

# 2024 Combinatorics Workshop

## 2024 조합론 학술대회

Wednesday, 28 August 2024 - Friday, 30 August 2024

Room 107, Bldg. S1-6 (자연대6호관)

Chungbuk National University (충북대학교)

<https://cw2024.combinatorics.kr/>

## Book of Abstracts

### Organizers

- Doowon Koh (고두원), Chungbuk National University
- Eunjeong Lee (이은정), Chungbuk National University
- Sang-il Oum (엄상일), IBS Discrete Mathematics Group
- Meesue Yoo (류미수), Chungbuk National University

### Hosts

- Chungbuk National University
- IBS Discrete Mathematics Group

### Advisory Committee

- Committee of Discrete Mathematics, The Korean Mathematical Society

# History

1. 2004 Combinatorics Workshop (August 16-17, 2004)
  - Venue: Yonsei University, Seoul
  - Host: Yonsei University BK21 Research Group
2. 2005 Combinatorial Mathematics Conference (August 8-10, 2005)
  - Venue: Yonsei University, Seoul
  - Host: Yonsei University BK21 Research Group
  - (Local) Organizers: Dongsu Kim, Seung Kyung Park, Sang-Gu Lee, Jaeun Lee
3. 2006 Combinatorics Workshop (August 9-10, 2006)
  - Venue: Yeongnam University, Gyeongsan
  - Host: Department of Mathematics, Yeongnam University
  - (Local) Organizers: Jaeun Lee, Young Soo Kwon
4. 2007 Combinatorics Workshop (August 6-8, 2007)
  - Venue: KAIST, Daejeon
  - Host: KAIST Combinatorics Lab
  - (Local) Organizers: Heesung Shin, Jang Soo Kim
5. 2008 Workshop on Combinatorics (August 7-8, 2008)
  - Venue: Sungkyunkwan University, Suwon
  - Host: Sungkyunkwan University BK21 Division
  - (Local) Organizers: Gi-Sang Cheon, Sang-Gu Lee
6. 2009 Combinatorics Workshop (August 20-21, 2009)
  - Venue: KAIST, Daejeon
  - Host: KAIST ASARC
  - (Local) Organizers: Soon-Yi Kang, Sang-il Oum
7. 2010 Combinatorics Workshop (August 20-21, 2010)
  - Venue: Yeongnam University, Gyeongsan
  - Host: Department of Mathematics, Yeongnam University
  - (Local) Organizers: Jaeun Lee, Young Soo Kwon
8. 2011 Combinatorics Workshop (August 17-19, 2011)
  - Venue: Kangwon National University, Chuncheon
  - Host: Department of Mathematics Education, Kangwon National University
  - (Local) Organizers: Seunghyun Seo, Heesung Shin
9. 2012 Combinatorics Workshop (August 9-10, 2012)
  - Venue: Chonnam National University, Gwangju
  - Host: Department of Mathematics, Chonnam National University
  - (Local) Organizers: Hyeong-Kwan Ju, Sangwook Kim
10. 2013 Combinatorics Workshop (August 7-9, 2013)
  - Venue: CAMP (Center for Applications of Mathematical Principles), NIMS, Daejeon
  - Host: NIMS (National Institute for Mathematical Sciences)
  - (Local) Organizers: Dongsu Kim, Boram Park, Seunghyun Seo, Heesung Shin, Ae Ja Yee
11. 2014 Combinatorics Workshop (October 31-November 1, 2014)
  - Venue: Ajou University, Suwon
  - Host: Department of Mathematics, Ajou University
  - (Local) Organizers: Soojin Cho, Suyoung Choi, Boram Park
12. 2015 Combinatorics Workshop (July 13-16, 2015)

