

Table 1: Theoretical results of the achieved 2-Wasserstein distance and the required gradient complexity for both log-concave (*italic*) and non-log-concave (**bold**) target distributions, where ϵ is any sufficiently small constant, Δ is the quantization error, and μ^* and λ^* denote the contraction rate of underdamped and overdamped Langevin dynamics respectively. Under non-log-concave target distributions, low-precision SGHMC achieves a better upper bound within shorter iterations compared with low-precision SGLD.

	Condition	Gradient Complexity	Achieved 2-Wasserstein
Full-precision gradient accumulators			
<i>SGLD/SGHMC</i> (Theorem 4)	<i>Strongly log-concave</i>	$\tilde{O}(\log(\epsilon^{-1}) \epsilon^{-2})$	$\tilde{O}(\epsilon + \Delta)$
SGLD (Theorem 7)	Non-log-concave	$\tilde{O}(\epsilon^{-4} \lambda^{*-1} \log^5(\epsilon^{-1}))$	$\tilde{O}\left(\epsilon + \log(\epsilon^{-1}) \sqrt{\Delta}\right)$
SGHMC (Theorem 1)	Non-log-concave	$\tilde{O}(\epsilon^{-2} \mu^{*-2} \log^2(\epsilon^{-1}))$	$\tilde{O}\left(\epsilon + \sqrt{\log(\epsilon^{-1}) \Delta}\right)$
Low-precision gradient accumulators			
<i>SGLD/SGHMC</i> (Theorem 5)	<i>Strongly log-concave</i>	$\tilde{O}(\log(\epsilon^{-1}) \epsilon^{-2})$	$\tilde{O}(\epsilon + \epsilon^{-1} \Delta)$
<i>VC SGLD/VC SGHMC</i> (Theorem 6)	<i>Strongly log-concave</i>	$\tilde{O}(\log(\epsilon^{-1}) \epsilon^{-2})$	$\tilde{O}(\epsilon + \sqrt{\Delta})$
SGLD (Theorem 8)	Non-log-concave	$\tilde{O}(\epsilon^{-4} \lambda^{*-1} \log^5(\epsilon^{-1}))$	$\tilde{O}\left(\epsilon + \log^5(\epsilon^{-1}) \epsilon^{-4} \sqrt{\Delta}\right)$
VC SGLD (Theorem 9)	Non-log-concave	$\tilde{O}(\epsilon^{-4} \lambda^{*-1} \log^3(\epsilon^{-1}))$	$\tilde{O}\left(\epsilon + \log^3(\epsilon^{-1}) \epsilon^{-2} \sqrt{\Delta}\right)$
SGHMC (Theorem 2)	Non-log-concave	$\tilde{O}(\epsilon^{-2} \mu^{*-2} \log^2(\epsilon^{-1}))$	$\tilde{O}\left(\epsilon + \log^{3/2}(\epsilon^{-1}) \epsilon^{-2} \sqrt{\Delta}\right)$
VC SGHMC (Theorem 3)	Non-log-concave	$\tilde{O}(\epsilon^{-2} \mu^{*-2} \log^2(\epsilon^{-1}))$	$\tilde{O}\left(\epsilon + \log(\epsilon^{-1}) \epsilon^{-1} \sqrt{\Delta}\right)$