Abstracts of 2012 Combinatorics Workshop

Combinatorics Workshop
August 9-10, 2012

Department of Mathematics,
Chonnam National University,
Gwangju, Korea

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Invited talk: August 9, 13:30 – 14:30

9A-1 Lattice paths and their generalizations

Seung Kyung PARK (Yonsei University)

Abstract

I will survey generalities of lattice paths enumeration first. Then several attempts to generalizing lattice paths will be discussed.

Finally, I will talk about recent progresses and results of generalized lattice paths.

9A-2 TBA

Seunghyun Seo (Kangwon National University)

Abstract

TBA

Invited talk: August 9, 15:30 – 16:30

9B-1 Polytopes and combinatorics in toric topology

Suyoung CHOI (Ajou University)

Abstract

Toric topology is the study of algebraic, combinatorial, geometric, and homotopy theoretic aspects of manifolds endowed by certain well behaved actions of the torus. One of the most interesting objects in toric topology is a manifold whose toric symmetry has a combinatorial structure isomorphic to that of a simplicial complex or a simple polytope.

A simple convex polytope P is (toric) *cohomologically rigid* if its combinatorial structure is determined by the cohomology ring of a quasitoric manifold over P, and is (toric) *combinatorially rigid* if its combinatorial structure is determined by its graded Betti numbers, which are important invariants coming from combinatorial commutative algebra. Not every P has these properties, but some important polytopes such as simplices or cubes are known to be cohomologically and combinatorially rigid. In general, it is known that if P is combinatorially rigid and it supports a quasitoric manifold, then P is cohomologically rigid.

In this talk, we survey results on toric rigidity of polytopes, and we provide two simple polytopes of dimension 3 having the discuss about the identical bigraded Betti numbers but non-isomorphic Tor-algebras. Furthermore, they turn out to be the first examples which are cohomologically rigid and not combinatorially. Moreover, one can see that the moment-angle manifolds arising from these two polytopes are homotopically different. Before this example, as far as I know, in all known examples of combinatorially different polytopes with same bigraded Betti numbers (such as vertex truncations of simplices), the moment-angle manifolds are all diffeomorphic.

Contributed talk: August 17, 16:30 – 17:00

9B-2 A new graph invariant rises in toric topology

Hanchul PARK (Ajou University)

Abstract

In this talk, we define an invariant, say the a-number, of any finite simple graph. The signed a-number of a graph G, denoted by $sa(G)$, is defined recursively as follows:

- $sa(\varnothing) = 1$.
- $sa(G) = 0$ if G has some connected component with odd vertices.
- $sa(G)$ is the product of signed a-numbers of components of G.
- If every component of G has even order, then $sa(G)$ is given by minus the sum of signed a-numbers of all induced subgraphs of G other than G itself.

The *a-number* of G, written by $a(G)$, is defined by the absolute value of $sa(G)$.

In fact, we have introduced the a-number for computing the homology of some kind of real toric manifolds, which are one of important objects of toric topology. For a given finite graph G, the graph associahedron P_G can be defined the Minkowski sum of all simplices determined by connected induced subgraphs of G. The class of graph associahedron includes some important families of simple polytopes, such as permutohedra Pe^n, associahedra As^n (or Stasheff polytopes), cyclohedra Cy^n and stellohedra St^n, corresponding to the complete graph, the path graph, the circle graph, and the star graph with n vertices respectively. Any graph associahedron P_G naturally corresponds to a real toric manifold $M(P_G)$. In topological viewpoint, it is very a natural approach to try to compute topological invariant, like the fundamental group or the cohomology ring, of a given manifold. Our main result says that i-th Betti number of $M(P_G)$ is the sum of a-numbers of induced subgraph of G of order $2i$. One remarkable corollary is that the sequence of Betti numbers of As^n is exactly the same with Catalan triangle, which provides new interpretation of Catalan numbers.

