Electrical Engineering 3BA3: Structure of Biological Materials

Solutions to Midterm Quiz #1 (2007)

1. **A synthetic biomaterial is a composite of Material #1 with an elastic modulus of** $E_1 = 80\,\text{GPa}$ **and Material #2 with an elastic modulus of** $E_2 = 1.3\,\text{GPa}$**. If the composite's volume is 20% Material #1 and 80% Material #2, then the elastic modulus** E **of the composite is:**

   a. less than 18 GPa,

   b. more than 1.6 GPa,

   c. dependent on how the two materials are arranged in the composite, or

   d. all of the above. **(5 pts)**

   The answer is **d. all of the above**. See notes from Tutorial #1 (or p. 359 or p. 381 of Berger et al.).

2. **The natural biomaterial keratin, found in human hair, is stiffer than collagen because:**

   a. it is a non-metallic mineral,

   b. it has more cross-links,

   c. it absorbs less water, or

   d. none of the above. **(5 pts)**

   The answer is **b. it has more cross-links**. See slide 13 from Lecture #3 (or p. 344 of Berger et al.).

3. **The *final* stage of wound repair is :**

   a. inflammation,

   b. remodelling,

   c. proliferation, or

   d. coagulation/haemostasis. **(5 pts)**

   The answer is **b. remodelling**. See slide 4 from Lecture #5 (or p. 346 of Enderle et al.).

4. **During running, the runner's *vertical velocity* is typically maximal:**

   a. at the middle of the stance phase,

   b. towards the end of the stance phase,

   c. just after the back foot leaves the ground, or

   d. when the runner's center of mass is at its maximal height. **(5 pts)**

   The answer is **b. towards the end of the stance phase**. See slide 9 from Lecture #6 (or p. 393 of Berger et al.).

5. **The stress-life diagram for a material is normally created by:**
   a. **cyclically applying a fixed load until the material fractures and repeating this procedure for a range of loads with a new sample of the material each time,**
   b. **applying a steadily increasing load until the material fractures,**
   c. **cyclically applying a load, starting with a high load and reducing the load whenever fracturing appears imminent, or**
   d. **applying a cyclic load and measuring when the material wears through.** **(5 pts)**

   The answer is **a. cyclically applying a fixed load until the material fractures and repeating this procedure for a range of loads with a new sample of the material each time**. See slides 17–20 from Lecture #8 (or p. 67 or p. 375 of Berger et al.).

6. **During running, the maximal joint power production is greater in the ankle than in the knee because:**
   a. **the ankle's maximal joint moment during running is much greater than the knee's,**
   b. **the time during which the knee's joint moment is positive is more spread out,**
   c. **the knee only absorbs power during running, or**
   d. **the ankle's rotational velocity is greater during the maximal joint moment.** **(5 pts)**

   The answer is **d. the ankle's rotational velocity is greater during the maximal joint moment**. The maximal joint moments are of similar magnitude (around 200 N·m), the durations of the positive joint moments are similar, and the knee produces power as well as absorbing it, so the answer must be that the ankle's rotational velocity is greater. See slide 11 from Lecture #8 (or p. 400 of Berger et al.).

7. **In function electrical stimulation for drop foot, the initial site of action potential generation is normally in:**
   a. **a motor neuron axon,**
   b. **the sarcoplasmic reticulum,**
   c. **a muscle fiber, or**
   d. **the motor cortex.** **(5 pts)**

   The answer is **a. a motor neuron axon**. See Student Presentation #3.

8. **The force that a muscle can generate as a function of its length is maximal when the muscle is at its natural length because:**

   a. **more fast-twitch fibers are recruited at this length,**

   b. **of the faster oxidative metabolism at this length,**

   c. **a greater number of motor units are activated at this length, or**

   d. **of the optimal overlap of actin and myosin filaments at this length.** **(5 pts)**

   The answer is **d. of the optimal overlap of actin and myosin filaments at this length**. See slide 14 from Lecture #7 (or p. 407 of Berger et al.).

