ECAM
BRUSSELS ENGINEERING SCHOOL

I410A Algorithm and Optimisation

Session 2

Problem Modelling and Solving

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Objectives

- Analyse how a problem can be modelled (semi-)formally
    - Express problem and its instance with common data structure
    - Describe a solution as operations on the model and data
- Analyse general algorithmic problem solving techniques
    - Problem decomposition and recursivity
    - Common algorithm patterns

Problem Modelling

Stable Temperature

- Looking for the longest period with a stable temperature

  Given successive temperature measures taken by an IoT device

- Problem can be modelled with a sequence

  Containing all the measured values, ordered by timestamps

- Similar to the longest plateau problem

  Consecutive segment of an array with equal values

Longest Plateau (1)

- Naive version that will run in $\mathcal{O}(n^2)$

Checking all the plateau starting at any position in the array

```python
def naive_longest_plateau(arr):
    n = len(arr)
    m = 0
    for i in range(n):
        l = 0
        j = i
        while j < n and arr[j] == arr[i]:
            j += 1
            l += 1
        m = max(l, m)
    return m
```

Longest Plateau (2)

- Smarter version that will run in $\mathcal{O}(n)$

  Elements of the array are only examined once each

```python
def smart_longest_plateau(arr):
    n = len(arr)
    if n == 0:
        return 0
    l = 1
    m = 1
    for i in range(1, n):
        if arr[i] == arr[i - 1]:
            l += 1
            m = max(m, l)
        else:
            l = 1
    return m
```

Three Glass Riddle (1)

- Three glasses with capacities 5L, 8L and 12L
  - Only the 12L container is full initially, two others are empty
  - The problem is to have exactly 6L in one jug
- The three glass riddle can modelled as a graph

  Solving the riddle is just a well-known algorithm on the graph

Modelling Problem as Graph

- First define what should be represented by the nodes

  What "thing" is manipulated? What is the system state? Etc.

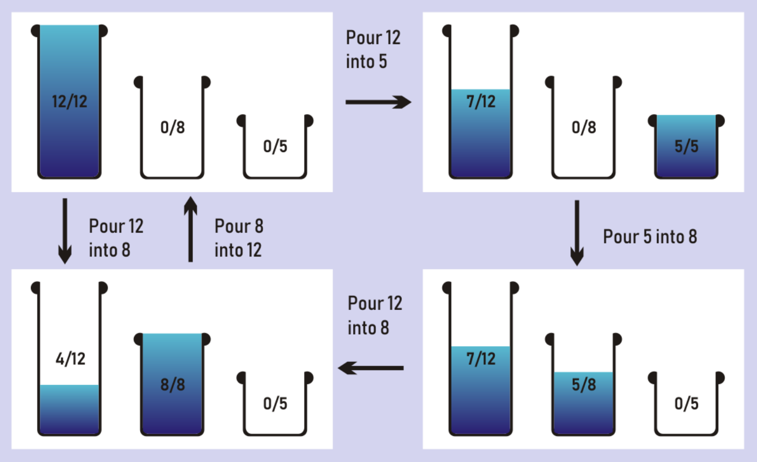
- Then define relationships between represented "things"

  Should the edges be weighted, directed?

- Is there a property of the graph that is the problem solution

  For which there exists an algorithm (or it can be found easier)

Three Glass Riddle (2)



Problem Solving Technique

Problem Decomposition

- Break a problem into smaller parts that are simpler

  Combine solutions of subproblems to solve main problem

- Two main ways to decompose a problem
  - Identifies different blocks solving heterogeneous problems
  - Transform problem to a smaller instance of the same problem

Multiple Pointer

- Iterating over an array or collection with several pointers

  Can be used to go from a $\mathcal{O}(n^2)$ time complexity to $\mathcal{O}(n)$

- Two common strategies use multiple pointers
  - **Two pointers**: run through an array from both ends
  - **Sliding window**: looking at several contiguous elements

Divide and Conquer

- Solving simpler problem instances to solve the problem

  Breaking down the collections into a set of smaller ones

- Same rule must be applied to the small pieces

  Complexity to solve the smallest pieces should be low

$\mathcal{O}(1)$ Lookup

- Use constant lookup time complexity data structures

  Hashtable, hashset, dictionary, etc.

- Used in specific situations to store information

  Counting frequencies, checking duplicates, etc.

References

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Credits

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