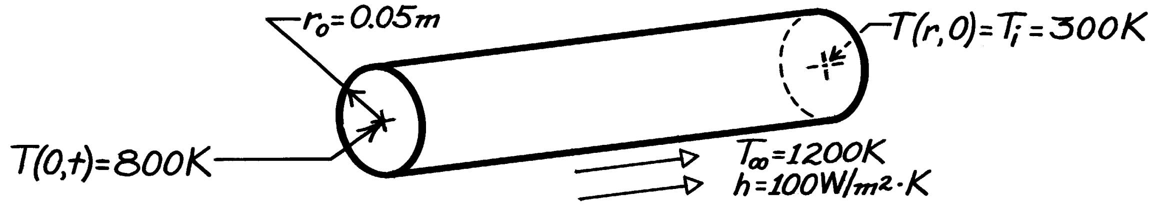
PROBLEM 5.15

KNOWN: Diameter and radial temperature of AISI 1010 carbon steel shaft. Convection coefficient and temperature of furnace gases.

FIND: Time required for shaft centerline to reach a prescribed temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Constant properties.

PROPERTIES: AISI 1010 carbon steel, *Table A.1* $\left(\overline{T} = 550\text{ K}\right)$: $\rho = 7832\text{ kg/m}^3$, k = 51.2 W/m·K, c = 541 J/kg·K, $\alpha = 1.21\times10^{-5}\text{ m}^2\text{/s}$.

ANALYSIS: The Biot number is

$$\text{Bi} = \frac{hr_o/2}{k} = \frac{100\text{ W/m}^2\cdot\text{K}\left(0.05\text{ m/2}\right)}{51.2\text{ W/m}\cdot\text{K}} = 0.0488.$$

Hence, the lumped capacitance method can be applied. From Equation 5.6,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hAs}{\rho Vc}\right)t\right] = \exp\left[-\frac{4h}{\rho cD}t\right]$$

$$\ln\left(\frac{800-1200}{300-1200}\right) = -0.811 = -\frac{4\times100\text{ W/m}^2\cdot\text{K}}{7832\text{ kg/m}^3\left(541\text{ J/kg}\cdot\text{K}\right)0.1\text{ m}}t$$

$$t = 859\text{ s.}$$ <

COMMENTS: To check the validity of the foregoing result, use the one-term approximation to the series solution. From Equation 5.52c,

$$\frac{T_o - T_\infty}{T_i - T_\infty} = \frac{-400}{-900} = 0.444 = C_1\exp\left(-\varsigma_1^2\text{Fo}\right)$$

For Bi = hr_o/k = 0.0976, Table 5.1 yields ς_1 = 0.436 and C_1 = 1.024. Hence

$$\frac{-(0.436)^2\left(1.2\times10^{-5}\text{ m}^2/\text{s}\right)}{(0.05\text{ m})^2}t = \ln(0.434) = -0.835$$

$$t = 915\text{ s.}$$

The results agree to within 6%. The lumped capacitance method underestimates the actual time, since the response at the centerline lags that at any other location in the shaft.