Principal curvatures

Let us consider a smooth surface shown in Figure 1. Let us consider point M at this surface. Let us choose such Cartesian system that:

1. Its origin is in M ;
2. xy plane coincides with the tangent plane passing through M .

In this coordinate system, the surface in the vicinity of M can be presented as $z = z(x,y)$. Since the origin of the Cartesian system is in M , $z(0,0)=0$. Due to our choice of the xy plane, the following is true: $\left(\partial z(x,y)/\partial x\right)_{\substack{x=0\\y=0}} = 0$, $\left(\partial z(x,y)/\partial y\right)_{\substack{x=0\\y=0}} = 0$.

Let us decompose $z = z(x,y)$ into the MacLoren's series in the vicinity of M :

$$z = \frac{1}{2}\left(\frac{\partial^2 z}{\partial x^2}\right)_{\substack{x=0\\y=0}} x^2 + \left(\frac{\partial^2 z}{\partial x \partial y}\right)_{\substack{x=0\\y=0}} xy + \frac{1}{2}\left(\frac{\partial^2 z}{\partial y^2}\right)_{\substack{x=0\\y=0}} y^2 + \text{higher-oder terms}$$

By a proper rotation of the coordinate system around the $z-$axis, it is always possible to attain the following expression:

$$z = \frac{1}{2}\left(k_1 x^2 + k_2 y^2\right) + \text{higher-oder terms}$$

k_1 and k_2 are called principal curvatures at point M .

$R_1 \equiv 1/k_1$ and $R_2 \equiv 1/k_2$ are called principal radii of curvature at point M .

$K \equiv k_1 k_2$ is the Gaussian curvature at point M .

$H \equiv (k_1 + k_2)/2$ is the mean (average) curvature at point M .

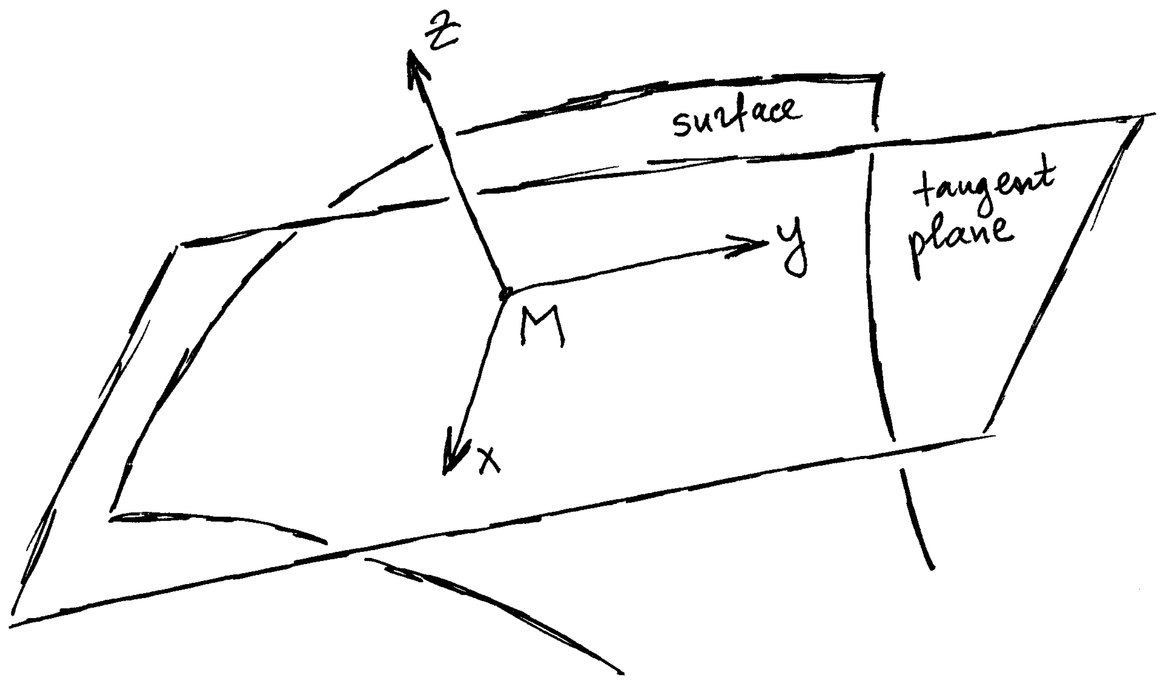

Figure 1