CHAPTER - 5

COMPLEX No. + QUAD. Eq.

EXERCISE :- 5.1

Q.1. $(5i)\left(-\frac{3}{5}i\right)$

soln. $(5i)\left(-3i/5\right)$

$= \cancel{5}\, i \times \frac{(-3)(i)}{\cancel{5}}$

$= -3i^2 = -3\times(-1) = 3$

$\Rightarrow \boxed{(5i)\left(\frac{-3}{5}i\right) = 3 + i0}$

Q.2. $i + i^{19}$

soln. $i + i^{19}$

$= i + i^{4.4+3}$

$= i + (-i)$

$= 0$

$\Rightarrow \boxed{i + i^{19} = 0 + i0}$

Q.3. i^{-39}

soln. i^{-39}

$= i^{4.(-10)+1}$

$= i$

$\Rightarrow \boxed{i^{-39} = 0 + i1}$

Q.4. $3(7+i7) + i(7+i7)$

soln. $3(7+i7) + i(7+i7)$

$= 21 + i21 + i7 + i^2 7$

$= 21 + i21 + i7 + (-7)$

$= 14 + (21+7)i$

$= 14 + i28$

Q.5. $(1-i) - (-1+i6)$

soln. $\Rightarrow (1-i) - (-1+i6)$

$= 1 - i + 1 - i6$

$= 2 - i(6+1)$

$= 2 - i7$

$\Rightarrow \boxed{(1-i)-(-1+i6) = 2+i(-7)}$

Q.6. $\left(\frac{1}{5} + i\frac{2}{5}\right) - \left(4 + i\frac{5}{2}\right)$

soln. $\frac{1}{5} - 4 + i\left(\frac{2}{5} - \frac{5}{2}\right)$

$= \frac{1-20}{5} + i\left(\frac{4-25}{10}\right)$

$= \frac{-19}{5} + i\left(\frac{-21}{10}\right)$

$= -\left(\frac{19}{5}\right) - \left(i\frac{21}{10}\right)$

Q.7. $\left[\left(\frac{1}{3} + i\frac{7}{3}\right) + \left(4 + i\frac{1}{3}\right)\right] - \left(\frac{-4}{3} + i\right)$

soln $= \left[4 + \frac{1}{3} + i\left(\frac{7}{3} + \frac{1}{3}\right)\right] - \left(\frac{-4}{3} + i\right)$

$= \left(\frac{13}{3} + i\frac{8}{3}\right) - \left(\frac{-4}{3} + i\right)$

$= \left(\frac{13}{3} + \frac{4}{3}\right) + i\left(\frac{8}{3} - 1\right)$

$= \frac{17}{3} + i\frac{5}{3}$

Q.8. $(1-i)^4$

soln. $(1-i)^4$

$= ((1-i)^2)^2$

$= (1+i^2-2i)^2$

$= (\cancel{1}+(\cancel{-1}) - 2i)^2$

$= (-2i)^2 = 4i^2$

$= 4\times(-1) = -4$

$\Rightarrow \boxed{(1-i)^4 = -4 + i0}$

Q.9. $\left(\frac{1}{3}+3i\right)^3$

soln. $(1/3 + 3i)^3$

$= \frac{1}{27} + 27i^3 + \cancel{3}\times\frac{1}{\cancel{3}}\cdot 3i\left(\frac{1}{3}+3i\right)$

$= \frac{1}{27} + 27i^3 + i + 9i^2$

$= \frac{1}{27} + 27i(-1) + i + 9(-1)$

$= \frac{1}{27} - 27i + i - 9$

$= \frac{-242}{27} + i(-26)$

$\Rightarrow \boxed{\left(\frac{1}{3}+3i\right)^3 = \frac{-242}{27} + i(-26)}$

Q.10. $\left(-2-\frac{1}{3}i\right)^3$

soln. $\left(-2-\frac{1}{3}i\right)^3$

$= (-2)^2 + \left(\frac{-1i}{3}\right)^3 + \cancel{3}\times(-2)\times\left(\frac{-1i}{\cancel{3}}\right)\left[-2-\frac{1i}{3}\right]$

$= -8 - \frac{1}{27}i^3 - 4i - \frac{2}{3}i^2$

$= -8 - \left(\frac{i + 108i}{27}\right) - \frac{2}{3}(-1)$

$= \frac{-24+2}{3} - \left(\frac{-1+108}{27}\right)i$

$= \frac{-22}{3} - \frac{107i}{27}$

$\rightarrow \boxed{\left(-2-\frac{1}{3}i\right)^3 = \frac{-22}{3} - \frac{107i}{27}}$

