stream aquarium for ecological studies. U.S. Dep. Agric. For. Serv. Res. Note PNW-403. Pacific Northwest Forest and Range Experiment Station, Portland, OR. 8 p.

REISER, D. W., AND T. C. BJORNN. 1979. Habitat requirements of anadromous salmonids. U.S. Dep. Agric. For. Serv. Gen. Tech. Rep. PNW-96. Pacific Northwest Forest and Range Experiment Station, Portland, OR. 54 p.

SAVINO, J. F., AND R. A. STEIN. 1982. Predator–prey interactions between largemouth bass and bluegills as influenced by simulated, submersed vegetation. Trans. Am. Fish. Soc. 111: 255–266.

SLANEY, P. A., AND T. G. NORTHCOTE. 1974. Effects of prey abundance on density and territorial behavior of young rainbow trout (*Salmo gairdneri*) in laboratory channels. J. Fish. Res. Board Can. 31: 1201–1209.

SYMONS, P. E. K. 1971. Behavioral adjustment of population density to available food by juvenile Atlantic salmon. J. Anim. Ecol. 40: 569–587.

WARE, D. M. 1973. Risk of epibenthic prey to predation by rainbow trout (*Salmo gairdneri*). J. Fish. Res. Board Can. 30: 787–797.

WERNER, E. E., J. F. GILLIAM, D. J. HALL, AND G. G. MITTELBACH. 1983. An experimental test of the effects of predation risk on habitat use in fish. Ecology 64: 1540–1548.

WILZBACH, M. A. 1984. Prey availability overrides cover in determining growth and abundance of stream-dwelling cutthroat trout. Ph.D. thesis, Oregon State University, Corvallis, OR. 105 p.

WILZBACH, M. A., AND J. D. HALL. 1985. Prey availability and foraging behavior of cutthroat trout in an open and forested section of stream. Verh. Int. Ver. Limnol. 22: 2516–2522.

Quadratic Discriminant Analysis with Covariance for Stock Delineation and Population Differentiation: A Study of Beaked Redfishes (*Sebastes mentella* and *S. fasciatus*)

R. K. Misra

Fisheries Research Branch, Department of Fisheries and Oceans, P.O. Box 550, Halifax, N.S. B3J 2S7

Misra, R. K. 1985. Quadratic discriminant analysis with covariance for stock delineation and population differentiation: a study of beaked redfishes (*Sebastes mentella* and *S. fasciatus*). Can. J. Fish. Aquat. Sci. 42: 1672–1676.

Stock delineation is of vital importance in fisheries management programs. Linear discriminant function (LDF) has been employed extensively in population differentiation studies but is of severely restricted usefulness when populations differ in their dispersion matrices. Quadratic discriminant function (QDF) is the appropriate analysis to employ in these situations. Here, I analyzed morphometric data of beaked redfishes (*Sebastes mentella* and *S. fasciatus*) by a recently developed conditional QDF.

L'identification des stocks revêt une importance vitale pour les programmes de gestion halieutique. Les études d'identification de populations font souvent appel à une fonction discriminante linéaire, mais l'utilité de cette dernière est grandement restreinte quand les matrices de dispersion des populations varient. Dans ces cas, l'analyse appropriée à utiliser est une fonction discriminante quadratique. L'auteur analyse des données morphométriques sur le sébaste (*Sebastes mentella* et *S. fasciatus*) à l'aide d'une fonction discriminante quadratique conditionnelle récemment mise au point.

