Heaps & Priority Queues

Computer Science Department
University of Central Florida

COP 3502 – Computer Science I

Binary Heaps

- Building a Heap from scratch (a Max heap)
  - Given: an unsorted list of n values
    - **54, 87, 27, 67, 19, 31, 29, 18, 32, 56, 7, 12, 31**
  - How can we build a heap from these values?
    - It is really just a series of “insertions”
    - Simply insert the n elements into the heap in the order that they arrive (in our case, from left to right)
    - WHILE there are more elements:
      1) Insert the next element
      2) Percolate Up to a suitable position
  - Once all elements are inserted, we have our heap

Binary Heaps

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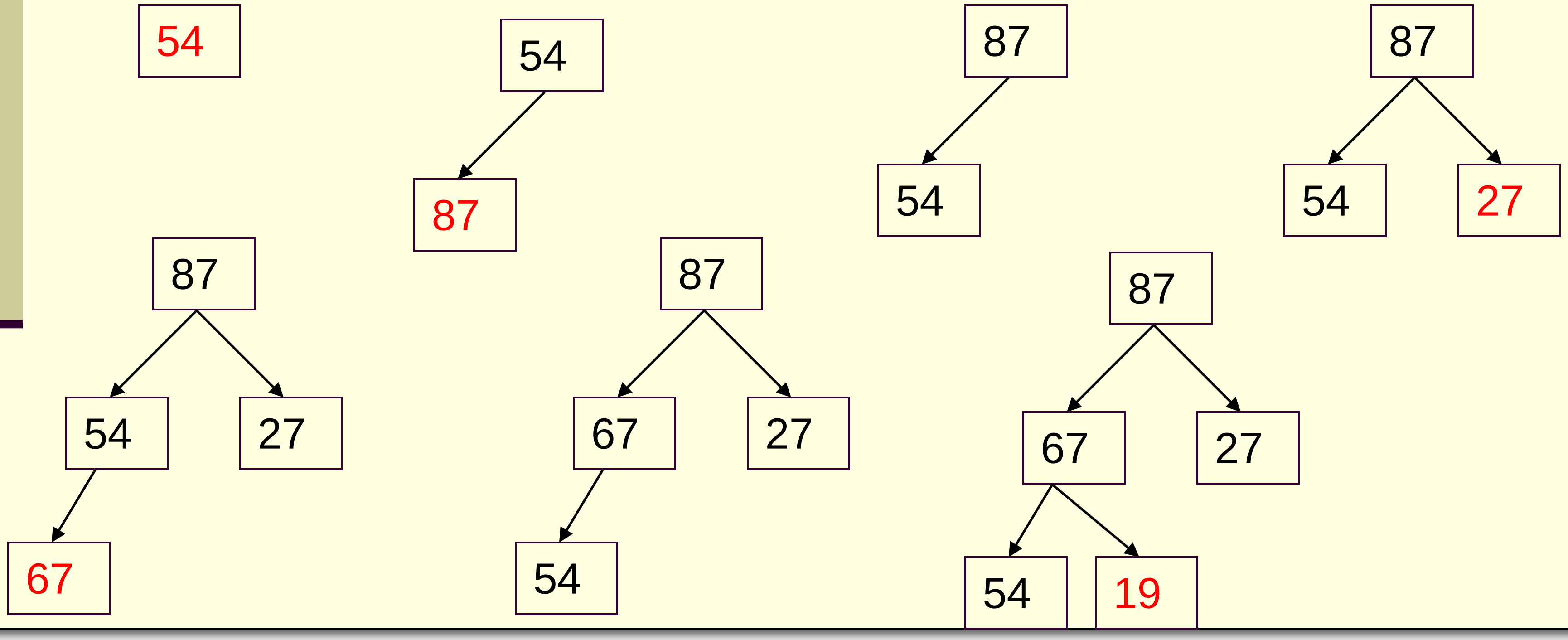

Binary Heaps

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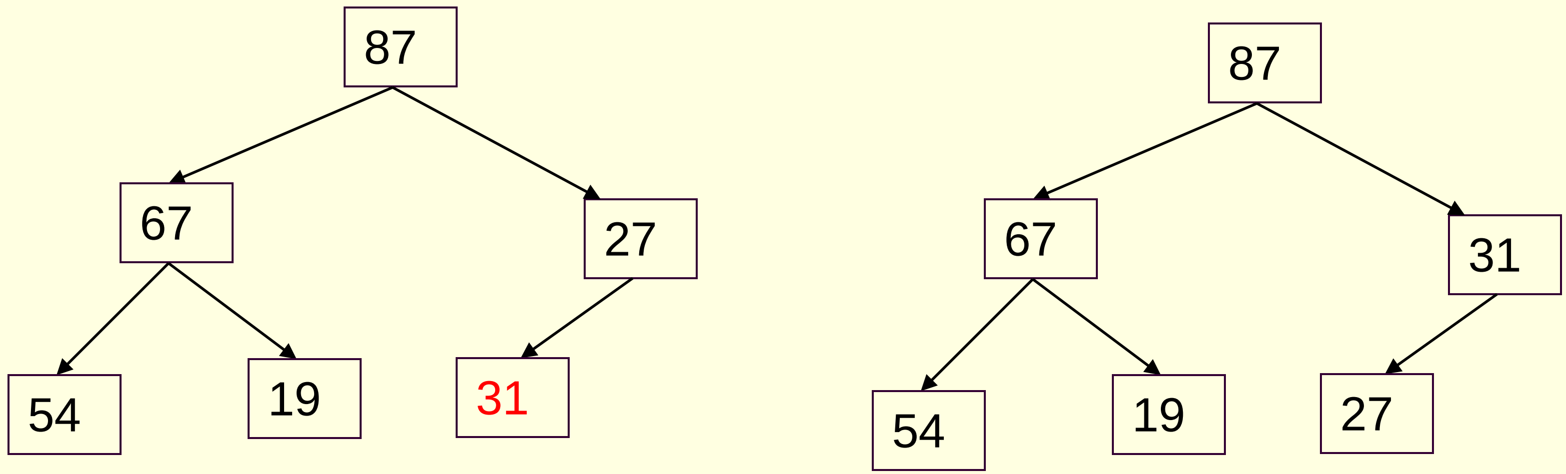

