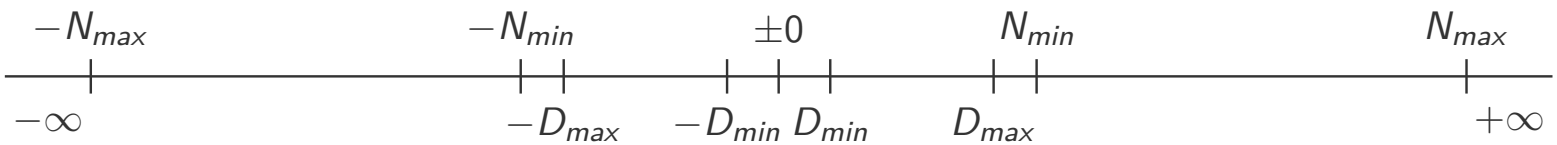
Floating-Point Number

- Floating-point numbers are equidistant between powers of two

  Same number of representable numbers between 2^k and 2^{k+1}

- Denormalised numbers used to get more precision close to 0

  Remembering that not all real numbers can be represented



Double Precision

- Same representation principle as with single precision

  With more bits for the exponent and fraction parts

- Real number n encoded with 64 bits in double precision
  - Sign bit S (1 bit): $s = \pm 1$
  - Exponent E (11 bits): $-1022 \leq e \leq 1023$
  - Fraction F (52 bits): $1 \leq 1.f < 2$

- More bits for the fraction part than for the exponent

  The precision is more important than the amplitude

Multiplication of Floating-Point

- Addition of exponents and multiplication of mantissas

  With possible (re)normalisation steps during calculation

- For example: $((1.10)_2 \times 2^{-3}) \times ((1.11)_2 \times 2^7)$
  - Exponent: $-3 + 7 = 4$
  - Mantissa: $(1.10)_2 \times (1.11)_2 = (10.1010)_2$
  - Result (before normalisation): $(10.1010)_2 \times 2^4$
  - Result (after normalisation): $(1.01010)_2 \times 2^5$

$\Rightarrow (1.5 \times 2^{-3}) \times (1.75 \times 2^7) = 1.3125 \times 2^5 = 42$

Division of Floating-Point

- Subtraction of exponents and division of mantissas

  With possible (re)normalisation steps during calculation

- For example: $((1.01010)_2 \times 2^5) \div ((1.11)_2 \times 2^7)$
  - Exponent: $5 - 7 = -2$
  - Mantissa: $(1.01010)_2 \div (1.11)_2 = (0.11)_2$
  - Result (before normalisation): $(0.11)_2 \times 2^{-2}$
  - Result (after normalisation): $(1.1)_2 \times 2^{-3}$

$$\Rightarrow (1.3125 \times 2^5) \div (1.75 \times 2^7) = 1.5 \times 2^{-3} = 0.1875$$

Addition of Floating-Point

- Addition of mantissas

  With denormalisation to bring back operands to same exponent

- For example: $((1.10)_2 \times 2^4) + ((1.11)_2 \times 2^6)$
  - Denormalisation: $(1.10)_2 \times 2^4 \rightarrow (0.011)_2 \times 2^6$
  - Mantissa: $(0.011)_2 + (1.11)_2 = (10.001)_2$
  - Result (before normalisation): $(10.001)_2 \times 2^6$
  - Result (after normalisation): $(1.0001)_2 \times 2^7$

$\Rightarrow (1.5 \times 2^4) + (1.75 \times 2^6) = 1.0625 \times 2^7 = 136$

Subtraction of Floating-Point

- Subtraction of mantissas
  - With denormalisation to bring back operands to same exponent
  - May need to invert the operands and then fixing the sign
- For example: $((1.11)_2 \times 2^3) - ((1.101)_2 \times 2^7)$
  - Denormalisation: $(1.11)_2 \times 2^3 \rightarrow (0.000111)_2 \times 2^7$
  - Mantissa: $(1.101)_2 - (0.000111)_2 = (1.100001)_2$
  - Result (before normalisation): $(1.100001)_2 \times 2^7$
  - Result (after normalisation): $(1.100001)_2 \times 2^7$

$$\Rightarrow (1.75 \times 2^3) - (1.625 \times 2^7) = 1.515625 \times 2^7 = -194$$

