PI APPROXIMATION DAY MATHQUIZ 2021

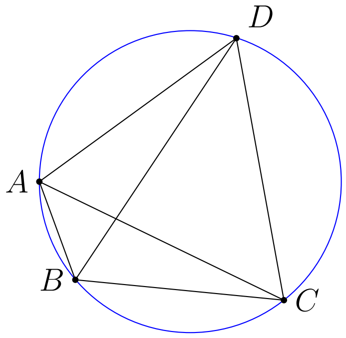
Questions

Section A

1. 'X', an ancient Indian mathematician gave the value of π as 62832/20000, which is equal to 3.1416 (correct upto four decimal places) and also mentioned that this value of π is approximate (*"asanna"*). Identify 'X'.

   (A) Ramanujan
   (B) Brahmagupta
   (C) Madhava
   (D) Aryabhatta
   (E) Lilabati

2. The theorem related to this figure is $AC.BD = AB.CD + BC.AD$. The theorem was given by:



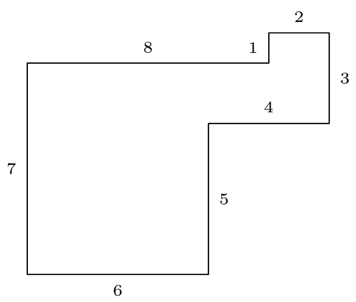
   (A) Brahmagupta
   (B) Ptolemy
   (C) Ceva

(D) Menelaus
(E) Thales

3. Which one of the following is a wrong statement?

(A) Pi(π) Approximation Day on July 22 honours the concept of pi(π)
(B) Pi(π) is occasionally referred to as the Archimedes constant
(C) Pi(π) is transcendental
(D) Pi(π) is equal to the ratio of two numbers 22 and 7 which makes it an irrational number
(E) In the approximate value of pi(π) the digits in the decimal part go on and on with no pattern, neither ends nor repeats

4. A 'X' is any polygon (see diagram below) with all right angles whose sides are consecutive integer lengths. Identify 'X'.

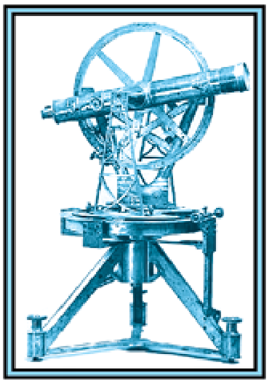


(A) Polygon
(B) Octagon
(C) Golygon
(D) Decagon
(E) Heptagon

5. 'X' was an Indian scientist and statistician who had a significant role in formulating India's strategy for industrialization in the 2nd Five-Year Plan (1956-61). 'X' was the founder of the reputed Indian Statistical Institute. 'X' was also a friend of Srinivasa Ramanujan at Cambridge. Identify 'X'.

(A) P.C. Ray
(B) P.C. Mahalanobis
(C) C.R. Rao
(D) C.S. Seshadri
(E) S.N. Bose

6. Below is an image of a surveying instrument which works on the principle of trigonometry. It is used for measuring angles with the help of a rotating telescope. Identify the instrument



(A) Ammeter
(B) Protractor
(C) Anemometer
(D) Theodolite
(E) Gauge

7. 'X' was an ancient Indian mathematician, who was a Vedic brahmin priest and an architect of very high standards. He stated the theorem now known as Pythagoras theorem in his book Sulbha Sutra. The picture of 'X' is given below. Identify 'X'.

(A) Sridhar Acharya
(B) Aryabhatta
(C) Brahmagupta
(D) Baudhayana
(E) Bhaskara

8. “Every even integer greater than 2 can be expressed as a sum of two primes”. It is known as:

   (A) Hodge conjecture
   (B) Twin prime conjecture
   (C) Legendre’s conjecture
   (D) Andrica’s conjecture
   (E) Goldbach’s conjecture

9. Suppose we are going to prove that a statement p is true, we firstly assume that the negation of the statement is true i.e., $\sim p$ is true. Then we prove that $\sim p$ is false. Hence the statement p is true. What is this method of proof called?

