Electrical Engineering 3BA3: Structure of Biological Materials

Solutions to Midterm Quiz #1 (2008)

1. **Tendons and ligaments:**
    - **a. have very rubbery stress-strain relationships,**
    - **b. contain a greater proportion of elastin than collagen,**
    - **c. have fairly crystalline stress-strain relationships, or**
    - **d. are manufactured by chondrocytes.** **(5 pts)**

    The answer is **c. have fairly crystalline stress-strain relationships**. Fig. 8.3 of Berger et al. shows that tendons and ligaments have very little curvature in their stress-strain relationships. When compared to other tissue such as cartilage and skin that exhibit very rubbery elasticity, then tendon and ligament must be considered to be fairly crystalline (see curves marked T and L in Fig. 8.6 on slide 22 of Lecture #3 and text on pp. 346–347 of Berger et al.).

2. **Maintenance of healthy bone requires the normal functioning of:**
    - **a. fibroblasts only,**
    - **b. osteoclasts only,**
    - **c. osteoblasts only, or**
    - **d. both osteoblasts and osteoclasts.** **(5 pts)**

    The answer is **d. both osteoblasts and osteoclasts**. See slide 16 of Lecture #3 or pp. 345–346 of Berger et al.

3. **The *first* stage of wound repair is :**
    - **a. inflammation,**
    - **b. remodelling,**
    - **c. proliferation, or**
    - **d. coagulation/haemostasis.** **(5 pts)**

    The answer is **d. coagulation/haemostasis**. See slide 4 of Lecture #5.

4. ***Carbon materials* have not been successful utilized in orthopaedic implants because:**
    - **a. they all have very poor biocompatibility,**
    - **b. nobody has thought of it yet,**
    - **c. it is not yet possible to obtain suitable mechanical properties, or**
    - **d. they are much too heavy.** **(5 pts)**

    The answer is **c. it is not yet possible to obtain suitable mechanical properties**. See slide 20 of Lecture #4 or pp. 357–358 of Berger et al.

5. **To prevent infection, synthetic biomaterials:**
   a. **must be sterilized using steam under pressure,**
   b. **must be sterilized using radiation,**
   c. **must be sterilized using ethylene oxide gas, or**
   d. **can be sterilized using any of the above.** **(5 pts)**

   The answer is **d. can be sterilized using any of the above**. See slide 16 of Lecture #5 or p. 349 of Berger et al.

6. **The chemical *acetylcholine* (ACh) is:**
   a. **the neurotransmitter used in neuromuscular junction (i.e., the synapse between a motor neuron and a muscle fiber),**
   b. **a constituent chemical of nylon,**
   c. **present in fast glycolytic muscle fibers but not in slow oxidative fibers, or**
   d. **the chemical that causes myosin filaments to move along actin filaments in myofibrils.** **(5 pts)**

   The answer is **a. the neurotransmitter used in neuromuscular junction (i.e., the synapse between a motor neuron and a muscle fiber)**. See slides 5 & 6 of Lecture #7.

7. **In a normal running motion, the *hip* joint:**
   a. **has a much higher maximum power than the maximum knee or ankle power,**
   b. **only produces power,**
   c. **only absorbs power, or**
   d. **is mainly involved in placing the leg in the correct position for each phase of the stance.** **(5 pts)**

   The answer is **d. is mainly involved in placing the leg in the correct position for each phase of the stance**. The hip's maximum power is lower than that of the knee and ankle during running, and the hip's power takes both positive and negative values (i.e., it both produces and absorbs power), so its main function is to move the leg to the correct position for each part of the stride, rather than producing or adsorbing large amounts of power (see slide 12 of Lecture #8 or pp. 400–401 of Berger et al.)

8. *Thin* split-thickness skin grafts:

a. damage the donor site more than do *full-thickness* grafts,

b. can be stretched to cover a greater recipient area than can *full-thickness* grafts,

c. include only a portion of the epidermis and none of the dermis, or

d. look like normal skin immediately after transplantation. (5 pts)

The answer is **b. can be stretched to cover a greater recipient area than can full-thickness grafts**. See slides 26–30 of Student Presentation #3.

