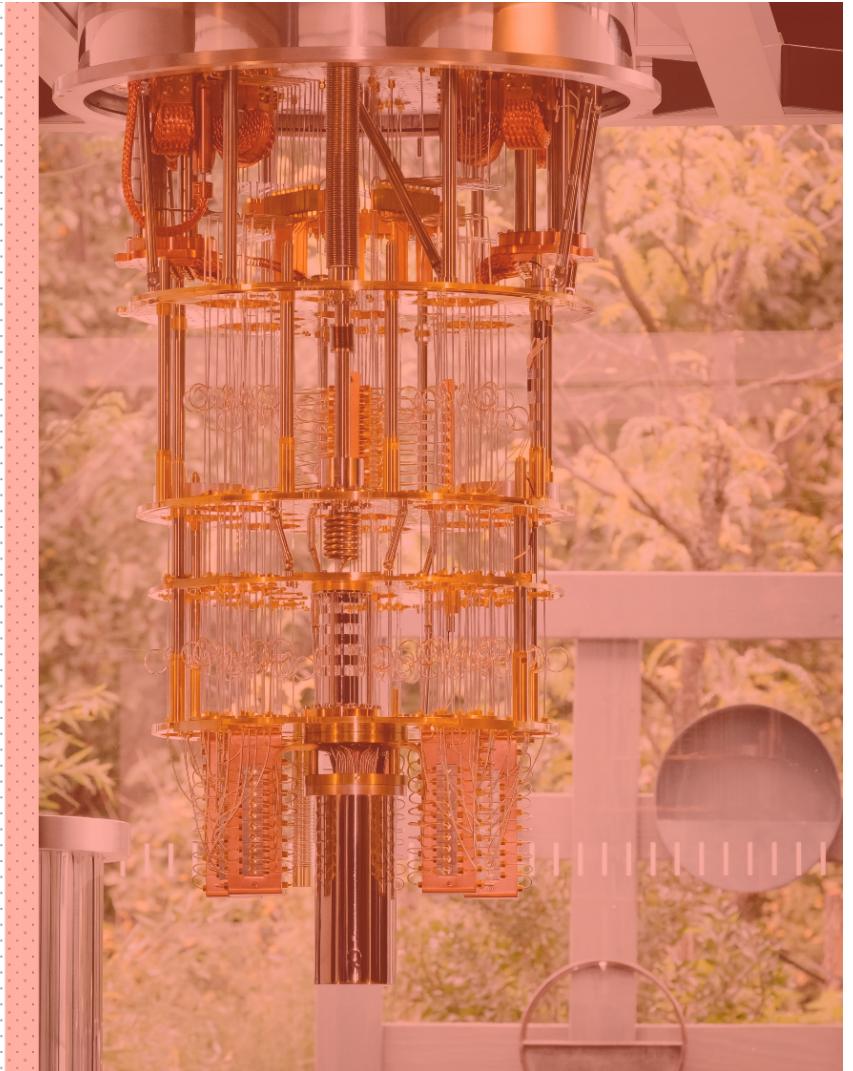


# **Towards Practical Hybrid Quantum / Classical Computing**

**M.Sc. Thesis Defense**

**Marcus Edwards**



# Conference Participation

MACHINE LEARNING FOR QUANTUM DESIGN





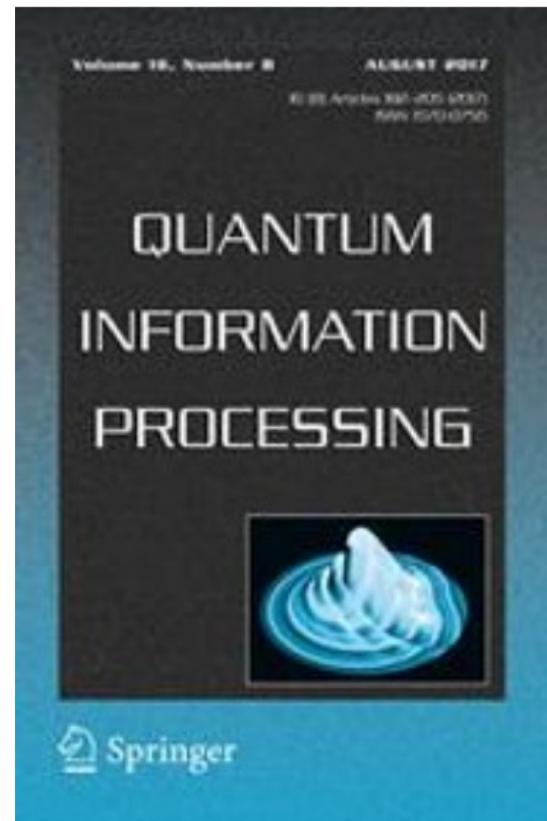
# Journal Publication

M. Edwards, A. Mashatan, and S. Ghose.

“A review of quantum and hybrid quantum / classical blockchain protocols”.

In: Quantum Information Processing 19.6 (2020).

DOI: 10.1007/s11128-020-02672-y.



# Defense Overview



## OPEN PROBLEMS

1. Point-to-Point Communication
2. Trusted Node Networks
3. Hybrid / Quantum Networks



## LITERATURE REVIEW

1. Blockchain fundamentals
2. Quantum blockchains
3. Hybrid blockchains



## METHODOLOGIES

1. Experimental Control Study
2. Experimental Technology Design
3. Theoretical Network Design



## RESULTS

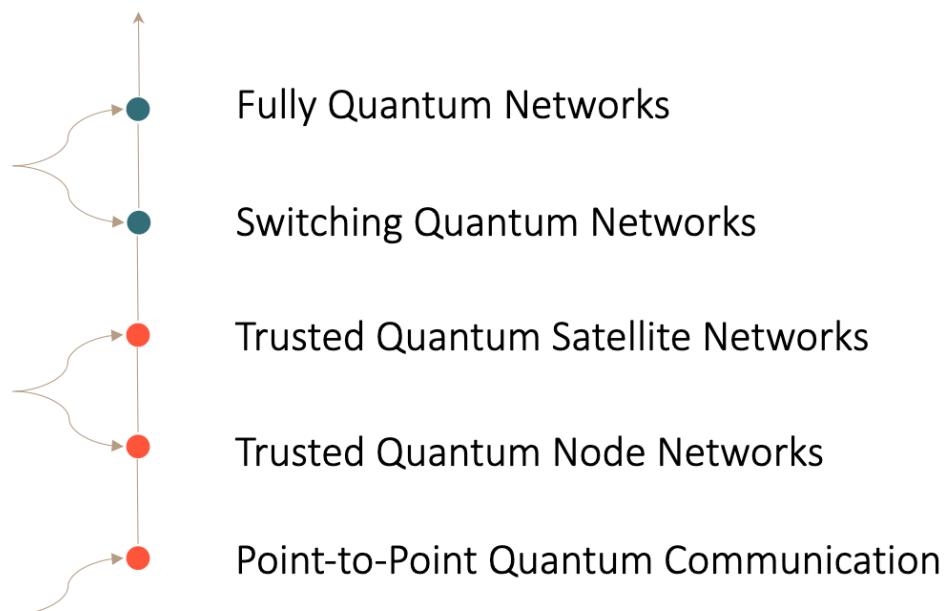
1. Experimental Study Results
2. Experimental Demonstration
3. Bounded Scalability

# Motivation and Context

Status: Basic Physics Research

Status: Engineering and Design

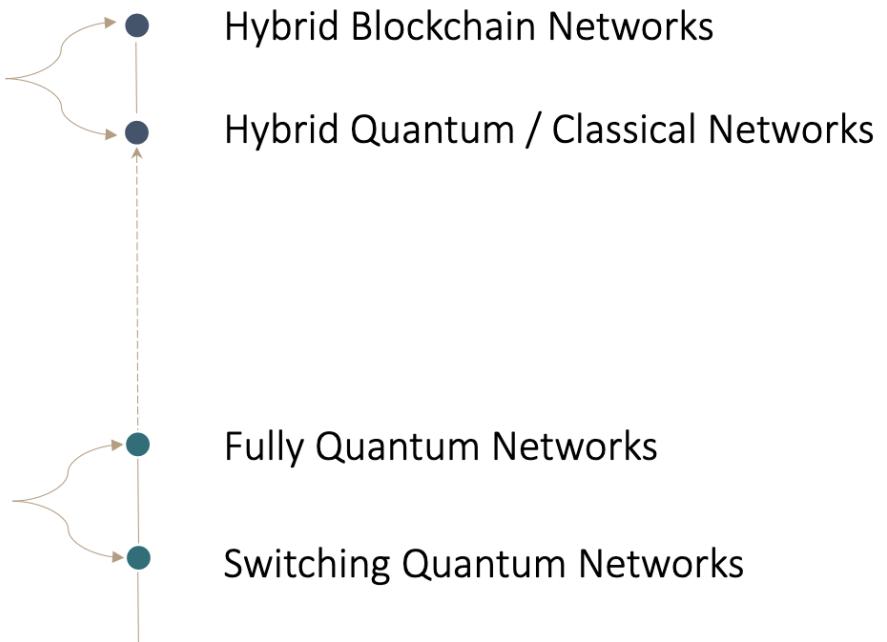
Status: Commercially Available



Norbert Lütkenhaus. "Lecture 22 - QKD Networks." QIC 890 - Applied Quantum Cryptography (Nov. 2020), Waterloo ON, University of Waterloo.

