McMaster University Math 1AA3/1ZB3 Summer 2013
PRACTICE Final Exam
August 7, 2013

Duration: 180 minutes

Instructor: Owen Baker

Name: ____

Student ID Number: ______________

This test paper is printed on both sides of the page. There are 25 (multiple choice) questions and 9 pages. You are responsible for ensuring that your copy of this test is complete. Bring any discrepancies to the attention of the invigilator. Detach page 9 (the bonus problem) and write your name and ID number there now.

Instructions

(1) Write your name, ID number, and **version number** on the computer card.
(2) All answers must be entered on the computer card with an HB pencil. Read the marking instructions on the card.
(3) Each multiple choice question is worth one mark, for a total of 25 marks. No marks will be deducted for wrong answers or blank answers.
(4) Any multiple choice question left blank will receive 0 marks, even if the correct answer is circled on the exam page. You **must enter your answers on the computer card.**
(5) Scratch paper is available for rough work; ask the invigilator.
(6) With the extra credit problem, your total mark may rise above 25. The number of marks awarded for this problem is at the discretion of the instructor and depends on the quality and completeness of your solution.

1) $\sin(\pi/2) + \sin(2\pi/2) + \sin(3\pi/2) + \sin(4\pi/2) + \sin(5\pi/2) + \cdots =$

A. doesn't converge **B.** 0 **C.** $1/2$ **D.** 1 **E.** ∞

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2) $\displaystyle\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^{2n} =$

A. 1 **B.** e **C.** 4 **D.** $2e^2$ **E.** e^4

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3) Evaluate the improper integral: $\displaystyle\int_{-1}^{1} x^{-2/3}\,dx =$

A. 0 **B.** 2 **C.** 3 **D.** 6 **E.** doesn't converge

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4) Which **one** of the following series **converges absolutely**?

A. $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{1}{n}$ **B.** $\displaystyle\sum_{n=2}^{\infty}\frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n+1)}{2\cdot 4\cdot 6\cdot\cdots\cdot(2n+2)}$ **C.** $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{1}{n(\ln(n))^2}$

D. $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{(2n)!}{n!}$ **E.** $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$

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5) Find the interval of convergence for $\displaystyle\sum_{n=0}^{\infty}\frac{(x+2)^n}{5n^2+1}$.

A. $[1,3)$ **B.** $(1,3]$ **C.** $[-3,-1]$ **D.** $[-3,-1)$ **E.** $[-3,3)$

6) Select the Maclaurin series for $\dfrac{\tan^{-1}(x) - x}{x}$.

A. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n}}{(2n+1)}$ **B.** $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n}}{(2n+1)!}$ **C.** $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{x^{2n}}{2n+1}$

D. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{x^{2n}}{(2n+1)!}$ **E.** $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{x^{2n+1}}{(2n+1)!}$

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7) Solve the **linear** differential equation: $xy' + \frac{1}{2}y = \sqrt{x}$.

A. $y = 2 + \dfrac{C}{\sqrt{x}}$ **B.** $y = \dfrac{2}{3}\sqrt{x} + C$ **C.** $y = \dfrac{2}{3}\sqrt{x} + \dfrac{C}{\sqrt{x}}$

D. $y = \sqrt{x} + C$ **E.** $y = \sqrt{x} + \dfrac{C}{\sqrt{x}}$

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8) Solve the **separable** initial value problem: $y' = xy^2 + y^2 + 4x + 4, \qquad y(\sqrt{\pi}/2) = 2.$

A. $y = \dfrac{\pi/2}{x^2 + 2x - \sqrt{\pi}}$ **B.** $y = \dfrac{8}{\pi}\tan^{-1}\left(\dfrac{4x^2}{\pi}\right)$

C. $y = 2\tan(x^2 + 2x - \sqrt{\pi})$ **D.** $y = \tan(x^2 + 2x - \sqrt{\pi})$

E. $y = \dfrac{x^2y^2}{2} + xy^2 + 2x^2 + 4x$

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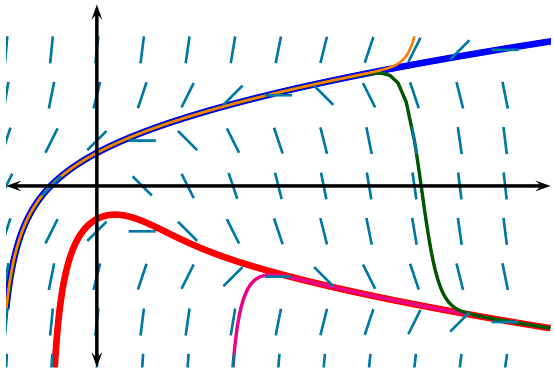
9) Find the **orthogonal trajectories** to the curve family $\{y = ke^{-x}\}$.

