

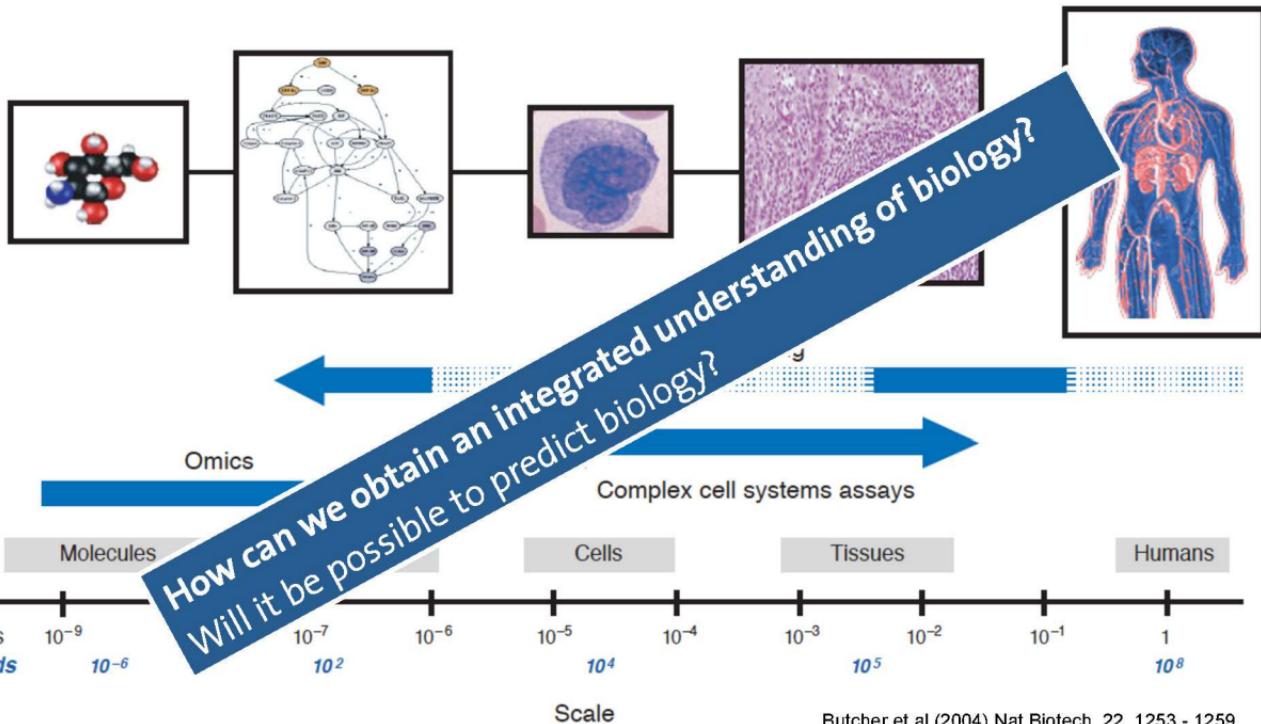
From Networks to Function – Computational Models of Organogenesis

Dagmar Iber

Computational Biology Group (CoBi)
ETH Zurich



Challenge: Integration across scales



DEVELOPMENTAL SYSTEMS BIOLOGY

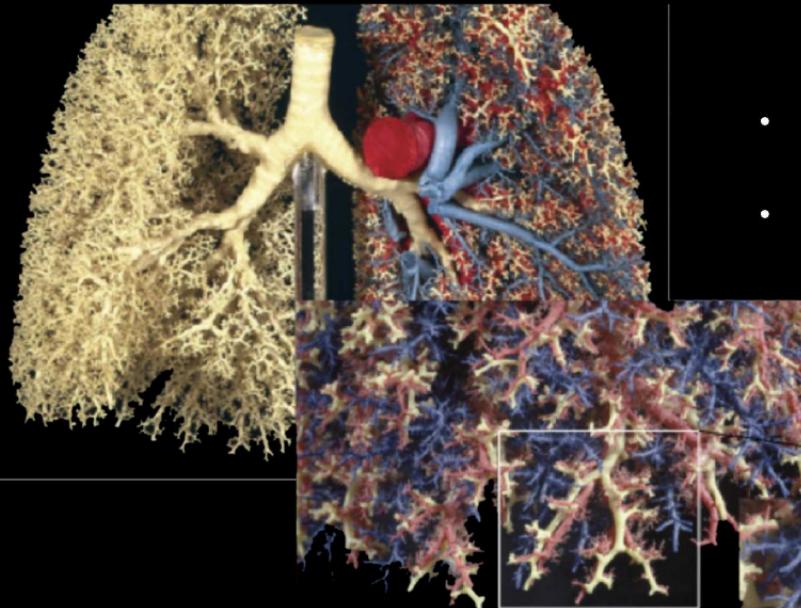
TOPICS

- I. How to break symmetry?
Building a lung and a kidney...
- II. How to pattern a growing embryo?
- III. How to terminate growth?



©1994
Brad Smith
Elwood Linney
Center for In Vivo Microscopy
Funding: National Center for Research Resources
North Carolina Biotechnology Center

BRANCHING MORPHOGENESIS



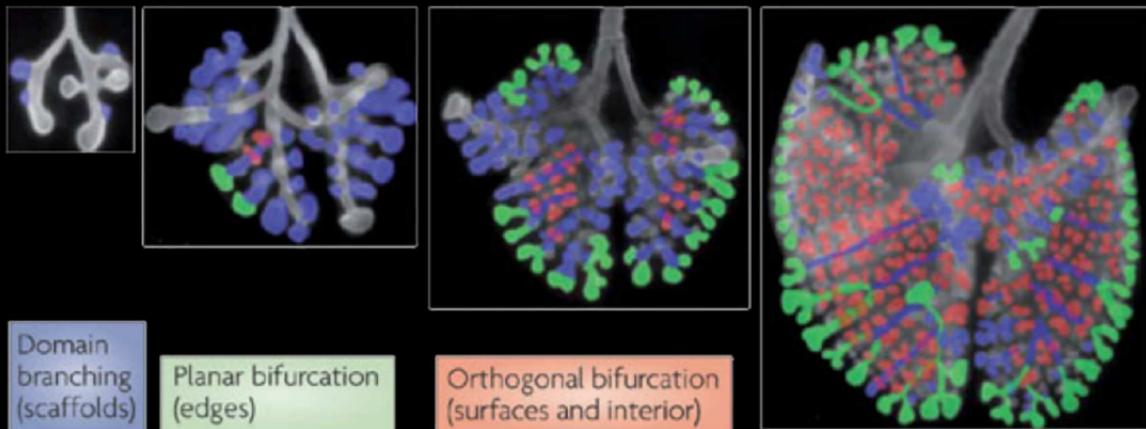
- Bronchial Tree: ~2400 km
(300-500 million alveolae)
- Surface: ~70 m²
(1 side of a tennis court)

The bronchial tree is so designed that the functions of the lung can be carried out with minimum entropy production.

Wilson, *Nature*, 1967

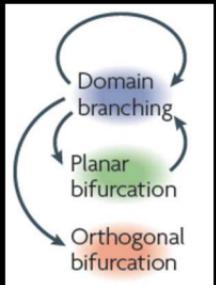
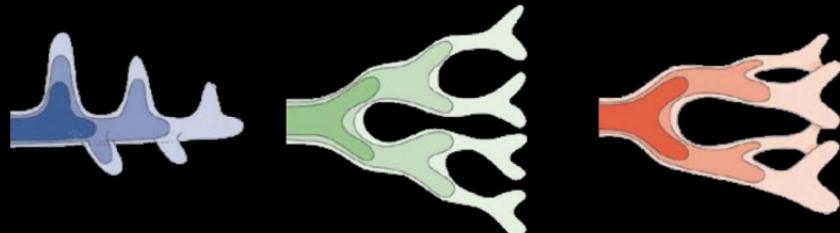
Glenny, *J Appl Physiol*, 2011

LUNG DEVELOPMENT



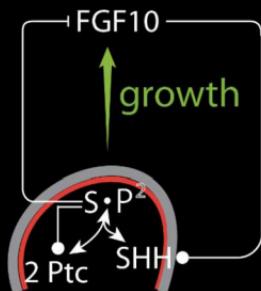
Metzger *et al.* *Nature*, 2008

Domain Branching Planar Bifurcation Orthogonal Bifurcation



Affolter *et al.* *Nature Reviews Mol Cell Biology* 2009

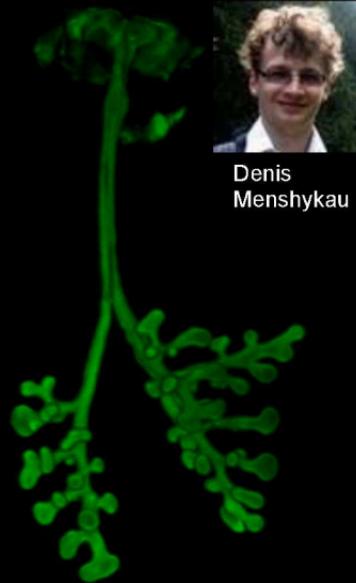
MODELLING OF THE REGULATORY NETWORK



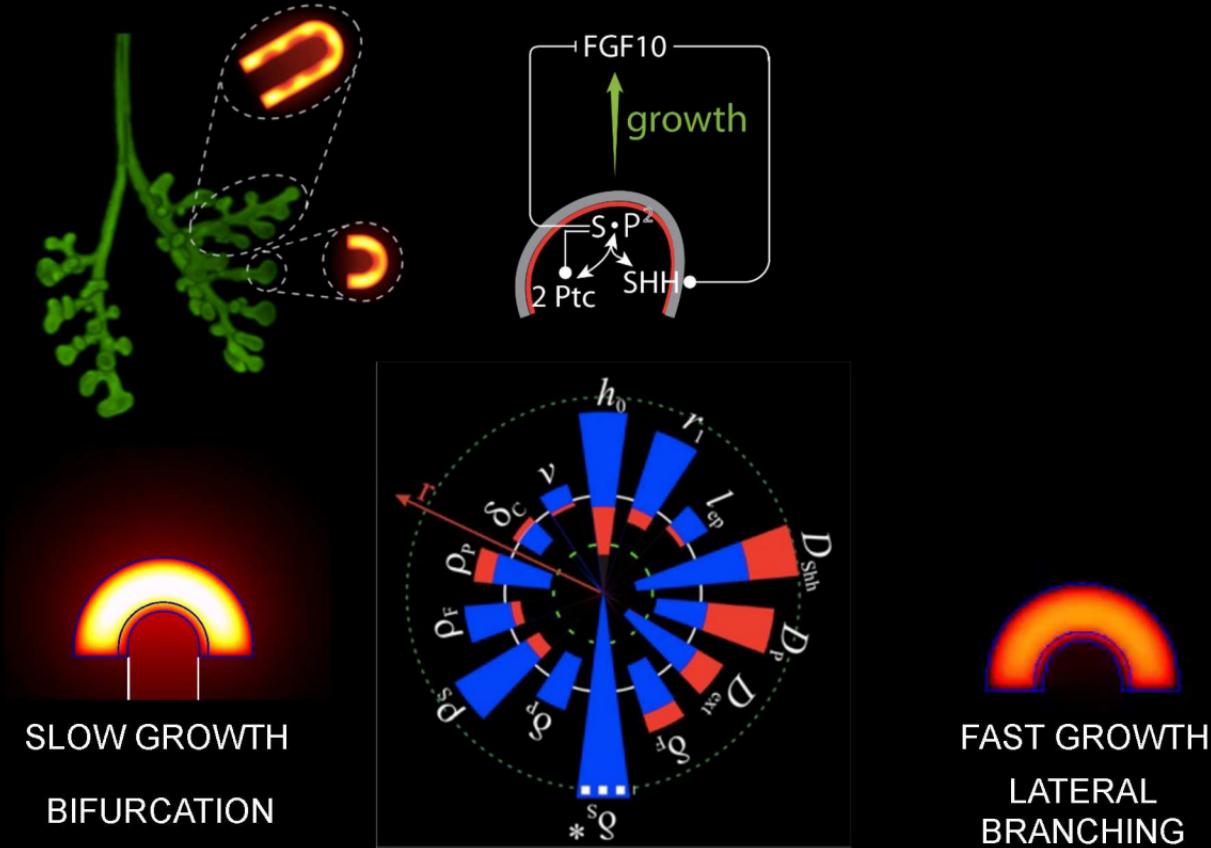
$$\dot{F} = \Delta F + \rho_F \frac{1}{(P^2 S)^n + 1} - \delta_F F$$

$$\dot{P} = D_P \Delta P + \rho_P - \delta_P P + (\nu - 2\delta_C) P^2 S$$

$$\dot{S} = D_S \Delta S + \rho_S \frac{F^n}{F^n + 1} - \overline{\delta_S S}_{400 \mu m} - \delta_C P^2 S$$

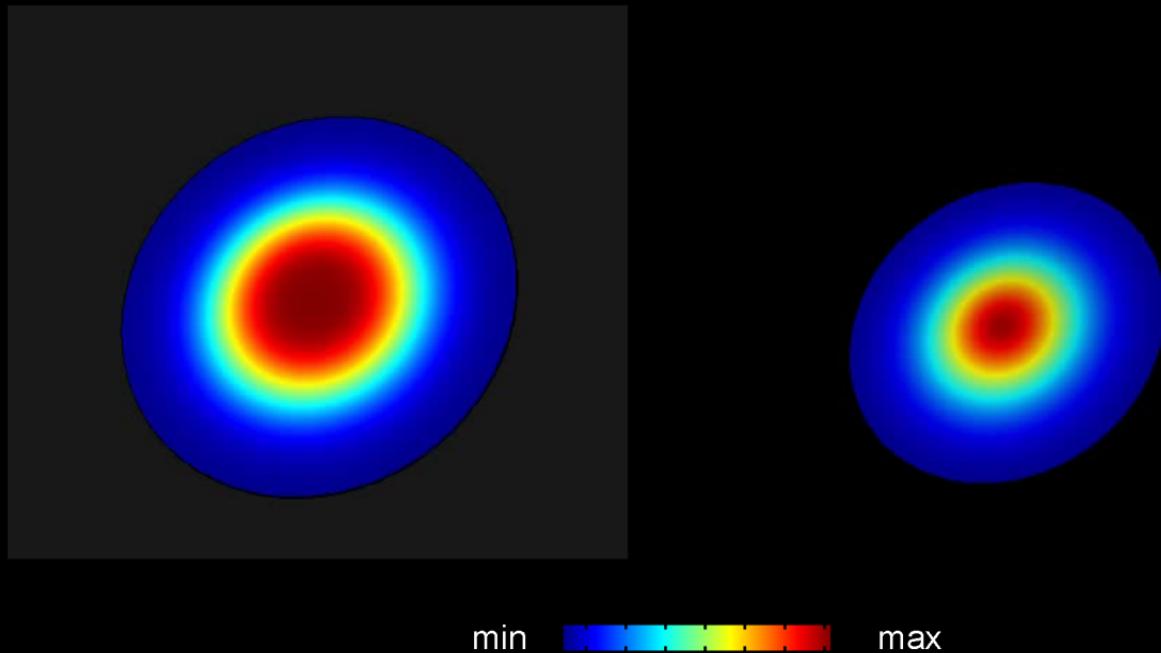


DETERMINATION OF LUNG BRANCH POINTS



PATTERNING & BRANCHING ON A GROWING DOMAIN

Turing mechanism based on a receptor-ligand Interaction permits branching.