# Research Trajectory

Status: Presented in this Thesis!



# Open Questions

## Hybrid / Quantum Networks

- Multi-party applications (not reduceable to point-to-point)
  - Anonymous channels
  - Multi-party arbitrary function evaluation
- 

## Trusted Node Networks

- Integration of QKD into security architecture
  - Reduction of trust assumptions
  - Authentication or proof of identity
- 

## Point-to-Point Communication

- Tools to evaluate protocols
- Abilities to handle imperfections
- More applications

Norbert Lütkenhaus. "Lecture 22 - QKD Networks." QIC 890 - Applied Quantum Cryptography (Nov. 2020), Waterloo ON, University of Waterloo.

# Practical Quantum / Classical Internet Goals

## VIABLE TECHNOLOGY

Requires quantum control.

## COMPLEMENTARY PARADIGMS

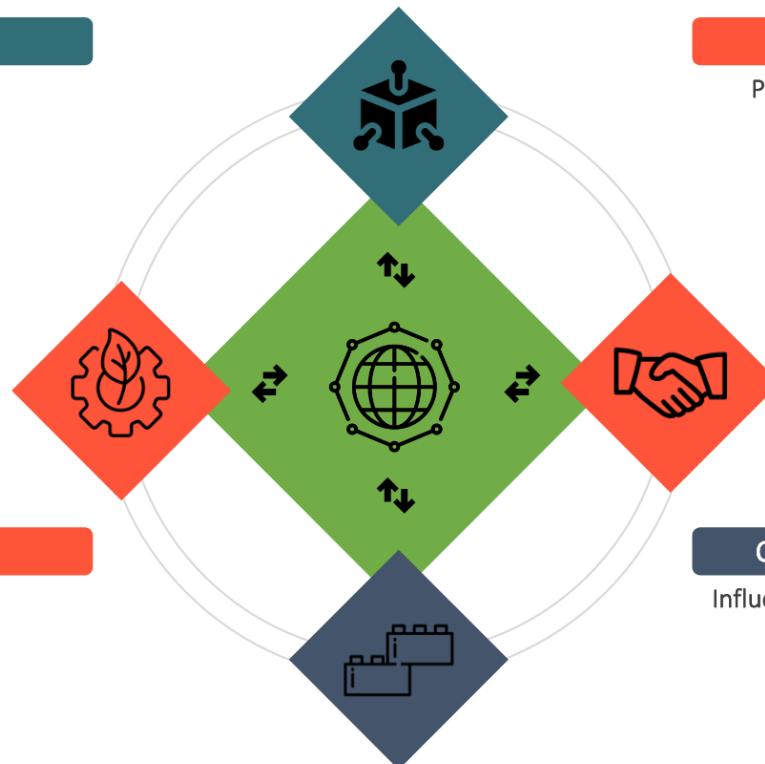
Paradigms need to be complementary.

## SUSTAINABLE ECOSYSTEM

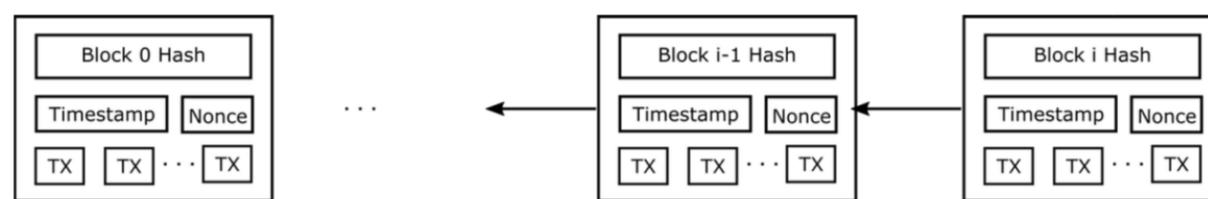
New tooling and security.

## CONSTRUCTIVE COLLABORATION

Influence between peers should be kept level.



# Blockchain – Data Structure



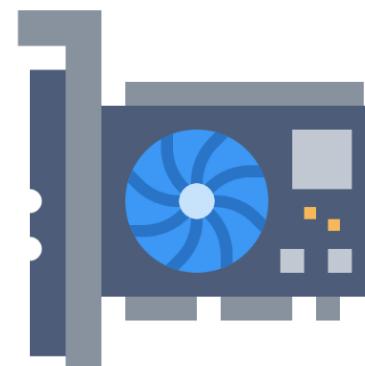
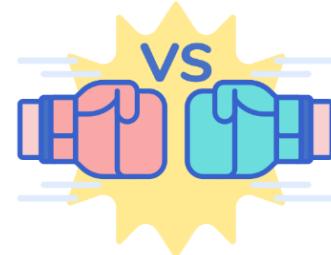
- TX = a transaction record
- Timestamp = the exact time of a block's publication
- Nonce = a unique, non-recurring identifier for a block
- Hash = the “proof of work” for the block

Satoshi Nakamoto. Bitcoin: A Peer-to-Peer Electronic Cash System. url: <https://bitcoin.org/bitcoin.pdf>.

# Blockchain – Consensus Algorithms



Proof of Stake



Proof of Work

Dylan J. Yaga et al. Blockchain Technology Overview. Nov. 2018. url: <https://www.nist.gov/publications/blockchain-technology-overview>.

# Blockchain – Smart Contracts



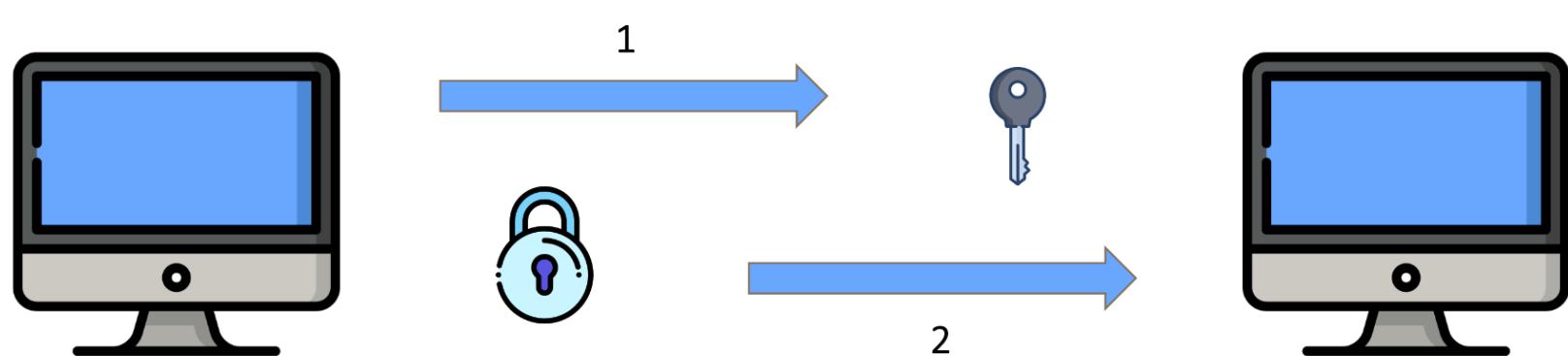
Vitalik Buterin. Ethereum whitepaper. 2013. url: <https://whitepaper.io/document/5/ethereum-whitepaper>.

