Math 1AA3/1ZB3 Test #2 (Version 1)
March 17th, 2014

Name:____
(Last Name) (First Name)

Student Number:______________________

This test consists of 19 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1. Find the sum of the following series.

$$\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n+1}}{3^{2n+1}(2n+1)!}$$

(a) $\sqrt{3}$ **(b)** $\frac{\sqrt{3}}{2}$ **(c)** $\frac{1}{2}$ **(d)** $e^{\pi/3}$ **(e)** 0

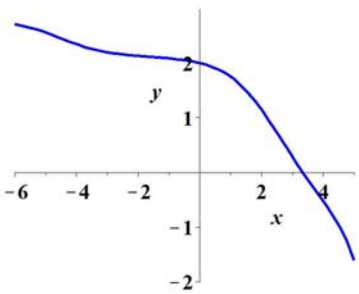
2. Find all values of r for which the function $y = e^{rx}$ satisfies the differential equation $y'' + 2y' + y = 0$.
(a) $r = 1, 2$ **(b)** $r = -2, 2$ **(c)** $r = -2$ **(d)** $r = -1$ **(e)** $r = -1, 1$

3. Find the Taylor polynomial $T_2(x)$ for $f(x) = \ln(1 + 4x)$, centered at $a = 1$.
(a) $\ln 5 + \frac{1}{5}(x-1) - \frac{1}{50}(x-1)^2$ **(b)** $\ln 5 + \frac{1}{5}(x-1) - \frac{2}{25}(x-1)^2$
(c) $\ln 5 + \frac{4}{5}(x-1) - \frac{8}{25}(x-1)^2$ **(d)** $\ln 5 + \frac{1}{5}(x-1) - \frac{1}{25}(x-1)^2$
(e) $\ln 5 + \frac{4}{5}(x-1) - \frac{16}{25}(x-1)^2$

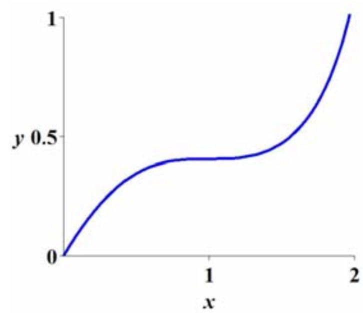
4. By inspecting the following differential equation and its properties, determine which of the given graphs could be the graph of a solution to the equation.

$$y' = -e^y(x-2)^2$$

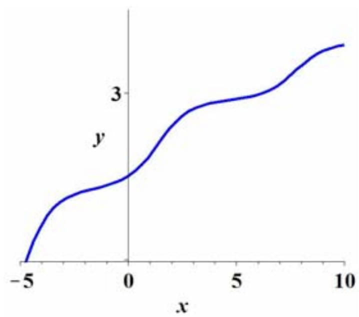
(a)



(b)



(c)



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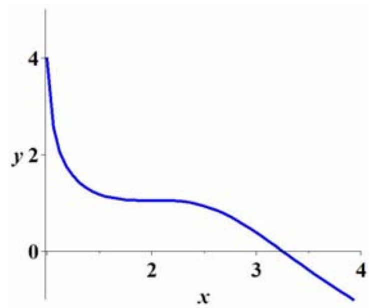

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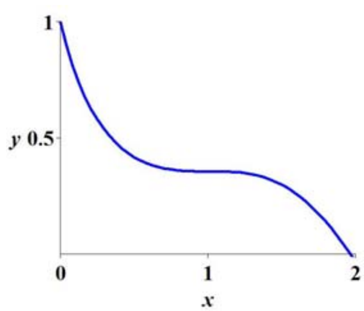


5. Suppose that we approximate $f(x) = x^{1/2}$ with a Taylor polynomial of degree 1 [i.e., $T_1(x)$] centered at $a = 4$ on the interval $2 \leq x \leq 6$. According to Taylor's inequality, what is the smallest possible value for the error involved in this approximation?