Q.11. $4-3i$

soln. let the multiplicative inverse be x,

then, $x = \bar{z}/|z|^2 = \frac{4+3i}{(\sqrt{25})^2} = \frac{4+3i}{25}$

~~$(4-3i) \times x = 1$~~

~~$x = (4-3i)^{-1}$~~ $= \frac{4}{25} + \frac{i3}{25}$

$\therefore$ M.I. of $(4-3i) = \left(\frac{4}{25}+\frac{3i}{25}\right)$ ~~ap~~ ~~or~~

Q.12. $\sqrt{5}+3i$

soln. let the M.I. be x,

then, $x = \bar{z}/|z|^2 = \frac{\sqrt{5}-3i}{(\sqrt{5+9})^2} = \frac{\sqrt{5}-3i}{16} = \frac{\sqrt{5}}{16} - \frac{3i}{16}$

~~$(\sqrt{5}+3i) \times x = 1$~~

~~$x = (\sqrt{5}+3i)^{-1}$~~

$\therefore$ MI of $(\sqrt{5}+3i) = \left(\frac{\sqrt{5}}{16} - \frac{3i}{16}\right)$ ~~ap~~ ~~or~~

Q.13. $-i$

soln let the M.I. be x,

then, $x = \bar{z}/|z|^2$

$\cancel{(-i) \times x = 1}$ $= \dfrac{i}{\sqrt{0+1^2}} = \dfrac{i}{1} = i$

$\cancel{x = (-i)^{0}}$

$\therefore$ M.I. of $(-i) = (i)$ ...

Q.14. Exp. in form of $a + ib$:

Soln. $\dfrac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\cancel{\sqrt{3}} + \sqrt{2}\,i) - (\cancel{\sqrt{3}} - i\sqrt{2})}$

$= \dfrac{9 - i^2 5}{2\sqrt{2}\,i}$

$= \dfrac{9 - (-1)5}{2\sqrt{2}\,i}$

$= \dfrac{9+5}{2\sqrt{2}\,i} = \dfrac{\cancel{14}\,7}{\cancel{2}\sqrt{2}\,i} = \dfrac{7\sqrt{2}}{2i}$

$= \dfrac{7\sqrt{2}\,(i^2)}{2\,\cancel{(i)}\,(i^2)}$

$= \dfrac{+7\sqrt{2}\;i}{2(-1)} = \dfrac{-7\sqrt{2}\,i}{2}$

$\Rightarrow \boxed{\dfrac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}\,i) - (\sqrt{3} - i\sqrt{2})} = \dfrac{-7\sqrt{2}\,i}{2}}$

EXERCISE :- 5.2

Q.1. $z = -1 - i\sqrt{3}$

soln. $|z| = \sqrt{a^2 + b^2}$ $\qquad a = -1, \ b = -\sqrt{3}$

$= \sqrt{(-1)^2 + (-\sqrt{3})^2}$

$= \sqrt{1+3} = \sqrt{4} = 2$

$\Rightarrow |z| = 2$

Q.2 $z = -\sqrt{3} + i$

soln. $|z| = \sqrt{(-\sqrt{3})^2 + 1^2}$

$= \sqrt{3+1}$

$= \sqrt{4} = 2$

Q.3 $1 - i$.

soln. $z = 1 - i \quad \ldots (i).$ $\qquad a = 1, \ b = -1$

$r = \sqrt{(1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\Rightarrow r = \sqrt{2}$

In polar form $\Rightarrow z = r(\cos\theta + i\sin\theta)$ ...(ii)

On comparing (i) & (ii),

we get,

$r\cos\theta = 1$ ...(iii). $r\sin\theta = -1$ ..(iv)

$\because$ $\cos\theta$ is +ve & $\sin\theta$ is −ve.

$\therefore$ θ lies in 4th quadrant.

solving (iii),

$$r\cos\theta = 1$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = 1/\sqrt{2}$$

$$-\cos 45^\circ = -1/\sqrt{2}$$

$$\Rightarrow \theta = -45^\circ \text{ or } \frac{-\pi}{4}$$

$\therefore$ in polar form,

$$z = \sqrt{2}\left(\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}\right)$$

Q.4. $-1 + i$

soln. $z = -1 + i$ ...(1). , $a = -1$, $b = 1$

$$r = \sqrt{a^2 + b^2} = \sqrt{1+1} = \sqrt{2}$$

In polar form,

$$z = r(\cos\theta + i\sin\theta) \quad \text{...(2)}$$

On comparing (1) & (2),

we get,

$r\cos\theta = -1$ ... (3) $r\sin\theta = 1$ ..(4)

$\because$ $\cos\theta$ is −ve & $\sin\theta$ is +ve.