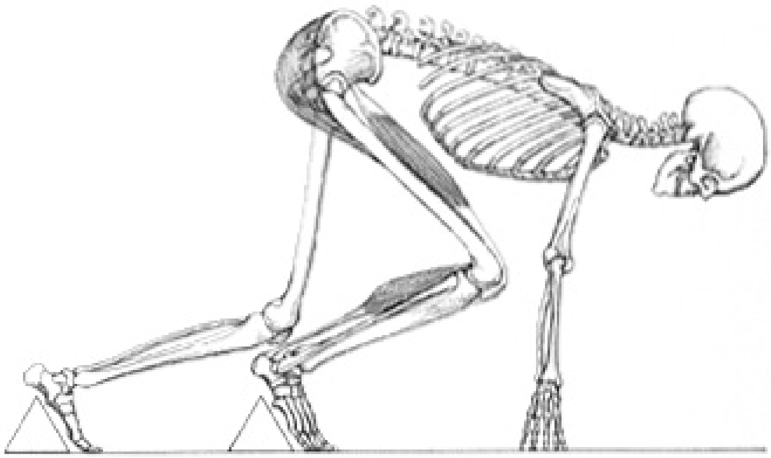
9. **Discuss the bioethical complexities of choosing subjects for clinical trials of biomedical technologies to treat life-threatening diseases.** **(15 pts)**

   The main complexities arise from the possibly-competing goals of obtaining the best measurement of benefit and minimizing risk for the patients (non-maleficence). If the technology has some substantial risk of (i) not working sufficiently to keep the patient alive or (ii) having harmful side-effects, then the only suitable subjects might be those patients for whom all other treatment options have failed and are likely to die soon. However, if such patients are chosen as subjects, then they may be so sick that (a) the full benefit of the technology is not clearly displayed or (b) they die anyway, and it may be difficult to determine whether the biomedical technology contributed to the death or not. Healthier subjects may better display the benefit of the technology, but the risks may be too great, or there may be other viable treatment options still available.

10. **Explain why platinum-iridium alloys are often the preferred material for implanted stimulating electrodes.** **(15 pts)**

   Platinum and iridium are non-toxic and very resistant to corrosion and consequently quite biocompatible. Furthermore, they can deliver fairly large currents to an electrolyte (such as the body's extracellular fluids) without electrode dissolution, production of gasses or increasing the pH of the electrolyte. Iridium is actually even less susceptible to corrosion and production of gasses than platinum, but it is more brittle than platinum and consequently cannot be easily machined into electrodes. Therefore, a platinum-iridium alloy with a greater percentage of platinum (typically 90:10) is preferred. (See slides 39 & 40 of Student Presentation #2.)

11. A sprinter is positioned in their starting blocks as illustrated below.



Assume the following:

- **i.** **The side cross-section of the starting blocks (as illustrated) is an equilateral triangle.**
- **ii.** **The runner is at rest for time $t < 0$ and then begins to propel themselves forward at time $t = 0$.**
- **iii.** **Once the runner begins propelling themselves forward, they lift their hands immediately (i.e., at time $t = 0$).**
- **iv.** **The total ground reaction force at the starting blocks is normal to the front surface of the blocks and acts through the center of mass (COM) of the sprinter.**
- **v.** **The total ground reaction force magnitude at the starting blocks has a constant value of $F_g = 5\,\text{kN}$ from time $t = 0$ to time $t = 0.1\,\text{s}$.**
- **vi.** **The runner has a mass of 90 kg, and the acceleration due to gravity is $g = 9.8\,\text{m/s}^2$.**

What is the increase in the total mechanical energy of the runner's COM from time $t = 0$ to time $t = 0.1\,\text{s}$? (15 pts)

The internal angles between the sides of an equilateral triangle are all 60°, and thus, the front surfaces of the starting blocks are inclined at an angle of 30° from vertical. The ground reaction force (GRF) F_g, which is normal to the front surface of the blocks, is consequently angled 30° from horizontal, as illustrated below. The horizontal and vertical components of the GRF are thus:

$$F_{gy} = F_g \cos\left(30^\circ\right) = 4.33\,\text{kN}, \quad \text{for } 0 \le t \le 0.1\,\text{s},$$

and:

$$F_{gz} = F_g \sin\left(30^\circ\right) = 2.5\,\text{kN}, \quad \text{for } 0 \le t \le 0.1\,\text{s},$$

respectively.