Received January 10, 1985
Accepted July 2, 1985
(J8067)

Reçu le 10 janvier 1985
Accepté le 2 juillet 1985

Stock delineation is of vital importance in fisheries management programs. Effective management of a species depends on the reliability of our knowledge of distribution and biological characteristics of individual stocks. The distinction between deepwater redfish (*Sebastes mentella*) and Labrador redfish (*S. fasciatus*), although vital for the effective management of redfish resources in the Northwest Atlantic, has been less than clear for decades (Misra and Ni 1983; Ni 1981a, 1981b). Macdonald and Pitcher (1979) developed a method of analyzing distribution mixtures, at the univariate level, and applied it to estimate age-group parameters from size–frequency data. Morphometric measurements have frequently been used to differentiate populations in general (see e.g. Blackith and Reyment 1971; Ihssen et al. 1981; Misra and Ni 1983) and stocks of fish in particular (see e.g. Almeida 1982; Casselman et al. 1981; Davidson et al. 1982; Ihssen et al. 1981; Sharp et al. 1978). In a stock discrimination analysis of capelin in the Gulf of St. Lawrence, Sharp et al. (1978) noted that morphometrics offered greater potential in separating capelin (*Mallotus villosus*) stocks than meristics.

Fisher's discriminant function has been employed extensively, by the most conservative standards, in behavioral, biological, business, fisheries, medical, and social research (Goldstein and Dillon 1978). "Even when two similar species can be identified with a single measurement, a combined criterion of two or more may increase the separation between them" (Bliss 1970). The

TABLE 1. Means and ranges (mm) of eight morphometric characters Y_i, $i = 1, \ldots, 8$, for *S. mentella* ($n = 99$) and *S. fasciatus* ($n = 99$). All measurements were transformed to common logarithms.

Y_i	Character description	*S. mentella* Mean	*S. mentella* Range	*S. fasciatus* Mean	*S. fasciatus* Range
1	Head length	2.0021	1.8325–2.1772	1.8211	1.7634–1.9685
2	Preanal length	2.2471	2.0846–2.4190	2.0991	2.0158–2.2480
3	Pectoral fin base	1.3462	1.1790–1.5198	1.2184	1.1335–1.3856
4	Anal fin base	1.6122	1.4330–1.7528	1.4438	1.3522–1.6107
5	Length of longest pelvic ray	1.6730	1.4914–1.8156	1.5486	1.4265–1.6776
6	Length of longest pectoral ray	1.8386	1.6484–1.9845	1.6712	1.6085–1.8312
7	Dorsal length of caudal peduncle	1.5957	1.3324–1.7679	1.4351	1.3263–1.5888
8	Standard length	2.4243	2.2742–2.5763	2.2725	2.2041–2.4133

following are noted:

(1) Discriminant functions are employed when the existence of reference samples can be assumed on the basis of an external criterion (Kendall and Stuart 1976).

(2) Discriminant analysis of morphometric data done in the past has generally been unsatisfactory due to sampling bias associated with the varying size of individuals and the large overlapping of measurements. These difficulties are almost invariably encountered in a stock delineation study. To overcome these, Misra and Ni (1983) employed linear discriminant function (LDF) with covariance to discriminate between *S. mentella* and *S. fasciatus*. Individuals vary within populations, as in standard length, to warrant correction as in a discriminant function which is adjusted by covariance (Bliss 1970).

(3) Morphometric measurements are particularly appropriate in a discriminant analysis, as these are taken on "continuous" variables and multivariate normality is closely approximated by their logarithms (Bliss 1967; Misra and Ni 1983; Pimentel 1979). Performance of LDF in non-normal situations is very misleading (Dillon 1979; Lachenbruch et al. 1973).

LDF is valid when the two populations are multivariate normal with different mean vectors of variates, but identical dispersion (or variance–covariance) matrices (Kendall and Stuart 1976). When two populations have unequal dispersions, discriminant score for each population will have quadratic terms in the variates. LDF then is of severely restricted usefulness and must, indeed, be replaced by quadratic discriminant functions (QDF) (Kendall and Stuart 1976).

Rawlings et al. (1985) examined the two-group discriminant analysis problem where each observation vector, consisting of the discriminators and covariates, comes from one of the two multivariate normal populations with unequal dispersion matrices. As inequality of group dispersions may be of a common occurrence as in the redfish data of Misra and Ni (1983), the method of Rawlings et al. (1985) will be very useful in population differentiation studies. My objectives were (1) to present the method of Rawlings et al. (1985) briefly, after reorganizing it following Li (1964) and Rao (1973), and (2) to reanalyze the redfish data of Misra and Ni (1983) by this new method.