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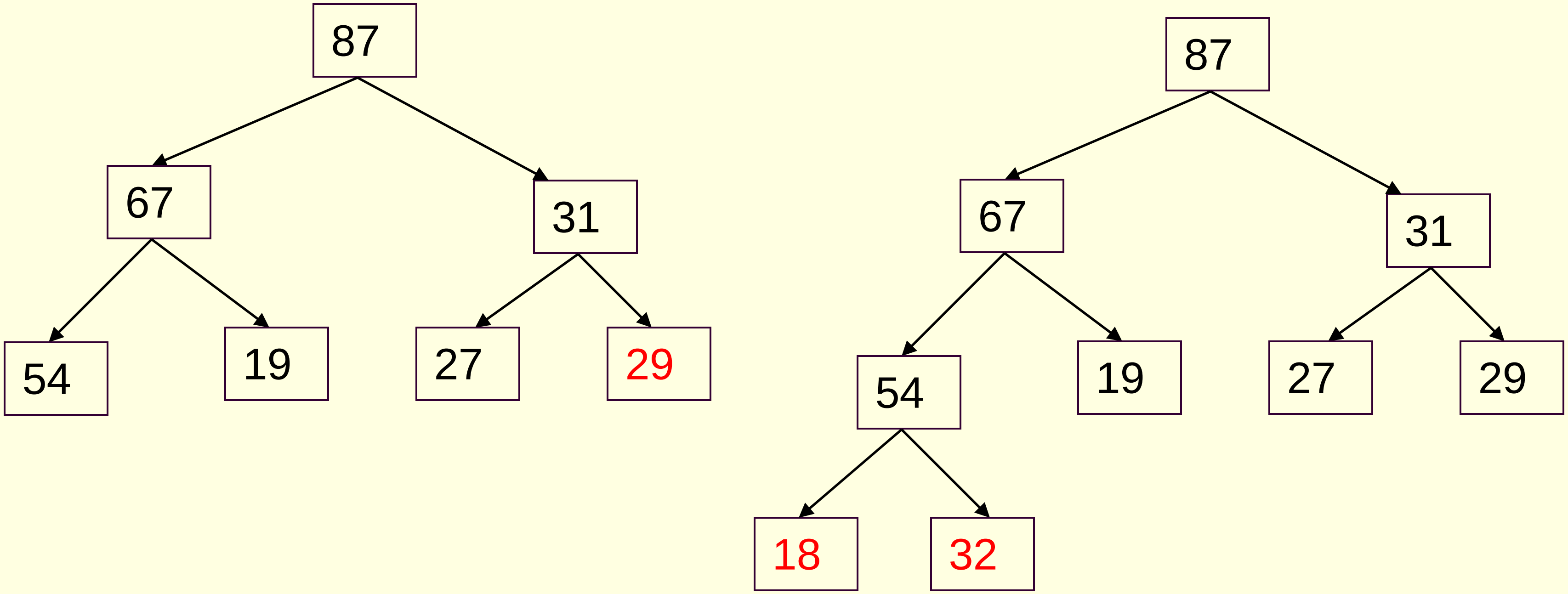


Binary Heaps

- Building a Heap from scratch
  - Running time:
    - How long does it take to do one insertion?
      - We just covered this!
      - An insertion takes O(logn)
        - As in the worst case, it has to Percolate all the way Up to root
    - And we have **n** elements to insert
    - **<u>Running time to make a heap from n elements is O(nlogn)</u>**

Binary Heaps

- Building a Heap from scratch
  - Can we do better than O(nlogn) time?
    - Turns out that we can
  - Start by arbitrarily placing your elements into a complete binary tree
  - Then, starting at the lowest level,
  - Perform a Percolate Down (if necessary)
  - So we work from the bottom and go up to the root
  - Performing a Percolate Down at each node
    - Only if necessary
  - This function is known as **Heapify**

Binary Heaps

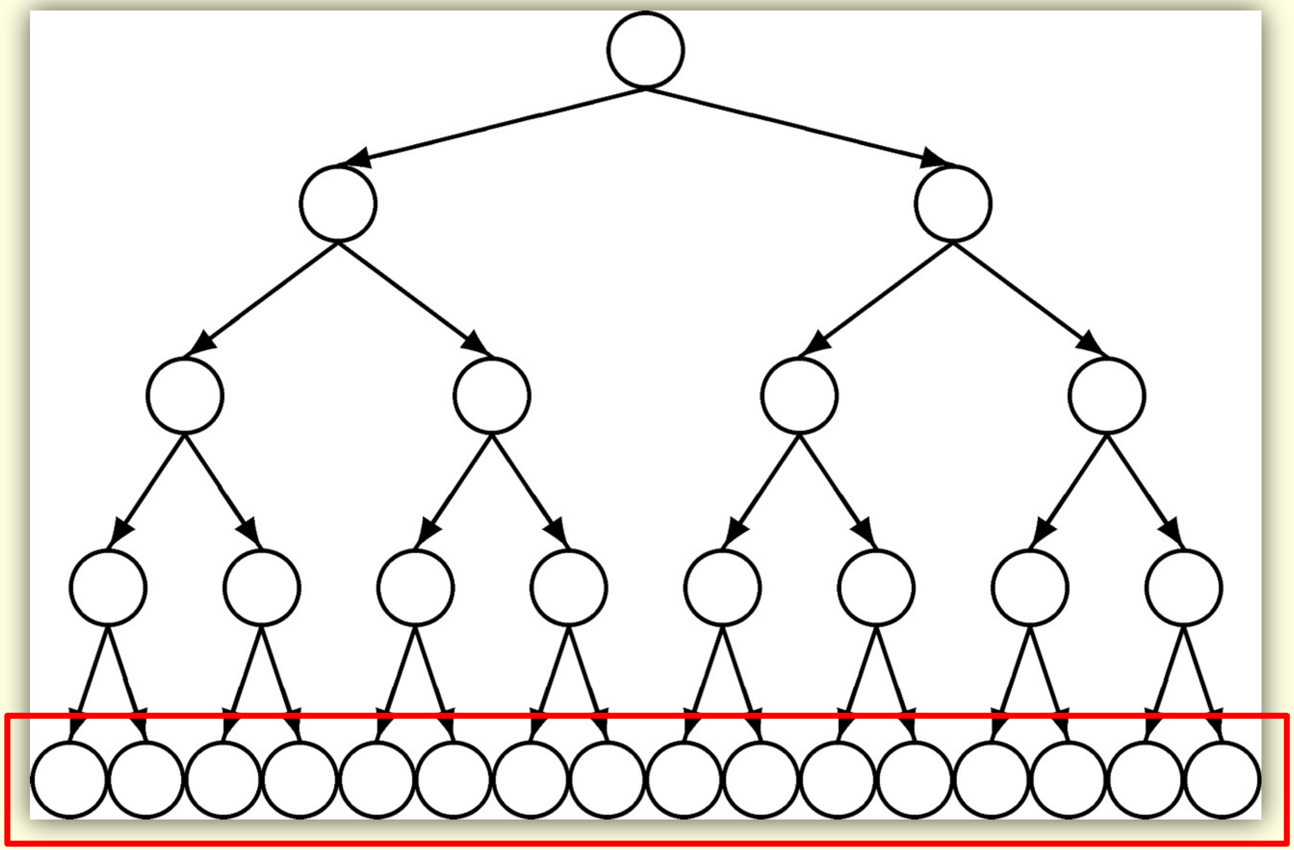
- Building a Heap from scratch
  - Running time:
    - Note:
      - Realize that for any given complete tree, that is completely filled, the lowest level has ½ of the total nodes in a tree
      - In a complete tree of 31 nodes, the lowest level has 16 nodes
        - And since they are already at the lowest level,
        - Those 16 nodes will NOT need to Percolate Down

Binary Heaps

- Building a Heap from scratch

These nodes do NOT have to Percolate Down!