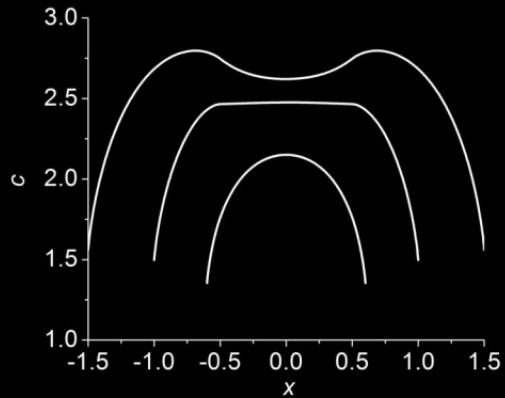
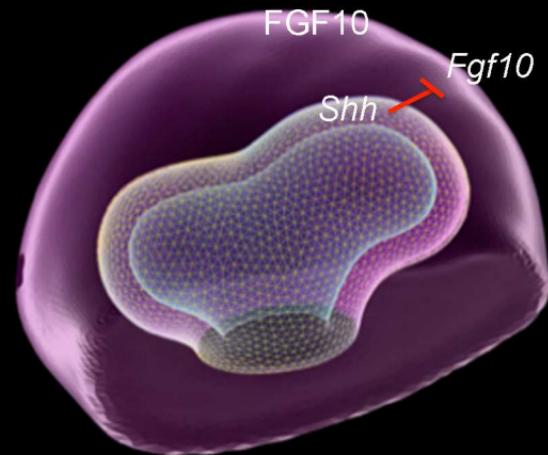
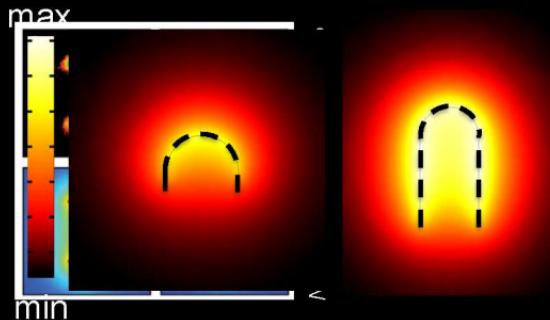


Alternative Mechanisms

1. EPITHELIUM – MESENCHYME DISTANCE-DEPENDENT REGULATION

- i. SHH -| Fgf10 results in bifurcating profile at the tips
- ii. Steeper Gradients at shorter distance

2. GEOMETRY-INDUCED GRADIENTS



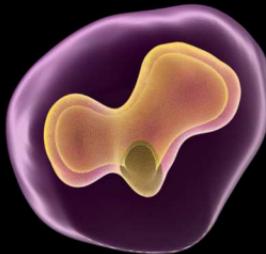
MODEL TESTING with EMBRYONIC DATASETS

A sequence of segmented lung bud geometries (from Blanc *et al* PLoS One 2012)

Stage 1



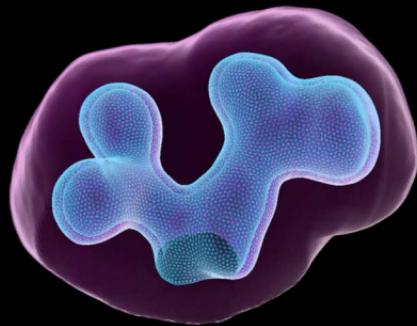
Stage 2



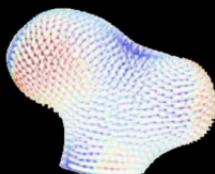
Stage 3



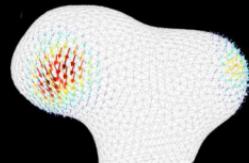
Stage 4



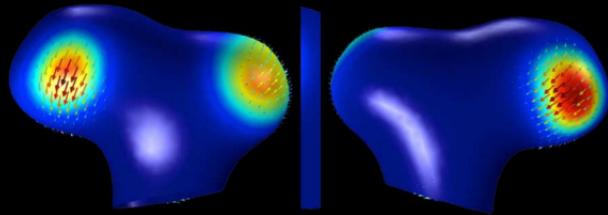
Displacement field



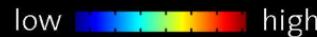
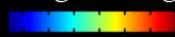
Growth field



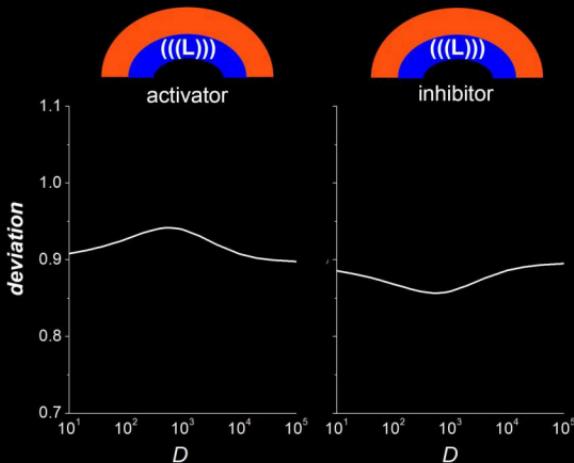
Computed Signalling (solid) & Growth (arrows) fields
Signalling field



shrinkage growth

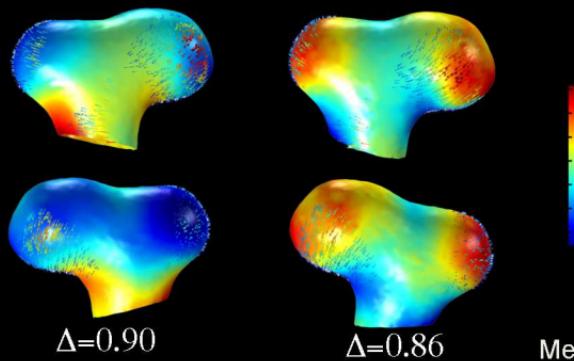


Geometry-based Mechanisms – Stage 1→2



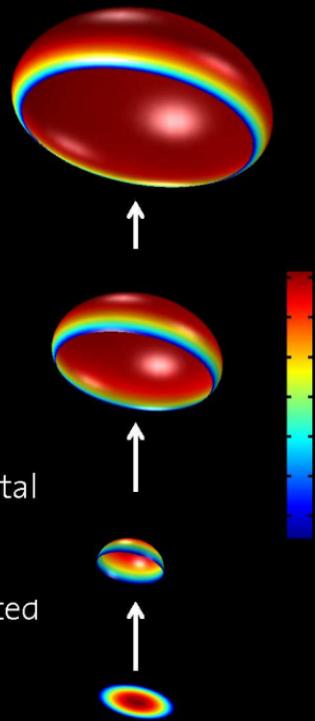
Deviation between
computed and
measured growth
fields:

$$\Delta = \sqrt{\sum_{\text{EM}} (|\bar{v}_n| - S)}$$

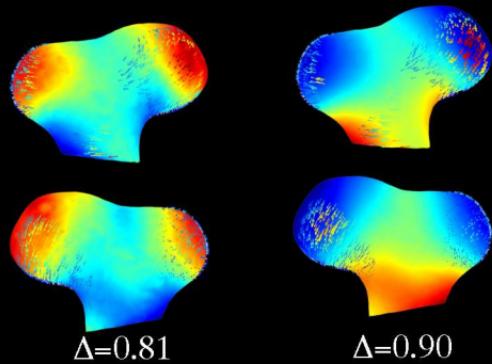
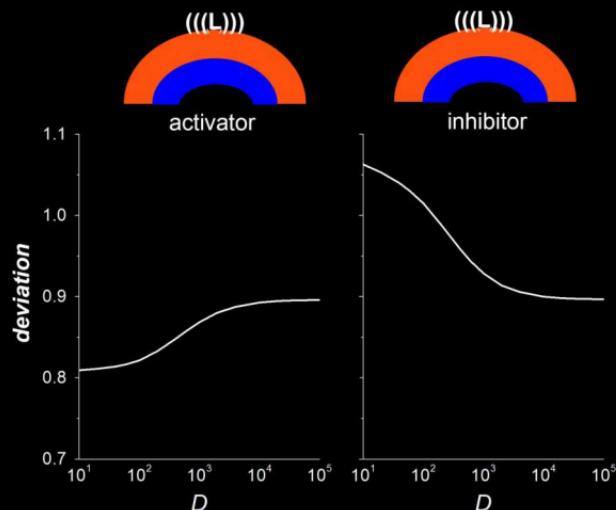


arrows – experimental
growth field

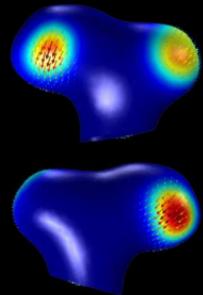
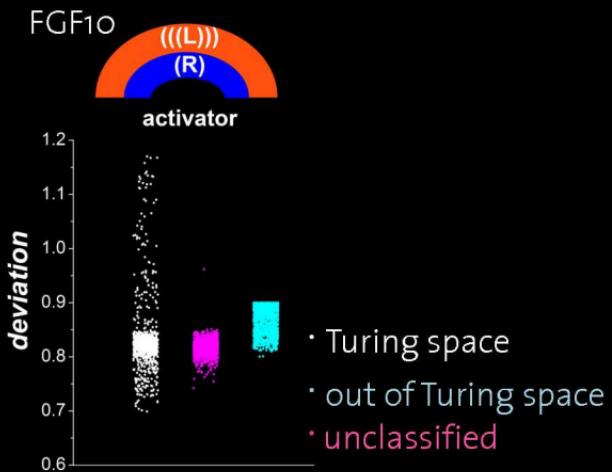
solid color – computed
growth field



Distance & Gradient Based Mechanisms – Stage 1->2

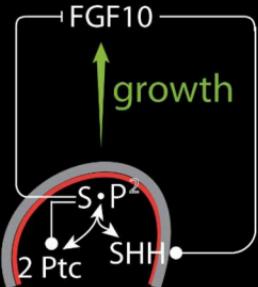


TURING-BASED MECHANISMS



$$\Delta=0.62$$

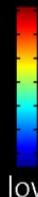
SHH



Deviation between computed and measured growth fields:

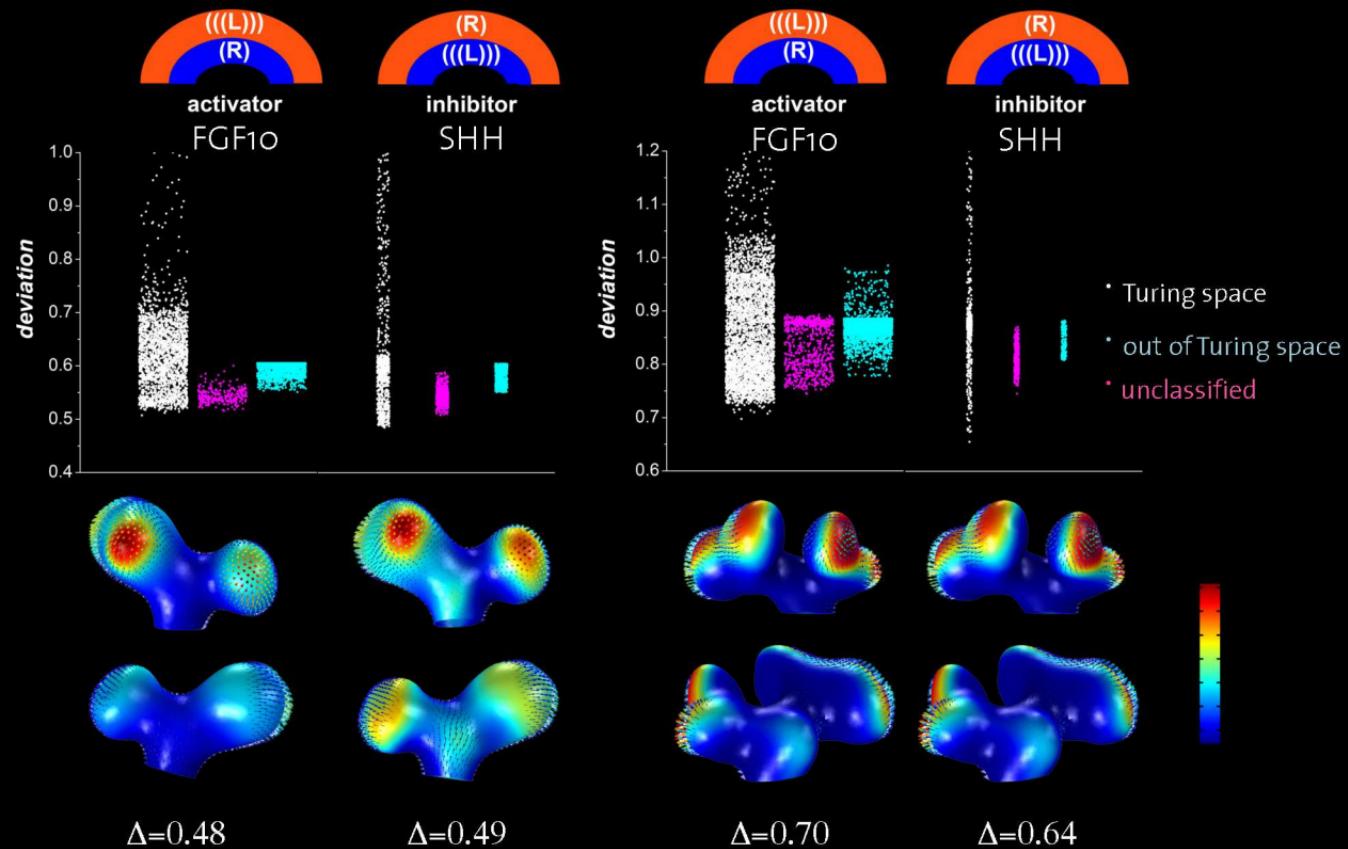
$$\Delta = \sqrt{\sum_{EM} (|\bar{v}_n| - S_n)^2}$$

