

2024 IBS-DIMAG workshop on combinatorics and geometric measure theory

Sunday, July 14, 2024

14:00-17:00

Meeting and discussion

Monday, July 15, 2024

9:00–9:30

Registration, coffee

9:30—9:40

Doowon Koh

Opening remarks

9:40—10:40

Alex Iosevich

Tutorial, session 1

10:40—11:00

Coffee break

11:00—12:00

János Pach

With strings attached

12:00—14:00

Lunch

14:00—15:00

Alex Iosevich

Tutorial, session 2

15:00—15:30

Coffee break

15:30—17:00

Open Problem Session

Tuesday, July 16, 2024

9:30—10:30

Alex Iosevich

Tutorial, session 3

10:30—11:00

Coffee break

11:00—11:30

Bochen Liu

Dimension of Diophantine approximation, orthogonal
projection, and sum-product

11:30—12:00

Neil Lyall

Some Problems in Geometric Ramsey Theory

12:00—14:00

Lunch

14:00—14:30

Eyvindur Palsson

Geometric averaging operators and point configurations

14:30—15:00

Steven Senger

Falconer-type problems for dot products

15:00—16:00

Coffee break

16:00—16:30

Izabella Laba

A short survey of integer tilings

16:30—17:00

Manik Dhar

TBA

Wednesday, July 17, 2024

9:30—10:30	Pertti Mattila	Hausdorff dimension of plane sections and general intersections
10:30—11:00		Coffee break
11:00—11:30	Giorgis Petridis	Products of differences in arbitrary finite fields
11:30—12:00	Jonathan Pakianathan	Random walks and the “Euclidean” association scheme in finite vector spaces
12:00—14:00		Lunch
14:00—19:00		Excursion
19:00		Banquet

Thursday, July 18, 2024

9:30—10:30	Hong Wang	Invited lecture
10:30—11:00		Coffee break
11:00—11:30	Alan Chang	TBA
11:30—12:00	Terry Harris	Subsets of vertical planes in the first Heisenberg group
12:00—14:00		Lunch
14:00—14:30	Charlotte Aten	TBA
14:30—15:00	Semin Yoo	Improved upper bounds for the largest size of Diophantine m -tuples
15:00—16:00		Coffee break
16:00—16:30	Olivine Silier	TBA
16:30—17:00	Matthew Kroecker	The Average Number of Points in a Spanned Plane

Friday, July 19, 2024

9:30—10:30	Cosmin Pohoata	TBA
10:30—11:00		Coffee break
11:00—11:30	Andreas Holmsen	TBA
11:30—12:00	Jinha Kim	Star clusters in independence complexes of hypergraphs
12:00—14:00		Lunch
14:00—14:30	Hai Long Dao	The combinatorics of syzygies
14:30—15:00	Wei-Hsuan Yu	TBA
15:00—16:00		Coffee break
16:00—16:30	Alexander Clifton	Covers of triangular grids
16:30—17:00	Mihalis Kolountzakis	Common fundamental domains

Monday, July 15

11:00–12:00 Plenary Lecture

Chair:

Speaker: János Pach

Title: With strings attached

Abstract: The intersection graph of a collection C of sets is the graph whose vertex set is C and in which two sets in C are connected by an edge if and only if they have nonempty intersection. String graphs, intersection graphs of continuous curves (“strings”) in the plane have been studied intensively since the 1960s, for their exciting algorithmic and combinatorial properties and their applications in chip design, network theory, graph drawing and elsewhere. After giving a whirlwind tour of string graph theory, I will present some recent results and annoying open problems. In particular, I will sketch the proof of the following theorem, joint with Jacob Fox and Andrew Suk. Given a set R of n red curves, and a set B of n blue curves in the plane such that any two of them meet at most once, there are subsets $R' \subset R$ and $B' \subset B$ with $|R'|, |B'| \geq \Omega(n)$ with the property that either every curve in R' crosses every curve in B' , or every curve in R' is disjoint from every curve in B' .

Tuesday, July 16

11:00–12:00 Contributed Talk Session

Chair:

Speaker: Bochen Liu

Title: Dimension of Diophantine approximation, orthogonal projection, and sum-product

Abstract: Given $0 < s < 1$, we construct a new family of sets in the line of Hausdorff dimension s via Diophantine approximation, in which the classical Jarnik example is the endpoint case. Then we are able to show a recent result of Ren and Wang, known sharp on orthogonal projection, is also sharp on $A + cB, c \in C$. This is joint work with Longhui Li.