The horizontal acceleration a_y, dependent only on F_{gy}, is given by:

$$a_y = F_{gy}/m = 4.33\times10^3/90 = 48.11 \text{ m}\cdot\text{s}^{-2}, \quad \text{for } 0 \le t \le 0.1\,\text{s}.$$

Consequently, the horizontal velocity $v_y(t)$ varies as a function of time according to:

$$v_y(t) = v_y(0) + \int_0^t a_y \cdot \mathrm{d}s = 0 + \int_0^t 48.11\cdot \mathrm{d}s = 48.11t \text{ m}\cdot\text{s}^{-1}, \quad \text{for } 0 \le t \le 0.1\,\text{s},$$

such that at time $t = 0.1\,\text{s}$, $v_y = 4.811\,\text{m}\cdot\text{s}^{-1}$.

The vertical acceleration a_z, dependent on F_{gz} and the force due to gravity, is given by:

$$a_z = F_{gz}/m - g = 2.5\times10^3/90 - 9.8 = 17.98 \text{ m}\cdot\text{s}^{-2}, \quad \text{for } 0 \le t \le 0.1\,\text{s}.$$

Consequently, the vertical velocity $v_z(t)$ varies as a function of time according to:

$$v_z(t) = v_z(0) + \int_0^t a_z \cdot \mathrm{d}s = 0 + \int_0^t 17.98\cdot \mathrm{d}s = 17.98t \text{ m}\cdot\text{s}^{-1}, \quad \text{for } 0 \le t \le 0.1\,\text{s},$$

such that at time $t = 0.1\,\text{s}$, $v_z = 1.798\,\text{m}\cdot\text{s}^{-1}$.

The vertical displacement $d_z(t)$, relative to the initial height of the COM, varies as a function of time according to:

$$d_z(t) = d_z(0) + \int_0^t v_z \cdot \mathrm{d}s = 0 + \int_0^t 17.98s\cdot \mathrm{d}s = 8.99t^2 \text{ m}, \quad \text{for } 0 \le t \le 0.1\,\text{s},$$

such that at time $t = 0.1\,\text{s}$, $d_z = 0.0899\,\text{m}$.

The initial horizontal and vertical velocities are zero, and hence the initial kinetic energy is zero. The increase in kinetic energy at time $t = 0.1\,\text{s}$ is thus:

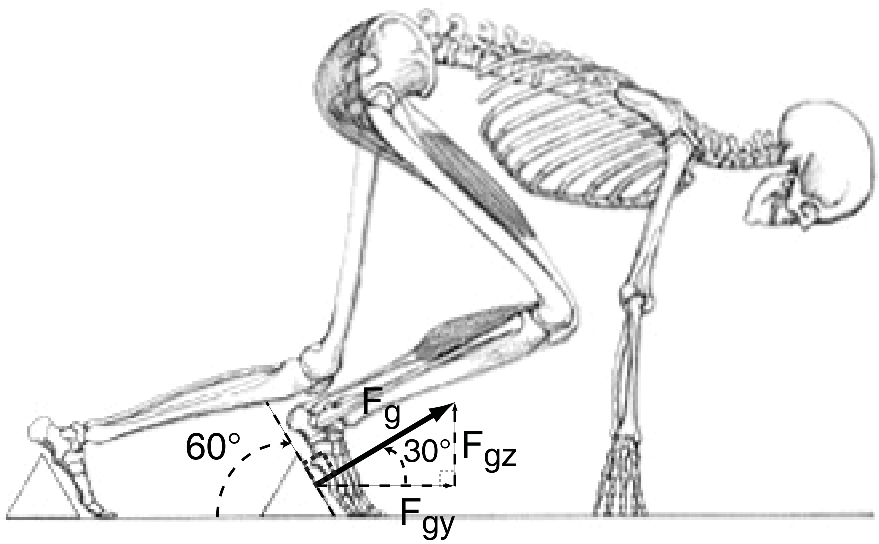
$$\begin{aligned} E_{\text{kin}}(t = 0.1\,\text{s}) &= \frac{1}{2}mv_z^2(t = 0.1\,\text{s}) + \frac{1}{2}mv_y^2(t = 0.1\,\text{s}) \\ &= \frac{1}{2}90\left(4.811^2 + 1.798^2\right) \\ &= 1.19\text{ kJ}, \end{aligned}$$

