

Question 1 (33 points)

Solution:

a)

$$Z_{L_1} = j2\pi fL_1 = j3.768 = 3.768\angle 90^\circ \Omega \quad [2]$$

$$Z_{L_2} = j2\pi fL_2 = j7.536 = 7.536\angle 90^\circ \Omega \quad [2]$$

$$Z_{C_1} = 1/j2\pi fC_1 = -j5.307 = 5.307\angle -90^\circ \Omega \quad [2]$$

$$Z_{eqc} = R_3 + Z_{L_2} + Z_{C_1} = 20 + j2.229 = 20.123\angle 6.36^\circ \Omega \quad [2]$$

$$\begin{aligned} Z_{in} &= Z_{L_1} + Z_{eqc} \parallel (R_1 + R_2) \\ &= Z_{L_1} + \frac{Z_{eqc} \cdot (R_1 + R_2)}{Z_{eqc} + R_1 + R_2} \\ &= 10.030 + j4.323 = 10.921\angle 23.33^\circ \Omega \end{aligned} \quad [3]$$

$$F_p = \cos(\angle 23.33^\circ) = 0.918 \quad (\text{Lagging}) \quad [2]$$

b)

$$v(t) = 848 \sin(\omega t + 30^\circ) \rightarrow V = 600\angle 30^\circ V \quad [2]$$

$$I = V / Z_{in} = 54.94\angle 6.67^\circ A \quad [2]$$

$$S = VI^* = 32964\angle 23.33^\circ VA \quad [2]$$

$$P = |S| \cdot \cos(\angle 23.33^\circ) = 30271.5 W \quad [2]$$

$$Q = |S| \cdot \sin(\angle 23.33^\circ) = 13048.4 VAR \quad [2]$$

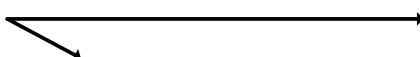
$$\begin{aligned} |S| &= 32964 [VA] \\ Q &= 13048.4 [VAR] \\ \theta &= 23.33^\circ \\ P &= 30271.5 [W] \end{aligned}$$


[4]

c)

$$I_{R_2} = \frac{Z_{eqc}}{Z_{eqc} + (R_1 + R_2)} \cdot I = 27.187 + j4.713 = 27.593\angle 9.84^\circ A \quad [2]$$

(Use $v(t)$ as the reference)

$$\begin{aligned} V &= 600\angle 0^\circ [V] \\ I_{R_2} &= 27.6\angle -20.16^\circ [A] \end{aligned}$$


[4]

Question 2 (30 points)

Solution:

a)

$$Z_1 = j2\pi fL_1 + 1/j2\pi fC_1 = j6.029 - j10.616 = -j4.587 = 4.587\angle -90^\circ \Omega \quad [2]$$

$$Z_2 = R_1 + 1/j2\pi fC_2 = 8 - j10.616 = 13.292\angle -53.03^\circ \Omega \quad [2]$$

$$Z_3 = R_2 + j2\pi fL_2 = 3 + j1.13 = 3.206\angle 20.65^\circ \Omega \quad [2]$$

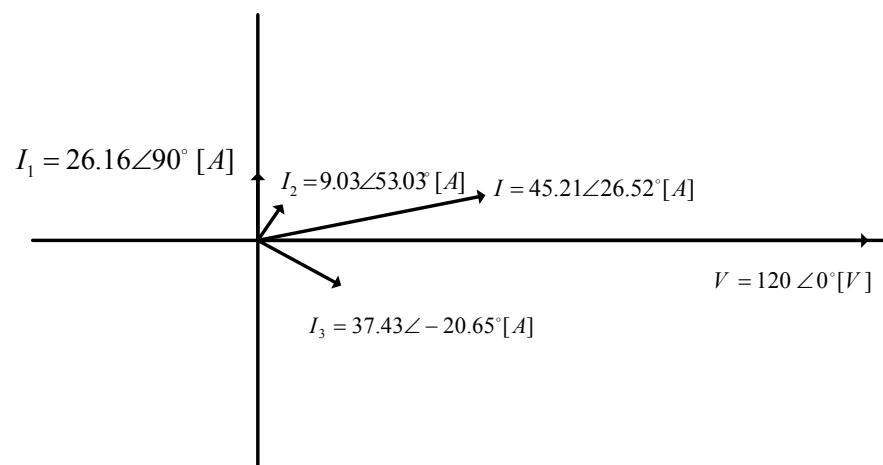
b)

