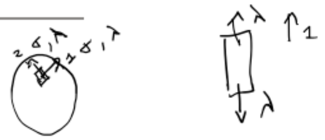
Taller 5



$x_1 = X_1 \lambda$

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \qquad \det(\mathbf{F}) = 1 \qquad \mathbf{C} = \mathbf{B} = \mathbf{F}^T \mathbf{F} = \begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & \lambda^{-4} \end{bmatrix}$$

$$\lambda^2 \lambda_3 = 1$$

$$\lambda_3 = \lambda^{-2}$$

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^{-2} \end{bmatrix}$$

$$W(\mathbf{C}) = c_1 \, tr(\mathbf{C}) + c_2 \left( \overbrace{\tfrac{1}{2}\left( tr(\mathbf{C})^2 - tr(\mathbf{C}^2) \right)}^{I_2} \right) - p \left( \det(\mathbf{F}) - 1 \right)$$

$$\mathbf{S} = 2 \frac{\partial W(\mathbf{C})}{\partial \mathbf{C}} \quad , \quad \frac{\partial tr(\mathbf{A})}{\partial \mathbf{A}} = \mathbf{I} \quad , \quad \frac{\partial I_2(\mathbf{A})}{\partial \mathbf{A}} = \underline{tr(\mathbf{A}) \mathbf{I} - \mathbf{A}^T}$$

$$\mathbf{S} = 2 \left( c_1 \mathbf{I} + c_2 ( tr(\mathbf{C}) \mathbf{I} - \mathbf{C} ) - p \frac{\partial \det(\mathbf{F})}{\partial \mathbf{C}} \right)$$

$$\frac{\partial \det(\mathbf{F})}{\partial \mathbf{C}} = \frac{1}{2} J \mathbf{C}^{-1} \qquad (\mathbf{A}\mathbf{B})^{-1} = \mathbf{B}^{-1} \mathbf{A}^{-1}$$

$$\mathbf{S} = 2 c_1 \mathbf{I} + 2 c_2 ( tr(\mathbf{B}) \mathbf{I} - \mathbf{C} ) - p J \mathbf{C}^{-1}$$

$$\mathbf{C}^{-1} = (\mathbf{F}^T \mathbf{F})^{-1} = \mathbf{F}^{-1} \mathbf{F}^{-T}$$

$$\boldsymbol{\sigma} = J^{-1} \mathbf{F} \mathbf{S} \mathbf{F}^T$$

$$\boldsymbol{\sigma} = J^{-1} \left( 2 c_1 \mathbf{F} \mathbf{F}^T + 2 c_2 \left( tr(\mathbf{B}) \mathbf{F} \mathbf{F}^T - \underbrace{\mathbf{F} \overbrace{\mathbf{F}^T \mathbf{F}}^{\mathbf{C}} \mathbf{F}^T}_{\mathbf{B} \cdot \mathbf{B} = \mathbf{B}^2} \right) - p J \underbrace{\mathbf{F} \mathbf{C}^{-1} \mathbf{F}^T}_{\mathbf{I}} \right)$$

$$\boldsymbol{\sigma}(\mathbf{B}) = J^{-1} \left( 2 c_1 \mathbf{B} + 2 c_2 ( tr(\mathbf{B}) \mathbf{B} - \mathbf{B}^2 ) - p J \mathbf{I} \right)$$

Considerando $J = 1$, $B_{11} = \lambda^2$, $B^2_{11} = \lambda^4$

$$\sigma_{11} = 2 c_1 \lambda^2 + 2 c_2 \{ (2 \lambda^2 + \lambda^{-4}) \lambda^2 - \lambda^4 \} - p = \sigma_{22} = \sigma$$

$$\sigma_{33} = 2 c_1 \lambda^{-4} + 4 c_2 \lambda^{-2} - p = 0$$

$$\sigma_{33} = 0 \Rightarrow p = 2 c_1 \lambda^{-4} + 4 c_2 \lambda^{-2}$$

Diferencia con la tarea 3

- compresible $\quad \det(\underline{F}) \neq 1$
- $\sigma_{22} = \sigma_{33} = 0$

$\lambda_{22} = \lambda_{33} = \lambda_T$

Problema 3) $\quad \sigma(\lambda), \quad \lambda \in [0{,}8 \,,\, 1{,}8]$

$$\sigma_{11}(\lambda, \lambda_T) = f_{11}(\lambda, \lambda_T)$$

$$\sigma_{22} = \sigma_{33} = \underline{f_{22}(\lambda, \lambda_T) = 0}$$

$\lambda_{T_0} = 1$

for $\lambda_i$ in $[0{,}8 \,,\, 1{,}8]$:

$\quad \lambda_T = \underline{\text{Newton}}(f_{22}(\lambda_i), Df_{22}, \lambda_{T_0})$

$\quad \sigma(\lambda) = f_{11}(\lambda_i, \lambda_T)$

$\quad \lambda_{T_0} = \lambda_T$