9. Discuss briefly:

a. why *cost-benefit analysis* is important for the application of technology to healthcare, and

b. how the cost-benefit analysis of technology to treat a non-life-threatening condition could be compared to that for a life-threatening condition. (15 pts)

a. Cost-benefit analysis is primarily used in healthcare to determine how to distribute the budget of a government healthcare program, an insurance scheme or a hospital across all the possible expenses in a fair and efficient manner. For example, it may be feasible to provide an expensive treatment if the number of patients requiring it is small, but a substantial benefit would need to be justified if an expensive treatment were to be proscribed for a large patient population. However, in a healthcare system with a limited budget, this would take away resources from other budget areas. Therefore, the relative cost and benefit of different treatments needs to be considered. In order to make comparisons across different forms of treatment, factors such as quality of life and life expectancy typically end up being given a monetary value in practice. While this may seem somewhat callous, it is required to equitably allocate resources amongst all possible healthcare costs.

   An important feature of cost-benefit analysis is to determine what may be hidden costs or benefits. For example, an expensive surgery may end up costing less in the long run than not performing the surgery and subsequently needing to hospitalize or provide drugs for the patient over a long period of time.

   Cost-benefit analysis is also performed by manufacturers of biomedical technologies to demonstrate how a new treatment would affect a healthcare budget.

b. While we might wish to save a human life at any cost, in practice it is necessary to balance the resources allocated to emergency rooms, intensive care units and specific staff and treatments for life-threatening conditions with those for non-life-threatening conditions. Consequently, it is necessary to derive an appropriate way of incorporating both quality of life and life expectancy into a single measure to determine the relative costs and benefits of different treatments that may primarily provide improved quality of life and those that could provide an extended life expectancy.

One way of doing this is to use a metric that is the product of a measure of life quality with the expected years of life remaining. The benefit of a treatment is then quantified by how much this product (quality of life × life expectancy) is increased by the treatment. Consequently, an increase in both life expectancy and quality of life scores highest. Either an increase in quality of life for a reasonable life expectancy or an increased life expectancy with a reasonable quality of life would score moderately well. Treatments that provide an increased quality of life but cannot provide a good life expectancy would not score well; nor would treatments that provide a long life expectancy but with very poor quality of life.

An alternative method is to poll the public to determine how important certain aspects of quality of life are to people in our society. Their value can be compared to the value of extending life expectancy by asking the individuals polled how many years of life they might be willing to give up in order to have a specific increase in quality of life. For example, how many years of life would you be willing to give up in order to be able to live in a mostly independent manner at home rather than being hospitalized for the remainder of your life.

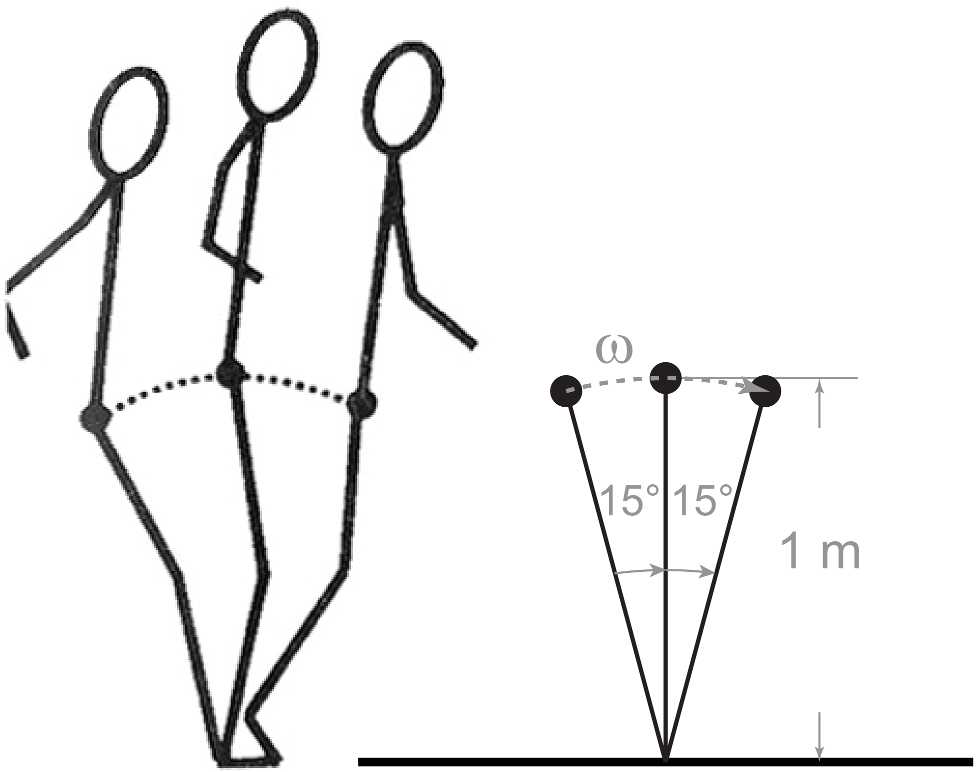
10. Briefly explain the difficulties involved with anchoring of metal alloy prostheses in bone and compare the advantages and disadvantages of using cements versus direct osseointegration. (15 pts)