They are already at the bottom most level.



Binary Heaps

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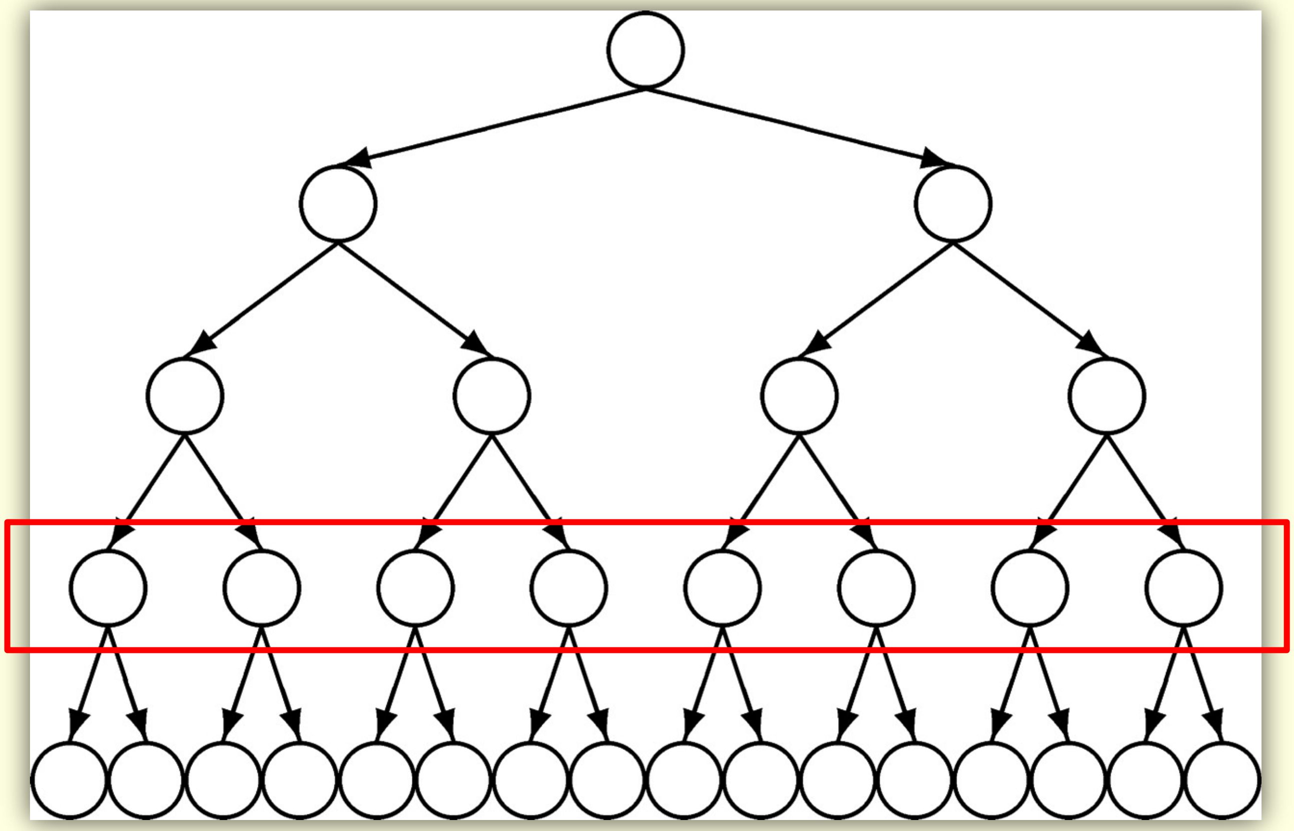
        - And since they are already at the lowest level,
        - Those 16 nodes will NOT need to Percolate Down
      - <u>The level above the 16 nodes has 8 nodes</u>
      - What can we say about those 8 nodes?
      - Notice that, at MOST, those 8 nodes will have to Percolate Down only one level

Binary Heaps

- Building a Heap from scratch

These nodes only have to Percolate Down one level.



Binary Heaps

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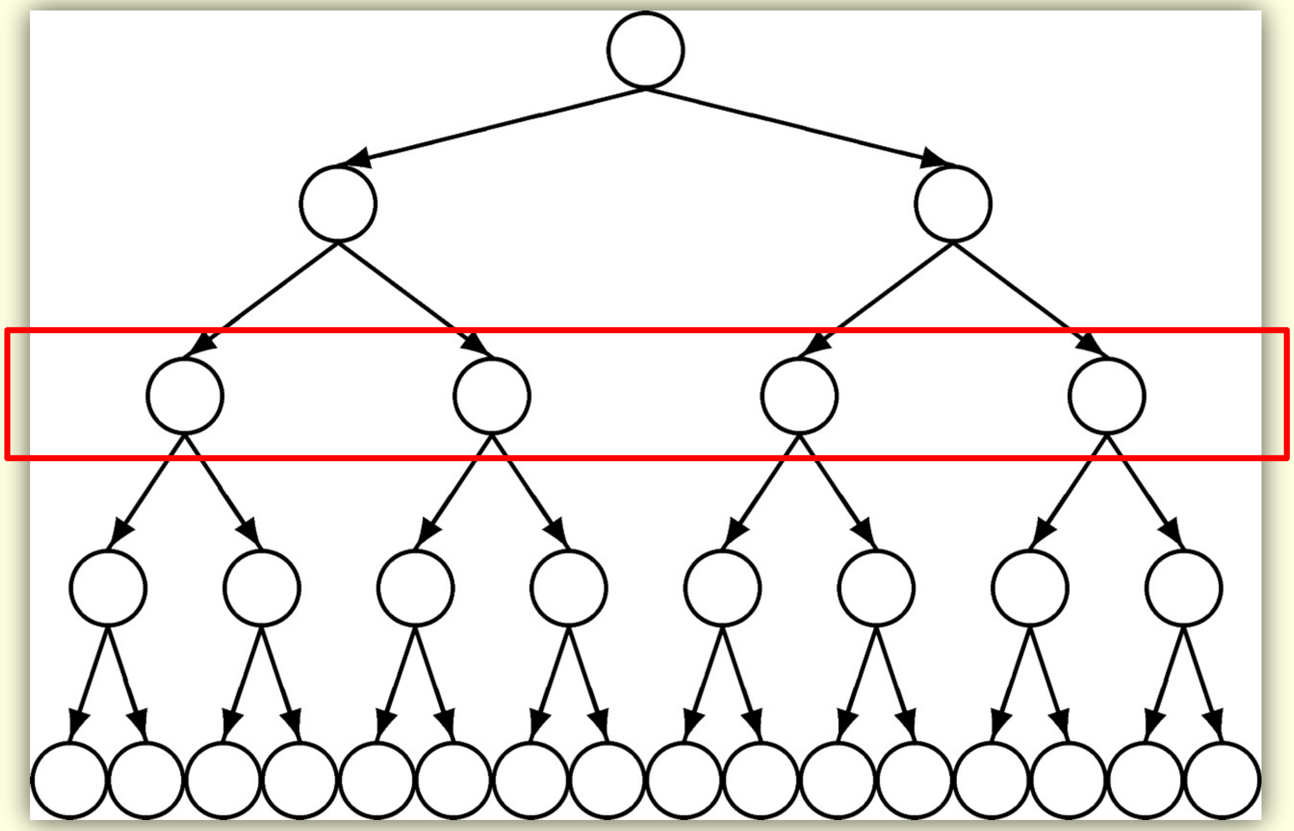
      - <u>The level above the 16 nodes has 8 nodes</u>
      - What can we say about those 8 nodes?
      - Notice that, at MOST, those 8 nodes will have to Percolate Down only one level
      - And the level above the 8 nodes has 4 nodes
      - Those 4 nodes, at most, percolate down 2 levels, etc, etc.

