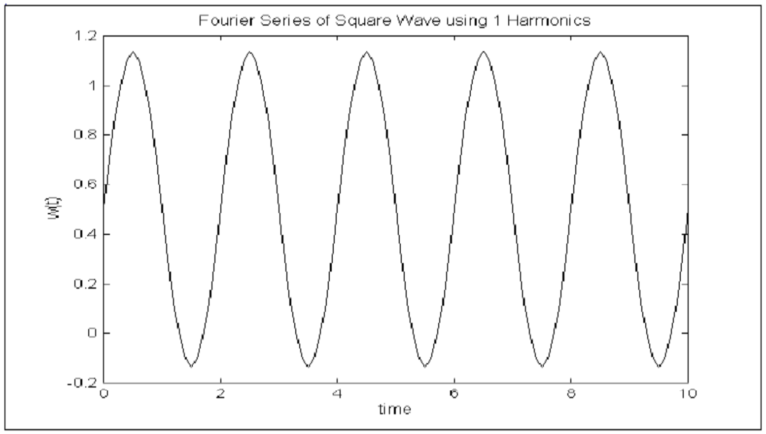
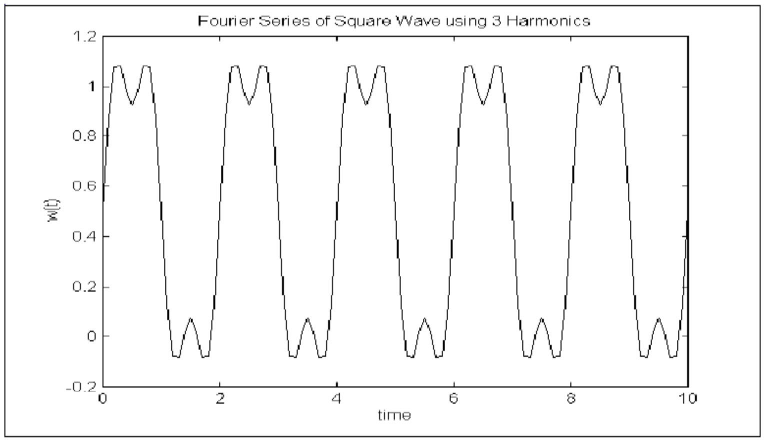
CHAPTER 6

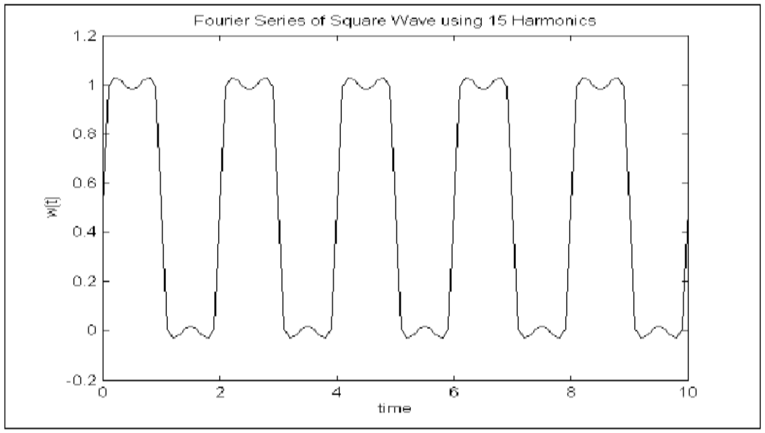
IMAGE TRANSFORMS

Frequency Domain

- The Fourier Transform
  - Any periodic signal could be represented by a series of sinusoids.







Frequency Domain

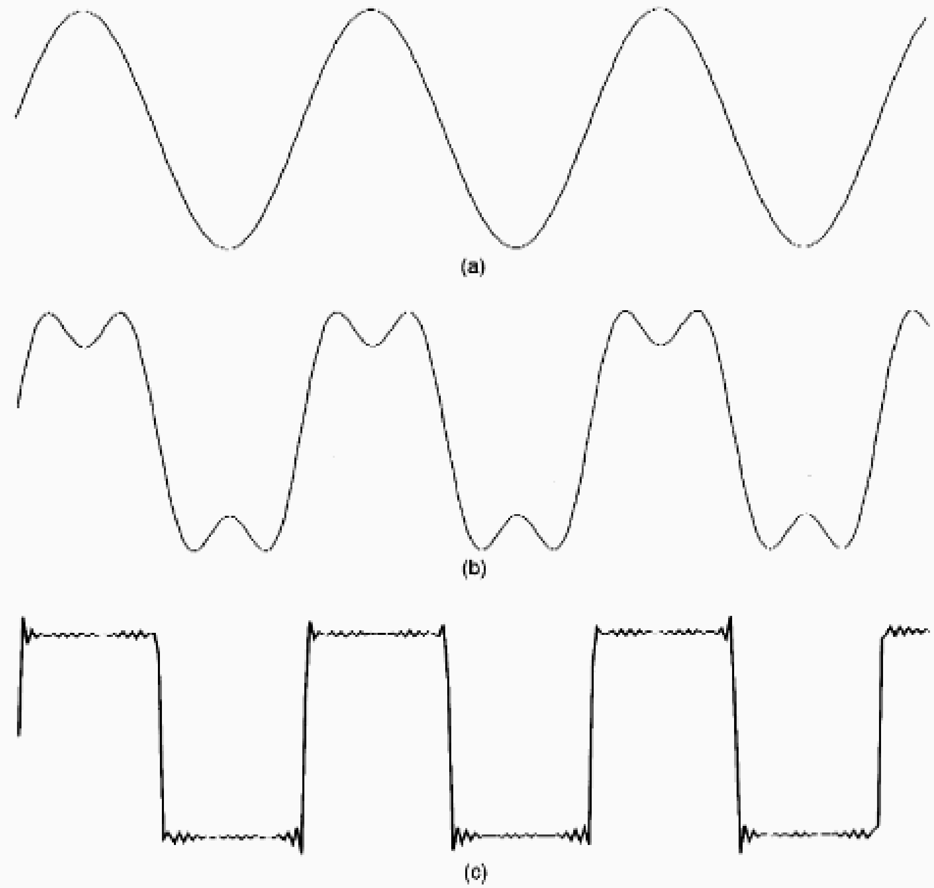
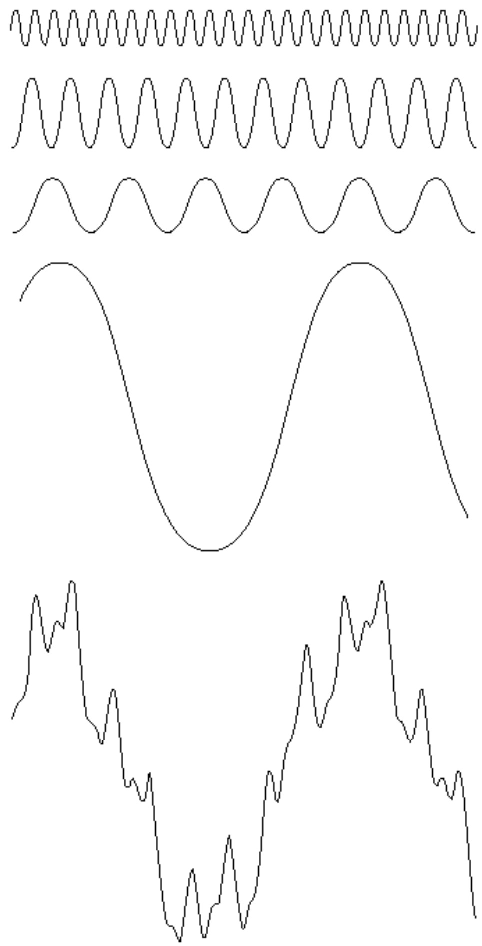


FIGURE 7.1 (a) Fundamental frequency: sine(x); (b) Fundamental plus one harmonic: sine(x) + sine(3x)/3; (c) Fundamental plus 16 harmonics: sine(x) + sine(3x)/3 + sine(5x)/5. . . .

Frequency Domain



The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

Frequency Domain

- Spatial Frequency
  - The spatial frequency of an image refers to the rate at which the pixel intensities change.