- Venue: CAMP (Center for Applications of Mathematical Principles), NIMS, Daejeon
  - Host: NIMS (National Institute for Mathematical Sciences)
  - (Local) Organizers: JiYoon Jung, Seunghyun Seo, Heesung Shin, Jiang Zeng
13. 2016 Combinatorics Workshop (July 21-23, 2016)
- Venue: KAIST, Daejeon
  - Host: KIAS CMC
  - (Local) Organizers: Soojin Cho, Dongsu Kim, Young Soo Kwon, Sang-il Oum
14. 2017 Combinatorics Workshop (August 16-17, 2017)
- Venue: Sungkyunkwan University, Suwon
  - Host: Applied Algebra and Optimization Research Center (AORC), Sungkyunkwan University
  - (Local) Organizers: Gi-Sang Cheon, Jang Soo Kim, Young Soo Kwon, Heesung Shin
15. 2018 Combinatorics Workshop (August 17-18, 2018)
- Venue: Seoul National University, Seoul
  - Host: IMDARC, Seoul National University
  - (Local) Organizers: Woong Kook, Seung Jin Lee
16. 2019 Combinatorics Workshop (August 13-15, 2019)
- Venue: Hotel Skypark Songdo, Incheon
  - Host: Combinatorics Incheon and IBS Discrete Mathematics Group
  - (Local) Organizers: O-joung Kwon, Suil O, Sang-il Oum, Heesung Shin
17. 2020 Combinatorics Workshop (August 24, 2020)
- Venue: Kyung Hee University, Seoul (online)
  - Host: Kyung Hee University and IBS Discrete Mathematics Group
  - (Local) Organizers: Jeong Han Kim, Sang June Lee, Sang-il Oum, Heesung Shin
18. 2021 Combinatorics Workshop (December 20-22, 2021)
- Venue: The Bloomvista, Yangpyeong
  - Host: IBS Discrete Mathematics Group
  - (Local) Organizers: Jeong-Ok Choi, Sang-il Oum, Heesung Shin
19. 2022 Combinatorics Workshop (September 30-October 1, 2022)
- Venue: GIST, Gwangju
  - Host: GIST and IBS Extremal Combinatorics and Probability Group
20. 2023 Combinatorics Workshop (August 16-18, 2023)
- Venue: Yonsei University, Seoul
  - Host: Yonsei University

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# 2024 Combinatorics Workshop

## Wednesday, August 28, 2024

**Opening/Registration** - 107 (2:00 PM - 2:30 PM)

**Invited Talk** - 107 (2:30 PM - 3:30 PM)

time	title	presenter
2:30 PM	Asymptotic bounds of Ramsey Numbers	KIM, Jeong Han (KIAS)

**Coffee Break** - 107 (3:30 PM - 4:00 PM)

**Contributed Talk** - 107 (4:00 PM - 5:30 PM)

time	title	presenter
4:00 PM	102-avoiding Inversion Sequences	SHIN, Heesung (Inha University)
4:30 PM	Random matchings in linear hypergraphs	LEE, Hyunwoo (KAIST & IBS ECOPRO)
5:00 PM	Towards a classification of $1$ -homogeneous graphs with positive intersection number $a_1$	LEE, Jae-Ho (University of North Florida & POSTECH)

**Dinner** (5:30 PM - 7:30 PM)

# Thursday, August 29, 2024

Invited Talk - 107 (9:30 AM - 10:30 AM)		
time	title	presenter
9:30 AM	Extensions of the colorful Helly theorem for d-collapsible and d-Leray complexes	KIM, Minki (GIST)

## Coffee Break - 107 (10:30 AM - 11:00 AM)

Contributed Talk - 107 (11:00 AM - 11:30 AM)		
time	title	presenter
11:00 AM	Transversal Hamilton paths and cycles of arbitrary orientations in tournaments	SEO, Jaehyeon (Yonsei University)

## Group Photo / Lunch (11:30 AM - 1:30 PM)

Invited Talk - 107 (1:30 PM - 2:30 PM)		
time	title	presenter
1:30 PM	Toric Colorability of Graphs of Simplicial $d$ -Polytopes with $d+4$ vertices	CHOI, Suyoung (Ajou University)

## Coffee Break - 107 (2:30 PM - 3:00 PM)

Contributed Talk - 107 (3:00 PM - 4:00 PM)		
time	title	presenter
3:00 PM	Alternating $\mathcal{B}$ -permutations arising from toric topology	YOON, Younghan (Ajou University)
3:30 PM	Partitions of ordered partitions and Bott manifolds	JEONG, Junho (Chungbuk National University)