high

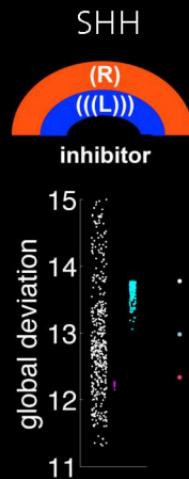
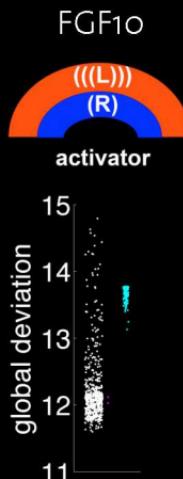
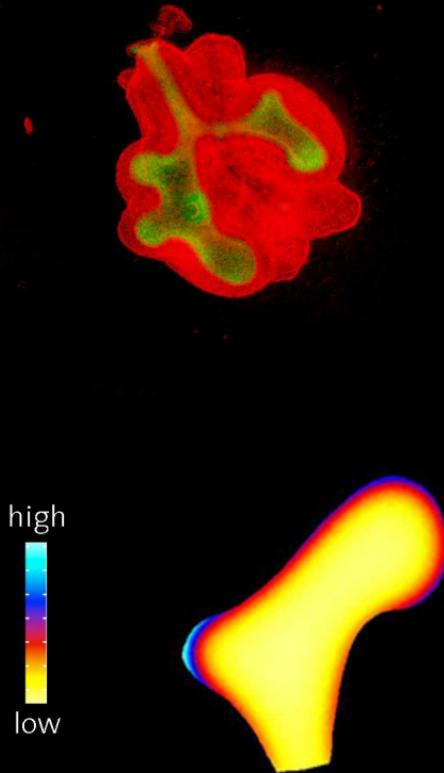


growth field:
arrows – experimental
solid color – computed

TURING-BASED MECHANISMS– LATER STAGES



DYNAMIC DATASET SUPPORTS TURING MECHANISM



- Turing space
 - out of Turing space
 - unclassified
- Menshykau et al. (2014) Development

BRANCHING IN DIFFERENT ORGANS

Lung



Kidney



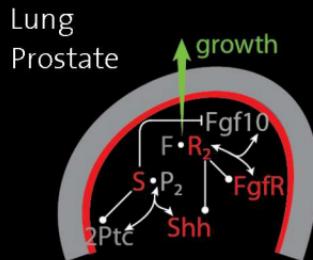
Salivary Gland



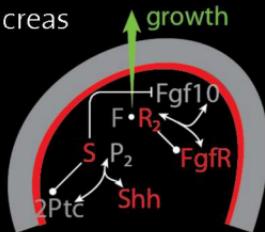
Prostate



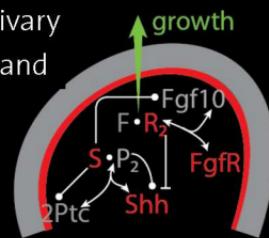
Lung



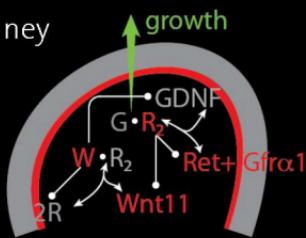
Pancreas



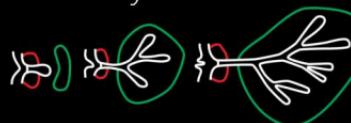
Salivary Gland



Kidney



Mammary Gland



Pancreas



CONCLUSION on BRANCHING MORPHOGENESIS

- PERIODIC PATTERNING: Ligand-receptor based Turing models (but none of the other proposed models) predict growth fields during branching morphogenesis
- DELAYED BRANCHING: The mesenchyme appears to locally delay the branching program
- SPACE-FILLING TREE: The Turing Mechanism recognizes open mesenchymal space
- GENERALITY: The Mechanism applies to Lung & Kidney Branching Morphogenesis despite different molecular networks



TURING PATTERNS

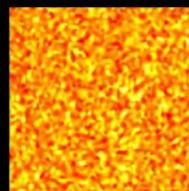
OPEN ISSUES

- ① What are the molecular Turing Components?
- ② Size of Turing Space
- ③ Pattern Robustness to Noisy Initial Conditions

Turing Pattern
Alan Turing 1952



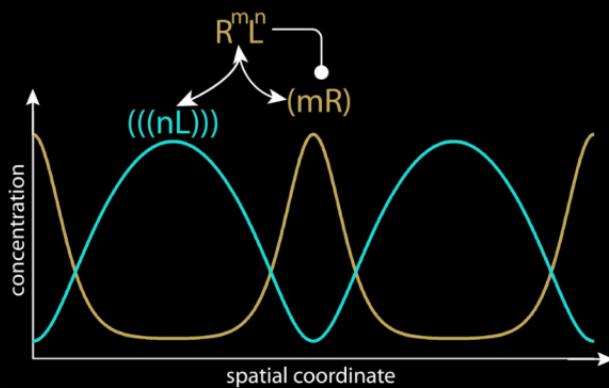
Stable System -
without Diffusion



Instable System -
with Diffusion



1. LIGAND-RECEPTOR BASED TURING PATTERNS



$$\dot{R} = \Delta R + \gamma(a - R + R^2 L)$$

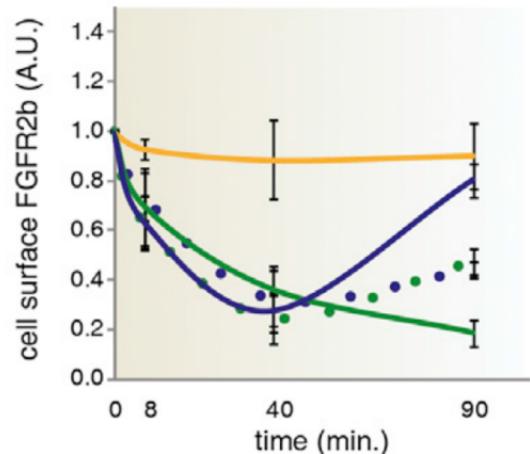
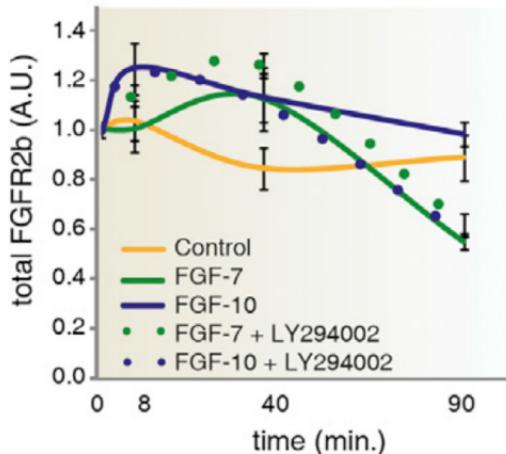
$$\dot{L} = D\Delta L + \gamma(b - R^2 L)$$

REQUIREMENTS:

1. Faster diffusion of ligands relative to receptors
2. Cooperative Binding
3. Positive Feedback of ligand-receptor signaling on receptor abundance

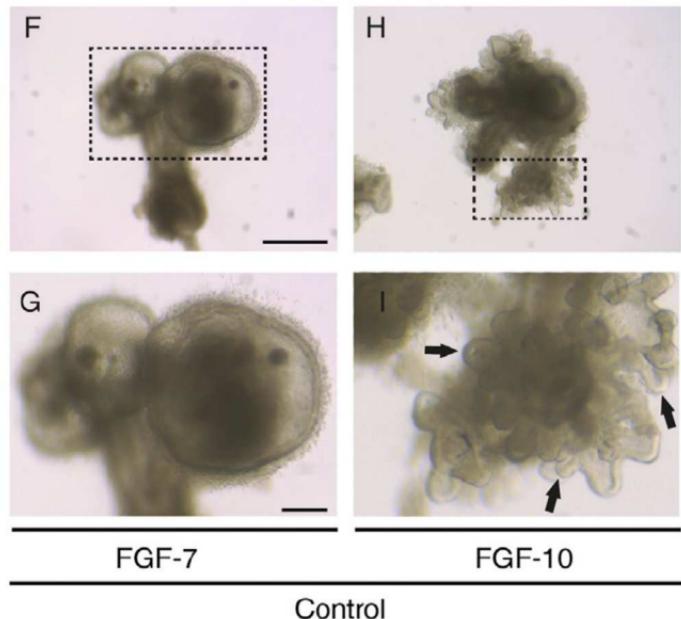
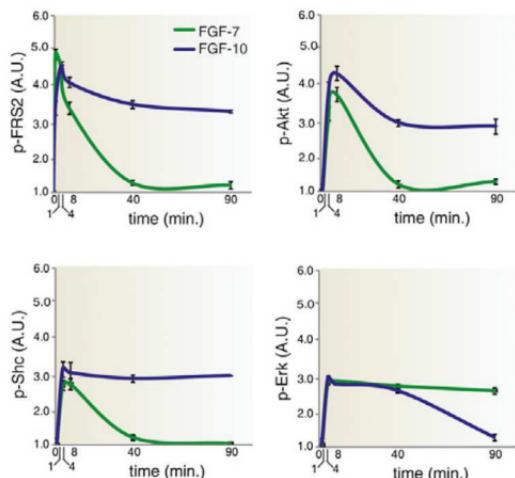
These conditions are met by many ligand-receptor systems.

FGF10, but not FGF7 meets the Turing conditions



Recycling of FGFR2b to the membrane in response to FGF10, but not FGF7 in FGFR2b-transfected HeLa cells.

FGF10, but not FGF7 enhances budding



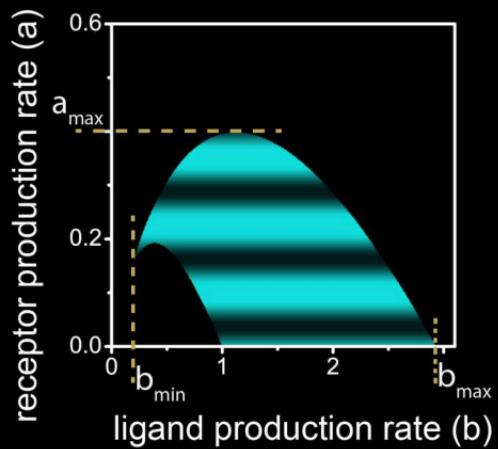
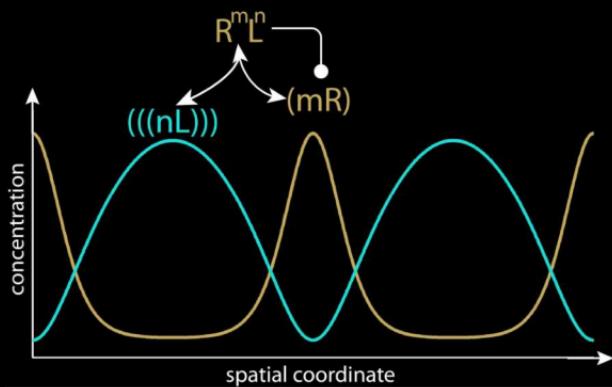
2. MECHANISMS TO ENLARGE TURING SPACE

$$\dot{R} = \Delta R + \gamma(a - R + R^2 L)$$

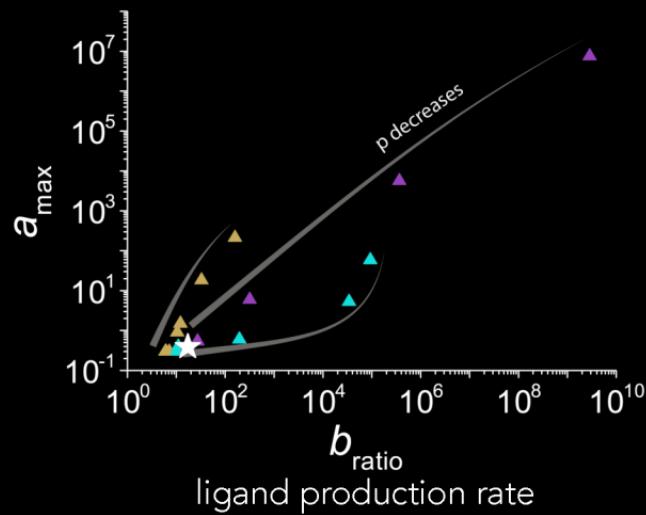
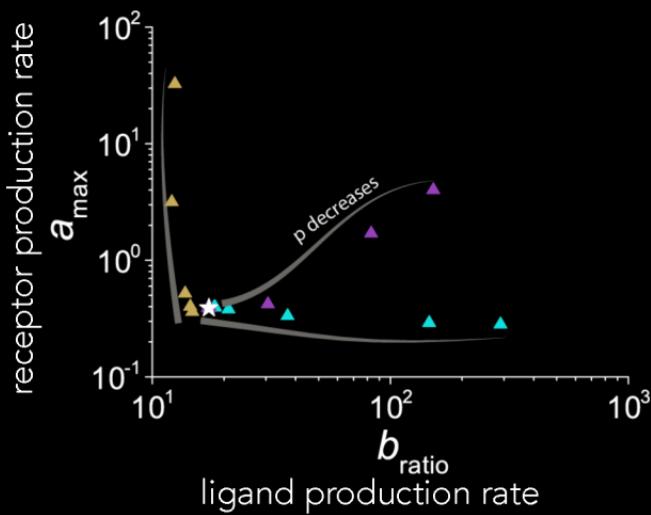
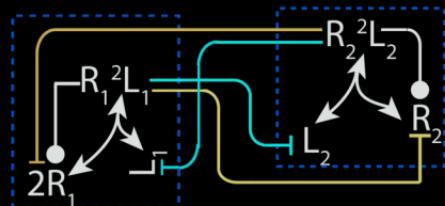
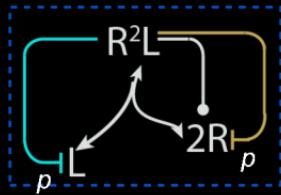
$$\dot{L} = D\Delta L + \gamma(b - R^2 L)$$