   (A) Method of induction
   (B) Method of substitution
   (C) Method of elimination
   (D) Method of contradiction
   (E) Method of partial fractions

10. Regarded as the highest honour in the field of mathematics, the ‘X’ is a prize awarded to two, three or four mathematicians every four years by the International Mathematical Union(IMU). It was first awarded to the Finnish mathematician Lars Ahlfors in 1936 and it has been awarded every four years since 1950. This prize has only one female recipient. Identify the prize ‘X’.

    (A) Abel Prize
    (B) Fields Medal
    (C) Chern Medal
    (D) Nobel Prize
    (E) Wolf Prize

Section B

1. If $2^a = 3^b = 7^c = 42$, then what is the value of $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}$?

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

2. The average score of boys in an examination of a school is 71 and that of girls is 73. The average score of the school in the examination is 71.8. Find the ratio of the number of boys to the number of girls that appeared in the examination.

   (A) 3:2
   (B) 2:3
   (C) 3:4
   (D) 4:3
   (E) 1:2

3. We write down all the digits from 1 to 9 side by side. Now we put '+' between as many digits as we wish to, so that the sum of the numbers become 513. It is explained below

   1 2 3 4 5 6 7 8 9 = 513

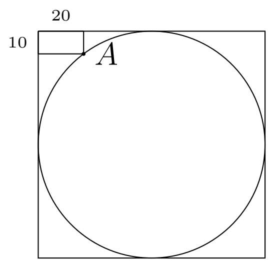
   Now suppose we put '+' signs at the following places:

   12+345+67+89=513

   Since there are four numbers, so the average can be calculated by dividing the sum by 4. What is the average, if the sum is 261?

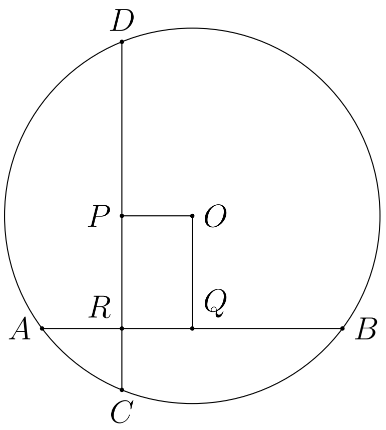
   (A) 52.2
   (B) 42.2
   (C) 62.2
   (D) 72.2
   (E) 32.2

4. In the figure given below, the rectangle at the corner measure 20 cm $\times$ 10 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?



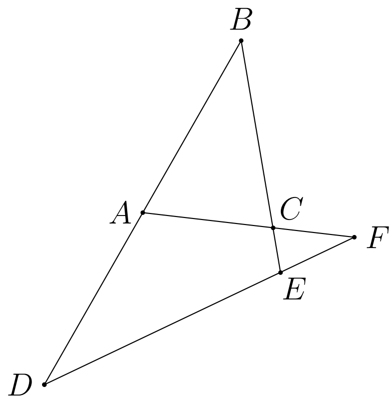
(A) 10
(B) 40
(C) 50
(D) 60
(E) none of the above

5. AB and CD are mutually perpendicular chords of a circle intersecting at a point R (see figure). OP and OQ are perpendiculars on chords CD and AB respectively. If $OP = 1$, $OQ = 4$ and $DP = 2BQ$, then what is the approximate value of AB? (figure not to scale)



(A) 1.732
(B) 2.236
(C) 2.828
(D) 3.464
(E) 4.472

6. In the figure below, BAD, BCE, ACF and DEF are straight lines. If $BA = BC$, $AD = AF$, $EB = ED$ and $\angle BED = x^\circ$, then what is the value of x?



(A) 96
(B) 102
(C) 108
(D) 115
(E) 120

7. A lady goes to 3 different temples for performing puja with 'X' flowers but she needs to offer 'Y' flowers to the idols in each temple. She goes to a magical pond that can double the number of flowers washed in it. She washes the 'X' flowers, doubles it and offers 'Y' flowers among them to the 1st temple. Then she goes to the pond again, doubles the remaining flowers and offers 'Y' among them to the 2nd temple. Again she doubles the remaining flowers and after offering 'Y' flowers to the 3rd temple, her task is accomplished and she has no flowers remaining with her. What could be the possible values of 'X' and 'Y'?

