Price-Based Resource Allocation in Spectrum-Sharing OFDMA Femtocell Networks

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***Abstract*—Deployment of femtocells have emerged as a promising technique to address the need for the exponentially increasing mobile traffic demand. They can also improve the capacity and coverage for the indoor wireless users. However, the cross-tier interference in such heterogeneous networks between the femtocells and the macrocells is a design challenge which should be answered before such deployment. In this paper, a spectrum-sharing OFDMA femtocell network is considered in which the macrocell base station protects itself and its users by pricing the interference from femtocell access points. A price-based resource allocation is formulated as a Stackelberg game model to jointly maximize the revenue of the macrocell and the utility of the femtocell users (FUEs) while a predetermined tolerable cross-interference constraint at the macrocell is met. In the proposed model, the macrocell determines the subchannel assignment and the interference prices for the FUEs. Then, the FUEs choose their optimal downlink power. To find the Stackelberg equilibrium, the optimization problems of the leader and the followers are solved. For the followers' sub-game, a closed form expression is obtained. To find the leader's sub-game solution, the non-convex mixed integer nonlinear (MINLP) problem is first converted to an equivalent convex MINLP. Then, we employ the outer approximation (OA) to iteratively and efficiently solve the leader problem. The numerical results validate the convergence of the OA algorithm and the operation of our price-based scheme.**

***Index Terms*—Spectrum-sharing OFDMA femtocell network, Stackelberg game, interference pricing, outer approximation.**

I. Introduction

To enhance the network coverage for indoor environments and improve the quality of service provisioning, the use of femtocell is widely adopted by the wireless operator [1]. The femtocell network can provide ubiquitous connectivity to the end users and offload traffic from the macrocell. A femtocell is enabled by a home base station that is connected to the service provider via a backhaul link. Not only the femtocell users, but also the operators benefit from deployment of the femtocell access point (FAPs). With the help of FAPs, FUEs can experience better indoor voice and data coverage and higher data rates. From the network operator's perspective, the FAPs alleviate the burden on the macro base stations (MBSs) by data offloading resulting in higher quality of service (QoS) [1], [2].

It is more preferable for operators to implement spectrum-sharing, rather than spectrum splitting, between femtocells and macrocells due to the spectrum scarcity and implementation difficulty [3]. However, for spectrum-sharing in two-tier femtocell networks, the cross-tier and co-tier interference can greatly degrade the network performance. Therefore, the interference mitigation in two-tier femtocell networks has become an active area of research. Design of resource allocation and interference mitigation strategies for spectrum-sharing OFDMA-based femtocell networks has received attention recently. Power allocation and subchannel assignment has been widely investigated in recent efforts to maximize the user's capacity subject to a predefined threshold for the cross-tier interference temperature in two tier networks [4], [5]. In [6], a joint power and subchannel allocation algorithm is proposed to maximize the total capacity of densely deployed femtocells.

Game theory based resource allocation and interference mitigation schemes in femtocell networks have been investigated in the literature. In [7], non-cooperative power allocation with signal-to-interference-plus-noise ratio (SINR) adaptation was used to alleviate the interference from femtocells to macrocells. In [3], a Stackelberg game based power control was formulated to maximize the femtocell capacity under the cross-tier interference constraint. In [8], a non-cooperative power and subchannel allocation scheme for co-channel femtocells was proposed, together with macrocell user transmission protection. Subchannel allocation for femtocells was solved using a correlated equilibrium game-theoretic approach aiming to minimize the interference to the primary MBS in [9]. To protect the primary macrocell and imperfect channel state information, the cooperative Nash bargaining resource allocation scheme under the constraint of the cross-tier interference temperature limit was studied in [10].

In this paper, we investigate a price-based joint subchannel assignment and power allocation problem for orthogonal frequency division multiple access (OFDMA) spectrum-sharing femtocell networks. We model the problem as a Stackelberg game with the consideration of cross-tier interference limitation. The macrocell BS is the leader and the FUEs are the followers. In the first sub-game, the macrocell announces the subchannel assignments and interference prices. In the second sub-game, based on the announced subchannel assignment and interference prices, the FUEs maximize their utility with respect to their downlink power. To find the Stackelberg equilibrium of the game, for the followers' sub-game a closed form solution is proposed while the leader problem is solved using outer approximation (OA) algorithm optimally. To the best of our knowledge, price-based resource allocation to jointly allocate the subchannel and the power using Stackelberg model for spectrum-sharing OFDMA femtocell networks has not been studied in previous works.

