MATH 1A03/1ZA3 Fall 2013
Midterm 2 - Version 3
November 14 2013
Duration: 90 minutes

Instructors: M. Bays, D. Haskell, E. Harper, C.McLean

Name: _____

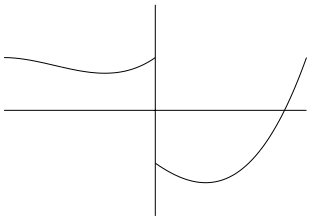
Student ID Number: _____

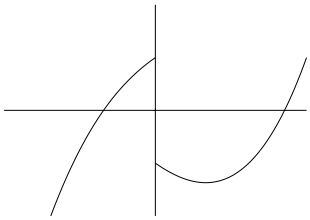
This test paper is printed on both sides of the page. There are 20 question on 10 pages. You are responsible for ensuring that your copy of this test is complete. Bring any discrepancies to the attention of the invigilator.

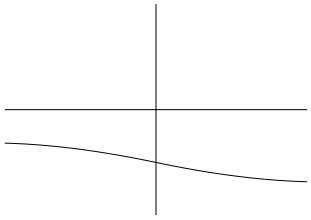
Instructions

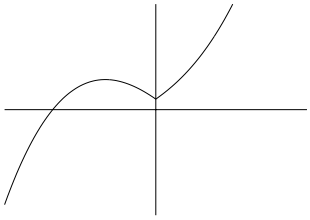
1. Write your name, ID number, and **version number** on the computer card.
2. All answers must be entered on the computer card with an HB pencil. Read the marking instructions on the card.
3. Each question is worth one mark. No marks will be deducted for wrong answers or blank answers.
4. Any question left blank will receive 0 marks, even if the correct answer is circled on the exam page. You **must enter your answers on the computer card.**
5. Scratch paper is available for rough work; ask the invigilator.
6. A calculator may be used for this exam. Only the official McMaster calculator, the Casio-fx-991, is allowed.

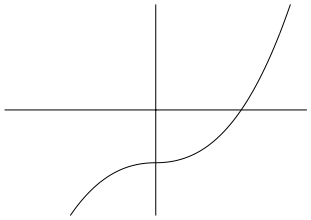
1. Suppose $f''(x) < -1$ for all $x < 0$, and $f''(x) > 1$ for $x > 0$. Which of the following is certainly **NOT** the graph of f?

A) 

B) 

C) 

D) 

E) 

---

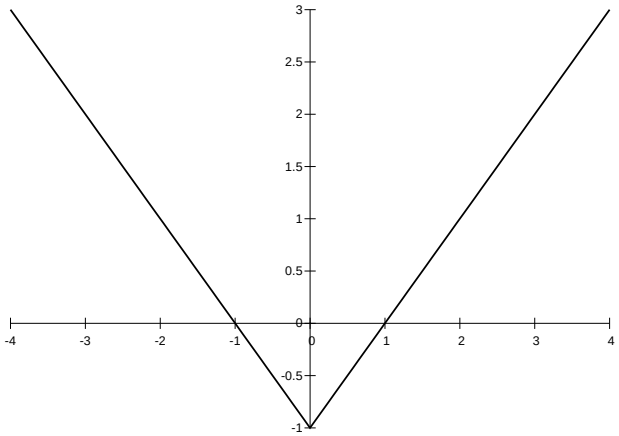
2. Find $\int_1^e \left(4x^{-1} + \cos(4x)\right) dx$.

A) $4 + \dfrac{\sin(4) - \sin(4e)}{4}$ B) $\dfrac{-2}{e^2} - 2 + \sin(4e) - \sin(4)$ C) $\dfrac{\sin(4e) - \sin(4)}{4}$

D) $\dfrac{16 + \sin(e)}{4}$ E) $4 + \dfrac{\sin(4e) - \sin(4)}{4}$

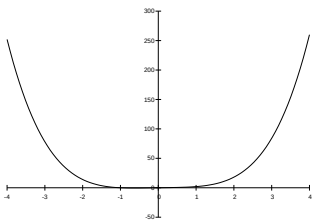
Continued on page 3

3. Suppose $f''(x)$ has the following graph:

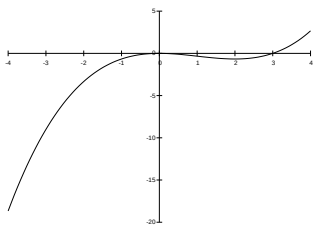


Which of the following could be the graph of $f(x)$?