## Coffee Break - 107 (4:00 PM - 4:30 PM)

Contributed Talk - 107 (4:30 PM - 5:30 PM)		
time	title	presenter
4:30 PM	Homotopy Types of Vietoris-Rips Complexes and Their Connection to Hyperconvexity	LIM, Sunhyuk (Sungkyunkwan University)
5:00 PM	On the extremal number of face-incidence graphs	BAEK, Jisun (Yonsei University)

## Banquet (5:30 PM - 7:30 PM)

# Friday, August 30, 2024

Invited Talk - 107 (9:30 AM - 10:30 AM)

time	title	presenter
9:30 AM	Lusztig $q$ -weight multiplicities via affine crystals	KIM, Donghyun

Coffee Break - 107 (10:30 AM - 11:00 AM)

Contributed Talk - 107 (11:00 AM - 11:30 AM)

time	title	presenter
11:00 AM	Combinatorics of orthogonal polynomials on the unit circle	SONG, Minho (Sungkyunkwan University)

Coffee Break (11:30 AM - 1:30 PM)

Invited Talk - 107 (1:30 PM - 2:30 PM)

time	title	presenter
1:30 PM	Enumeration of multiplex juggling card sequences using generalized $q$ -derivatives	KIM, Jang Soo (Sungkyunkwan University)

Contributed Talk - 107 (2:30 PM - 3:00 PM)

time	title	presenter
2:30 PM	Two ways to generalize matroids with coefficients	KIM, Donggyu (KAIST & IBS DIMAG)

Closing - 107 (3:00 PM - 3:30 PM)

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**Invited Talk 1 / Aug. 28, 14:30 - 15:30**

## Asymptotic bounds of Ramsey Numbers

**Speaker:** Jeong Han Kim<sup>1</sup>

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<sup>1</sup> *KIAS*

**Abstract:** Ramsey numbers, denoted as  $R(s, t)$ , are fundamental in graph theory, representing the smallest number of vertices  $n$  such that every graph on  $n$  vertices either contains a clique of size  $s$  or an independent set of size  $t$ . Recent developments in Ramsey theory have focused on finding asymptotic bounds for Ramsey numbers. In this talk, we survey asymptotic bounds of Ramsey Numbers  $R(3, t)$  and  $R(4, t)$ , including significant contributions of Sam Mattheus and Jacques Verstraete on  $R(4, t)$ .

**Contributed Talk 1 / Aug. 28, 16:00 - 16:30**

## 102-avoiding Inversion Sequences

**Speaker:** Heesung Shin<sup>4</sup>

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**Authors:** JiSun Huh<sup>1</sup>; Sangwook Kim<sup>2</sup>; Seunghyun Seo<sup>3</sup>; Heesung Shin<sup>4</sup>

<sup>1</sup> *Ajou University*

<sup>2</sup> *Chonnam National University*

<sup>3</sup> *Kangwon National University*

<sup>4</sup> *Inha University*

**Abstract:** A sequence  $(e_1, e_2, \dots, e_n)$  is an inversion sequences if  $0 \leq e_i < i$  for all  $i = 1, \dots, n$ . We say that an inversion sequences  $e = (e_1, e_2, \dots, e_n)$  *contains* the pattern 102 if there exist some indices  $i < j < k$  such that  $e_j < e_i < e_k$ . Otherwise,  $e$  is said to *avoid* the pattern 102.

In this talk, we will construct a correspondence between the set of 2-Schröder paths without peaks and valleys ending with a diagonal step and the set of 102-avoiding inversion sequences.

This is joint work with JiSun Huh, Sangwook Kim, and Seunghyun Seo.