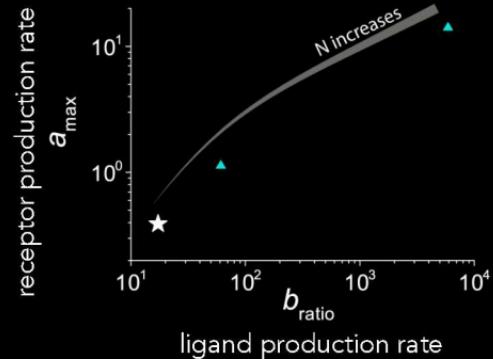
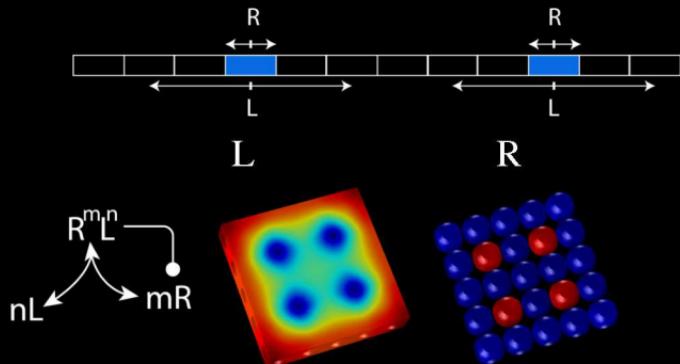
Tamas Kurics



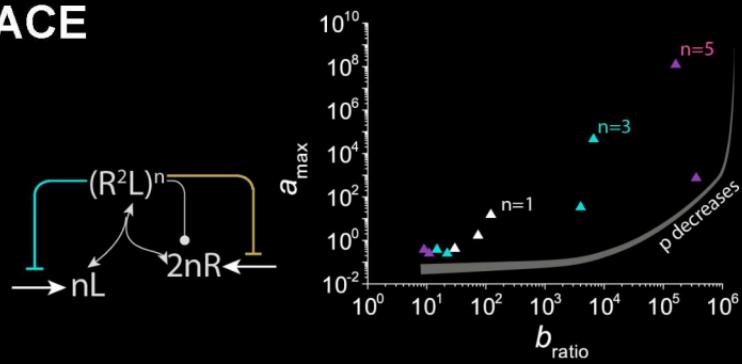
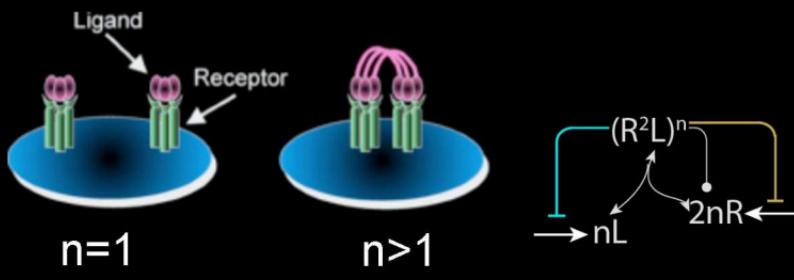
NEGATIVE FEEDBACKS & COUPLING OF TURING MODULES ENLARGE TURING SPACE



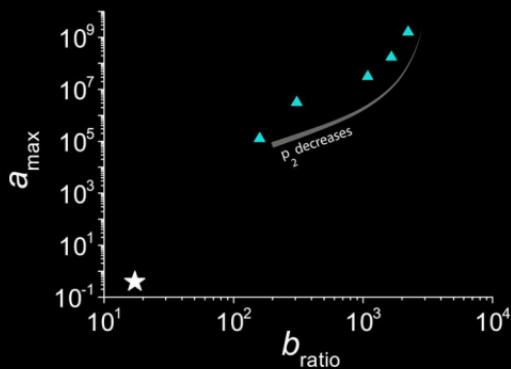
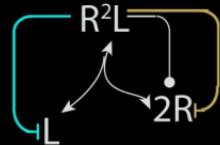
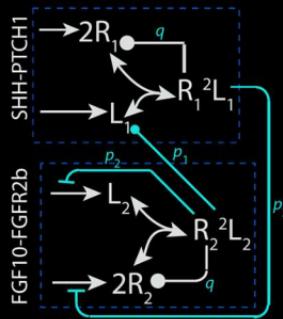
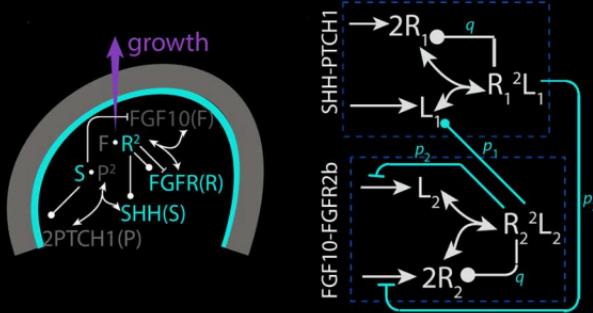
TURING PATTERNS EXIST ON CELLULAR DOMAINS & LARGER TURING SPACE AT HIGHER CELL DENSITY



RECEPTOR CLUSTERING ENLARGES TURING SPACE

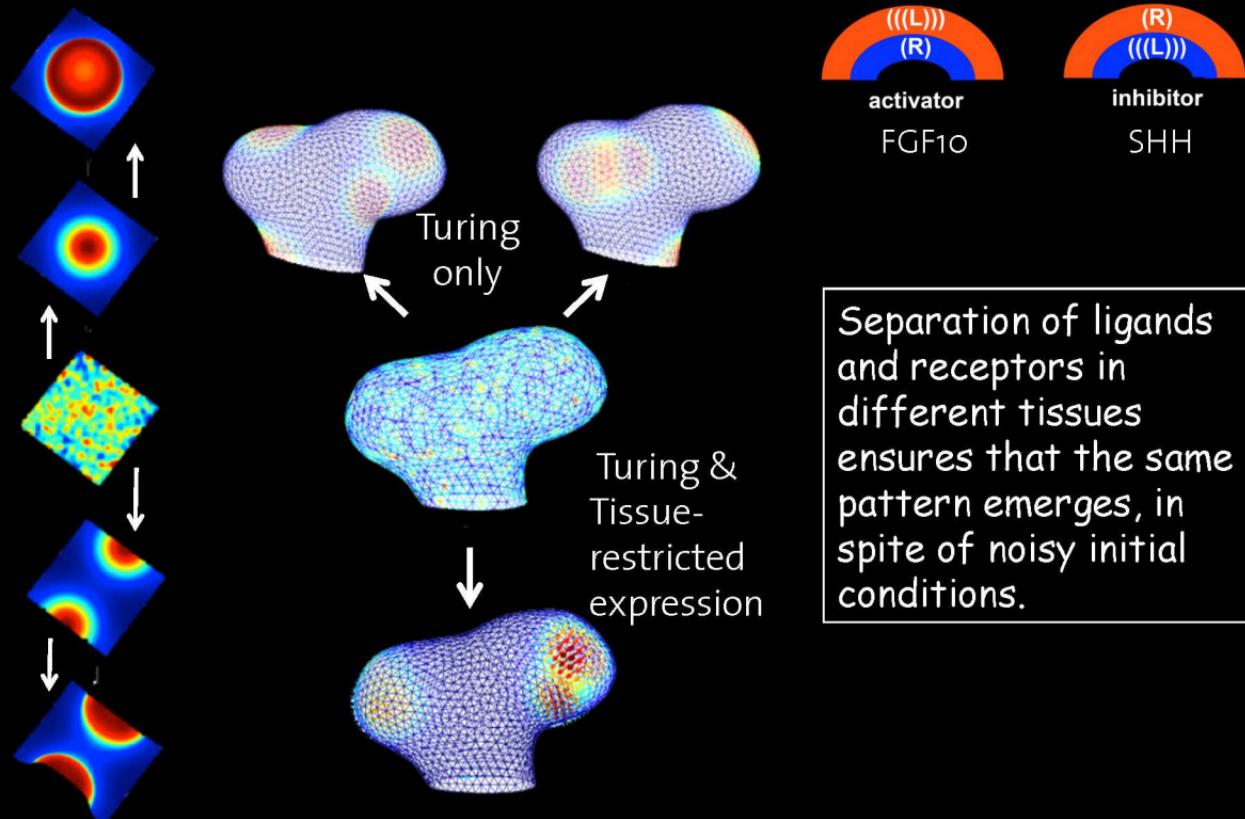


2. TURING SPACE FOR MODELS OF BRANCHING MORPHOGENESIS



Physiological Models of Branching Morphogenesis have huge Turing Space.

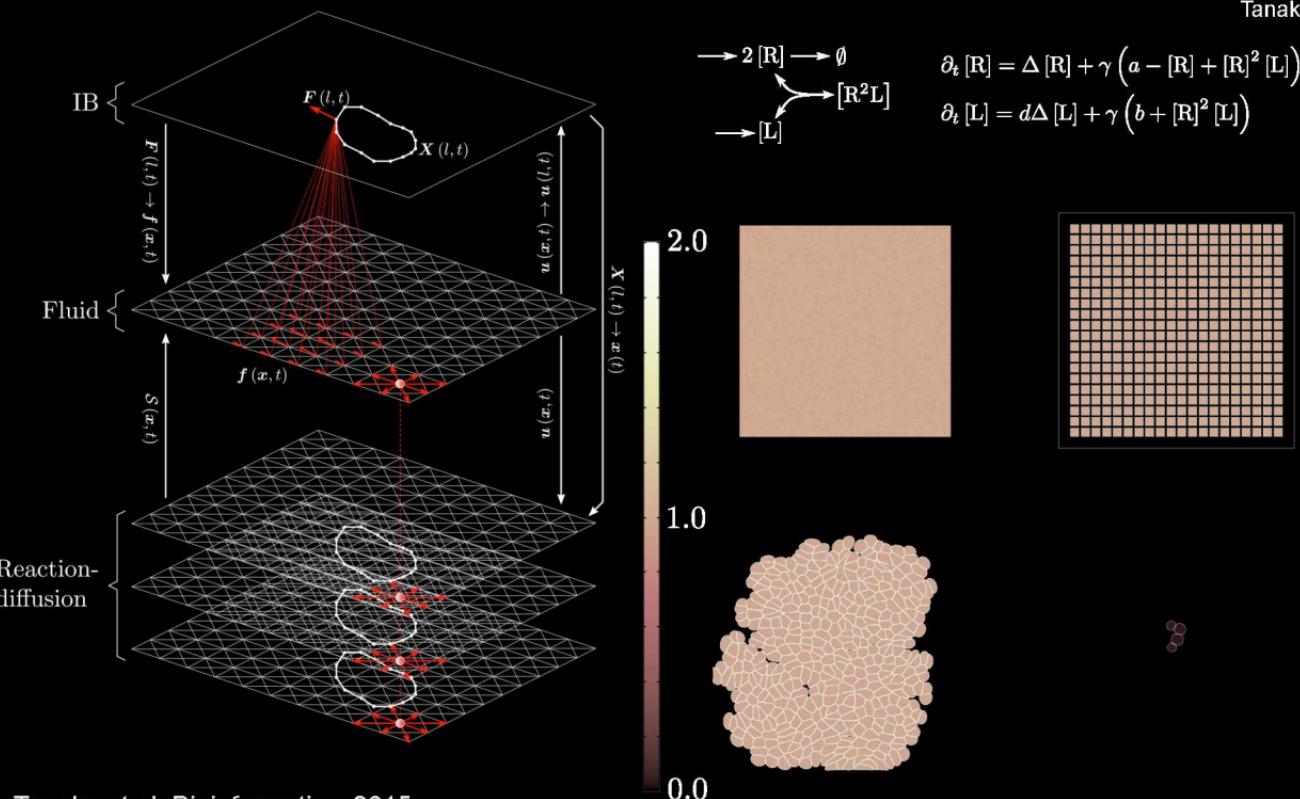
3. ROBUST BRANCHING MORPHOGENESIS



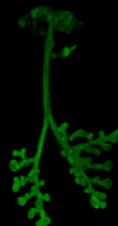
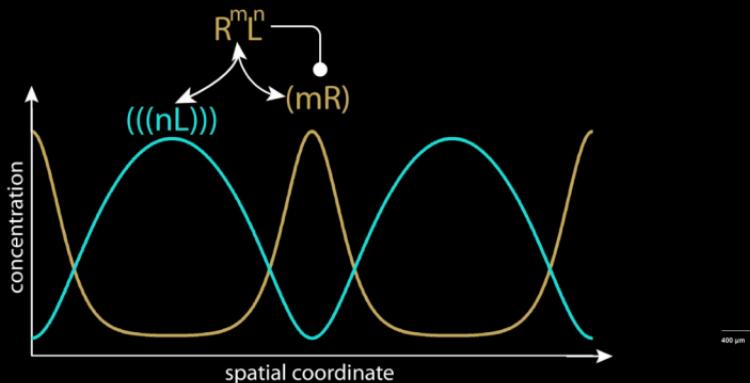
LBIBCell: A SIMULATION FRAMEWORK FOR TISSUE MORPHOGENESIS