and the increase in gravitational potential energy at time $t = 0.1\,\text{s}$ is:

$$\begin{aligned} E_{\text{grav}}(t = 0.1\,\text{s}) &= mgd_z(t = 0.1\,\text{s}) \\ &= 90\cdot 9.8\cdot 0.0899 \\ &= 79.28\text{ J}, \end{aligned}$$

giving a total increase in mechanical energy at time $t = 0.1\,\text{s}$ of $E_{\text{tot}} = 1.27\text{ kJ}$.

The calculation of the GRF angle is illustrated below for a single block. Note however that the GRF magnitude for the two blocks combined was given in the question, and the combined GRF was assumed to act directly through the sprinter's COM.



12. A new metal alloy being developed for orthopaedic implants has a tensile stress-strain relationship given by:

$$\tau = \frac{2}{1+e^{-500\varepsilon}} - 1 \quad \text{GPa}, \quad \text{for } 0 \le \varepsilon \le \varepsilon_c,$$

where τ is the tensile stress (in units of GPa), ε is the tensile strain, and ε_c is the strain at failure.

a. If the ultimate tensile strength is 950 MPa, what is the strain at failure ε_c?

b. What is the elastic modulus E for the linear portion of the stress-strain curve?

c. Find the yield stress, if it is defined as the stress at which the slope of the stress-strain curve is only 40% of the slope of the linear portion. (15 pts)

a. According to the given equation, the stress at failure, referred to as the ultimate tensile strength (UTS), is:

$$0.95 = \frac{2}{1+e^{-500\varepsilon_c}} - 1 \quad \text{GPa}.$$

Solving for ε_c gives:

$$\begin{aligned}
&0.95 = \frac{2}{1+e^{-500\varepsilon_c}} - 1 \\
&\Rightarrow 1.95 = \frac{2}{1+e^{-500\varepsilon_c}} \\
&\Rightarrow 1.95\left(1+e^{-500\varepsilon_c}\right) = 2 \\
&\Rightarrow 1+e^{-500\varepsilon_c} = \frac{2}{1.95} \\
&\Rightarrow e^{-500\varepsilon_c} = \frac{2}{1.95} - 1 = \frac{2-1.95}{1.95} = \frac{0.05}{1.95} \\
&\Rightarrow -500\varepsilon_c = \log_e\left(\tfrac{0.05}{1.95}\right) \\
&\Rightarrow \varepsilon_c = -\log_e\left(\tfrac{0.05}{1.95}\right)\Big/500 = 7.327\times10^{-3}.
\end{aligned}$$

b. The stress-strain curve is linear for small ε and becomes non-linear at higher strains. Therefore, the elastic modulus E for the linear portion of the stress-strain curve is the slope of the curve at $\varepsilon = 0$. The derivate of the given equation is:

$$\begin{aligned}
\frac{d\tau}{d\varepsilon} &= \frac{2\cdot(-1)\left(-500\cdot e^{-500\varepsilon}\right)}{\left(1+e^{-500\varepsilon}\right)^2} \\
&= \frac{1000\cdot e^{-500\varepsilon}}{\left(1+e^{-500\varepsilon}\right)^2} \quad \text{GPa}, \quad \text{for } 0 \le \varepsilon \le \varepsilon_c.
\end{aligned}$$