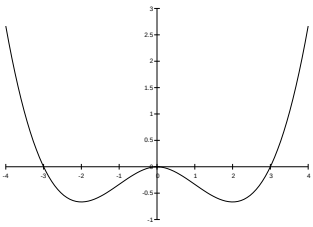
A)



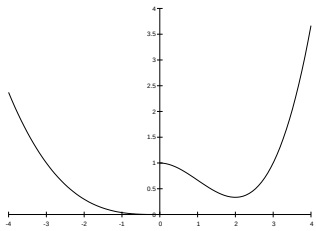
B)



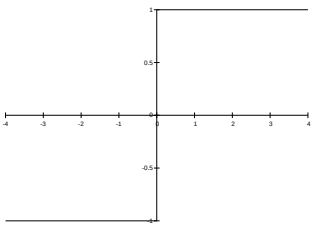
C)



D)



E)



Continued on page 4

4. Which of the following represents the sum $-\frac{3}{2}+\frac{5}{4}-\frac{7}{6}+\frac{9}{8}-\frac{11}{10}$?

A) $\sum_{i=0}^{9}(-1)^{i+1}\frac{2i+1}{2i+2}$ B) $\sum_{i=0}^{4}(-1)^{i}\frac{2i+1}{2i+2}$ C) $\sum_{i=1}^{4}(-1)^{i}\frac{2i+1}{2i+2}$

D) $\sum_{i=1}^{5}(-1)^{i+1}\frac{2i-1}{2i}$ E) $\sum_{i=1}^{5}(-1)^{i}\frac{2i+1}{2i}$

---

5. Which of the following formulas is correct?

(I) $\int \ln(x)\,dx = \frac{1}{|x|}+C$

(II) $\int \cos^3(x)\,dx = \frac{1}{4}\cos^4(x)\sin(x)+C$

(III) $\int \sin(x)\cos(x)\,dx = \frac{1}{2}\sin^2(x)+C$

A) (II) and (III) B) (III) only C) (II) only

D) (I) and (III) E) (I) only

---

6. Evaluate $\int_{1/4}^{3/4}\cos(2\pi x)\,dx$.

A) $-\frac{1}{\pi}$ B) 0 C) -2

D) $\frac{1}{\pi}$ E) 2

Continued on page 5

7. Suppose f is a differentiable function, $f(0) = 0$, and $|f(x)| \leq 1$ for all x in $[0, 1]$. Which of the following must be true?

(I) $|f'(x)| \leq 1$ for some x in $[0, 1]$

(II) $|f'(x)| \leq 1$ for all x in $[0, 1]$

(III) $|f'(x)| = 1$ for some x in $[0, 1]$

A) (I) and (III) only B) (I) only C) (III) only
D) (I) and (II) only E) (I), (II) and (III)

---

8. Which point or points on the parabola with equation $y = x^2$ are closest to the point $(x, y) = (0, 2)$?

A) $\left(\frac{3}{2}, \frac{9}{4}\right)$ and $\left(-\frac{3}{2}, \frac{9}{4}\right)$ B) $\left(\sqrt{\frac{3}{2}}, \frac{3}{2}\right)$ and $\left(-\sqrt{\frac{3}{2}}, \frac{3}{2}\right)$
C) $(1, 1)$ and $(-1, 1)$ D) $(0, 0)$
E) All points on the parabola are equally close to this point.

---

9. Which of the following are true of $f(x) = \dfrac{x}{(x^2 - 1)\sin(x)}$

(I) f has a vertical asymptote at $x = 0$

(II) f has a vertical asymptote at $x = -1$

(III) f has a vertical asymptote at $x = \pi$

A) (I) only B) (I) and (II) only C) (II) and (III) only
D) (III) only E) (I),(II) and (III)

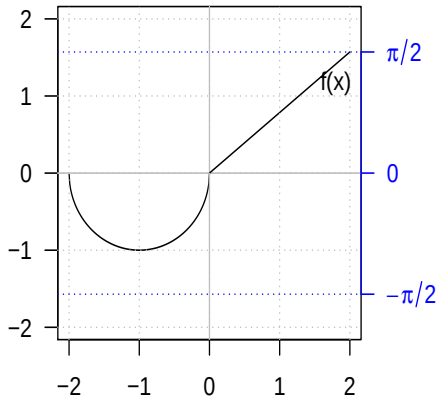
Continued on page 6

10. Which of the following functions satisfy $\int_{-2}^{2} f(x)\,dx = 0$?

(I) $f(x) = e^{x^2+4}$

(II) $f(x) = (4x)^7 - \sin(x)$

(III) $f(x)$ as plotted to the right:



A) (II) & (III) B) (I) & (III) C) (I), (II), & (III)
D) (I) & (II) E) none of (I), (II), or (III)

---

11. Evaluate $\int_{-\pi/2}^{\pi/2} |\sin(x)|\,dx$.

A) 0 B) -2 C) 1 D) 2
E) The integral does not exist because $|\sin(x)|$ is not differentiable.