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The rest of this paper is organized as follows. In Section II, we introduce the system model. In Section III, the problem is formulated using Stackelberg game. In Section IV, the Stackelberg equilibrium of the game is investigated through outer approximation. The numerical results are presented in Section V. Finally, Section VI concludes the paper.

II. System Model

We consider a scenario where subchannels are shared between femtocells and a macrocell. The co-channel femtocells are overlaid on a macrocell as shown in Figure 1. We focus on the resource allocation problem on the downlink of the femtocells. We assume that the macrocell and femtocells can exchange information. Equal power allocation and round-robin scheduling schemes are used for macrocell users [4]. The number of femtocell users of FAP k is denoted by F_k for $k = 1, \ldots, K$. Let M denote the number of active macrocell users in the coverage area of the macrocell.

The OFDMA system has a bandwidth of B, which is divided into N subchannels. Let $g^{FM,k}_{u,n}$ and $g^k_{u,n}$ be the channel gains on subchannel n from MBS and from FAP k to the FUE u in femtocell k, respectively, where $k \in \{1, 2, \ldots, K\}$, $u \in \{1, 2, \ldots, F_k\}$, $n \in \{1, 2, \ldots, N\}$. Let $g^{MF,k}_{w,n}$ be the channel gain of subchannel n from FAP k to macrocell user $w \in \{1, 2, \ldots, M\}$ who is occupying subchannel n. Let $p^k_{u,n}$ and $p^M_{w,n}$ denote femtocell k's transmit power to femto user u on subchannel n and macrocell's transmit power to macro user w on subchannel n, respectively, and $\boldsymbol{P} = [p^k_{u,n}]_{K\times F_k\times N}$ be the power allocation matrix of the K femtocells. Denote the subchannel indication matrix as $\boldsymbol{A} = [a^k_{u,n}]_{K\times F_k\times N}$, where $a^k_{u,n} = 1$ means that subchannel n is assigned to femtocell user u in femtocell k, and $a^k_{u,n} = 0$ otherwise. Then, the received SINR at femtocell user u in the k-th FAP occupying the n-th subchannel is given by

$$\gamma^k_{u,n} = \frac{p^k_{u,n} g^k_{u,n}}{p^M_{w,n} g^{FM,k}_{u,n} + \sigma^2} = p^k_{u,n} h^k_{u,n}, \quad (1)$$

where $p^M_{w,n} g^{FM,k}_{u,n}$ is the interference from the macrocell, $h^k_{u,n} = \frac{g^k_{u,n}}{p^M_{w,n} g^{FM,k}_{u,n} + \sigma^2}$, and σ^2 is the additive white Gaussian noise (AWGN) power, where the inter-femtocell interference is also considered as part of the noise power assuming a sparsely deployed scenario. Based on Shannons capacity formula, the downlink capacity on subchannel n of femtocell user u in femtocell k is given by

$$C^k_{u,n} = \log_2(1 + \gamma^k_{u,n}). \quad (2)$$

III. Problem Formulation

We consider a price-based resource allocation scheme where each FUE pays for the power that the macrocell allocates to it. The design parameters are the powers that FAP allocates to FUEs, the allocated subcchannel to the FUEs, and the price the FUEs pay for interference to the macrocell. Both the FUEs and the macrocell aim to maximize their utility. We employ a Stackelberg game model to formulate the joint problem of maximizing the utility of the FUEs and the macrocell. The macrocell BS works as the leader and the FUEs are the followers. In the first stage, the macrocell specifies the subchannel allocation scheme for the FUEs and also announces the set of interference prices. In the second stage, based on the subchannel allocation and the given interference prices, each FUE optimizes its utility over the downlink power from FAP to each FUE.

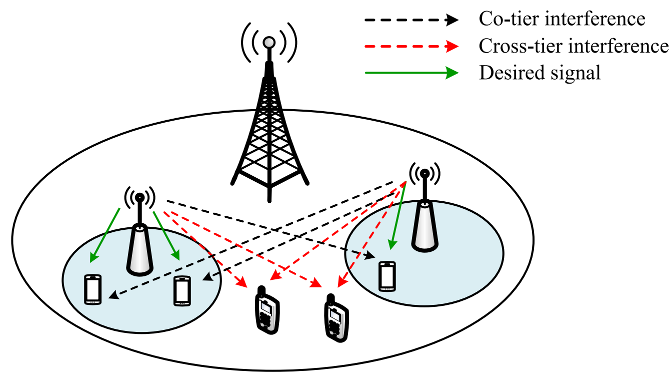

Fig. 1. System Model

The macrocell aims to announce the subchannel allocation and the interference prices in a way that the total cross-tier interference of all the FUEs to the macrocell does not exceed from a predefined threshold. Let I^{th}_n denote the maximum tolerable interference level on subchannel n for the macrocell. Then, we have

