

CIRCLE-ROPE: CONE-LIKE DECOUPLED ROTARY POSITIONAL EMBEDDING FOR LARGE VISION-LANGUAGE MODELS

Chengcheng Wang^{1*} Jianyuan Guo^{2*} Hongguang Li^{1*}

Yuchuan Tian⁴ Ying Nie¹ Chang Xu^{3†} Kai Han^{1†}

¹ Huawei Noah’s Ark Lab. ² City University of Hong Kong. ³ University of Sydney.

⁴ State Key Lab of General AI, School of Intelligence Science and Technology, Peking University.

{wangchengcheng20, lihongguang6, ying.nie, kai.han}@huawei.com;
jianyguo@cityu.edu.hk; tianyc@stu.pku.edu.cn; c.xu@sydney.edu.au

ABSTRACT

Rotary Position Embedding (RoPE) is a widely adopted technique for encoding relative positional information in large language models (LLMs). However, when extended to vision-language models (VLMs), RoPE and its variants enforce relative positional dependencies separately within text and image tokens, introducing unintended cross-modal positional biases. For example, image tokens depicting semantically consistent content are assigned distinct positional encodings solely due to spatial location variations. As a result, such tokens exhibit entirely different relative positional relationships with their corresponding text tokens, ultimately leading to misaligned cross-modal representations. To address this, we propose Per-Token Distance, a simple yet effective metric for quantifying the independence of positional encodings across modalities. Informed by this analysis, we introduce Circle-RoPE, a novel encoding scheme designed to eliminate spurious cross-modal biases. Our key idea is to project image token indices onto a *ring* that is orthogonal to the linear axis of text token indices, thereby forming a cone-like structure in the positional encoding space. In this configuration, each text token (point on the linear text axis) becomes the apex of a cone and maintains an equal distance to all image tokens (points on the circular image *ring*), reducing artificial cross-modal biases while preserving intra-image spatial information. To further enhance performance, we propose a staggered strategy that applies different RoPE variants across layers. Extensive experiments demonstrate that our method effectively preserves spatial information from images while reducing relative positional bias, offering a more robust and flexible positional encoding framework for VLMs. The code is available at <https://github.com/lose4578/CircleRoPE>.

1 INTRODUCTION

In the rapidly evolving transformer landscape, Rotary Position Embedding (RoPE) [19] has emerged as the de facto standard for encoding relative positional information in large language models (LLMs). When extending models to handle both textual and visual inputs, as in Vision-Language Models (VLMs), a challenge emerges: how to effectively encode positional information across disparate modalities. Text is inherently sequential, while visual data is spatially structured, characterized by attributes such as location, orientation, viewpoint, and scale—properties that are fundamentally different and largely uncorrelated with textual order.

*Equal contribution. † Corresponding author.

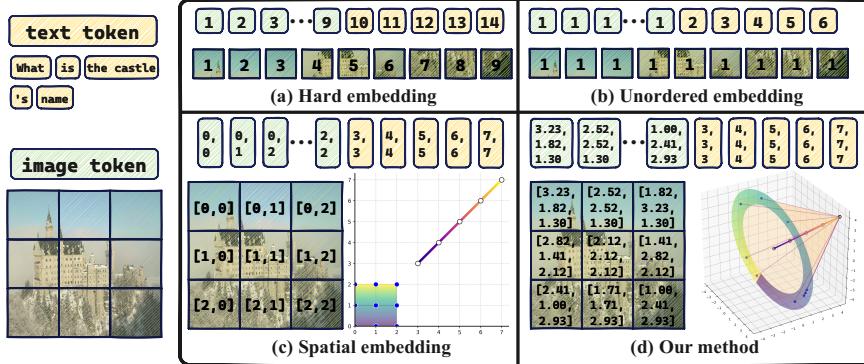


Figure 1: Text (yellow) and image (green) tokens are labeled with their position indices under different RoPE-based encoding schemes. (a) hard embedding method, which encodes image tokens by their flattened sequence; (b) unordered embedding method, assigning the same index to all image tokens within an image; (c) spatial embedding method, where image tokens are indexed according to their 2D positions in the original image; (d) our method, which remaps image token index onto a circle orthogonal to the text index direction, achieving a decoupled encoding.

Different approaches have been explored to tackle this issue. For instance, Figure 1(a) illustrates models like LLaVA [13], Emu3 [21], InternLM-VL [4], and DeepSeek-VL2 [23], which flatten image tokens into a 1D sequence and concatenated them with text tokens, directly applying the standard 1D RoPE from LLMs to multimodal encoding. Figure 1(b) depicts the strategy used in mPLUG-Owl3 [27], where all image patches are simply assigned with the same positional index. Figure 1(c) depicts the positional encoding in M-RoPE [20] (Qwen2-VL), which preserves the spatial layout of images while modeling textual sequentiality, though it still concatenates image and text tokens in the same sequence as in Figure 1(a).

All existing RoPE variants either flatten visual tokens into a 1D sequence or arrange them on a 2D grid before concatenating with text tokens. Both approaches, however, **introduce spurious cross-modal positional biases**—not from actual data relationships but from the hard-coded design of positional embeddings—which can undermine multimodal understanding. Figure 2 illustrates this issue with a visual question answering (VQA) example: “What type of religion is displayed high on the clock tower?” The phrase *high on* requires spatial reasoning, and *clock tower* requires object recognition. Yet their relationships to the correct image regions are distorted by index-based encoding. Two common biases emerge: (i) **semantic misalignment**—*high on* should align with the top of the tower (index 1) but is instead placed near index 8; and (ii) **inconsistent multi-token distances**—*clock tower* corresponds to multiple image tokens of the tower, but their relative distances to the text vary, leading to inconsistency.

In this work, we directly address the problem of positional bias by proposing **Circle Rotary Position Embedding (Circle-RoPE)**, a flexible positional encoding scheme that preserves intrinsic spatial relationships while maintaining consistent cross-modal alignment. At its core, our approach applies geometric transformations to the original coordinate indices of visual tokens before computing RoPE rotation factors. This ensures a fully decoupled encoding of text and image tokens, effectively mitigating cross-modal positional biases.

Specifically, we extend the M-RoPE mechanism, which represents image token indices by height-width coordinates, with two key innovations. First, we propose *Circular Image Token Index Projection (CIP, Sec. 4.1)* which projects 2D grid coordinates onto a circle in 3D space whose normal vector is aligned with the text vector. This transformation ensures orthogonal separation: each text token index lies along the normal vector and maintains equal Euclidean distance and consistent RoPE distances to all points on the circle, forming a cone-like structure. Meanwhile, relative spatial relationships among image tokens are preserved, as shown in Figure 1(d). This design effectively disentangles positional dependencies across modalities.

Second, we propose an *Alternating Geometry Encoding (AGE, Sec. 4.2)* strategy which cyclically switches between M-RoPE and our proposed Circle-RoPE across layers, leveraging their complementary strengths for more robust multimodal representations.