Floating-Point Precision

- Not all reals can be represented with floating-point numbers

  Approximations are made with literals and on computations

- One solution is to make computations with cents

  Stored in integer number variables such as `int` or `long`

```c
#include <stdio.h>

int main()
{
    float price = 3.26;
    float paid = 100.00;
    float change = paid - price;

    printf("Change: %f\n", change);    // Prints "Change: 96.739998"

    return 0;
}
```

Floating-Point Comparison (1)

- Never compare floating-point numbers for (non-)equality

  Due to rounding to closest representable floating-point number

```c
#include <stdio.h>

int main()
{
    float result = 1.0 / 10.0;
    printf("Value: %f\n", result);    // Prints "Value: 0.100000"
    if (result == 0.1) {
        printf("Okay!\n");
    }

    return 0;
}
```

Floating-Point Comparison (2)

- Test floating-point numbers closeness given an ε

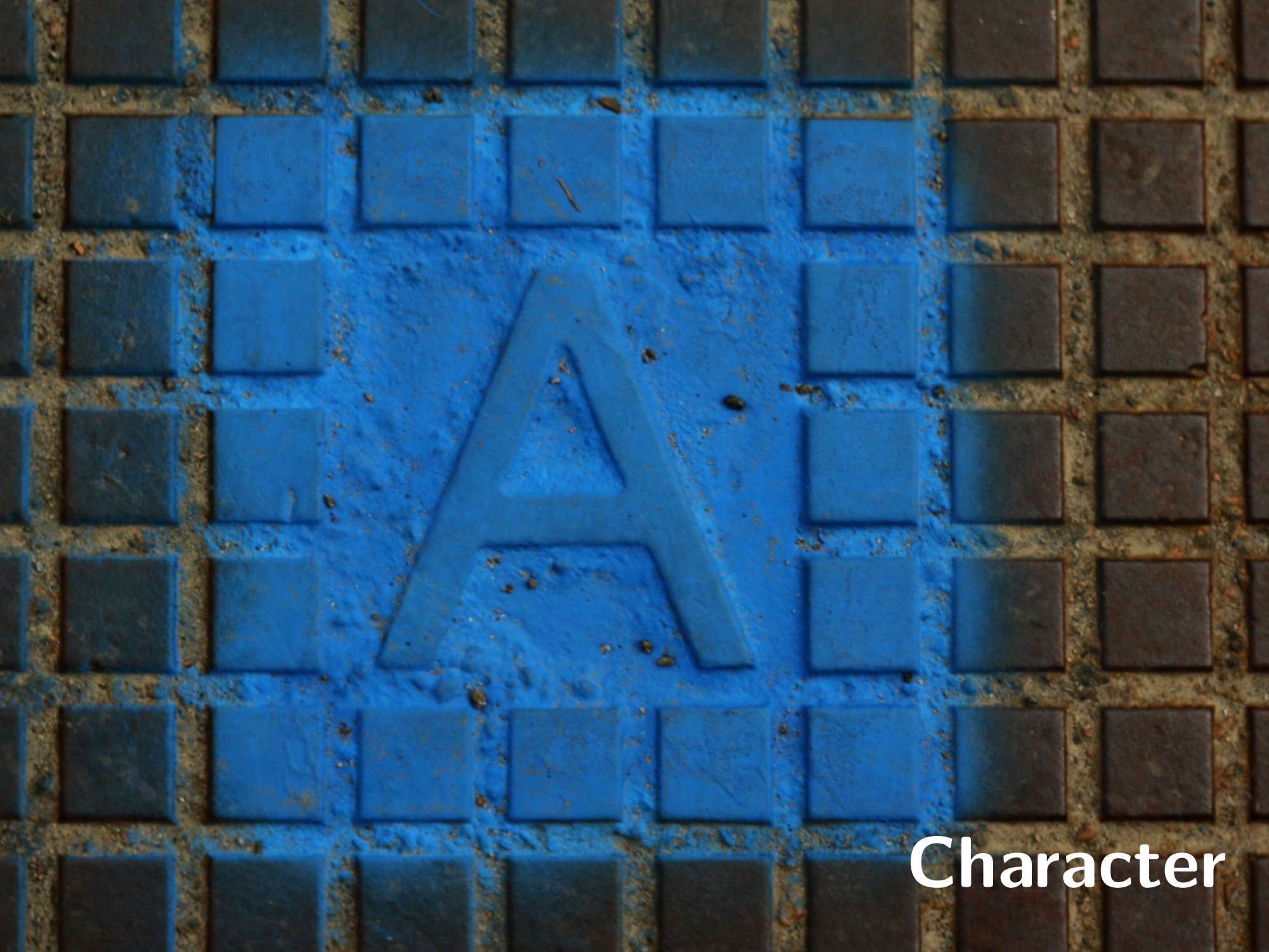
  Use the `fabs` function and the `FLT_EPSILON` constant

```c
#include <float.h>
#include <math.h>
#include <stdio.h>

int almostEqual(float a, float b)
{
    return fabs(a - b) <= FLT_EPSILON;
}

int main()
{
    float result = 1.0 / 10.0;
    printf("Value: %f\n", result);    // Prints "Value: 0.100000"
    if (almostEqual(result, 0.1)) {
        printf("Okay!\n");            // Prints "Value: Okay!"
    }

    return 0;
}
```