**Contributed Talk 2 / Aug. 28, 16:30 - 17:00****Random matchings in linear hypergraphs****Speaker:** Hyunwoo Lee<sup>1</sup>

hyunwo9216@gmail.com

<sup>1</sup> KAIST & IBS ECOPRO

**Abstract:** For a given hypergraph  $H$  and a vertex  $v \in V(H)$ , consider a random matching  $M$  chosen uniformly from the set of all matchings in  $H$ . In 1995, Kahn conjectured that if  $H$  is a  $d$ -regular linear  $k$ -uniform hypergraph, the probability that  $M$  does not cover  $v$  is  $(1 + o_d(1))d^{-1/k}$  for all vertices  $v \in V(H)$ . This conjecture was proved for  $k = 2$  by Kahn and Kim in 1998.

We disprove this conjecture for all  $k \geq 3$ . For infinitely many values of  $d$ , we construct  $d$ -regular linear  $k$ -uniform hypergraph  $H$  containing two vertices  $v_1$  and  $v_2$  such that  $\mathcal{P}(v_1 \notin M) = 1 - \frac{(1+o_d(1))}{d^{k-2}}$  and  $\mathcal{P}(v_2 \notin M) = \frac{(1+o_d(1))}{d+1}$ . The gap between  $\mathcal{P}(v_1 \notin M)$  and  $\mathcal{P}(v_2 \notin M)$  in this  $H$  is best possible. In the course of proving this, we also prove a hypergraph analog of Godsil's result on matching polynomials and paths in graphs, which is of independent interest.

**Contributed Talk 3 / Aug. 28, 17:00 - 17:30**

## **Towards a classification of 1-homogeneous graphs with positive intersection number $a_1$**

**Speaker:** Jae-Ho Lee<sup>2</sup>

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**Authors:** Jack Koolen<sup>1</sup>; Mamoon Abdullah<sup>1</sup>; Brhane Gebremichel<sup>1</sup>; Jae-Ho Lee<sup>2</sup>

<sup>1</sup> *University of Science and Technology of China*

<sup>2</sup> *University of North Florida & POSTECH*

**Abstract:** Let  $\Gamma$  be a graph with diameter at least two. Then  $\Gamma$  is said to be 1-homogeneous (in the sense of Nomura) whenever for every pair of adjacent vertices  $x$  and  $y$  in  $\Gamma$ , the distance partition of the vertex set of  $\Gamma$  with respect to both  $x$  and  $y$  is equitable, and the parameters corresponding to equitable partitions are independent of the choice of  $x$  and  $y$ . Assume  $\Gamma$  is 1-homogeneous distance-regular with intersection number  $a_1 > 0$  and diameter  $D \geq 5$ . Define  $b = b_1/(\theta_1 + 1)$ , where  $b_1$  is the intersection number and  $\theta_1$  is the second largest eigenvalue of  $\Gamma$ . In this talk, we show that if intersection number  $c_2 \geq 2$ , then  $b \geq 1$  and one of the following (i)–(vi) holds:

- (i)  $\Gamma$  is a regular near  $2D$ -gon,
- (ii)  $\Gamma$  is a Johnson graph  $J(2D, D)$ ,
- (iii)  $\Gamma$  is a halved  $\ell$ -cube where  $\ell \in \{2D, 2D + 1\}$ ,
- (iv)  $\Gamma$  is a folded Johnson graph  $\bar{J}(4D, 2D)$ ,
- (v)  $\Gamma$  is a folded halved  $(4D)$ -cube,
- (vi) the valency of  $\Gamma$  is bounded by a function of  $b$ .

Moreover, we characterize 1-homogeneous graphs with classical parameters and  $a_1 > 0$ , as well as tight distance-regular graphs.

This is a joint work with J. Koolen, M. Abdullah, B. Gebremichel.

**Invited Talk 2 / Aug. 29, 9:30 - 10:30**

## **Extensions of the colorful Helly theorem for $d$ -collapsible and $d$ -Leray complexes**

**Speaker:** Minki Kim<sup>1</sup>

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**Authors:** Minki Kim<sup>1</sup>; Alan Lew<sup>2</sup>

<sup>1</sup> *GIST*

<sup>2</sup> *Carnegie Mellon University*

**Abstract:** We present extensions of the colorful Helly theorem for  $d$ -collapsible and  $d$ -Leray complexes, providing a common generalization to the topological colorful Helly theorem by Kalai and Meshulam, the very colorful Helly theorem by Arocha et al., and the semi-intersecting colorful Helly theorem by Karasev and Montejano. As an application, we obtain a strengthened version of Tverberg's theorem. This is joint work with Alan Lew.