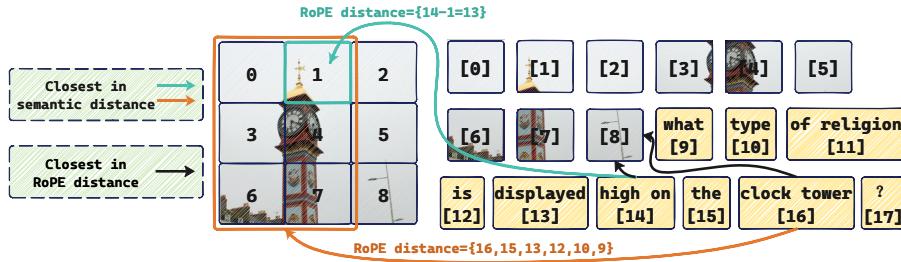


Figure 2: A VQA Example where image and text tokens are sequentially concatenated. The image token at index 8 exhibits the smallest RoPE distance to all text tokens, despite semantically closer image tokens being located elsewhere. The text token at index 16 exhibits varying distances to the six image patches that correspond to the same semantic content. These misalignments highlights how conventional RoPE methods introduce unintended relative positional biases.

In summary, our contributions are twofold: (i) we identify and address cross-modal relative positional biases in existing RoPE variants through the design of Circle-RoPE; (ii) we validate its effectiveness across multiple LVMs and diverse multimodal tasks, achieving improved spatial consistency and visual reasoning.

2 RELATED WORK

VLMs unify visual and textual representations within a single transformer, yet effectively integrating modality-specific positional encodings remains a fundamental challenge. A common strategy is to apply RoPE [19] uniformly across the combined token sequence. However, its naive application to concatenated image and text tokens introduces cross-modal positional bias: the attention becomes sensitive to arbitrary positional offsets between the two modalities. This bias distorts cross-modal alignment, particularly since visual tokens often reside in distant segments of the sequence from relevant text, resulting in impaired information fusion.

Recent advances in multimodal LLMs and pixel-level understanding highlight the significance of unified architectures and position encoding strategies [7, 31, 10, 28, 11, 22]. Many VLMs, *e.g.*, Emu3 [21], InternLM-VL [4], Baichuan-Omni [12], Eve [18], DeepSeek-VL2 [24], and LLaVA series [13, 14], adopt a simple strategy of flattening all tokens in a 1D sequence and using shared position encoding such as RoPE for both text and image tokens.

A distinct research direction assigns a shared positional index to all visual tokens. For example, mPLUG-Owl3 [27] assigns all patches of an image the same position index (via a placeholder token) when applying RoPE. This interleaved scheme preserves the image’s insertion position in the text context and reduces index disparity among image patches, alleviating some bias due to modality mixing.

A third strategy is to introduce spatial positional embeddings tailored to the 2D structure of images. Qwen2-VL [20] exemplifies this by decomposing RoPE into separate dimensions (height, width, and temporal indices) for images, *i.e.*, Multimodal RoPE (M-RoPE). This approach encodes image patches with 2D coordinates instead of large 1D indices, thereby better aligning visual tokens with textual positions.

Each method partially mitigates cross-modal positional issues, yet none completely eliminates bias: shared-index approaches discard intra-image spatial structure, while both flattened 1D sequences and spatial embeddings may retain subtle cross-modal misalignment.

3 PRELIMINARIES AND PROBLEM ANALYSIS

Recent work has extended RoPE from LLMs to multimodal settings, yet often overlooks a fundamental issue: the inherent misalignment between text token indices and image token positions. For example, while Qwen-VL’s M-RoPE [20, 2] introduces 3D encoding for video (width, height, time) and improves performance; however, like other methods, it fails to decouple positional mappings

across modalities. This failure forces unnatural relative position relationships between semantically related text and image tokens during RoPE encoding (as illustrated in Figure 2), introducing cross-modal bias in training and inference. Figure 1 shows some common approaches for implementing multimodal position embeddings:

- Hard embedding (Figure 1(a)): The image tokens are flattened into a 1D sequence and concatenated directly with the text tokens. While straightforward and intuitive, this method does not guarantee that each text token interacts independently with image tokens, often introducing unwanted positional biases instead of relying solely on high-level semantic understanding.
- Unordered embedding (Figure 1(b)): All image tokens are assigned the same index, thus the distance between any text token and all image tokens in the same image is identical. However, this approach ignores the relative positions containing spatial information among the image tokens themselves, leading to a loss of fine-grained visual structure.
- Spatial embedding (Figure 1(c)): Tokens are assigned 2D indices based on their positions in the image, providing more accurate spatial information among image tokens but still failing to guarantee independence between text and image token positions.

Existing approaches predominantly focus on encoding spatial information for images and sequential information for text independently, overlooking the potential interference caused by intertwined positional embeddings. This oversight can introduce unintended biases, distorting cross-modal alignment. Ideally, to eliminate such biases, the "distance" in RoPE index between each text token and all image tokens should remain consistent, ensuring positional independence across modalities.

Table 1: PTD values of different RoPE methods.

Embedding method	Hard	Unordered	Spatial	Ours
Relative position information	✓	✗	✓	✓
PTD	2.22	0	0.64	0

Per-Token Distance (PTD) Metric. To quantify and compare how different RoPE-based methods affect the relative position relationship between text and image tokens, we design a metric called *Per-Token Distance* (PTD). PTD evaluates the independence between the text token index and the image token index after the application of positional encoding. Its formal definition is as follows: suppose the index list of image tokens is $I = \{i_1, i_2, \dots, i_{N_{\text{image}}}\}$ with size N_{image} , and the index list of text tokens is $T = \{t_1, t_2, \dots, t_{N_{\text{text}}}\}$ with size N_{text} . The PTD is calculated as:

$$\text{PTD} = \frac{1}{N_{\text{image}} N_{\text{text}}} \sum_{t \in T} \sum_{i \in I} |d(t, i) - \bar{D}_t|, \quad \bar{D}_t = \frac{1}{N_{\text{image}}} \sum_{i \in I} d(t, i) \quad (1)$$

where $d(x, y)$ denotes the Euclidean distance between x and y . A smaller PTD value indicates a lower variance in the distances from each text token to the set of image tokens. This uniformity signals a higher degree of disentanglement between the text and image token indices. We compute PTD for three typical multimodal encoding methods, *i.e.*, hard embedding (Figure 1(a)), unordered embedding (Figure 1(b)), and spatial embedding (Figure 1(c)). For convenience, we set $N_{\text{image}} = 9$ and $N_{\text{text}} = 5$. The PTD values are show in the Table 1. A non-zero PTD value after applying existing RoPE methods directly indicates the presence of cross-modal relative positional bias. This bias can hinder further performance improvements in VLMs.

Therefore, we propose to map all image token indices to positions equidistant from every text token index, aiming to minimize the PTD metric (ideally achieving 0) and mitigate cross-modal positional bias.

4 METHOD

We propose a novel positional encoding method for VLMs, **Circle Rotary Position Embedding (Circle-RoPE)**. Its core idea is to transform image token indices (w, h) through a series of coordinate projections before applying the rotary matrix [20], thereby removing undesired cross-modal relative positional biases while preserving spatial relationships among image tokens. Circle-RoPE consists of two components: *Circular Image Token Index Projection (CIP, Sec. 4.1)* and *Alternating Geometry Encoding (AGE, Sec. 4.2)*, with the details elaborated in the following sections.