$$\sum_{k=1}^{K} \sum_{u=1}^{F_k} a^k_{u,n} p^k_{u,n} g^k_{w,n} \le I^{th}_n, \ \forall n. \quad (3)$$

The total transmit power of each femtocell should be less than or equal to the maximum transmit power denoted by $P^k_{\max}$. This is the summation over all subchannels and users. That is,

$$\sum_{u=1}^{F_k} \sum_{n=1}^{N} a^k_{u,n} p^k_{u,n} \le P^k_{\max}, \forall k. \quad (4)$$

Considering these constraints, we model the utility maximization problem using the Stackelberg game as the following.

A. Utility of the Femtocell Users

For femtocell user u in FAP k and for a given subchannel assignment and given interference prices by the macrocell, the optimal utility of each FUE can be obtained from

Problem 1. (FUEs Sub-game):

$$\begin{aligned} \max_{p^k_{u,n}} \quad & \sum_{n=1}^{N} a^k_{u,n} (\lambda^k_{u,n} C^k_{u,n} - \mu^k_{u,n} g^k_{w,n} p^k_{u,n}) \\ \text{s.t.} \quad & p^k_{u,n} \ge 0, \forall k, u, n, \end{aligned} \quad (5)$$

where $\mu^k_{u,n}, \forall k, u$ and $\lambda^k_{u,n}, \forall k, u$ are the interference price and utility gain for FUE u in FAP k at subchannel n, respectively. It is observed that the FUEs benefit from the transmit rate from FAP, and they should pay for the cross-tier interference to the macrocell. Each FUE maximizes its utility for the given interference price and subchannel assignment by choosing $p^k_{u,n}$.

B. Utility of the Macrocell

The MBS benefits from leasing the spectrum to FUEs. The optimal utility of the MBS or the owner of the spectrum for given power variables $p^k_{u,n}, \forall k, u, n$ can be obtained from

Problem 2. (Macrocell Sub-game):

$$
\begin{aligned}
\max_{\boldsymbol{A},\boldsymbol{\mu}} \quad & \sum_{k=1}^{K}\sum_{u=1}^{F_k}\sum_{n=1}^{N} a^k_{u,n}\mu^k_{u,n}g^k_{w,n}p^k_{u,n} \\
\text{s.t.} \quad & C1: \mu^k_{u,n} \geq 0,\ \forall k,u,n \\
& C2: \sum_{k=1}^{K}\sum_{u=1}^{F_k} a^k_{u,n}p^k_{u,n}g^k_{w,n} \leq I^{th}_n,\ \forall n \\
& C3: \sum_{u=1}^{F}\sum_{n=1}^{N} a^k_{u,n}p^k_{u,n} \leq P^k_{\max}, \forall k \\
& C4: \sum_{u=1}^{F_k} a^k_{u,n} \leq 1,\ \forall k,n \\
& C5: a^k_{u,n} \in \{0,1\},\ \forall k,u,n,
\end{aligned}
$$

where constraint $C1$ represents the non-negative price constraint for the interference on each subchannel; $C2$ expresses the tolerable cross-tier interference temperature level on each subchannel of the macrocell; $C3$ limits the transmit power of each FAP to be below $P^k_{\max}$; $C4$ and $C5$ are imposed to guarantee that each subchannel can only be assigned to at most one FUE in each FAP.

IV. Equilibrium of the Proposed Game

The Stackelberg equilibrium (SE) is the point where the utility of the leader is maximized given that the followers play their best response or adopting a Nash equilibrium [11]. In this section, using the backward induction method, we aim to find the SE of the Stackelberg game. First, the followers sub-game, i.e., Problem 1 is solved based on a given interference price vector and subchannel assignment. After substituting the optimal solution of the followers' sub-game into the leader sub-game, i.e., Problem 2, the optimal subchannel assignment and interference price vector is obtained. In the following we obtain the best responses for both sub-games.