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$\therefore$ θ lies in second quadrant

on solving (4).,

$$r \sin \theta = 1$$

$$\sin \theta = \frac{1}{\sqrt{2}}$$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \Rightarrow \sin\left(\pi - \frac{\pi}{4}\right) = \sin \theta$$

$$\Rightarrow \sin \frac{3\pi}{4} = \sin \theta$$

$$\Rightarrow \theta = \frac{3\pi}{4}$$

$\therefore$ in polar form,

$$z = \sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$$

Q.5. $-1 - i$

soln. $z = -1 - i$ ....(i) , $a = -1$, $b = -1$

$$r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

In polar form,

$$z = r(\cos \theta + i \sin \theta) \quad \text{...(ii)}$$

On comparing (i) & (ii),

we get,

$r \cos\theta = -1$ -- (iii) $r \sin \theta = -1$ --(iv)

$\because$ $\cos\theta$ is -ve & $\sin\theta$ is -ve.

$\therefore$ θ lies in ~~first~~ 3rd quadrant.

on solving (iv),

$$r \sin\theta = -1$$
$$\sqrt{2} \sin\theta = -1$$
$$-\sin\theta = \frac{1}{\sqrt{2}}$$
$$\sin(-\theta) = \sin\frac{\pi}{4}$$
$$\sin(-\theta) = \sin\left(\pi - \frac{\pi}{4}\right)$$
$$-\theta = 3\pi/4$$
$$\theta = -3\pi/4$$

$\therefore$ In polar form,

$$z = \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\cos\frac{-3\pi}{4}\right)$$

Q.6. -3

soln. $z = -3 + i0$ ...(i), $a = -3$, $b = 0$.

$$r = \sqrt{a^2+b^2} = \sqrt{(-3)^2+0^2} = \sqrt{9} = 3$$

In polar form,

$$z = r(\cos\theta + i\sin\theta) \quad \text{...(ii)}.$$

comparing (i) with (ii), we get,

$r\cos\theta = -3$

$\Rightarrow \cos\theta = -1$

$\Rightarrow \cos(\theta) = -\cos 0$

$\Rightarrow \cos\theta = \cos(\pi - \theta) \Rightarrow \theta = 180°$

$\sin\theta = 0$

$\sin\theta = \sin 0$

$\Rightarrow \sin\theta = \sin(\pi - 0)$

$\therefore \theta = \pi$

$\because \theta$ is in 3rd quad

$\therefore$ In polar form,

$$z = 3(\cos\pi + i\sin\pi)$$

Q.7. $\sqrt{3} + i$

soln. $z = \sqrt{3} + i$ ...(i) , $a = \sqrt{3}$, $b = 1$

$$r = \sqrt{a^2+b^2} = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

In polar form,

$z = r(\cos\theta + i\sin\theta)$ ... (ii)

On comparing (i) and (ii),
we get,

$r\cos\theta = \sqrt{3}$ ...(iii), $r\sin\theta = 1$ ...(iv).

$\because$ Both $\cos\theta$ & $\sin\theta$ are +ve.
$\therefore$ θ lies in first quadrant.

On solving (iv),

$r\sin\theta = 1$

$\sin\theta = 1/2$

$\sin\theta = \sin\pi/6$

$\Rightarrow \theta = \pi/6$

$\therefore$ In Polar form,

$z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

Q.8. i

soln. $z = 0 + i$ ...(i). $a = 0$, $b = 1$

$$r = \sqrt{a^2+b^2} = \sqrt{0+1} = \sqrt{1} = 1$$

In Polar form,

$$z = r(\cos\theta + i\sin\theta) \quad \ldots (ii)$$

On comparing (i) and (ii),
we get,
$r\cos\theta = 0 \quad \ldots (iii) \qquad r\sin\theta = 1$

$\because$ Both $\cos\theta$ & $\sin\theta$ are not $-ve$.
$\therefore$ θ lies in first quadrant.