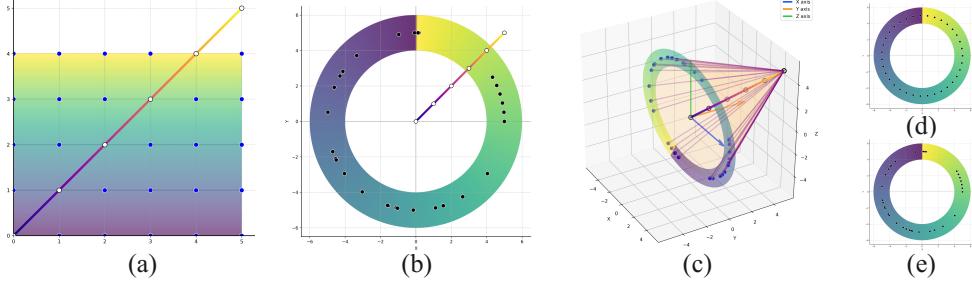


Figure 3: Transformation steps for *Circular Image Token Index Projection (CIP)*: (i) coordinate centralization, (ii) mixed-angle circular mapping, and (iii) target plane rotation as described in Sec 4.1. For clarity, the starting points of text and image indices are aligned in above figure, preserving their relative positional distances without loss of generality. (a) Initial M-RoPE [20] index in step (i); (b) 2D circular structure after steps (i) and (ii); (c) 3D circular structure after step (iii); (d) Grid-index angle (GA) in step (ii); (e) Spatial-origin angle (SA) in step (ii).

4.1 CIRCULAR IMAGE TOKEN INDEX PROJECTION

We begin by designing Circular Image Token Index Projection (CIP) to fully decouple image token indices from text token indices, *i.e.*, achieve $\text{PTD} = 0$. The key idea of CIP is to project image token indices onto a structured geometric space, ensuring uniform RoPE distances to any text token and eliminating unintended positional biases. The CIP process consists of three key steps:

- (i) *Coordinate Centralization*: Shift the geometric center of all image token indices to the origin, standardizing the coordinate reference.
- (ii) *Mixed-Angle Circular Mapping*: Project the centralized image token indices onto a 2D circular trajectory. The angular position of each index is determined by a combination of its spatial-origin angle and its grid-index angle, with a defined radius for structural consistency.
- (iii) *Target Plane Rotation*: Rotate the 2D circular structure from previous step onto a specific plane in 3D space. The orientation of this plane is determined by the text token indices, ensuring orthogonality between the image token index plane and the text token index direction.

In M-RoPE [20], image token indices are represented separately by width and height coordinates, text tokens use 1D positional index equivalent to standard RoPE. As shown in Figure 3(a), given the original M-RoPE index, we obtain the image token index based on a regular grid, denoted as $C = \{(x_{ij}, y_{ij})\}_{i \in W, j \in H}$, where $W = \{0, 1, \dots, w - 1\}$ and $H = \{0, 1, \dots, h - 1\}$. Here, w and h correspond to the width and height of the image after tokenization. For clarity, we let W correspond to the x -axis and H to the y -axis. The goal of CIP is to transform the original image token index $C = \{(x_{ij}, y_{ij})\}$ into decoupled indices from the text tokens, resulting in $C_{\text{proj}} = \{(x_{ij}^{\text{proj}}, y_{ij}^{\text{proj}})\}$. These transformed indices are then directly used for RoPE computation.

4.1.1 COORDINATE CENTRALIZATION

To facilitate subsequent transformations, we first center the image token index coordinates. Specifically, the geometric center $P_{\text{center}} \in \mathbb{R}^2$ of the image token indices is calculated as follows:

$$P_{\text{center}} = \frac{1}{2} \left(\max_i(C_i) + \min_i(C_i) \right) \quad (2)$$

We then subtract this center point from all original coordinates to obtain the centered coordinates:

$$C' = C - P_{\text{center}} \quad (3)$$

This ensures that the geometric center of $C' = \{(x'_{ij}, y'_{ij})\}$ is located at the origin $(0, 0)$, providing a natural reference frame for subsequent projection and rotation.

4.1.2 MIXED-ANGLE CIRCULAR MAPPING

To construct a cone-like structure that effectively decouples the text token indices from the image token indices, we first transform the centered image token coordinates C' into polar coordinates and

project them onto a 2D circle. During this transformation, the angular position of each point on the circle is determined by a combination of its spatial-origin angle (SA) and grid-index angle (GA), while the radius R remains flexible. The resulting 2D circular structure is illustrated in Figure 3(b). We detail the calculation of these two angles and the radius in the following.

Angle Calculation: We combine two complementary angles to balance spatial structure with index information, determining the transformed angle for each image token index:

- (1) *Spatial-Origin Angle* θ_{ij}^{SA} (*SA*): we first compute the polar angle of each centered point (x'_{ij}, y'_{ij}) :

$$\theta_{ij}^{\text{atan}2} = \text{atan2}(y'_{ij}, x'_{ij}) \quad (4)$$

where function $\text{atan2}(y, x)$ returns the angle between the point (x, y) and the positive x -axis, in $(-\pi, \pi]$. Then, we normalize these angles to the range $[0, 2\pi)$:

$$\theta_{\min} = \min_{i,j}(\theta_{ij}^{\text{atan}2}), \quad \theta_{\max} = \max_{i,j}(\theta_{ij}^{\text{atan}2}), \quad \Delta\theta = \theta_{\max} - \theta_{\min} \quad (5)$$

thus, as illustrated in Figure 3(e), the SA is given by:

$$\theta_{ij}^{\text{SA}} = \begin{cases} \frac{\theta_{ij}^{\text{atan}2} - \theta_{\min}}{\Delta\theta} \times 2\pi & \text{if } \Delta\theta > 0 \\ 0 & \text{if } \Delta\theta \leq 0 \end{cases} \quad (6)$$

- (2) *Grid-Index Angle* θ_{ij}^{GA} (*GA*): We flatten the $H \times W$ grid into a 1D sequence with $N = H \times W$ points, assigning each point a uniformly spaced angle based on its flattened index $k \in \{0, \dots, N - 1\}$:

$$\theta_k^{\text{GA}} = \frac{k}{N} \times 2\pi \quad (7)$$

mapping the index k back to the grid position (i, j) yields θ_{ij}^{GA} , ensuring the angles are equally spaced around the circle, as shown in Figure 3(d).

- (3) *Angle Mixing*: The final mixed angle θ_{ij}^{mix} is computed by a weighted average of the two strategies:

$$\theta_{ij}^{\text{mix}} = \alpha \cdot \theta_{ij}^{\text{SA}} + (1 - \alpha) \cdot \theta_{ij}^{\text{GA}} \quad (8)$$

the coefficient $\alpha \in [0, 1]$ controls the balance between preserving spatial information and enhancing the uniqueness of each position. While the SA retains more spatial structure, the GA leads to a clearer separation between positions, making it easier for the model to distinguish between them.

