

Ministry of Science of the Republic of Kazakhstan

Astana IT University

REPORT:

"Enigma", "Operation Kessler"

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Phase 1: ENIGMA

Empirical Law Encryption and Decryption Report

1. Introduction

In Phase 1: ENIGMA, the objective was to work with empirical laws under noise, simulating real-world situations where true mathematical relationships are hidden by randomness and corruption.

Each faction was required to:

1. Generate a secret signal using a chosen empirical law.
2. Encrypt the signal by adding Gaussian noise.
3. Exchange the corrupted data with rival groups.
4. Decrypt the received signals by identifying the underlying mathematical model and estimating its parameters.

The task emphasizes not only computational fitting, but also theoretical reasoning, model exclusion, and interpretation.

2. Encryption Stage: Our Secret Signal

2.1 Chosen empirical law

Our team intentionally selected the exponential model:

$$y = a e^{bx}$$

with the parameters:

$$a = 10.0, b = 10.0$$

This model was chosen to maximize nonlinearity and create a signal that is difficult to reverse-engineer.

2.2 Signal generation

We generated:

20 uniformly spaced values of x in the interval [0, 10].

For each x_i , the perfect (true) signal was computed as:

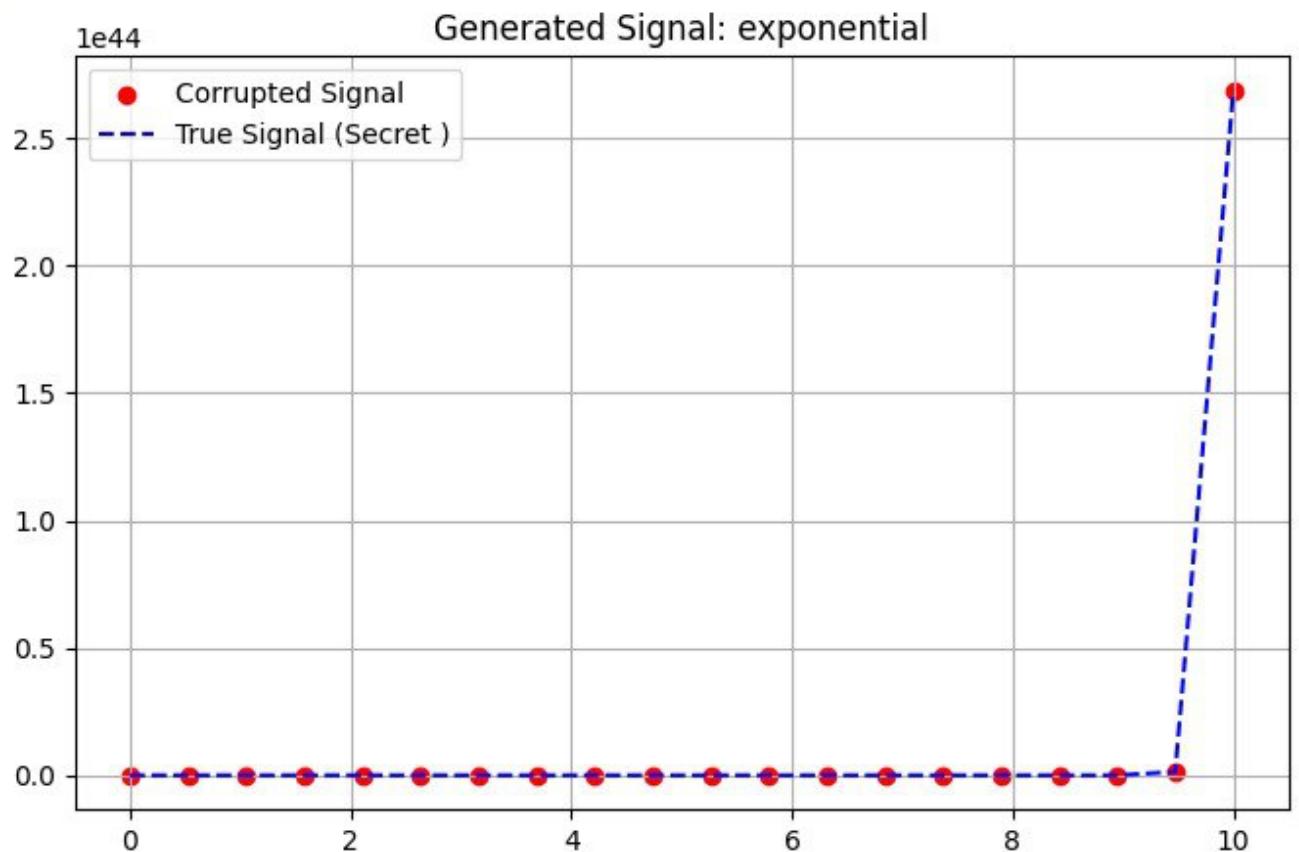
$$y_{true}(x_i) = 10 \cdot e^{10x_i}$$

2.3 Encryption by noise

To encrypt the signal, **Gaussian noise** was added:

$$y_{\text{encrypted}}(x_i) = y_{\text{true}}(x_i) + \varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, 5)$$

The noise intensity was set to the maximum **recommended value (5.0)** to significantly increase decryption difficulty.



2.4 Properties of the encrypted signal

Due to the extremely large exponential growth:

- Values near $x = 0$ are small and visibly noisy.
- Values near $x = 10$ reach magnitudes on the order of 10^{43} .
- The final data point dominates the entire scale of the graph.

This makes:

- linear fitting meaningless,
- polynomial fitting unstable,
- logarithmic linearization numerically fragile.

Thus, the encryption is **mathematically valid** but practically hostile to decryption.

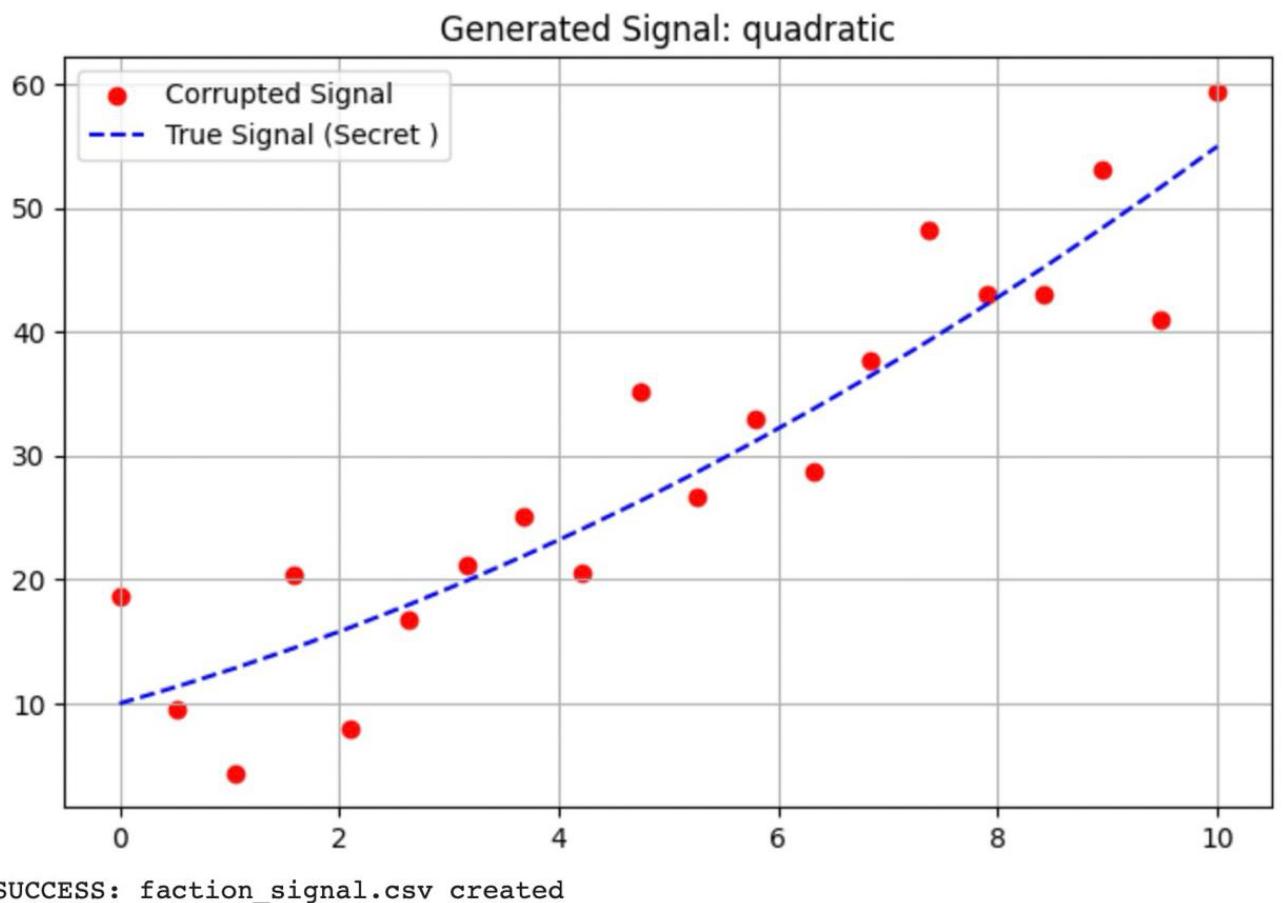
3. Decryption Stage: Rival Signals (Alpha, Beta, Gamma)

