McMaster University Math 1A03/1ZA3 Summer 2013
Midterm 1
May 22, 2013

Duration: 75 minutes

Instructor: R. Conlon

Name: ____

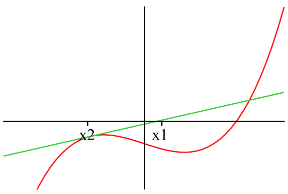
Student ID Number: ________________

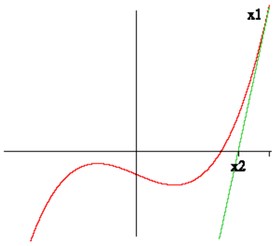
This test paper is printed on both sides of the page. There are 15 questions on 5 pages. You are responsible for ensuring that your copy of this test is complete. Bring any discrepancies to the attention of the invigilator.

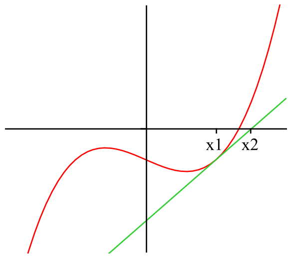
Instructions

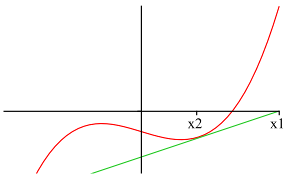
(1) No calculators are allowed.
(2) Write your name and ID number on the computer card.
(3) All answers must be entered on the computer card with an HB pencil. Read the marking instructions on the card.
(4) Each question is worth one mark. No marks will be deducted for wrong answers or blank answers.
(5) Any question left blank will receive 0 marks, even if the correct answer is circled on the exam page. You **must enter your answers on the computer card.**
(6) Scratch paper is available for rough work; ask the invigilator.

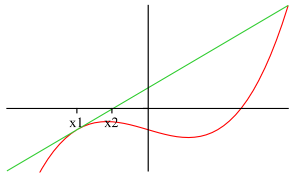
1) Consider the function $f(x) = x^3 - x - 1$. We use Newton's method with $x_1 = 1$ to approximate the solution to $f(x) = 0$. Which of the following graphs correctly shows the calculation of x_2 from x_1?

A.


B.


C.


D.


E.


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2) If f and g are continuous functions with $f(1) = 5$ and $\lim_{x\to 1}(2f(x) - g(x)) = 4$, then $g(1)$ equals

A. 14 **B.** 5 **C.** 3 **D.** 6 **E.** 0

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3) $\cosh(2x) - \sinh(2x) =$

A. $2e^{-2x}$ **B.** $2e^{2x}$ **C.** e^{-2x} **D.** e^{2x} **E.** $\dfrac{e^{2x}}{2}$

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4) The function $f(x) = |x^2 - 1|$ is NOT differentiable at the points

A. $x = 1$ and $x = -1$ only **B.** $x = 1$ only **C.** $x = -1$ only
D. $x = 0$ only **E.** $x = -1$, $x = 1$, and $x = 0$

5) The equation of the tangent line to the graph of $y = \dfrac{1}{1+x^2}$ at the point $(-1, \frac{1}{2})$ is:

A. $x = \dfrac{1}{2}y - \dfrac{5}{4}$ **B.** $y = -x - \dfrac{1}{2}$ **C.** $y = \dfrac{1}{2}x - 1$

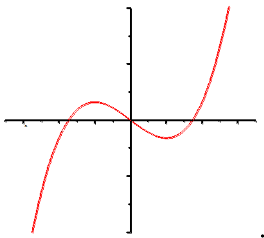
D. $y = -\dfrac{1}{2}x$ **E.** $y = \dfrac{1}{2}x + 1$

6) Given the function $h(x) = \dfrac{e^{2x}}{1+e^{2x}}$, which of the following is the inverse function?

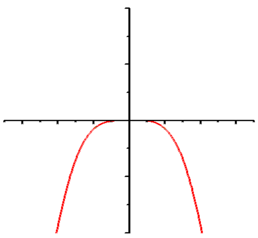
A. $h^{-1}(x) = \dfrac{x\ln(2)}{1-2\ln(x)}$ **B.** $h^{-1}(x) = \dfrac{1}{2}\ln\left(\dfrac{x}{1-x}\right)$ **C.** $h^{-1}(x) = \dfrac{-\ln(x)\cdot\ln(2)}{1-\ln(x)}$

D. $h^{-1}(x) = \dfrac{1+e^{2x}}{e^{2x}}$ **E.** $h^{-1}(x) = 2\ln\left(\dfrac{x}{x-1}\right)$

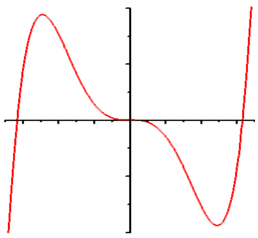
7) The graph of the derivative of a function is shown.