Simon
Tanaka



CONCLUSION



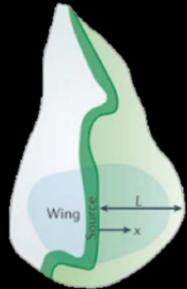
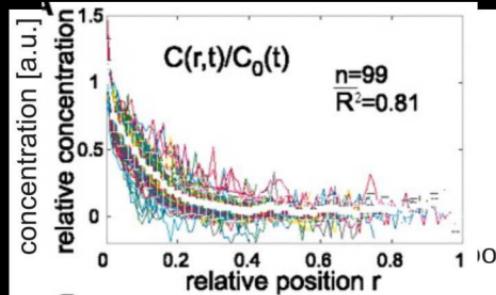
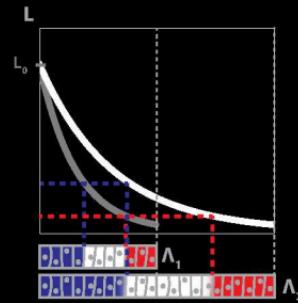
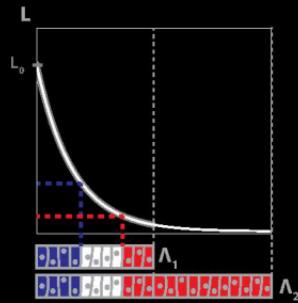
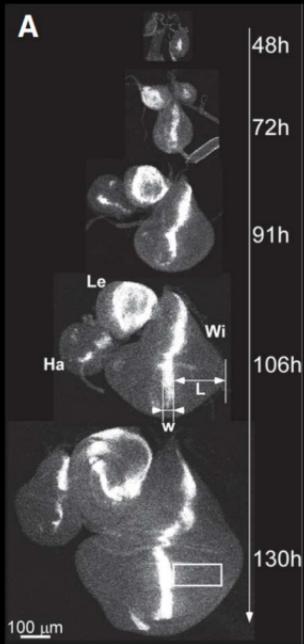
- **PARAMETER SPACE:** Negative Feedbacks, Coupling, and Receptor clustering and restriction to single cells increases size of feasible parameter space
- **ROBUSTNESS:** Separation of ligand and receptors in different tissue layers makes Turing mechanisms robust to noisy initial conditions
- **TISSUE GROWTH:** Cell-based simulations of growing tissue shows that different patterns can emerge than on continuous static domains



PART II: Scaled Morphogen Read-Out on Growing Domains

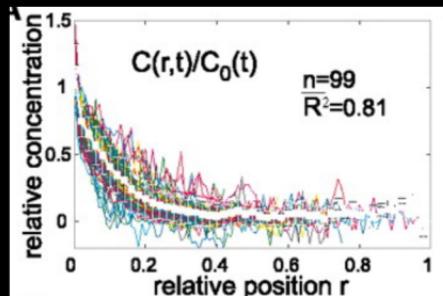
HOW DO PATTERNS SCALE WITH DOMAIN SIZE?

$$c(x) = c(0) \exp\left(\frac{-x}{\lambda}\right)$$



Data from Wartlick et al., *Science*, 2011

HOW DO PATTERNS SCALE WITH DOMAIN SIZE?



The length of the Dpp gradient scales with the length of the growing domain:

How does the Dpp gradient "know" the size of the domain?

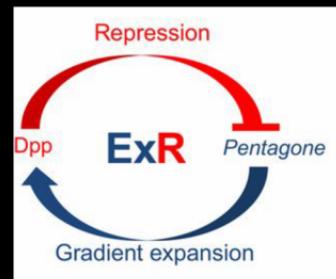
Steady-state model:

$$c(x) = c(0) \exp\left(\frac{-x}{\lambda}\right)$$

$$\lambda = \sqrt{\frac{D}{k_{\text{deg}}}}$$

Wartlick et al., *Science*, 2011.

Expansion-repression model:



Averbukh et al., *Development*, 2014.

Advection-Diffusion system scales almost perfectly

Diffusion equation on a 1D growing domain:

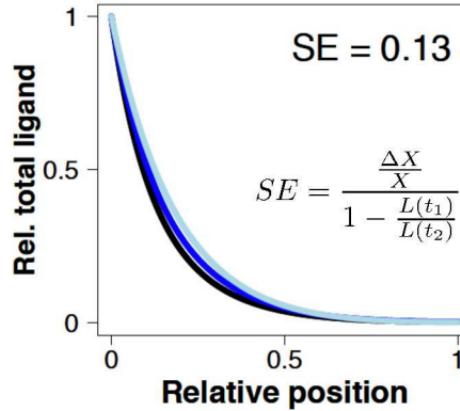
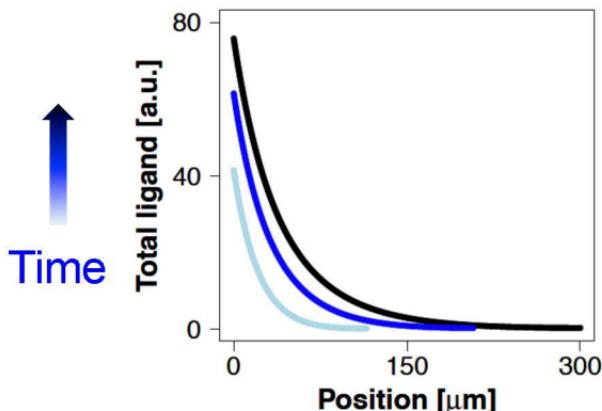
$$\frac{\partial c}{\partial t} = \underbrace{D \frac{\partial^2 c}{\partial x^2}}_{\text{diffusion}} - \underbrace{u \frac{\partial c}{\partial x}}_{\text{advection}} - \underbrace{c \frac{\partial u}{\partial x}}_{\text{dilution}}$$

- Transport by Diffusion & Advection
- Dilution of Dpp because of growth
- Constant influx of Dpp from LHS

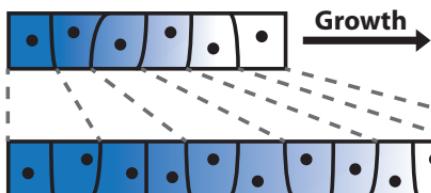


Patrick
Fried

$$L(t) = L(0) + v \cdot t \quad \text{Linear uniform growth (speed } v \text{ determined by Wartlick et. al.)}$$



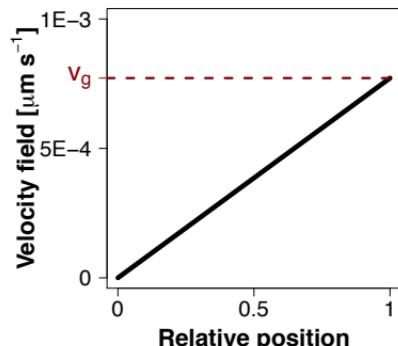
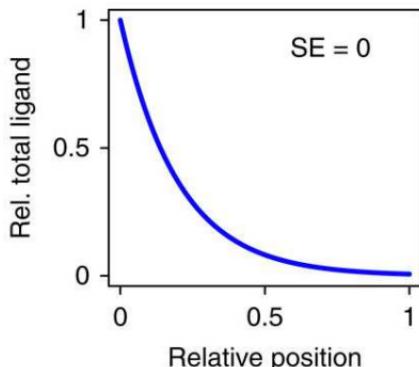
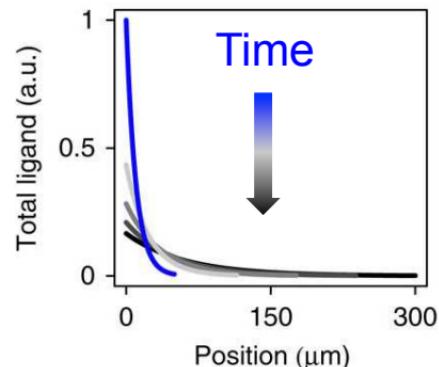
Perfect Scaling by Advection & Dilution



$$\frac{\partial c}{\partial t} = \underbrace{-u \frac{\partial c}{\partial x}}_{\text{advection}} - \underbrace{c \frac{\partial u}{\partial x}}_{\text{dilution}}$$

VELOCITY FIELD for linear uniform growth
(speed v determined by Wartlick et. al.)

$$L(t) = L(0) + v \cdot t$$



Diffusion expands gradient with square-root of time

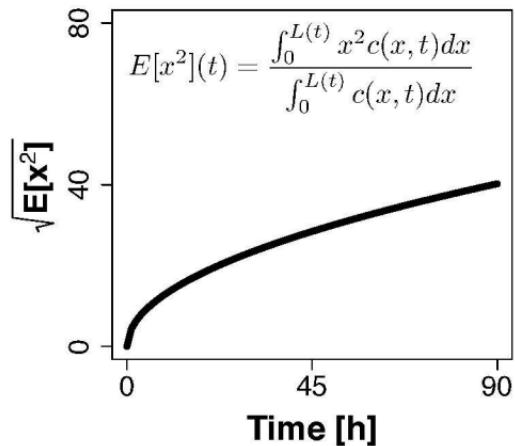
$$\frac{\partial c}{\partial t} = D \frac{\partial^2}{\partial x^2}$$

Dirac Delta function as IC:

$$c(x, t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right)$$

Flux Boundary condition:

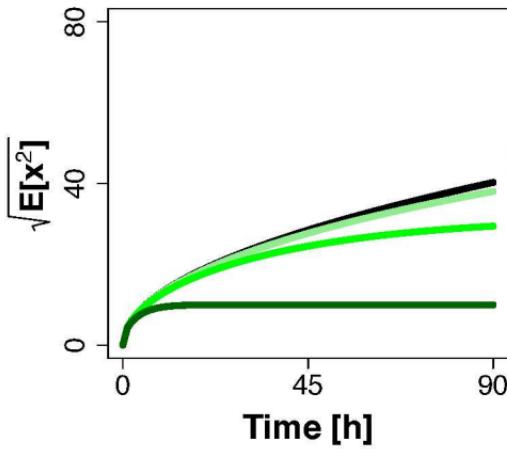
$$\sigma^2 = Dt$$



Diffusion: $\lambda \sim \sqrt{t} = \sqrt{\frac{L(t)}{v} - \frac{L_0}{v}}$

Perfect scaling: $\lambda \sim L(t)$

Degradation limits diffusion-driven gradient expansion



Increasing
degradation

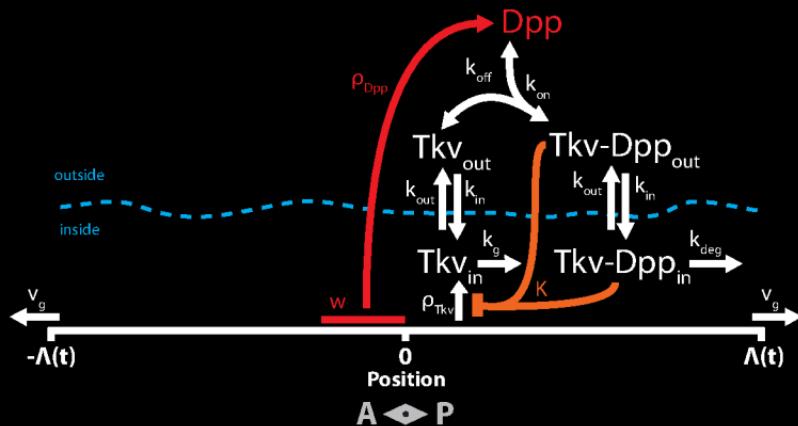
Pre-steady state effect

Degradation limits diffusion-driven
gradient expansion

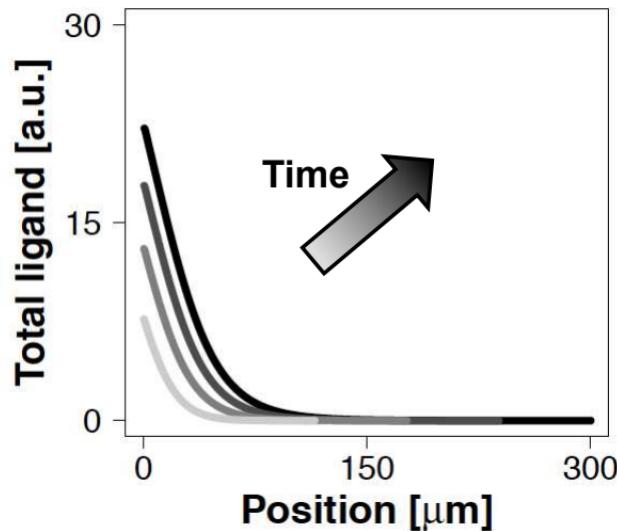
Diffusion: $\lambda \sim \sqrt{t} = \sqrt{\frac{L(t)}{v} - \frac{L_0}{v}}$

Perfect scaling: $\lambda \sim L(t)$

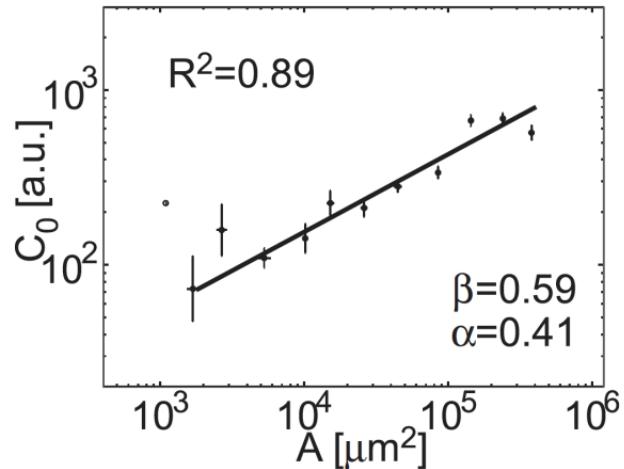
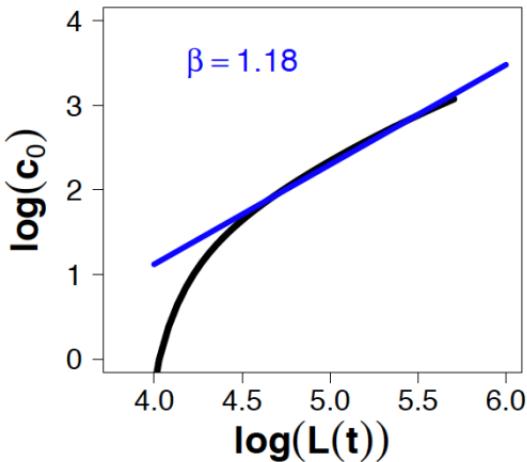
THE DPP SIGNALLING MODEL



Model shows temporal increase of concentration



Power-law relation fits temporal D_{pp} increase

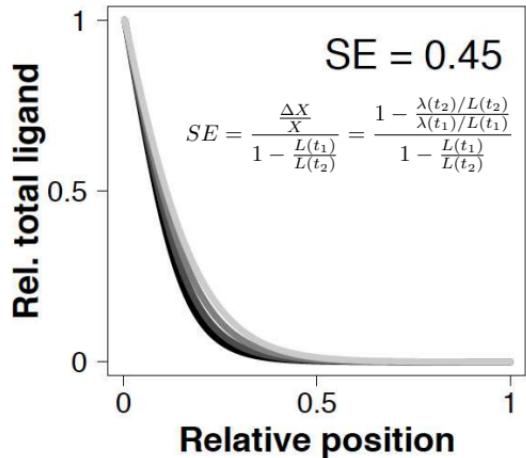
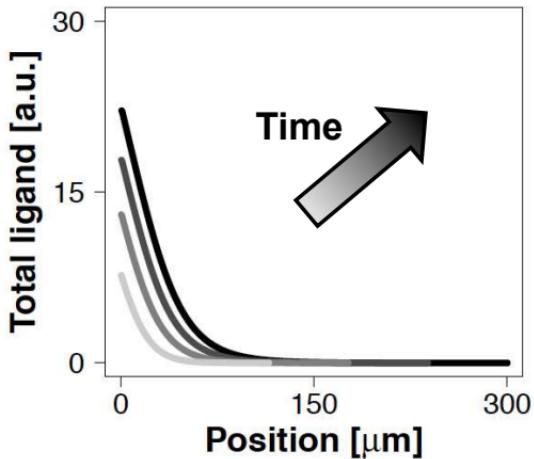


Power-law exponents are the same if $A \sim L^2$

Fried and Iber, *Nat. Commun.*, 2014.

