Last Day (in 1.2 (3))

Linear Systems (continued)

eg. $3x + 2y = 3$

$x - y = 2$

$$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

- $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ → coefficient matrix
- $\begin{bmatrix} x \\ y \end{bmatrix}$ → variables
- $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ → constants

$A\vec{x} = \vec{b}$ Form.

Augmented Matrix form

$[A|b]$ ← often lazy ppl. forget the bar.

$$= \left[\begin{array}{cc|c} 3 & 2 & 3 \\ 1 & -1 & 2 \end{array}\right]$$

Elementary Row Operations

Each row op. preserves the solution set: Has same and all solution before & after operation.

1) Swap any 2 rows eg. $R_1 \leftrightarrow R_2$
2) Multiply by non-zero constant 1 row. eg $R_2 \to 3R_2$

3) Take a multiple of row, add it to another.

$R_1 \to R_1 + 2R_2$

ie Take row 1 & replace with Row 1 added to 2 Row 2

eg. Ops in Action

$$\left.\begin{aligned} 3x + 2y &= 3 \\ x - y &= 2 \end{aligned}\right\} \Rightarrow \left[\begin{array}{cc|c} 3 & 2 & 3 \\ 1 & -1 & 2 \end{array}\right]$$

Now $R_1 \leftrightarrow R_2 \Rightarrow$ $\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 3 & 2 & 3 \end{array}\right]$

$R_2 \to R_2 - 3R_1 \Rightarrow$

$$\begin{bmatrix} 1 & -1 & | & 2 \\ 0 & 5 & | & -3 \end{bmatrix}$$

$\hookrightarrow 5y = -3$

$R_3 \to \frac{1}{5}R_3 \Rightarrow$

$$\begin{bmatrix} 1 & -1 & | & 2 \\ 0 & 1 & | & -3/5 \end{bmatrix}$$

$\Leftarrow$

$y = -3/5$

$x - y = 2 \Rightarrow$

$x = 2 - 3/5$

$= 7/5$

Gaussian Elimination

Perform row operations to reduce our augmented matrix into Row Eschelon Form (RE)

What is Row Ech. Form?

1) each non-zero row
   First non-zero entry an 1
   ie. "leading 1"
2) Each row has it's leading 1 to right of row above.

3) All ~~zero~~ zero rows at the bottom of matrix.

eg. $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & -3/5 \end{bmatrix}$

eg. $\begin{bmatrix} \textcircled{1} & 0 & 2 \\ 0 & \textcircled{1} & 3 \\ 0 & 0 & \textcircled{1} \end{bmatrix}$

eg. $\begin{bmatrix} \textcircled{1} & 2 & 3 & 4 \\ 0 & 0 & \textcircled{1} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

} R.E. Form.

eg. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ Not R.E.

eg. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ✓ RE

eg. $\begin{bmatrix} 0 \end{bmatrix}$ ✓ RE
(Goofy!)

eg. $\begin{bmatrix} 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ✓ RE ✓

Again: Gaussian Elimination

1) Put system into Aug. Matrix
2) Use Row Ops to put into RE Form.

* Start from bottom up & back-substitute.

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eg. $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 2 & 1 & 1 & 3 \\ 3 & 0 & 2 & 2 \end{array}\right]$

↳ Same as:

$$\begin{cases} x - y = 1 \\ 2x + y + z = 3 \\ 3x + 2z = 2 \end{cases}$$

Solve this system!

$$\left[\begin{array}{ccc|c} \textcircled{1} & -1 & 0 & 1 \\ 2 & 1 & 1 & 3 \\ 3 & 0 & 2 & 2 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ \text{then} \\ R_3 \to R_3 - 3R_1 \end{array}$$

("kill" 1st column below leading 1!)

$$\Rightarrow \left[\begin{array}{ccc|c} \textcircled{1} & -1 & 0 & 1 \\ 0 & 3 & 1 & 1 \\ 0 & 3 & 2 & -1 \end{array}\right] \quad R_2 \to R_2/3$$

$$\Rightarrow \left[\begin{array}{ccc|c} \textcircled{1} & -1 & 0 & 1 \\ 0 & \textcircled{1} & 1/3 & 1/3 \\ 0 & 3 & 2 & -1 \end{array}\right] \quad R_3 - 3R_2$$

$$\Rightarrow \left[\begin{array}{ccc|c} \textcircled{1} & -1 & 0 & 1 \\ 0 & \textcircled{1} & 1/3 & 1/3 \\ 0 & 0 & \textcircled{1} & -2 \end{array}\right]$$

REF form! ✓

Last eqn: $z = -2$

2nd eqn

$$y + \frac{1}{3}z = \frac{1}{3}$$

$$\equiv$$

$$y = \frac{1}{3} - \frac{1}{3}(-2) = 1$$

1st eqn

$$x - y = 1$$

$$\equiv$$

$$x = 1 + (1) = 2$$

$$x = 2,\ y = 1,\ z = -2$$

Gaussian Elimination

Eg.

$$\left[\begin{array}{ccc|c} \textcircled{1} & 3 & 0 & 1 \\ 2 & -1 & 1 & 2 \\ 3 & 2 & 1 & 3 \end{array}\right]$$

Solve via Gaussian Elimination!

Solution

$R_2 \to R_2 - 2R_1$ & then $R_3 \to R_3 - 3R_1$

(Note: each changes a different row, & does not depend on results of other!

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & -7 & 1 & 0 \\ 0 & -7 & 1 & 0 \end{array}\right]$$

$R_3 - R_2$

(Get it over with!)

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 7 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_2\left(\tfrac{1}{7}\right)$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & 1 \\ 0 & 1 & -\tfrac{1}{7} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{RE form!}$$

Last Eqn $\quad 0 = 0$ whoa!

2nd Eqn $\quad y - \frac{1}{7}z = 0$

$$y = z/7$$

1st. Eqn $\quad x + 3y + 0z = 1$

$$x = 1 - 3(z/7)$$

Let $z = t$, a parameter.

$$\begin{cases} x = 1 - \frac{3}{7}t \\ y = \frac{1}{7}t \\ z = t \end{cases}$$

(∞ solutions) parametric eqn. of a line!