Metal alloys are very different in chemical structure to natural biological materials, so biological tissues such as bone do not easily adhere to metal alloy prostheses, and the body's immune response tends to encapsulated the metal with fibrous tissue (slide 11 of Lecture #5).

Cements provide a quick and fairly reliable way to attach some metal alloy prostheses to bone (slides 35 & 46 of Student Presentation #6), but when setting, cements such as PMMA polymerize with an exothermic reaction that may kill surrounding bone cells (slide 11 of Lecture #5). In addition, cements are more brittle than bone and metal alloy implants, so they are liable to fracture (slide 19 of Lecture #9) allowing migration of the implant. Fractured or worn cement particles may also aggravate bone tissue and weaken it (slide 46 of Student Presentation #6).

Direct osseointegration can provide a stronger bond between the implant and the bone, but it takes a lot longer to develop (slide 47 of Student Presentation #6). In addition, a more precise fitting of the implant to the bone is required (slide 47 of Student Presentation #6). To promote attachment, the surface of the implant can be treated by roughening it or making it porous (slides 6 & 7 of Lecture #5), but ingrowth of fibrous tissue can hinder osseointegration and cause the attachment to fail (slides 6 & 7 of Lecture #5 and slides 18 & 19 of Student Presentation #8).

11. Consider the inverted-pendulum model of walking illustrated below.



Assume the following:

i. **The person has a mass of 70 kg, and the acceleration due to gravity is** $g = 9.8\ \mathrm{m/s^2}$ **.**

ii. **The person's leg joints move such that their centre of mass (COM) moves in an arc with a constant radius of 1 m around the pivot point in their ankle throughout the single-stance phase of walking, as depicted above.**

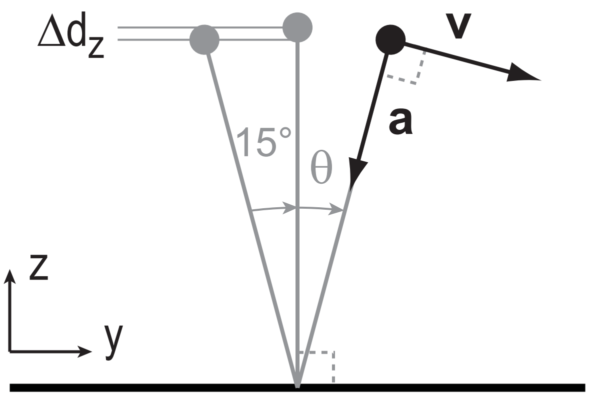
iii. **A single-foot stance phase begins at time** $t = 0$ **with the COM at 15° from vertical (relative to the pivot point in the ankle) and ends with the COM at 15° past vertical at time** $t = 0.5\ \mathrm{s}$**.**

iv. **The person is walking in such a way that their COM has a constant angular velocity** $\omega = 60\ {}^{\circ}/\mathrm{s}$ **for** $0 \le t \le 0.5\ \mathrm{s}$ **.**