A. FUEs Sub-game

Lemma 1: For a given $a^k_{u,n}$ and $\mu^k_{u,n}, \forall k, u, n$, the FUEs sub-game is a convex optimization problem.

Proof: We define the objective function of Problem 1 as

$$f(\mathbf{p}) = \sum_{n=1}^{N} a^k_{u,n}(\lambda^k_{u,n}C^k_{u,n} - \mu^k_{u,n}g^k_{w,n}p^k_{u,n}).$$

We find the Hessian matrix of $f(\mathbf{p})$. For all $n \neq m$, we have

$$\frac{\partial^2 f(\mathbf{p})}{\partial p^k_{u,n}\partial p^k_{u,m}} = 0. \tag{6}$$

The diagonal elements of the Hessian matrix are

$$\frac{\partial^2 f(\mathbf{p})}{\partial (p^k_{u,n})^2} = -\frac{\tilde{\lambda}^k_{u,n}(h^k_{u,n})^2}{(1+p^k_{u,n}h^k_{u,n})^2}, \tag{7}$$

where $\tilde{\lambda}^k_{u,n} = \lambda^k_{u,n}/\ln 2$. The Hessian of $f(\mathbf{p})$ is a diagonal matrix. Since all the diagonal elements of the Hessian matrix are non-positive and the rest of the elements are zero, the function $f(\mathbf{p})$ is concave. ■

One can find the optimal solution of Problem 1 which is the optimal power for FUE u in FAP k on subchannel n as

$$p^{k*}_{u,n} = \left(\frac{\tilde{\lambda}^k_{u,n}}{\mu^k_{u,n}g^k_{w,n}} - \frac{1}{h^k_{u,n}}\right)^+,\ \forall k,u,n. \tag{8}$$

From (8), it is observed that if the interference price is higher than a threshold, FUE u in FAP k at subchannel n is not willing to pay for the interference to the macrocell. Therefore, the FUE leaves the game if the power price proposed by the macrocell is high. Consequently, the positivity condition for each FUE u in FAP k at subchannel n can be written as

$$\mu^k_{u,n} \leq \frac{\tilde{\lambda}^k_{u,n}h^k_{u,n}}{g^k_{w,n}},\ \forall k,u,n. \tag{9}$$

B. Macrocell Sub-game

Substituting the optimal powers in (8) into the macrocell sub-game (i.e., Problem 2) and imposing an additional constraint for positivity of the powers, we obtain

Problem 3:

$$
\begin{aligned}
\max_{\boldsymbol{A},\boldsymbol{\mu}} \quad & \sum_{k=1}^{K}\sum_{u=1}^{F_k}\sum_{n=1}^{N} a^k_{u,n}\left(\tilde{\lambda}^k_{u,n} - \frac{\mu^k_{u,n}g^k_{w,n}}{h^k_{u,n}}\right) \\
\text{s.t.} \quad & C1: \mu^k_{u,n} \geq 0,\ \forall k,u,n \\
& C2: \sum_{k=1}^{K}\sum_{u=1}^{F} a^k_{u,n}\left(\frac{\tilde{\lambda}^k_{u,n}}{\mu^k_{u,n}} - \frac{g^k_{w,n}}{h^k_{u,n}}\right) \leq I^{th}_n,\ \forall n \\
& C3: \sum_{u=1}^{F}\sum_{n=1}^{N} a^k_{u,n}\left(\frac{\tilde{\lambda}^k_{u,n}}{\mu^k_{u,n}g^k_{w,n}} - \frac{1}{h^k_{u,n}}\right) \leq P^k_{\max}, \forall k \\
& C4: \sum_{u=1}^{F} a^k_{u,n} \leq 1,\ \forall k,n \\
& C5: a^k_{u,n} \in \{0,1\},\ \forall k,u,n \\
& C6: \mu^k_{u,n} \leq \frac{\tilde{\lambda}^k_{u,n}h^k_{u,n}}{g^k_{w,n}},\ \forall k,u,n.
\end{aligned}
$$

This is a non-convex mixed integer non-linear programming (MINLP) optimization problem. The non-convexity arises due to the product of $a^k_{u,n}$ and the price variables, $\mu^k_{u,n}, \forall k, u, n$, in the objective function and the constraints.