# Quantum Blockchain

- Quantum coins
  1. Generate two random bit strings  $M$  and  $N$  of length  $l$
  2. Prepare a quantum state  $|\$> = |0>^{\otimes l}$
  3. For each bit  $i < l$ :
    - If  $M_i = 0$  and  $N_i = 0$ , do nothing to the  $i^{th}$  qubit
    - If  $M_i = 0$  and  $N_i = 1$ , rotate the  $i^{th}$  qubit state to  $|1>$
    - If  $M_i = 1$  and  $N_i = 0$ , rotate the  $i^{th}$  qubit state to  $|+>$
    - If  $M_i = 1$  and  $N_i = 1$ , rotate the  $i^{th}$  qubit state to  $|->$

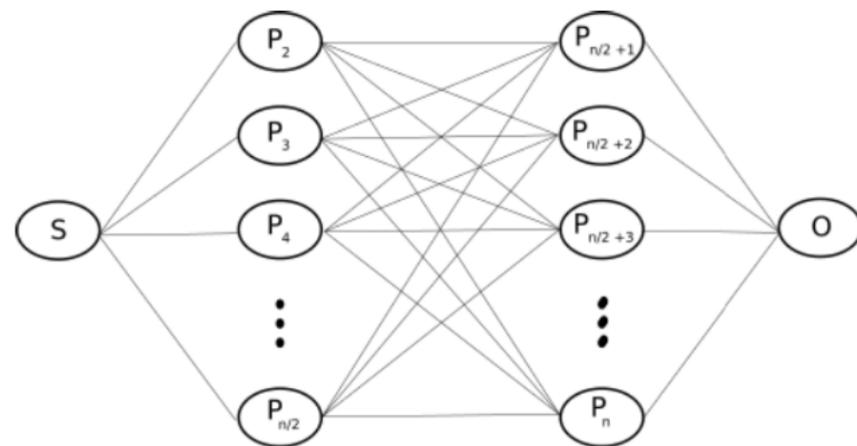
Stephen Wiesner. "Conjugate coding". In: ACM SIGACT News 15.1 (1983), pp. 78–88. doi: 10.1145/1008908.1008920.

# Quantum Binding Commitments



Dominique Unruh. "Collapse-Binding Quantum Commitments Without Random Oracles". In: Advances in Cryptology - ASIACRYPT 2016 Lecture Notes in Computer Science (2016), pp. 166–195. doi: 10.1007/978-3-662-53890-6\_6.

# Quantum Honest Byzantine Agreement



- S = Sender of vote
- P = other network participant
- n = number of participants
- L = verification lists distributed to all P
- b = Boolean vote value
- ID = indexes of b in L

Xin Sun et al. "Quantum-enhanced Logic-based Blockchain I: Quantum Honest-success Byzantine Agreement and Qulogicoind". In: arXiv e-prints, arXiv:1805.06768 (May 2018), arXiv:1805.06768. arXiv: 1805.06768 [quant-ph].

# Collision Free Quantum Money

- equal superposition of exponentially many unrelated terms

$$|\$l\rangle = \frac{1}{\sqrt{N_l}} \sum_{x \in t, L(x)=l} |x\rangle$$

- Valid quantum money states

$$M^r \doteq \sum_l |\$l\rangle \langle \$l| \quad M = \frac{1}{N} \sum_{i=1}^N P_i$$

- Verification Kraus operators

$$(I \otimes \frac{1}{\sqrt{N}} \sum_{i=1}^N |i\rangle) U (I \otimes \frac{1}{\sqrt{N}} \sum_{i=1}^N |i\rangle)^\dagger = \frac{1}{N} \sum_{i=1}^N P_i = M$$

- Approximately verified state

$$\sum_l |\$l\rangle \langle \$l|$$

Andrew Lutomirski et al. "Breaking and making quantum money: toward a new quantum cryptographic protocol". In: arXiv e-prints, arXiv:0912.3825 (Dec. 2009), arXiv:0912.3825. arXiv: 0912.3825 [quant-ph].

# Quantum Lightning



Quantum money



Verification using a classical serial number



Verification does not tamper with money

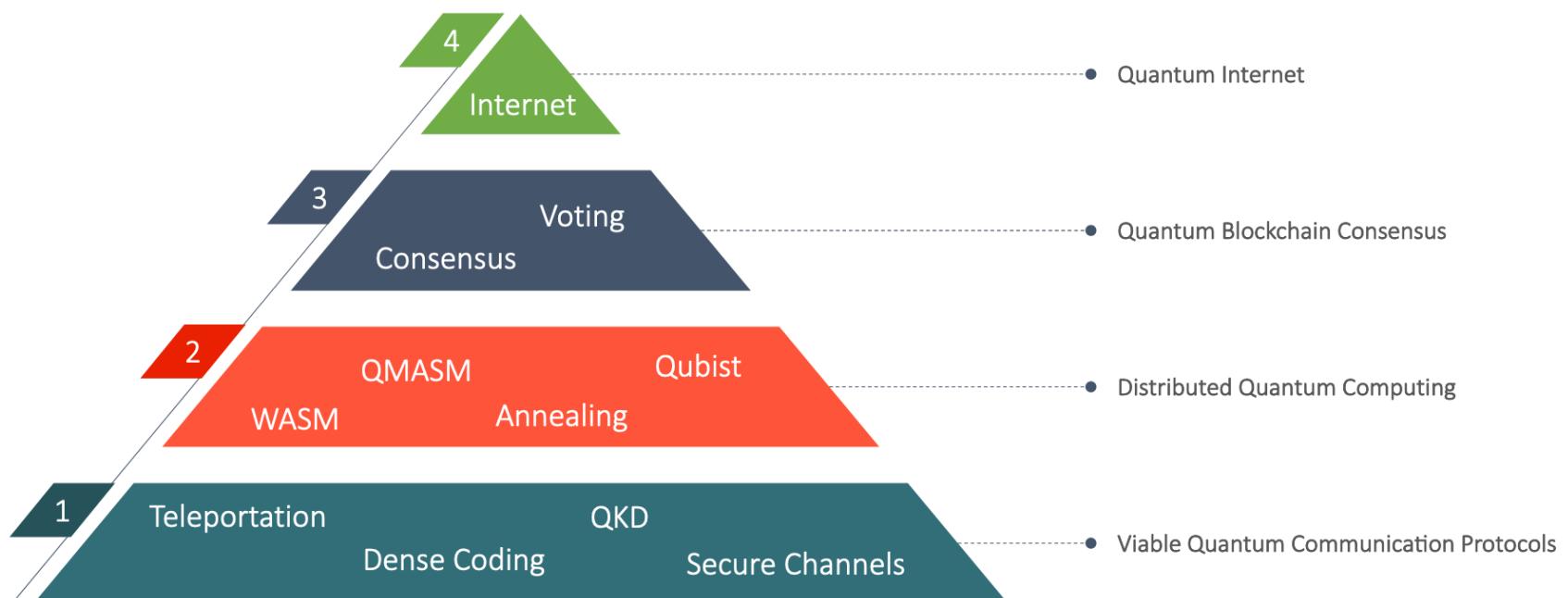
Mark Zhandry. "Quantum Lightning Never Strikes the Same State Twice". In: Advances in Cryptology - EUROCRYPT 2019 Lecture Notes in Computer Science (2019), pp. 408–438. doi: 10.1007/978-3-030-17659-4\_14.

# Hybrid Quantum Classical Blockchain

- Payments
  - P sends  $|$>$ , a contract id *cid*, and serial number *serial* to P'
  - P' sends a *Retrieve Contract* message to the ledger, retrieving the contract *cid*.
  - P' accepts the payment if *cid* and  $\text{Verify}(|$>) = \text{serial}$
- Coin Recovery
  - *Trigger* message to cause a smart contract to execute a circuit *BanknoteLost*
  - P' has the chance to challenge for a fixed time by demonstrating ownership of the matching classical serial number
  - If not successfully challenged, P is given the new quantum coin

Andrea Coladangelo. "Smart contracts meet quantum cryptography". In: arXiv e-prints, arXiv:1902.05214 (Feb. 2019), arXiv:1902.05214. arXiv: 1902.05214 [quant-ph].

