Pages to follow: 6

ENGINEER 2P04
ENGINEERING MECHANICS "A"

Term I
September - December 2009

Section: 1 Dr. K.S.Sivakumaran
Section: 2 Dr. Dean Inglis

Total Marks: 60

TERM TEST: I

SOLUTIONS

14 October 2009

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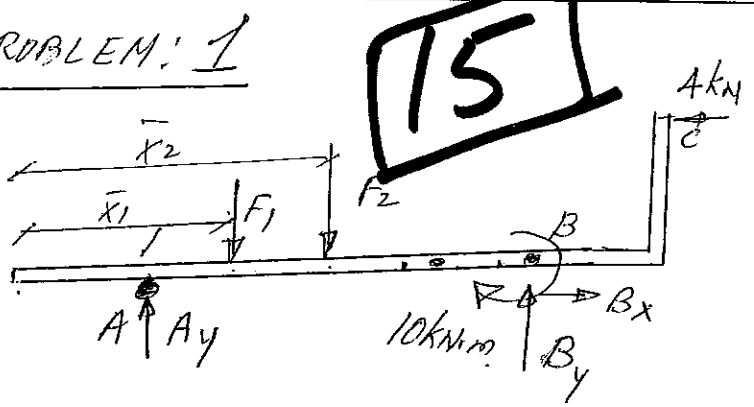
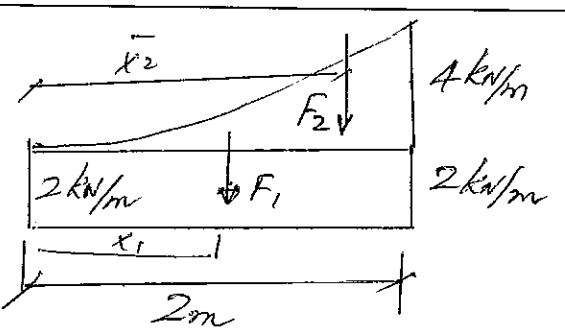
Term Test I Solutions

PROBLEM: 1

15



Free Body diagram for bent-beam ABC



$F_1 = 2m \times 2kN/m = 4kN$

$\bar{x}_1 = \frac{2}{2} = 1m.$

$F_2 = \frac{1}{3} \times 2m \times 4m = 2.67kN$

$\bar{x}_2 = 2m - \frac{1}{4} \times 2m = 1.5m.$

7

8

$\rightarrow$ $B_x - 4kN = 0$ — (1) $\therefore B_x = 4kN.$

$\uparrow$ $A_y + B_y - F_1 - F_2 = 0$ — (2)

$\circlearrowleft M_B$ $-1.8m \times A_y + (2.4 - \bar{x}_1) \times F_1 + (2.4 - \bar{x}_2) \times F_2 + 0.5m \times 4 - 10 = 0.$ — (3)

$\therefore A_y = \dfrac{(2.4-1) \times 4 + (2.4-1.5) \times 2.67 + 0.5 \times 4 - 10}{1.8}$

$\therefore A_y = 0\ kN$

(2) $\Rightarrow$ $B_y = F_1 + F_2 - A_y = 4 + 2.67 - 0 = 6.67kN$

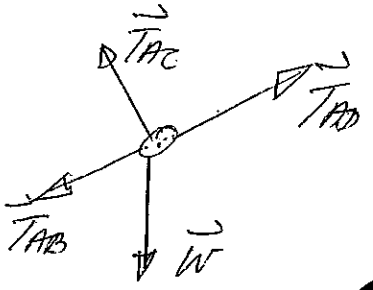
$\therefore$ Support Reaction at A = 0 kN

Support Reaction at B = $B_x = 4kN$

$B_y = 6.67kN.$

PROBLEM: 2. (15)