Radius Calculation: The choice of radius R affects the scale of the transformed coordinates and influence the effective frequency range used by RoPE [19]. We provide two strategies here:

- (1) Fixed: Use a predefined constant value R_{fix} .
- (2) Automatic (auto- k): Scale R based on a measure of the spread of the centered coordinates C' , such as the maximum L_2 norm:

$$R_{\text{auto}} = k \times \max_{i,j} \|(x'_{ij}, y'_{ij})\|_2 \quad (9)$$

where k is a predefined scaling factor (e.g., $k = 1$ or $k = 2$).

Mapping to the Circle: Based on the computed angle θ_{ij}^{mix} and radius R , the new coordinates of each image token index on the XY -plane are given by $x_{ij}^{\text{circ}} = R \cos(\theta_{ij}^{\text{mix}})$ and $y_{ij}^{\text{circ}} = R \sin(\theta_{ij}^{\text{mix}})$, which collectively form a circle $C_{\text{circ}} = \{(x_{ij}^{\text{circ}}, y_{ij}^{\text{circ}})\}$, as illustrated in Figure 3(b).

4.1.3 TARGET PLANE ROTATION

After above transformation, visual token index points are mapped to C_{circ} on the XY -plane. To decouple them from the text token index (*i.e.*, achieve PTD=0), we further rotate the circle in 3D space so that its plane is perpendicular to vector V_{text} defined by the text token index, *i.e.*, V_{text} serves as the normal vector of the circle. For computational convenience, we extend C_{circ} to 3D space by initializing the third (z) coordinate to zero for all points. The specific conversion process is as follows:

- (1) Define the target plane normal: normalize the V_{text} to obtain a unit normal vector \mathbf{n} :

$$\mathbf{n} = \frac{V_{\text{text}}}{\|V_{\text{text}}\|_2} = (n_x, n_y, n_z) \quad (10)$$

- (2) Construct an orthonormal basis for the target plane: then define two orthonormal vectors $\{\mathbf{u}, \mathbf{v}\}$ lying in the target plane and orthogonal to \mathbf{n} :

$$\mathbf{u}' = (-n_y, n_x, 0) \quad \mathbf{u} = \frac{\mathbf{u}'}{\|\mathbf{u}'\|_2}, \quad \mathbf{v} = \mathbf{n} \times \mathbf{u} \quad (11)$$

where \mathbf{u} is a unit vector lying in the target plane and orthogonal to \mathbf{n} , while \mathbf{v} is also orthogonal to both \mathbf{n} and \mathbf{u} , ensuring that $\mathbf{u}, \mathbf{v}, \mathbf{n}$ forms a right-handed orthonormal basis.

- (3) Coordinate transformation: for each point $P_{ij}^{\text{circ}} = (x_{ij}^{\text{circ}}, y_{ij}^{\text{circ}}, 0)$ on C_{circ} , compute its new coordinate on the target plane as a linear combination:

$$P_{ij}^{\text{proj}} = x_{ij}^{\text{circ}} \mathbf{u} + y_{ij}^{\text{circ}} \mathbf{v} \quad (12)$$

Applying this transformation to all points yields the final projected set $C_{\text{proj}} = \{(x_{ij}^{\text{proj}}, y_{ij}^{\text{proj}}, z_{ij}^{\text{proj}})\}$, as illustrated in Figure 3(c). These points lie on a circle in 3D space, with its normal vector aligned to V_{text} , ensuring PTD=0 and preserving spatial information relative to the image.

4.2 ALTERNATING GEOMETRY ENCODING (AGE)

In Transformer-based LVLMs, different layers tend to capture distinct geometric patterns, where lower layers focus on local details and higher layers emphasize global structure. Therefore, we propose **Alternating Geometry Encoding (AGE)**, which cyclically switches between the M-RoPE [20] index and the Circle-RoPE index across different Transformer layers, allowing the model to capitalize on the complementary strengths of multiple geometric representations.

4.3 ENCODING TEMPORAL ORDER IN MULTI-IMAGE SEQUENCES

When the input contains multiple images, we explicitly encode their sequential order by translating each image’s circular-encoding center along a fixed global axis. Concretely, let c_i denote the center of the circular positional encoding for the i -th image in the sequence (indexed from $i=1$). We define a constant direction vector $g = [1, 1, 1]^T$ and a stride $\Delta=1$ (default), and set

$$c_i^{\text{final}} = c_i + (i - 1) \Delta g.$$

This translation assigns each image a unique location in the 3D positional space while keeping the within-image geometric structure determined by Circle-RoPE intact.

For example, when we have a sequence with three images `image1`, `image2`, `image3` whose original centers are at 0, the final centers become

$$c_1^{\text{final}} = 0 + [0, 0, 0], \quad c_2^{\text{final}} = 0 + [1, 1, 1], \quad c_3^{\text{final}} = 0 + [2, 2, 2].$$

5 EXPERIMENT

In this section, we first introduce our model configuration and parameter details. We then compare our proposed method with mainstream models. Finally, we conduct ablation studies to validate the effectiveness of our approach and analyze the contributions of different components.

5.1 TRAINING SETTING

To evaluate the effectiveness of our method, we employ Qwen2.5-VL [2] and LLaVA [13] as baseline models for our experiments. The only modification introduced is in the implementation of the positional encoding method; all other configurations are retained from the baseline model. During training, we exclusively update the parameters of the LLM component while keeping the parameters of

the Vision-Language projection layers and the Vision Encoder frozen. All experiments are conducted under a unified training setup. The complete set of hyperparameter configurations are provided in appendix's Table 7. For training, we randomly sample one-tenth of the MAmmoTH-VL Instruct dataset (**12M**) [8] and exclude all video data, resulting in a subset named MAmmoTH-VL-Sub (**1M**). Our experiments demonstrate that even with this reduced data size, our method achieves significant performance improvements compared to the baseline.

5.2 COMPARISON WITH OTHER MODELS

This section evaluates the performance of Circle-RoPE on a diverse range of datasets, benchmarking it against state-of-the-art models such as SAIL-VL [5], InternVL2.5 [4], Ovis2 [16], Phi-3.5-vision [1], and various scales of MiniCPM-V-2 [26] and Qwen2.5-VL [2].

To ensure a comprehensive and fair comparison of the open-source models listed in Table 2, we employed VLMEvalKit [6] to evaluate all models under a unified protocol. As we utilized a third-party open-source toolkit, and the version of GPT used for evaluation differs from those reported in the original papers of some models, the results presented in the table may not be entirely consistent with the official results.

Table 2: Performance of VLM Instruct models and our method (improvement over Qwen2.5-VL shown in parentheses).