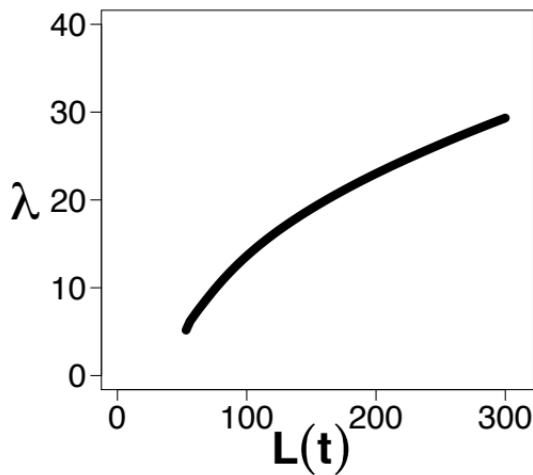
Wartlick et al., *Science*, 2011.

Imperfect scaling is observed in the wing disc model



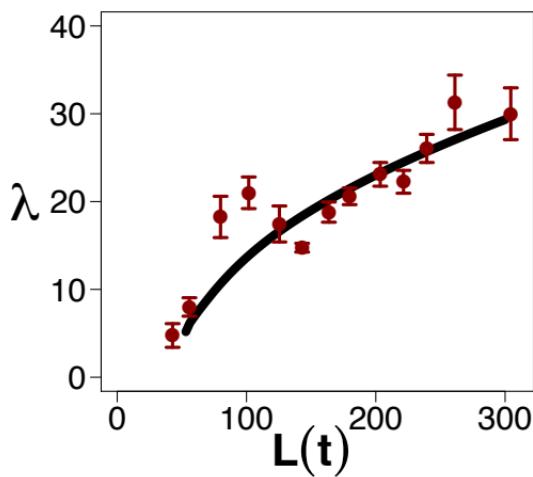
The gradients scale to some extent but clearly not perfectly.

Imperfect scaling explains gradient expansion data

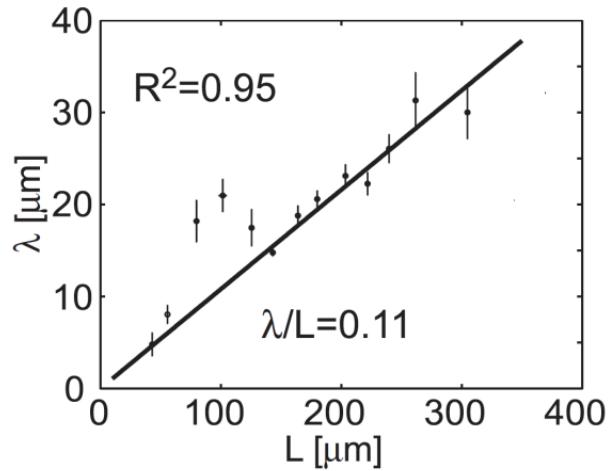


Fried and Iber, *Nat. Commun.*, 2014.

Imperfect scaling explains gradient expansion data

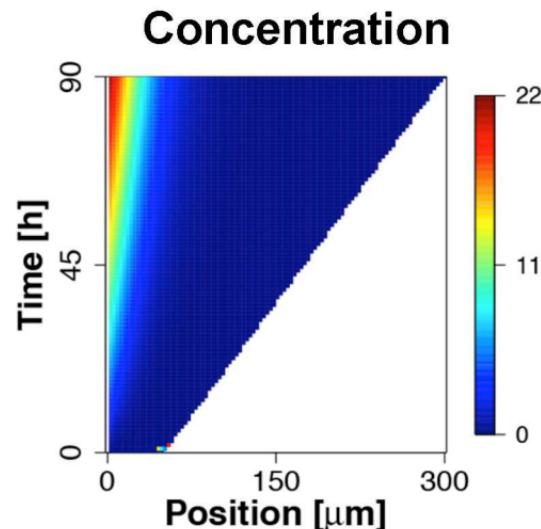
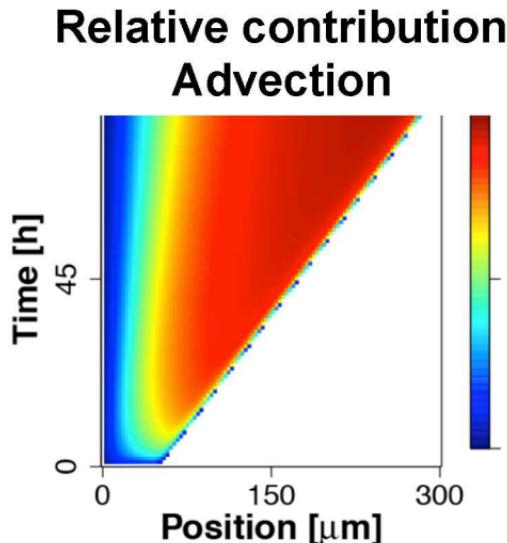


Fried and Iber, *Nat. Commun.*, 2014.



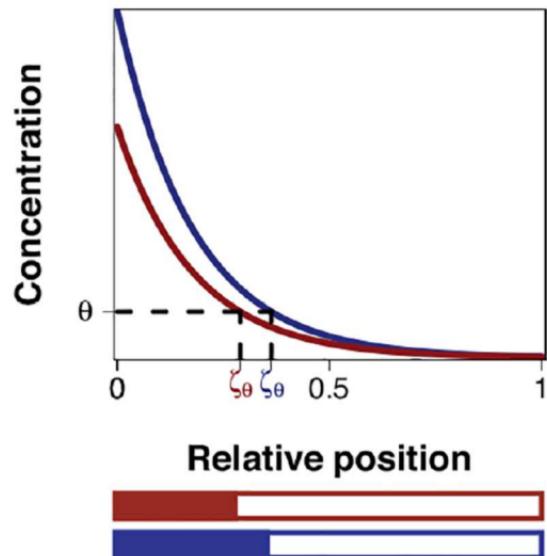
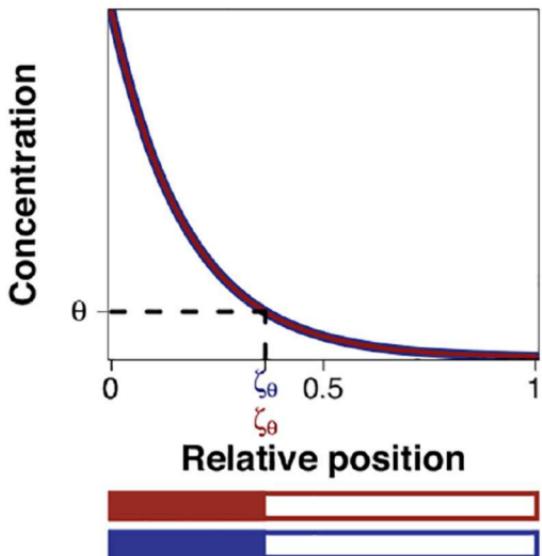
Wartlick et al., *Science*, 2011.

Imperfect scaling is dominated by diffusion



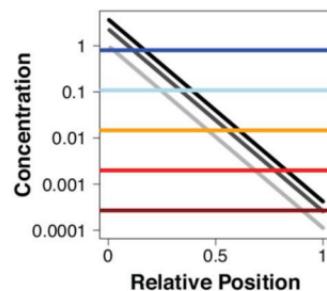
- Diffusion dominates in the front of the tissue
- Advection dominates at the end of the tissue

How to read out an increasing, scaled gradient?

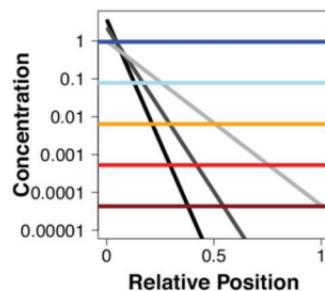


Threshold-based Read-Out

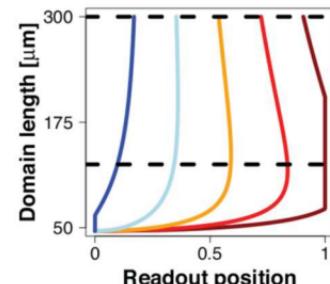
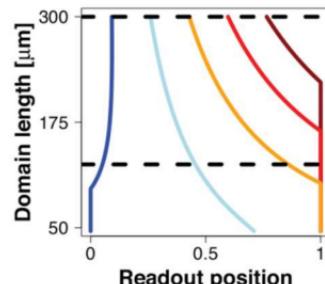
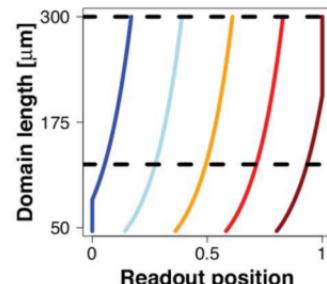
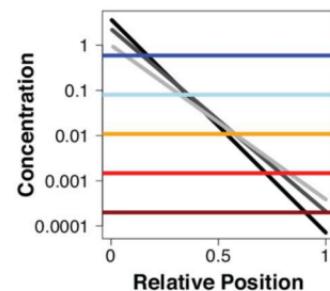
Perfect scaling



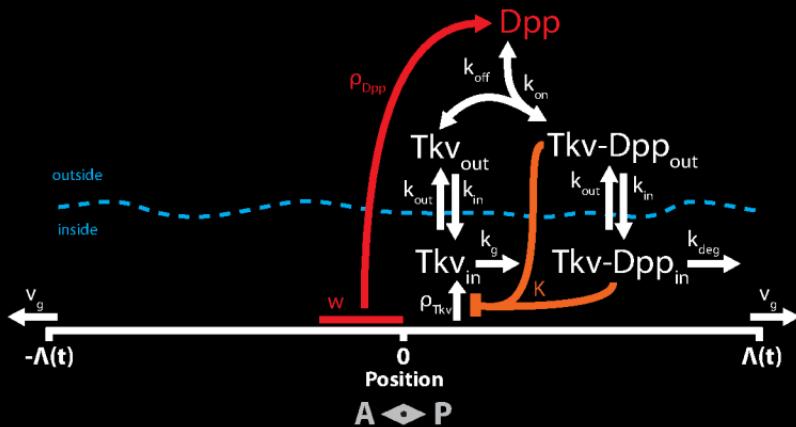
No scaling



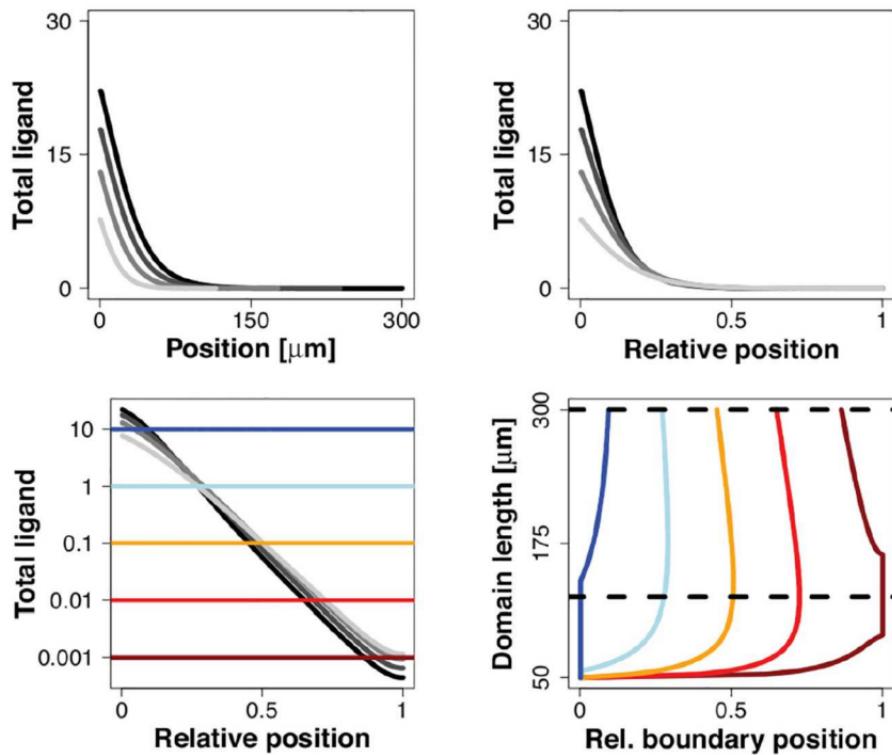
Imperfect scaling



THE DPP SIGNALLING MODEL

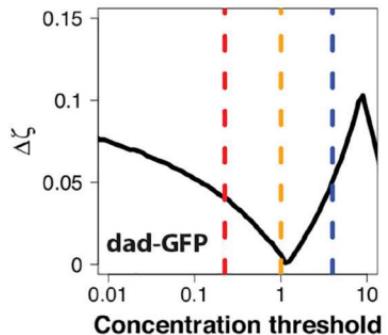
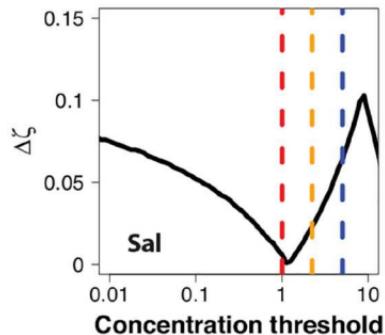
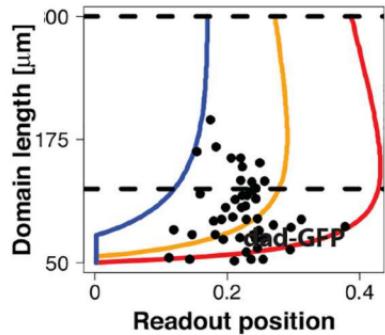
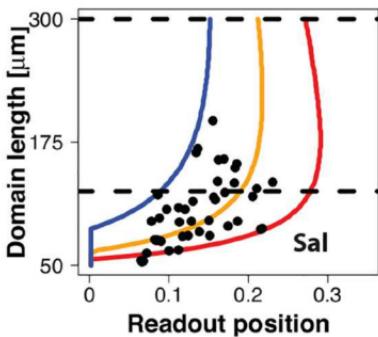