# Complementary Research Areas



# Effectiveness of Quantum Control for Networks

			
<h3>Algorithm Selection</h3> <p>Looked at entanglement based communication</p> <p>Chose controlled quantum teleportation</p> <p>Chose controlled Dense Coding</p>	<h3>Translation to Code</h3> <p>Converted circuits to low-level specifications in OpenQASM</p> <p>Converted OpenQASM to Python programs</p> <p>Made compatible with IBM Q application programming interface</p>	<h3>Experimentation</h3> <p>Designed and implemented case-based fidelity measurement circuits</p> <p>Designed and implemented tests for the impact of the controller on result fidelity</p> <p>Automated batch testing for each algorithm and controller behaviour</p>	<h3>Results Analysis</h3> <p>Collected statistical results from batches of results</p> <p>Performed analysis of statistical significance of the controller</p> <p>Performed state tomography to verify fidelity results</p>

# Open Questions Addressed

## Hybrid / Quantum Networks

- Multi-party applications (not reduceable to point-to-point)
  - Anonymous channels
  - Multi-party arbitrary function evaluation
- 

## Trusted Node Networks

- Integration of QKD into security architecture
  - Reduction of trust assumptions
  - Authentication or proof of identity
- 

## Point-to-Point Communication

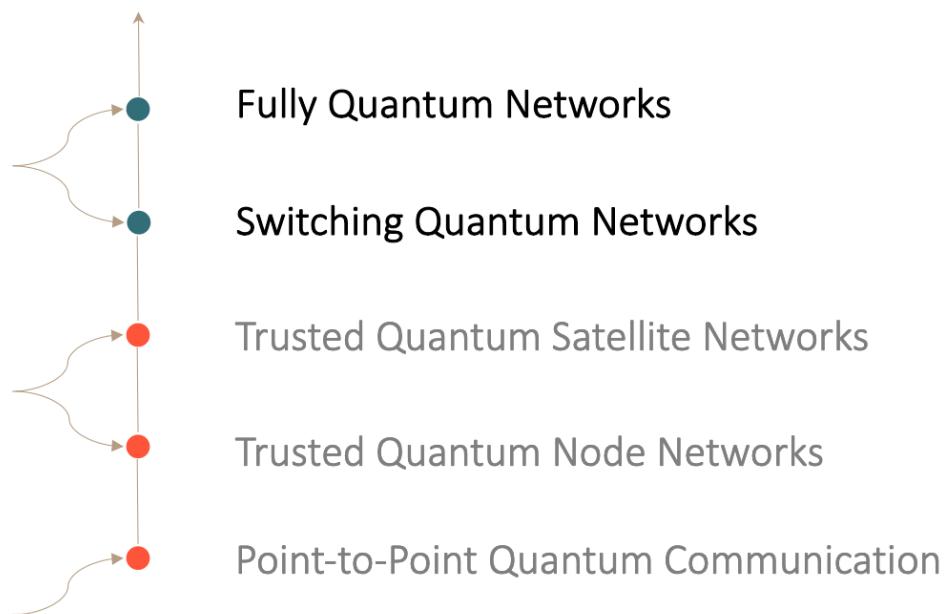
- Tools to evaluate protocols
- Abilities to handle imperfections
- More applications

# Motivation and Context

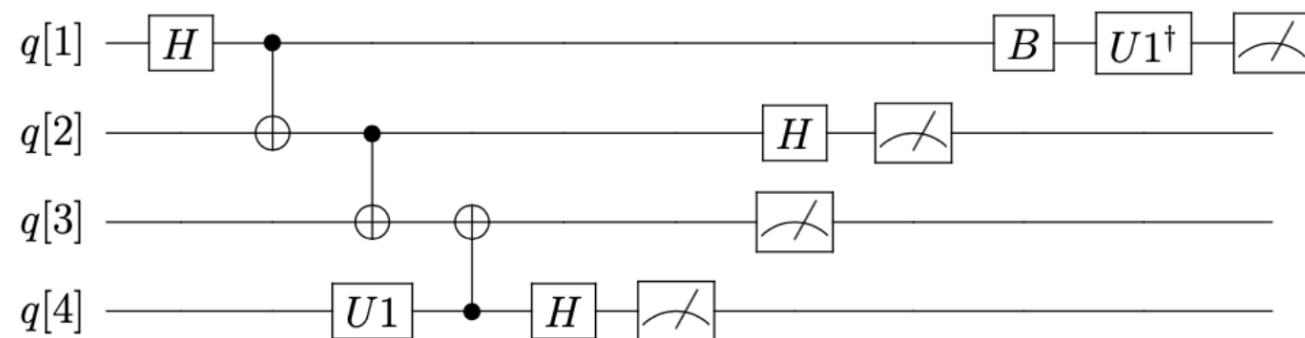
Status: Basic Physics Research

Status: Engineering and Design

Status: Commercially Available



# Controlled Teleportation



Xihan Li and Fuguo Deng. "Controlled teleportation". In: Frontiers of Computer Science in China 2.2 (2008), pp. 147–160. doi: 10.1007/s11704-008-0020-0.

# Bob's Decoding Operations

Bell State	Charlie's Result	Bob's Operation
$ \phi^+>_{xA}$	$ +x>$	I
$ \phi^+>_{xA}$	$  -x >$	Z
$ \phi^->_{xA}$	$  +x >$	Z
$ \phi^->_{xA}$	$  -x >$	I
$ \psi^+>_{xA}$	$  +x >$	X
$ \psi^+>_{xA}$	$  -x >$	XZ
$ \psi^->_{xA}$	$  +x >$	XZ
$ \psi^->_{xA}$	$  -x >$	X

Xihan Li and Fuguo Deng. "Controlled teleportation". In: Frontiers of Computer Science in China 2.2 (2008), pp. 147–160. doi: 10.1007/s11704-008-0020-0.

# Experimental Methodology

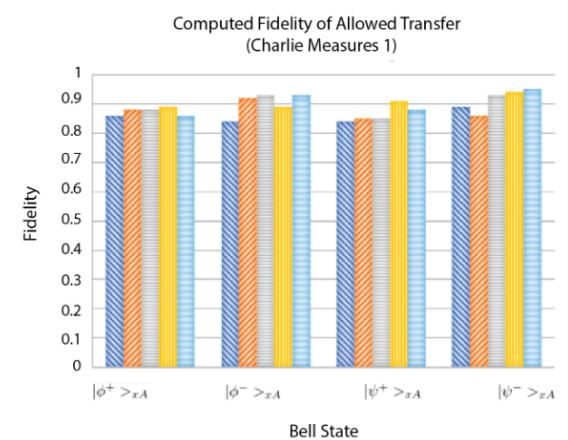
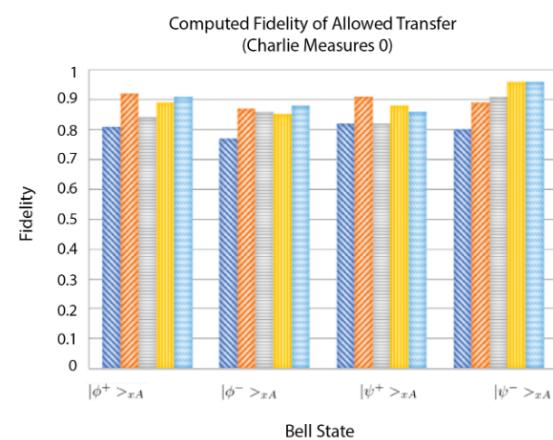
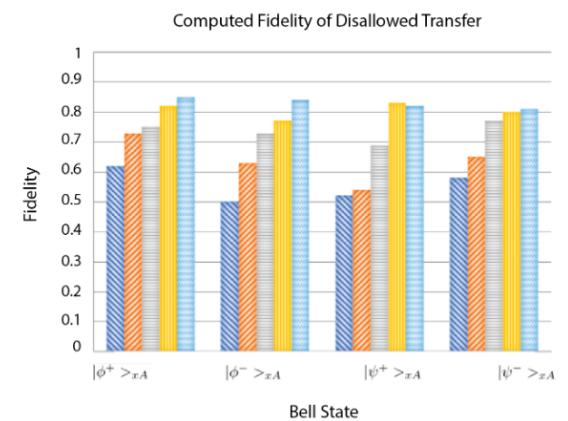
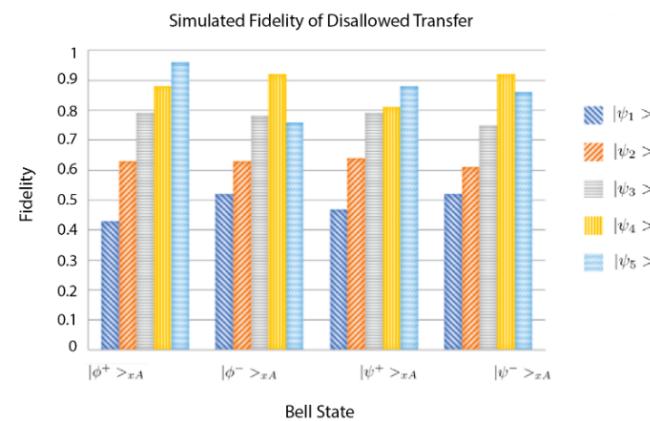
- Limitations