Materials and Methods

As this information is provided in detail in Misra and Ni (1983) and Ni (1981a, 1981b), it is given here only briefly. Prior to doing the LDF analysis of Misra and Ni (1983), specimens of *S. mentella* and *S. fasciatus* were separated into two groups on the basis of the extrinsic gas bladder musculature. Identification by extrinsic gas bladder musculature requires the tedious and technical work of dissecting a fish. Use of extrinsic gas bladder musculature as a direct measure of misclassification is, therefore, not feasible where several (hundreds) fish need to be identified. Twelve morphometric characters (listed in table 1 of Misra and Ni 1983) were measured on an individual specimen. All measurements were transformed to (common) logarithms for reasons explained earlier in the text. Misra and Ni (1983) noted that parsimony in the number of variates to be included in a discriminant analysis is desirable when sample sizes are small. This is, however, not necessary when analyzing large samples (as in this study) although omitting variables with nonsignificant discriminant coefficients will increase the stability of the discriminant function (Bliss 1970). Misra and Ni (1983) demonstrated that for their redfish data only seven discriminators were effective. Their LDF was therefore based on eight characters, i.e. the seven discriminators and the covariate, standard length. Those characters are listed in Table 1.

For their LDF, Misra and Ni (1983) employed only "complete" specimens, i.e. specimens for which all eight measurements were available because "missing observations virtually destroy morphometrics" (Pimentel 1979). Each sample had 99 complete specimens. Table 1 gives means and ranges of these characters Y_i, $i = 1, \ldots, 8$. Based on eight characters, there were 99 complete specimens in *S. mentella*, as opposed to 97 in table 1 of Misra and Ni (1983) which were identified on the basis of 12 characters. Consequently, some of the mean values shown in Table 1 here do not match with those shown in table 1 of Misra and Ni (1983).

Let (1) n_i, $i = 1, 2$, be the size of the sample drawn from population P_i and (2) $\boldsymbol{Y}_i$ be the $p \times 1$ vector of observations on individual k, $k = 1, \ldots, n_i$, from P_i. For the redfish study of Misra and Ni (1983), $p = 8$. For the p-variate normal distribution of the observation vector from P_i, let the mean vector be denoted as $\boldsymbol{\mu}_i$ and the dispersion matrix as $\boldsymbol{A}_i$. The discriminant score for the ith population is

$$S_i = -\tfrac{1}{2}\ln|\boldsymbol{A}_i| - \tfrac{1}{2}(\boldsymbol{Y} - \boldsymbol{\mu}_i)'\boldsymbol{A}_i^{-1}(\boldsymbol{Y} - \boldsymbol{\mu}_i) + \ln \pi_i$$

where π_i is the prior probability for the ith population. In a discriminant analysis when the prior probabilities are not known and population parameters $\boldsymbol{\mu}_i$ and $\boldsymbol{A}_i$ are substituted by their sample values, the prior probabilities are taken to be proportional to observed numbers of the two species in the sample that was classified on the basis of extrinsic gas bladder musculature (Rao 1973). The conditional QDF of Rawlings et al. (1985) leads to a classification rule that is used to discriminate between two populations by computing the difference $S_1 - S_2$, after adjusting for differences in the covariate.