A. $\left\{y = C \pm \sqrt{2x}\right\}$ **B.** $\left\{y = \pm\sqrt{2x + C}\right\}$ **C.** $\{y = C\sqrt{x}\}$

D. $\{y = Ce^{x}\}$ **E.** $\{y = Ce^{-x}\}$

10) Which differential equation has some solutions depicted here?



A. $\dfrac{dy}{dx} = y^2 - x$ **B.** $\dfrac{dy}{dx} = y^2 + x$ **C.** $\dfrac{dy}{dx} = y^2$

D. $\dfrac{dy}{dx} = x^3 + 2x$ **E.** $\dfrac{dy}{dx} = y^2 - x^2$

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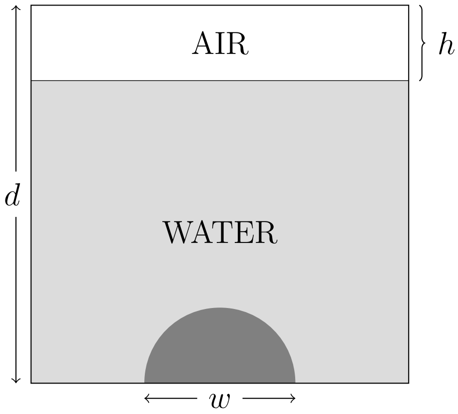
11) A vertical dam has a **semicircular gate** as shown below. h is the height of air above the water level, d is the height of the entire figure, and w is the width of the gate.



Which integral represents the hydrostatic force on the **semicircular gate**?
(Notation: $\delta = \rho g$ is the weight density of water.)

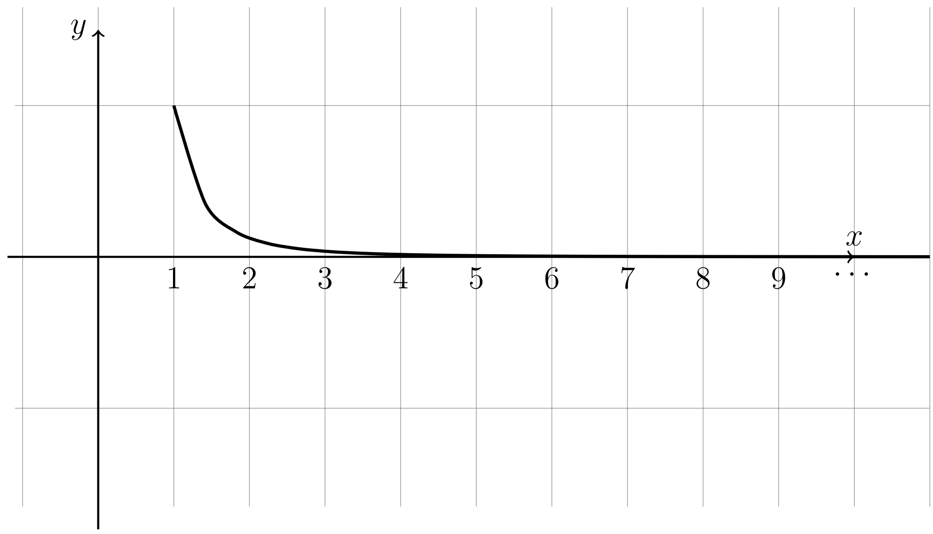
A. $\displaystyle\int_0^{w/2} 2\delta(d-h-y)\sqrt{(w/2)^2 - y^2}\,dy$ **B.** $\displaystyle\int_0^{w/2} \delta(d-h-y)\sqrt{(w/2)^2 - y^2}\,dy$

C. $\displaystyle\int_h^{d} 2\delta y\sqrt{(w/2)^2 - y^2}\,dy$ **D.** $\displaystyle\int_h^{d} \delta y\sqrt{(w/2)^2 - y^2}\,dy$ **E.** $\displaystyle\int_h^{d} \delta(y-h)\sqrt{w^2 - y^2}\,dy$

12) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+3y^2} =$

A. 1 **B.** $2/3$ **C.** $1/2$ **D.** 0 **E.** doesn't exist

13) The curve below ($y = 1/x^3$, $x \geq 1$) is **revolved** about the y**-axis**.



Select the integral that represents the **surface area** and tell if this area is **finite or infinite**.

A. $\displaystyle\int_1^\infty 2\pi x\sqrt{1+\left(\frac{3}{x^4}\right)^2}\,dx,$ FINITE

B. $\displaystyle\int_1^\infty 2\pi x\sqrt{1+\left(\frac{3}{x^4}\right)^2}\,dx,$ INFINITE

C. $\displaystyle\int_1^\infty \frac{2\pi}{x^3}\sqrt{1+\left(\frac{3}{x^4}\right)^2}\,dx,$ FINITE

D. $\displaystyle\int_1^\infty \frac{2\pi}{x^3}\sqrt{1+\left(\frac{3}{x^4}\right)^2}\,dx,$ INFINITE

E. $\displaystyle\int_0^1 \frac{2\pi}{\sqrt[3]{y}}\sqrt{1+\left(\frac{1}{3}y^{-4/3}\right)^2}\,dy,$ FINITE

14) Identify the conic section: $25x^2 - 600x - 144y^2 = 0$.