**Contributed Talk 4 / Aug. 29, 11:00 - 11:30**

## Transversal Hamilton paths and cycles of arbitrary orientations in tournaments

**Speaker:** Jaehyeon Seo<sup>4</sup>

jaehyeonseo@yonsei.ac.kr

**Authors:** Hyunwoo Lee<sup>1</sup>; Debsoumya Chakraborti<sup>2</sup>; Jaehoon Kim<sup>3</sup>; Jaehyeon Seo<sup>4</sup>

<sup>1</sup> KAIST & IBS ECOPRO

<sup>2</sup> University of Warwick

<sup>3</sup> KAIST

<sup>4</sup> Yonsei University

**Abstract:** It is well-known that a tournament always contains a directed Hamilton path. Rosenfeld conjectured that if a tournament is sufficiently large, it contains a Hamilton path of any given orientation. This conjecture was approved by Thomason, and Havet and Thomassé completely resolved it by showing there are exactly three exceptions.

We generalized this result into a transversal setting. Let  $\mathbf{T} = \{T_1, \dots, T_{n-1}\}$  be a collection of tournaments on a common vertex set  $V$  of size  $n$ . We showed that if  $n$  is sufficiently large, there is a Hamilton path on  $V$  of any given orientation which is obtained by collecting exactly one arc from each  $T_i$ . Such a path is said to be *transversal*.

It is also a folklore that a strongly connected tournament always contains a directed Hamilton cycle. Rosenfeld made a conjecture for arbitrarily oriented Hamilton cycles in tournaments as well, which was approved by Thomason (for sufficiently large tournaments) and Zein (by specifying all the exceptions). We also showed a transversal version of this result. Together with the aforementioned result, it extends our previous research, which is on transversal generalizations of existence of directed paths and cycles in tournaments.

This is a joint work with Debsoumya Chakraborti, Jaehoon Kim, and Hyunwoo Lee.

**Invited Talk 3 / Aug. 29, 13:30 - 14:30**

## **Toric Colorability of Graphs of Simplicial $d$ -Polytopes with $d + 4$ vertices**

**Speaker:** Suyoung Choi<sup>1</sup>

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<sup>1</sup> *Ajou University*

**Abstract:** The 1-skeleton of a convex polytope  $P$  is called the graph of  $P$ .

A graph of a simplicial  $d$ -polytope is said to be toric colorable if there is a vertex coloring  $\lambda: V(G) \rightarrow \mathbb{Z}^d$  such that  $\{v_1, \dots, v_d\}$  forms a face of  $P$  implies that  $\{\lambda(v_1), \dots, \lambda(v_d)\}$  is unimodular.

In this talk, we discuss the toric colorability of graphs of simplicial  $d$ -polytopes with  $d + 4$  vertices.

**Contributed Talk 5 / Aug. 29, 15:00 - 15:30**

## **Alternating $\mathcal{B}$ -permutations arising from toric topology**

**Speaker:** Younghan Yoon<sup>1</sup>

younghan300@ajou.ac.kr

**Authors:** Suyoung Choi<sup>1</sup>; Younghan Yoon<sup>1</sup>

<sup>1</sup> *Ajou University*

**Abstract:** In this talk, we focus on the rational Betti numbers of real toric manifolds associated with chordal nestohedra. We introduce an explicit description for the Betti numbers using alternating  $\mathcal{B}$ -permutations for a chordal building set  $\mathcal{B}$ . We provide detailed computations for interesting cases of chordal nestohedra, including permutohedra, associahedra, stellohedra, Stanley-Pitman polytopes, and Hochschild polytopes.

This is joint work with Suyoung Choi.