(a) $\frac{1}{8 \cdot 6^{3/2}}$ **(b)** $\frac{1}{2 \cdot 6^{3/2}}$ **(c)** $\frac{1}{2^{9/2}}$ **(d)** $\frac{1}{2^{5/2}}$ **(e)** $\frac{1}{2^{7/2}}$

6. Find a power series representation of the following function.

$$f(x) = \frac{1}{1-3x^2}$$

(a) $\sum_{n=0}^{\infty}(-1)^n 3^n x^{2n}$ **(b)** $\sum_{n=0}^{\infty}\frac{1}{3^n}x^{2n}$ **(c)** $\sum_{n=0}^{\infty}3^n x^{2n}$ **(d)** $\sum_{n=0}^{\infty}\frac{(-1)^n}{3^n}x^{2n}$
(e) $\sum_{n=0}^{\infty}(-1)^n 3^{2n} x^{2n}$

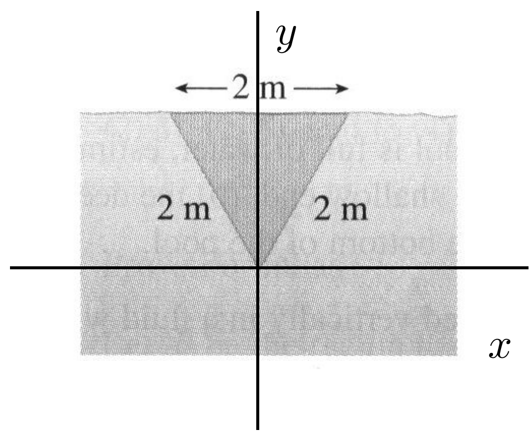
7. A cube with 1m long sides is sitting on the bottom of an aquarium in which the water (whose density is ρ) is 2m deep. Find the hydrostatic force on one of the sides of the cube.
(a) $\frac{3}{2}\rho g$ **(b)** $3\rho g$ **(c)** $\frac{1}{2}\rho g$ **(d)** $2\rho g$ **(e)** $\frac{1}{3}\rho g$

8. Find the Maclaurin series for the following function.

$$f(x) = \frac{1}{\sqrt{1-x^2}}$$

(a) $1+\sum_{n=1}^{\infty}\frac{3\cdot 5\cdot 7\cdot\cdots\cdot(2n-1)}{n!\,2^n}x^{2n}$ **(b)** $1+\sum_{n=1}^{\infty}\frac{2\cdot 5\cdot 8\cdot\cdots\cdot(3n-1)}{n!\,2^n}x^{2n}$
(c) $1+\sum_{n=1}^{\infty}\frac{2\cdot 4\cdot 6\cdot\cdots\cdot(2n)}{n!\,2^{2n}}x^{2n}$ **(d)** $1+\sum_{n=1}^{\infty}(-1)^n\frac{3\cdot 5\cdot 7\cdot\cdots\cdot(2n-1)}{n!\,2^{n+1}}x^{2n}$
(e) $1+\sum_{n=1}^{\infty}(-1)^n\frac{2\cdot 5\cdot 8\cdot\cdots\cdot(3n-1)}{n!\,2^{n+1}}x^{2n}$

9. A triangular vertical plate is submerged in water (whose density is ρ), as shown in the diagram to the right. Using the coordinate system given in the diagram, find an integral which represents the hydrostatic force against one side of the plate.



(a) $\rho g\int_0^{\sqrt{3}}\frac{y}{3}(\sqrt{3}-y)dy$ **(b)** $\rho g\int_0^{\sqrt{3}}\frac{2y}{\sqrt{3}}(\sqrt{3}-y)dy$ **(c)** $\rho g\int_0^{2}y(2-y)dy$
(d) $\rho g\int_0^{\sqrt{3}}2y(2-y)dy$ **(e)** $\rho g\int_0^{2}2y(2-y)dy$

10. A 98 L tank is filled with pure water. Water which has a concentration of 4g of salt per litre flows into the tank at a rate of 7 L/min, and the mixture is stirred to a uniform concentration. Water also leaks from the tank at the same rate, 7 L/min. Let $y(t)$ be the amount of salt (in grams) in the tank at time t. Find a differential equation describing the rate of change of salt in the tank.