1. Measurements taken during an execution cannot affect the gates that are applied as a part of the circuit.
2. API returns a probability distribution of measurement results over a number of executions.
3. Direct measurement results provide no insight since they can't discriminate between contexts.

- Strategies

1. A post-selection algorithm was implemented that filtered out bad choices by Bob.
2. Each of the four circuits was run in a batch of 1000 executions each.
3. The dagger of the input state was applied to qubit B at the end of the procedure.

# Fidelities

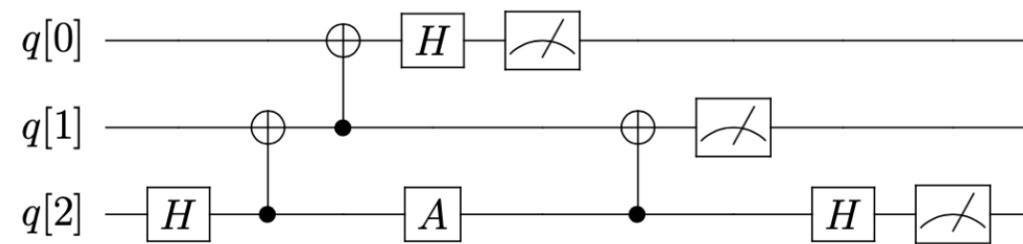


# Significance



$$\frac{\bar{F}_{allowed} - \bar{F}_{disallowed}}{max(Q3_{allowed}) - min(Q1_{disallowed})} = \frac{0.87 - 0.71}{0.92 - 0.62} = 57\%$$

# Superdense Coding



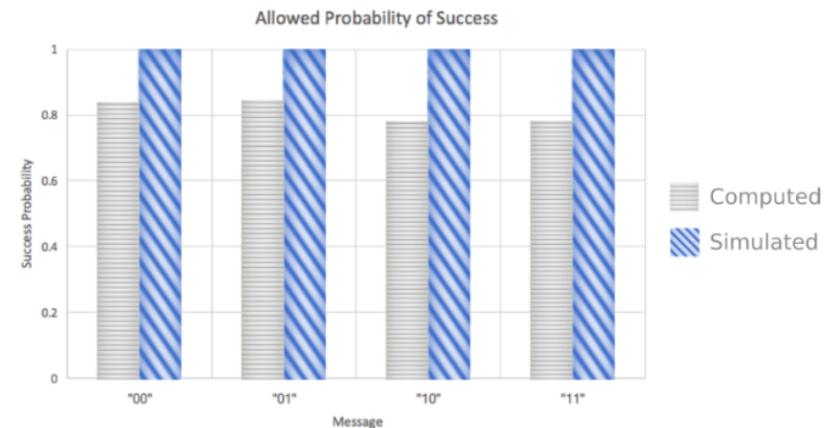
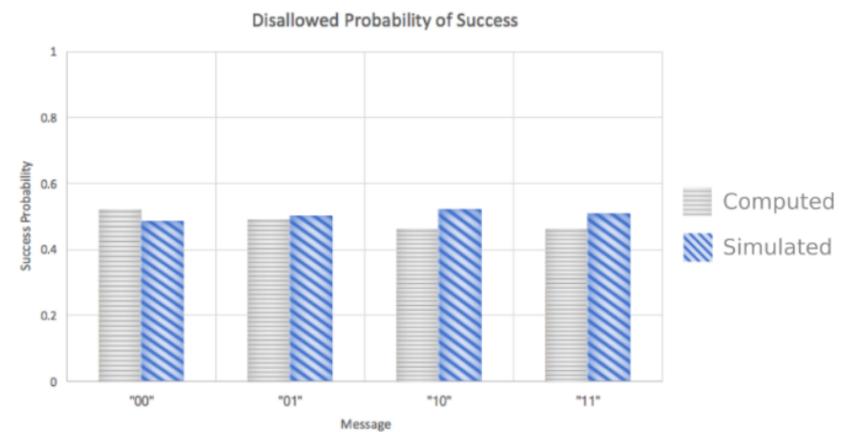
Jiu-Cang Hao, Chuan-Feng Li, and Guang-Can Guo. "Controlled dense coding using the Greenberger-Horne-Zeilinger state". In: Physical Review A 63.5 (Nov. 2001). doi: 10.1103/physreva.63.054301.

# Data Encoding Operations

Message	Gates
00	I
01	Z
10	X
11	Y

Jiu-Cang Hao, Chuan-Feng Li, and Guang-Can Guo. "Controlled dense coding using the Greenberger-Horne-Zeilinger state". In: Physical Review A 63.5 (Nov. 2001). doi: 10.1103/physreva.63.054301.

# Fidelities



# State Tomography Verification

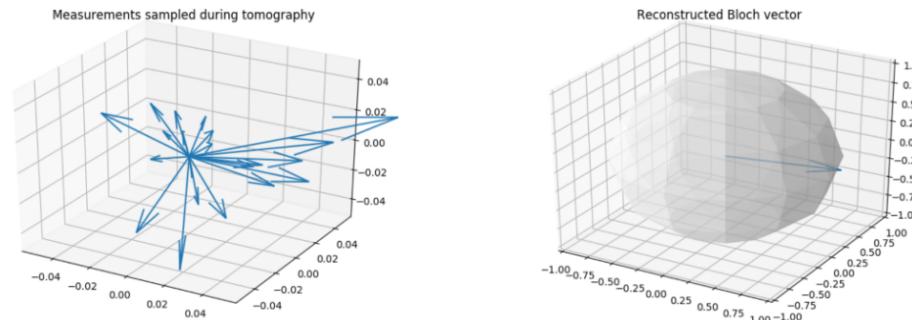
- Relative phases
- Axis rotations
- Bloch vector reconstruction
- Results (Alice sends Z)