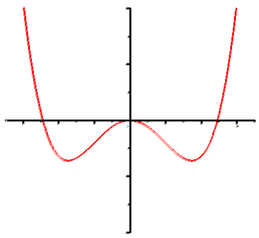
Which of the following could be the graph of the actual function?



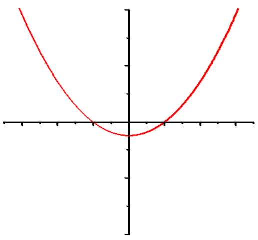
A.



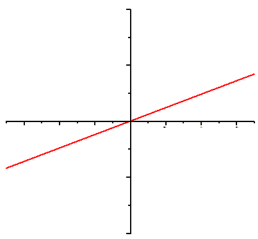
B.



C.



D.



E.

8) Let f be a function such that $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = 2$. Which of the following statements must always be true?

I. f is continuous at $x = 1$.
II. f is differentiable at $x = 1$.
III. The derivative of f is continuous at $x = 1$.

A. I only
B. II only
C. III only
D. I and II only
E. II and III only

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9) If $f(x) = e^{\arccos(x)}$, $-1 < x < 1$, then $f'(x)$ equals

A. $e^{\arccos(x)}\left(\frac{1}{\sqrt{1-x^2}}\right)$ **B.** $-e^{\arccos(x)}\left(\frac{1}{\sqrt{1-x^2}}\right)$ **C.** $e^{\arccos(x)}$

D. $e^{\arccos(x)}\left(\frac{1}{\sqrt{1+x^2}}\right)$ **E.** $-e^{\arccos(x)}\left(\frac{1}{\sqrt{x^2-1}}\right)$

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10) The domain of the function $f(x) = \arcsin(2x+1)$ is equal to:

A. $[-1, 1]$ **B.** $[-1, 0]$ **C.** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **D.** $[0, 1]$ **E.** $(-\infty, \infty)$

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11) If $y = \sin(x^2)$, then $xy'' - y' =$

A. $-4x^3 \sin(x^2)$ **B.** $4x\cos(x^2) - 2x^2 \sin(x^2)$ **C.** $-x\sin(x^2) - \cos(x^2)$

D. $-2x^2 \sin(x^2)$ **E.** $-x\sin(2) - \cos(2x)$

12) Suppose that f and g are twice differentiable functions everywhere on $\mathbb{R}$. Which of the following is the correct formula for $(f \cdot g)''$, the second derivative of the pointwise product of the functions f and g?

A. $f'' \cdot g + f \cdot g''$ **B.** $f'' \cdot g + f' \cdot g' + f \cdot g''$ **C.** $f'' \cdot g - 2f' \cdot g' + f \cdot g''$
D. $f'' \cdot g - f' \cdot g' + f \cdot g''$ **E.** $f'' \cdot g + 2f' \cdot g' + f \cdot g''$

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13) Which of the following expressions is equal to $\sin(2\arccos(x))$?

A. $2x\sqrt{1+x^2}$ **B.** $\dfrac{x}{1+x^2}$ **C.** $\dfrac{2x^2}{\sqrt{1-x^2}}$ **D.** $2x\sqrt{1-x^2}$
E. $\dfrac{x^2}{1-x^2}$

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14) Find y', given that $e^{xy} - 2y = (x+y)^2$.

A. $\dfrac{ye^{xy} - y^2 - 2y}{x^2 + 2 + xe^{xy}}$ **B.** $\dfrac{ye^{xy} - 2x - 2y}{2x + 2y + 2 - xe^{xy}}$ **C.** $x + y - \dfrac{1}{2}e^{xy}$
D. $\dfrac{y^2 - e^{xy}}{2}$ **E.** $\dfrac{ye^{xy}}{2x + 2y + 2}$

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15) Given that $f(x) = (\ln(x))^{\ln(x)}$ for $x > 1$, which of the following is the correct expression for $f'(x)$?

A. $\ln(x) \cdot (\ln(x))^{\ln(x)-1}$ **B.** $(\ln(x))^{\ln(x)}\left(\ln(x) \cdot (\ln(x))^{\ln(x)-1}\right)$
C. $(\ln(x))^{\ln(x)}\left(\ln(\ln(x)) + \dfrac{1}{x}\right)$ **D.** $\ln(x)^{\ln(x)}\left(\dfrac{\ln(\ln(x))}{x} + 1\right)$
E. $(\ln(x))^{\ln(x)}\left(\dfrac{\ln(\ln(x))}{x} + \dfrac{1}{x}\right)$

Answer key

#1	#2	#3	#4	#5
C	D	C	A	E
#6	**#7**	**#8**	**#9**	**#10**
B	C	D	B	B
#11	**#12**	**#13**	**#14**	**#15**
A	E	D	B	E