$$\Delta w_j = \gamma \left\langle \left(r_{out}(t) - \overline{r_{out}} \right) \left(r_j(t) - \bar{r}_j \right) \right\rangle_t$$

$$r_{out}(t) = \sum_{k=1}^{N} w_k \, r_k(t)$$

$$\overline{r_{out}} = \sum^{N} w_k \langle r_k(t) \rangle_t = \sum w_k \bar{r}_k$$

expand $r_{out}(t)$

$$\Delta w_j = \gamma \left\langle \left(\sum_{k=1}^{N} w_k r_k(t) - \sum_{k=1}^{N} w_k \bar{r}_k \right) \left(r_j(t) - \bar{r}_j \right) \right\rangle_t$$

Take weights out of time average

$$\Delta w_j = \gamma \left\langle \sum_k w_k \left(r_k(t) - \bar{r}_k \right) \left(r_j(t) - \bar{r}_j \right) \right\rangle_t = \gamma \sum_k w_k \left\langle \left(r_k(t) - \bar{r}_k \right) \left(r_j(t) - \bar{r}_j \right) \right\rangle_t =$$

$$= \gamma \sum_k C_{jk} w_k \quad \text{with} \quad C_{jk} = \left\langle \left(r_j(t) - \bar{r}_j \right) \left(r_k(t) - \bar{r}_k \right) \right\rangle_t$$

covariance

$$= \gamma \left\{ C \cdot \vec{w} \right\}_j$$

matrix-vector product

$$\boxed{\Delta \vec{w} = C \, \vec{w}}$$