**Generalized Conjunctions and Disjunctions**

Define by *primitive recursion*.

$$C_n = \bigcap_{i=1}^{n} p_i$$

$$D_n = \bigcap_{i=1}^{n} p_i$$

Base case:

$$D_0 \equiv F$$

$$[D_1 \equiv p_1]$$

Recursive Step:

$$D_{n+1} \equiv D_n \vee p_{n+1}$$

$$D_1 \equiv D_0 \vee p_1$$

$$D_1 \equiv F \vee p_1 |==| p_1$$

Empty conjunction: $C_0 \equiv F$

$$[C_1 \equiv p_1]$$

$$C_{n+1} \equiv C_n \wedge p_{n+1}$$

$$C_1 \equiv C_0 \wedge p_1$$

$$C_1 \equiv T \wedge p_1 |==| p_1$$

Suppose you had a set:

$$S = \{p, q, \ldots\}$$

What does it mean if the disjunction of the set is true? There is at least one term that is true.

$$If\ S = \varnothing$$

$$\bigcup S = False$$

$$\bigcap S = True$$

The conjunction of an empty set is true because you can't show a term that is false.

Generalized Distributive Law:

$$\left(\bigcup_{i=1}^{n} p_i\right) \wedge q |==| \bigcap_{i=1}^{n} p_i \wedge q$$

Generalized Demorgan's Law:

$$\neg\left(\bigcup_{i=1}^{n} p_i\right) |==| \bigcap_{i=1}^{n} \neg p_i$$