Binary Heaps

- Building a Heap from scratch

These nodes only have to Percolate Down two levels.



Binary Heaps

- Building a Heap from scratch
  - Running time:
    - So only ½ of the nodes in a tree may need to be percolated down one level or more
    - Only ½ of those (1/4 of the total) may have to be percolated down two or more levels
    - Only ½ of those (1/8 of the total) may have to be percolated down three or more levels, etc., etc.
    - So if we add up the total number of swaps, we get:
    - $(1/2)*n + (1/4)*n + (1/8)*n + \ldots \approx n$
    - **So this Heapify function runs in O(n) time**

Brief Interlude: FAIL Picture

Daily UCF Bike FAIL

Courtesy of
Kyle Perez

Binary Heaps

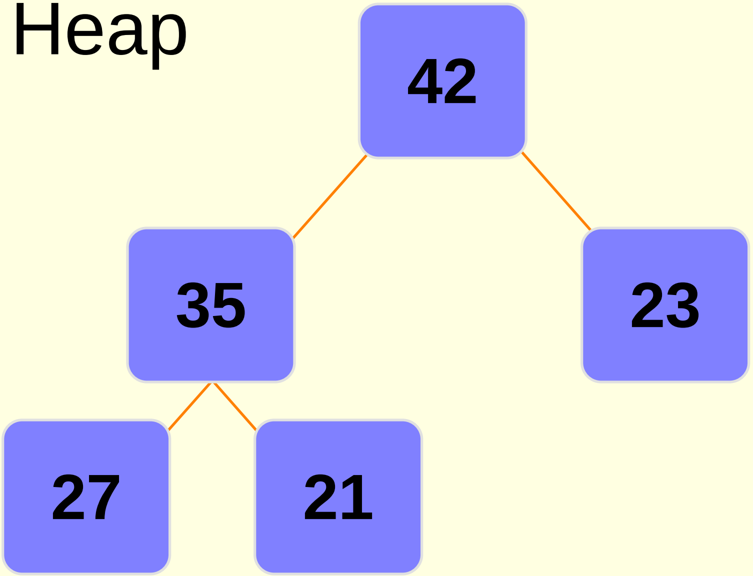
- Implementing a Binary Heap
  - Remember:
    - a binary heap is a complete binary tree
  - <u>So we can implement this binary tree as an array!</u>
  - How?
    - If a tree is “complete”,
      - The root would be the 1st position of the array (index 1)
      - The two children of the node would be in index 2 and 3
      - The 4 nodes on the next level would be in index 4 – 7
      - The 8 nodes on the next level would be in index 8 - 15
      - and so on

Binary Heaps

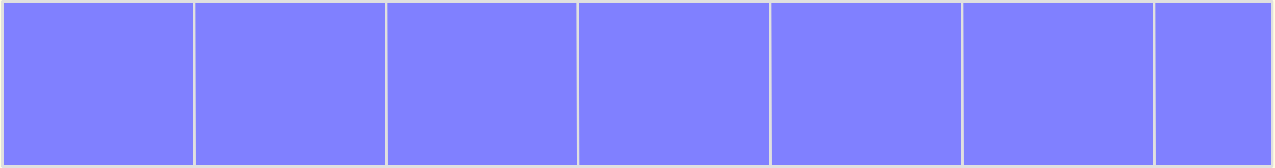
- Implementing a Binary Heap
  - Notes:
    - So we are wanting to implement one ADT
      - A Priority Queue
    - To do so, we utilize another ADT
      - A Heap
    - And to implement the actual Heap, which, in turn, implements the Priority Queue
      - <u>**We use an array!**</u>
    - So after all of this, we simply use an array
    - <u>And the way we dereference the array and manipulate the data is what makes “the array a tree”</u>

Binary Heaps

- Implementing a Binary Heap



- We store the data from the nodes in a partially-filled array.

