Lecture 2015-01-26

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  - Representing a Translational Mechanical System
  - Free Body Diagrams
    - 1) for M_1
    - 2) for M_2
      - A) M_1 is frozen
      - B) M_2 is frozen

Representing a Translational Mechanical System

Find State Space Representation, where $y = x_2(t)$.

State variables:

> - position
> - velocity of each of the masses

Assume a frictionless system.

$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = \frac{d^2x}{dt}$$

Free Body Diagrams

1) for M_1

$M_1 s^2 x_1(s)$ <-out-
$kx_1(s)$ <-in-
$Dsx_1(s)$ <-out-

$$M_1 s^2 x_1(s) + Dsx_1(s) + kx_1(s) = kx_2(s)$$
$$M_1 s^2 x_1(s) + Dsx_1(s) + kx_1(s) - kx_2(s) = 0$$

2) for M_2

A) M_1 is frozen

$kx_2(s)$<–
$f(s)$–>
$M_2 s^2 X_2(s)$<–

B) M_2 is frozen

$$M_2 s^2 X_2(s) + KX_2(s) = f(s) + KX_1(s)$$
$$M_2 s^2 X_2(s) + KX_2(s) - KX_1(s) = f(s)$$

> Inverse Laplace Transforms

$$M_1 \frac{d^2 x_1}{dt^2} + D\frac{dx_1}{dt} + KX_1 - KX_2 = 0$$
$$M_2 \frac{d^2 X_2}{dt^2} + KX_2 - KX_1 = f(t)$$

$$\begin{bmatrix} X_1 \\ V_1 \\ X_2 \\ V_2 \end{bmatrix}$$

$\frac{d^2 x_1}{dx^2} = \frac{dv_1}{dt}$
$\frac{d^2 x_2}{dt^2} = \frac{dv_2}{dt}$

$\frac{dx_1}{dt} = v_1$
$\frac{d^2 x_2}{dt} = v_2$

$M_1 \frac{dv_1}{dt} + DV_1 + KX_1 - KX_2 = 0$
$M_2 \frac{dv_2}{dt} + kX_2 - KX_1 = f(t)$

$\frac{dv_1}{dt} = -\frac{D}{M_1} V_1 - \frac{K}{M_1} X_1 + \frac{K}{M_1} X_2$
$\frac{dv_2}{dt} = -\frac{k}{M_2} X_2 + \frac{K}{M_2} X_1 + \frac{1}{M_2} f(t)$

$\frac{dv_1}{dt} = \dot{v}_1$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{V}_1 \\ \dot{X}_2 \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{K}{M_1} & -\dfrac{D}{M_1} & \dfrac{K}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{K}{M_2} & 0 & -\dfrac{K}{M_2} & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{M_2} \end{bmatrix} f(t)$$

$$y = X_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} + [0] f(t)$$