Threshold-based Read-Out of Dpp

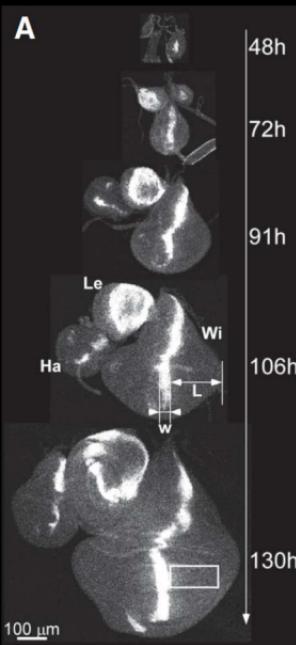


Dpp Gradient Read-Out

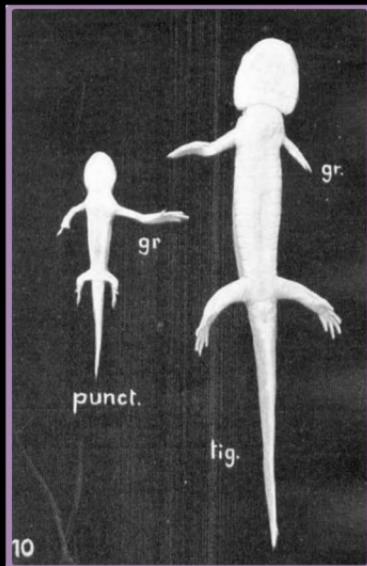
Data points from
Hamaratoglu et al.
PLoS Biol. 2011



Conclusion on Scaling Mechanism



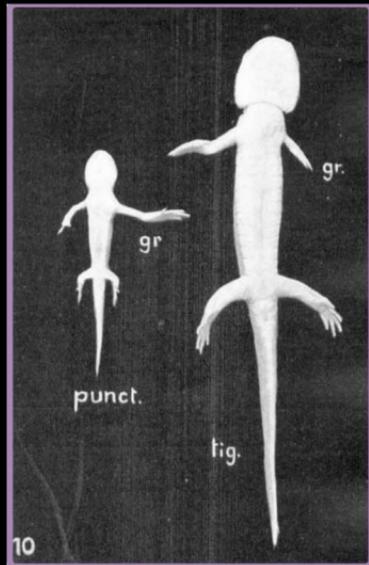
- A general mechanism for **imperfect scaling**
- This mechanism requires pre-steady state kinetics
→ **low degradation rate of internalized Dpp**
- **Drosophila wing disc:**
 - Imperfect scaling is consistent with biological data
 - Threshold-based read-out of Dpp enables precise definition of **gene expression domains** despite continuous increase of Dpp concentration
- In the wing disc transport is dominated by diffusion. In **other tissues** it could be dominated by **advection**.



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THE JOURNAL OF EXPERIMENTAL ZOOLOGY
59, 1 (FEBRUARY 1931) © 1931 WILEY-LISS
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PART III: What determines size?

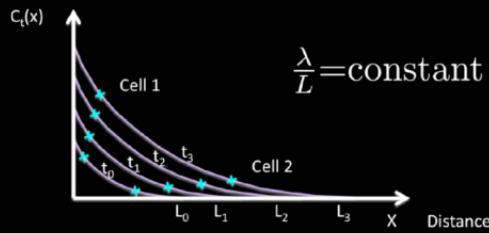
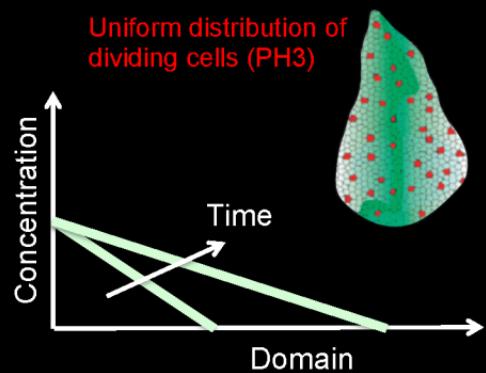
GROWTH TERMINATION



CREDIT: V. C. TWITTY AND J. L. SCHWIND,
THE JOURNAL OF EXPERIMENTAL ZOOLOGY
59, 1 (FEBRUARY 1931) © 1931 WILEY-LISS
INC., A WILEY COMPANY

- I. **GENERAL FACTORS**
 - **surrounding tissue / larvae:** same size if discs develop outside larvae
 - **developmental time:** developmental delays do not matter for final size
 - **counting of cell divisions:** enhancing or blocking cell divisions does not alter final wing disc size

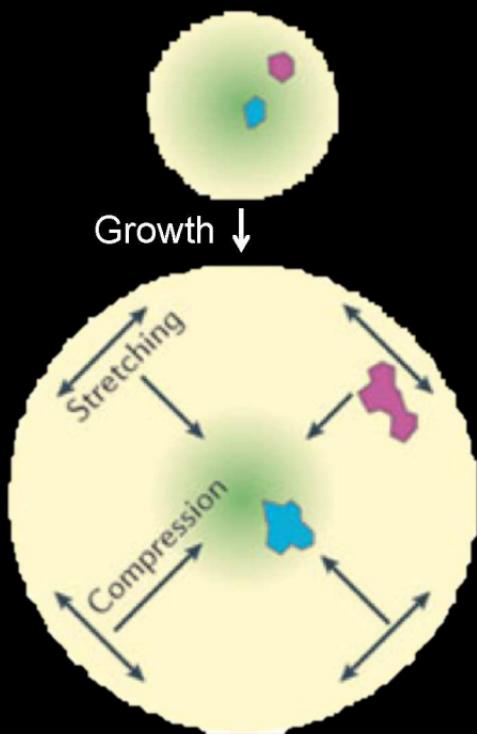
GROWTH CONTROL



II. MORPHOGENS

- Proliferation proportional to steepness of (linear) morphogen gradient: Dpp gradient is of exponential shape
- Proliferation rate dependent on relative change of Dpp amplitude: Dpp signalling not necessary for normal growth; imperfect scaling of the Dpp gradient

GROWTH CONTROL



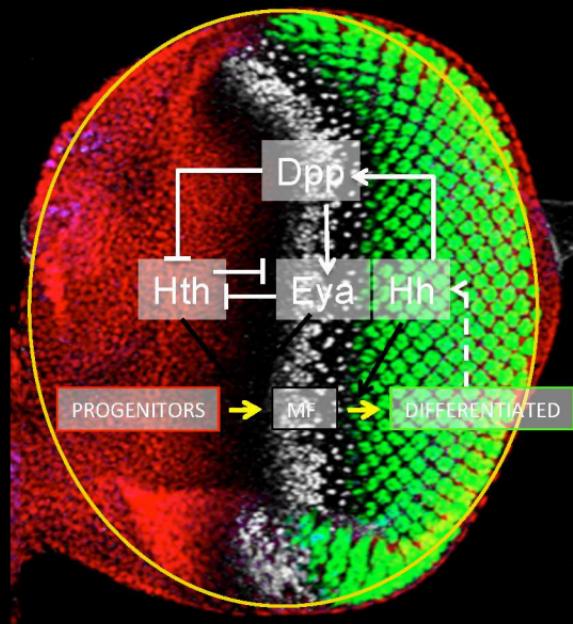
III. TISSUE MECHANICS

- based on combined Dpp morphogen signalling and mechanical feedback: Dpp signalling not necessary for normal growth.

IV. CELL DIFFERENTIATION

- Example: loss of growth plate terminates long bone growth

GROWTH CONTROL IN DROSOPHILA EYE

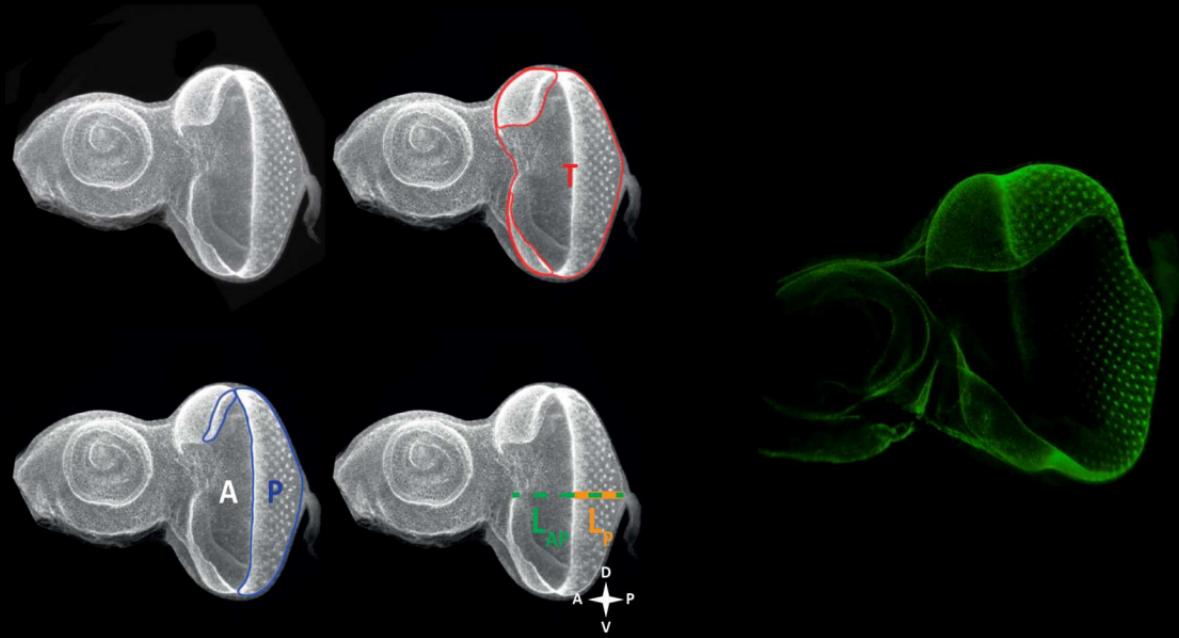


pictures courtesy of Gunilla Ståhls-Mäkelä, Helsinki

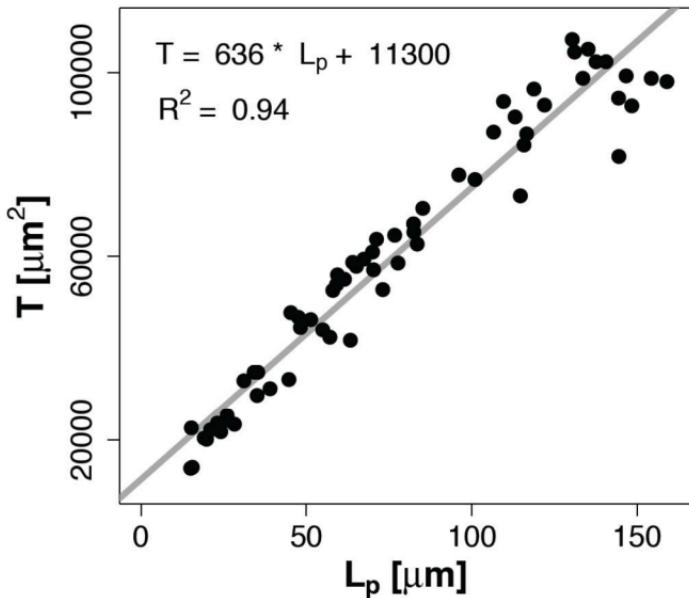
CABD (Sevilla): Max Sanchez,
Daniel Aguilar, Fernando Casares

ETH Zürich (CH): Patrick Fried,
Jannik Vollmer

3D Quantitative Growth Measurements



The Eye Disc Area expands linearly with time



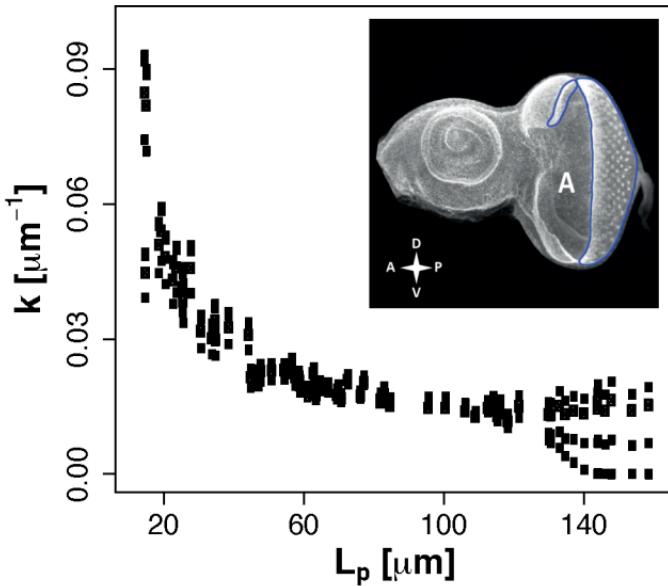
Jannik Vollmer



Patrick Fried

The Area Growth Rate Declines over dev. time

$$\frac{dT}{dL_p} = \frac{dA}{dL_p} + \frac{dP}{dL_p} = k(L_p) \cdot A + L_{MF}(L_p) \cdot (\alpha - 1)$$



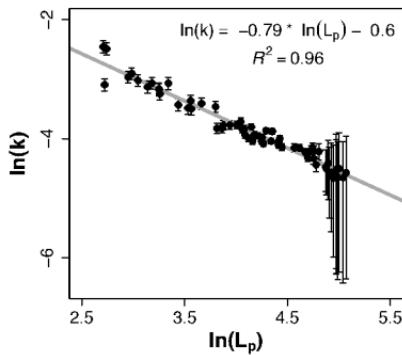
The area growth rate on the anterior side declines continuously.