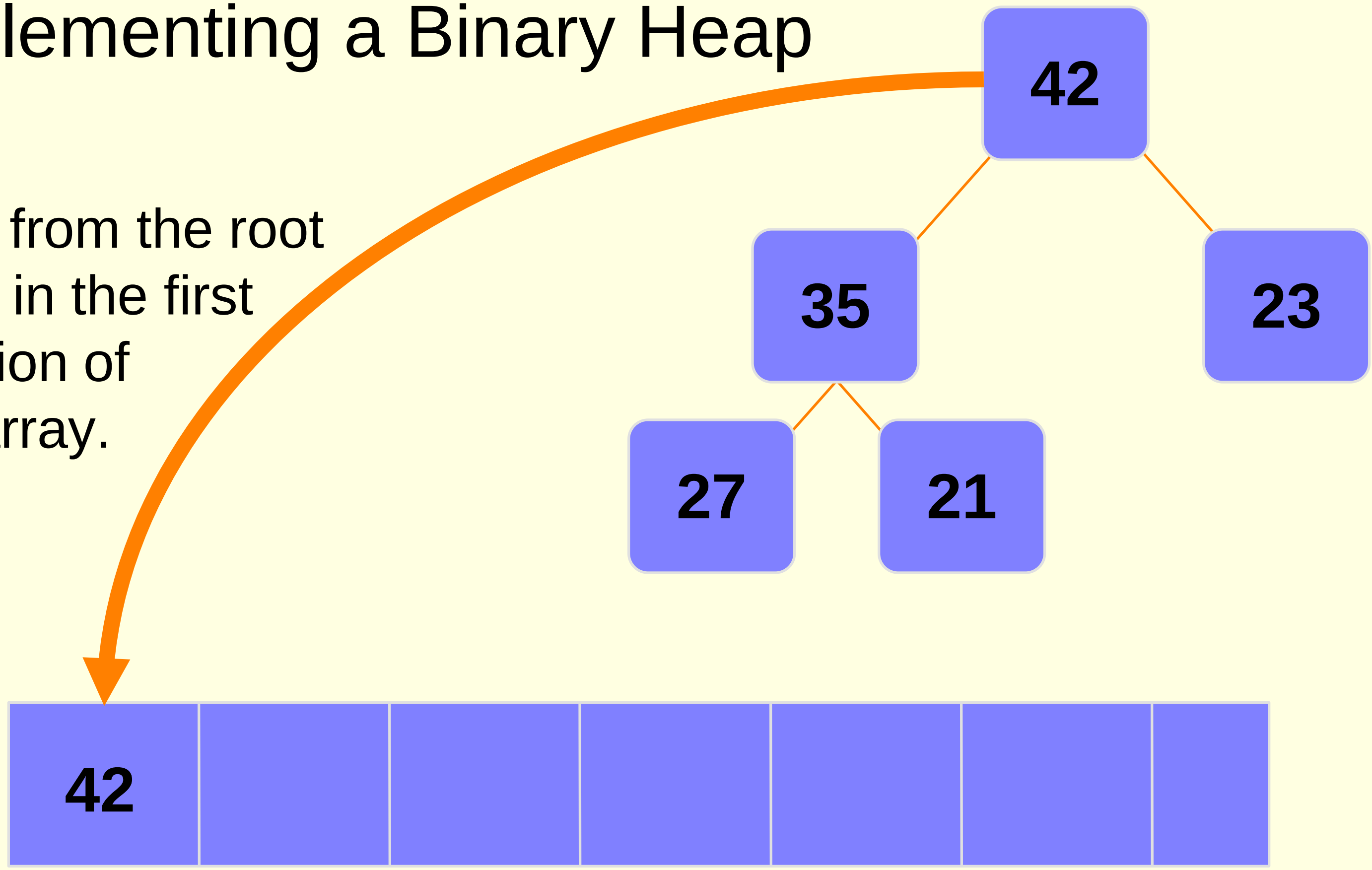


An array of data

Binary Heaps

- Implementing a Binary Heap

- Data from the root goes in the first location of the array.