Determine the following:

a. **The change in gravitational potential energy** ΔE_{grav} **from the lowest point in the single-stance phase (i.e., at** $t = 0$**) to the highest point (i.e., when the COM is directly above the pivot point in the ankle).**

b. **Expressions for the vertical ground reaction force** $F_{gz}(t)$ **and horizontal ground reaction force** $F_{gy}(t)$ **that would be measured during the period** $0 \le t \le 0.5\ \mathrm{s}$ **according to this model and the given assumptions.** **(15 pts)**



a. The change in gravitational potential energy ΔE_{grav} is dependent on the change in vertical height Δd_z of the COM according to:

$$\Delta E_{\text{grav}} = mg\Delta d_z \,.$$

As depicted in the figure above, at time $t = 0$ the COM is at a height of:

$$d_z\left(t=0\right) = 1\cos\left(-15^\circ\right)\text{ m} = 0.9659\text{ m} \,.$$

Therefore, the change in vertical height is $\Delta d_z = 1 - 0.9659\text{ m} = 0.0341\text{ m}$ and the change in gravitational potential energy is:

$$\Delta E_{\text{grav}} = 70\cdot 9.8\cdot 0.0341 = 23.37\text{ J} \,.$$

b. One approach to determining the ground reaction force components is to first calculate the centripetal acceleration for motion with a constant angular velocity in an arc, as shown in the figure above. For an angular velocity $\omega = 60\ {}^\circ\!/\text{s}$ at a radius of 1 m, the COM is moving with a linear velocity of magnitude:

$$V = \left|\mathbf{V}\right| = \frac{60}{180}\cdot\pi\cdot 1\ \text{ m/s} = 1.0472\ \text{ m/s} \,.$$

Consequently, the centripetal acceleration is:

$$a = \left|\mathbf{a}\right| = \frac{V^2}{r} = \frac{\left(1.0472\right)^2}{1}\ \text{ m/s}^2 = 1.0966\ \text{ m/s}^2 \,.$$

The vertical component of the acceleration a_z changes with the angle θ of the COM (with positive θ corresponding to clockwise rotation past vertical) according to:

$$a_z\left(\theta\right) = -1.0966\cos\left(\theta\right)\ \text{ m/s}^2 \,.$$

Note that the negative sign is required in the front of this equation because the centripetal acceleration is downwards while the positive z direction is upwards.

Likewise the horizontal component a_y varies according to:

$$a_y\left(\theta\right) = 1.0966\sin\left(-\theta\right)\ \text{ m/s}^2 \,.$$

Note that the negative sign is required within the parentheses of the latter equation because the horizontal component of the centripetal acceleration is positive (i.e., forwards for the positive y direction being forwards) for negative θ and negative (i.e., backwards) for positive θ.

In order to calculate the ground reaction force components, it is necessary to determine how θ changes as a function of time. For a constant angular velocity of $\omega = 60\ {}^\circ\!/\mathrm{s}$, the angle changes with t according to:

$$\theta(t) = -15 + 60t\ {}^\circ = -\frac{\pi}{12} + \frac{\pi}{3}t \text{ rad},$$

giving vertical and horizontal acceleration, respectively, of:

$$a_z(t) = -1.0966\cos\left(-\frac{\pi}{12} + \frac{\pi}{3}t\right)\ \mathrm{m/s^2} \quad \text{and} \quad a_y(t) = 1.0966\sin\left(\frac{\pi}{12} - \frac{\pi}{3}t\right)\ \mathrm{m/s^2}.$$

Thus, the ground reaction forces components will be:

$$F_{gz}(t) = mg + ma_z(t) = 70\cdot 9.8 - 70\cdot 1.0966\cos\left(-\frac{\pi}{12} + \frac{\pi}{3}t\right)\ \mathrm{N}$$
$$= 686 - 76.7636\cos\left(-\frac{\pi}{12} + \frac{\pi}{3}t\right)\ \mathrm{N}$$

and

$$F_{gy}(t) = ma_y(t) = 70\cdot 1.0966\sin\left(\frac{\pi}{12} - \frac{\pi}{3}t\right)\ \mathrm{N}$$
$$= 76.7636\sin\left(\frac{\pi}{12} - \frac{\pi}{3}t\right)\ \mathrm{N}.$$