$$I_1 = \frac{120\angle 0^\circ}{Z_1} = j26.16 = 26.16\angle 90^\circ A \quad [2]$$

$$I_2 = \frac{120\angle 0^\circ}{Z_2} = 5.434 + j7.212 = 9.03\angle 53.03^\circ A \quad [2]$$

$$I_3 = \frac{120\angle 0^\circ}{Z_3} = 35.03 - j13.19 = 37.43\angle -20.65^\circ A \quad [2]$$

$$I = I_1 + I_2 + I_3 = 40.46 + j20.18 = 45.21\angle 26.52^\circ A \quad [2]$$



[4]

c)

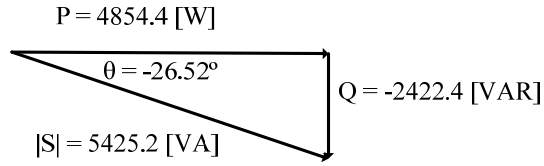
$$S = VI^* = 5425.2\angle -26.52^\circ VA \quad [2]$$

$$P = |S| \cdot \cos(\angle -26.52^\circ) = 4854.4 W \quad [2]$$

$$Q = |S| \cdot \sin(\angle -26.52^\circ) = -2422.4 VAR \quad [2]$$

$$F_p = \cos(\angle -26.52^\circ) = 0.895 \text{ (Lagging)} \quad [2]$$

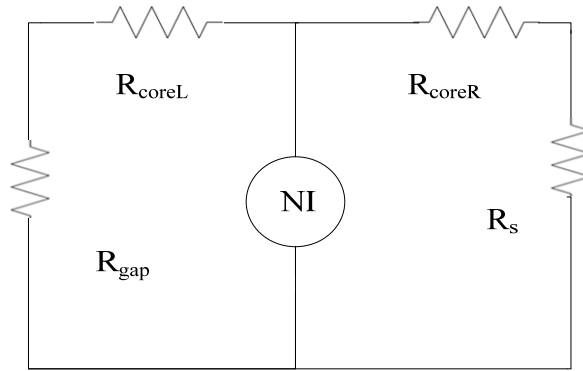
d)



[4]

Question 3 (35points)

Solution:



[4]

b)

$$R_{gap} = \frac{l_{gap}}{\mu_0 A_g} = 7.96 \times 10^6 \text{ A} \cdot t / Wb \quad [3]$$

$$R_s = \frac{l_s}{\mu_0 \mu_{r,s} A_s} = 239 \text{ A} \cdot t / Wb \quad [3]$$

$$R_{coreL} = \frac{l_{coreL}}{\mu_0 \mu_{r,c} A_c} = 3.98 \times 10^3 \text{ A} \cdot t / Wb \quad [3]$$

$$R_{coreR} = \frac{l_{coreR}}{\mu_0 \mu_{r,c} A_c} = 3.98 \times 10^3 \text{ A} \cdot t / Wb \quad [3]$$

$$R_{in} = (R_{coreL} + R_{gap}) \parallel (R_{coreR} + R_s) = 4216.8 \text{ A} \cdot t / Wb \quad [3]$$

c)

$$\Phi = \frac{NI}{R_{in}} = 0.3557 \text{ Wb} \quad [2]$$

$$\Phi_s = \frac{R_{coreL} + R_{gap}}{R_{coreL} + R_{gap} + R_{coreR} + R_s} \Phi = 0.3555 \text{ Wb} \quad [3]$$

$$\Phi_{gap} = \frac{R_{coreR} + R_s}{R_{coreL} + R_{gap} + R_{coreR} + R_s} \Phi = 0.0002 \text{ Wb} \quad [3]$$

d)

$$B_{gap} = \frac{\Phi_{gap}}{A_g} = 0.1 \text{ Wb} / m^2 \quad [2]$$

$$B_s = \frac{\Phi_s}{A_s} = 177.75 \text{ Wb} / m^2 \quad [2]$$

$$\text{e) } H_{gap} = \frac{B_{gap}}{\mu_0} = 8 \times 10^4 \text{ A} \cdot t / m \quad [2]$$

$$H_s = \frac{B_s}{\mu_0 \mu_{r,s}} = 1.4152 \times 10^3 \text{ A} \cdot t / m \quad [2]$$