Lecture Cancelled-handed out tests

(April Fools!)

Test 2 Solutions

1) DFT

$$X(n) = \delta(n) + \delta(n-1)$$

$$X_k = \sum_{n=0}^{N-1} x(n) e^{-i\omega_0 nk}$$

$$N = 4$$

$$\omega_0 = \frac{2\pi}{N} = \frac{\pi}{2}$$

$$X_0 = 1 + 1 = 2$$

$$X_1 = 1 + e^{-i\frac{\pi}{2}} = 1 - i$$

$$X_2 = 1 + e^{-i2\frac{\pi}{2}} = 0$$

$$X_3 = 1 + e^{-i3\frac{\pi}{2}} = 1 + i$$

2π-periodic

2) Block Diagram

No need to convert to difference equation.

⊕ ALWAYS has 2 inputs, one output. If anything else, ignore.

Serial construction

$$\frac{.3}{1-(0.3)(0.5)e^{-i\omega}}$$

$$\frac{\left(1+(0.2)e^{-i\omega}\right)(0.3)}{1-(0.3)(0.5)e^{-i\omega}}$$

$$y(n) = x(n-1)$$

$$\hat{y}(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}, m = n-1$$

$$= \sum_{n=-\infty}^{\infty} x(n-1) z^{-n}$$

$$= z^{-1} \sum_{m=-\infty}^{\infty} x(m) z^{-n} = z^{-1}\hat{x}(z)$$

3) Continuous

$$x(t)=\delta(t)-\delta(t-2)$$

$$X(\omega)=\int_{-\infty}^{\infty}x(t)e^{-i\omega t}dt$$

$$X(\omega)=\int_{-\infty}^{\infty}\left(\delta(t)-\delta(t-2)\right)e^{-i\omega t}dt$$

$$=\int_{-\infty}^{\infty}\delta(t)e^{-i\omega t}dt-\int_{-\infty}^{\infty}\delta(t-2)e^{-i\omega t}dt$$

$$=1-e^{-i2\omega}$$

4) Synthesis

$$\omega_0=\tfrac{\pi}{3}$$

$$X_0=1\leftarrow DC\ term\,so\,X(t)=+1$$

$$X_1=X_{-1}=\tfrac{1}{2}\leftarrow\cos$$

$$X_3=X_{-3}=\tfrac{1}{2}\leftarrow\cos$$

$$X(t)=1+\cos\left(\tfrac{\pi}{3}t\right)+\cos\overbrace{(\pi t)}^{3\cdot\omega_0=3\cdot\frac{\pi}{3}}$$

5)

$$H(\omega)=\begin{cases}1 & \frac{-\pi}{4}\le\omega\le\frac{\pi}{4}\\ 0 & else\end{cases}$$

$$2\pi\,periodic,discrete\,system$$

$$h(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\omega)e^{i\omega n}d\omega$$

$$h(n)=\frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}e^{i\omega n}d\omega$$

$$=\frac{1}{2\pi}\frac{1}{i\cdot n}e^{i\omega n}\Big|_{\frac{-\pi}{4}}^{\frac{\pi}{4}}$$

$$=\frac{1}{2\pi}\frac{1}{i\cdot n}\left(e^{in\frac{\pi}{4}}-e^{-in\frac{\pi}{4}}\right)$$

$$=\frac{1}{\pi n}\sin\left(\frac{\pi}{4}n\right)$$

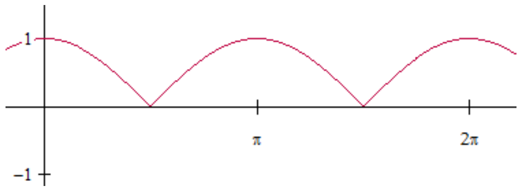
$$\boxed{4\cdot\sin c\left(\tfrac{\pi}{4}n\right)}\leftarrow bread\,\&\,butter\,of\ radar\,industry$$

6)

Discrete, so no complex

$$H\left(\tfrac{\pi}{2}\right)=0$$

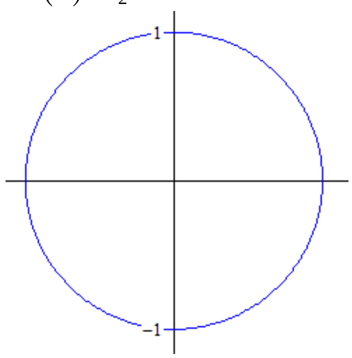
$$H(0)=1$$



$$\tilde{H}(\omega)=\left(e^{-i\omega}\overbrace{-e^{i\frac{\pi}{2}}}^{+i}\right)\left(e^{-i\omega}\overbrace{-e^{i\frac{\pi}{2}}}^{-i}\right)$$

$$=e^{-i2\omega}+1$$

$$\tilde{H}(0)=\tfrac{1}{2}$$



$$H(\omega)=\tfrac{1}{2}+\tfrac{1}{2}e^{-i2\omega}$$

$$y(n)=\tfrac{1}{2}x(n)+\tfrac{1}{2}x(n-2)$$

Oh wow! So tests actually ARE being handed out.