An alternative approach to determine the vertical and horizontal components of the acceleration is first to calculate the vertical and horizontal components of the linear velocity $\mathbf{V}$. As shown in the figure on the previous page, the vertical velocity changes with the angle θ of the COM according to:

$$v_z(\theta) = 1.0472\sin(-\theta)\ \mathrm{m/s}.$$

Note that the negative sign is required within the parentheses of this equation because the vertical component of the velocity is positive (i.e., upwards for the positive z direction being upwards) for negative θ and negative (i.e., downwards) for positive θ.

Likewise, the horizontal velocity is:

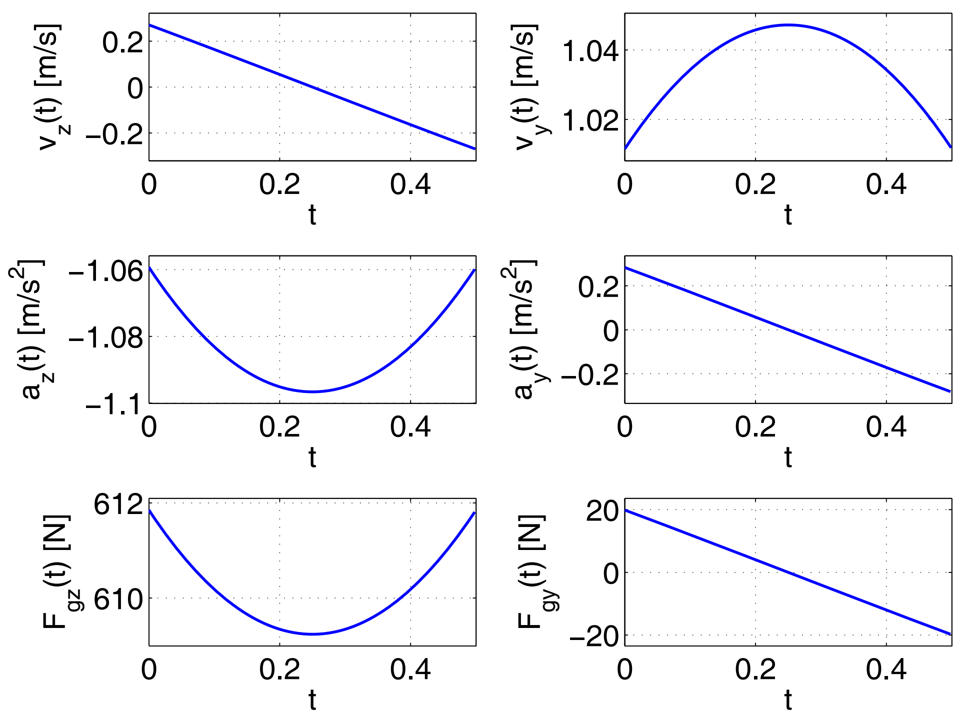
$$v_y(\theta) = 1.0472\cos(\theta)\ \mathrm{m/s}.$$

Given the change in θ with t derived above, these velocities vary with time according to:

$$v_z(t) = 1.0472\sin\left(\frac{\pi}{12} - \frac{\pi}{3}t\right)\ \mathrm{m/s} \text{ and } v_y(t) = 1.0472\cos\left(-\frac{\pi}{12} + \frac{\pi}{3}t\right)\ \mathrm{m/s}.$$

The vertical and horizontal components of the COM's acceleration can be found by taking the derivates of the velocity equations with respect to time, which gives equations for $a_z(t)$ and $a_y(t)$ that are identical to those found using the first method, and consequently identical equations for the ground reaction force components.