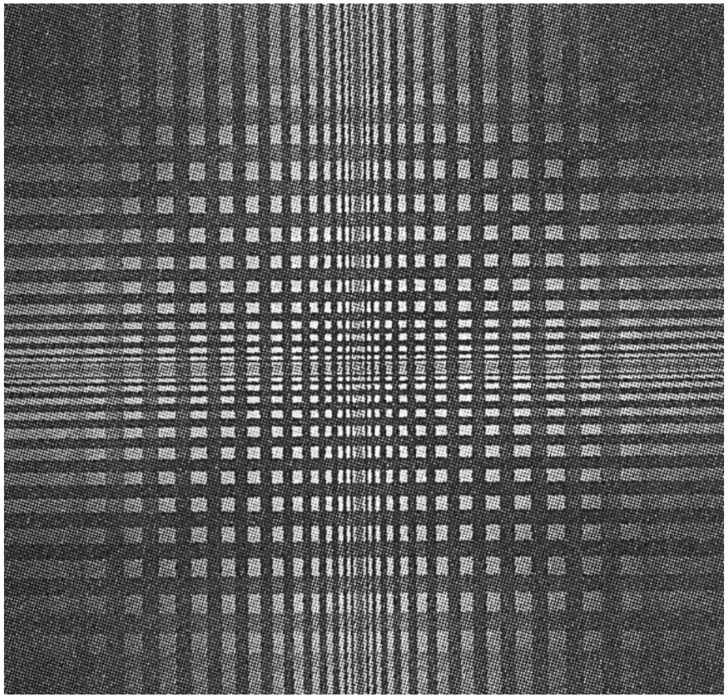
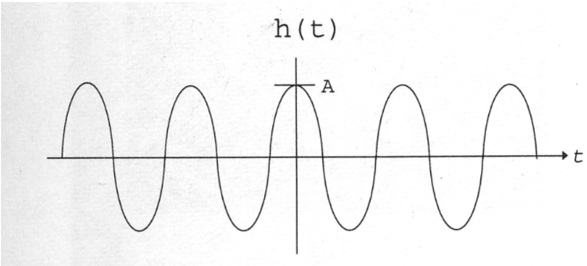


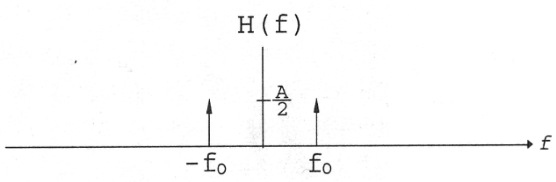
Image of varying frequencies

Fourier Transform

- Cosine wave and its frequency



Time Domain



Frequency Domain

Fourier Transform

$$H(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$h(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H(u,v)e^{j2\pi(ux+vy)}dudv$$

$$j = \sqrt{-1}, \quad e^{jx} = \cos x + j\sin x, \quad \left|e^{jx}\right| = 1$$

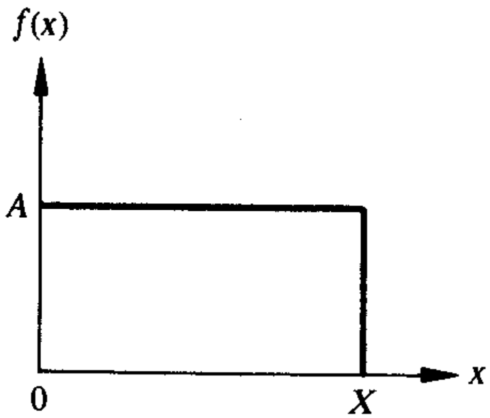
$$Let \quad F(u,v) = R(u,v) + jI(u,v)$$

$$\left|F(u,v)\right| = \sqrt{R^2(u,v) + I^2(u,v)} \ : \text{ Fourier spectrum}$$

$$\Theta(u,v) = \tan^{-1}\frac{I(u,v)}{R(u,v)} \ : \text{ Phase angle}$$

Fourier Transform

- Example (1-D Fourier transform)



$$F(u) = \int_{-\infty}^{\infty} f(x)\exp[-j2\pi ux]dx$$

$$= \int_{0}^{X} A\exp[-j2\pi ux]dx$$

$$= \frac{-A}{j2\pi ux}[e^{-j2\pi ux}]_0^X = \frac{-A}{j2\pi ux}[e^{-j2\pi uX} - 1]$$

$$= \frac{A}{j2\pi u}[e^{j\pi uX} - e^{-j\pi uX}]e^{-j\pi uX}$$

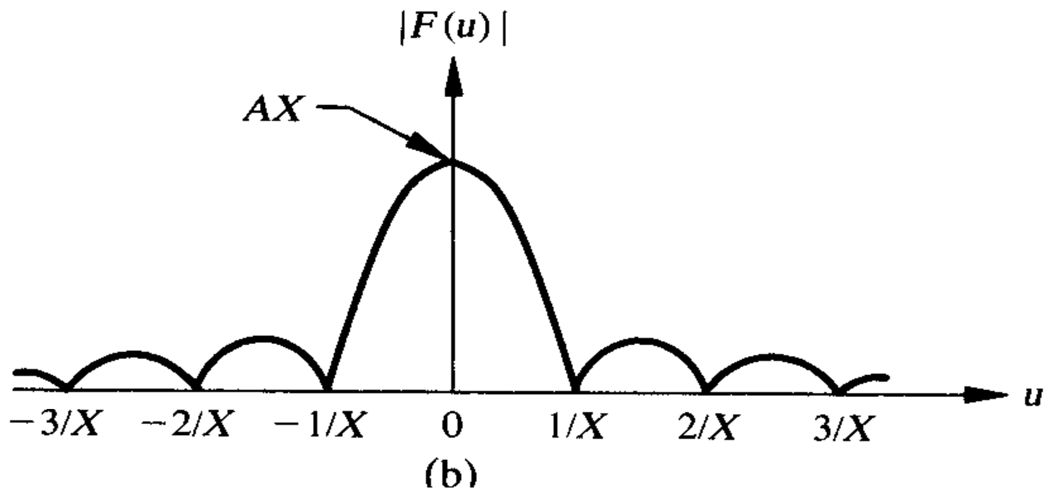
$$= \frac{A}{\pi u}\sin(\pi uX)e^{-j2\pi uX}$$

Fourier Transform

- Example (1-D Fourier transform)

$$|F(u)| = \left|\frac{A}{\pi u}\right| \left|\sin(\pi u X)\right| \left|e^{-j2\pi u X}\right|$$

$$= AX \left|\frac{\sin(\pi u X)}{\pi u X}\right|$$

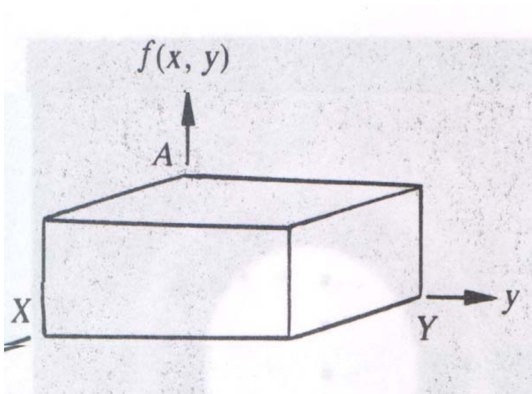


Fourier spectrum

Fourier Transform

- Example (2-D Fourier transform)

$$F(u,v)=\int_{-\infty}^{\infty}\int f(x,y)\exp[-j2\pi(ux+vy)]dxdy$$



$$=A\int_0^X \exp[-j2\pi ux]dx\int_0^Y \exp[-j2\pi vy]dy$$

$$=A\left[\frac{e^{-j2\pi ux}}{j2\pi u}\right]_0^X\left[\frac{e^{-j2\pi uy}}{j2\pi v}\right]_0^Y$$

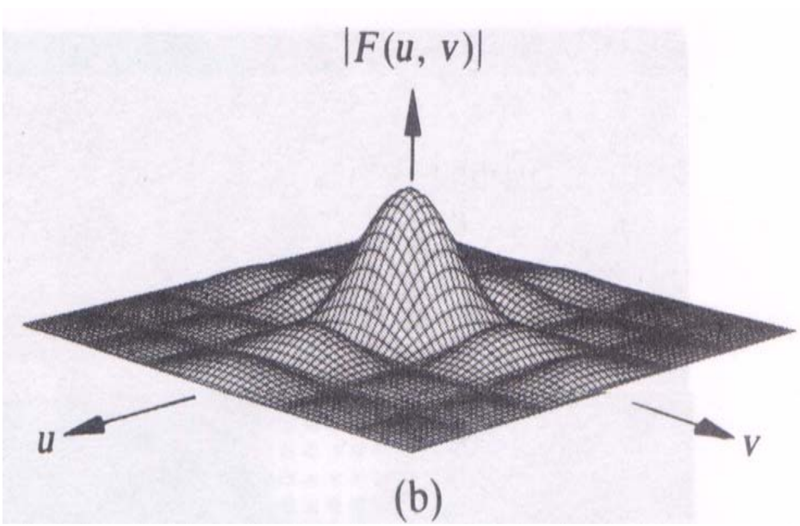
$$=\frac{A}{-j2\pi u}\left(e^{-j\pi uX}-1\right)\frac{1}{-j2\pi v}\left(e^{-j\pi vY}-1\right)$$

$$=AXY\left(\frac{\sin(\pi uX)e^{-j2\pi uX}}{(\pi uX)}\right)\left(\frac{\sin(\pi vY)e^{-j2\pi vY}}{(\pi vY)}\right)$$

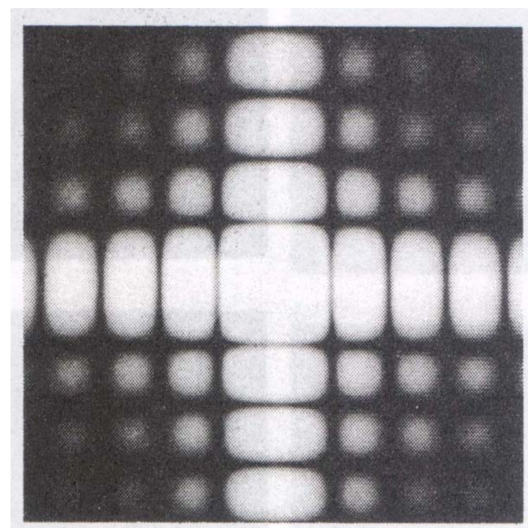
Fourier Transform

- Example (2-D Fourier transform)

$$|F(u,v)| = AXY \left|\frac{\sin(\pi u X)}{\pi u X}\right| \left|\frac{\sin(\pi v Y)}{\pi v Y}\right|$$



