SNR Analysis For SAR Imaging From Raw Data Via Compressed Sensing

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Abstract

Compressed sensing (CS) is a theory that guarantees the quality of reconstruction of a sparse signal from limited samples. The state-of-art CS radar imaging algorithms are all working on the compressed range data. They are feasible since matched filtering increases the signal to noise ratio (SNR). On the other hand, imaging from the raw data directly can reduce the hardware complexity of a radar system. However, according to the radar equation, the SNR in the radar echo signal is usually at a low level (e.g., -5dB). In this paper, we formulate a method of synthetic aperture radar (SAR) imaging from raw data via compressed sensing, and analyze SNR in the echo signal based on CS. Combining the traditional radar equation with the theory of compressed sensing, we provide an expression of SNR based on CS. The simulation results demonstrate the validity of our expression. An experiment of spaceborne stripmap SAR raw data is carried out, which shows the feasibility of SAR imaging from raw data via the method proposed in this paper.

1 Introduction

The synthetic aperture radar (SAR) is a kind of remote sensing device which can image the ground targets with high resolution. It is widely used in many military and civil applications. The need of high resolution radar imaging results in the requirement of a dense spatial sampling of the echo signal, which brings high data rate to the radar system. However, the limited onboard memory and downlink data rate become major constraints to high data rate imaging for the most current spaceborne SAR systems [1].

Compressed sensing [2] is a new technique for efficiently sampling signals that are sparse in certain transform domains. The number of measurements it needs is typically far below the one expected from the requirements of the Nyquist sampling theorem.

The technique of compressed sensing is proved to be feasible to radar imaging in recent years. Currently, this technology is applied to inversed synthetic aperture radar (ISAR) and spotlight SAR imaging in [3], and stripmap SAR imaging in [4]. CS is also introduced into random noise SAR imaging in [5]. A novel CS SAR imaging method with the compressed range data is proposed in [6].

In this paper, we analyze the SNR in the echo signal by combining the traditional radar equation and the theory of CS. From the analysis we discover that the SNR in the echo signal has a connection with the sparsity of the observed scene. The simulation results confirm the validity of the new SNR expression. We also provide a feasible CS stripmap SAR imaging method from the raw data. The experimental result based on the RADARSAT-1 raw data shows the feasibility of the proposed imaging method.

The remainder of the paper is organized as follows. Section 2 reviews the basic CS theory, and Section 3 presents a CS spaceborne stripmap SAR imaging algorithm from raw data. The analysis of CS SNR in the echo signal is provided in Section 4. The result of RADARSAT-1 is shown in Section 5, and the conclusion is given in Section 6.

2 Theory of CS

This section exposes briefly the theoretical fundamentals of CS theory. Consider a time-domain signal $x \in \mathbb{C}^{N\times 1}$, which has a sparse representation in some basis $\Psi = [\phi_1 \mid \phi_2 \mid \cdots \mid \phi_N]$,

$$x = \sum_{k=1}^{N} \psi_k \alpha_k = \Psi\alpha \tag{1}$$

where α is an $N \times 1$ column vector of weighting coefficients $\alpha_k = \langle x, \psi_k \rangle$. If there are only $K (K \ll N)$ of the coefficients α_k to be nonzero, x is called sparse in the Ψ domain with K sparsity. If the measurement of x is acquired in the time domain also, that is,

$$y = \Phi x + n \tag{2}$$

where $\Phi \in \mathbb{C}^{M\times N}$ is the observation matrix, $y \in \mathbb{C}^{M\times 1}$ is the measurement, and $n \in \mathbb{C}^{M\times 1}$ is the measurement noise. Since $M < N$, x cannot be recovered directly from y. But by substituting x with (1), y can be written as,

$$y = \Phi x + n = \Phi\Psi\alpha + n = \Theta\alpha + n \tag{3}$$

Because α in (3) has K sparsity, and $K < M < N$, the coefficients in α could be solved from an optimization problem,

$$\min_{\alpha} \|\alpha\|_1 \;\; s.t. \;\; \|y - \Theta\alpha\|_2 \leq \varepsilon \tag{4}$$

The signal x can then be reconstructed from α. Problem (4) can be solved with different kinds of optimization methods, such as IST [7]. When the measurement matrix $\Theta = \Phi\Psi$ satisfies certain restricted isometry property (RIP) [8] conditions, the sparse vector α can be reconstructed perfectly.