$$\text{phase}_j = \frac{2\pi j}{\text{phases} - 1}$$

Axis Rotations

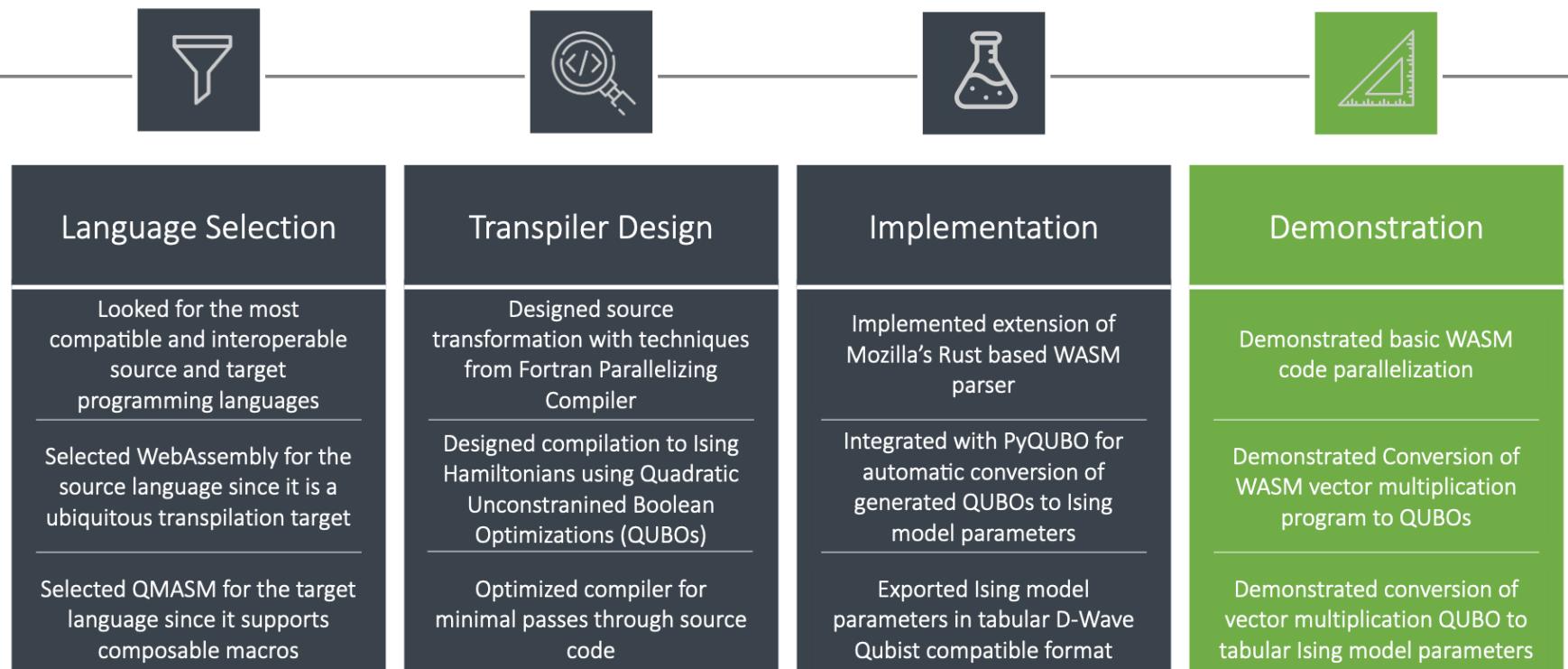
X-Axis	Y-Axis	Z-Axis
<b>M H</b>	<b>M H S<sup>†</sup></b>	<b>M</b>

```
# sum all observed vectors
x, y, z = zip(*bloch_vectors)
x = functools.reduce(lambda pre, curr: pre + curr, x)
y = functools.reduce(lambda pre, curr: pre + curr, y)
z = functools.reduce(lambda pre, curr: pre + curr, z)
```



Shukla, Abhishek, et al. "Complete Characterization of the Directly Implementable Quantum Gates Used in the IBM Quantum Processors." 2018.

# Hybrid Streaming Web Code Execution



# Open Questions Addressed

## Hybrid / Quantum Networks

Multi-party applications (not reduceable to point-to-point)  
Anonymous channels  
Multi-party arbitrary function evaluation

---

## Trusted Node Networks

Integration of QKD into security architecture  
Reduction of trust assumptions  
Authentication or proof of identity

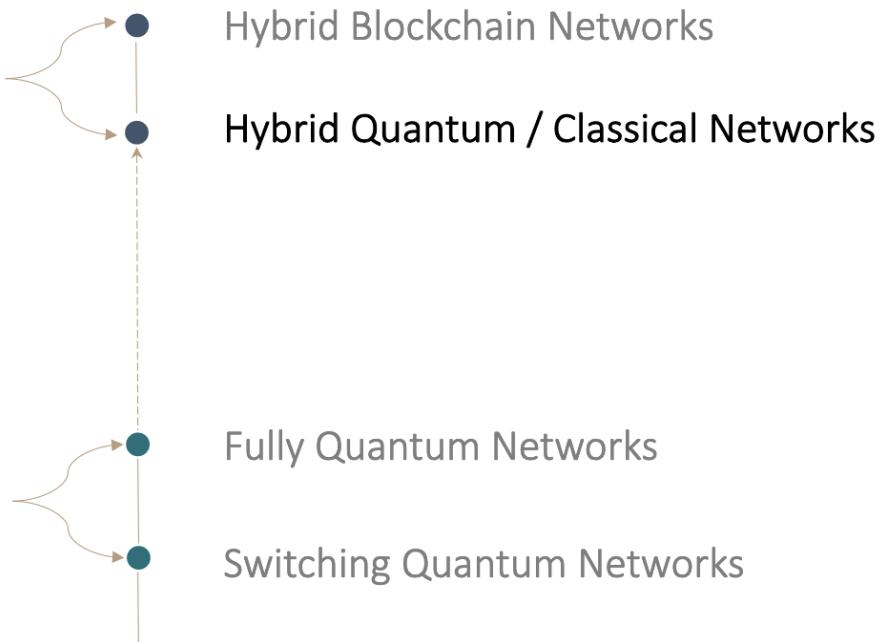
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## Point-to-Point Communication

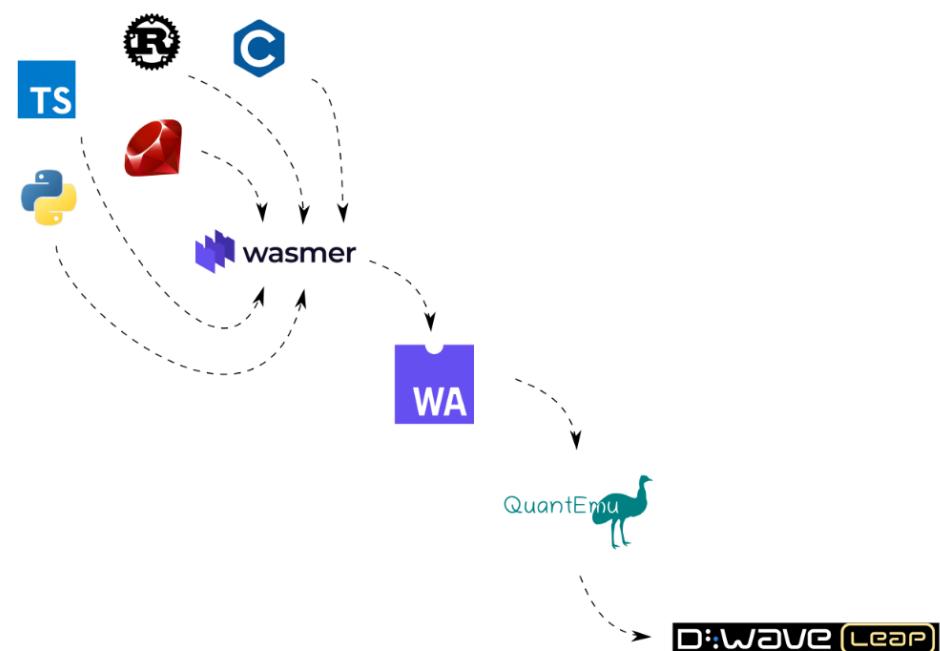
Tools to evaluate protocols  
Abilities to handle imperfections  
More applications

# Motivation and Context

Status: Presented in this Thesis!

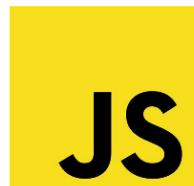


# WebAssembly



# WebAssembly Text Format

```
function accel(vi, vf, t){  
    return (vi - vf)/t;  
}  
  
(func $accel (param $vi i32) (param $vf i32) (param $t i32) (result i32)  
  (i32.div_u  
   (i32.sub  
    (get_local $vf)  
    (get_local $vi)  
   )  
   (get_local $t)  
  )  
)
```



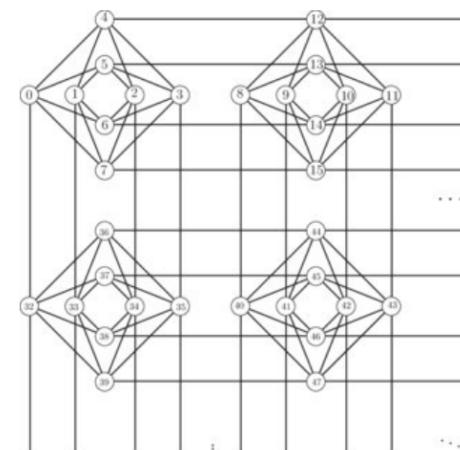
MDN Web Docs. Understanding WebAssembly text format. url: [https://developer.mozilla.org/en-US/docs/WebAssembly/Understanding\\_the\\_text\\_format#S-expressions](https://developer.mozilla.org/en-US/docs/WebAssembly/Understanding_the_text_format#S-expressions).