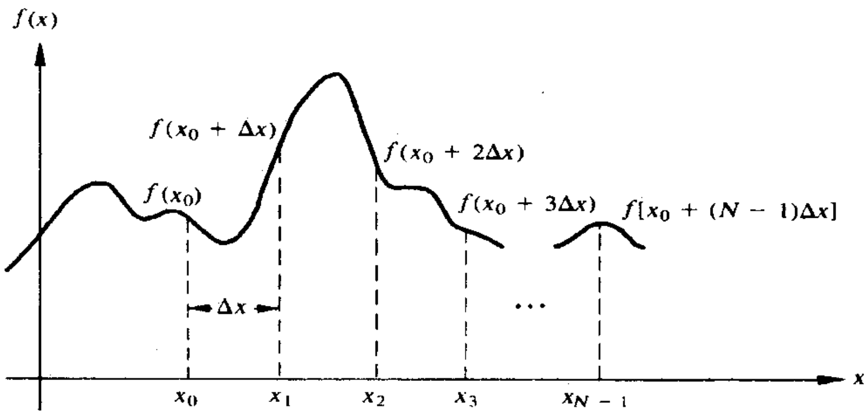
(Fourier spectrum)

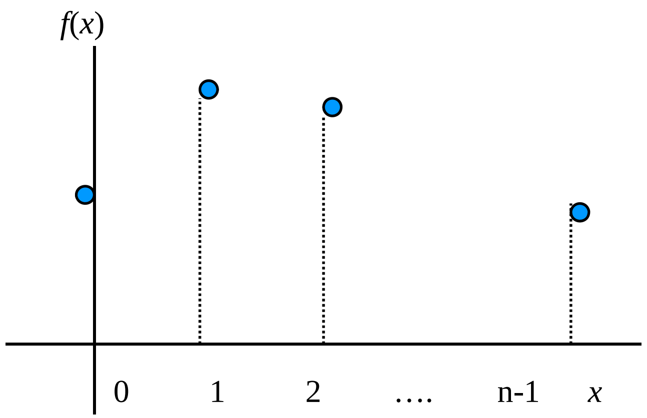


(spectrum as an intensity)

Discrete Fourier Transform (DFT)

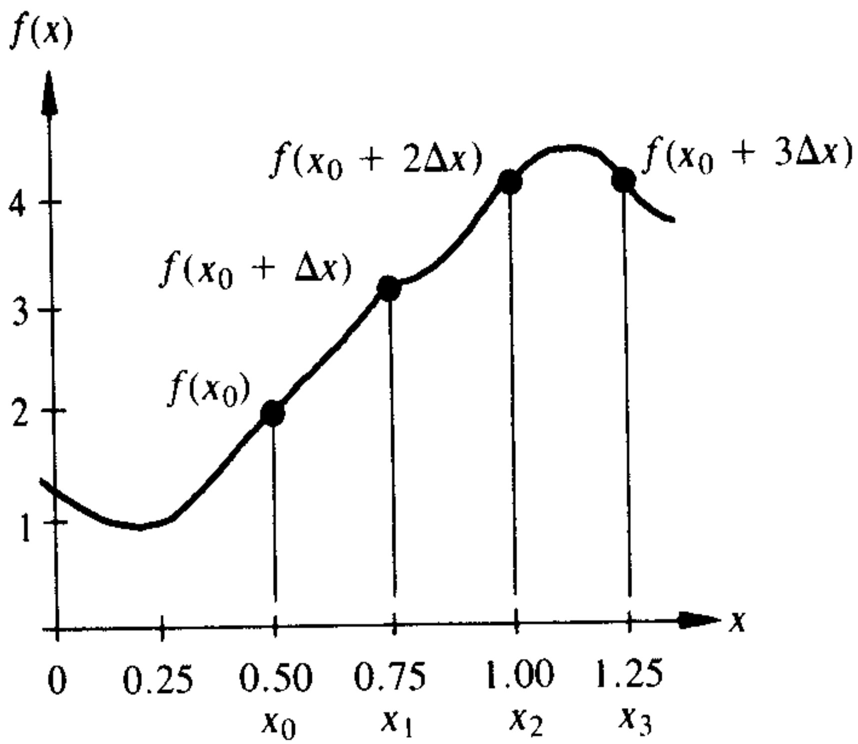
- Suppose that a continuous function $f(x)$ is discretized into a sequence by taking N samples Δx units apart.

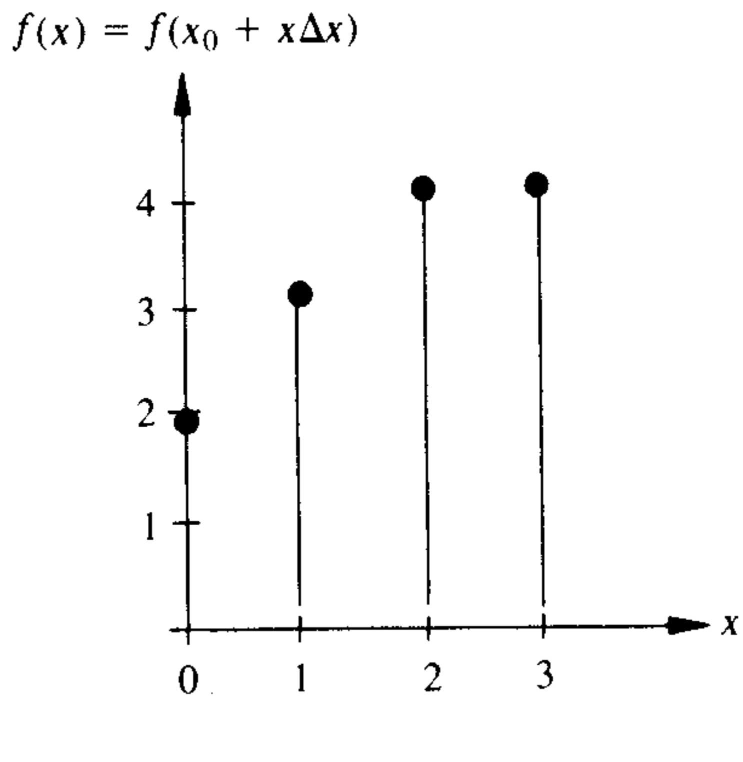




Discrete Fourier Transform

- Example





Discrete Fourier Transform

- Example (continued)

$$F(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)\exp[-j2\pi ux/N]$$

$$F(0) = \frac{1}{4}\sum_{x=0}^{3} f(x)\exp[0]$$

$$= \frac{1}{4}[f(0)+f(1)+f(2)+f(3)]$$

$$= \frac{1}{4}(2+3+4+4)$$

$$= 3.25$$

Discrete Fourier Transform

- Example (continued)