3 The Framework of CS SAR Imaging From Raw Data

In the case of a point target scene, we assume that the transmitted signal is a chirp signal

$$s(\tau) = \mathrm{rect}\left(\frac{\tau}{T_p}\right) \exp\left(j2\pi f_c \tau + j\pi K_r \tau^2\right) \tag{5}$$

where τ is the fast time; f_c is the carrier frequency; K_r is the chirp rate; T_p denotes time width of chirp pulse; and $\mathrm{rect}(\cdot)$ stands for the unit rectangular function. The echo from a point target can be written as

$$\begin{aligned} s_0(\tau, \eta) &= x_0 \cdot \mathrm{rect}\left(\frac{\tau - R_0(\eta)/c}{T_p}\right) \cdot \mathrm{rect}\left(\frac{p_0/v_a - \eta}{T_a}\right) \\ &\cdot \exp\left\{-j\frac{4\pi f_0 R_0(\eta)}{c} + j\pi K_r \left(\tau - \frac{R_0(\eta)}{c}\right)^2\right\} \end{aligned} \tag{6}$$

where x_0 is the backscatter coefficient of the point target, p_0 is the azimuth position of the target, $R_0(\eta) = \sqrt{R_0^2 + (v_a\eta)^2}$ is the instantaneous range between the target and the radar platform, R_0 is the shortest slant range of target at $\eta = 0$, v_a is the platform velocity, c is speed of light and T_a is the synthetic aperture time. Discretizing the target scene as N scattering centers, assuming that it only contains K strong scattering centers with different locations and deeming other weak scattering centers as noise, the echo signal is as follows:

$$s(\tau, \eta) = \sum_{i=0}^{N-1} s_i(\tau, \eta) + n(\tau, \eta), \tag{7}$$

where $n(\tau, \eta)$ represents the synthetic additive noise.
After random down-sampling in the range and azimuth directions respectively, (7) can be written as

$$Y = \Phi X + N = \Theta H X + N \tag{8}$$

where $\Theta \in \mathbb{C}^{M\times N}$ is the random down-sampling matrix, $H \in \mathbb{C}^{N\times N}$ is the system measurement matrix, and $N \in \mathbb{C}^{M\times 1}$ denotes the additive noise vector, $X \in \mathbb{C}^{N\times 1}$ is the target vector whose nonzero components correspond to the complex amplitudes of the K strongest scattering centers

$$X = [x_0, x_1, \cdots, x_{N-1}]^T \tag{9}$$

When Φ satisfies certain conditions and SNR is high enough, the K-sparse target vector X can be exactly reconstructed by solving the following regularization problem with convex methods

$$\min_{X} \lambda\|X\|_q^q + \|Y - \Phi X\|_2^2 \tag{10}$$

where q is in $(0, 1]$. In most cases, the statistics of noise are unknown and need to be estimated. So we use a denoising method based on IST [7] to solve (10). The l_q algorithm consists of the following steps:
i) Initialization: Set the iteration counter $k = 1$, $X^{(k)} = 0$, $X^{(k+1)} = 0$, $\lambda = 0$ and $L = \|\Phi\|_2$.
ii) Estimation: $X^{(k+1)} = X^{(k)} + \frac{1}{L}\Phi^H\left\{Y - \Phi X^{(k)}\right\}$.
iii) Shrinkage/Thresholding: If $\|X^{(k+1)}\|_2 \geq f(\lambda)$ then set $X^{(k+1)} = X^{(k+1)} - f(\lambda)$, else set $X^{(k+1)} = 0$.
iv) Updating: If $\|X^{(k+1)} - X^{(k)}\|_2 \leq$ max_err or $k \geq$ max_iter then terminate; else set $X^{(k)} = X^{(k+1)}$, and then update the tolerance $f(\lambda)$ by set $\lambda = \max(\|X^{(k)}\|_2, K)$.
v) Iteration: Increase k by 1 and iterate from Step (ii).