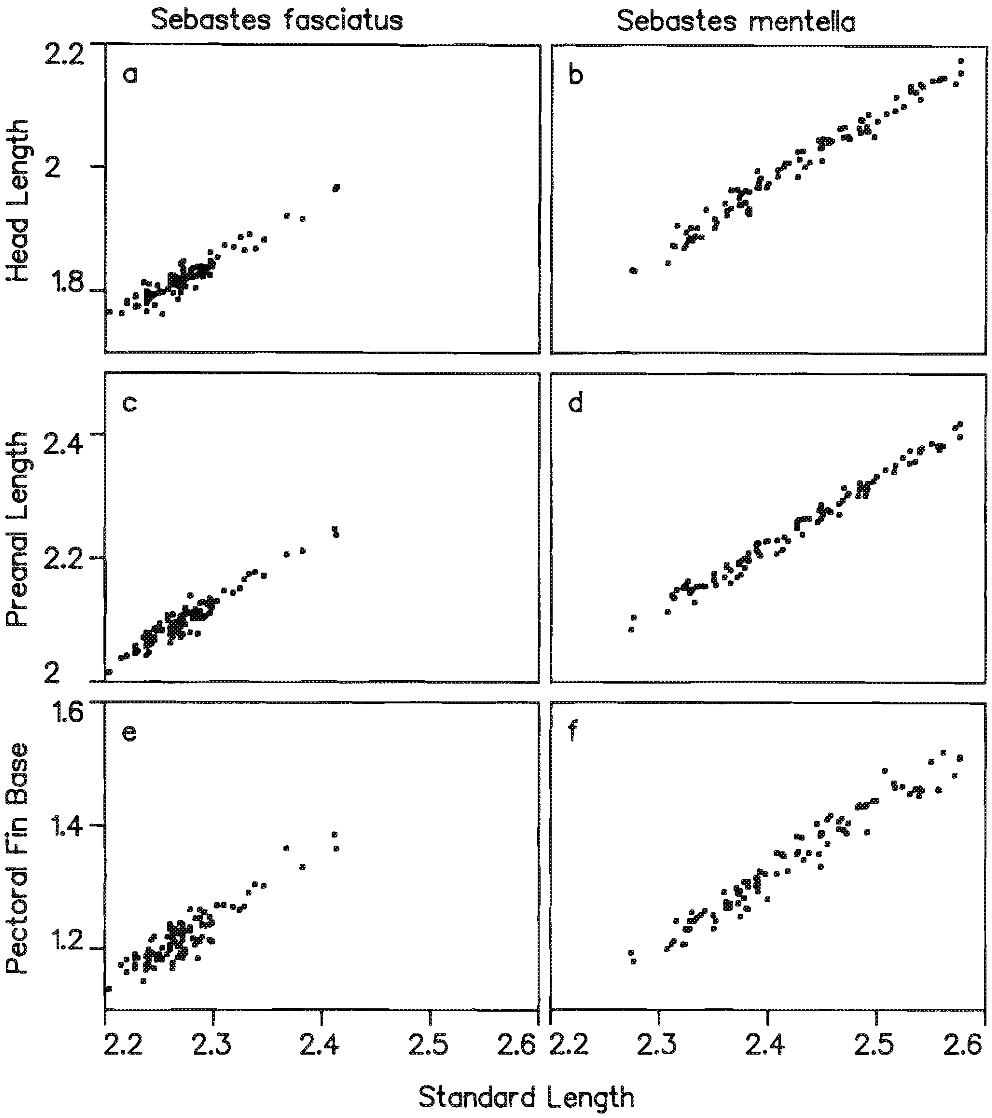

FIG. 1. Bivariate scatter plots of the covariate and the variates to compare dispersions for *S. fasciatus* and *S. mentella*. All measurements were transformed to common logarithms of millimetres.