Generalized Bender's decomposition (GBD) and outer approximation (OA) are two methods to solve MINLP optimization problems optimally. These algorithms consist of solving an alternating sequence of relaxed problems (MILP) and nonlinear programming problems (NLPs) [12], [13]. We employ the OA algorithm to solve Problem 3 optimally. The basic idea of the OA algorithm is that at each iteration, an upper bound and a lower bound on the MINLP solution are generated. The upper bound results from the solution of the problem called a *primal problem*. The lower bound results from the solution of the MILP which is called a *master problem*. The master problem is derived based upon an outer approximation (linearization) of the nonlinear objective and constraints around the primal solution. As the iterations proceed, two sequences of updated upper bounds and lower bounds are generated which converge to the optimal solution in a finite number of iterations [12], [14].

To be able to use OA algorithm, the MINLP is required to be separable in continuous and integer variables, and the integer variables should be appeared linearly in the optimization problem and constraints. Also, for the case that the integer variables are fixed, the problem should be a convex optimization problem over the continuous variables. We note that the non-convexity is due to the product terms between $a_{u,n}^k$ and the price variables, $\mu_{u,n}^k$. We adopt the big-M formulation [14] to decompose the product terms, i.e., $a_{u,n}^k \mu_{u,n}^k$. The big-M formulation linearizes such terms due to the binary nature of $a_{u,n}^k$. The optimization Problem 3 can be recast as

Problem 4:

$$
\begin{aligned}
\max_{\boldsymbol{A},\boldsymbol{\mu}} \quad & \sum_{k=1}^{K}\sum_{u=1}^{F_k}\sum_{n=1}^{N}\left(\tilde{\lambda}_{u,n}^k - \frac{\mu_{u,n}^k g_{w,n}^k}{h_{u,n}^k}\right) \\
\text{s.t.} \quad & C1, C4, C5, C6 \\
& \tilde{C}2: \sum_{k=1}^{K}\sum_{u=1}^{F}\left(\frac{\tilde{\lambda}_{u,n}^k}{\mu_{u,n}^k} - \frac{g_{w,n}^k}{h_{u,n}^k}\right) \le I_n^{th},\ \forall n \\
& \tilde{C}3: \sum_{u=1}^{F}\sum_{n=1}^{N}\left(\frac{\tilde{\lambda}_{u,n}^k}{\mu_{u,n}^k g_{w,n}^k} - \frac{1}{h_{u,n}^k}\right) \le P_{\max}^k, \forall k \\
& C7: \frac{\tilde{\lambda}_{u,n}^k}{\mu_{u,n}^k} - \frac{g_{w,n}^k}{h_{u,n}^k} \le a_{u,n}^k I_n^{th},\ \forall k, u, n \\
& C8: \frac{\tilde{\lambda}_{u,n}^k}{\mu_{u,n}^k g_{w,n}^k} - \frac{1}{h_{u,n}^k} \le a_{u,n}^k P_{\max}^k,\ \forall k, u, n.
\end{aligned}
$$

It is observed that when $a_{u,n}^k = 1$, the corresponding constraints in $C7$ and $C8$ does not affect in the feasible set. On the other hand, if for any k, u, n, the integer variable becomes zero ($a_{u,n}^k = 0$), due to the constraints $C7$ and $C8$, we have $\mu_{u,n}^k \ge \frac{\tilde{\lambda}_{u,n}^k h_{u,n}^k}{g_{w,n}^k}$. Considering this and the constraint $C6$, the only possible value for the interference price variable is $\mu_{u,n}^k = \frac{\tilde{\lambda}_{u,n}^k h_{u,n}^k}{g_{w,n}^k}$. Therefore, the corresponding term in the objective function and also the interference constraint $C2$ and the power constraint $C3$ will automatically be equal to zero. Consequently, the Problems 3 and 4 are equivalent.

To apply the OA algorithm, the MINLP should be a separable convex optimization problem for the fixed values of the integer variables [12], [13]. The following lemma shows that this condition is satisfied for our reformulated MINLP in Problem 4.

Lemma 2: For fixed integer variables $a_{u,n}^k, \forall k, u, n$, Problem 4 is a convex optimization problem with respect to $\mu_{u,n}^k, \forall k, u, n$.

Proof: For fixed integer variable $a_{u,n}^k, \forall k, u, n$ in Problem 4, it is observed that the objective function is affine, and the constraints are convex, so Problem 4 is a convex optimization problem. ■

Since Problem 4 is separable in continuous and integer variables, and integer variables appear linearly in the optimization problem, according to Lemma 2, the OA algorithm can be employed to solve Problem 4 optimally.