**Contributed Talk 6 / Aug. 29, 15:30 - 16:00****Partitions of ordered partitions and Bott manifolds****Speaker:** Junho Jeong<sup>1</sup>

junhojeong@chungbuk.ac.kr

**Authors:** Junho Jeong<sup>1</sup>; Jang Soo Kim<sup>2</sup>; Eunjeong Lee<sup>1</sup><sup>1</sup> *Chungbuk National University*<sup>2</sup> *Sungkyunkwan University*

**Abstract:** Bott manifolds are smooth projective toric varieties providing interesting avenues among topology, geometry, representation theory, and combinatorics. They are used to understand the geometric structure of Bott–Samelson–Demazure–Hansen (BSDH) varieties, which provide desingularizations of Schubert varieties. However, not all Bott manifolds originate from BSDH varieties. Those that do are specifically referred to as Bott manifolds of *Bott–Samelson–Demazure–Hansen type*. In this talk, we explore a relationship between Bott manifolds of BSDH type and partitions of ordered partitions. This talk is based on joint work with Jang Soo Kim and Eunjeong Lee.

**Contributed Talk 7 / Aug. 29, 16:30 - 17:00**

## **Homotopy Types of Vietoris-Rips Complexes and Their Connection to Hyperconvexity**

**Speaker:** Sunhyuk Lim<sup>1</sup>

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<sup>1</sup> *Sungkyunkwan University*

**Abstract:** The Vietoris-Rips complex, originally introduced by Leopold Vietoris in the early 1900s to develop a homology theory for metric spaces, has since found applications in various areas of mathematics. Eliyahu Rips and Mikhail Gromov further utilized it in their studies of hyperbolic groups. More recently, classifying the homotopy types of Vietoris-Rips complexes has become a significant problem in Topological Data Analysis and Global Metric Geometry. Understanding these complexes can enhance our grasp of the persistence barcode's strength and provide lower bounds for the Gromov-Hausdorff distance between manifolds. In this talk, we will delve into these motivations and introduce the precise connections between Vietoris-Rips complexes, hyperconvex metric spaces, and their homotopy types.

**Contributed Talk 8 / Aug. 29, 17:00 - 17:30**

## On the extremal number of face-incidence graphs

**Speaker:** Jisun Baek<sup>2</sup>

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**Authors:** David Conlon<sup>1</sup>; Jisun Baek<sup>2</sup>; Joonkyung Lee<sup>2</sup>

<sup>1</sup> *California Institute of Technology*

<sup>2</sup> *Yonsei University*

**Abstract:** The  $(k, r)$ -incidence graph of a regular polytope  $\mathcal{P}$  is the bipartite incidence graph between  $k$ -faces and  $r$ -faces of  $\mathcal{P}$ . We obtain a general upper bound and a corresponding supersaturation result for the extremal number of the  $(k, r)$ -incidence graph of any regular polytope.

This generalises recent results of Janzer and Sudakov, who obtained the same bound for hypercubes and bipartite Kneser graphs, and confirms the conjecture of Conlon and Lee on the extremal number of  $K_{d,d}$ -free bipartite graphs for certain  $(k, r)$ -incidence graphs.

Our proof, based on the reflection group method developed by Conlon and Lee, presents the method in a purely algebraic manner.

As a consequence, this puts a number of results, including the Janzer-Sudakov theorem, the Conlon-Lee theorem on weakly norming graphs, and Coregliano's theorem on Sidorenko's conjecture, in the unified framework and simplifies all the proofs.

Joint work with David Conlon and Joonkyung Lee.

**Invited Talk 4 / Aug. 30, 9:30 - 10:30**

## **Lusztig $q$ weight multiplicities via affine crystals**

**Speaker:** Donghyun Kim<sup>1</sup>

hyun920310@snu.ac.kr

**Authors:** Hyeonjae Choi<sup>1</sup>; Donghyun Kim<sup>1</sup>; Seung Jin Lee<sup>1</sup>

<sup>1</sup> *Seoul National University*

**Abstract:** Lusztig  $q$  weight multiplicity is a polynomial in  $q$  whose positivity has been verified by linking it to a specific affine Kazhdan-Lusztig polynomial. However, a combinatorial formula beyond type A has not been known until recently.