A. ellipse with vertices $(\pm 12, 0), (0, \pm 5)$ and foci $(\pm 13, 0)$.
B. ellipse with vertices $(0, 0), (24, 0), (12, \pm 5)$ and foci $(11, 0), (13, 0)$.
C. hyperbola with vertices $(\pm 12, 0)$ and foci $(\pm 13, 0)$.
D. hyperbola with vertices $(0, 0), (24, 0)$ and foci $(-1, 0), (25, 0)$.
E. hyperbola with vertices $(12, \pm 5)$ and foci $(12, \pm 13)$.

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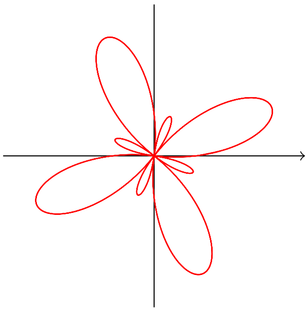
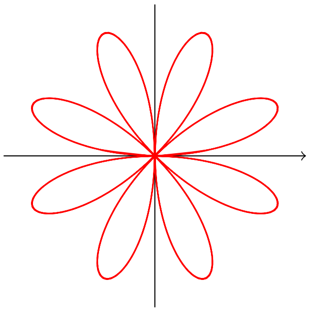
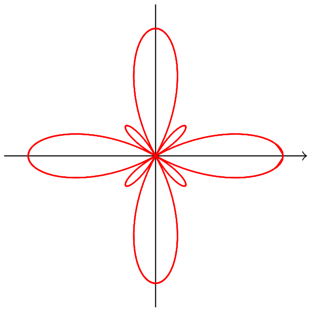
15) Which of the below is a plot of the **polar curve** $r = 1 + 2\cos(4\theta)$?



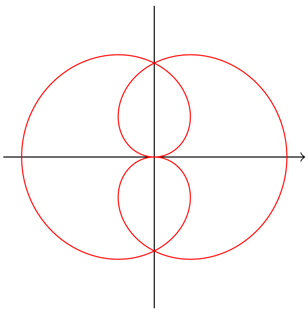
A.



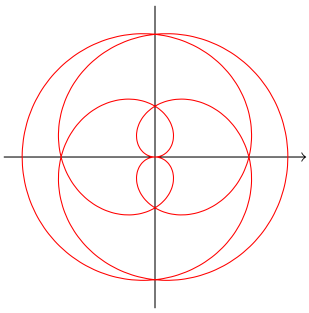
B.



C.



D.



E.

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16) $\dfrac{\partial}{\partial y}\left(\dfrac{\partial}{\partial x}\left(\tan(\sec(x)) + \tan(\sec(y)) + x^2y^2\right)\right) =$

A. $2x + 2y$ **B.** $x + y$ **C.** $4xy$ **D.** $2xy$ **E.** xy

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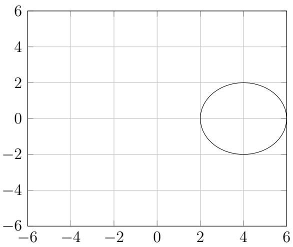
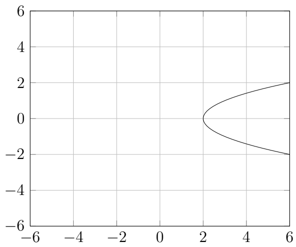
17) Find the slope of $x(t) = e^t + e^{-t}, \quad y(t) = e^t - e^{-t}$ when $t = 1$.

A. 0 **B.** $\dfrac{e^2+1}{1-e^2}$ **C.** $\dfrac{1-e^2}{e^2+1}$ **D.** $\dfrac{e^2+1}{e^2-1}$ **E.** $\dfrac{e^2-1}{e^2+1}$

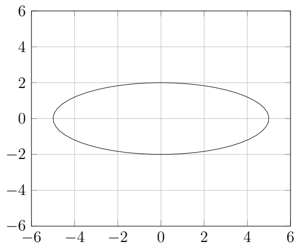
18) Which of the below is a **parametric plot** of $x(t) = e^t + e^{-t}, \quad y(t) = e^t - e^{-t}$?



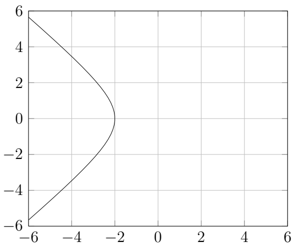
A.