Dataset	SAIL-VL [5]	InternVL2.5 [4]	Ovis2 [16]	MiniCPM [26] V-2	MiniCPM [26] V-2.6	Phi-3.5 [1] vision	Qwen2.5-VL [2]	Ours
	2B	4B	2B	2.8B	8B	4.2B	3B	3B
MMMU _{val} [29]	41.44	<u>51.56</u>	43.78	37.00	43.44	44.44	50.22	52.11 (+1.89)
MMMU-Prooverall [30]	14.51	26.65	21.21	14.77	20.26	16.42	27.92	28.44 (+0.52)
MathVista _{mini} [15]	60.70	60.60	64.50	40.80	60.20	43.70	62.40	63.40 (+1.00)
MMStar [3]	56.47	<u>58.53</u>	58.67	41.00	57.53	47.40	54.13	58.20 (+4.07)
AI2D [9]	77.72	81.38	82.77	64.77	81.28	77.59	78.14	81.80 (+3.66)
RealWorldQA [25]	63.01	64.97	67.06	55.03	65.62	53.99	65.75	66.54 (+0.79)
InfoVQA [17]	62.86	72.27	71.65	40.20	64.86	35.18	<u>77.25</u>	77.42 (+0.17)
Avg Score	53.82	59.42	58.52	41.94	56.17	45.53	59.40	61.13 (+1.73)

5.3 EXPERIMENT ON CIRCULAR MAPPING

We conducted ablation studies on the parameters used in Circular Image Token Index Projection (CIP). To validate the effectiveness of angle mixing and to select the optimal radius, we designed a series of ablation experiments. Specifically, we varied the angle mixing parameter α and explored different strategies for calculating the radius. As shown in Table 3, the model achieves the most balanced performance when $\alpha = 0.5$ and the radius is set to 10.

Additionally, we provide results for the baseline model after supervised fine-tuning (SFT) on the MAmmoTH-VL-Sub (**1M**) dataset. This allows for a direct comparison of how different parameter configurations affect model performance under the same conditions.

5.4 EXPERIMENT ON ALTERNATING GEOMETRY ENCODING

To thoroughly assess the impact of utilizing different geometry encoding strategies across various model layers, we systematically designed and evaluated four distinct encoding configurations. Specifically, the strategies we explored include: (1) applying Circle-RoPE consistently in all layers, thereby maintaining a uni-

Table 3: Performance comparison across different CIP configurations.

α	Radius	MMMU _{val} [29]	MMMU-Prooverall [30]	MMStar [3]	MathVista _{mini} [15]	Avg Score
baseline		50.22	27.92	54.13	62.40	48.67
$\alpha = 0$	auto	52.38	28.12	57.50	61.70	49.93
$\alpha = 0$	5	51.32	<u>29.01</u>	<u>58.32</u>	62.40	50.26
$\alpha = 0$	10	51.49	29.13	58.57	62.70	50.47
$\alpha = 0.3$	10	52.05	28.50	58.22	<u>63.30</u>	<u>50.52</u>
$\alpha = 0.5$	10	52.11	28.44	58.20	63.40	50.54
$\alpha = 0.7$	10	52.03	28.39	58.13	62.90	50.36
$\alpha = 1$	10	<u>52.16</u>	28.35	57.70	63.40	50.40
$\alpha = 0.5$	auto	50.04	26.64	57.30	62.20	49.05

form encoding approach throughout the network; (2) adopting Circle-RoPE only in the upper layers, from layer 19 to 36; and (3) employing Circle-RoPE exclusively in the lower layers, specifically layers 1 through 18, to evaluate the impact of introducing relative position bias at different depths of the model.

We also include (4) implementing an Alternating Geometry Encoding strategy, in which Circle-RoPE and M-RoPE are alternated at every successive layer to maximize the complementary strengths of both encoding methods. As illustrated in Table 4, the experimental results clearly demonstrate that the alternating strategy achieves the most robust performance among all tested configurations. This finding confirms that alternating between the two encoding methods enables the model to leverage the strengths of both approaches simultaneously. This finding suggests that leveraging the unique advantages of both encoding methods at different stages of the model can lead to enhanced overall effectiveness and more expressive geometric representations.

Table 4: Performance comparison across different AGE configurations.

Strategy	MMMU (val)	MMMU_Pro	MMStar	MathVista_MINI	A12D_TEST	ChartQA_TEST	InfoVQA	Avg score
strategy 1	51.32	28.41	55.93	65.20	80.39	84.15	76.92	63.19
strategy 2	<u>52.66</u>	<u>28.51</u>	59.87	65.20	79.81	81.96	76.87	63.55
strategy 3	53.48	28.62	<u>59.30</u>	<u>64.50</u>	79.30	82.61	<u>77.35</u>	<u>63.59</u>
strategy 4	52.11	28.44	58.20	63.40	81.80	<u>84.12</u>	77.42	63.64

5.5 GENERALIZABILITY VERIFICATION ON DIFFERENT ARCHITECTURES

To validate the generalizability of our proposed Circle-RoPE, we conducted a rigorous ablation study on LLaVA [13] with a distinct architecture from the one primarily used in our work. We selected Llava-onevision-qwen2-0.5b as the base model, and performed experiments on the MAmmoTH-VL-Sub dataset. This setup provides a robust testbed for evaluating the adaptability and effectiveness of our method.

We compared four variants of the model to isolate the impact of our contributions: **Llava [1D-RoPE] (base)**: The original Llava-onevision-qwen2-0.5b model, serving as a foundational reference. **Llava [M-RoPE]**: we replaced Llava’s 1D-RoPE with M-RoPE from Qwen2.5-VL. **Llava [Circle-RoPE]**: Our proposed Circle-RoPE was integrated into the Llava architecture, replacing its original 1D-RoPE.

The experimental results are summarized in Table 5. Our proposed Circle_RoPE consistently outperforms all other variants across every metric.

Table 5: Ablation study on the Llava-0.5B model to verify the generalizability of Circle-RoPE. Our method achieves the best performance across all benchmarks, demonstrating its effectiveness on a different model architecture.