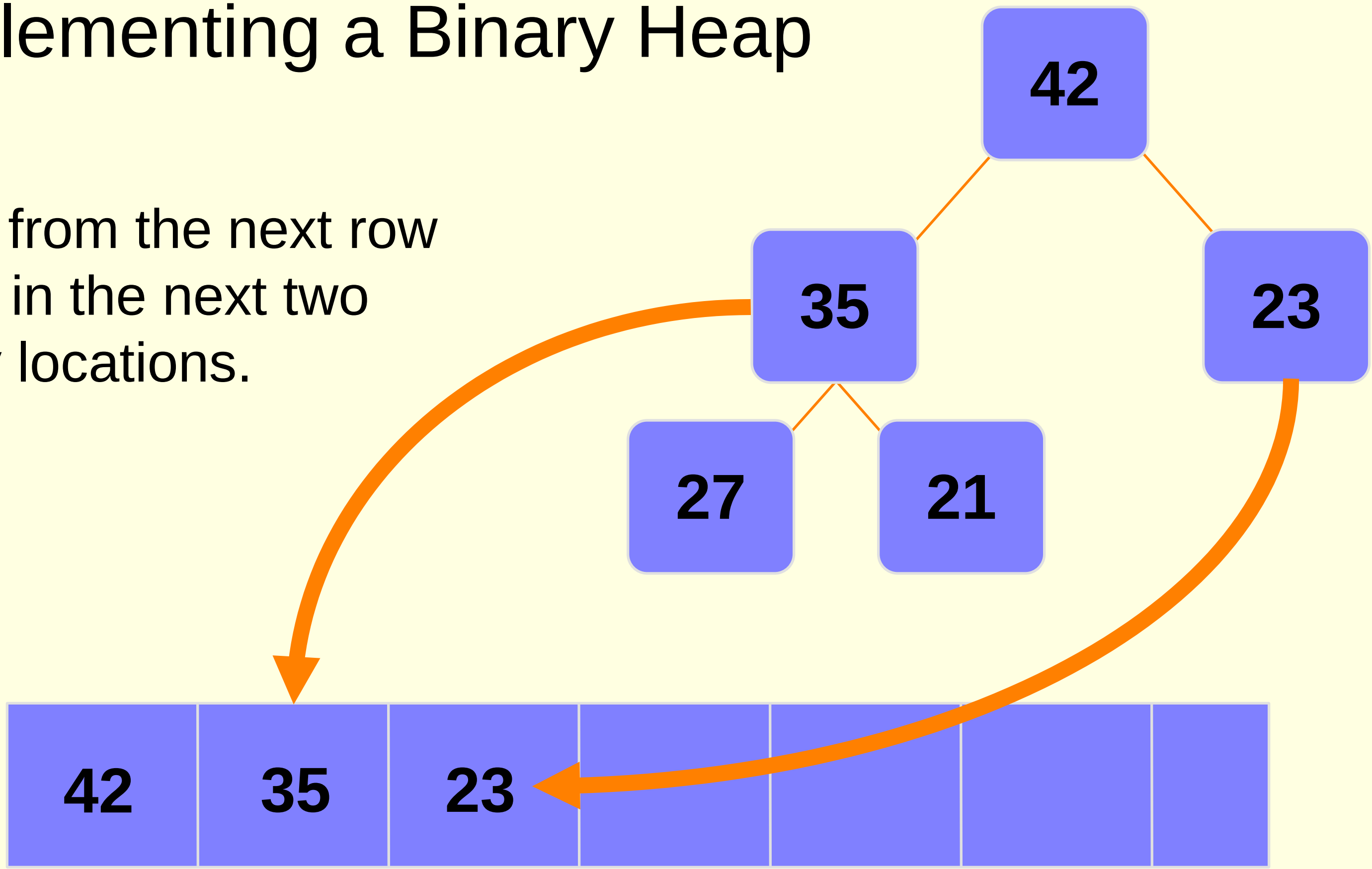


An array of data

Binary Heaps

- Implementing a Binary Heap

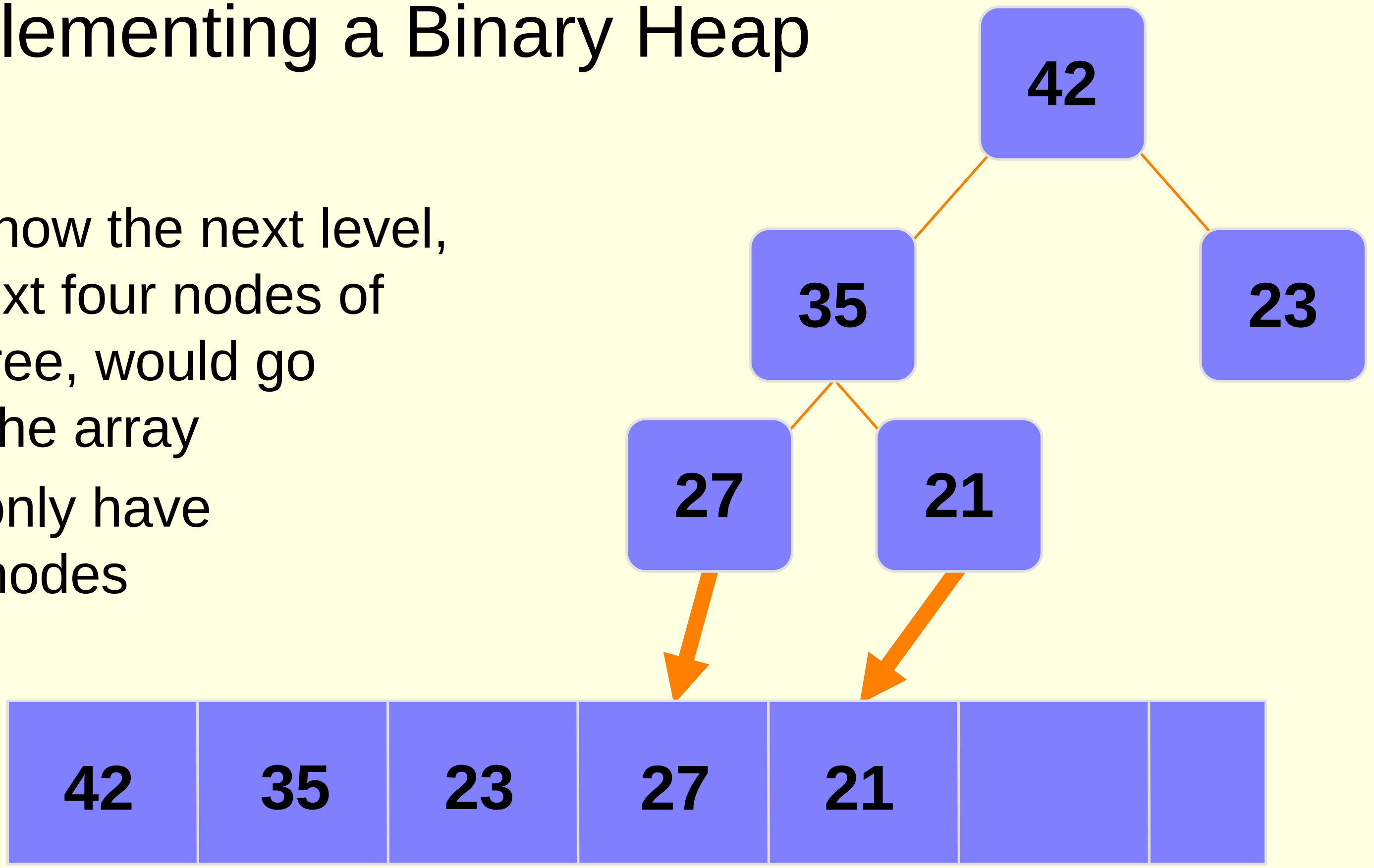
- Data from the next row goes in the next two array locations.



An array of data

Binary Heaps

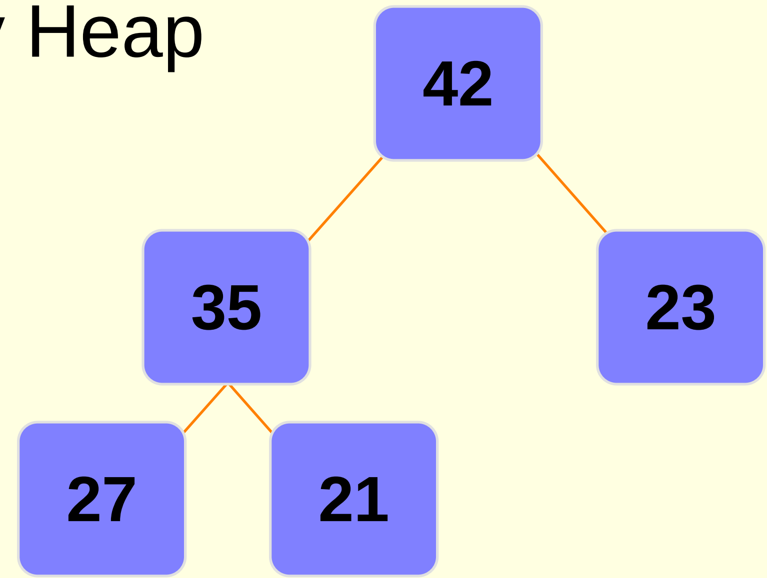
- Implementing a Binary Heap
- And now the next level, or next four nodes of the tree, would go into the array
- We only have two nodes



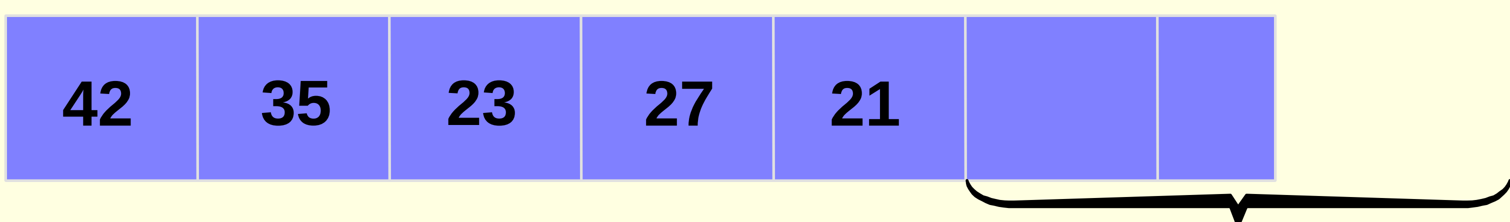
An array of data

Binary Heaps

- Implementing a Binary Heap



- We are only concerned with the front part of the array
- If the tree has 5 nodes, then we only care about the first five spots of the array

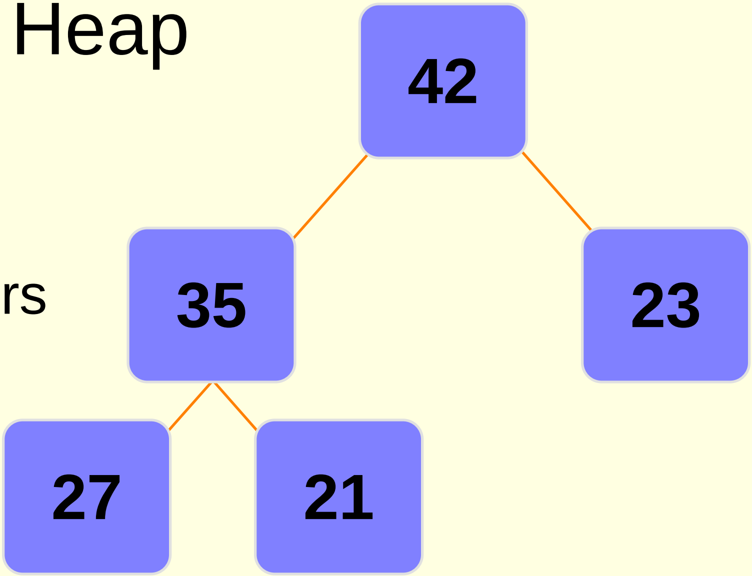


An array of data

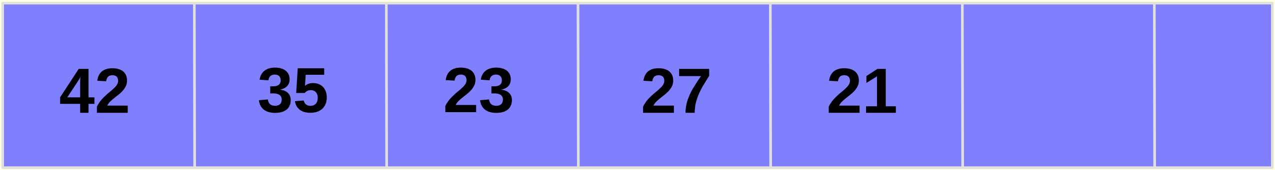
We don't care what's in this part of the array.

