REGRESSION PROBLEM

DATA : $y_1, y_2, \dots y_\ell \in \mathbb{R}$ RELATED TO THE POINTS $x_1, x_2, \dots, x_\ell \in \mathbb{R}$

FIND THE BEST APPROXIMATION WITH A POLYNOMIAL p WITH A DEGREE $(n-1) \le \ell$

$$p(x) = z_1 + z_2 x + z_3 x^2 + \dots + z_n x^{n-1}$$

RESIDUAL $r^i = p(x_i) - y_i \quad \rightarrow \quad \|r\| \downarrow$

$$\begin{cases} \min\limits_z \|Az - y\| \\ z \in \mathbb{R}^n \end{cases}$$

IF $\|\cdot\|_2$ THEN THE PROBLEM HAS A UNIQUE SOLUTION

$$A^T \cdot A z = A^T y \quad \rightarrow \quad z = \text{inv}(A^T \cdot A) A^T y$$

$\|\cdot\|_1, \|\cdot\|_\infty \rightarrow$ LINEAR PROGRAMMING PROBLEM

$\|\cdot\|_1$

$$\begin{cases} \min\limits_z \|Az - y\|_1 \\ z \in \mathbb{R}^n \end{cases} \equiv \begin{cases} \min\limits_z \sum\limits_{i=1}^{\ell} |A_i z - y_i| = \min\limits_{z,u} \sum\limits_{i=1}^{\ell} u_i \\ z \in \mathbb{R}^n \end{cases} \equiv \begin{cases} \min\limits_{z,u} \sum\limits_{i=1}^{\ell} u_i \\ u_i \ge A_i z - y_i & \forall i = 1, \dots \ell \\ u_i \ge y_i - A_i z & \forall i = 1, \dots \ell \end{cases}$$

$\|\cdot\|_\infty$

$$\begin{cases} \min\limits_z \|Az - y\|_1 \\ z \in \mathbb{R}^n \end{cases} \equiv \begin{cases} \min\limits_z \max\limits_{i=1,\dots,\ell} |A_i z - y_i| = \min\limits_{z,u} \underset{\in \mathbb{R}}{u} \\ u \ge A_i z - y_i \\ u \ge y_i - A_i z \end{cases}$$

ε-SV

GIVEN A SET OF DATA $\{(x_1, y_1), \dots (x_\ell, y_\ell)\}$, $x^i \in \mathbb{R}^n$, $y_i \in \mathbb{R}$ AND $\varepsilon > 0$, WE WANT TO FIND A FUNCTION f THAT $|f(x_i) - y_i| \le \varepsilon \quad \forall i = 1, \dots \ell$

$f = w^T x + b \longrightarrow$ FLAT $\rightarrow$ CONVEX QUADRATIC PROBLEM

$$\begin{cases} \min\limits_{w,b} \frac{1}{2} \|w\|^2 \\ y_i \le w^T x^i + b + \varepsilon & \forall i = 1, \dots, \ell \\ y_i \ge w^T x^i + b - \varepsilon & \forall i = 1, \dots, \ell \end{cases}$$

SLACK VARIABLES

$$(P)\begin{cases} \min\limits_{w,b} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{\ell}(\varepsilon^+ + \varepsilon^-) \\ y_i \le w^T x^i + b + \varepsilon + \varepsilon^+ \quad \forall i = 1, \dots, \ell \\ y_i \ge w^T x^i + b - \varepsilon - \varepsilon^- \quad \forall i = 1, \dots, \ell \\ \varepsilon^+ \ge 0 \\ \varepsilon^- \ge 0 \end{cases}$$

$$(D)\begin{cases} -\min \frac{1}{2}\sum_{i=1}^{\ell}\sum_{j=1}^{\ell}(\lambda_i^+ - \lambda_i^-)(\lambda_j^+ - \lambda_j^-)(x_i)^T x_j + \varepsilon\sum_{i=1}^{\ell}(\lambda_i^+ + \lambda_i^-) - \sum_{i=1}^{\ell} y_i(\lambda_i^+ - \lambda_i^-) \\ \sum_{i=1}^{\ell}(\lambda_i^+ - \lambda_i^-) = 0 \\ \lambda_i^+ \in [0, C] \quad \forall i = 1, \dots, \ell \\ \lambda_i^- \in [0, C] \quad \forall i = 1, \dots, \ell \end{cases}$$

$$Q = \begin{pmatrix} X & -X \\ -X & X \end{pmatrix} \qquad X = [X(i)^T X(j)] \quad \forall i = 1 \dots, \ell$$

CONVEX QUADRATIC PROBLEM

$\lambda_i^+, \lambda_i^- > 0 \rightarrow x_i$ SUPPORT VECTOR

$$w = \sum_{i=1}^{\ell}(\lambda_i^+ - \lambda_i^-)\, x_i$$

FOR $0 < \lambda_i^+ < C \qquad b = y_i - w^T x_i - \varepsilon$

FOR $0 < \lambda_i^- < C \qquad b = y_i - w^T x_i + \varepsilon$

KERNEL FUNCTION

- NON LINEAR REGRESSION
- $\Phi(x) : \mathbb{R}^n \rightarrow \mathcal{H}$

$$X = [k(x_i, x_j)]_{i = 1, \dots, \ell}$$

FOR $0 < \lambda_i^+ < C \qquad b = y_i - \varepsilon - \sum_{j=1}^{\ell}(\lambda_j^+ - \lambda_j^-)\, k(x_i, x_j)$

FOR $0 < \lambda_i^- < C \qquad b = y_i + \varepsilon - \sum_{j=1}^{\ell}(\lambda_j^+ - \lambda_j^-)\, k(x_i, x_j)$

REGRESSION FUNCTION $\quad f(x) = w^T \Phi(x) + b = \sum_{i=1}^{\ell}(\lambda_i^+ - \lambda_i^-)\, k(x_i, x) + b$

KERNEL FUNCTIONS

$x^T y$; $(x^T y + 1)^p$ POLYNOMIAL; $e^{-\gamma \|x-y\|^2}$ $\parallel$ $e^{-\|x-y\|^2/(2\delta)^2}$ GAUSSIAN ; $\tanh(\beta x^T y + \gamma)$