$$F(1) = \frac{1}{4}\sum_{x=0}^{3} f(x)\exp[-j2\pi ux/4]$$

$$= \frac{1}{4}\left(2e^{0} + 3e^{-j\pi/2} + 4e^{-j\pi} + 3e^{-j3\pi/2}\right)$$

$$= \frac{1}{4}\left(-2 + j\right)$$

$$F(2) = -\frac{1}{4}\left(1 + j0\right)$$

$$F(3) = -\frac{1}{4}\left(2 + j\right)$$

Discrete Fourier Transform

- Example (continued)
  - Fourier spectrum

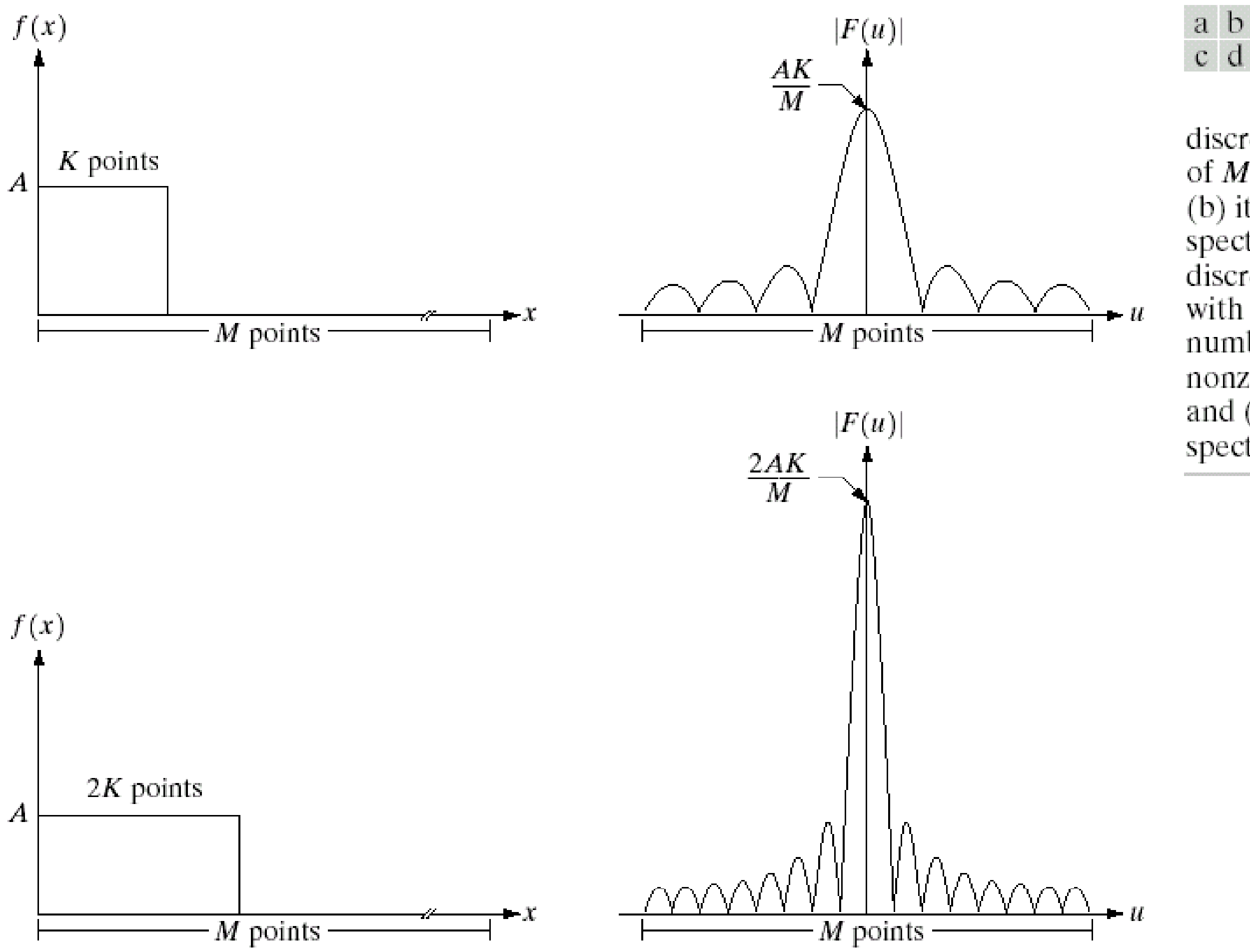
$$|F(0)| = 3.25$$

$$|F(1)| = \left[\left(\frac{2}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]^{\frac{1}{2}} = \frac{\sqrt{5}}{4}$$

$$|F(2)| = \left[\left(\frac{1}{4}\right)^2 + \left(\frac{0}{4}\right)^2\right]^{\frac{1}{2}} = \frac{1}{4}$$

$$|F(3)| = \left[\left(\frac{2}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]^{\frac{1}{2}} = \frac{\sqrt{5}}{4}$$

Discrete Fourier Transform

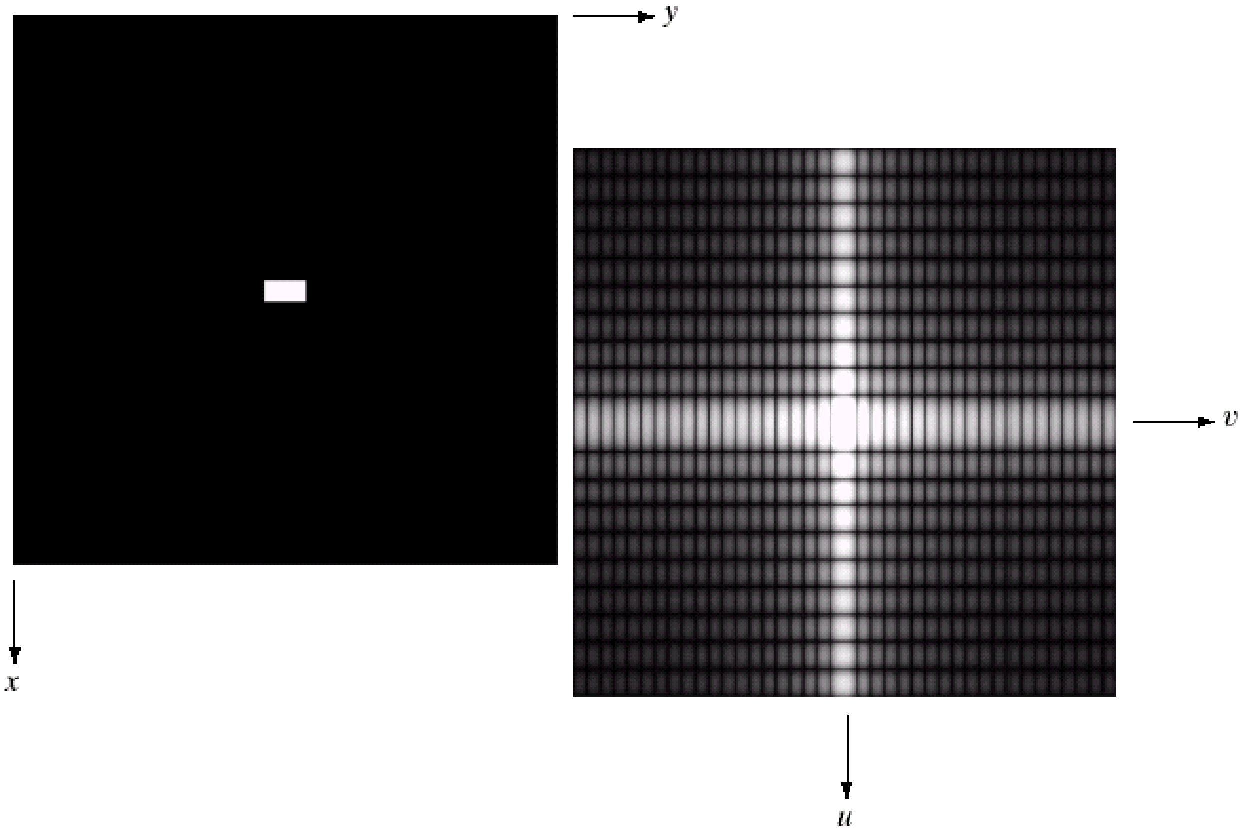


(a) A discrete function of *M* points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

Discrete Fourier Transform

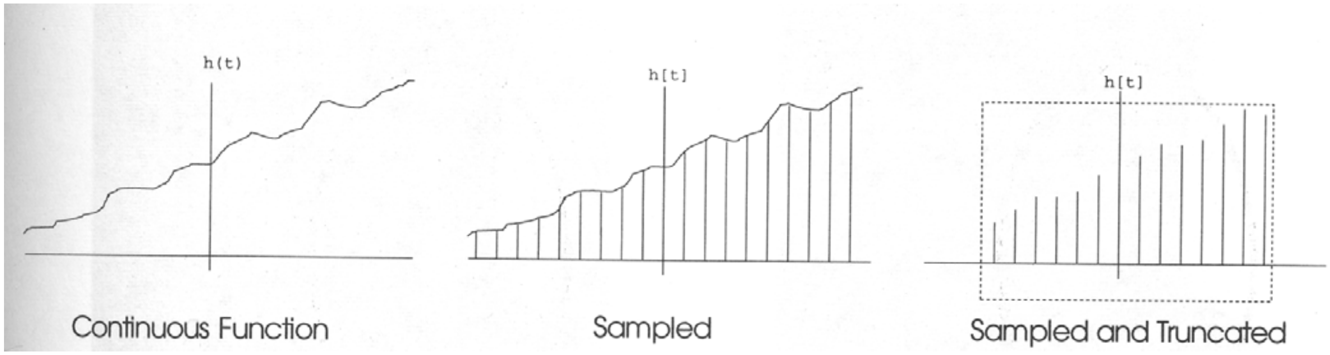
a b

(a) Image of a 20×40 white rectangle on a black background of size 512×512 pixels.
(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.