Tutorial: Assignment 7

BTW, the solutions were given to us at 8:30AM today, the assignment was given to us at 12:30PM today and we went over it at 3:30PM today. It's like a 3-hour test…

1)

$$f_s=8000Hz$$

$$(f_1,f_2)=(400,1000Hz)$$

$$f=\frac{400}{8000}=\frac{1}{20}$$

$$pitch=f\times f_s$$

$$\omega_0=2\pi f=\frac{2\pi}{20}=\frac{\pi}{10}$$

$$\omega_{0_2}=2\pi f=\frac{2\pi}{8}=\frac{\pi}{4}$$

2)

Poles:

$0.8e^{i\frac{\pi}{4}}, 0.8e^{i\frac{\pi}{10}}, 0.8e^{-i\frac{\pi}{4}}, 0.8e^{-i\frac{\pi}{10}}$

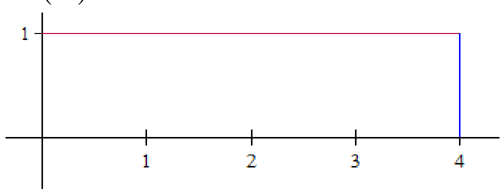
So zeroes <π/10, π/10 < π/4, >π/4, so let's choose complex points 0, $\left(\frac{\frac{\pi}{10}+\frac{\pi}{4}}{2}\right)$, and π. Thus the

Zeroes are: $e^{i0}, e^{i\frac{\frac{\pi}{10}+\frac{\pi}{4}}{2}}, e^{i\pi}$.

3)

$$x(t)=\begin{cases}1 & 0\le t\le 4\\ 0 & else\end{cases}$$

$X(\omega)$



$$x(t)=\begin{cases}1 & \frac{-\tau}{2}\le t\le \frac{\tau}{2}\\ 0 & else\end{cases} \Rightarrow \tau \sin c\left(\frac{\omega\tau}{2\pi}\right)$$

$$x(t)=\begin{cases}1 & -2\le t\le 2\\ 0 & else\end{cases} \Rightarrow x(\omega)=9\sin c\left(\frac{\omega\cdot 4}{2\pi}\right)$$

$$X_1(t)=x(t-k)$$

$$X_2(\omega)=e^{-i\omega k}, k=2$$

$$X(\omega)=\boxed{4\sin c\left(\frac{\cancel{4}^{2}\omega}{\cancel{2}\pi}\right)\cdot e^{-i\omega 2}}$$

$$H_1(\omega)=\begin{cases}1 & \overbrace{|\omega|\le 2}^{-2\le\omega\le 2}\\ 0 & else\end{cases}$$

$$H_2(\omega)=1-e^{-i\omega}$$

$$h_1(t)=\frac{i}{2\pi}\int_{-2}^{2}1\cdot e^{i\omega t}d\omega \leftarrow CTFT$$

$$=\frac{1}{2\pi}\left(\frac{-ie^{it\omega}}{t}\right)\Bigg|_{-2}^{2}$$

$$=\frac{1}{2\pi t}\left(e^{-2t}-e^{2t}\right)=\frac{\sin(2t)}{\pi t}$$

$$H_2(\omega) = 1 - e^{-i\omega}$$
$$= \delta(n) - \delta(n-1)$$
$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(1 - e^{-i\omega}\right)e^{i\omega n}$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-i\omega n} - e^{i\omega(n-1)} d\omega$$
$$= \delta(n) - \delta(n-1)$$

3)

$$y(n) = x(n) - x(n-1)$$
$$H(\omega) = 1 - e^{-i\omega}$$
$$H(\omega) = \frac{\Sigma\alpha\left(e^{-i\omega}\right)^n}{1 - \Sigma\beta\left(e^{-i\omega}\right)^n}$$
$$\left|H(\omega)\right|$$