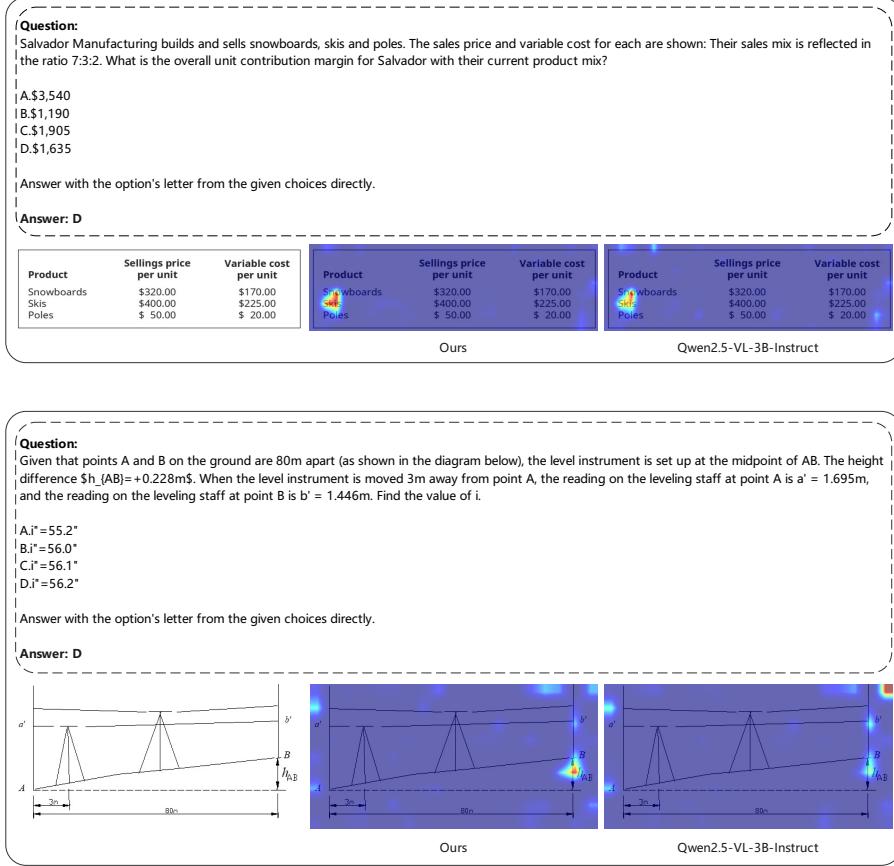
Model	MMMU-val	MMMU_Pro-avg	MMStar	MathVistamini	Avg Score
Llava [1D-RoPE]	32.22	12.92	37.07	35.70	29.48
Llava [M-RoPE]	32.59	12.81	37.18	35.40	29.50
Llava [Circle-RoPE]	32.77	13.21	37.22	36.10	29.83

As shown in Table 5, Circle-RoPR demonstrates strong performance, surpassing both the baseline model LLaVA and the version adapted with M-RoPE. This demonstrates that the benefits of Circle-RoPE are not confined to the Qwen-VL architecture but are generalizable to other LVLMs. For the experiments on the Llava model, we directly applied the optimal hyperparameters (α and R) discovered on Qwen2.5-VL without any architecture-specific tuning. The consistent performance gains prove that Circle-RoPE is a versatile and stable module that can be readily integrated into different models.

5.6 VISUALIZATION OF ATTENTION MAP

To further evaluate the impact of our proposed method, we provide the visualization of attention distributions. The proposed methodology enables the visualization of cross-modal attention for Circle-RoPE and Qwen2.5-VL-3B-Instruct [2], with evaluations performed on the MMMUtest

benchmark [29]. Concretely, we first isolate and extract the attention matrix from the final decoder layer. The average attention from all text tokens to their corresponding image regions is then computed, projected back to the image domain, and reconstructed into a coarse-grained grid. This grid is subsequently transformed into a heatmap, followed by smoothing and enlargement through bilinear interpolation. Finally, a power-law contrast enhancement is applied to highlight salient points. The visualization results show that our method is able to concentrate more effectively on the regions relevant to the given question while exhibiting fewer attentional allocations to irrelevant areas.



6 CONCLUSION

In this paper, we address the challenges of directly applying RoPE to multimodal VLM settings. Existing methods primarily focus on extending RoPE to the vision modality while neglecting the critical interplay between the positional indices of vision and text tokens. To evaluate this overlooked aspect, we first introduce the per-token distance metric to quantify the misalignment. Building on these insights, we propose Circle-RoPE, a novel framework consisting of three transformation steps. Our key idea is to preserve the relative positional information within the vision modality while simultaneously mitigating erroneous relative position biases between text and image tokens. This decoupled positional encoding enhances cross-modal alignment, paving the way for more robust multimodal understanding.

REFERENCES

- [1] Marah Abdin, Jyoti Aneja, Hany Awadalla, Ahmed Awadallah, Ammar Ahmad Awan, Nguyen Bach, Amit Bahree, Arash Bakhtiari, Jianmin Bao, Harkirat Behl, et al. Phi-3 technical report: A highly capable language model locally on your phone, 2024. URL <https://arxiv.org/abs/2404.14219>, 2024.

- [2] Shuai Bai, Keqin Chen, Xuejing Liu, Jialin Wang, Wenbin Ge, Sibo Song, Kai Dang, Peng Wang, Shijie Wang, Jun Tang, et al. Qwen2. 5-vl technical report. *arXiv preprint arXiv:2502.13923*, 2025.
- [3] Lin Chen, Jinsong Li, Xiaoyi Dong, Pan Zhang, Yuhang Zang, Zehui Chen, Haodong Duan, Jiaqi Wang, Yu Qiao, Dahua Lin, et al. Are we on the right way for evaluating large vision-language models? *arXiv preprint arXiv:2403.20330*, 2024.
- [4] Zhe Chen, Weiyun Wang, Yue Cao, Yangzhou Liu, Zhangwei Gao, Erfei Cui, Jinguo Zhu, Shenglong Ye, Hao Tian, Zhaoyang Liu, et al. Expanding performance boundaries of open-source multimodal models with model, data, and test-time scaling. *arXiv preprint arXiv:2412.05271*, 2024.
- [5] Hongyuan Dong, Zijian Kang, Weijie Yin, Xiao Liang, Chao Feng, and Jiao Ran. Scalable vision language model training via high quality data curation. *arXiv preprint arXiv:2501.05952*, 2025.
- [6] Haodong Duan, Junming Yang, Yuxuan Qiao, Xinyu Fang, Lin Chen, Yuan Liu, Xiaoyi Dong, Yuhang Zang, Pan Zhang, Jiaqi Wang, Dahua Lin, and Kai Chen. Vlmevalkit: An open-source toolkit for evaluating large multi-modality models, 2024. URL <https://arxiv.org/abs/2407.11691>.
- [7] Hao Fei, Yuan Zhou, Juncheng Li, Xiangtai Li, Qingshan Xu, Bobo Li, Shengqiong Wu, Yaoting Wang, Junbao Zhou, Jiahao Meng, Qingyu Shi, Zhiyuan Zhou, Liangtao Shi, Minghe Gao, Daoan Zhang, Zhiqi Ge, Weiming Wu, Siliang Tang, Kaihang Pan, Yaobo Ye, Haobo Yuan, Tao Zhang, Tianjie Ju, Zixiang Meng, Shilin Xu, Liyu Jia, Wentao Hu, Meng Luo, Jiebo Luo, Tat-Seng Chua, Shuicheng Yan, and Hanwang Zhang. On path to multimodal generalist: General-level and general-bench, 2025. URL <https://arxiv.org/abs/2505.04620>.
- [8] Jarvis Guo, Tuney Zheng, Yuelin Bai, Bo Li, Yubo Wang, King Zhu, Yizhi Li, Graham Neubig, Wenhui Chen, and Xiang Yue. Mammoth-vl: Eliciting multimodal reasoning with instruction tuning at scale. *arXiv preprint arXiv:2412.05237*, 2024.
- [9] Aniruddha Kembhavi, Mike Salvato, Eric Kolve, Minjoon Seo, Hannaneh Hajishirzi, and Ali Farhadi. A diagram is worth a dozen images. In *Computer Vision–ECCV 2016: 14th European Conference, Amsterdam, The Netherlands, October 11–14, 2016, Proceedings, Part IV 14*, pp. 235–251. Springer, 2016.
- [10] Weixian Lei, Jiacong Wang, Haochen Wang, Xiangtai Li, Jun Hao Liew, Jiashi Feng, and Zilong Huang. The scalability of simplicity: Empirical analysis of vision-language learning with a single transformer, 2025. URL <https://arxiv.org/abs/2504.10462>.
- [11] Xiangtai Li, Henghui Ding, Haobo Yuan, Wenwei Zhang, Jiangmiao Pang, Guangliang Cheng, Kai Chen, Ziwei Liu, and Chen Change Loy. Transformer-based visual segmentation: A survey. *IEEE transactions on pattern analysis and machine intelligence*, 2024.
- [12] Yadong Li, Jun Liu, Tao Zhang, Song Chen, Tianpeng Li, Zehuan Li, Lijun Liu, Lingfeng Ming, Guosheng Dong, Da Pan, et al. Baichuan-omni-1.5 technical report. *arXiv preprint arXiv:2501.15368*, 2025.
- [13] Haotian Liu, Chunyuan Li, Qingyang Wu, and Yong Jae Lee. Visual instruction tuning, 2023. URL <https://arxiv.org/abs/2304.08485>.
- [14] Shilong Liu, Hao Cheng, Haotian Liu, Hao Zhang, Feng Li, Tianhe Ren, Xueyan Zou, Jianwei Yang, Hang Su, Jun Zhu, Lei Zhang, Jianfeng Gao, and Chunyuan Li. Llava-plus: Learning to use tools for creating multimodal agents, 2023. URL <https://arxiv.org/abs/2311.05437>.
- [15] Pan Lu, Hritik Bansal, Tony Xia, Jiacheng Liu, Chunyuan Li, Hannaneh Hajishirzi, Hao Cheng, Kai-Wei Chang, Michel Galley, and Jianfeng Gao. Mathvista: Evaluating mathematical reasoning of foundation models in visual contexts. *arXiv preprint arXiv:2310.02255*, 2023.
- [16] Shiyin Lu, Yang Li, Qing-Guo Chen, Zhao Xu, Weihua Luo, Kaifu Zhang, and Han-Jia Ye. Ovis: Structural embedding alignment for multimodal large language model. *arXiv preprint arXiv:2405.20797*, 2024.

