

Week 7 Class notes

Section 7.3 - Trigonometric Substitution

In this section, we seek to solve integrands containing the following algebraic expressions using trigonometric terms:

1. $\sqrt{a^2 - x^2} \rightarrow$ use $x = a\sin\theta$.
Comes from identity: $\sin^2\theta + \cos^2\theta = 1$
2. $\sqrt{a^2 + x^2} \rightarrow$ use $x = a\tan\theta$.
Comes from identity: $1 + \tan^2\theta = \sec^2\theta$
3. $\sqrt{x^2 - a^2} \rightarrow$ use $x = a\sec\theta$.
Comes from identity: $\sec^2\theta - 1 = \tan^2\theta$

Now let us go through some examples.

Example 1: textbook page 495 number 26

Evaluate $\int \frac{x^2}{\sqrt{4x - x^2}} dx$

Solution:

Perform complete the square for $4x - x^2 \rightarrow 4 - 4 + 4x - x^2 = 4 - (x - 2)^2$. Use rule number 3, $a = 2$ and let $x - 2 = 2\sin\theta$. $dx = 2\cos\theta d\theta$

$$\int \frac{x^2}{\sqrt{4x - x^2}} = \int \frac{4 + 8\sin\theta + 4\sin^2\theta}{2\cos\theta} 2\cos\theta d\theta$$

$$= 6\theta - 8\cos\theta - \sin 2\theta$$

Example 2: textbook page 495 number 27

Evaluate $\int \frac{dx}{(x^2 + 2x + 2)^2}$

Solution:

$$\int \frac{dx}{(x^2 + 2x + 2)^2} = \int \frac{dx}{((x + 1)^2 + 1)^2}$$

Let $x + 1 = \tan\theta$. Then, $dx = \sec^2\theta d\theta$. Use rule number 2, we get:

$$\begin{aligned}\int \frac{dx}{((x+1)^2+1)^2} &= \int \frac{\sec^2\theta d\theta}{\sec^4\theta} = \int \cos^2\theta d\theta \\ &= \int \frac{(1 - \cos(2\theta))}{2} d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4}\end{aligned}$$