

# MATLS 3T04 “Phase Transformations” 2013-2014

DM's solutions of Test 3 problems

## Problem 1

$a$	$b$	$c$	$d$
$C^{L/\beta}(\infty)$	$C^{L/\beta}(\lambda)$	$C^{L/\alpha}(\lambda)$	$C^{L/\alpha}(\infty)$

$\Delta T_1$  is supercooling needed to create interfaces;  $\Delta T_2$  is supercooling needed to drive diffusion.

$$\underbrace{\Delta C_1}_{C^{L/\alpha}(\lambda) - C^{L/\beta}(\lambda)} = \underbrace{\Delta C_2}_{C^{L/\alpha}(\infty) - C^{L/\beta}(\infty)} \left(1 - \frac{\lambda_{\min}}{\lambda}\right)$$

## Problem 2

If GP zones are spherical, then the lattice parameters of two components are close, which means

that  $\eta = \frac{1}{a_0} \left( \frac{da}{dx} \right)_{a=a_0}$  in  $\frac{\partial^2 G}{\partial x^2} + 2\eta^2 \frac{\frac{E}{\text{Young's modulus}}}{1 - \frac{\nu}{\text{Poisson's ratio}}} \frac{V}{\text{molar volume}} = 0$  is small, which suggests that the

gaps are closer.

If, however, Young's (pictured) modulus of the Al–Ag FCC is much greater than that of Al–Cu FCC, then a smallness of  $\eta$  as such does not mean that the whole term is smaller.

## Problem 3

Even if you did not remember that

$$v = \frac{1}{2} \frac{C^\gamma - C_0}{\sqrt{(C_0 - C^\alpha)(C^\gamma - C^\alpha)}} \sqrt{\frac{D^\gamma}{t}} \quad (1)$$

you had enough time to derive (1). By making a usual assumption about molar volumes, you can write (1) as

$$v = \frac{1}{2} \frac{x^\gamma - x_0}{\sqrt{(x_0 - x^\alpha)(x^\gamma - x^\alpha)}} \sqrt{\frac{D^\gamma}{t}} \quad (2)$$

	First alloy ( $T = 700^\circ\text{C}$ )	Second alloy ( $T = 600^\circ\text{C}$ )
$x_0$	0.2	
$x^\gamma$	0.3	0.4
$x^\alpha$	0.12	0.16
$\frac{x^\gamma - x_0}{\sqrt{(x_0 - x^\alpha)(x^\gamma - x^\alpha)}}$	0.83	2.04

Apparently,  $\frac{x^\gamma - x_0}{\sqrt{(x_0 - x^\alpha)(x^\gamma - x^\alpha)}}$  is not very different for the two alloys, but the difference

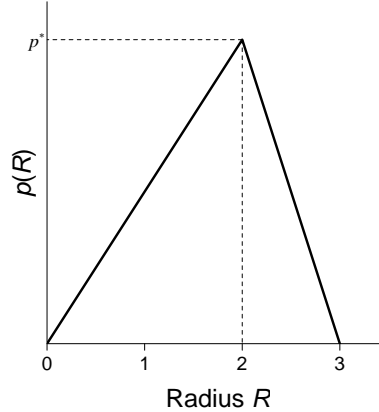
between  $D^\gamma$  at  $700^\circ\text{C}$  and  $600^\circ\text{C}$  would be significant. We can thus conclude that  $\alpha$ -precipitates were growing faster in the first alloy.

### Problem 4

Two possible reasons:

1. Decarburization may change the carbon concentration in steel quite dramatically.
2. Grain size after a prolonged austenitization maybe much greater than that for which the TTT diagram was constructed. Porter and Easterling forgot to transfer info on the average grain size into their book. What a shame, eh?

### Problem 5 (bonus) – 25 marks



Let us use the normalizing condition, which is valid for any probability density function:

$$\int_{-\infty}^{\infty} p(R) dR = \int_0^3 p(R) dR = \int_0^2 p(R) dR + \int_2^3 p(R) dR = \frac{1}{2} \times 2 \times p^* + \frac{1}{2} \times 1 \times p^* = \frac{3}{2} p^* = 1 \quad (3)$$

It is clear from (3) that  $p^* = \frac{2}{3}$ . Consequently, an explicit expression can be written for our probability density function:

$$p(R) = \begin{cases} \frac{R}{3}, & 0 \leq R \leq 2 \\ 2 - \frac{2R}{3}, & 2 < R \leq 3 \end{cases}$$

The average radius of the precipitates is:

$$\bar{R} = \int_0^3 R p(R) dR = \int_0^2 R \frac{R}{3} dR + \int_2^3 R \left( 2 - \frac{2R}{3} \right) dR = \frac{5}{3} \approx 1.67$$

The average volume of the precipitates is:

$$\bar{V} = \int_0^3 \frac{4\pi R^3}{3} p(R) dR = \int_0^2 \frac{4\pi R^3}{3} \frac{R}{3} dR + \int_2^3 \frac{4\pi R^3}{3} \left( 2 - \frac{2R}{3} \right) dR = \frac{524\pi}{135} \approx 12.19$$

$\bar{V}$  is different from  $\frac{4\pi(\bar{R})^3}{3} = \frac{500\pi}{81} \approx 19.39$ , which is not surprising, indeed. If it is not “indeed” for you, then please read again about probability density functions.