- [17] Minesh Mathew, Viraj Bagal, Rubèn Pérez Tito, Dimosthenis Karatzas, Ernest Valveny, and C. V Jawahar. Infographicvqa, 2021. URL <https://arxiv.org/abs/2104.12756>.
- [18] Miao Rang, Zhenni Bi, Chuanjian Liu, Yehui Tang, Kai Han, and Yunhe Wang. Eve: Efficient multimodal vision language models with elastic visual experts. *arXiv preprint arXiv:2501.04322*, 2025.
- [19] Jianlin Su, Murtadha Ahmed, Yu Lu, Shengfeng Pan, Wen Bo, and Yunfeng Liu. Roformer: Enhanced transformer with rotary position embedding. *Neurocomputing*, 568:127063, 2024.
- [20] Peng Wang, Shuai Bai, Sinan Tan, Shijie Wang, Zhihao Fan, Jinze Bai, Keqin Chen, Xuejing Liu, Jialin Wang, Wenbin Ge, et al. Qwen2-vl: Enhancing vision-language model's perception of the world at any resolution. *arXiv preprint arXiv:2409.12191*, 2024.
- [21] Xinlong Wang, Xiaosong Zhang, Zhengxiong Luo, Quan Sun, Yufeng Cui, Jinsheng Wang, Fan Zhang, Yueze Wang, Zhen Li, Qiying Yu, et al. Emu3: Next-token prediction is all you need. *arXiv preprint arXiv:2409.18869*, 2024.
- [22] Xilin Wei, Xiaoran Liu, Yuhang Zang, Xiaoyi Dong, Pan Zhang, Yuhang Cao, Jian Tong, Haodong Duan, Qipeng Guo, Jiaqi Wang, et al. Videorope: What makes for good video rotary position embedding? *arXiv preprint arXiv:2502.05173*, 2025.
- [23] Zhiyu Wu, Xiaokang Chen, Zizheng Pan, Xingchao Liu, Wen Liu, Damai Dai, Huazuo Gao, Yiyang Ma, Chengyue Wu, Bingxuan Wang, Zhenda Xie, Yu Wu, Kai Hu, Jiawei Wang, Yaofeng Sun, Yukun Li, Yishi Piao, Kang Guan, Aixin Liu, Xin Xie, Yuxiang You, Kai Dong, Xingkai Yu, Haowei Zhang, Liang Zhao, Yisong Wang, and Chong Ruan. Deepseek-vl2: Mixture-of-experts vision-language models for advanced multimodal understanding, 2024. URL <https://arxiv.org/abs/2412.10302>.
- [24] Zhiyu Wu, Xiaokang Chen, Zizheng Pan, Xingchao Liu, Wen Liu, Damai Dai, Huazuo Gao, Yiyang Ma, Chengyue Wu, Bingxuan Wang, et al. Deepseek-vl2: Mixture-of-experts vision-language models for advanced multimodal understanding. *arXiv preprint arXiv:2412.10302*, 2024.
- [25] X.AI. Grok-1.5 vision preview. <https://x.ai/blog/grok-1.5v>, 2024.
- [26] Yuan Yao, Tianyu Yu, Ao Zhang, Chongyi Wang, Junbo Cui, Hongji Zhu, Tianchi Cai, Haoyu Li, Weilin Zhao, Zhihui He, et al. Minicpm-v: A gpt-4v level mllm on your phone. *arXiv preprint arXiv:2408.01800*, 2024.
- [27] Jiabo Ye, Haiyang Xu, Haowei Liu, Anwen Hu, Ming Yan, Qi Qian, Ji Zhang, Fei Huang, and Jingren Zhou.mplug-owl3: Towards long image-sequence understanding in multi-modal large language models. *arXiv preprint arXiv:2408.04840*, 2024.
- [28] Haobo Yuan, Xiangtai Li, Tao Zhang, Zilong Huang, Shilin Xu, Shunping Ji, Yunhai Tong, Lu Qi, Jiashi Feng, and Ming-Hsuan Yang. Sa2va: Marrying sam2 with llava for dense grounded understanding of images and videos, 2025. URL <https://arxiv.org/abs/2501.04001>.
- [29] Xiang Yue, Yuansheng Ni, Kai Zhang, Tianyu Zheng, Ruqi Liu, Ge Zhang, Samuel Stevens, Dongfu Jiang, Weiming Ren, Yuxuan Sun, et al. Mmmu: A massive multi-discipline multimodal understanding and reasoning benchmark for expert agi. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*, pp. 9556–9567, 2024.
- [30] Xiang Yue, Tianyu Zheng, Yuansheng Ni, Yubo Wang, Kai Zhang, Shengbang Tong, Yuxuan Sun, Botao Yu, Ge Zhang, Huan Sun, et al. Mmmu-pro: A more robust multi-discipline multimodal understanding benchmark. *arXiv preprint arXiv:2409.02813*, 2024.
- [31] Tao Zhang, Xiangtai Li, Zilong Huang, Yanwei Li, Weixian Lei, Xueqing Deng, Shihao Chen, Shunping Ji, and Jiashi Feng. Pixel-sail: Single transformer for pixel-grounded understanding, 2025. URL <https://arxiv.org/abs/2504.10465>.