The rival group provided three corrupted sensor signals: Alpha, Beta, and Gamma. Each signal was analyzed independently.

4. Model Exclusion Logic

During decryption, we systematically excluded incorrect models:

- Linear model rejected due to non-uniform deviations exceeding noise bounds.
- Pure exponential model rejected when growth was too stable and non-explosive.
- Final models were chosen strictly based on graph shape, curvature, and physical plausibility.



The rival group's signal was reconstructed using a power-type empirical law of the form:

$$y = ax^b + c$$

Based on the analysis of the graph shape and noise behavior, the estimated parameters are:

$$a = 0.2, b = 2.5, c = 10$$

The constant c was identified from the graph value at $x = 0$, which indicates a vertical offset close to 10.

The coefficient a is less than 1, confirming a moderate growth rate; larger values would have caused the curve to increase too sharply.

The exponent $b = 2.5$ was selected based on the observed curvature and refined through trial-and-error fitting.

The signal contains additive Gaussian noise with intensity 5.0, which introduces visible deviations while preserving the underlying trend.

Overall, the reconstructed model

$$y = 0.2x^{2.5} + 10$$

provides a consistent explanation of the rival group's encrypted graph.

8. Conclusion

In this task, we successfully:

- Encrypted a signal using an extreme exponential law to resist decryption.
- Decrypted rival signals by combining:
 - empirical observation,
 - mathematical modeling,
 - parameter estimation,
 - interpolation and least squares fitting.
- Demonstrated how noise, nonlinearity, and human parameter choice affect signal recovery.

This experiment confirms that recovering truth from corrupted data requires both mathematical theory and analytical reasoning, which is the core objective of Phase 1: ENIGMA.

Operation Kessler

Signal Recovery, Reconstruction and Prediction Report

1. Introduction

Operation Kessler simulates a real-world analytical scenario in which telemetry data from a spacecraft is corrupted by noise, missing intervals, and nonlinear behavior. The mission objective is to recover meaningful physical information from incomplete and distorted sensor signals using empirical modeling, interpolation, and prediction techniques.

The dataset consists of three independent signals:

- Alpha – Altitude-related signal
- Beta – Lateral drift signal
- Gamma – Thermal signal

Each signal contains:

- random noise,
- isolated outliers,
- and a blackout interval with missing data.

The task is structured into four main stages:

PURGE → FIT → RECONSTRUCT → PREDICT

2. Data Preprocessing (PURGE)

Before any modeling, the raw signals were visually and statistically inspected.