The declining growth rate is not the result of cell differentiation.

What is the Underlying Mechanism?

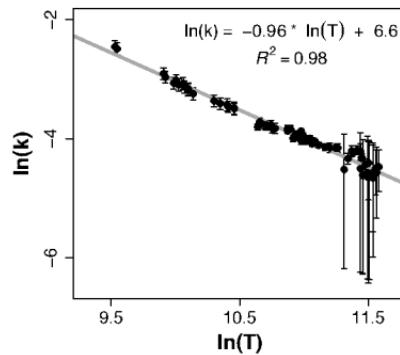
Power law

$$k = k_0 \left(\frac{L_P(0)}{L_P} \right)^\delta$$



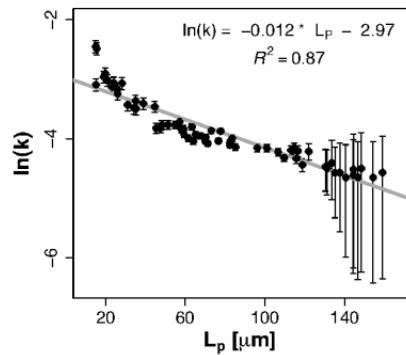
Area-dependent growth law

$$k = k_0 \left(\frac{T(0)}{T(L_P)} \right)$$

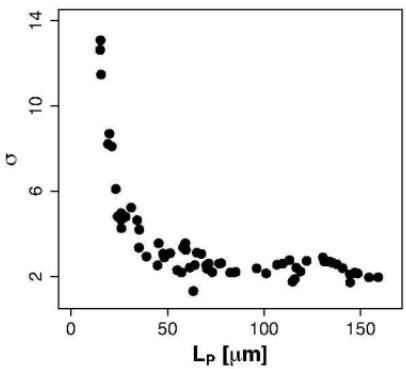
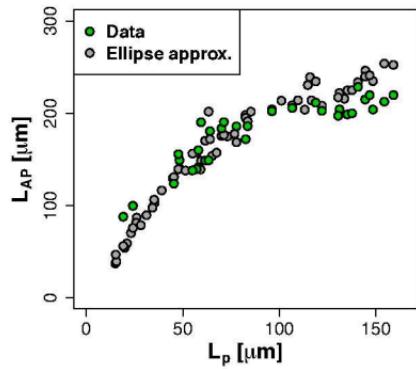
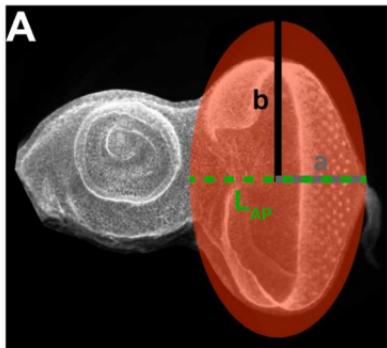


Exponential growth law

$$k = k_0 e^{-\delta \cdot \Delta L_P} + k_1$$



Ellipse Approximation of the Eye Disc

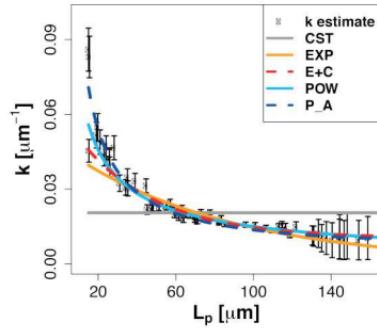


$$T = \pi \sigma a^2$$

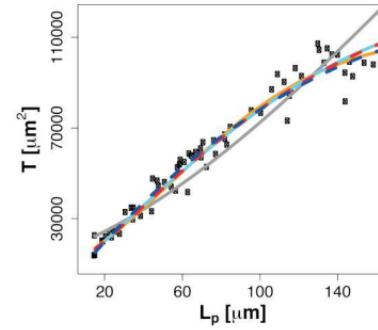
$$P = \frac{T}{\pi} \left(\cos^{-1} \left(1 - \frac{2L_p}{L_{AP}} \right) - 2 \left(1 - \frac{2L_p}{L_{AP}} \right) \sqrt{\frac{L_p}{L_{AP}} \left(1 - \frac{L_p}{L_{AP}} \right)} \right)$$

AREA-DEPENDENT GROWTH RATE FITS BEST

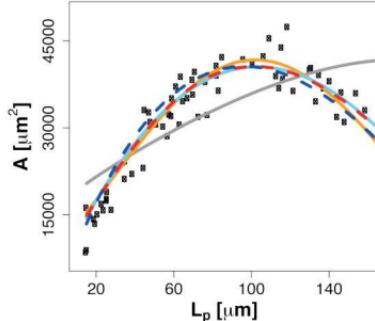
Growth
Rate



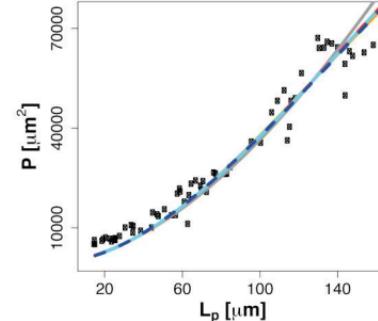
Total
Area



Anterior
Area

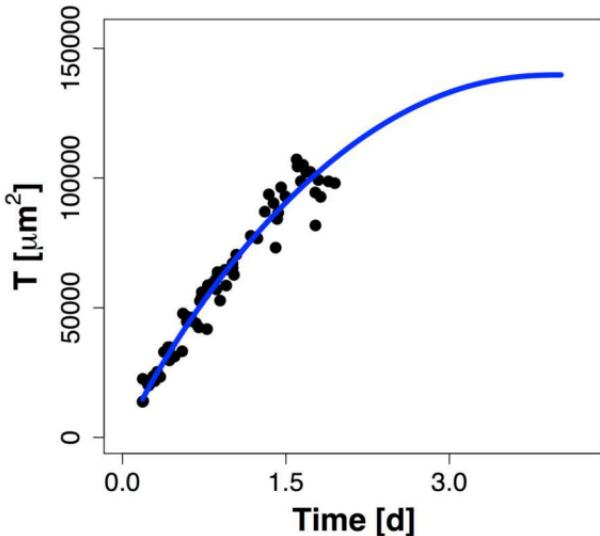


Posterior
Area

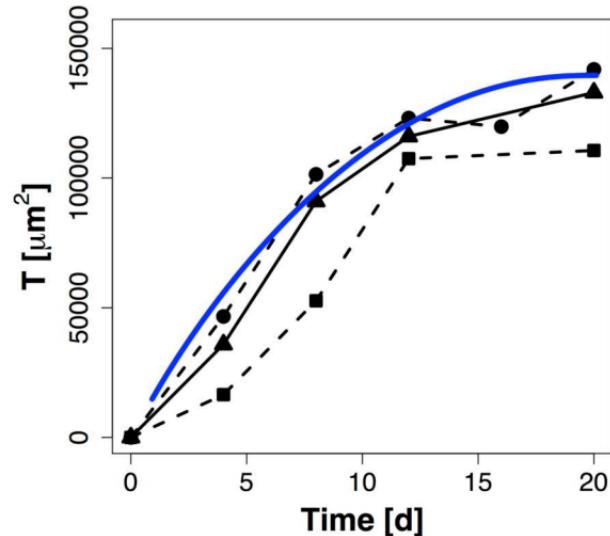


Size Preservation At Lower Dev. Speeds

Larvae



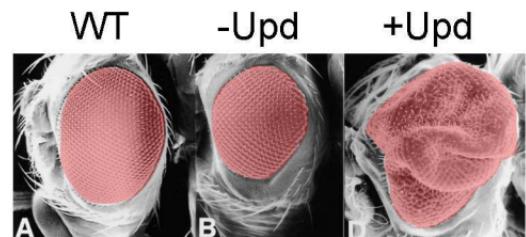
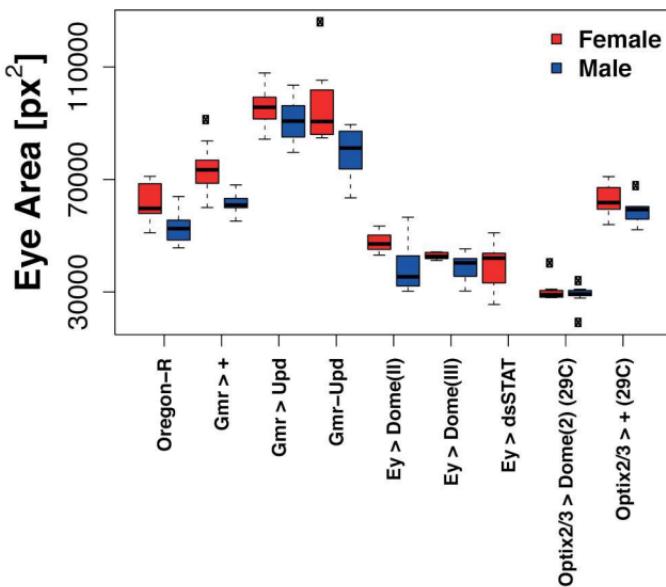
Grafted Eye Discs



Area-Dependent growth rate naturally preserves final size at lower developmental speeds.

CANDIDATE: Cytokine Unpaired (Upd)

Production restricted to initial stages of eye development (before MF starts)



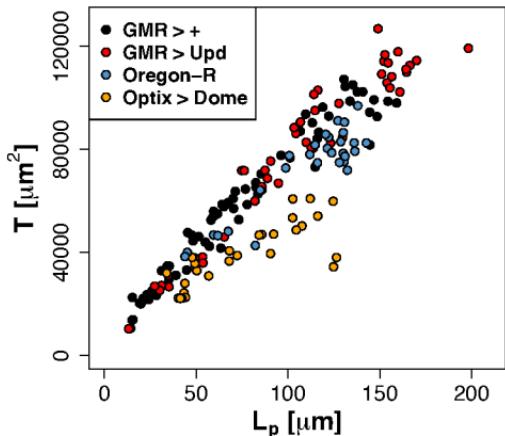
Bach et al., 2003

LARGEST mutant (130% of wt):
Gmr > Upd

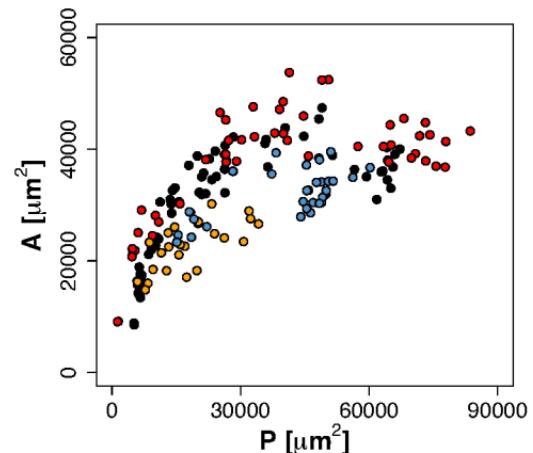
SMALLEST mutant (40% of wt):
Optix > Dome

GROWTH KINETICS IN MUTANTS DIFFER

TOTAL AREA



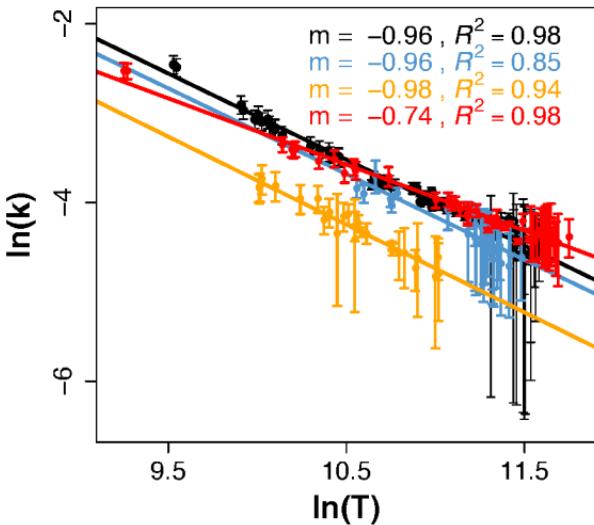
ANTERIOR AREA



Growth Kinetics depend on Upd Production Rate

Area-dependent

$$k = k_0 \left(\frac{T(0)}{T(L_P)} \right)$$



- Control strains have slope -1
- Optix > Dome has substantially lower growth rate, but slope -1
- Continuous expression of Upd in Gmr > Upd leads to slower decay of the growth rate

Conclusion: Eye Disc Growth Control

- Area growth rate declines inversely proportional to area-growth
- Area-dependent growth rate explains size preservation at lower developmental rate
- Upd as candidate for growth control by dilution



pictures courtesy of Gunilla Ståhls-Mäkelä, Helsinki

Everything should be made as simple as possible, but no simpler.
Albert Einstein

THANKS!!



Computational Biology Group (CoBi)



CISD

COMSOL Engineers

Biological Collaborators

BODY AXIS FORMATION

- Technau group (Vienna)

BRAIN DEVELOPMENT

- Taylor Group (Basel)

BRANCHING MORPHOGENESIS

- McMahon group (USC, US)
- Sapin / Blanc (France)

DROSOPHILA EYE DISC

- Casares lab (Sevilla, Spain)

LIMB & BONE DEVELOPMENT

- Mariani group (USC, US)
- Zeller group (Basel)

OVARIAN FOLLICLES & IVF

- De Geyter (Basel)