4 SNR Analysis

In this section, we present an overview of the radar equation, combine it with the theory of CS, and provide an SNR expression for the CS based method.

4.1 Overview of the radar equation

According to the traditional radar equation [1], in the point target case, the traditional SNR of the echo signal is

$$\mathrm{SNR}_{\mathrm{trad}} = \frac{P_s}{kF_{op}T_sB_n} = \frac{P_tG^t\sigma A_e}{(4\pi R^2)^2 kF_{op}T_sB_n} \tag{11}$$

where P_s is the received power, k is Boltzmann's constant, $k = 1.38 \times 10^{-23} J/K$; F_{op} is the operating noise factor and T_s is the total source equivalent noise temperature, B_n stands for the bandwidth of the receiver which is chosen wide enough to pass the signal, but no wider. σ is the radar scattering cross section of a target. The inferred cross section σ is the product of the relative backscatter coefficient x and the geometry area ΔS, $\sigma = x \cdot \Delta S$. For a receiving antenna, the effective aperture is $A_e = \lambda^2 G^r/4\pi$.
For the distributed target, the traditional radar equation turns into

$$\mathrm{SNR}_{\mathrm{trad}} = \frac{P_t \int [G^t(\theta,\phi)\sigma(\theta,\phi)A_e(\theta,\phi)/(4\pi R^2)^2]d\theta d\phi}{kF_{op}T_sB_n} \tag{12}$$

It is useful to assume the distributed target with isotropic mean backscatter coefficient over the scene and that it keeps the same for every position of the antenna. So the cross section becomes an constant $\overline{\sigma}$, and (12) can be simplified to

$$\mathrm{SNR_{trad}} = \frac{P_t \overline{\sigma} \lambda^2}{(4\pi)^3 k F_{op} T_s B_n} \int \frac{G^t(\theta,\phi) G^r(\theta,\phi)}{R^4} d\theta d\phi \tag{13}$$

In a single cell case, the $\mathrm{SNR_{trad}}$ is expressed as

$$\mathrm{SNR_{trad}^{cell}} = \frac{P_t G^2 \overline{\sigma} \lambda^2 \Delta p \Delta R_g}{(4\pi)^3 R^4 k F_{op} T_s B_n} \tag{14}$$

where Δp is the extension distance in azimuth and ΔR_g is the extension distance in ground range.

4.2 Analysis of the SNR based on CS

According to (8), the SNR based on compressed sensing can be defined as $\mathrm{SNR_{cs}} = \frac{\|\Phi X\|_2^2}{\|N\|_2^2}$. It is useful to assume that the antenna receives an entire aperture of the echo signal, and there is no down-sampling, which means $\Theta = I$. So in a single cell case, the SNR can be express as

$$\overline{\mathrm{SNR}}_{\mathrm{cs}}^{\mathrm{j}} = \frac{E\left(\|\Phi_{ij} x_j\|_2^2\right)}{E\left(\|N_i\|_2^2\right)} = \frac{P_t G^t \sigma_j A_e}{(4\pi R^2)^2 k F_{op} T_s B_n} \tag{15}$$

and in average meaning,

$$\overline{\mathrm{SNR}}_{\mathrm{cs}}^{\mathrm{cell}} = \frac{P_t G^t \overline{\sigma} A_e}{(4\pi R^2)^2 k F_{op} T_s B_n} \tag{16}$$

$$\overline{\mathrm{SNR}}_{\mathrm{cs}} = K \cdot \frac{P_t G^t \overline{\sigma} A_e}{(4\pi R^2)^2 k F_{op} T_s B_n} = K \cdot \mathrm{SNR_{trad}} \tag{17}$$

For a K-point target scene, if all of the points are independent, and they expectations equal zero, then

$$\begin{aligned}\overline{\mathrm{SNR}}_{\mathrm{cs}} &= \frac{E\left(\|\Phi X\|_2^2\right)}{E\left(\|N\|_2^2\right)} = \frac{E\left(\sum\limits_i \left\|\sum\limits_j \Phi_{ij} x_j\right\|_2^2\right)}{E\left(\sum\limits_i \|N_i\|_2^2\right)} \\ &= \sum_j \frac{\sum\limits_i E\left(\|\Phi_{ij} x_j\|_2^2\right)}{\sum\limits_i E(\|N_i\|_2^2)} = \sum_j \overline{\mathrm{SNR}}_{\mathrm{cs}}^{\mathrm{j}}\end{aligned} \tag{18}$$