A third method is to derive expressions for the vertical and horizontal displacement of the COM as a function of time, take derivates with respect to time to obtain the vertical and horizontal velocity and then take derivates again to obtain the vertical and horizontal acceleration.



(Note that the horizontal velocity increases then decreases, corresponding to a positive horizontal ground reaction force then a negative force. This is the opposite of what is seen as a result of normal frictional forces according to slides 5 and 9 of Lecture #6. The horizontal ground reaction force observed in this problem would only be obtained if the foot were anchored to the ground, rather than being restricted by frictional forces. Consequently, it is not possible to maintain a constant angular velocity of your COM when walking normally.)

12. The total force F_T produced by a muscle as a function of muscle length is the sum of two components, the activate force F_A produced by myofibril contraction and the passive force F_P produced by the stress-strain characteristics of the muscle.

Consider contraction of a particular muscle for which the activate force F_A depends on the muscle length according to the relationship:

$$F_A = \begin{cases} 0, & \text{for } L < 0.5, \\ 20 - 80\left(L-1\right)^2, & \text{for } 0.5 \le L \le 1.5, \\ 0, & \text{for } L > 1.5, \end{cases}$$

where F_A has units of N and L is the muscle length relative to (i.e., divided by) its resting length.

The muscle's passive force F_P depends on the muscle's relative length according to the relationship:

$$F_P = \begin{cases} 0, & \text{for } L < 0.75, \\ 40\left(L-0.75\right)^3, & \text{for } L \ge 0.75. \end{cases}$$

where F_P has units of N.

a. At what relative length L *within the muscle's normal operating range of* $0.5 \le L \le 1.2$ is the total force $F_T = F_A + F_P$ maximal?

b. What is the maximal total force (i.e., the total force at the relative length L found in part a. above)? (15 pts)

a. The total force within the muscle's normal operating range of $0.5 \le L \le 1.2$ is:

$$F_T = \begin{cases} 20 - 80\left(L-1\right)^2, & \text{for } 0.5 \le L < 0.75, \\ 20 - 80\left(L-1\right)^2 + 40\left(L-0.75\right)^3, & \text{for } 0.75 \le L \le 1.2. \end{cases}$$

The force within the range $0.75 \le L \le 1.2$ must be greater than the force within the range $0.5 \le L < 0.75$, so it is only necessary to determine the maximum of the equation $20 - 80\left(L-1\right)^2 + 40\left(L-0.75\right)^3$ for $0.75 \le L \le 1.2$.

Taking the derivative of this expression gives:

$$\begin{aligned} \frac{\mathrm{d}F_T}{\mathrm{d}L} &= -160\left(L-1\right) + 120\left(L-0.75\right)^2 \quad \text{for } 0.75 \le L \le 1.2 \\ &= -160L + 160 + 120\left(L^2 - 2\cdot 0.75L + 0.75^2\right) \\ &= 120L^2 - 340L + 227.5, \end{aligned}$$

and equating this quadratic equation to zero gives solutions of:

$$\frac{dF_T}{dL} = 120L^2 - 340L + 227.5 = 0 \quad \text{for } 0.75 \le L \le 1.2$$

$$\Rightarrow L = \frac{340 \pm \sqrt{340^2 - 4 \cdot 120 \cdot 227.5}}{2 \cdot 120} = \frac{13}{12} \text{ or } \frac{7}{4}.$$

The first solution corresponds to a local *maximum* at $L \approx 1.0833$, which is within the normal operating range. The second solution corresponds to a local *minimum* at $L = 1.75$, which is outside the range of values of L for which this expression is valid—the actual local minimum is at $L = 1.5$, as shown on the plot below. Therefore the total force is maximal when the muscle is 13/12 times its resting length.

b. The total force at $L = 13/12$ is:

$$F_T = 20 - 80\left(\frac{13}{12} - 1\right)^2 + 40\left(\frac{13}{12} - 0.75\right)^3 = 20.93 \text{ N}.$$

Note that the maximum active force in 20 N, so the passive stress-strain characteristics of the muscle have added another 0.93 N to the maximum total force and shifted the length at which the maximum is achieved by one-twelfth of the resting length.