(a) $\frac{dy}{dt} = \frac{2}{49} - \frac{y(t)}{98}$ **(b)** $\frac{dy}{dt} = \frac{4}{7} - \frac{y(t)}{98}$ **(c)** $\frac{dy}{dt} = 28 - \frac{y(t)}{98}$ **(d)** $\frac{dy}{dt} = \frac{2}{49} - \frac{y(t)}{14}$
(e) $\frac{dy}{dt} = 28 - \frac{y(t)}{14}$

11. Which of the below integrals represents the area of the surface obtained by rotating the curve $y = e^x$, $0 \leq x \leq 1$ about the x-axis?
(a) $2\pi \int_0^1 \sqrt{1+u^2}\, du$ **(b)** $2\pi \int_0^1 e^x \sqrt{1+e^x}\, dx$ **(c)** $2\pi \int_0^1 x\sqrt{1+e^{x^2}}\, dx$
(d) $2\pi \int_0^1 x\sqrt{1+e^{2x}}\, dx$ **(e)** $2\pi \int_1^e \sqrt{1+u^2}\, du$

12. Let $y(x)$ be the solution to the initial value problem

$$y' = x^2 y, \quad y(0) = 2.$$

Find $y(1)$.

(a) $2e^{1/3}$ **(b)** $3e^{1/2}$ **(c)** $2e^{3/2}$ **(d)** $3e^{2/3}$ **(e)** $e^{2/3}$

13. Find a Cartesian equation of the following paramatric curve.

$$x = e^{2t}, \quad y = t + 1$$

(a) $y = 2 - \ln x$ **(b)** $y = 1 - \frac{1}{2}\ln x$ **(c)** $y = 2 + \ln x$ **(d)** $y = 1 + \frac{1}{2}\ln x$
(e) $y = \frac{1}{2} + \ln x$

14. Consider the following paramtric curve.

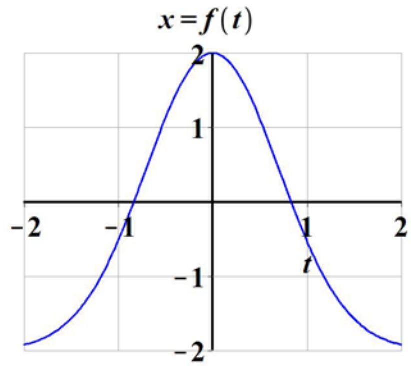
$$x = 1 + \sin t, \quad y = t + \cos t, \quad 0 \leq t \leq 2\pi$$

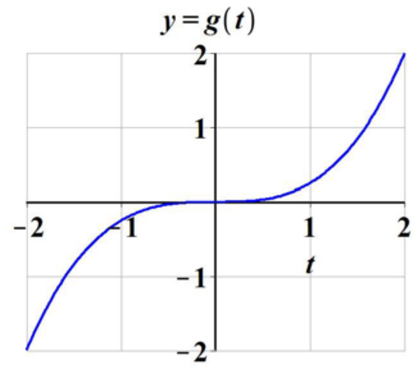
For what value(s) of t is the tangent line vertical?
(a) $0, 2\pi$ **(b)** $\frac{3\pi}{2}$ **(c)** $\frac{\pi}{2}, \frac{3\pi}{2}$ **(d)** $0, \pi, 2\pi$ **(e)** π

15. The half life of cesium-137 is 30 years. How long will it take a sample of cesium-137 to decay to 10% of its original amount?