Li (1964) stated that (1) adjustment may be done at any meaningful value of the covariate, Y_8 (Table 1), and (2) the value most frequently chosen would be the overall mean of Y_8. For convenience of expressing the conditional QDF in a pleasant form, random vector of discriminators Y_i, $i = 1, \ldots, 7$, is denoted as $\boldsymbol{X}$ and the covariate Y_8 as $\boldsymbol{Z}$. Also, mean vector $\boldsymbol{\mu}_i$ is expressed as $(\mu'_{x_i}, \mu_{z_i})'$ and the dispersion matrix $\boldsymbol{A}_i$ as

$$\begin{bmatrix} A_{xx_i} & A_{xz_i} \\ A_{zx_i} & A_{zz_i} \end{bmatrix}.$$

The conditional QDF is then given as follows:

$$\text{QDF} = \tfrac{1}{2} \ln (|\boldsymbol{B}_2|/|\boldsymbol{B}_1|) - \tfrac{1}{2}(X - \alpha_1)'\boldsymbol{B}_1^{-1}(X - \alpha_1) + \tfrac{1}{2}(X - \alpha_2)'\boldsymbol{B}_2^{-1}(X - \alpha_2)$$

where $\boldsymbol{B}_i = A_{xx_i} - A_{xz_i} \cdot A_{zz_i}^{-1} \cdot A_{zx_i}$ and $\boldsymbol{\alpha}_i = \mu_{x_i} - A_{xz_i} \cdot A_{zz_i}^{-1}(z - \mu_{z_i})$ for $i = 1, 2$ are conditional dispersion matrices and covariance adjusted vector of means, respectively. An individual is assigned to that population for which the QDF score is larger, i.e. assign an individual to P_1 when conditional QDF > $\ln (\pi_2|\pi_1)$ and to P_2 otherwise.

Rawlings et al. (1985) employed the leave-one-out estimator (also see Kshirsagar 1972) for the actual error rate for the sample based conditional QDF. First, observations are removed, one at a time. Then the conditional QDF is estimated based on the remaining observations. This is followed by a record of whether it is misclassified. The estimated expected actual error rate is calculated as

$$\sum_{i=1}^{2} (\pi_i f_i / n_i)$$

where f_i is the number of misclassified observations for the ith sample.

Analysis and Results

Specimens of *S. fasciatus* were generally smaller than those of *S. mentella* and the range of morphometric characters overlapped broadly between groups (Table 1). In their analysis of the redfish data, Misra and Ni (1983) employed the LDF where