# Quantum Annealing

- D-Wave's annealers are capable of solving problems of a form

$$\mathbf{H}(\hat{\sigma}) = \sum_{i=0}^{N-1} h_i \sigma_i + \sum_{i=0}^{N-2} \sum_{j=i+1}^{N-1} J_{i,j} \sigma_i \sigma_j$$

- Qubit connectivity “Chimera” graph



D-Wave. url: <https://www.dwavesys.com/p-res-releases/d-wave-previews-next-generation-quantum-computing-platform>.

# Quadratic Unconstrained Optimization

- A quadratic unconstrained Boolean expression

$$-x - z + 2xz$$

- Unpenalized solution example

$$\mathbf{P} = 2xz - x - z + 1 = 2 \times 0 \times 1 - 0 - 1 + 1 = -1 + 1 = 0$$

- Penalized solution example

$$\mathbf{P} = 2xz - x - z + 1 = 2 \times 0 \times 0 - 0 - 0 + 1 = 1$$

D-Wave. Wave Ocean Software Documentation. url: <http://docs.ocean.dwavesys.com/>.

# QMASM Macros

Cell	Logic	Quadratic pseudo-Boolean function representation
NOT	$Y = \neg A$	$\mathbb{H}_{\neg}(\sigma_{\gamma}, \sigma_A) = \sigma_A \sigma_{\gamma}$
AND	$Y = A \wedge B$	$\mathbb{H}_{\wedge}(\sigma_{\gamma}, \sigma_A, \sigma_B) = -\frac{1}{2}\sigma_A - \frac{1}{2}\sigma_B + \sigma_{\gamma} + \frac{1}{2}\sigma_A \sigma_B - \sigma_A \sigma_{\gamma} - \sigma_B \sigma_{\gamma}$
OR	$Y = A \vee B$	$\mathbb{H}_{\vee}(\sigma_{\gamma}, \sigma_A, \sigma_B) = -\frac{1}{2}\sigma_A - \frac{1}{2}\sigma_B - \sigma_{\gamma} + \frac{1}{2}\sigma_A \sigma_B - \sigma_A \sigma_{\gamma} - \sigma_B \sigma_{\gamma}$
NAND	$Y = A \uparrow B$	$\mathbb{H}_{\uparrow}(\sigma_{\gamma}, \sigma_A, \sigma_B) = -\frac{1}{2}\sigma_A - \frac{1}{2}\sigma_B - \sigma_{\gamma} + \frac{1}{2}\sigma_A \sigma_B + \sigma_A \sigma_{\gamma} + \sigma_B \sigma_{\gamma}$
NOR	$Y = A \downarrow B$	$\mathbb{H}_{\downarrow}(\sigma_{\gamma}, \sigma_A, \sigma_B) = \frac{1}{2}\sigma_A + \frac{1}{2}\sigma_B + \sigma_{\gamma} + \frac{1}{2}\sigma_A \sigma_B + \sigma_A \sigma_{\gamma} + \sigma_B \sigma_{\gamma}$
XOR	$Y = A \oplus B$	$\begin{aligned} \mathbb{H}_{\oplus}(\sigma_{\gamma}, \sigma_A, \sigma_B, \sigma_a) = & \frac{1}{2}\sigma_A - \frac{1}{2}\sigma_B - \frac{1}{2}\sigma_{\gamma} + \sigma_a - \frac{1}{2}\sigma_A \sigma_B \\ & - \frac{1}{2}\sigma_A \sigma_{\gamma} + \sigma_A \sigma_a + \frac{1}{2}\sigma_B \sigma_{\gamma} - \sigma_B \sigma_a - \sigma_{\gamma} \sigma_a \end{aligned}$
XNOR	$Y = A \Leftrightarrow B$	$\begin{aligned} \mathbb{H}_{\Leftrightarrow}(\sigma_{\gamma}, \sigma_A, \sigma_B, \sigma_a) = & \frac{1}{2}\sigma_A - \frac{1}{2}\sigma_B + \frac{1}{2}\sigma_{\gamma} + \sigma_a - \frac{1}{2}\sigma_A \sigma_B \\ & + \frac{1}{2}\sigma_A \sigma_{\gamma} + \sigma_A \sigma_a - \frac{1}{2}\sigma_B \sigma_{\gamma} - \sigma_B \sigma_a + \sigma_{\gamma} \sigma_a \end{aligned}$
2:1 MUX	$Y = (S \wedge B) \vee (\neg S \wedge A)$	$\begin{aligned} H_{MUX}(\sigma_{\gamma}, \sigma_S, \sigma_A, \sigma_B, \sigma_a) = & \frac{1}{2}\sigma_S + \frac{1}{4}\sigma_A - \frac{1}{4}\sigma_B \\ & + \frac{1}{2}\sigma_{\gamma} + \sigma_a + \frac{1}{4}\sigma_S \sigma_A - \frac{1}{4}\sigma_S \sigma_B + \frac{1}{2}\sigma_S \sigma_{\gamma} + \sigma_S \sigma_a \\ & + \frac{1}{2}\sigma_A \sigma_B - \frac{1}{2}\sigma_A \sigma_{\gamma} + \frac{1}{2}\sigma_A \sigma_a - \sigma_B \sigma_{\gamma} - \frac{1}{2}\sigma_B \sigma_a + \sigma_{\gamma} \sigma_a \end{aligned}$

Scott Pakin. "A quantum macro assembler". In: 2016 IEEE High Performance Extreme Computing Conference (HPEC) (2016). doi: 10.1109/hpec.2016.7761637.

# QMASM TypeScript

QMASM, Los Alamos National Laboratory's macro assembler for D-wave's quantum annealer, implemented in TypeScript.

Language documentation is provided by Scott Pakin [here](#).

## New in Version 1.0.0

- Support for the following QMASM features:
  - Specifying 2-local Ising Hamiltonian parameters via: qubit weights, coupling strengths
  - Relating qubits via: chains, anti-chains, equivalences
  - Relating qubits to classical values via pins
  - Importable and parameterizable macro system via: !begin\_macro, include, !use\_macro
  - Macro chaining via: !next
  - Assertions
  - Logical and mathematical expressions involving the following elements: +, -, \*, /, \*\*, =, /=, ||, |, &&, &, ~, !, ^, %, <, >, <<, >>, <=, >=
  - For loops
  - If/else conditionals
  - Support for the following classical types: Iterator, Range, Int, Float, Bool
  - Support for the following quantum data types: Qubit, Ancillary, QubitArray, Register

### Install

```
> npm i qmasm-ts
```

### Weekly Downloads

16



### Version

1.0.5

### License

BSD-3-Clause-Att...

### Unpacked Size

88.6 kB

### Total Files

11

### Issues

0

### Pull Requests

0

### Homepage

[🔗 github.com/MackEdweise/qmasm-ts#re...](https://github.com/MackEdweise/qmasm-ts#readme)

### Repository

[🔗 github.com/MackEdweise/qmasm-ts](https://github.com/MackEdweise/qmasm-ts)

