Diff Eqns Jan–Apr 2024 Extra Credit 2024-05-06 12:40-13:10

(1) (5 marks) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a linear transformation without any real eigenvalues. Show that there is a basis v_1, v_2 with respect to which the matrix of T looks like

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

with $a, b \in \mathbb{R}$.

(2) (5 marks) Give an example (with a proof) of a two-dimensional system such that the origin is in the closure of every phase curve and every phase curve passes through all the four quadrants.