Using the OA algorithm, Problem 4 is decomposed into two sub-problems, namely primal problem and master problem. The continuous variables, $\mu_{u,n}^k$, are obtained by solving the primal problem for fixed values of the integer variables, $a_{u,n}^k$. Using the solution of the primal problem, the integer variables are obtained by solving the master problem. For the first iteration of the primal problem, a feasible initial value is chosen for the binary variables, $a_{u,n}^k \forall k, u, n$. The OA algorithm iteratively solves the primal problem and the master problem until their solutions converge to the optimal value. Let L denote the number of iterations required for the convergence of the OA algorithm. The primal and master problems are described at a given iteration $l \in \{1, 2, ..., L\}$ below.

Primal Problem (l-th iteration) For a given optimal $a_{u,n}^k, \forall k, u, n$ at iteration $l-1$, $a_{u,n}^{k*}(l-1)$, the primal problem can be formulated as

$$
\begin{aligned}
\min_{\boldsymbol{\mu}} \quad & -\sum_{k=1}^{K}\sum_{u=1}^{F}\sum_{n=1}^{N}\left(\tilde{\lambda}_{u,n}^k - \frac{\mu_{u,n}^k g_{w,n}^k}{h_{u,n}^k}\right) \qquad (10) \\
\text{s.t.} \quad & C1, \tilde{C}2, \tilde{C}3, C6 \\
& \tilde{C}7: \frac{\tilde{\lambda}_{u,n}^k}{\mu_{u,n}^k} - \frac{g_{w,n}^k}{h_{u,n}^k} \le a_{u,n}^{k*}(l-1) I_n^{th},\ \forall k, u, n \\
& \tilde{C}8: \frac{\tilde{\lambda}_{u,n}^k}{\mu_{u,n}^k g_{w,n}^k} - \frac{1}{h_{u,n}^k} \le a_{u,n}^{k*}(l-1) P_{\max}^k,\ \forall k, u, n.
\end{aligned}
$$

This problem is a convex optimization problem in continuous variables $\mu_{u,n}^k, \forall k, u, n$, and can be solved optimally using well-known convex optimization algorithms such as interior point efficiently. Let $\mu_{u,n}^{k*}(l), \forall k, u, n$ denote the optimal power prices obtained from primal problem at the l-th iteration.

Master Problem (l-th iteration) In order to form the master problem, the non-linear functions in the constraints are approximated by an affine function around the optimal solution of the primal problem. Therefore, the master problem can be

Algorithm 1: The Outer Approximation

Initialize $a_{u,n}^{k}(0), \forall k, u, n,$

Set convergence error ϵ, $l \leftarrow 1$, $\mathrm{UB}^{(l)} = \infty$, $\mathrm{LB}^{(l)} = 0$.

while $|\mathrm{UB}^{(l)} - \mathrm{LB}^{(l)}| \leq \epsilon$ **do**

  Solve primal problem and obtain $\mu_{u,n}^{k*}(l), \forall k, u, n$ and the upper bound, $\mathrm{UB}^{(l)}$.

  Solve the master problem and obtain δ^*, $a_{u,n}^{k*}(l), \forall k, u, n$, and the lower bound, $\mathrm{LB}^{(l)}$.

  $l \leftarrow l + 1$

formulated as

$$\min_{\boldsymbol{\mu}, \boldsymbol{A}, \delta} \quad \delta \tag{11}$$

$$\text{s.t.} \quad -\sum_{k=1}^{K}\sum_{u=1}^{F_k}\sum_{n=1}^{N}\left(\tilde{\lambda}_{u,n}^{k} - \frac{\mu_{u,n}^{k} g_{w,n}^{k}}{h_{u,n}^{k}}\right) \leq \delta$$

$$C1, C4, C5, C6$$

$$\sum_{k=1}^{K}\sum_{u=1}^{F}\left(\frac{2\tilde{\lambda}_{u,n}^{k}}{\mu_{u,n}^{k*}(l)} - \frac{g_{w,n}^{k}}{h_{u,n}^{k}} - \frac{\tilde{\lambda}_{u,n}^{k}\mu_{u,n}^{k}}{(\mu_{u,n}^{k*}(l))^2}\right) \leq I_n^{th}, \ \forall n$$

$$\sum_{u=1}^{F_k}\sum_{n=1}^{N}\left(\frac{2\tilde{\lambda}_{u,n}^{k}}{\mu_{u,n}^{k*}(l) g_{w,n}^{k}} - \frac{1}{h_{u,n}^{k}} - \frac{\tilde{\lambda}_{u,n}^{k}\mu_{u,n}^{k}}{(\mu_{u,n}^{k*}(l))^2 g_{w,n}^{k}}\right) \leq P_{\max}^{k}, \forall k$$