On solving (iii),
$r\cos\theta = 0$
$\cos\theta = 0$
$\cos\theta = \cos 90°$
$\theta = 90° = \frac{\pi}{2}$

$\therefore$ In polar form,
$$z = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

EXERCISE:- 5.3

Q.1. $x^2 + 3 = 0$

soln. $x^2 + 3 = 0$

$x^2 = -3$

$x = \sqrt{-3} \Rightarrow x = \pm\sqrt{3}\, i$

Q.2. $2x^2 + x + 1 = 0$

soln. $2x^2 + x + 1 = 0$

By the quadratic formula (for $(b^2 - 4ac) < 0$), we get,

$$x = \frac{-b \pm \sqrt{4ac - b^2}\, i}{2a}$$

$$= \frac{-1 \pm \sqrt{4 \times 2 \times 1 - 1^2}\, i}{2a}$$

$$= \frac{-1 \pm \sqrt{7}\, i}{2a}$$

Q.3. $x^2 + 3x + 9 = 0$

soln. $x^2 + 3x + 9 = 0$

By the quadratic formula,

$$x = \frac{-3 \pm \sqrt{4 \times 1 \times 9 - 3^2}\, i}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{27}\, i}{2}$$

$$= \frac{-3 \pm 3\sqrt{3}\, i}{2}$$

Q.4. $-x^2 + x - 2 = 0$

soln. $-x^2 + x - 2 = 0$

By the quadratic formula,

$$x = \frac{-1 \pm \sqrt{4 \times (-1) \times (-2) - (-1)^2}\, i}{2 \times (-1)}$$

$$= \frac{-1 \pm \sqrt{8 - 1}\, i}{-2}$$

$$= \frac{-1 \pm \sqrt{7}\, i}{2}$$

Q. 5. $x^2 + 3x + 5 = 0$

soln. $x^2 + 3x + 5 = 0$

By the quadratic formula,

$$x = \frac{-3 \pm \sqrt{4 \times 1 \times 5 - (3)^2}\, i}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{20 - 9}\, i}{2}$$

$$= \frac{-3 \pm \sqrt{11}\, i}{2}$$

Q. 6. $x^2 - x + 2 = 0$

soln. $x^2 - x + 2 = 0$

By the quadratic formula,

$$x = \frac{-(-1) \pm \sqrt{4 \times 1 \times 2 - (-1)^2}\, i}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{8 - 1}\, i}{2}$$

$$= \frac{1 \pm \sqrt{7}\, i}{2}$$

Q. 7. $\sqrt{2}\, x^2 + x + \sqrt{2} = 0$

soln. $\sqrt{2}\, x^2 + x + \sqrt{2} = 0$

By the quadratic formula,

$$x = \frac{-\sqrt{2} \pm \sqrt{4 \times \sqrt{2} \times \sqrt{2} - (1)^2}\, i}{2 \times \sqrt{2}}$$

$$= \frac{-\sqrt{2} \pm \sqrt{4 \times 2 - 1}\, i}{2\sqrt{2}}$$

$$= \frac{-\sqrt{2} \pm \sqrt{7}\, i}{2\sqrt{2}}$$

Q.8. $\sqrt{3}\, x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

soln. $\sqrt{3}\, x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

By the quadratic formula,

$$x = \frac{-(-\sqrt{2}) \pm \sqrt{4 \times \sqrt{3} \times 3\sqrt{3} - (\sqrt{2})^2}\, i}{2\sqrt{3}}$$

$$= \frac{\sqrt{2} \pm \sqrt{12 \times 3 - 2}\, i}{2\sqrt{3}}$$

$$= \frac{\sqrt{2} \pm \sqrt{34}\, i}{2\sqrt{3}}$$

Q.9. $x^2 + x + 1/\sqrt{2} = 0$

soln. $x^2 + x + 1/\sqrt{2} = 0$

$\cancel{2x^2 + x + \sqrt{2}} = 0$

By the quadratic formula,

$$x = \frac{-1 \pm \sqrt{4 \times 1 \times \sqrt{2} - 1^2}\, i}{2.1}$$

$$= \frac{-1 \pm \sqrt{4\sqrt{2} - 1}\, i}{2}$$

Q.10. $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$

soln. $x^2 + x/\sqrt{2} + 1 = 0$

$\sqrt{2}x^2 + x + \sqrt{2} = 0$

By the quadratic formula,

$$x = \frac{-(1) \pm \sqrt{4 \times \sqrt{2}\,\sqrt{2} - (1)^2}\; i}{2 \cdot \sqrt{2}}$$

$$= \frac{-1 \pm \sqrt{8 - 1}\; i}{2\sqrt{2}}$$

$$= \frac{-1 \pm \sqrt{7}\; i}{2\sqrt{2}}$$