F.B.D for A



Forces in vector form

$$\vec{T}_{AD} = \left(\frac{-1i + 2j + 2k}{\sqrt{1^2+2^2+2^2}}\right) T_{AD}$$

$$= -0.33\,T_{AD}\,i + 0.67\,T_{AD}\,j + 0.67\,T_{AD}\,k$$

$$\vec{T}_{AC} = (\cos 120^\circ i + \cos 135\, j + \cos 60\, k)\,T_{AC}$$

$$= -0.5\,T_{AC}\,i - 0.707\,T_{AC}\,j + 0.5\,T_{AC}\,k$$

(5)

$$\vec{T}_{AB} = T_{AB}\,i \quad ; \qquad \vec{W} = -W\,k$$

For the equilibrium point O

(4)

$$\vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AD} + \vec{W} = 0$$

$$(-0.33(T_{AB}\,i) + (-0.5\,T_{AC}\,i - 0.707\,T_{AC}\,j + 0.5\,T_{AC}\,k) + (-0.33\,T_{AD}\,i + 0.67\,T_{AD}\,j + 0.67\,T_{AD}\,k) - W\,k = 0$$

(i) $\Rightarrow$ $T_{AB} - 0.5\,T_{AC} - 0.33\,T_{AD} = 0$ — (1)

(j) $\Rightarrow$ $-0.707\,T_{AC} + 0.67\,T_{AD} = 0$ — (2)

(k) $\Rightarrow$ $0.5\,T_{AC} + 0.67\,T_{AD} - W = 0$ — (3)

(2) − (3): $+1.207\,T_{AC} = W$ $\therefore T_{AC} = +0.8285\,W$

(2) $\Rightarrow$ $T_{AD} = \dfrac{0.707\,T_{AC}}{0.67}$ $\therefore T_{AD} = +0.8743\,W$

(6)

(1) $\Rightarrow$ $T_{AB} = 0.5\,T_{AC} + 0.33\,T_{AD}$ $\therefore T_{AB} = 0.7028\,W$

Cable force $\leq 800\,N$

$T_{AC} \Rightarrow 0.8285\,W \leq 800$ $\therefore W \leq 965.6\,N.$

$T_{AD} \Rightarrow 0.8743\,W \leq 800$ $\therefore W \leq 915\,N.$

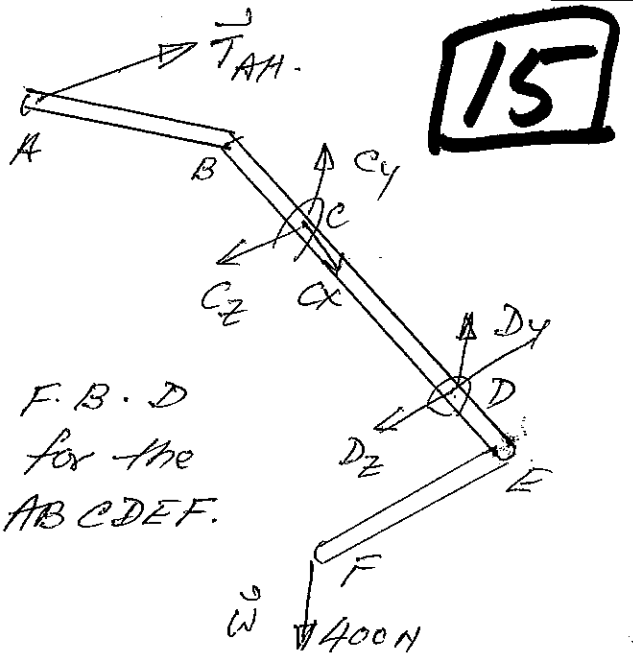
Extension $\Delta = \dfrac{T_{AB}}{k}$

$\therefore \dfrac{0.7028\,W}{1500} \leq 400\times10^{-3}$

$\therefore W \leq 853.7\,N.$

$\therefore$ SPRING GOVERNS and largest weight supported $853.7\,N$

15



F.B.D for the ABCDEF.

Force Reaction at $\vec{C}$

$\vec{F}_c = C_x i + C_y j + C_z k$

Force Reaction at $\vec{D}$ (D does not exert Axial Thrust)

$\vec{F}_D = 0i + D_y j + D_z k.$

Since there are parallel reactions support C and D do not produce moment reactions.

