Centre of Mass

- Centre of Mass, Centre of Gravity
- Rigid-body equilibrium

Serway 12.1-12.3; and parts of 9.5

Practice problems: Chapter 9, problems 37, 38
Chapter 12, problems 1, 3, 6, 9, 11, 17.

Equilibrium of a Particle

Equilibrium: no motion (no *acceleration*).

Newton's 2nd Law, $\sum \mathbf{F} = m\mathbf{a} = 0$

The vector sum of forces acting on a body in equilibrium is zero.

This leads to 3 independent equations (e.g., for x, y, z components); or 2 equations in 2-D problems. For a particle, this is the whole story.

Equilibrium of a Rigid Body

1) **a** = 0 (no translational acceleration), so

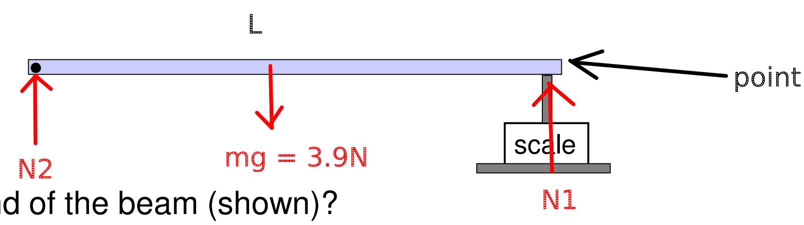
$$\boxed{\Sigma \mathbf{F} = 0} \quad \text{(no net force)}$$

2) $\alpha = 0$ (no angular acceleration), so

$$\boxed{\Sigma \boldsymbol{\tau} = 0} \quad \text{(no net torque)}$$

Net external forces *and* net external torque must be zero for a body in equilibrium.

The uniform beam (weight 4.00 N, length L) is supported by a pin at one end, and a scale that can be placed anywhere along the beam. What will the scale read if it is



a) at the end of the beam (shown)?
b) at the centre of the beam?
c) at distance ¼ L from the pin?
d) at distance ¾ L from the pin?

1) N1 + N2 = mg
2) ΣT = 0 --> mg (L/2) - N2L = 0 --> N2 = (1/2)mg

N2 = 2N1
2N1 + N1 = mg
3N1 = mg
N1 = mg/3
N2 = 2mg/3

D) w L/2 = N2 x 3/4 L
N2 = 4/3 x 1/2 x w

Where should we choose the "pivot point" when calculating torques?

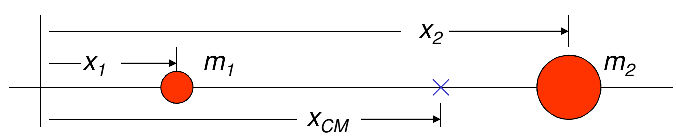
*(Answer: **In Statics problems**, it doesn't matter.)*

Theorem: If the **net force on a body is zero**, then the *total torque due to all forces* will not depend on the choice of "pivot point".

In particular, if $\mathbf{F}_{net} = 0$*, then, if* $\tau_{net} = 0$ *for one "pivot point",* τ_{net} *is also zero for **every other** pivot point.*

Centre of Mass, Centre of Gravity

Two particles on the x axis; total mass, $M = m_1 + m_2$.



The *centre of mass (CM)* is defined as the location

$$x_{CM} = (m_1 x_1 + m_2 x_2)/M$$

- Weighted average of the position

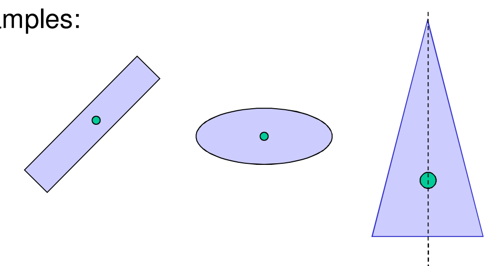
The centre of mass is an "average position" of the mass.

Many particles, three dimensions:

$$\mathbf{r}_{\mathrm{CM}} = \frac{\Sigma m_{\mathrm{i}}\mathbf{r}_{\mathrm{i}}}{\Sigma m_{\mathrm{i}}} = \frac{\Sigma m_{\mathrm{i}}\mathbf{r}_{\mathrm{i}}}{M}$$

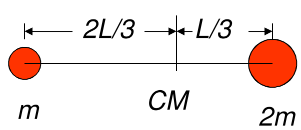
(Recall the *position vector* **r** has components x, y, z.)