Specially, in average meaning, (18) turns into:

$$\overline{\mathrm{SNR}}_{\mathrm{cs}} = K \cdot \mathrm{SNR_{cs}^{cell}} \tag{19}$$

Else if the K scatters do not distribute independently, then we use RIP to analyse SNR. If Φ satisfies $\mathrm{RIP}(N, K, \delta)$ for every K-sparse vector X, then

$$(1-\delta)\|X\|_2^2 \leq \|\Phi X\|_2^2 \leq (1+\delta)\|X\|_2^2 \tag{20}$$

Moreover, if the amplitudes of the K scattering points are independent, and the noise is white and independent with the signal as well, then

$$(1-\delta)\frac{E(\|X\|_2^2)}{E(\|N\|_2^2)} \leq \overline{\mathrm{SNR}}_{\mathrm{cs}} \leq (1+\delta)\frac{E(\|X\|_2^2)}{E(\|N\|_2^2)} \tag{21}$$

For a distributed target, it is appropriate to define the radar cross section per unit geometrical area of the scene as a random variable. The conclusion of the SNR of a distributed target is the same with the K-point target.

4.3 Simulation

In order to check out the analysis of SNR based on CS, two sets of simulations have been carried out. The key simulation parameters are as follows. Slant range is 15000 m, radar velocity is 150 m/s, pulse during time is 1 μs, carrier frequency is 1 GHz, range resolution is 2 m, azimuth resolution is 2 m, and the discrete scene is 31×31 in grid. In the first test, we place a 50-point target, where the scatters are i.i.d. and their amplitudes satisfy $U(0.2, 1)$. A white Gaussian noise is then added to the scene, which satisfies $N(0, 1)$. The SNR in the traditional radar equation is $\mathrm{SNR_{trad}} = -4.437$ dB. According to (19), the theoretic SNR is $\mathrm{SNR_{cs}}^{theory} = 12.55$ dB, while the SNR calculated from the simulation is $\mathrm{SNR_{cs}}^{sim} = 12.36$ dB. The result is shown in Fig. 1.

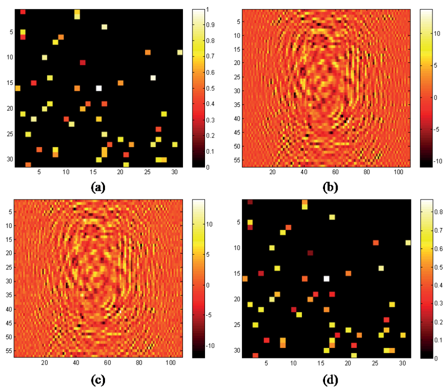

Figure 1: Simulation results of a 50-point target. (a) True target scene, (b) Real part of the echo with no noise, (c) Real part of the echo with noise, (d) the l_1 recovered scene

The second test is an extended experiment of the first one. We change the number of points from 1 to 100. The point targets are i.i.d. and satisfy $U(0.2, 1)$, and the added white Gaussian noise satisfies $N(0, 1)$. For each point-target scene, we repeat 100 times to get the statistic curve of the SNR based on CS. The theoretic curve of SNR is also given as a comparison. We can see that the statistic curve also satisfies (19). The result is shown in Fig. 2.

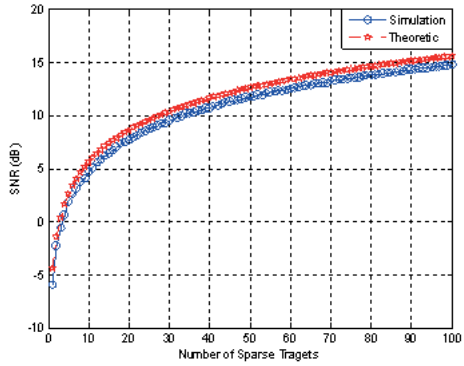

Figure 2: Simulation result of the SNR change curve

5 Experimental Results

In this section, we provide some experimental results from the raw data of RADARSAT-1 sea area. The details of target and data parameters are provided in [9].