Character

Text

- Alphanumeric characters and special characters

  Association of a numeric identity to each character

- Several mapping tables do exist and can used
  - ASCII *(American Standard Code for Information Interchange)* on 7 bits
  - EBCDIC *(Extended Binary Coded Decimal Internal Code)* on 8 bits
  - Unicode on 8, 16 then 32 bits

- Mapping tables are implemented with an encoding
  - Coding/decoding a (sequence of) character(s) into bits
  - Way to represent numeric identities with a sequence of bits

ASCII

- The ASCII character table (iso-646) contains 128 characters

  Enough characters in this table for texts in English

	0	1	2	3	4	5	6	7	8	9	*A*	*B*	*C*	*D*	*E*	*F*
0	NUL	SOH	STH	ETH	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	CD2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	spc	!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	‘	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

For example, $A \leftrightarrow (41)_{16} = (65)_{10}$

- Extended ASCII on 8 bits or more to have more characters

  Basic ASCII extensions such as EBCDIC, for example

`char` Type

- Smallest unit of storage is the `char` occupying one byte

  Typically following an extended ASCII encoding with 8 bits

- Three categories of characters can be defined
  - 95 human readable: letters, digits, punctuation marks
  - 33 not printable but still visible: space, line feed, tab, etc.
  - 128 special codes used by computer: start/end of text, etc.

- Can use some integer number operations with characters

  Typically arithmetic and comparison operators

Print Alphabet

- Letters and digits follow each other in ASCII table

  Alphabet can be printed with a loop, starting with 'A' to 'Z'

```c
#include <stdio.h>

int main()
{
    char c;
    for (c = 'A'; c <= 'Z'; c++)
        printf("%c", c);
    printf("\n");

    return 0;
}
```


Data Conversion

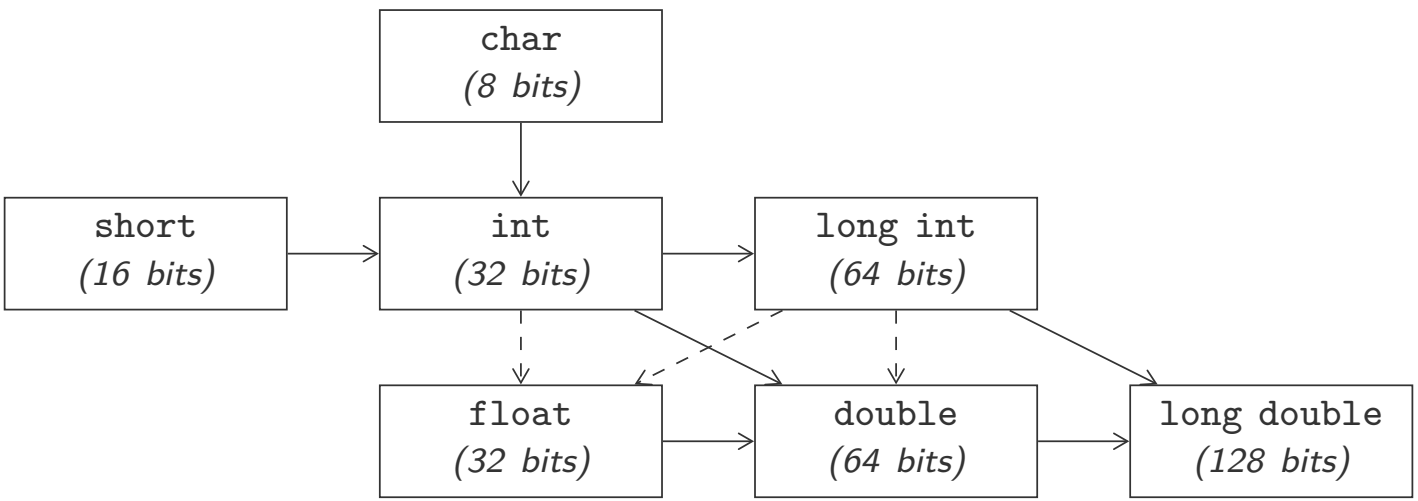
Data Conversion (1)