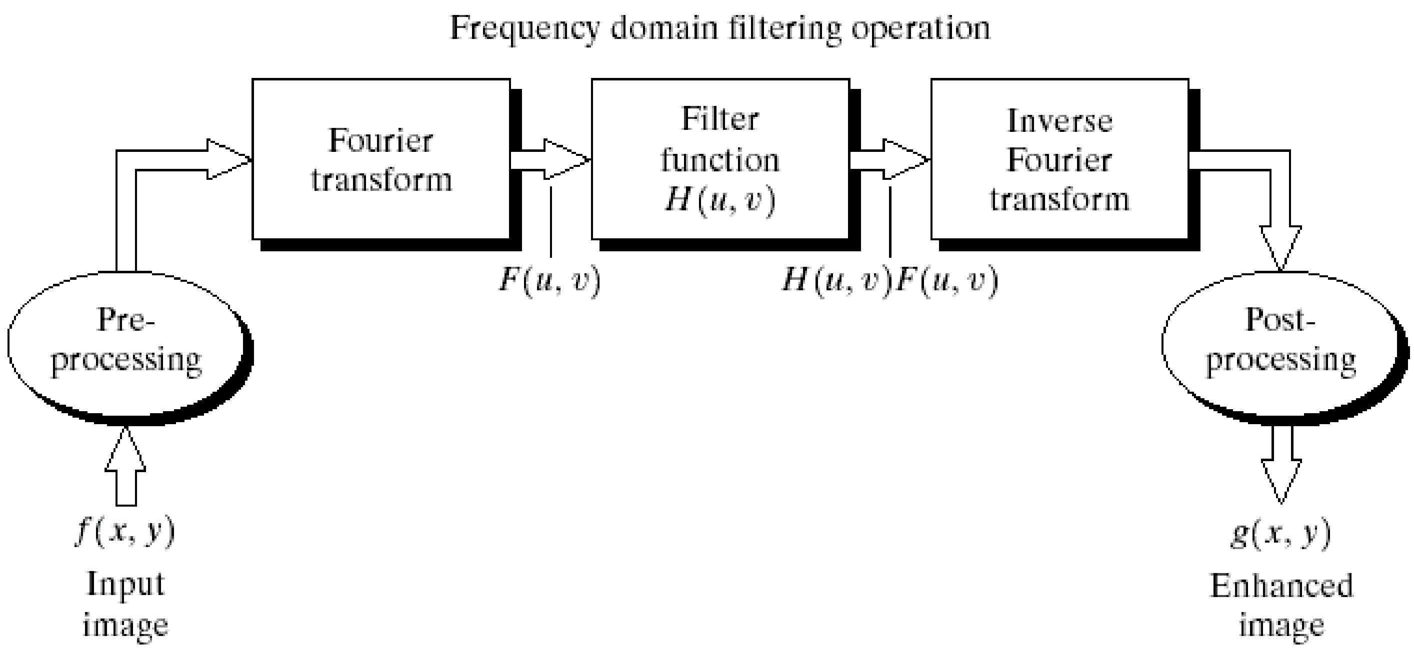
Discrete Fourier Transform

- DFT expects input to be periodic



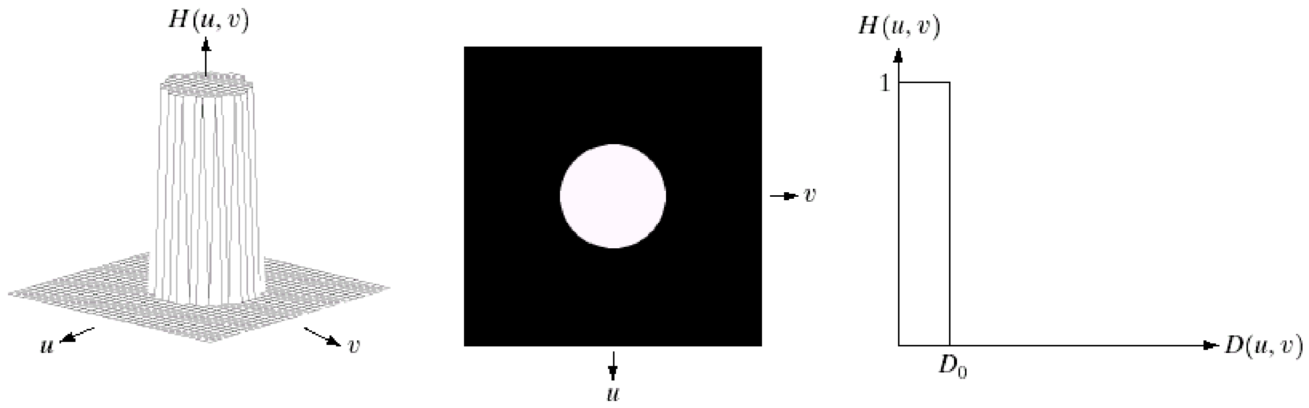
- Gibbs phenomenon
  - ringing effect caused by sampling & truncation
  - can reduce width of ringing by increasing the number of data samples
  - amplitude of ringing is proportional to difference between amplitude of first and last sample
    - can reduce it by multiplying data by windowing function

Image Enhancement in Frequency Domain



Basic steps for filtering in the frequency domain.

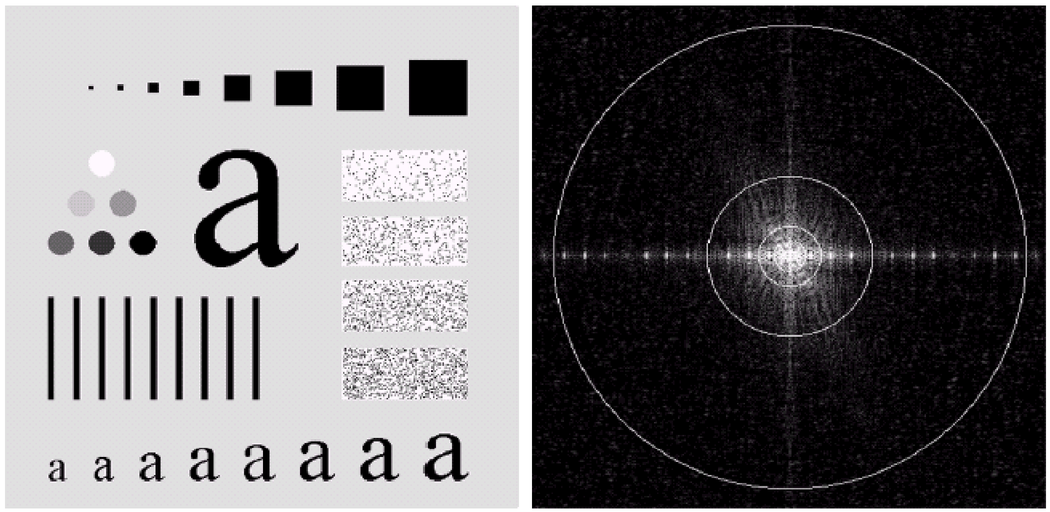
Ideal Lowpass Filter (LPF)



a b c

(a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

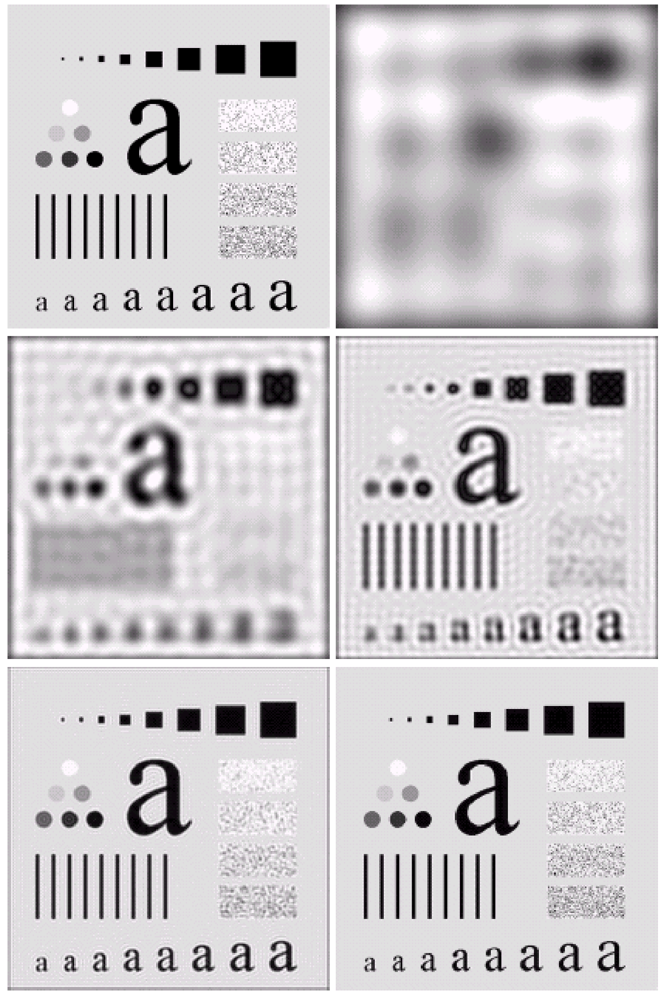
Fourier Spectrum



a b

FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Ideal Lowpass Filter (Examples)



a b
c d
e f

FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

Ringing Artifacts (The Gibbs Phenomenon)