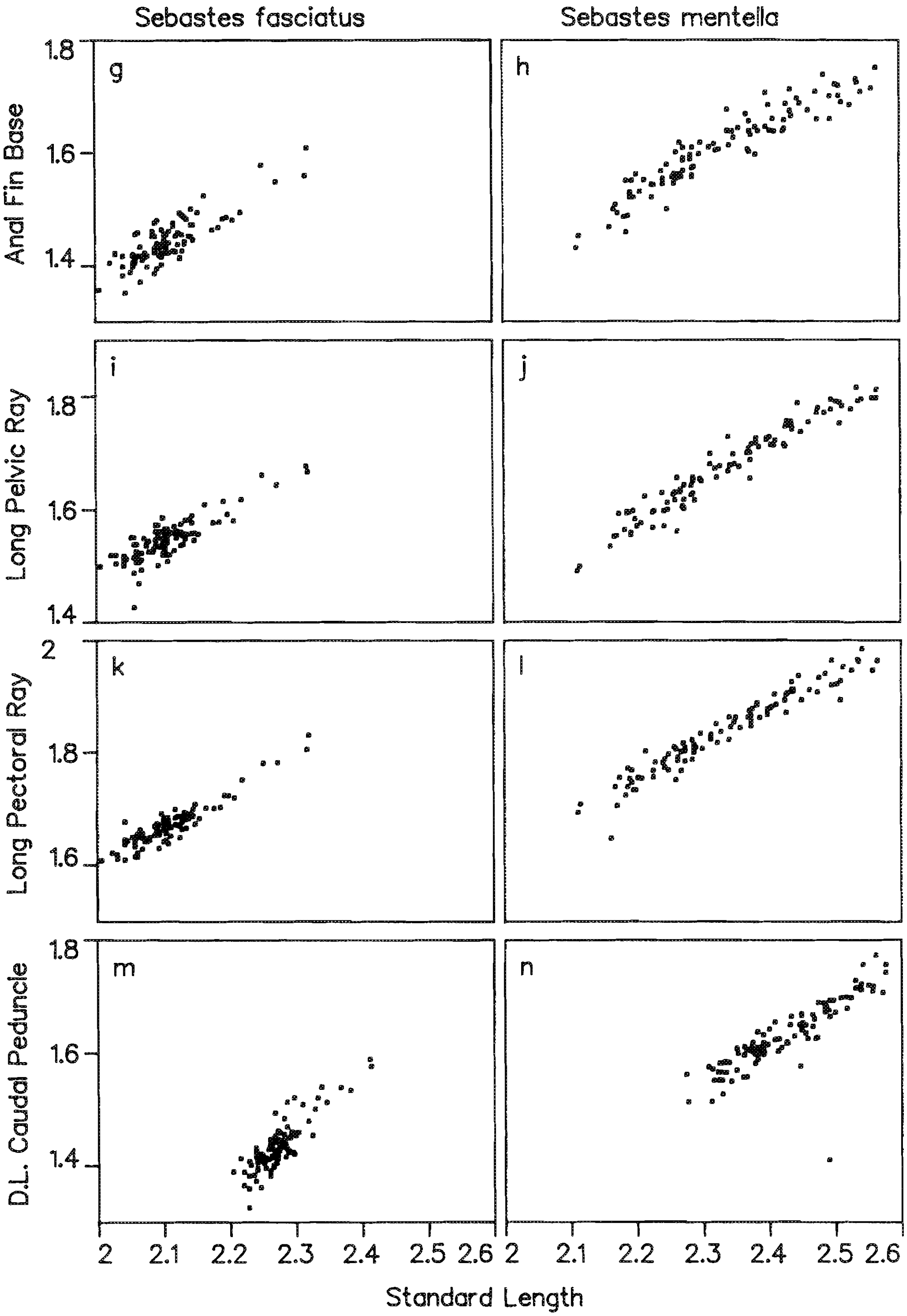

FIG. 1. (*Concluded*)

vectors of means of discriminators were adjusted for inequalities in standard length. They also noted that large overlaps of ranges (46% in *S. mentella* and 67% in *S. fasciatus*) of Y_8 or Z and its inadequacy to discriminate between the two species effectively provided additional support for using standard length as a covariate.

Figure 1 shows the bivariate scatter plots of the covariate (abscissa) and each of the variates (ordinate) Y_i, $i = 1, \ldots 7$, for *S. fasciatus* and *S. mentella*. It is recalled that the two samples here had exactly the same number (viz. 99) of observations. To further facilitate visual comparison of the two dispersions, I employed same scales of measurement for the two samples in each plot.

Unequal group dispersions are obvious in the figure; the ellipse (of scatter of points) for *S. mentella* is elongated and thinner whereas that for *S. fasciatus* is shorter and wider. These observations were confirmed, statistically, as follows: null hypothesis of equality of the conditional dispersion matrices B_i, $i = 1, 2$, of discriminators was tested by the modified generalized likelihood-ratio statistic explained (e.g. in Morrison 1976). This null hypothesis was rejected ($p \leq 0.0001$), thus indicating the following: (1) The assumption of the equality of dispersion matrices underlying the use of the LDF with covariance by Misra and Ni (1983) was not justified. (2) Conditional QDF, and not the LDF with covariance that Misra and Ni (1983) employed, would provide the appropriate discriminant analysis for the redfish data. Analysis of the redfish data by the conditional QDF was therefore done following the procedure explained earlier in the text. This analysis was followed by an estimate of the actual error rate for the sample based conditional QDF. The leave-one-out estimator identified a total of 24 misclassified individuals, which included 20 *S. mentella* and 4 *S. fasciatus*, from 198 specimens. The estimated expected error rate was thus 12% thus indicating that classification by the conditional QDF had been quite useful.

Acknowledgements

I thank the Director, Fisheries Research Branch, Department of Fisheries and Oceans, St. John's, Nfld., for his permission to use the redfish data in this presentation. Figures were drawn by Mr. G. M. Blake.

