Concurrency Theory, Assignment Lecture 4

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We construct a program `Prog` in PGLEc with MPP for the function $y = \mathsf{Prog}(x1, x2) = x1 * (x2 + 1)$. We first construct a program `MultAccum` for the function $y = \mathsf{MultAccum}(x1, x2, x3) = x1 * x2 + x3$. The program `MultAccum` reflects the following recursive definition of $\mathsf{MultAccum}$:

$$\mathsf{MultAccum}(x1, x2, x3) = \begin{cases} x3 & \text{if } x1 = 0 \\ \mathsf{MultAccum}(x1 - 1, x2, x2 + x3) & \text{otherwise.} \end{cases}$$

```
MultAccum with comments:
L1; +x1==Z{;
  // If x1 = 0 return the accumulated value of x3.
  y=x3;
}{;
  // At this point x1 > 0, so x1 has a field p and
  // can be safely decremented.
  x1=x1.p;
  // Save x1 and x2 into temporaties.
  t1 = x1;
  t2 = x2;
  // Add t2 and t3 and put the result in x3.
  x1 = t2;
  x2 = t3;
  Add;
  x3 = y;
  // Restore the old values of x1 and x2 and continue.
  x1 = t1;
  x2 = t2;
  ##L1;
}
```

Note that $\mathsf{Prog}(x1, x2) = \mathsf{MultAccum}(x1, x2 + 1, 0)$.

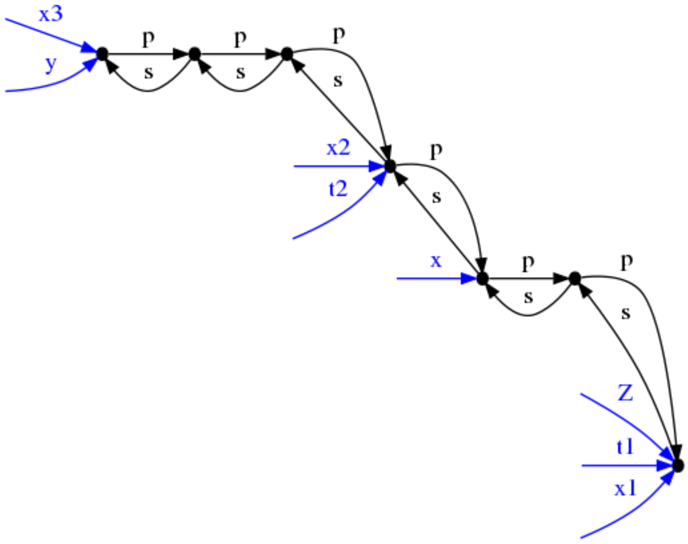
```
Prog with comments:
// Increment x2.
x = x2;
S;
x2 = y;
// Initialize x3 = 0.
x3 = Z;
// Continue with MultAccum.
MultAccum;
```

The comments on the annotated versions of the programs constitute a short argument on the general correctness of the programs. Let's prove that the recursive definition of MultAccum is correct. We use induction on $x1$. Suppose $x1 = 0$. Then $x1 * x2 + x3 = x3$. Now suppose $x1 > 0$ and $\mathsf{MultAccum}(x1 - 1, x2, x3) = (x1 - 1) * x2 + x3$ for all natural numbers $x2$ and $x3$. Now $\mathsf{MultAccum}(x1, x2, x3) = \mathsf{MultAccum}(x1-1, x2, x3+x2) = (x1-1)*x2+(x3+x2) = x1*x2+x3$.

Following is a full listing of an evolution of `Prog` on $x1 = x2 = 2$.

```
Evolution for x1 = x2 = 2:
Z=new; y=Z;
x=y;
-x/s{; x.+s; y=new; y.+p; y.p=x; x.s=y; }{; y=x.s; };
x=y;
-x/s{; x.+s; y=new; y.+p; y.p=x; x.s=y; }{; y=x.s; };
x1=y; x2=y;

x=x2;
-x/s{; x.+s; y=new; y.+p; y.p=x; x.s=y; }{; y=x.s; };
x2=y;
x3=Z;
L1; +x1==Z{;
  y=x3;
}{;
  x1=x1.p;
  t1 = x1;
  t2 = x2;
  x1 = t2;
  x2 = x3;
  L0; +x1/p{;
    x1=x1.p;
    +x2/s{; x2=x2.s; }{;
      x2.+s; y=new; y.+p; y.p=x2; x2.s=y; x2=x2.s; };
    ##L0;
  }{; y=x2; };
  x3 = y;
  x1 = t1;
  x2 = t2;
  ##L1;
}
```



Here is the resulting fluid.

The source code of this assignment along with the programs can be found at `github.com/comco/concurrency-theory-assignments`.