In 2019, Lee proposed a combinatorial formula for type C using a novel combinatorial concept known as semistandard oscillating tableaux. We will outline the proof of Lee's conjecture and discuss how it can be extended to type B spin weights case.

Based on joint work with Hyeonjae Choi and Seung Jin Lee.

**Contributed Talk 9 / Aug. 30, 11:00 - 11:30**

## Combinatorics of orthogonal polynomials on the unit circle

**Speaker:** Minh Song<sup>1</sup>

smh3227@skku.edu

**Authors:** Jihyeug Jang<sup>1</sup>; Minh Song<sup>1</sup><sup>1</sup> *Sungkyunkwan University*

**Abstract:** Orthogonal polynomials on the unit circle (OPUC) are a family of polynomials orthogonal with respect to integration on the unit circle in the complex plane. The values of these integrals can be obtained by calculating moments. Numerous combinatorial studies have explored the moments of various types of orthogonal polynomials, including classical orthogonal polynomials, Laurent biorthogonal polynomials, and orthogonal polynomials of type  $R_I$ .

In this talk, we first explain how OPUC relate to these other variations. Next, we study the moments of OPUC from a combinatorial perspective, providing three path interpretations: Łukasiewicz paths, gentle Motzkin paths, and Schröder paths. Using these combinatorial interpretations, we derive explicit formulas for the generalized moments of some examples of OPUC, including the circular Jacobi polynomials and the Rogers–Szegő polynomials. Furthermore, we introduce several types of generalized linearization coefficients and provide combinatorial interpretations for each of them.

**Invited Talk 5 / Aug. 30, 13:30 - 14:30**

## **Enumeration of multiplex juggling card sequences using generalized $q$ -derivatives**

**Speaker:** Jang Soo Kim<sup>2</sup>

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**Authors:** Yumin Cho<sup>1</sup>; Jaehyun Kim<sup>1</sup>; Jang Soo Kim<sup>2</sup>; Nakyung Lee<sup>1</sup>

<sup>1</sup> *Gyeonggi Science High School for the Gifted*

<sup>2</sup> *Sungkyunkwan University*

**Abstract:** In 2019, Butler, Choi, Kim, and Seo introduced a new type of juggling card that represents multiplex juggling patterns in a natural bijective way. They conjectured a formula for the generating function for the number of multiplex juggling cards with capacity.

In this paper we prove their conjecture. More generally, we find an explicit formula for the generating function with any capacity. We also find an expression for the generating function for multiplex juggling card sequences by introducing a generalization of the  $q$ -derivative operator. As a consequence, we show that this generating function is a rational function.

**Contributed Talk 10 / Aug. 30, 14:30 - 15:00**

## Two ways to generalize matroids with coefficients

**Speaker:** Donggyu Kim<sup>1</sup>

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<sup>1</sup> KAIST & IBS DIMAG

**Abstract:** Dress (1986) introduced matroids with coefficients offering a unified approach to ordinary matroids, representations of matroids over fields, and oriented matroids. Baker and Bowler (2019) extended this theory, whose result includes a partial field representation by Semple and Whittle (1996).

I will present two generalizations of matroids with coefficients. One is about skew-symmetric matrices and even delta-matroids, based on joint work with Tong Jin. We deduce several results on the representability of even delta-matroids as applications. The other concerns symmetric matrices and new matroid-like objects called antisymmetric matroids. It extends old results on the representability of matroids by Tutte (1958) and basis graphs of matroids by Maurer (1973). These two generalizations involve an interesting interplay between Lagrangian orthogonal/symplectic Grassmannians and combinatorics.

## Miscellaneous Information

- Campus map



- Venue: 자연대 6호관 (S1-6) 107호
- Wifi: TBA
- 8/29 (Thur.), 8/30 (Fri.) Lunch: 은하수 식당 (N21동)
- 8/28 (Wed.) Dinner: 나능이 능이백숙 (충북 청주시 서원구 성봉로 220번길 94 1층)
- 8/29 (Thur.) Dinner: 갈비명가 연웅 (충북 청주시 서원구 성봉로 220번길 154)