$\vec{r}_A = 0i + 250 \sin 30 j + 250 \cos 30 k$

$= 0i + 125 j + 216.5 k$

Load $\vec{w} = -400 j$;

$\therefore \vec{T}_{AH} = \dfrac{0i + (250-125)j - 216.5)k}{\sqrt{(250-125)^2 + 216.5^2}} T_{AH}$ ← 250

$\vec{T}_{AH} = 0i + 0.5 T_{AH} j - 0.866 T_{AH} k$

For the equilibrium $\sum \vec{F} = 0$

$\therefore \vec{T}_{AH} + \vec{F}_c + \vec{F}_D + \vec{w} = 0$

$0 + 0.5 T_{AH} j - 0.866 T_{AH} k$
$C_x i + C_y j + C_z k$
$0i + D_y j + D_z k$
$-400 j = 0$

16

$C_x = 0$ ——— (1)

$0.5 T_{AH} + C_y + D_y - 400 = 0$ ——— (2)

$-0.866 T_{AH} + C_z + D_z = 0$ ——— (3)

For the moment equilibrium $\sum \vec{M}_c = 0$

$\therefore \vec{r}_{CH} \times \vec{T}_{AH} + \vec{r}_{CD} \times \vec{F}_D + \vec{r}_{CE} \times (-400 j) = 0.$

$\vec{r}_{CH} = -50i + 250 j + 0k$; $\vec{r}_{CD} = 300 i$; $\vec{r}_{CE} = 350i + 0j + 250k$

$$\begin{vmatrix} i & j & k \\ -50 & 250 & 0 \\ 0 & 0.5T_{AH} & -0.866T_{AH} \end{vmatrix} + \begin{vmatrix} i & j & k \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} i & j & k \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

$$[(250 \times -0.866 T_{AH})i - (-50 \times -0.866 T_{AH})j + (-50 \times 0.5 T_{AH})k]$$

$$+ [0i - 300 D_z j + 300 D_y k]$$

$$+ [-(250 \times -400)i - 0j + (350 \times -400)k] = 0$$

$\therefore$ (2) $\Rightarrow -250 \times 0.866 T_{AH} + 250 \times 400 = 0$ ——— (4) $\therefore T_{AH} = 461.9\ N$

(j) $\Rightarrow -50 \times 0.866 T_{AH} - 300 D_z = 0$ ——— (5) $\therefore D_z = -66.7\ N$

(k) $\Rightarrow -50 \times 0.5 T_{AH} + 300 D_y - 350 \times 400 = 0$ ——— (6)

16

$\therefore D_y = \dfrac{350 \times 400 + 50 \times 0.5 \times 461.9}{300}$ $\therefore D_y = 505.2\ N$

(3)

(2) $\Rightarrow C_y = 400 - 0.5 T_{AH} - D_y$ $\therefore C_y = -336.2\ N$

$= 400 - 0.5 \times 461.9 - 505.2$

(3) $\Rightarrow C_z = 0.866 T_{AH} - D_z$ $\therefore C_z = 466.7\ N$

$= 0.866 \times 461.9 - (-66.7)$

$\therefore$ Tension in wire AH $T_{AH} = 461.9\ N$

Reactions at C $C_x = 0$; $C_y = -336.2\ N$, $C_z = 466.7\ N$

Reactions at D $D_x = 0$; $D_y = 505.2\ N$, $D_z = -66.7\ N$

PROBLEM-4

Consider the equilibrium of whole truss.

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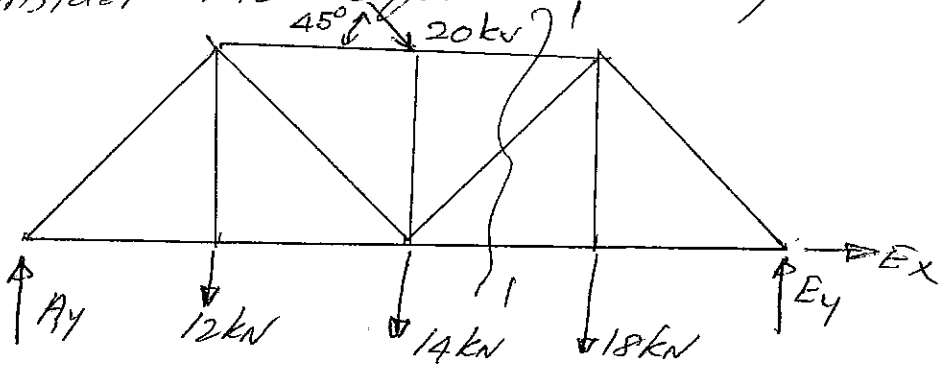


F.B.D for whole truss

$\rightarrow \quad E_x + 20\cos 45^\circ = 0 \quad -(1)$ $\quad E_x = -14.14\ kN$