Continued on page 7

12. If $f(x)$ is differentiable and always strictly increasing on $[a, b]$, what must be true about $\int_a^b f'(x)\,dx$?

A) $\int_a^b f'(x)\,dx = 0$ B) $\int_a^b f'(x)\,dx < 0$

C) $\int_a^b f'(x)\,dx > 0$ D) $\int_a^b f'(x)\,dx = f(b)$

E) Nothing can be said about the definite integral.

---

13. Find $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

A) $\ln(\sqrt{x}) + C$ B) $\sqrt{x}e^{\sqrt{x}} + C$ C) $2\sqrt{x}e^{\sqrt{x}} + C$

D) $2e^{\sqrt{x}} + C$ E) $e^{\sqrt{x}} + C$

---

14. Find $\int \cot(x)dx$.

A) $-\csc^2(x) + C$ B) $\tan(x) + C$ C) $\ln(\sin(x)) + C$

D) $\sec^2(x) + C$ E) $\ln(\cos(x)) + C$

---

15. Suppose f is a differentiable function and $\lim_{x\to+\infty} f(x) = +\infty$. What is $\lim_{x\to+\infty} f(x)e^{-f(x)}$?

A) e B) $-\infty$ C) $+\infty$ D) $\frac{1}{f'(0)}$ E) 0

Continued on page 8

16. Which of the following expresses the right-endpoint Riemann sum approximation to $\int_2^4 e^x\,dx$ with four endpoints?

A) $R_4 = \sum_{i=0}^{3} e^{\frac{i}{2}} \frac{1}{2}$ B) $R_4 = \sum_{i=0}^{3} e^{2+\frac{i}{2}} \frac{1}{2}$ C) $R_4 = \frac{1}{2} \sum_{i=1}^{4} e^{2+\frac{i}{2}}$

D) $R_4 = \sum_{i=0}^{3} e^{i} \frac{1}{2}$ E) $R_4 = \sum_{i=1}^{4} e^{i+\frac{1}{2}}$

---

17. On which of the following intervals does the Mean Value Theorem hold for the function $f(x) = |x^2 - 1|$?

(I) $[-2, 0]$ (II) $[-1, 1]$ (III) $[0, 2]$

A) (III) only B) (I) and (III) only C) (II) only D) (I), (II), (III)
E) (I) only

Continued on page 9

18. You are given a very nasty function g as a Maple function, and you want to find its limiting behavior at infinity. You enter the Maple command

```
limit(g(x),x=infinity);
```

and get the response

```
Float(undefined)
```

What can you do?

(I) Forget Maple and apply L'Hôpital's rule; this will always work.

(II) Use Maple to evaluate g at increasingly larger inputs to try to find a limiting value.

(III) Use Maple to plot a graph of g, with domain including very large inputs, to try to observe the limiting value.

A) (II) only B) (I) and (II) C) (III) only
D) (I) only E) (II) and (III)

---

19. A marble is dropped into a deep lake of thick syrup. It enters the lake with a vertical velocity of -1 metres per second. Once in the lake, it is pulled down by gravity - but this is partially counteracted by drag effect of the syrup, resulting in a vertical acceleration of $a(t) = -5e^{-5t}$ metres per second per second t seconds after entering the lake.

At what depth is the marble 1 second after entering the lake?

A) $\frac{9}{5} + \frac{e^{-5}}{5}$ B) $2 - e^{-5}$ C) $\frac{1}{5}e^{-5} + \frac{1}{5}$ D) $5e^{-5}$ E) $125e^{-5}$

---

20. A cylindrical steel oil drum must be made to hold 32L of oil. The steel used to manufacture the lid and the base of the oil drum costs \$10 per square metre, while the steel used for the rest of the drum costs \$5 per square metre. What is the radius that minimizes the cost of manufacturing one oil drum?

A) $2\sqrt{2}\pi$ B) $2\sqrt{2}\pi^{-1/3}$ C) $2\pi^{-1/3}$ D) 2π E) 0

END OF QUESTIONS

Continued on page 10

Summation formulae

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

END OF TEST PAPER