(A) 5 and 6
(B) 6 and 5
(C) 6 and 7
(D) 7 and 6
(E) 7 and 8

8. The product of three integers X, Y and Z is 192. Given that Z is equal to 4 and P is equal to the average of X and Y. What is the minimum possible value of P?

(A) 6
(B) 6.5
(C) 7
(D) 8
(E) 9.5

9. Given that the positive integers $x > 1$ and y satisfies the equation $2007x - 21y = 1923$. Find the minimum value of $3x + 2y$.

(A) 1342
(B) 1370
(C) 2013
(D) 2021
(E) 2035

10. If x is a real number and

$$A = \frac{-1+3x}{1+x} - \frac{\sqrt{|x|-2} - \sqrt{2-|x|}}{|2-x|}$$

then what is the value of A?

(A) 3
(B) 5
(C) 7
(D) 9
(E) 11

Section C

1. Find the number of positive integral values of n for which $n^2 + 96$ is a perfect square.

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

2. $1011\left[1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+\cdots+2021}\right]=?$

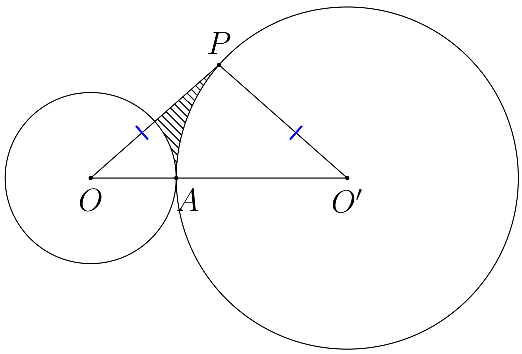
(A) 2019
(B) 2020
(C) 2021
(D) 2022
(E) 2023

3. How many triplets of prime numbers (p, q, r) satisfy the equation

$$15p + 7pq + qr = pqr$$

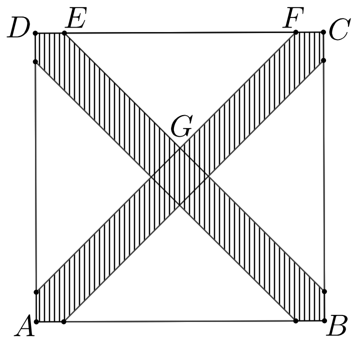
(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

4. Two circles with centres O and O' intersect at a point A such that $OA = \sqrt{2} - 1$. If OP is a tangent to the circle with centre O' and $OP = O'P$, then find the approximate area of the shaded region.



(A) 0.6
(B) 0.3
(C) 0.04
(D) 0.03
(E) 0.02

5. The diagonals of a square tile are painted symmetrically with a brush of width 1 unit as shown in the figure. Exactly half of the area of this tile is covered with paint. What is the length of the side of this tile?



(A) $\sqrt{2}-2$
(B) $\sqrt{2}-1$
(C) $\sqrt{2}$
(D) $\sqrt{2}+1$
(E) $\sqrt{2}+2$

Hints and Solutions

Section A

1. (D)
2. (B)
3. (D)
4. (C)
5. (B)
6. (D)
7. (D)
8. (E)
9. (D)
10. (B)

Section B

1. (A)

   Given, $2^a = 3^b = 7^c = 42$

   $\therefore 42^{\frac{1}{a}} = 2$, $42^{\frac{1}{b}} = 3$, $42^{\frac{1}{c}} = 7$

   Multiplying, we get $42^{\frac{1}{a}}.42^{\frac{1}{b}}.42^{\frac{1}{c}} = 2.3.7$

   or, $42^{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}} = 42$

   or, $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} = 1$

2. (A)

   Let the number of boys be x and the number of girls be y.

   Given, average score of boys= 71, $\therefore$ total score of boys= $71x$

   Average score of girls= 73, $\therefore$ total score of girls= $73y$

   Now, Average score $= \frac{\text{Total score}}{\text{Total no. of students}}$

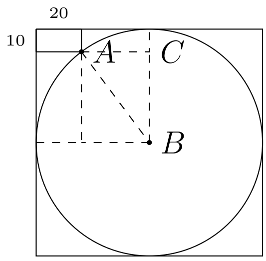
   or, $71.8 = \frac{71x+73y}{x+y}$

   Solving we get $8x = 12y \Rightarrow x : y = 3 : 2$.