Speaker: Neil Lyall

Title: Some Problems in Geometric Ramsey Theory

Abstract: We discuss the existence of certain point configurations in sets of positive density in both the continuous and discrete settings.

14:00–15:00 Contributed Talk Session

Chair:

Speaker: Eyvindur Palsson

Title: Geometric averaging operators and point configurations

Abstract: Two classic questions focus on distance: the Erdős distinct distance problem, which asks about the least number of distinct distances determined by points in the plane, and its continuous analog, the Falconer distance problem. When studying the Falconer distance problem, a geometric averaging operator, namely the spherical averaging operator, arises naturally. Questions similar to the Erdős distinct distance problem and the Falconer distance problem can also be posed for more complicated point configurations and correspondingly new geometric averaging operators appear. In this talk I will give a brief introduction to the motivating point configuration questions and then report on some novel geometric averaging operators and their mapping properties.

Speaker: Steven Senger

Title: Falconer-type problems for dot products

Abstract: The celebrated Falconer distance problem asks for a threshold on the Hausdorff dimension of a subset $E \subseteq \mathbb{R}^d$ that will guarantee the set of distances determined by pairs of points in E has positive Lebesgue measure. We discuss a variant of this problem involving dot products in place of distances.

16:00–17:00 Contributed talk session

Chair:

Speaker: Izabella Łaba

Title: A short survey of integer tilings

Abstract: A set $A \subset \mathbb{Z}$ tiles the integers by translations if there is a set $T \subset \mathbb{Z}$ such that every integer $n \in \mathbb{Z}$ has a unique representation $n = a + t$ with $a \in A$ and $t \in T$. The main open question regarding integer tilings is the Coven-Meyerowitz conjecture, providing a tentative characterization of finite tiles. We will survey some of the recent developments and open questions in this area, including a very recent joint result with Itay Londner where we prove the Coven-Meyerowitz tiling conditions for a new class of tilings.

Speaker: Manik Dhar

Title:

Abstract:

Wednesday, July 17

9:30–10:30 Plenary lecture

Chair:

Speaker: Pertti Mattila

Title: Hausdorff dimension of plane sections and general intersections

Abstract: I shall discuss conditions on a general family $P_\lambda : \mathbb{R}^n \rightarrow \mathbb{R}^m, \lambda \in \Lambda$, of orthogonal projections and a measure ω on Λ which guarantee that the Hausdorff dimension formula $\dim A \cap P_\lambda^{-1}\{u\} = s - m$ holds for ω almost all λ for measurable sets $A \subset \mathbb{R}^n$ with positive and finite s -dimensional Hausdorff measure, $s > m$. I shall present some families of projections where this applies. This leads to some new results on the Hausdorff dimension of intersections $\dim A \cap (g(B) + z)$ for almost all rotations g and for positively many $z \in \mathbb{R}^n$.

11:00–12:00 Contributed Talk Session

Chair:

Speaker: Giorgis Petridis

Title: Products of differences in arbitrary finite fields.

Abstract: It is not too hard to prove that any subset A of a finite field with q elements that has size $q^{2/3}$ the set $(A - A)(A - A)$ contains at least $q/2$ elements. We describe the ingredients needed to relax the condition to $q^{2/3-c}$ for a small absolute $c > 0$. Old joint work with Brendan Murphy.

Speaker: Jonathan Pakianathan

Title: Random walks and the “Euclidean” association scheme in finite vector spaces

Abstract: We will discuss the theory of finite association schemes and talk about a particular association scheme arising from “distance- t ” graphs over finite vector spaces V of odd order equipped with the “Euclidean” quadratic form. This “Euclidean” association scheme was studied in prior work of W.M.Kwok, E. Bannai, O. Shimabukuro and H. Tanaka as well as in Brittenham’s Ph.D. Thesis.

We will explain how vertical equidistribution of Kloosterman sums, is equivalent to certain asymptotics in the probability of return to origin in a k -step distance- t random walk in these vector spaces as well as outline how Delsarte’s linear programming bound can constrict distance sets of subsets E of the vector space.

This is based on joint work with Charles Brittenham.

Thursday, July 18

9:30–10:30 Invited lecture

Chair:

Speaker: Hong Wang

Title:

Abstract:

11:00–12:00 Contributed talk session

Chair:

Speaker: Alan Chang

Title:

Abstract:

Speaker: Terry Harris

Title: Subsets of vertical planes in the first Heisenberg group

Abstract: I will present the following result: If A is a subset of the first Heisenberg group which is contained in a vertical plane, then the Hausdorff dimension of A almost surely does not decrease under vertical projection. This is a (very) special case of a Conjecture of Balogh, Durand-Cartagena, Fassler, Mattila, and Tyson, from 2012. The proof uses Fourier analysis. I will discuss some of the methods used, and why the generalisation to arbitrary sets (not contained in a vertical plane) is difficult.

