First Principle of Finite Induction

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1 Introduction

In this paper, we will be doing a step-by-step solution to a common induction problem. Also, it is shown and given in many books in the study of number theory and reasoning and proofs.

2 Formula

We are given the formula

$$\sum_{n=1}^{\infty} n^2 = \frac{n(2n+1)(n+1)}{6}$$

with n being an element of the natural numbers.

3 Proof:

Now we begin by running off a few terms to help see the pattern emerge.

$$\sum_{n=1}^{\infty} 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

Let's assume that $n = 1$.

$$1 = \frac{1(2(1)+1)(1+1)}{6} = 1$$

$$1 = \frac{(2+1)(1+1)}{6} = 1$$

$$1 = \frac{6}{6} = 1$$

Next, we assume that $n = k$ and that k is an element of the natural numbers. An equation we will use in the induction hypothesis is denoted as Equation 1.

$$\sum_{k=1}^{\infty} 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \cdots + k^2 = \frac{k(2k+1)(k+1)}{6}$$