3. (A)

   261 is possible only if we take 123+45+6+78+9. So, average will be $\frac{261}{5} = 52.2$.

4. (C)



Let the radius be $AB = r$, then $AC = r - 20$ and $BC = r - 10$

$\therefore$ By Pythagoras theorem in ΔABC, we have $AC^2 + BC^2 = AB^2$

i.e., $(r - 20)^2 + (r - 10)^2 = r^2$

Solving we get $r^2 - 60r + 500 = 0 \Rightarrow (r - 50)(r - 10) = 0$

$\therefore r = 50$ or $r = 10$.

Note that r would be 10 if the corner of the rectangle had been lying on the inner circumference. But as per the given figure, the radius of the circle should be 50 cm.

5. (E)

   Let $BQ = x$ and $DP = 2x$. Clearly, $OPRQ$ is a rectangle and hence $PR = OQ = 4$ and $OP = QR = 1$. Since *perpendicular drawn from centre to a chord also bisects the chord*, therefore $DP = CP = 2x$ and $AQ = BQ = x$. Hence, $DR = DP + PR = 2x + 4, CR = CP - PR = 2x - 4, BR = BQ + QR = x + 1, AR = AQ - QR = x - 1$.
   By Intersecting Chords Theorem, $AR.BR = CR.DR \Rightarrow (x-1)(x+1) = (2x-4)(2x+4) \Rightarrow x^2 - 1 = 4x^2 - 16 \Rightarrow 3x^2 = 15 \Rightarrow x = \sqrt{5}$. Therefore, $AB = 2x = 2\sqrt{5} = 4.472$ (approx.)

6. (C)

   Let $\angle ADF = \alpha$. Since $AD = AF$, therefore $\angle ADF = \angle AFD = \alpha$. So, $\angle BAC = \angle ADF + \angle AFD = 2\alpha$. Since $BA = BC$, therefore $\angle BAC = \angle BCA = 2\alpha$. So, $\angle ABC = 180° - 4\alpha$. Also since $EB = ED$, therefore $\angle EBD = \angle EDB \Rightarrow 180° - 4\alpha = \alpha \Rightarrow 5\alpha = 180° \Rightarrow \alpha = 36°$.

Hence, $\angle BED = 180° - 2\alpha = 180° - 72° = 108°$. So, $x = 108$.

7. (E)

$7 \times 2 - 8 = 14 - 8 = 6$

$6 \times 2 - 8 = 12 - 8 = 4$

$4 \times 2 - 8 = 8 - 8 = 0$

So she had $X = 7$ flowers initially and had offered $Y = 8$ flowers in each temple.

8. (C)

Given that the product of three integers X, Y and Z is 192. Also Z = 4 and $\mathrm{P} = \frac{\mathrm{X+Y}}{2}$.

Now, $\mathrm{XYZ} = 192 \Rightarrow \mathrm{XY}.4 = 192 \Rightarrow \mathrm{XY} = 48 = 1 \times 48 = 2 \times 24 = 3 \times 16 = 4 \times 12 = 6 \times 8$.

Also given $\mathrm{P} = \frac{\mathrm{X+Y}}{2}$.

$(\mathrm{X}, \mathrm{Y}) = (1, 48) \Rightarrow \mathrm{P} = \frac{1+48}{2} = \frac{49}{2} \notin \mathbb{Z}$

$(\mathrm{X}, \mathrm{Y}) = (2, 24) \Rightarrow \mathrm{P} = \frac{2+24}{2} = 13 \in \mathbb{Z}$

$(\mathrm{X}, \mathrm{Y}) = (3, 16) \Rightarrow \mathrm{P} = \frac{3+16}{2} = \frac{19}{2} \notin \mathbb{Z}$

$(\mathrm{X}, \mathrm{Y}) = (4, 12) \Rightarrow \mathrm{P} = \frac{4+12}{2} = 8 \in \mathbb{Z}$

$(\mathrm{X}, \mathrm{Y}) = (6, 8) \Rightarrow \mathrm{P} = \frac{6+8}{2} = 7 \in \mathbb{Z}$

$\therefore$ Minimum possible value of P is 7.