Examples:



The CM is on the symmetry axes.

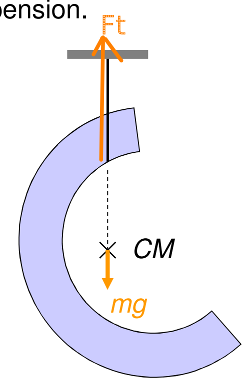
The CM can be outside the object. (It is on the line between the centres of the two rectangles which make up the “L”.)



The CM is closer to the more massive object

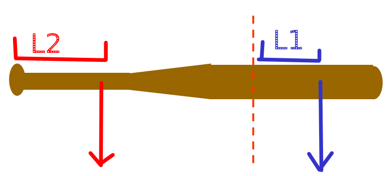
Centre of Gravity

The *CM* is also the location of the *centre of gravity*. When we consider the rotational equilibrium of a rigid body, we can treat the gravitational force as if it were a single force applied at the centre of mass. A suspended object will hang with its CM vertically below the point of suspension.



Quiz: Baseball bat

A baseball bat is sawn in half at its centre of mass. Which piece is heavier?



A) The short piece
B) The long piece
C) Both pieces have the same mass.

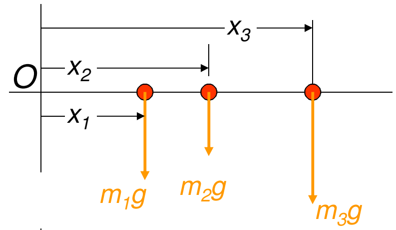
Although they have equal masses, the shorter piece has more torque because it has a shorter length.

Centre of Gravity

***Why** can we place the gravitational force at the CM?*

Calculate the torque (about *O)* due to the three weights on the x axis:

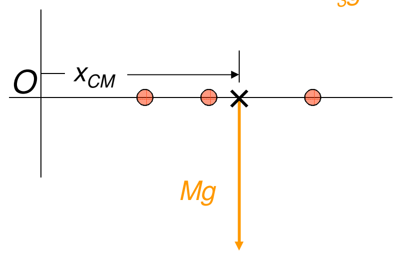
$$\tau = x_1 m_1 g + x_2 m_2 g + x_3 m_3 g$$



Now calculate the torque as if a single, total weight were placed at the *CM*:

$$\tau = x_{CM} Mg$$

But $x_{CM} = \dfrac{x_1 m_1 + x_2 m_2 + x_3 m_3}{M}$



The two methods give *the same torque*.

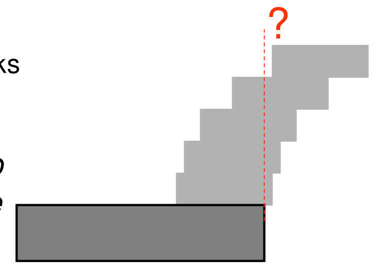
Replacing the *real* gravitational forces by a single force, equal to the total weight, and placed at the centre of gravity, will not change the total gravitational torque (about *any* pivot).

This means that the *external forces* needed to hold a rigid body in equilibrium can be calculated as if gravity were a single force applied at the centre of gravity.

In a uniform gravitational field, the centre of gravity is at the centre of mass.

Example: how far can a pile of bricks lean without falling over?

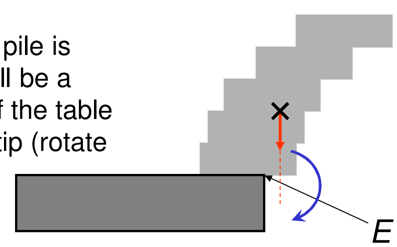
Is it possible for the top brick to be entirely past the edge of the table?



movement can be 1/2n, where n= 1 is the top brick

As long as the centre of gravity of the system does not exceed the edge. The centre of gravity is the middle of the middle brick.

If the centre of gravity of the entire pile is past the edge of the table, there will be a clockwise torque about the edge of the table (point E), and the whole stack will tip (rotate clockwise) about *E*.



If the centre of gravity of the pile is not past the edge of the table, the gravitational torque (about *E*) will be counterclockwise, and the stack will be stable (at least it will not tip at *E*).