$$\frac{2\tilde{\lambda}_{u,n}^{k}}{\mu_{u,n}^{k*}(l)} - \frac{g_{w,n}^{k}}{h_{u,n}^{k}} - \frac{\tilde{\lambda}_{u,n}^{k}\mu_{u,n}^{k*}(l)}{(\mu_{u,n}^{k*}(l))^2} \leq a_{k,u,n} I_n^{th}, \ \forall k, u, n$$

$$\frac{2\tilde{\lambda}_{u,n}^{k}}{\mu_{u,n}^{k*}(l) g_{w,n}^{k}} - \frac{1}{h_{u,n}^{k}} - \frac{\tilde{\lambda}_{u,n}^{k}\mu_{u,n}^{k}}{(\mu_{u,n}^{k*}(l))^2 g_{w,n}^{k}} \leq a_{u,n}^{k} P_{\max}^{k}, \ \forall k, u, n.$$

The master problem is a mixed integer linear problem (MILP) with respect to $a_{u,n}^{k}, \forall k, u, n$, δ, and $\mu_{u,n}^{k}, \forall k, u, n$. Therefore, it can be optimally solved by any standard MILP optimization toolbox such as MOSEK [15].

At iteration l, the optimal value of the master problem, δ^*, which is a lower bound for the optimal value of Problem 4 is obtained. In addition to the constraints in the previous iteration, a new constraint is added to the master problem in the current iteration. Based on this, the optimal solution of the master problem is always larger than or equal to that of the previous iteration and this lower bound is always non-decreasing. Moreover, the optimal solution of the primal problem is always larger than or equal to the optimal solution of Problem 4. Thus, the solution of the primal problem always gives an upper bound for the solution of the original problem. We set the upper bound at each iteration equal to the minimum of the upper bound of the previous iteration and the upper bound of the current iteration. At each iteration, we solve the primal problem with the given solution of the master problem obtained from the previous iteration. Then, we solve the master problem with the given solution of the primal problem. This process continues and within a finite number of iterations, the OA algorithm converges to the optimal solution. We summarize the OA algorithm in Alg. 1 pseudo code.

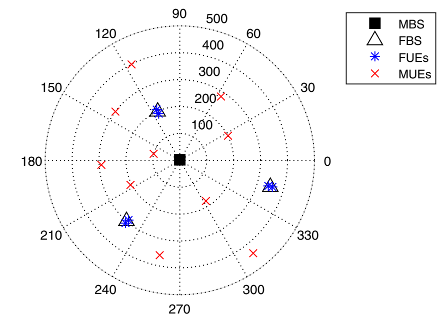

Fig. 2. Network topology in simulation setup for $K = 3$, $F_k = 2$, $M = 10$.

V. Numerical Results

In this section, we present the simulation results to evaluate the performance of the proposed price-based resource allocation for downlink transmission of the OFDMA femtocell network. The number of subchannels in the macrocell is $N = 10$, and all subchannels can be reused by each femtocell user. The simulation parameters are set as $B = 10$ MHz and $\sigma^2 = \frac{B}{N} N_0$, where $N_0 = -174$ dBm/Hz is the AWGN power spectral density. We also consider $P_{\max}^{k} = 20$ dBm, the cross-tier interference limit $I_n^{th} = 7.5 \times 10^{-11}$ W, and the utility gains $\lambda_{u,n}^{k} = 1, \forall k, u, n$. The downlink transmit power of the macrocell for MUEs is assumed to be 20 dBm. The co-channel femtocells are uniformly distributed in the macrocell coverage area. Macrocell coverage radius is 500m and the femtocell coverage radius is 10 m. The number of macro users M is 10. MUEs are uniformly distributed in the coverage area of the MBS. We assume that the Rayleigh fading channel gains are modeled as i.i.d. unit-mean exponentially distributed random variables. We also model the path loss by $d^{-\beta}$ in which d is the distance between the nodes and β is the path loss exponent for $\beta = 2$. A typical network scenario in our numerical examples is shown in Fig. 2 for $K = 3$, $F_k = 2, \forall k$.

In Fig. 3, the convergence of the OA algorithm is illustrated. It is observed that within few iterations, the upper bound and the lower bound of the optimal solution converge to each other and the optimal solution is obtained.