9. (B)

Given equation is $2007x - 21y = 1923$.
Dividing by 3, we have
$669x - 7y = 641 \Rightarrow 669x - 7y = 669 - 28 \Rightarrow 669(x-1) = 7(y-4)$

Therefore, x and y will be minimum if $x - 1 = 7$ and $y - 4 = 669$ i.e., $x_{\text{min}} = 8$ and $y_{\text{min}} = 673$.
Therefore, $(3x + 2y)_{\text{min}} = 3 \times 8 + 2 \times 673 = 1370$.

10. (C)

Since $|x| - 2$ and $2 - |x|$ are inside square roots, so $|x| - 2 \geq 0$ and $2 - |x| \geq 0$ simultaneously. Thus, $|x| = 2 \Rightarrow x = \pm 2$. If $x = 2$, then the denominator of 2^{nd} term (i.e., $|2-x|$) becomes 0, which is not possible. Hence, $x = -2$ only. Putting $x = -2$ in the given expression, we have $A = 7$.

Section C

1. (E)

Let $n^2 + 96 = m^2$, where $m \in \mathbb{Z}$.

This implies $m^2 - n^2 = 96$

or, $(m-n)(m+n) = 96 = 2 \times 48 = 3 \times 32 = 4 \times 24 = 6 \times 16 = 8 \times 12$

If $m - n = 2, m + n = 48$, solving we get $n = 23 \in \mathbb{Z}$

If $m - n = 3, m + n = 32$, solving we get $n = \frac{29}{2} \notin \mathbb{Z}$

If $m - n = 4, m + n = 24$, solving we get $n = 10 \in \mathbb{Z}$

If $m - n = 6, m + n = 16$, solving we get $n = 5 \in \mathbb{Z}$

If $m - n = 8, m + n = 12$, solving we get $n = 2 \in \mathbb{Z}$

Hence there are four values of n (2,5,10 and 23) for which n^2+96 is a perfect square.

2. (C)

$$1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+\cdots+2021}$$

$$=\sum_{n=1}^{2021}\frac{1}{1+2+\cdots n}$$

$$=\sum_{n=1}^{2021}\frac{1}{\frac{n(n+1)}{2}}$$

$$=2\sum_{n=1}^{2021}\frac{1}{n(n+1)}$$

$$=2\sum_{n=1}^{2021}\frac{(n+1)-n}{n(n+1)}$$

$$=2\sum_{n=1}^{2021}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=2\left[\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\cdots+\frac{1}{2021}-\frac{1}{2022}\right]$$

$$=2\left[1-\frac{1}{2022}\right]=2\times\frac{2021}{2022}$$

$$\therefore 1011\left[1+\frac{1}{1+2}+\frac{1}{1+2+3}+\cdots+\frac{1}{1+2+\cdots+2021}\right]$$

$$=1011\times 2\times\frac{2021}{2022}=2021.$$

3. (D)

The given equation can be written as $qr=p(qr-79-15)$. Since p,q,r are all primes, so there arise two cases:

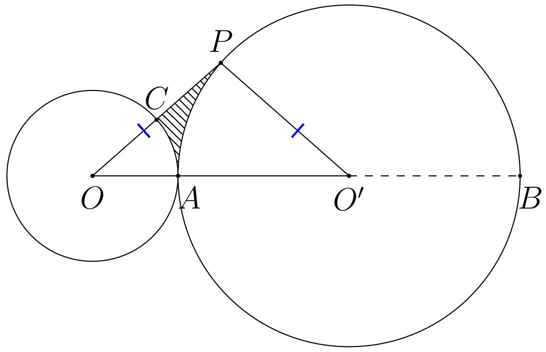
Case 1: Considering $p=q$, we have $r=qr-7q-15 \Rightarrow qr-7q-r=15 \Rightarrow q(r-7)-(r-7)=15+7 \Rightarrow (q-1)(r-7)=22$. Now, 22 can be factored as $1\times 22=22\times 1=2\times 11=11\times 2$. So we get $q=2,r=29$ or $q=23,r=8$ or $q=3,r=18$ or $q=12,r=9$ but since q,r are primes, so only $q = 2, r = 29$ is possible. Thus, $p = q = 2$. Hence, one such triplet is $(p, q, r) = (2, 2, 29)$.