14:00–15:00 Contributed Talk Session

Chair:

Speaker: Semin Yoo

Title: Improved upper bounds for the largest size of Diophantine m -tuples

Abstract: A set $\{a_1, a_2, \dots, a_m\}$ of distinct positive integers is a *Diophantine m -tuple* with property $D_k(n)$ if the product of any two distinct elements in the set is n less than a k -th power. One may wonder what is the largest size, $M_k(n)$, of such a tuple. In this talk, we provide a substantial improvement on a recent result by Dixit, Kim, and Murty (2022) on the upper bound of $M_k(n)$. In particular, we show $M_k(n) = o(\log n)$ for a specially chosen sequence of k and n tending to infinity, breaking the $\log n$ barrier unconditionally. Our proof is a combination of Stepanov's method and Gallagher's larger sieve. This is joint work with Seoyoung Kim, and Chi Hoi Yip.

16:00–17:00 Contributed Talk Session

Chair: Rutger Campbell

Speaker: Matthew Kroeker

Title: The Average Number of Points in a Spanned Plane

Abstract: In 1941, Melchior proved that, for any finite arrangement of points in the plane which is not contained in a line, the average number of points in a spanned line is less than three. In this talk, we discuss the problem of extending Melchior's Theorem to higher dimensions. We show that, for any finite arrangement of points in space which is not contained in a plane nor in the union of two lines, the average number of points in a spanned plane is at most an absolute constant. We also present a generalization of this result to hyperplanes of any dimension.

This talk is based on joint work with Rutger Campbell, Jim Geelen and Ben Lund.

Friday July 19

11:30–12:00

Speaker: Jinha Kim

Title: Star clusters in independence complexes of hypergraphs

Abstract: In 2013, Barmak introduced the concept of star clusters in independence complexes of graphs. We generalize Barmak's result, providing an analogous result for the independence complexes of hypergraphs. This implies that for a hypergraph H with a vertex v that is not isolated and not contained in a Berge cycle of length 3, there exists a hypergraph H' with fewer vertices than H such that the independence complex of H is homotopy equivalent to the suspension of the independence complex of H' . As an application, we also prove that if a hypergraph H has no Berge cycle of length divisible by 3, then the sum of all reduced Betti numbers of its independence complex is at most 1.

14:00–15:00 Contributed talk session

Chair:

Speaker: Hai-Long Dao

Title: The combinatorics of syzygies

Abstract: A fundamental problem in commutative algebra and algebraic geometry is to understand complexity of syzygies, the set of relations between a collection of polynomials. When the polynomials are just monomials, the problems can be reinterpreted combinatorially. I will discuss some fascinating features in this line of research, highlighting unexpected connections to fractals, geodesics and Hirsch-type bounds on diameter of graphs arising from algebraic constructions.

Speaker:

Title:

Abstract:

16:00–17:00 Contributed talk session

Chair:

Speaker: Alexander Clifton

Title: Covers of triangular grids

Abstract: We consider the minimum number, $f(n, d, k)$, of affine hyperplanes in \mathbb{R}^d needed to cover all the points of the triangular grid

$T_d(n) := \{(x_1, \dots, x_d) \in \mathbb{Z}_{\geq 0}^d \mid x_1 + \dots + x_d \leq n - 1\}$ at least k times each. Combining linear programming with combinatorial techniques, we determine the exact answer for $d = 2$ and $k \leq 4$, as well as an asymptotic formula in terms of n whenever $d \geq k - 2$. Joint work with Abdul Basit and Paul Horn.

Speaker: Mihalis Kolountzakis

Title: Common fundamental domains

Abstract: A fundamental domain T of a group H in a larger, abelian, group G is a selection of one representative from each coset of H in G . In other words $G = H \oplus T$ with the sum being direct. The main theme of this talk is when a collection H_1, \dots, H_n of subgroups of G , all of the same index, admit a common fundamental domain T

$$H_1 \oplus T = H_2 \oplus T = \dots = H_n \oplus T = G.$$

In tiling language we are seeking a set $T \subseteq G$ that will tile G when translated by any of the subgroups H_1, \dots, H_n

This problem has many different aspects: analytic, geometric, combinatorial and purely algebraic. We hope to be able to show several of these, explain some of the recent progress.