A combinatorial and non-recursive description of the a-number remains a question. This work is jointly with Professor Suyoung Choi, Ajou University.

Invited talk: August 10, 10:00 – 11:00

10A-1 On A-magic labeling of graphs

Jeong Ok CHOI (Gwangju Institute of Science and Technology)

Abstract

Let $G = (V, E)$ be a finite graph and let $(A, +)$ be an abelian group with identity 0. Then G is *A-magic* if and only if there exists a function ϕ from E into $A - \{0\}$ such that for some $c \in A$, $\sum_{e \in E(v)} \phi(e) = c$ for every $v \in V$, where $E(v)$ is the set of edges incident to v. Additionally, G is *zero-sum A-magic* if and only if ϕ exists such that $c = 0$. Let $zim(G)$ denote the subset of natural numbers such that $1 \in zim(G)$ if and only if G is zero-sum $\mathbb{Z}$-magic and $k \geq 2 \in zim(G)$ if and only if G is zero-sum $\mathbb{Z}_k$-magic. In this talk, two main subjects will be discussed: (1) we establish that if G is 3-regular, then $zim(G) = \mathbb{N} - \{2\}$ or $\mathbb{N} - \{2, 4\}$, (2) we relate the property of $\mathbb{Z}_2^k$-magic graphs with even edge-coverings, graph parity, factorability, and nowhere-zero 4-flows. Finally, we establish equivalent conditions for graphs of even order with bridges to be $\mathbb{Z}_2^k$-magic for all $k \geq 4$. This is joint work with J. Georges and D. Mauro.

10A-2 First order logic and random geometric graphs

Tobias MÜLLER (Utrecht University, Netherlands)

Abstract

We say that a graph property is first order expressible if it can be written as a logic sentence using the universal and existential quantifiers with variables ranging over the nodes of the graph, the usual connectives AND, OR, NOT, parentheses and the relations $=$ and $\sim$, where x $\sim$ y means that x and y share an edge. For example, the property that G contains a triangle can be written as

Exists x,y,z : (x $\sim$ y) AND (x $\sim$ z) AND (y $\sim$ z).

Starting from the sixties, first order expressible properties have been studied extensively on the most commonly studied model of random graphs, the Erdos-Renyi model. A number of very attractive and surprising results have been obtained, and by now we have a fairly full description of the behaviour of first order expressible properties on this model.

The Gilbert model of random graphs is obtained as follows. We take n points uniformly at random from the d-dimensional unit torus, and join two points by an edge if and only their distance is at most r.

In this talk I will discuss joint work with S. Haber which tells a nearly complete story on first order expressible properties of the Gilbert random graph model. In particular we settle several conjectures of McColm and of Agarwal-Spencer.

(Joint with S. Haber)

Contributed talk: August 10, 11:30 – 12:00

10A-3 Even Cycle Decompositions of Graphs with no odd-K_4-minor

Tony HUYNH (KAIST)

Abstract

An *even cycle decomposition* of a graph G is a partition of $E(G)$ into cycles of even length. Evidently, every Eulerian bipartite graph has an even cycle decomposition. Seymour proved that every 2-connected loopless Eulerian planar graph with an even number of edges also admits an even cycle decomposition. Later, Zhang generalized this to graphs with no K_5-minor.

We propose a conjecture involving signed graphs which contains all of these results. Our main result is a weakened form of this conjecture. Namely, we prove that every 2-connected loopless Eulerian odd-K_4-minor free signed graph with an even number of odd edges has an even cycle decomposition.

Our main technical tool is an exact structural description of the class of signed graphs with no odd-K_4-minor, initially stated by Lovász, Seymour, Schrijver, and Truemper. We will describe this structure theorem and give a brief sketch of our proof.

This is joint work with Sang-il Oum (KAIST) and Maryam Verdian-Rizi (KAIST).

