POLYTOPES AND COMBINATORICS IN TORIC TOPOLOGY

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Toric topology is the study of algebraic, combinatorial, geometric, and homotopy theoretic aspects of manifolds endowed by certain well behaved actions of the torus. One of the most interesting objects in toric topology is a manifold whose toric symmetry has a combinatorial structure isomorphic to that of a simplicial complex or a simple polytope.

A simple convex polytope P is (toric) *cohomologically rigid* if its combinatorial structure is determined by the cohomology ring of a quasitoric manifold over P, and is (toric) *combinatorially rigid* if its combinatorial structure is determined by its graded Betti numbers, which are important invariants coming from combinatorial commutative algebra. Not every P has these properties, but some important polytopes such as simplices or cubes are known to be cohomologically and combinatorially rigid. In general, it is known that if P is combinatorially rigid and it supports a quasitoric manifold, then P is cohomologically rigid.

In this talk, we survey results on toric rigidity of polytopes, and we provide two simple polytopes of dimension 3 having the discuss about the identical bigraded Betti numbers but non-isomorphic Tor-algebras. Furthermore, they turn out to be the first examples which are cohomologically rigid and not combinatorially. Moreover, one can see that the moment-angle manifolds arising from these two polytopes are homotopically different. Before this example, as far as I know, in all known examples of combinatorially different polytopes with same bigraded Betti numbers (such as vertex truncations of simplices), the moment-angle manifolds are all diffeomorphic.

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