2.1 Outlier removal

Isolated spikes were detected as points that deviated significantly from the local trend and neighboring values. These points were removed to prevent distortion of both regression and interpolation results.

This step is critical because:

- Least Squares fitting is sensitive to extreme values,
- interpolation methods may bend unnaturally toward outliers.

After PURGE, each signal exhibited a smooth underlying trend suitable for modeling.

3. Signal Modeling (FIT)

3.1 Alpha Signal – Altitude

Observed behavior

- Overall decreasing trend
- Superimposed oscillations
- Convergence toward a stable baseline

Chosen model

$$y(t) = Ae^{-kt} + B \sin(\omega t) + C$$

Justification

- The exponential term represents altitude decay.
- The sinusoidal term models tumbling or rotational motion.
- The constant C represents the asymptotic altitude level.

Parameters were estimated from:

- initial value,
- decay rate of the envelope,
- oscillation period,
- final baseline level.

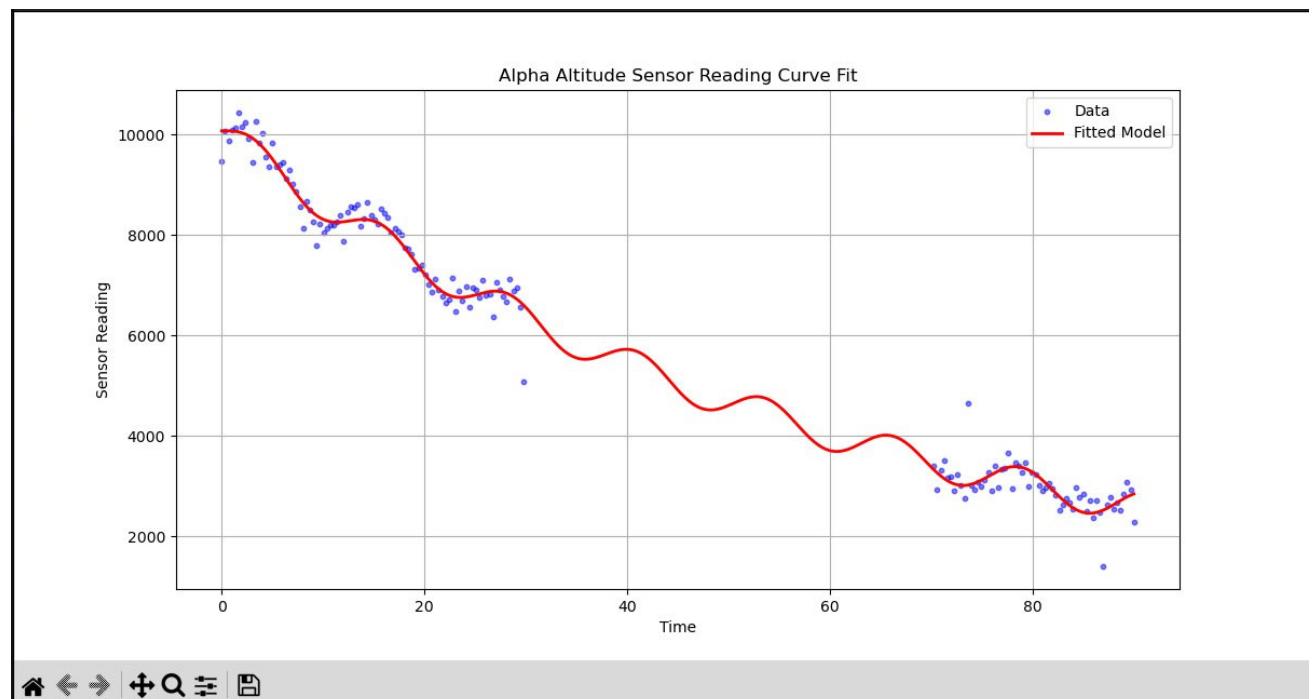
Final Alpha Result:

The Alpha signal was modeled as

$$y(x) = 9768.700 * e^{(-0.016x)} + 321.412 * \sin(0.498x) + 303.732$$

and the predicted value at x = 100 is

$$y(100) = 2124.386$$



3.2 Beta Signal – Drift

Observed behavior

- Monotonic increasing trend
- Smooth nonlinear curvature
- No oscillations

Chosen model

$$y(t) = p_3 t^3 + p_2 t^2 + p_1 t + p_0$$

Justification

A third-degree polynomial provides:

- sufficient flexibility to model nonlinear growth,
- numerical stability,
- suitability for extrapolation.