References

ALMEIDA, F. P. 1982. An analysis of the stock structure of silver hake, *Merluccius bilinearis*, in NAFO subareas 5 and 6. NAFO SCR Doc. 82/IX/81, Ser. No. N587, 36 p.

BLACKITH, R. E., AND R. A. REYMENT. 1971. Multivariate morphometrics. Academic Press, London and New York. 412 p.

BLISS, C. I. 1967. Statistics in biology. Vol. 1. McGraw-Hill Book Company, New York, NY. 558 p.

1970. Statistics in biology. Vol. 2. McGraw-Hill Book Company, New York, NY. 639 p.

CASSELMAN, J. M., J. J. COLLINS, E. J. CROSSMAN, P. C. IHSSEN, AND G. R. SPANGLER. 1981. Lake whitefish (*Coregonus clupeaformis*) stocks of the Ontario waters of Lake Huron. Can. J. Fish. Aquat. Sci. 38: 1771–1789.

DAVIDSON, K., R. W. ELNER, AND J. ROFF. 1982. Discrimination of Atlantic snow crab, *Chionoecetes opilio*, populations: a problem of management applications. NAFO SCR Doc. 82/IX/86, Ser. No. N595, 13 p.

DILLON, W. R. 1979. The performance of the linear discriminant function in non-optimal situations and the estimation of classification error rates; a review of recent findings. J. Mark. Res. XVI: 370-381.

GOLDSTEIN, M., AND W. R. DILLON. 1978. Discrete discriminant analysis. John Wiley & Sons, New York, NY. 186 p.

IHSSEN, P. E., H. E. BODRA, J. M. CASSELMAN, J. M. MCGLADE, N. R. PAYNE, AND F. M. ULTER. 1981. Stock identification: materials and methods. Can. J. Fish. Aquat. Sci. 38: 1838–1855.

KENDALL, M. G., AND A. STUART. 1976. The advanced theory of statistics. Vol. 3. Hafner Publishing Company, New York, NY. 585 p.

KSHIRSAGAR, A. M. 1972. Multivariate analysis. Marcel Dekker, Inc., New York, NY. 534 p.

LACHENBRUCH, P. A., C. SNEERINGER, AND L. T. REVO. 1973. Robustness of the linear and quadratic discriminant functions to certain types of non-normality. Commun. Stat. 1: 39–56.

LI, J. C. R. 1964. Statistical inference. Vol. 1. Edwards Brothers, Inc., Ann Arbor, MI. 658 p.

MACDONALD, P. D. M., AND T. J. PITCHER. 1979. Age-groups from size-frequency data: a versatile and efficient method of analyzing distribution mixtures. J. Fish. Res. Board Can. 36: 987–1001.

MISRA, R. K., AND I-H. NI. 1983. Distinguishing beaked redfishes (deepwater redfish, *Sebastes mentella*, and Labrador redfish, *S. fasciatus*) by discriminant analysis (with covariance) and multivariate analysis of covariance. Can. J. Fish. Aquat. Sci. 40: 1507–1511.

MORRISON, D. F. 1976. Multivariate statistical methods. McGraw-Hill Book Company, New York, NY. 415 p.

NI, I-H. 1981a. Separation of sharp-beaked redfishes, *Sebastes fasciatus* and *S. mentella*, from northeastern Grand Bank by morphology of extrinsic musculature. J. Northwest Atl. Fish. Sci. 2: 7–12.

1981b. Numerical classification of sharp-beaked redfishes, *Sebastes mentella* and *S. fasciatus*, from northeastern Grand Bank. Can. J. Fish. Aquat. Sci. 38: 873–879.

PIMENTEL, R. A. 1979. Morphometrics. Kendall/Hunt Publishing Company, Dubuque, IA. 276 p.

RAO, C. R. 1973. Linear statistical inference and its applications. John Wiley & Sons, New York, NY. 625 p.