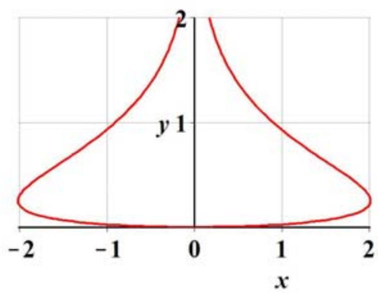
(a) $30\frac{\ln 2}{\ln 10}$ **(b)** $30\frac{\ln 10}{\ln 2}$ **(c)** $10\ \frac{\ln 30}{\ln 2}$ **(d)** $2\ \frac{\ln 30}{\ln 10}$ **(e)** $10\ \frac{\ln 2}{\ln 30}$

16. Use the given graphs to sketch the parametric curve $x = f(t)$, $y = g(t)$.





(a)



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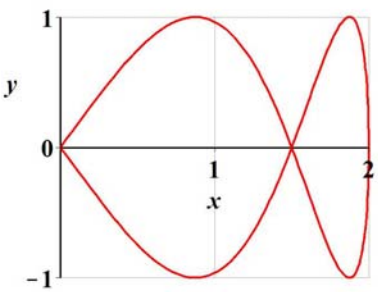

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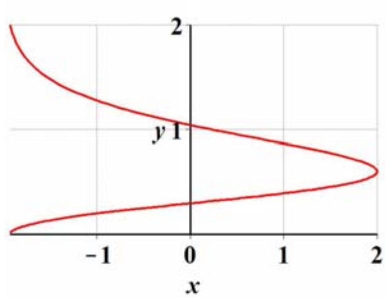

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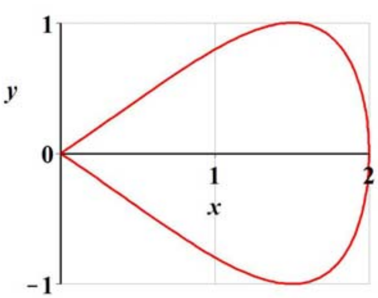
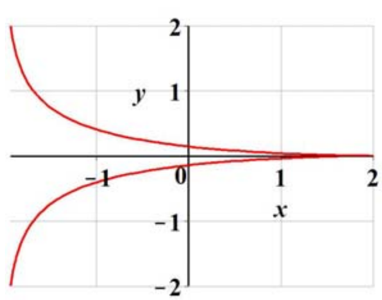


(e)



17. Solve the following differential equation.

$$x\,\frac{dy}{dx} - 4y = x^5 e^x$$

(a) $y = x^3(e^x + C)$ **(b)** $y = x^3 + Ce^x$ **(c)** $y = x^4(e^x + C)$ **(d)** $y = e^x(x^4 + C)$
(e) $y = x^5 + Ce^x$

18. Find an equation of the tangent line to the following parametric curve,

$$x = e^{t^2}, \quad y = \sqrt{t} + \ln t$$

at the point $(e, 1)$.
(a) $y = \frac{3}{4e}x + \frac{1}{4}$ **(b)** $y = \frac{1}{4e}x + \frac{3}{4}$ **(c)** $y = 3x - \frac{1}{4e}$ **(d)** $y = \frac{3}{4e}x + \frac{1}{2}$ **(e)** $y = \frac{1}{4e}x + \frac{1}{2}$

19. Evaluate the following integral as an infinite series.

$$\int e^{x^2}\,dx$$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1}$ **(b)** $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(n+1)!} x^{2n+2}$ **(c)** $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(n+1)!} x^{2n+1}$
(d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+1)n!} x^{2n+1}$ **(e)** $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(2n+2)n!} x^{2n+2}$

20. Correctly fill out the bubbles corresponding to your student number and the version number of your test in the correct places on the computer card.
Note: You are writing **Version 1**.

Answers (Version 1)**:**

1. b **2.** d **3.** c **4.** d **5.** d **6.** c **7.** a **8.** a **9.** b **10.** e
11. e **12.** a **13.** d **14.** b **15.** b **16.** e **17.** c **18.** a **19.** d