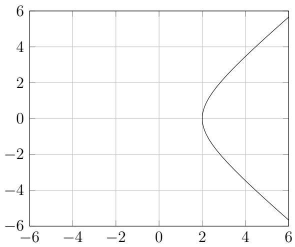
B.



C.



D.



E.

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19) Using the Taylor series $\ln(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots$, centred at $x = 1$, you approximate:

$$\begin{aligned}\ln(0.9) &\approx (-.1) - \frac{(-.1)^2}{2} + \frac{(-.1)^3}{3} - \frac{(-.1)^4}{4} + \cdots + (-1)^{n-1}\frac{(-.1)^n}{n} \\ &= -.1 - \frac{(.1)^2}{2} - \frac{(.1)^3}{3} - \frac{(.1)^4}{4} - \cdots - \frac{(.1)^n}{n}.\end{aligned}$$

You wish to know the **error** in your approximation (the difference between the true value for ln(0.9) and your approximate value). State the conclusion to **Taylor's Inequality**.

A. $|\text{error}| \le \frac{(.1)^n}{n}$ **B.** $|\text{error}| \le \frac{(.1)^{n+1}}{n+1}$ **C.** $|\text{error}| \le \frac{(.9)^{n+1}}{n+1}$

D. $|\text{error}| \le \frac{(.1)^{n+1}}{(n+1)(.9)^{n+1}}$ **E.** $|\text{error}| \le \frac{(.1)^{n+1}}{(n+1)!}$

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20) Suppose u and v are functions of x and y and $f(u,v) = \sin(u^2 v)$. Given the values:

$u(0,1) = \sqrt{\pi}$, $v(0,1) = -1$, $\frac{\partial u}{\partial x}(0,1) = 2$, $\frac{\partial u}{\partial y}(0,1) = 1$, $\frac{\partial v}{\partial x}(0,1) = {}^2/_\pi$, $\frac{\partial v}{\partial y}(0,1) = -{}^1/_{\sqrt{\pi}}$,

use the **chain rule** to compute $\frac{\partial f}{\partial y}(0,1)$.

A. 0 **B.** -1 **C.** $3\sqrt{\pi}$ **D.** $\frac{1}{\sqrt{\pi}}$ **E.** $\frac{-1}{\sqrt{\pi}}$

21) Approximate the value of $5(1.05)^2 + (2.1)^2$. Use the **tangent plane** approximation to $f(x, y) = 5x^2 + y^2$ based at the point $(x, y) = (1, 2)$.

A. 9.9 **B.** 9.92 **C.** 9.70 **D.** 0.90 **E.** 0.92

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22) You place a cafeteria tray at $(x, y, z) = (1, 1, 2)$ on the snow-covered graph of $z = x^2 + y^2$ and you hop aboard. You begin to sled downwards in the direction of steepest descent. Using the **gradient vector**, find the **magnitude** of this slope.

A. 1 **B.** $\sqrt{2}$ **C.** $2\sqrt{2}$ **D.** 2 **E.** 4

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23) $\displaystyle\int_0^1 \int_{y^3}^{y^2} 144xy\; dx\; dy =$

A. 7 **B.** 6 **C.** 5 **D.** 4 **E.** 3

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24) Find the **volume** of the finite region bounded above by the surface $z = \frac{xy^2}{1+x^2}$, below by the xy-plane, on the right by the plane $x = 2$, and in back by the plane $y = 1$.

A. $\dfrac{1}{6}(\ln(5) - 1)$ **B.** $\dfrac{1}{6}\ln(5)$ **C.** $\dfrac{1}{3}(\ln(5) - 1)$ **D.** $\dfrac{1}{3}\ln 5$ **E.** π

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25) Compute the double integral. **Hint**: reverse the integration order.

$$\int_0^{(\pi/3)^{1/4}} \int_{y^2}^{\sqrt{\pi/3}} y\cos(x^2)\; dx\; dy =$$

A. $\sqrt{3}/8$ **B.** $1/8$ **C.** $\pi/4$ **D.** $\sqrt{3}/4$ **E.** $1/4$

Name: ____

Student ID Number: ______________

Instructions: You may ask an invigilator for scratch paper on which to organize your solution, but only this sheet (and the reverse) will be marked.

Extra Credit Problem: Maclaurin Series for e^{-1/x^2}

Show that $f^{(n)}(0) = 0$ for all n where f is the function: $f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

[This means the Maclaurin series for $f(x)$ is 0, which does not converge to $f(x)$!]
If you use induction, clearly identify the induction hypothesis by placing it in a box.

Hint: you might start by showing that for *every* rational function $P(x)$, the function $g(x) = \begin{cases} P(x)e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ has derivative 0 at $x = 0$.