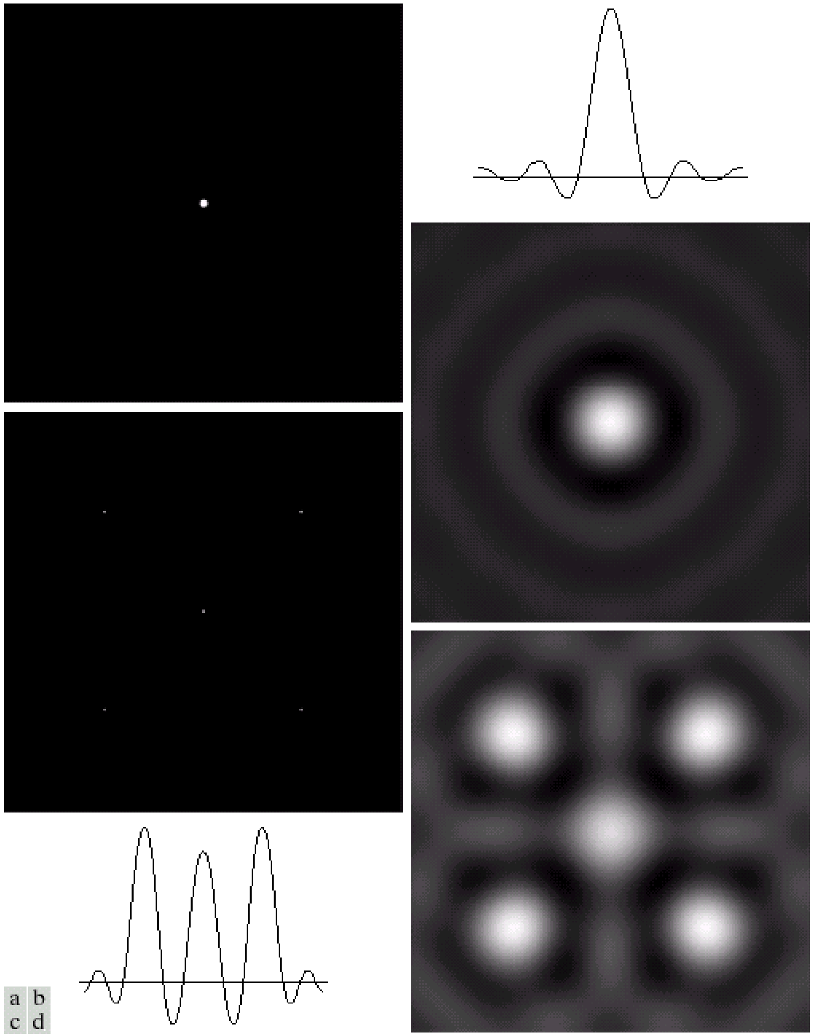
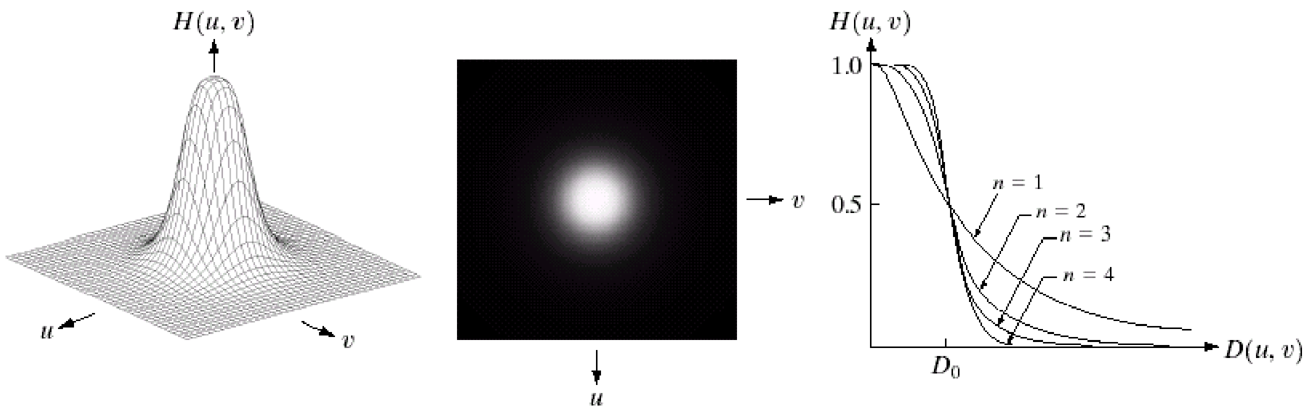


FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

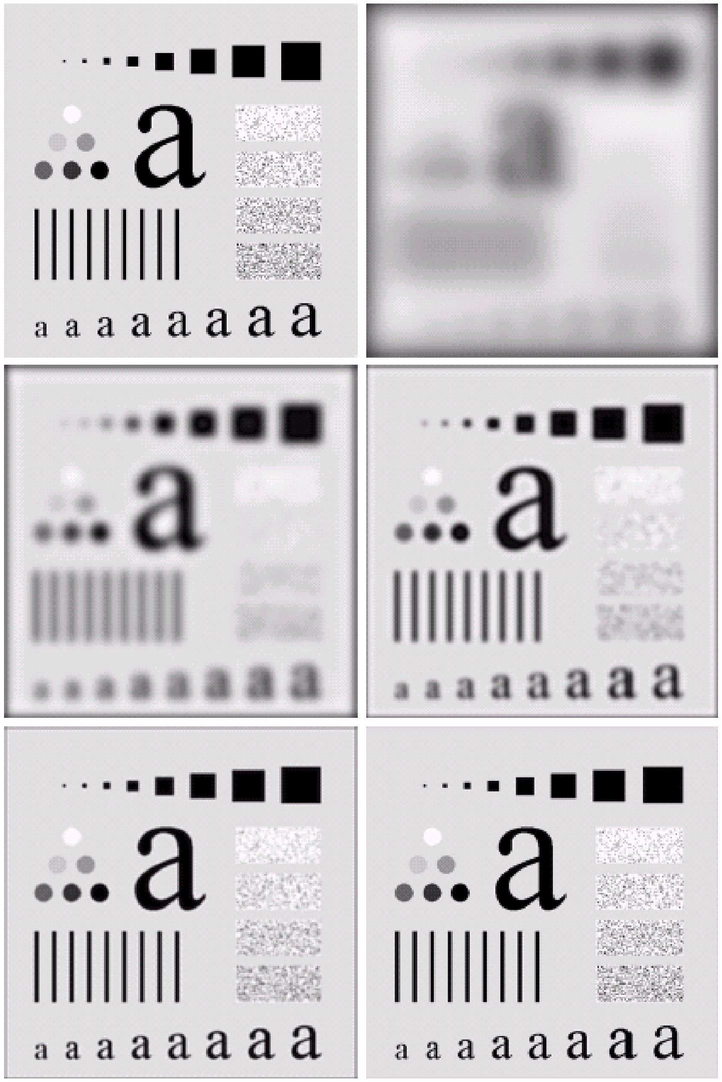
Butterworth Lowpass Filters



a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Butterworth Lowpass Filtering



a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Ringing Artifacts in Butterworth LPFs

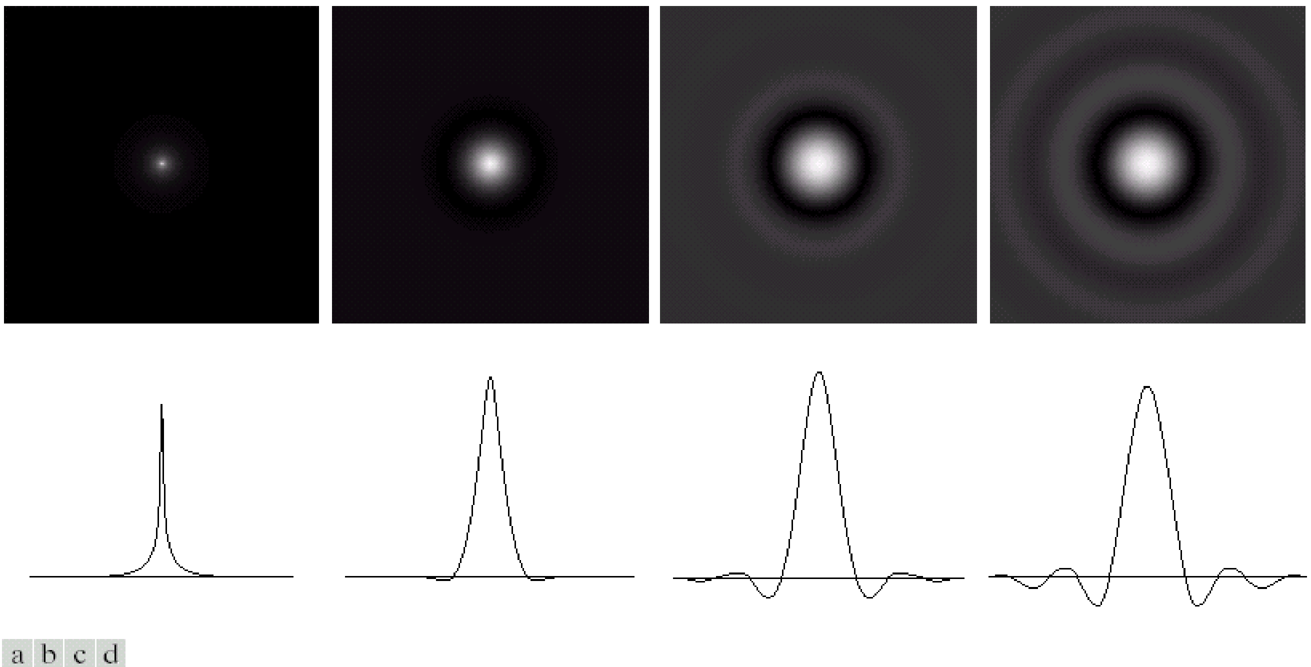
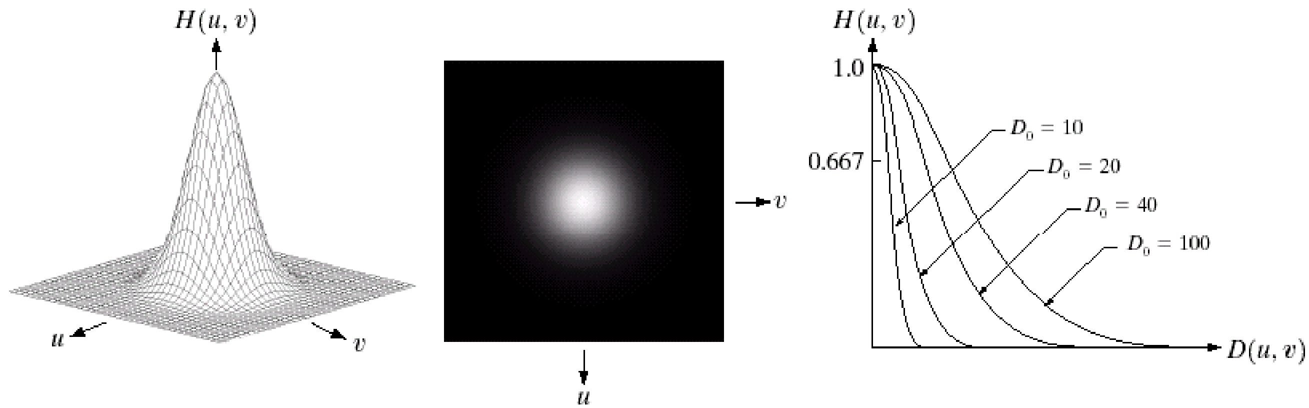


FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Gaussian Lowpass Filters



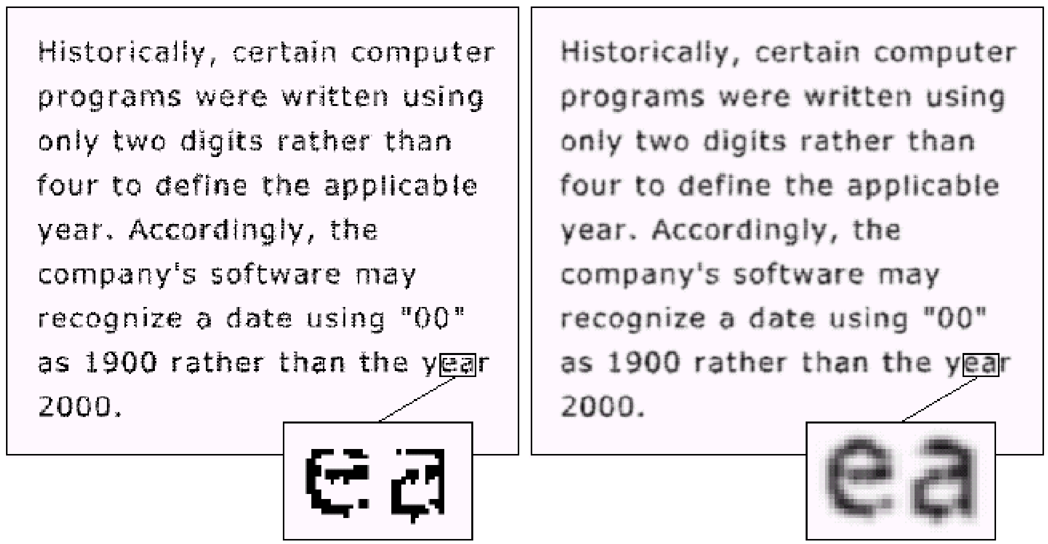
a b c

(a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

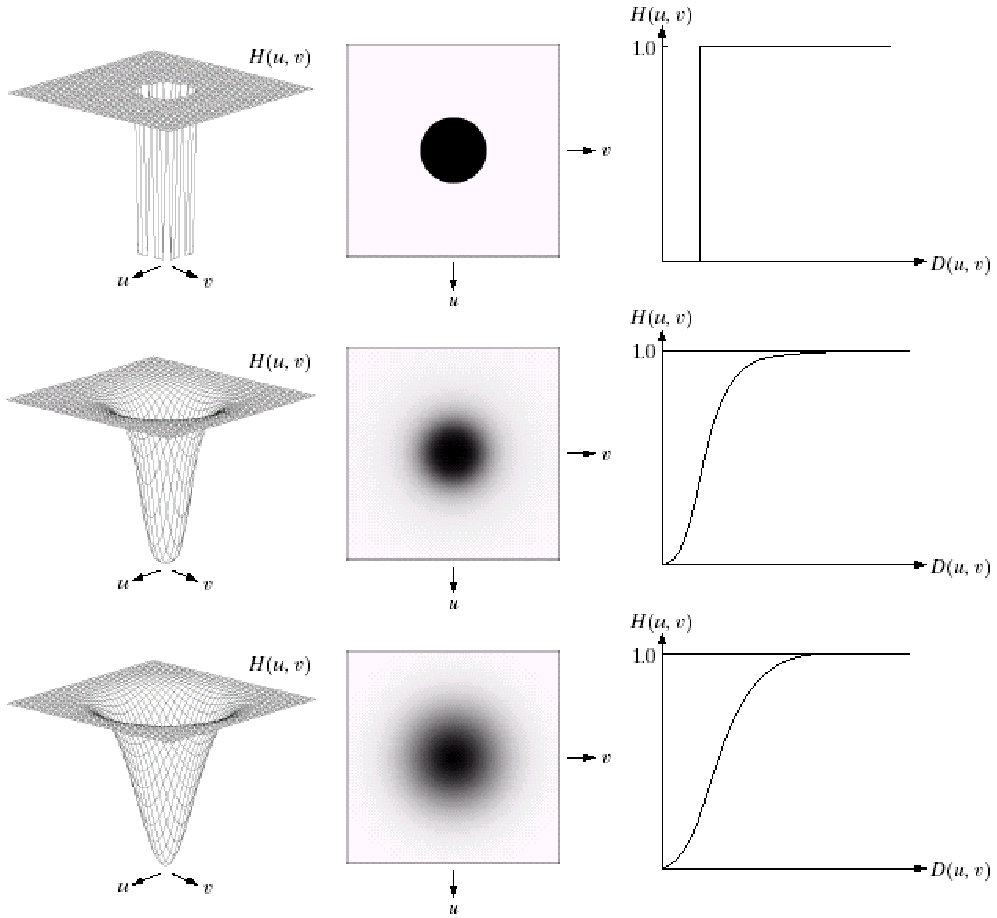
Gaussian Lowpass Filtering

a b

(a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



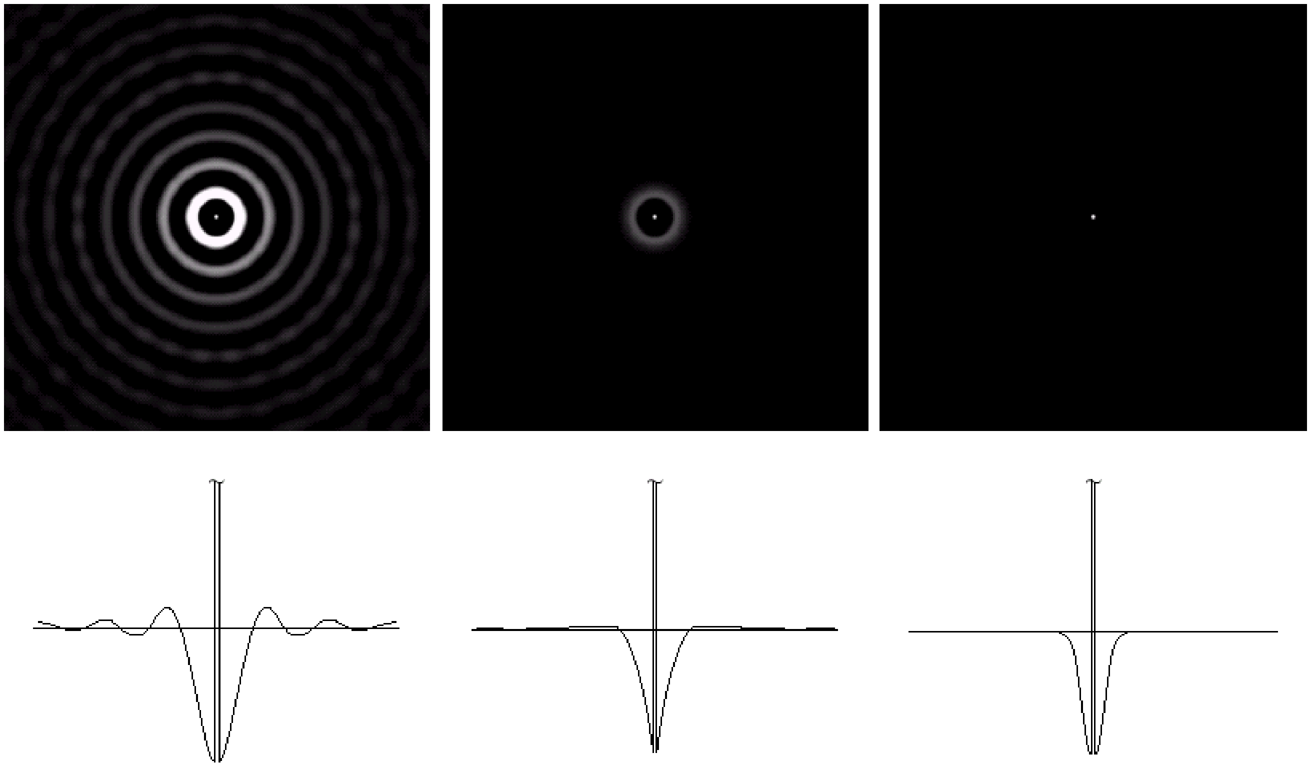
Highpass Filters (HPFs)