Binary Heaps

- Implementing a Binary Heap



- The links between the tree's nodes are not stored as pointers
- The only way we "know" that the "array is a tree" is based on how we choose to manipulate the array

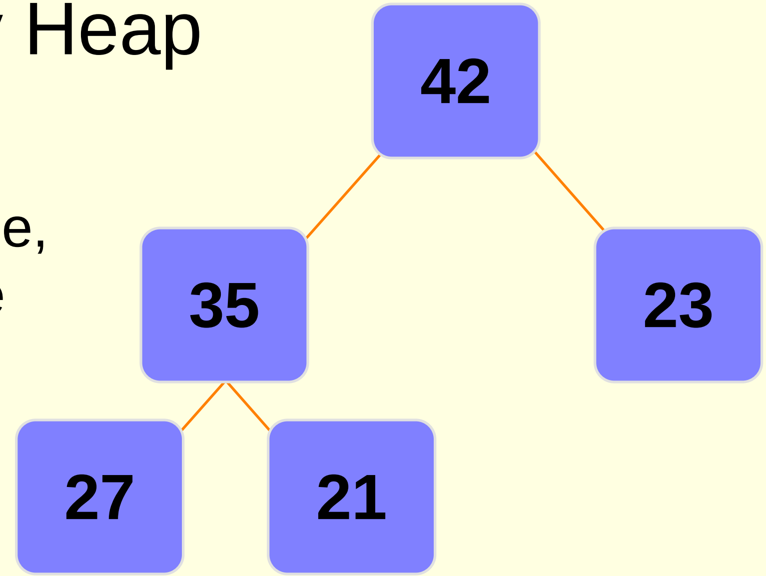


An array of data

Binary Heaps

- Implementing a Binary Heap

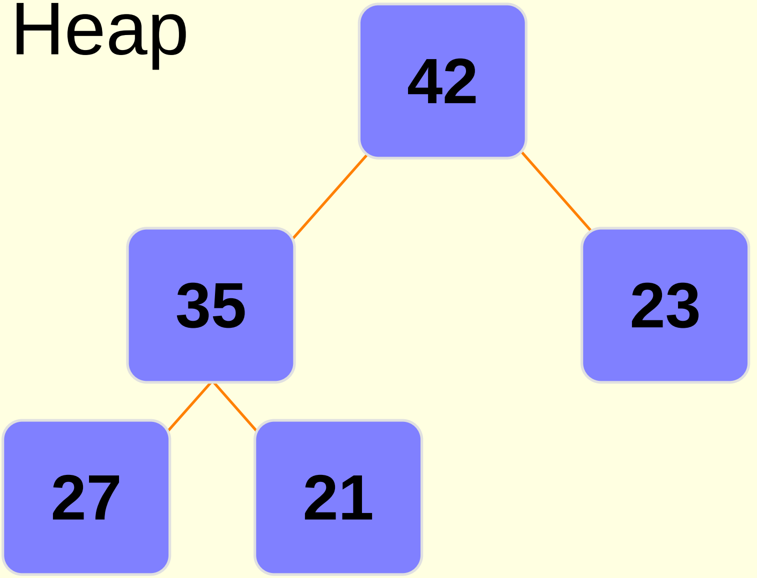
- If you know the index of a node, then it is easy to figure out the index of that node's parent or children



42	35	23	27	21		
[1]	[2]	[3]	[4]	[5]	[6]	

Binary Heaps

- Implementing a Binary Heap
  - The name of our array is A[]
  - Root is at position A[1]
  - Left child of A[i] = A[2i]
  - Right child of A[i] = A[2i+1]
  - Parent of A[i] = A[i/2]



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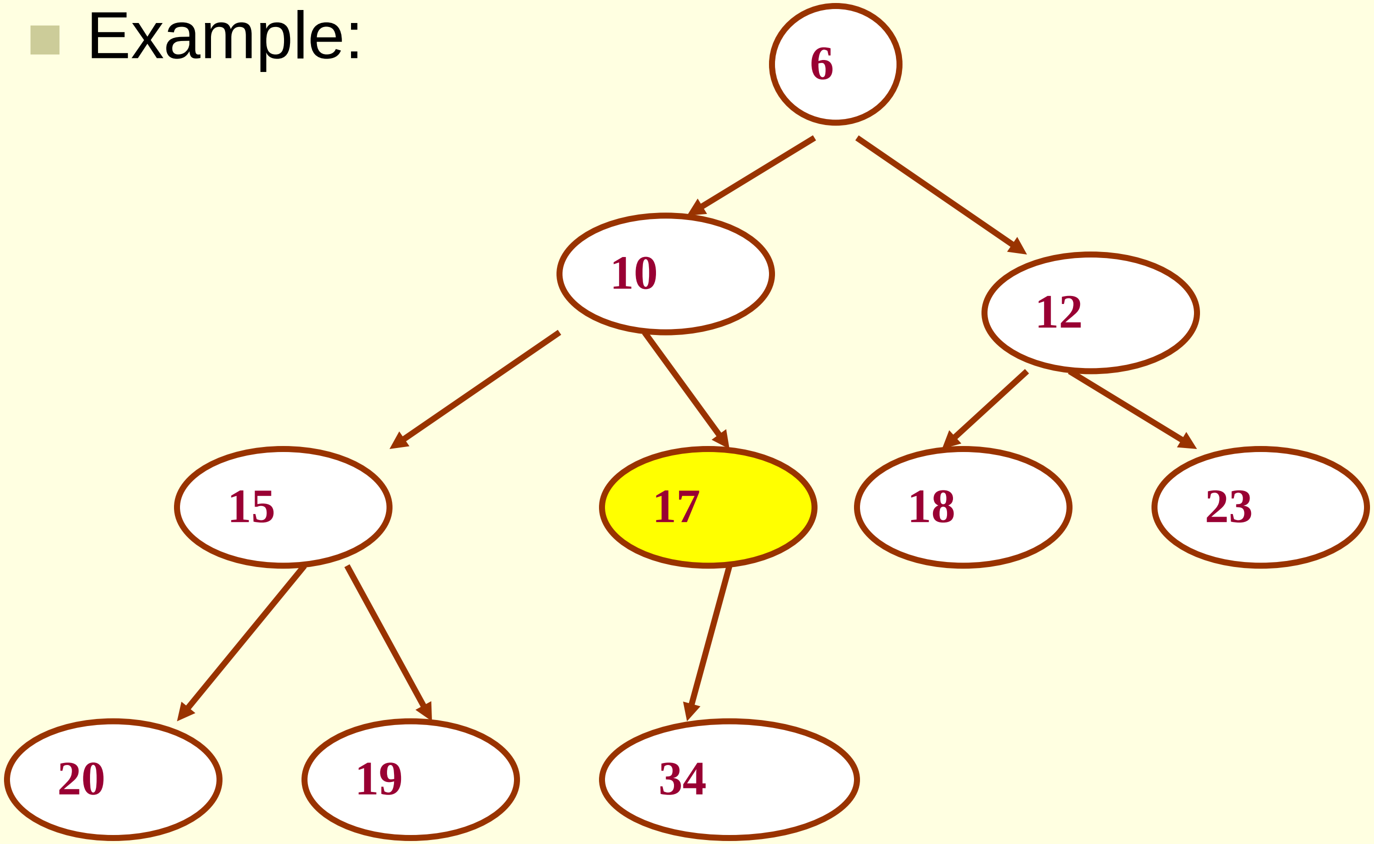
Binary Heaps