$\uparrow \quad A_y + E_y - 12 - 14 - 18 - 20\sin 45^\circ = 0 \quad -(2)$

$(\circlearrowleft M_E) \quad -12m \times A_y + 9m \times 12 + 6m \times 14 + 3m \times 18 + 3m \times [20\sin 45^\circ] = 0 \quad -(3)$

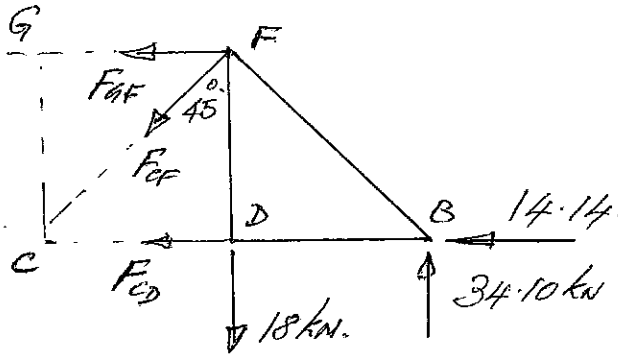
6

(3) => $A_y = 24.04\ kN$. (2) => $B_y = 34.10\ kN$.

9

METHOD OF SECTION.

Consider section 1-1



F.B.D for R.H.S of Section 1-1

$\rightarrow \quad -F_{GF} - F_{CD} - F_{CF}\cos 45^\circ - 14.14 = 0 \quad -(3)$

($F_{GF} = -50.2$)

$\therefore F_{CD} = 19.94\ kN$ (Tension)

$(\circlearrowleft M_C) \quad 3m \times F_{GF} - 3m \times 18 + 6m \times 34.10 = 0 \quad -(4)$

$\therefore F_{GF} = -50.2\ kN$. (Compression)

$\uparrow \quad -F_{CF}\cos 45^\circ - 18 + 34.10 = 0 \quad -(5)$

$\therefore F_{CF} = 22.8\ kN$ (Tension)

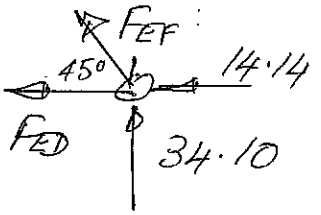
$\therefore$ Forces in Members;

CD = 19.94 kN (Tension); CF = 22.8 kN (Tension); GF = 50.2 kN (Compression)

OR (9)

METHOD OF JOINTS

Joint E

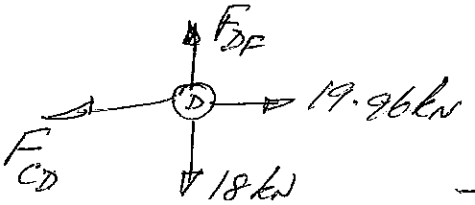


$\uparrow F_{EF}\sin 45^\circ + 34\cdot10 = 0$

$\therefore F_{EF} = -48\cdot22\ kN.$

$\rightarrow -F_{ED} - 14\cdot14 - F_{EF}\cos 45^\circ = 0 \quad \therefore F_{ED} = 19\cdot96\ kN.$

Joint D



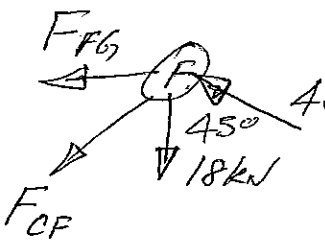
$\uparrow F_{DF} - 18 = 0$

$\therefore F_{DF} = 18\ kN$

$\rightarrow -F_{CD} + 19\cdot96 = 0$

$\therefore F_{CD} = 19\cdot96\ kN.$

Joint F



$\uparrow -F_{CF}\cos 45^\circ - 18 + 48\cdot22\cos 45^\circ = 0$

$\therefore F_{CF} = 22\cdot8\ kN.$

$\rightarrow -F_{FG} - F_{CF}\cos 45^\circ - 48\cdot22\sin 45^\circ = 0$

$\therefore F_{FG} = -50\cdot2\ kN.$

$\therefore$ Forces in Members

CD = 19.96 kN (Tension)

CF = 22.8 kN (Tension)

GF = 50.2 kN (Compression)