Lecture 2014-01-29

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Math objects made using [MathType](); graphs made using [Winplot]().

The POWER of the Equations

You are not ready to inherit its power.

LTI

$x(n) = e^{i\omega n}$

$y(n) = H(\omega) e^{i\omega n}$

$x(n) = \cos(\omega n)$

$y(n) = |H(\omega)| \cos(\omega n + \phi + \sphericalangle H(\omega))$

$$H(\omega) = \frac{\sum_{k=0}^{N} \alpha_k \left(e^{-i\omega}\right)^k}{1 - \sum_{k=1}^{M} \beta_k \left(e^{-i\omega}\right)^k}$$

FIR

A finite number of values that don't converge to 0

$y(n) = \sum_{k=0}^{N} \alpha_k x(n-k)$

e.g. 1)

$y(n) = x(n-1)$

$H(\omega) = e^{-i\omega}$

$x(n) = e^{i\omega n}$

$x(n-1) = e^{-i\omega} e^{i\omega n}$

$y(n) = x(n)$

$x(n) = e^{i\omega n}$

$y(n) = "$

e.g. 2)

$y(n) = \frac{1}{2} x(n) + \frac{1}{2} x(n-1)$

$H(\omega) = \frac{1}{2} + \frac{1}{2} e^{-i\omega}$

$x(n) = 1 + \cos\left(\frac{\pi}{2}n\right) + \cos(\pi n)$

ω	H(ω)
0	1←Gain
π/2	½ + ½i $= \frac{1}{2}e^{-i\frac{7\pi}{4}}$
π	0

Conjugate Complex Symmetry

$H(-\omega) = \overline{H(\omega)}$

$x^2 - 2 = 0$

$x = \pm\sqrt{2}$

$\left|H(-\omega)\right| = \left|\overline{H(\omega)}\right|$

$\sphericalangle H(-\omega) = -\sphericalangle H(\omega)$

Sampling Frequency: $f_s = \dfrac{1}{\Delta T}$

e.g. 3)

$f = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

Pitch = $f \cdot f_s$

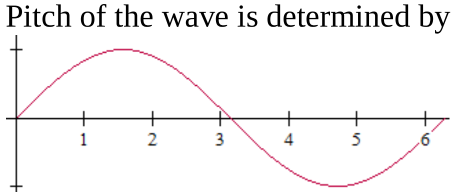
$\omega = 2\pi f = \pi$

Fourier

You’re not alone; people have been learning this since ~1880, while trying to figure out the heat equation

$x(t)$ is p-periodic

Pitch of the wave is determined by the length of the string



$f = \frac{1}{p}$

Fundamental frequency: $\omega_0 = \dfrac{2\pi}{p}$

Timbre/harmonics: determined by combination of frequencies

$$x(t) \approx \underbrace{A_0}_{\text{DC term}} + \sum_{k=1}^{\infty} A_k \cos\left(k\omega_0 t + \phi_k\right)$$

Harmonics: $k\omega_0, k > 1$

Coordinate transformation

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\overbrace{\boxed{A_k \cos(\phi_k)}}^{\tilde{A}_k}\cos(\omega_0 kt) + \overbrace{\boxed{-A_k \sin(\phi_k)}}^{\tilde{B}_k}\sin(k_0\omega_0 t)$$

$$x(t) \approx A_0 + \sum_{k=1}^{\infty} \tilde{A}_k \cos(\omega_0 kt) + \sum_{k=1}^{\infty} \tilde{B}_k \sin(\omega_0 kt)$$

$$A_k = \frac{1}{P}\int_0^P x(t)\cos(\omega_0 kt)\,\mathrm{d}t$$

Even a square wave is a sum of sinusoidals

$$\frac{1}{2\pi}\left(\int_0^{\pi}\cos(\omega_0 kt)\,\mathrm{d}t - \int_{\pi}^{2\pi}\cos(\omega_0 kt)\,\mathrm{d}t\right)$$

Complex numbers will be cleaner than partial differential equations, so we’ll just stick with that.

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(\omega_0 kt + \phi_k)$$

$$= A_0 + \sum_{k=1}^{\infty} \boxed{A_k \tfrac{1}{2} e^{i\phi k}} e^{i\omega_0 kt} + \boxed{A_k \tfrac{1}{2} e^{-i\phi k}} e^{-i\omega_0 kt}$$

$$= A_0 + \sum_{k=1}^{\infty} X_k e^{i\omega_0 kt} + \sum_{k=-1}^{-\infty} X_k e^{i\omega_0 kt}$$

The phase shifts go into our complex exponentials