- Implementing a Binary Heap
  - Example:



Binary Heaps

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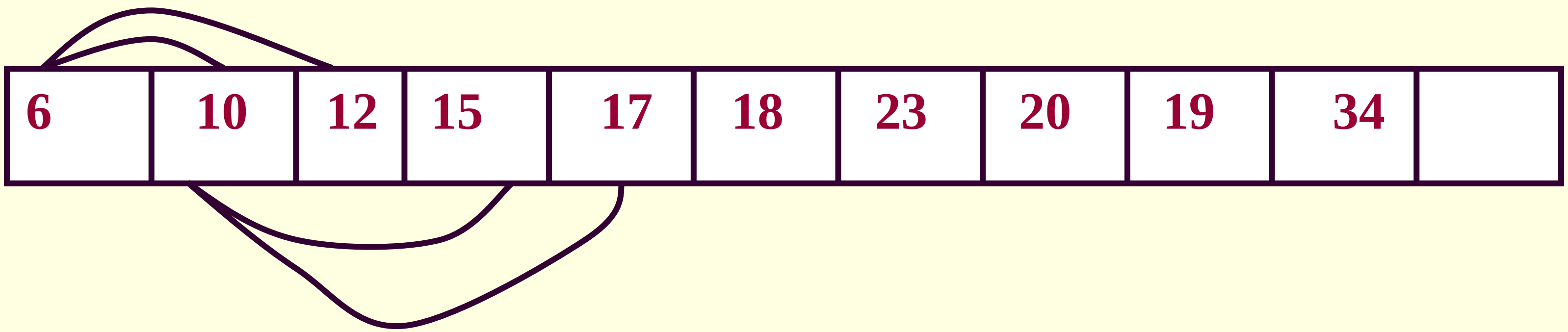
  - Example:



  - <u>Consider node 17:</u>
    - Position in the array: 5
    - It's parent is 10, and is located at position [5/2] = 2
    - 17's left child is node 34, and located at position 5*2 = 10
    - 17 has no right child. Position (2*5 + 1) = 11 (empty)

Binary Heaps

- Heapsort
  - We can use heaps to sort our data
  - Here's the algorithm:
    - Build a heap with all the n items
      - Takes O(n) time (cuz we add to a binary tree and run **Heapify**)
    - Extract the minimum item (if a Min-heap)
      - O(1)
    - Fix the heap after extraction
      - O(logn)
    - Perform this extraction n times for all the elements
    - Store these removed items, sequentially, in an array
    - Running time: O(nlogn)

Binary Heaps

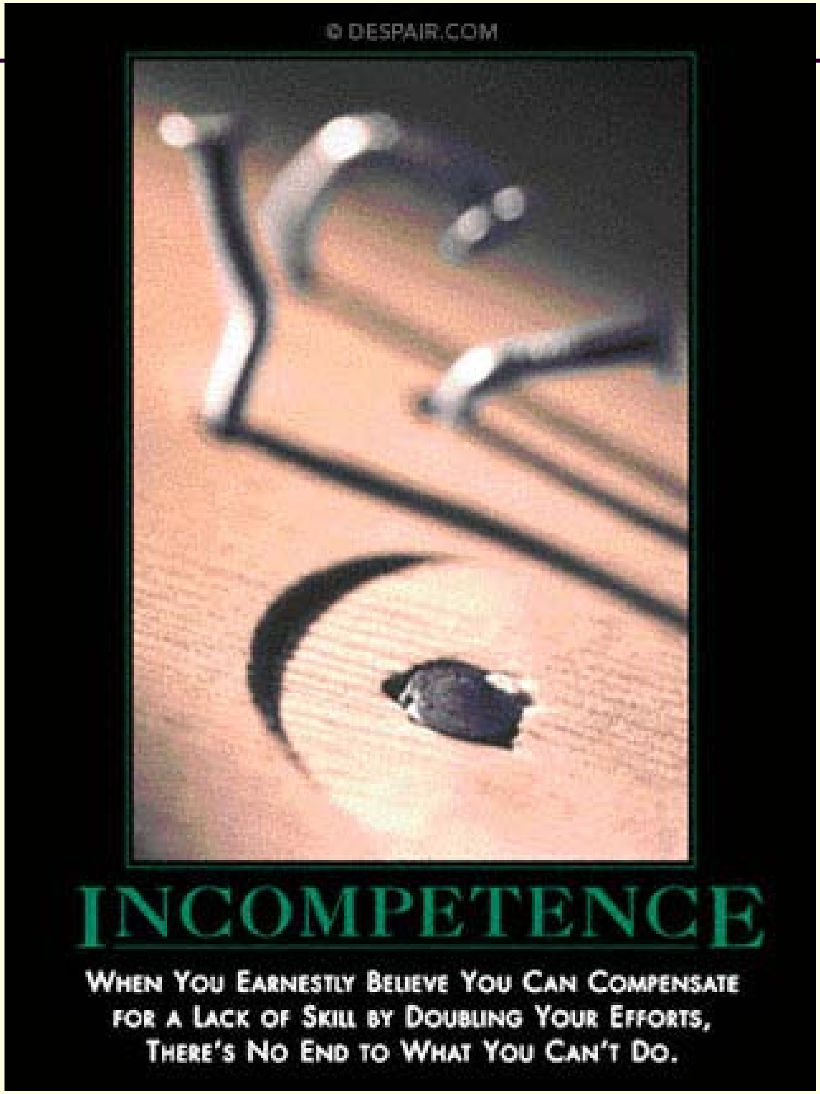
- Summary:
  - A binary heap is a tree that satisfies 2 properties:
    - The Heap Property
      - Max-heap OR Min-heap
    - The Shape Property
      - Must be a complete binary tree
  - To add elements to a heap
    - Place element at next available spot and Percolate Up
  - To remove elements from a heap,
    - Delete root, as it is always the one you want to remove
    - Then copy last element to root's position
    - Finally, Percolate Down

Binary Heaps

- Sumary:
  - The purpose of a heap is essentially to implement a Priority Queue
  - So we use one ADT to implement another ADT
  - And then, at the end of it all, we simply implement the Heap as an array!
    - We know our array is a Heap (a tree) based on how we dereference the array and how we choose to manipulate the data

WASN'T THAT PRODIGIOUS!

Daily Demotivator



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