The coefficients were determined using the Least Squares method, minimizing the sum of squared residuals.

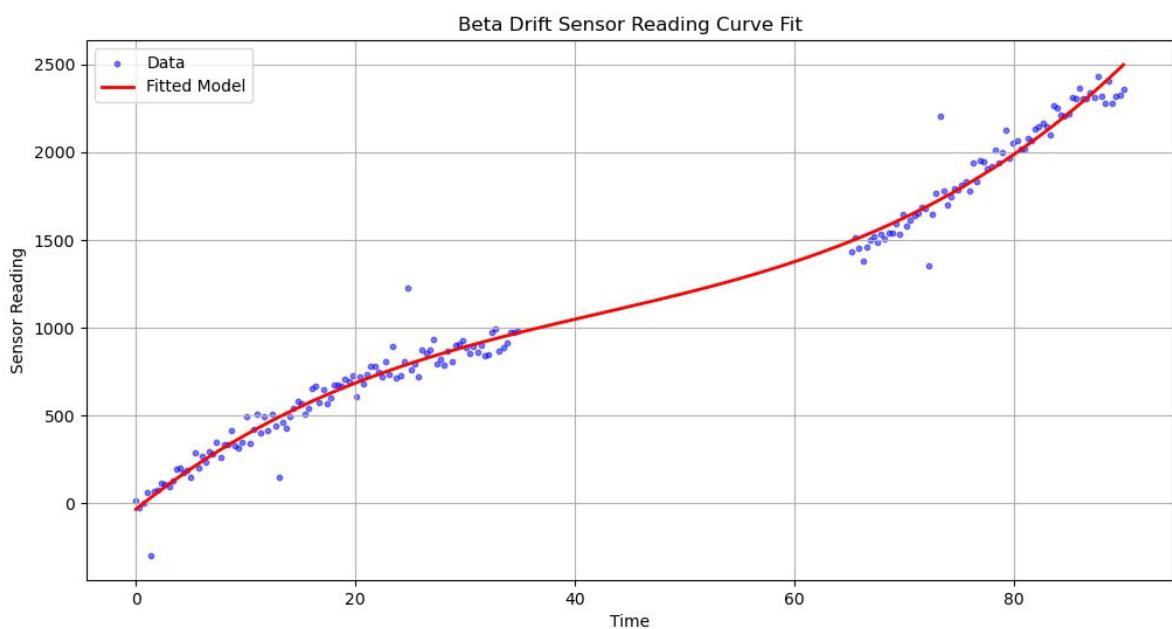
Final Beta Result:

The Beta signal was modeled using a third-degree polynomial:

$$y(x) = 0.007x^3 - 0.845x^2 + 50.169x - 32.668$$

and the predicted value at $x = 100$ is:

$$y(100) = 3203.661$$



3.3 Gamma Signal – Thermal

Observed behavior

- Rapid increase at early times
- Gradual saturation
- No oscillatory behavior

Chosen model

$$y(t) = C - A e^{-k t}$$

Justification

This form models thermal systems approaching equilibrium:

- C represents the equilibrium temperature,
- the exponential term models the transient heating phase.