APPENDIX

A FURTHER ANALYSIS AND DISCUSSION

A.1 THE ADAPTATION COST OF INTRODUCING CIRCLE-ROPE

We instantiate Circle-RoPE on the architecturally closest backbone, *Qwen2.5-VL*, and monitor step-wise training dynamics under SFT. We observed that even minor architectural modifications—such as altering the positional encoding—require substantial retraining with large-scale data for the model to adapt to the new positional distribution. We refer to this phenomenon as the adaptation cost.

Table 6: Step-wise training dynamics illustrating the *adaptation cost* when introducing Circle-RoPE on Qwen2.5-VL under SFT. At 3k steps, Circle-RoPE lags slightly behind; after ~ 8.5 k steps it surpasses the baseline on both benchmarks. Best per column is in **bold**.

Model	Step	Loss ↓	MMStar ↑	MathVision ↑
Qwen2.5-VL (SFT)	3000	0.7997	57.94	20.16
Circle-RoPE (SFT)	3000	0.8077	57.53	20.13
Qwen2.5-VL (SFT)	8463	0.7666	58.07	20.56
Circle-RoPE (SFT)	8463	0.7725	58.20	20.95

Even on the most similar backbone, Circle-RoPE exhibits a measurable *adaptation cost*: at early training (3k steps) its performance is slightly below the SFT baseline (Table 6). With continued optimization (~ 8.5 k steps), the advantages emerge and eventually surpass the baseline on both MMStar (+0.13) and MathVision (+0.39). This indicates that even minor positional-encoding changes require non-trivial optimization to re-stabilize the representation geometry. Under limited compute and a relatively small SFT set, these gains are conservative rather than inflated. Choosing Qwen2.5-VL was thus the most pragmatic and reliable validation setting given our constraints; adopting a more dissimilar backbone would likely incur a larger adaptation cost that is computationally prohibitive. The fact that Circle-RoPE achieves improvements despite the initial dip and limited data provides evidence of robustness and headroom; we expect further gains with larger-scale pre-training or extended SFT schedules.

A.2 EFFECTIVENESS OF ALTERNATING GEOMETRY ENCODING (AGE)

We introduce Alternating Geometry Encoding (AGE) into our method primarily for the following reasons:

(1) Complementary strengths and preservation of spatial information. While Circle-RoPE achieves image–text decoupling, it inevitably alters the strong grid-based spatial prior of image patches provided by the original RoPE. By alternating the two encoding methods, the model benefits from both: it reduces cross-modal positional bias (from Circle-RoPE) and fully utilizes the fine-grained internal spatial structure of the image (from RoPE), achieving a “1+1>2” effect.

(2) Compatibility with pre-trained knowledge and smooth transition. Our models are fine-tuned from Qwen2.5-VL, whose weights are deeply adapted to the original RoPE. Compared with applying a completely new encoding scheme to one contiguous part of the network, an alternating strategy minimizes the “shock” to the existing weight distribution. This enables smoother and more data-efficient convergence under limited SFT data, better integrating the pre-trained knowledge with the new capabilities introduced by Circle-RoPE.

In summary, AGE serves as an optional but effective mechanism that (i) fuses complementary geometric biases to preserve spatial reasoning while reducing cross-modal positional bias, and (ii) eases optimization by providing a gentler transition from RoPE-adapted weights to Circle-RoPE-enhanced representations. Empirically, our ablations reflect these stability and performance benefits.

B HYPERPARAMETERS

Table 7: Training Hyperparameter Configuration for our method.

Hyperparameter	Value
Base Model	Qwen2.5-VL-3B
Image Resolution	512×512
Global Batch Size	128
Learning Rate	1e-6
Optimizer	AdamW
LR Schedule	Cosine Decay
Number of Epochs	1
Warmup Ratio	0.1
Max Sequence Length	4096

C PSEUDOCODE IMPLEMENTATION OF CIRCLE-ROPE

```

1 import torch
2
3 def circular_image_token_projection(C: torch.Tensor, alpha: float, R: float, V_text:
4     torch.Tensor):
5     """
6         Circular Image Token Projection in PyTorch style.
7
8     Args:
9         C (torch.Tensor): Original image token grid coordinates (N, 2).
10        alpha (float): Angle mixing weight.
11        R (float): Circle radius.
12        V_text (torch.Tensor): Text vector direction, shape (3,).
13
14    Returns:
15        torch.Tensor: Projected coordinates (N, 3).
16
17    # =====
18    # Step 1: Coordinate Centralization
19    # =====
20    P_center = 0.5 * (C.max(dim=0).values + C.min(dim=0).values) # (2,)           # (N, 2)
21    C_prime = C - P_center
22
23    # =====
24    # Step 2: Mixed-Angle Circular Mapping
25    # =====
26
27    # 2a. Calculate Spatial-Origin Angle (SA)
28    raw_angles = torch.atan2(C_prime[:, 1], C_prime[:, 0])           # (N,)           # (N, 2)
29    min_angle = raw_angles.min()
30    max_angle = raw_angles.max()
31    delta_theta = max_angle - min_angle
32
33    if delta_theta > 0:
34        theta_SA = (raw_angles - min_angle) / delta_theta * 2 * torch.pi
35    else:
36        theta_SA = torch.zeros_like(raw_angles)
37
38    # 2b. Calculate Grid-Index Angle (GA)
39    N = C.shape[0]
40    k = torch.arange(N, device=C.device)                                # (N,)           # (N, 2)
41    theta_GA = (k.float() / N) * 2 * torch.pi
42

```

```

43     # 2c. Mix Angles
44     theta_mix = alpha * theta_SA + (1 - alpha) * theta_GA
45
46     # 2d. Map to 2D circle and expand to 3D
47     x_circ = R * torch.cos(theta_mix)
48     y_circ = R * torch.sin(theta_mix)
49     C_circ = torch.stack([x_circ, y_circ, torch.zeros_like(x_circ)], dim=-1)  # (N, 3)
50
51     # =====
52     # Step 3: Target Plane Rotation
53     # =====
54
55     # 3a. Construct orthonormal basis from text vector
56     n = V_text / V_text.norm()                                     # (3,)
57     u_prime = torch.tensor([-n[1], n[0], 0.0], device=C.device)
58     if u_prime.norm() < 1e-6:
59         u_prime = torch.tensor([1.0, 0.0, 0.0], device=C.device)
60     u = u_prime / u_prime.norm()
61     v = torch.cross(n, u)
62
63     # 3b. Project points from 2D circle to 3D target plane
64     #      This is a linear combination of basis vectors u and v.
65     C_proj = C_circ[:, 0].unsqueeze(-1) * u + C_circ[:, 1].unsqueeze(-1) * v  # (N, 3)
66
67     return C_proj

```