Evaluating the derivate at $\varepsilon = 0$ gives:

$$E = \left.\frac{\mathrm{d}\tau}{\mathrm{d}\varepsilon}\right|_{\varepsilon=0} = \frac{1000\cdot \mathrm{e}^{-500\cdot 0}}{\left(1+\mathrm{e}^{-500\cdot 0}\right)^2} = \frac{1000\cdot 1}{\left(1+1\right)^2} = 250 \quad \text{GPa.}$$

Alternatively, the elastic modulus can be estimated from the ratio of τ to ε **for extremely small values of ε**:

$$E \approx \frac{\tau}{\varepsilon} = \frac{2}{\varepsilon\left(1+\mathrm{e}^{-500\varepsilon}\right)} - \frac{1}{\varepsilon}, \quad \text{for } \varepsilon \ll 1\times 10^{-3}.$$

Evaluating this expression for $\varepsilon = 1\times 10^{-5}$ gives:

$$E \approx \frac{2}{1\times 10^{-5}\left(1+\mathrm{e}^{-500\times 10^{-5}}\right)} - \frac{1}{1\times 10^{-5}} = 249.9995 \approx 250 \text{ GPa.}$$

c. The slope at the yield strength (YS) is 40% of 250 GPa, i.e., 100 GPa. The yield strain ε_y, the strain at which the slope is 100 GPa, can be found according to:

$$\begin{aligned}
&100 = \frac{1000\cdot \mathrm{e}^{-500\varepsilon_y}}{\left(1+\mathrm{e}^{-500\varepsilon_y}\right)^2} \quad \text{GPa} \\
&\Rightarrow \left(1+\mathrm{e}^{-500\varepsilon_y}\right)^2 = 10\cdot \mathrm{e}^{-500\varepsilon_y} \\
&\Rightarrow 1+2\mathrm{e}^{-500\varepsilon_y} + \left(\mathrm{e}^{-500\varepsilon_y}\right)^2 = 10\cdot \mathrm{e}^{-500\varepsilon_y} \\
&\Rightarrow 1-8\mathrm{e}^{-500\varepsilon_y} + \left(\mathrm{e}^{-500\varepsilon_y}\right)^2 = 0.
\end{aligned}$$

This is a quadratic equation with the solutions:

$$\begin{aligned}
\mathrm{e}^{-500\varepsilon_y} &= \frac{-\left(-8\right)\pm\sqrt{\left(-8\right)^2 - 4\cdot 1\cdot 1}}{2\cdot 1} \\
&= \frac{8\pm\sqrt{64-4}}{2} \\
&= 7.873 \quad \text{or} \quad 0.1270.
\end{aligned}$$

Solving for ε_y gives:

$$\begin{aligned}
-500\varepsilon_y = \log_e 7.873 \quad &\text{or} \quad \log_e 0.1270 \quad \Rightarrow \\
\varepsilon_y = \frac{-\log_e 7.873}{500} \quad &\text{or} \quad \frac{-\log_e 0.1270}{500} \\
= \frac{-\log_e 7.873}{500} \quad &\text{or} \quad \frac{-\log_e 0.1270}{500} \\
= \pm 4.127\times 10^{-3}.&
\end{aligned}$$

Since ε_y is a tensile strain, it must be the positive value. Solving for the stress at $\varepsilon = 4.127\times10^{-3}$ gives a yield strength of:

$$\tau = \frac{2}{1+\mathrm{e}^{-500\cdot 4.127\times10^{-3}}} - 1 = 0.7746\ \mathrm{GPa} = 774.6\ \mathrm{MPa}.$$

The results are shown graphically in the figure below.