a	b	c
d	e	f
g	h	i

Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

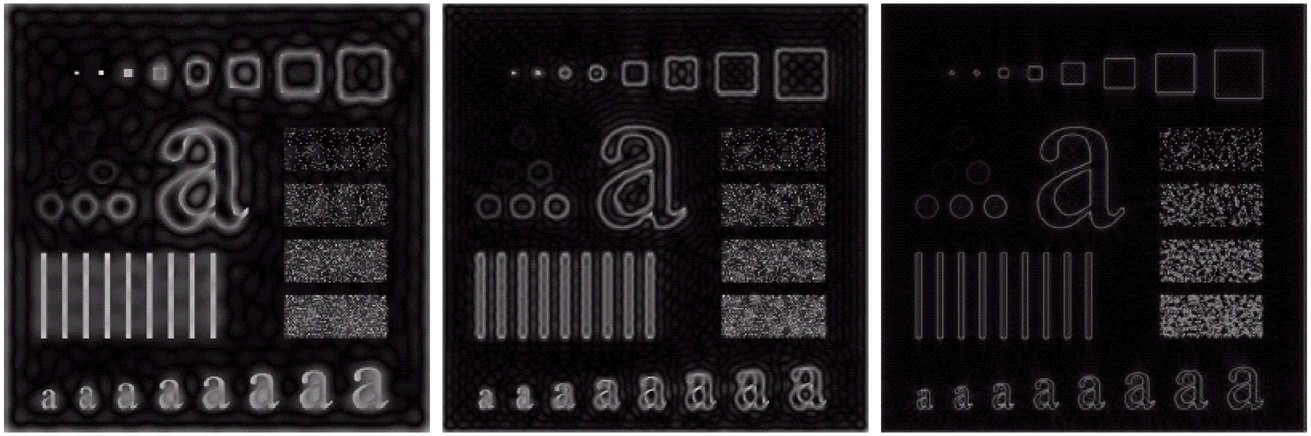
Spatial Representations of HPFs



a b c

Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

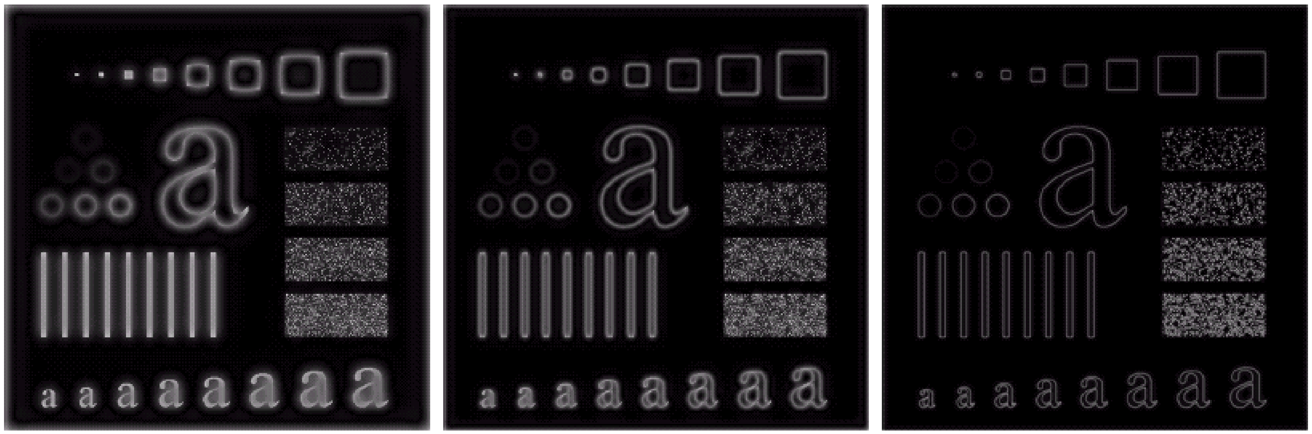
Ideal Highpass Filtering



a b c

Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

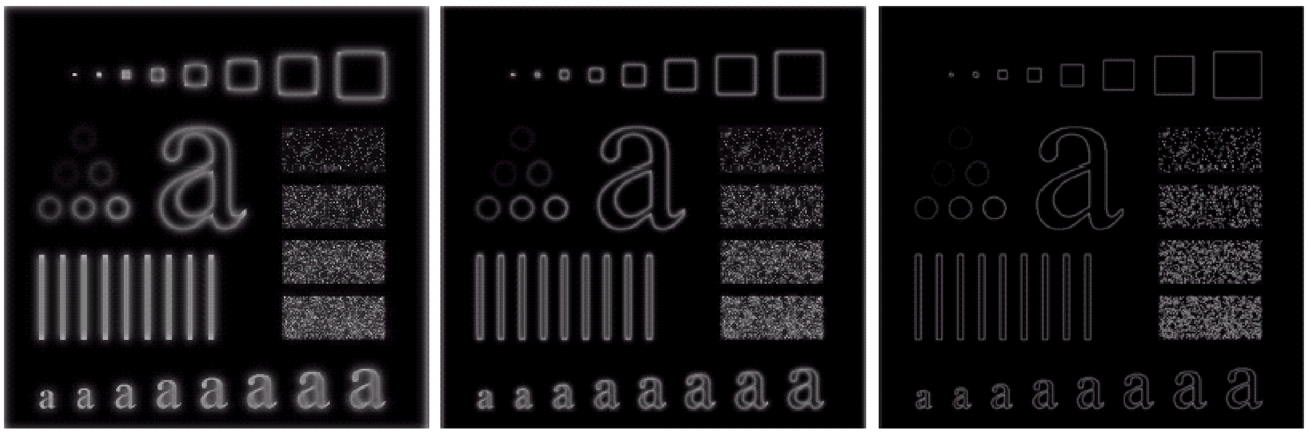
Butterworth Highpass Filtering



a b c

Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

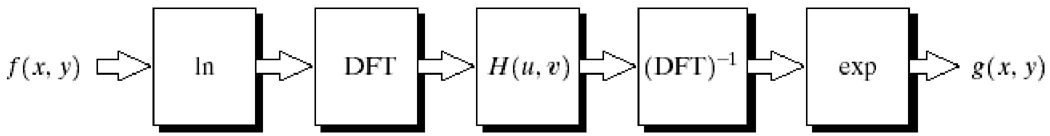
Gaussian Highpass Filtering



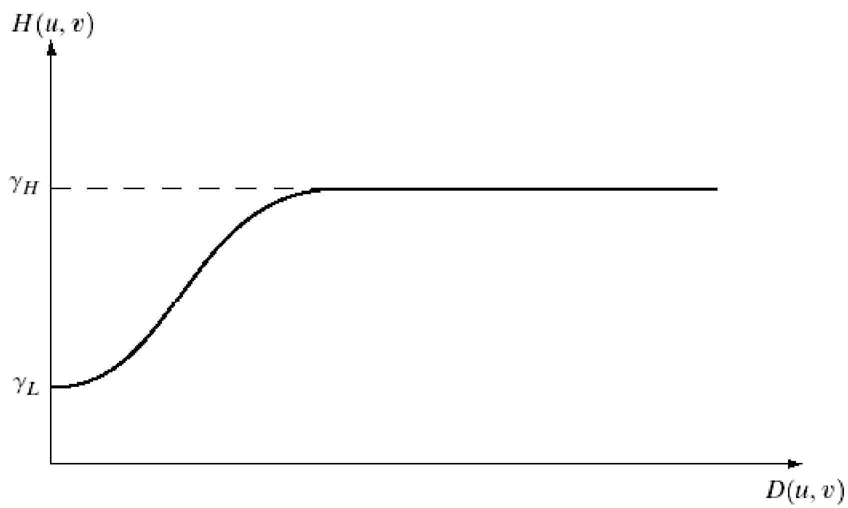
a b c

Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

Homomorphic Filtering



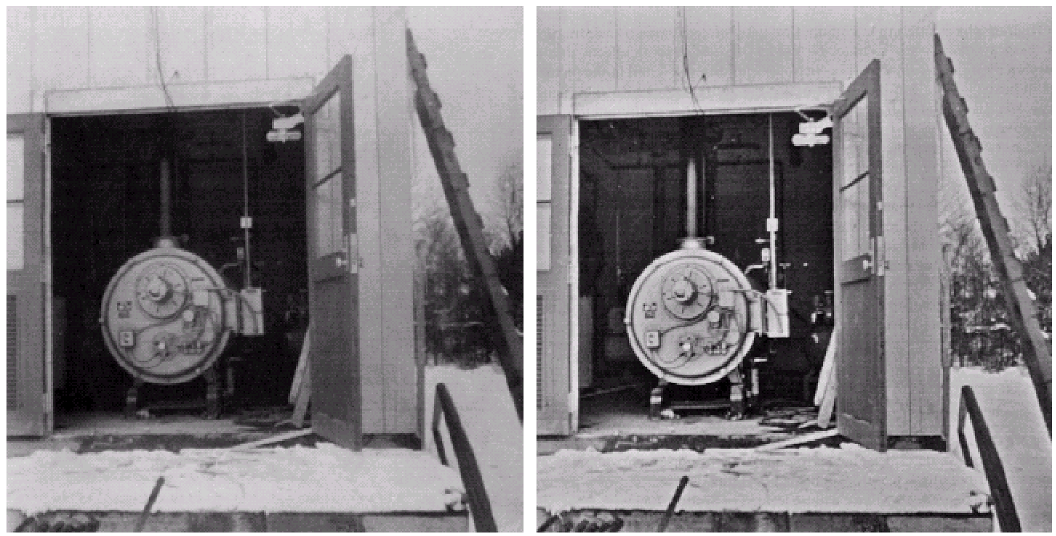
Homomorphic filtering approach for image enhancement.



Cross section of a circularly symmetric filter function. $D(u, v)$ is the distance from the origin of the centered transform.

Homomorphic Filtering (Example)

a b



(a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)