- Data can be converted from one type to another type

  Same value but represented with another data type

- Only some conversions are possible and make sense

  Depending on involved data types and size occupied in memory



Data Conversion (2)

- Two types of conversion between types are possible
  - Conversion without loss of information to a broader type
  - Conversion with loss of information to a narrower type
- Possible loss of precision converting to floating-point number

  From `int` to `float` and from `long int` to `double`
- Two ways to convert data from one type to another one

  Implicit conversion in several situations and explicit conversion

Implicit Conversion (1)

- Data can be implicitly converted in three different situations
  - By assigning a value to a variable
  - By arithmetic promotion during an operation
  - By compiler conversion
- Implicit conversion occurs without programmer intervention

  Programmer is not always aware that such conversion occurred

```
1  short a = 10L;          // 10 converted from long to short
2  char b = 65;            // 65 converted from int to char
3  double c = 10 * b;      // b converted from char to int and then
4                          // result of 10 * b converted from int to
                           double
```

Implicit Conversion (2)

- Implicit conversion if compiler has information about type
  - Parameters on procedure/fonction calls
  - Returned values in functions
- Two operands converted to the same type before operation
  - The result of the operation is of the larger type
  - Smallest type used is `int` by integer promotion

Explicit Conversion

- Data can be explicitly converted with the cast operator

  Possible to convert from any type to any other type

- Value after conversion can be completely changed
  - The sequence of bits is simply truncated for integer numbers
  - The closest value is chosen for floating-point numbers

- Cast operator is the destination type between parentheses

  To be placed in front of the value to convert

```
1  int i = 1000;
2  char c = (char) i;     // i converted from int to char: (result -24)
```

Information Loss

- Information can be lost when converting to a narrower type

  Not always possible to represent same information with less bits

- The sequence of bits is truncated to fit the new types

  Depending on the conversion, even the sign can change

```
    int i = 1000;
char c = (char) i;
```

00000000 00000000 00000011 11101000

11101000

Precision Loss

- Precision can be lost when converting to floating-point

  Because not all real numbers can be represented

- The order of magnitude after conversion is the same

  But the converted number can lose precision

```
1  int j = 123456789;
2  float f = j;          // 123456792.000000
```

Hidden Cast

- Hidden cast with compound operators on some situations

  If type of left operand narrower than type of right one

- A cast is implicit and some information loss can occur

  Avoid narrower type to the left or use full assignment

```
1  short x = 0;
2  int i = 123456;
3
4  x += i;
5  printf("%d\n", x);     // Prints "-7616"
```

Bit Manipulation

Bit Manipulation Operator

- Three logical operators at the binary level
  - `&`: binary *"AND"*
  - `|`: binary *"OR"* (inclusive or)
  - `^`: binary *"XOR"* (exclusive or)
- Two bit shift operators
  - `<<`: shift bits to the left with 0 padding
  - `>>`: shift bits to the right with 0 padding for `unsigned`

    For `signed`, 0 padding if logical and 1 padding if arithmetical
- The `~` operator computes the ones' complement

Multiplication and Division

- Shifting bits of an integer multiplies/divides it by a power of 2

  Left shift is a multiplication and right shift a division

- Only true with arithmetic shifts but not with logical shifts
  - Some issues may occur with negative integer numbers
  - Shifters are often way faster than dividers

```
1  int i = 13;
2
3  printf("%d\n", i << 2);      // Prints 52 (that is, 13 * 2^2)
4  printf("%d\n", i >> 1);      // Prints 6 (that is, 13 / 2)
5  printf("%d\n", -i << 2);     // Prints -52
6  printf("%d\n", -i >> 1);     // Prints -7
```