10B-1 Maps and their related different fields of mathematics

Young Soo KWON (Yeungnam University)

Abstract

Regular maps are highly symmetric 2-cell graph embeddings like 5 Platonic soids. The interest in regular maps extends to their connection to Dyck's triangle groups, Riemann surfaces, algebraic curves, Galois groups and other areas including Grothendieck theory. In this talk, we show some relationship of maps to above mentioned fields of mathematics. Also some recent results related to classification of regular maps are given.

Contributed talk: August 19, 14:30 – 15:00

10B-2 The number of Sidon sets in $[n]$ and the maximum size of Sidon subsets contained in a random subset of $[n]$

Sang June LEE (KIAS)

Abstract

A set A of positive integers is called a *Sidon set* if all the sums $a_1 + a_2$, with $a_1 \leq a_2$ and $a_1, a_2 \in A$, are distinct. In this talk we deal with results on the number of Sidon sets in $[n]$ and the maximum size of Sidon sets in sparse random subsets of $[n]$.

The first question in this talk was suggested by Cameron–Erdős in 1990. They proposed the problem of estimating the number of Sidon sets contained in $[n]$. Results of Chowla, Erdős, Singer, and Turán from the 1940s imply that the maximum size of Sidon sets in $[n]$ is $\sqrt{n}(1+o(1))$. From this result, one can trivially obtain that the number of Sidon sets contained in $[n]$ is between $2^{(1+o(1))\sqrt{n}}$ and $n^{c\sqrt{n}}$ for some absolute constant c. We obtain an upper bound $2^{c\sqrt{n}}$ on the number of Sidon sets which is sharp up to a constant factor in the exponent when compared to the previous lower bound $2^{(1+o(1))\sqrt{n}}$.

Next, we investigate the maximum size of Sidon sets contained in sparse random sets $R \subset [n]$. Let $R = [n]_m$ be a uniformly chosen, random m-element subset of $[n]$. Let $F([n]_m) = \max\{|S| \colon S \subset [n]_m \text{ is Sidon}\}$. Fix a constant $0 \leq a \leq 1$ and suppose $m = (1+o(1))n^a$. We show that there is a constant $b = b(a)$ for which

$$F([n]_m) = n^{b+o(1)} \tag{1}$$

almost surely and we obtain what $b = b(a)$ is. Surprisingly, between two points $a = 1/3$ and $a = 2/3$, the function $b = b(a)$ is constant.

This is joint work with Kohayakawa, Rödl, and Samotij.

Invited talk: August 10, 15:30 – 16:30

10C-1 Moments of Askey-Wilson polynomials

Jang Soo KIM (University of Minnesota)

Abstract

Askey-Wilson polynomials are a family of orthogonal polynomials that are at the top of the hierarchy in the Askey scheme. In this talk, we give new formulas for the moment $\mu_n(a, b, c, d; q)$ of Askey-Wilson polynomials. As a corollary we obtain a symmetric polynomial expressions for $\mu_n(a, b, c, 0; q)$. We give a combinatorial proof of the formula for $\mu_n(a, b, 0, 0; q)$. We also give the first combinatorial proof of the formula for the moments of q-Laguerre polynomials due to Corteel, Josuat-Vergès, Prellberg, and Rubey. If time permits, we will see that our formula can be used to derive various results in the literature. This is joint work with Dennis Stanton.

Contributed talk: August 10, 16:30 – 17:00

10C-2 Some Latin squares with intercalate changes

Bokhee IM (Chonnam National University)

Abstract

Considering some special Latin squares with intercalate changes which can be associated with a directed graph on the symmetric group of degree 3, we compare certain weak compatibility graphs related to those Latin squares.

This is joint work with Ji-Young Ryu.

Registered participants

Students are denoted by $^{\varepsilon}$.

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Yonsei University

(2) Jeong-Ok Choi (최정옥)
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(3) Jihoon Choi (최지훈)$^{\varepsilon}$
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