RAWLINGS, R. R., B. I. GRAUBARD, S. TEPER, R. S. RYBACK, AND M. J. ECKARDT. 1985. Conditional quadratic discrimination in the identification of biological markers for disease screening. Biom. J. (In press)

SHARP, J. C., K. W. ABLE, W. C. LEGGETT, AND J. E. CARSCADDEN. 1978. Utility of meristics and morphometric characters for identification of capelin (*Mallotus villosus*) stocks in Canadian Atlantic waters. Can. J. Fish. Aquat. Sci. 35: 124–130.

Short-term Effect on the Metabolism of Lotic Benthic Communities Following Experimental Acidification

Martine Allard and Guy Moreau[1]

Département de biologie, Université Laval, Québec, Qué. G1K 7P4

Allard, M., and G. Moreau. 1985. Short-term effect on the metabolism of lotic benthic communities following experimental acidification. Can. J. Fish. Aquat. Sci. 42: 1676–1680.

Short-term quantitative change in whole benthic community metabolism was observed at low pH. From the beginning until 38 d after acidification (with one exception), oxygen uptake was significantly lower in the acidified channels (pH 4.0, pH 4.0 + 0.4 mg Al $\cdot$ L^{-1}) than in the control (pH 6.3–6.9, 0.05–0.13 mg Al $\cdot$ L^{-1}). By the midpoint of acidification (45 d) until the end (85 d), oxygen uptake of the communities in treated channels did not differ from the control. However, at the end of the experiment in acidified channels, the biomass of macroinvertebrates and algae were higher and the number of macroinvertebrates was lower than in the control. The lower total community metabolism may result in a lower decomposition rate, as organic matter in sediment was higher in acidified channels with or without aluminum (58 and 54%, respectively) than in the control (46%).

[1]Author to whom correspondence should be addressed.

This article has been cited by:

1. Steven E Campana, Alexandra Valentin, Jean-Marie Sévigny, Don Power. 2007. Tracking seasonal migrations of redfish (Sebastes spp.) in and around the Gulf of St. Lawrence using otolith elemental fingerprints. *Canadian Journal of Fisheries and Aquatic Sciences* **64**:1, 6-18. [Abstract] [PDF] [PDF Plus]
2. Michael H. Prager, Kyle W. Shertzer. An Introduction to Statistical Algorithms Useful in Stock Composition Analysis 499-516. [Crossref]
3. A. Valentin, J.-M. Sevigny, J.-P. Chanut. 2002. Geometric morphometrics reveals body shape differences between sympatric redfish Sebastes mentella, Sebastes fasdatus and their hybrids in the Gulf of St Lawrence. *Journal of Fish Biology* **60**:4, 857-875. [Crossref]
4. Peter J. Rubec, Jacquie M. McGlade, Bertin L. Trottier, André Ferron. 1991. Evaluation of Methods for Separation of Gulf of St. Lawrence Beaked Redfishes, Sebastes fasciatus and S. mentella: Malate Dehydrogenase Mobility Patterns Compared with Extrinsic Gasbladder Muscle Passages and Anal Fin Ray Counts. *Canadian Journal of Fisheries and Aquatic Sciences* **48**:4, 640-660. [Abstract] [PDF] [PDF Plus]
5. Michael H. Prager, Mary C. Fabrizio. 1990. Comparison of Logistic Regression and Discriminant Analyses for Stock Identification of Anadromous Fish, with Application to Striped Bass (Morone saxatilis) and American Shad (Alosa sapidissima). *Canadian Journal of Fisheries and Aquatic Sciences* **47**:8, 1570-1577. [Crossref]
6. Margaret A. Wilzbach. 1985. Relative Roles of Food Abundance and Cover in Determining the Habitat Distribution of Stream-Dwelling Cutthroat Trout (Salmo clarki). *Canadian Journal of Fisheries and Aquatic Sciences* **42**:10, 1668-1672. [Crossref]