The radar equation indicates that the traditional SNR of the relative echo signal is approximately -10 dB, in such condition, exactly recovering from the raw data by l_1 seems desperately.

Fig. 3(a) shows the imaging result via conventional Range Doppler (RD) algorithm, and Fig. 3(b) shows the image of the same area which was imaged via spaceborne stripmap CS SAR algorithm with only 11.1% of the full data (15.4% of the azimuth data and 72.0% of the range data). We can see that there are good focus and less side-lobe. The experiment result shows that CS can be used in real spaceborne stripmap SAR imaging, and it is feasible to image from the raw data using l_1 algorithm.

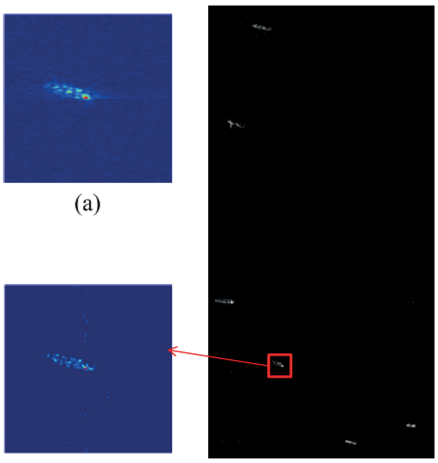

Figure 3: RADARSAT-1 raw data imaging results. (a) Partial RD imaging result, (b) Partial CS imaging result, (c) CS imaging result.

6 Conclusion

In this paper, we propose a method of spaceborne stripmap SAR imaging from raw data via compressed sensing. Combining the traditional radar equation with the theory of compressed sensing, we provide an SNR expression in the echo signal. From the equation, we can see that, in a scene with a proper sparsity, the SNR in the echo signal can be high enough for exact recovery of the target scene with high possibility. The simulation results demonstrate the validity of our equation, and the experiment result using the RADARSAT-1 data shows that it is feasible to image from raw data via the proposed method. Future works will extend this method to the other SAR modes, the speckle noise would be considered in the SNR analysis, and more experiments on testing the relationship between K and the SNR of the echo signal will be carried out in the future.

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References

[1] J.C. Curlander and R.N. McDonough, *Synthetic aperture radar: Systems and signal processing*, 1991.

[2] D. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, pp. 1289-1306, 2006.

[3] V. M. Patel, G. R. Easley, J. Healy and R. Chellappa, "Compressed synthetic aperture radar," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, pp. 244-254, 2010.

[4] B.C. Zhang, H. Jiang, W. Hong, and Y.R. Wu, "Synthetic aperture radar imaging of sparse targets via compressed sensing," *8th European Conference on Synthetic Aperture Radar*, Aachen Germany, 2010.

[5] H. Jiang, B.C. Zhang, Y.G. Lin, W. Hong, Y.R. Wu, and J. Zhan, "Random Noise Imaging SAR Based on Compressed Sensing," *Proceeding of IEEE International Geoscience and Remote Sensing Symposium*, Hawaii, USA, 2010, pp. 4624-4627.

[6] M.T. Alonso, P.L. Dekker and J.J. Mallorquí, "A Novel Strategy for Radar Imaging Based on Compressive Sensing," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 48, pp. 4285-4295, 2010.

[7] I. Daubechies, M. Defriese and C. DeMol, "An iterative thresholding algorithm for linear inverse problems with a sparsity constraint," *Communication Pure Application Mathematics*, vol. LVII, 2004, pp. 1413-1457.

[8] E. J. Candes and T. Tao, "Decoding by linear programming," *IEEE Transactions on Information Theory*, vol. 51, pp.: 4203-4215, 2005.

[9] I. Cumming and F. Wong, *Digital Processing of Synthetic Aperture Radar Data: Algorithms and Implementation*, Artech House Inc., 2005.