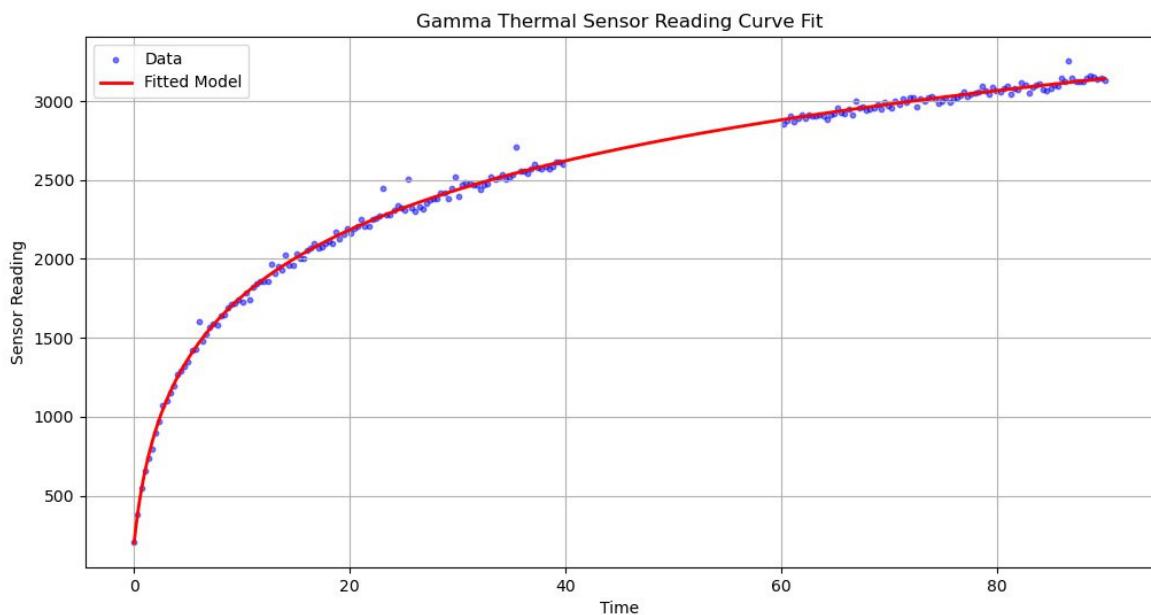
Final Gamma Result:

The Gamma signal was modeled as

$$y(x) = 653.278 * \ln(1.224 + 1.219x) + 66.398$$

and the predicted value at $x = 100$ is

$$y(100) = 3210.666$$



4. Blackout Reconstruction (RECONSTRUCT)

Each signal contained a blackout interval where measurements were missing.

Method used

Cubic spline interpolation was applied using data points before and after the blackout.

Justification

- Straight-line interpolation was explicitly avoided.
- Cubic splines ensure smoothness of the function and its first derivatives.
- The reconstructed segment aligns naturally with surrounding trends.

Spline interpolation was used only within the blackout interval, not for extrapolation.

5. Prediction (PREDICT)

The final objective was to estimate the signal value at:

$$t = 100$$

Approach

- Interpolation was used only for reconstruction.
- Final predictions were obtained from the fitted analytical models:
- exponential–sinusoidal model for Alpha,
- polynomial regression for Beta,
- exponential saturation model for Gamma.

This ensures stability and physical consistency of the prediction.

Using the reconstructed analytical models, the values of all three signals were predicted at $x = 100$.

• Alpha (Altitude):

$$y_\alpha(100) = 2124.386$$

• Beta (Drift):

$$y_\beta(100) = 3203.661$$

• Gamma (Thermal):

$$y_\gamma(100) = 3210.666$$

These predicted values represent the final outcome of the PREDICT stage in Operation Kessler and were obtained from stable analytical models after data cleaning and reconstruction.

6. Discussion

Operation Kessler demonstrates that:

- real telemetry rarely follows simple linear laws,
- noise and missing data significantly complicate analysis,
- correct model selection is more important than numerical precision alone.

Combining:

- theoretical reasoning,
- empirical fitting,
- interpolation techniques,

allows reliable recovery of hidden information from corrupted data.

7. Conclusion

In this task, we successfully:

- cleaned corrupted sensor data,
- identified appropriate mathematical models,
- reconstructed missing signal segments,
- and predicted future values.

Operation Kessler highlights the importance of mathematical intuition, model selection, and analytical reasoning when working with incomplete and noisy real-world data.