In Fig.4, the effect of maximum cross-tier interference threshold on the revenue of the macrocell is investigated. It shows that by increasing the maximum cross-tier interference threshold, the revenue of the macrocell increases. This is reasonable because the FAPs are allowed to transmit with higher power resulting more cross-interference and more revenue for the macrocell as well. It is also observed that by increasing the number of FUEs, the revenue of the macrocell increases.

In Fig. 5, the revenue of the MBS is depicted versus the maximum transmit power of the FAPs. By increasing the maximum transmit power of the FAPs, the revenue of the MBS increases due to the increased interference from the FAPs at the macrocell. It is observed that for sufficiently large maximum transmit power of the FAPs, the revenue of the

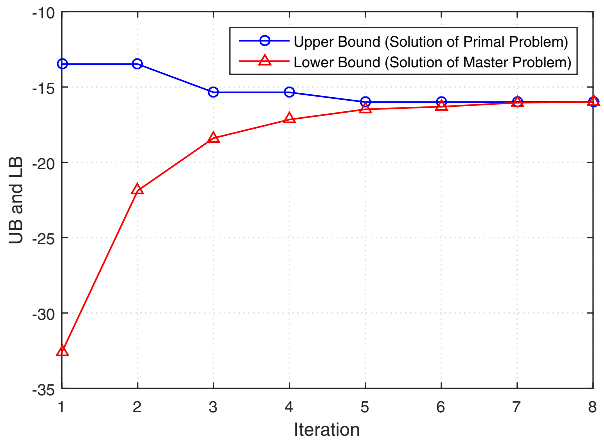

Fig. 3. Convergence of the OA algorithm

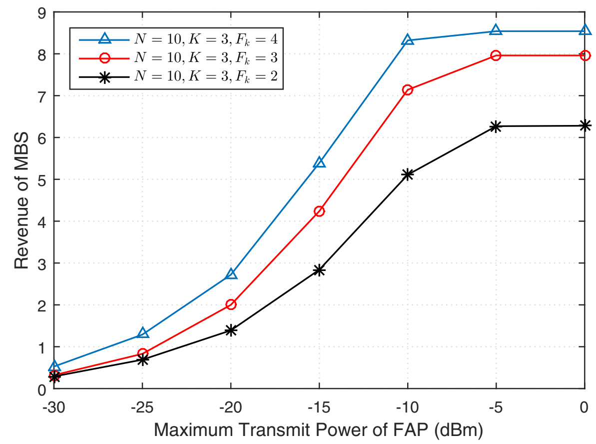

Fig. 5. Revenue of the macrocell versus maximum transmit power of FAP.

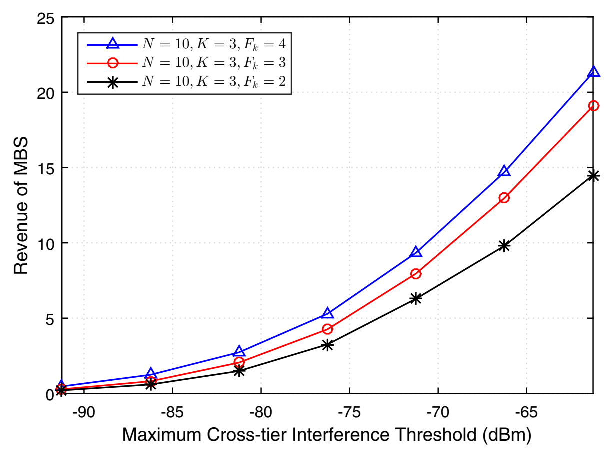

Fig. 4. Revenue of the macrocell versus cross-tier interference threshold.

macrocell does not increase any more. This is because the maximum cross-tier interference threshold constraint is active and limits the macrocell revenue.

VI. Conclusion

In this paper, we investigated the price-based resource allocation for joint subchannel and power allocation problems in spectrum-sharing OFDMA femtocell networks. The resource allocation problem was formulated as a Stackelberg game, where a cross-tier interference temperature limit is imposed to protect the primary macrocell. The macrocell plays as the leader and the FUEs are considered as the followers. For the FUEs sub-game, a closed form solution was proposed while the leader sub-game was solved using the outer approximation (OA) algorithm after reformulation of the non-convex mixed integer non-linear programming (MINLP) into a separable MINLP problem. The results of the paper are useful to practically design price-based spectrum-sharing OFDMA femtocell networks.

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