Case 2: Considering $p = r$, we have $q = qr - 7q - 15 \Rightarrow qr - 8q = 15 \Rightarrow q(r-8) = 15$. Now 15 can be factored as $1 \times 15 = 15 \times 1 = 3 \times 5 = 5 \times 3$. So we get $q = 1, r = 23$ or $q = 15, r = 9$ or $q = 3, r = 13$ or $q = 5, r = 11$ but since q, r are primes, so only $q = 3, r = 13$ and $q = 5, r = 11$ are possible. Using $p = r$, two more triplets are $(p, q, r) = (13, 3, 13), (11, 5, 11)$.

Therefore, there are **3** triplets of primes (p, q, r) satisfying the equation $15p + 7pq + qr = pqr$.

4. (C)

Extend AO' to intersect the circle at B. Let the point of intersection of OP and the circle with centre O be C. Let $O'A = O'P = OP = x$.



By tangent-secant theorem,

$$OP^2 = OA \cdot OB$$

$$\Rightarrow x^2 = (\sqrt{2} - 1)(\sqrt{2} - 1 + 2x)$$

$$\Rightarrow x^2 = (\sqrt{2} - 1)^2 + 2x(\sqrt{2} - 1)$$

$$\Rightarrow x^2 - 2x(\sqrt{2} - 1) + (\sqrt{2} - 1)^2 = 2(\sqrt{2} - 1)^2$$

$$\Rightarrow \{x - (\sqrt{2} - 1)\}^2 = 2(\sqrt{2} - 1)^2$$

$\Rightarrow x - (\sqrt{2} - 1) = \sqrt{2}(\sqrt{2} - 1)$

$\Rightarrow x - \sqrt{2} + 1 = 2 - \sqrt{2}$

$\Rightarrow x = 1$

Therefore, area of $\triangle OO'P = \frac{1}{2}x^2 = \frac{1}{2}(1)^2 = \frac{1}{2}$ sq. units

Since $OP = O'P$ and OP is a tangent, i.e., $\angle OPO' = 90°$, so $\angle POO' = \angle PO'O = 45°$. Therefore, the areas of sectors OAC and $O'AP$ are $\frac{45}{360} = \frac{1}{8}$ times the areas of their respective circles.

So, area of shaded region

$=\text{ar}(\triangle OO'P) - \{\text{ar(sector } OAC) + \text{ar(sector } O'AP)\}$

$= \frac{1}{2} - \frac{1}{8}\pi\{(\sqrt{2} - 1)^2 + 1^2\}$

$= \frac{1}{2} - \frac{1}{8}\pi(4 - 2\sqrt{2})$
$= 0.04$ (approx.)

5. (D)

Let $DE = x$ and $EG = y$.

By Pythagoras theorem, $1^2 = x^2 + x^2 = 2x^2 \Rightarrow x = \dfrac{1}{\sqrt{2}}$

$EF = CD - 2DE = CD - 2 \times \dfrac{1}{\sqrt{2}} = CD - \sqrt{2}$

Now, $EF^2 = EG^2 + GF^2 = y^2 + y^2$

$\therefore (CD - \sqrt{2})^2 = 2y^2$

$\Rightarrow CD - \sqrt{2} = \sqrt{2}y$

$\Rightarrow y = \frac{CD-\sqrt{2}}{\sqrt{2}}$

According to question,

Area of shaded region = Area of unshaded region = $\frac{1}{2}$(Area of square)

$\Rightarrow 4\left(\frac{1}{2}y^2\right) = \frac{1}{2}CD^2$

$\Rightarrow CD = 2y = 2 \cdot \frac{CD-\sqrt{2}}{\sqrt{2}}$

$\Rightarrow CD = \sqrt{2}CD - 1$

$\Rightarrow (\sqrt{2}-1)CD = 1$

$\Rightarrow CD = \dfrac{1}{\sqrt{2}-1} = \sqrt{2}+1$ units is the length of the tile.