Bit Masking

- Binary "AND" operator can be used to mask bits

  We have (0 & x) always gives 0 and (1 & x) always gives x

- Retrieve `n` bits from `x` starting at position `p`
  - First removing undesired bits to the right (`x >> (p+1-n)`)
  - Mask with `n` 1's to the right, then one zero and again 1's

```
1  unsigned char get_bits(unsigned char x, int p, int n)
2  {
3      return (x >> (p+1-n)) & ~(~0 << n);
4  }
```

Display Binary Representation

- Display the sequence of bits of the data in a variable

  Analysing the value bit by bit, with bit manipulation operator

- Combination of masking and bit shifting operators
  - Shift a bit to 1 under the desired position (`1 << i`)
  - Apply the mask to extract it (`1 << i & n`)
  - Shift the obtained value to the right to recover the bit

```
1  void print_bits(unsigned char n)
2  {
3      char i;
4      for (i = 8 * sizeof(unsigned char) - 1; i >= 0 ; i--)
5          printf("%d", (1 << i & n) >> i);
6  }
```

Bit Vector (1)

- A bit vector stores boolean values on single bits
  - Saves memory space when it is limited
  - Stores multiple flags in a single variable
- Reading the value of a bit as in the previous example

  Shifting a 1 *to the right position of the bit and mask with* &

```
1  unsigned char read_bit(unsigned char x, int n)
2  {
3      return (1 << n & x) >> n;
4  }
```

Bit Vector (2)

- Modification of a bit setting it to 0 or 1
  - Set to 1 with binary "OR": (1 | x) is always 1
  - Set to 0 with binary "AND": (0 & x) is always 0
- Combination of the operators & or | with shifting of bits

  So that the modification only changes one bit

```
1  unsigned char set_bit(unsigned char x, int n)
2  {
3      return x | (1 << n);
4  }
5
6  unsigned char reset_bit(unsigned char x, int n)
7  {
8      return x & ~(1 << n);
9  }
```

Flag Variable

- Define constants whose values are distinct powers of two

  Each constant only has one bit set to one

- Select multiple flags by "OR"-ing the desired flags

  Reading, modifying and clearing flags by manipulating bits

```c
#include <stdio.h>

#define ALARM    1
#define HEATING  2
#define COOLING  4
#define LIGHTS   8
#define DOORLOCK 16

int main()
{
   char config = HEATING | LIGHTS | DOORLOCK;
   printf("%d\n", config);          // Prints "26"
}
```

Short-Circuit Property

- Logical operator has the short-circuit property
  - Right operand is only evaluated if it necessary
  - $(0$ && $x)$ is always 0 and $(1$ || $x)$ is always 1
- Bit manipulation operator can be used as logical operator

  Faster, but does not have the short-circuit property

```
1  int x = 1, y = 2;
2
3  printf("%d\n", x & y);          // Prints "0"
4  printf("%d\n", x && y);         // Prints "1"
```

References

- Bradley Mitchell, *The Use of Octets in Computers and Networking: Octets are bytes ... sometimes*, June 21, 2019. https://www.lifewire.com/what-is-octet-818391
- IEEE, *IEEE Standard for Prefixes for Binary Multiples*, September 18, 2009. (ISBN: 978-0-7381-3386-7)
- Safier v. Western Digital Corporation, Case No. 05-03353 BZ (N.D. Cal. Mar. 17, 2006).
- Vedraw, *Why Unary is the Best Number System*, November 29, 2015. https://medium.com/@veedrac/why-unary-is-the-best-number-system-cc1e0edfb928
- Steve Hollasch, *IEEE Standard 754 Floating Point Numbers*, August 24, 2018. https://steve.hollasch.net/cgindex/coding/ieeefloat.html
- David C. Zentgraf, *What Every Programmer Absolutely, Positively Needs To Know About Encodings And Character Sets To Work With Text*, April 27, 2015. http://kunststube.net/encoding
- Sean Eron Anderson, *Bit Twiddling Hacks*, May 5, 2005. https://graphics.stanford.edu/~seander/bithacks.html

Credits

- https://www.flickr.com/photos/duncan/35981681216
- http://xkcd.com/394
- http://www.xkcd.com/953
- https://www.flickr.com/photos/entoropi/25488292585
- https://en.wikipedia.org/wiki/File:Binary_clock.svg
- https://www.flickr.com/photos/kay_prior/3977139291
- https://www.flickr.com/photos/blondinrikard/17265715421
- https://www.flickr.com/photos/tombass59/34155286024
- https://www.flickr.com/photos/zimpenfish/1880717616