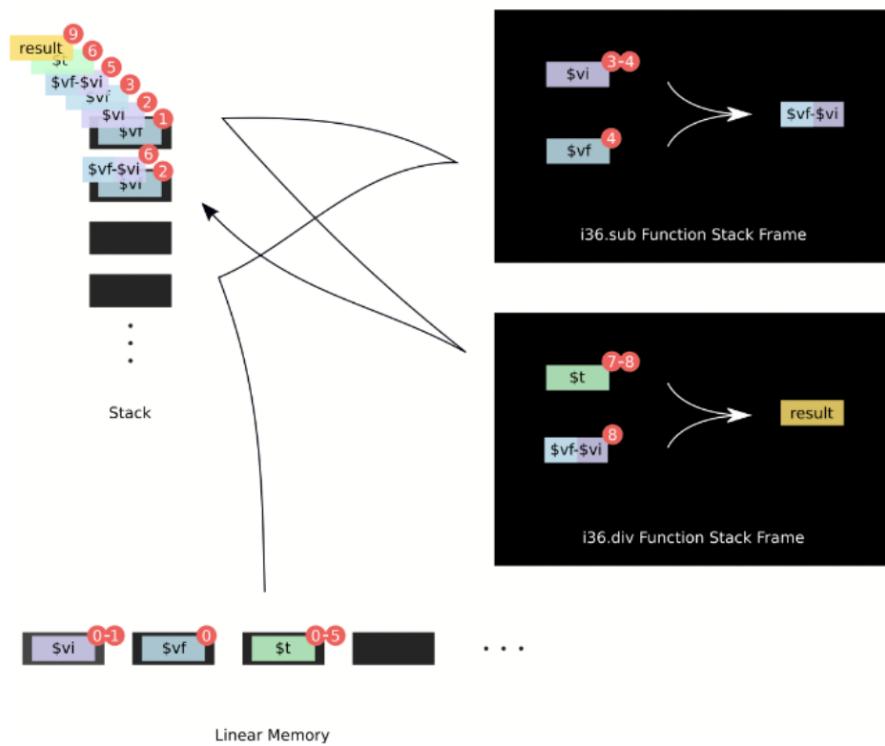
### Last publish

7 months ago

### Collaborators



# Dependency Tracing



# Interactive Transpiling

```
BeginWasm { version: 1 }
BeginSection { code: Type, range: Range { start: 10, end: 29 } }
TypeSectionEntry(FuncType { form: Func, params: [I32, I32, I32, I32, I32, I32], returns: [I32] })
TypeSectionEntry(FuncType { form: Func, params: [I32, I32, I32, I32], returns: [I32] })
EndSection
BeginSection { code: Function, range: Range { start: 31, end: 34 } }
EndSection
BeginSection { code: Export, range: Range { start: 36, end: 59 } }
ExportSectionEntry { field: "dot_three", kind: Function, index: 0 }
ExportSectionEntry { field: "dot_two", kind: Function, index: 1 }
EndSection
BeginSection { code: Code, range: Range { start: 61, end: 95 } }
BeginFunctionBody { range: Range { start: 63, end: 81 } }
1. Unreachable
2. GetLocal { local_index: 0 }
3. GetLocal { local_index: 3 }
4. I32Mul
5. GetLocal { local_index: 1 }
6. GetLocal { local_index: 2 }
7. GetLocal { local_index: 4 }
8. GetLocal { local_index: 5 }
9. Call { function_index: 1 }
10. I32Add
11. End
BeginFunctionBody { range: Range { start: 82, end: 95 } }
1. Unreachable
2. GetLocal { local_index: 0 }
3. GetLocal { local_index: 2 }
4. I32Mul
5. GetLocal { local_index: 1 }
6. GetLocal { local_index: 3 }
7. I32Mul
8. I32Add
9. End
EndSection
First pass found 2 functions:
[0, 1]
```

```
Parallelize function 1 (yes/no)?
y
Analyzing function 1...
Found 0 blocks in function 1
Found 0 calls to other functions from function 1
Parallelize function 0 (yes/no)?
y
Analyzing function 0...
Found 0 blocks in function 0
Found 1 calls to other functions from function 0
Registering call to function 1 from function 0
Found 0 blocks in function 1
Found 0 calls to other functions from function 1
```

# Lowering Expressions to Constraints

```
/// The physical expression enum represents the valid
/// operations and data types that can be understood by PyQUBO.
#[derive(Clone, Debug)]

pub enum PhysicalExpression {
    Not{ operand: Box<PhysicalExpression> },
    Add{ operand_one: Box<PhysicalExpression>, operand_two:
        Box<PhysicalExpression> },
    Mul{ operand_one: Box<PhysicalExpression>, operand_two:
        Box<PhysicalExpression> },
    Spin{ val: bool }, // 0 represents -1
    Num{ val: usize },
    Binary{ val: bool }
}

/// The abstract operation enum represents logical operations
/// that can be compiled to simulatable transfer functions
/// for quantum annealers.
#[derive(Clone, Debug)]

pub enum AbstractExpression {
    Spin { id: usize },
    Num { val: usize },
    Add { ty: Type },
    Mul { ty: Type }
}
```

# 8-bit Multiplier Circuit - Constraints

```
(Spin("A5") * Spin("B5"))

(Not(Spin("A4") * Spin("B5")) + Not(Spin("A5") * Spin("B4")))

(Not(Spin("A3") * Spin("B5")) + Spin("A4") * Spin("B4") + Not(Spin("A5") *
Spin("B3")))

(Not(Spin("A2") * Spin("B5")) + Spin("A3") * Spin("B4") + Spin("A4") * Spin("B3") +
Not(Spin("A5") * Spin("B2")))

(Num(1) + Not(Spin("A1") * Spin("B5")) + Spin("A2") * Spin("B4") + Spin("A3") *
Spin("B3") + Spin("A4") * Spin("B2") + Not(Spin("A5") * Spin("B1")))

(Not(Spin("A0") * Spin("B5")) + Spin("A1") * Spin("B4") + Spin("A2") * Spin("B3") +
Spin("A3") * Spin("B2") + Spin("A4") * Spin("B1") + Not(Spin("A5") * Spin("B0")))

(Spin("A0") * Spin("B4") + Spin("A1") * Spin("B3") + Spin("A2") * Spin("B2") +
Spin("A3") * Spin("B1") + Spin("A4") * Spin("B0"))
```

# 8-bit Multiplier Circuit – Ising Parameters

```
((('A2', 'A2'): -6.0
  ('A0*B2', 'A0*B2'): 27.0
  ('A0', 'A0*B2'): -10.0
  ('A1*B1', 'B0'): -16.0
  ('A1', 'B2'): 8.0,
  ('A0', 'B0'): 8.0
  ('B2', 'B2'): -6.0
  ('A1*B1', 'B1'): -10.0
  ('A2', 'A2*B0'): -10.0
  ('A0*B2', 'B1'): -16.0
  ('A1', 'B1'): 5.0
  ('A2*B0', 'A2*B0'): 27.0
  ('A1', 'A2'): 8.0
  ('B1', 'B2'): 8.0
  ('A0*B2', 'A1*B1'): 32.0,
  ('A1*B1', 'A2'): -16.0
  ('A1', 'A2*B0'): -16.0
  ('A1', 'A1'): -6.0
  ('A0', 'A2'): 8.0
  ('A0', 'A0'): -6.0,
  ('A2*B0', 'B2'): -16.0
  ('A1', 'A1*B1'): -10.0
  ('A0', 'B1'): 8.0
  ('A1*B1', 'A1*B1'): 27.0,
  ('A0', 'A1*B1'): -16.0
  ('A2', 'B2'): 8.0
  ('A1', 'B0'): 8.0
  ('A0*B2', 'B0'): -16.0
  ('B0', 'B0'): -6.0}, 7.0)
```

# A Scalable Hybrid Consensus Network Architecture

			
<b>Problem Selection</b>	<b>Algorithm Formulation</b>	<b>Architecture Design</b>	<b>Bounding</b>
Chose to address the complexity of scalability issue with p2p consensus	Designed a voting scheme that benefits from a quadratic quantum speedup	Designed to be demonstrable now with limited quantum resources	Determined order of quantum speedup
Chose to address the issue of fairness within blockchain	Designed for a sum-of-squares distribution of influence like that used in quadratic voting	Designed to gradually incorporate powerful quantum resources fairly	Determined scaling limitations based on current quantum device characteristics
Chose to address the issue of dishonest collusion within blockchain networks	Based consensus on a novel combination of distributed QML and QHBA	Designed for a hybrid cloud environment	Determined scaling limitations based on future projections for quantum devices

# Open Questions Addressed

## Hybrid / Quantum Networks

- Multi-party applications (not reduceable to point-to-point)
  - Anonymous channels
  - Multi-party arbitrary function evaluation
- 

## Trusted Node Networks

- Integration of QKD into security architecture
  - Reduction of trust assumptions
  - Authentication or proof of identity
- 

## Point-to-Point Communication

- Tools to evaluate protocols
- Abilities to handle imperfections
- More applications

# Research Trajectory

Status: Presented in this Thesis!



Hybrid Blockchain Networks

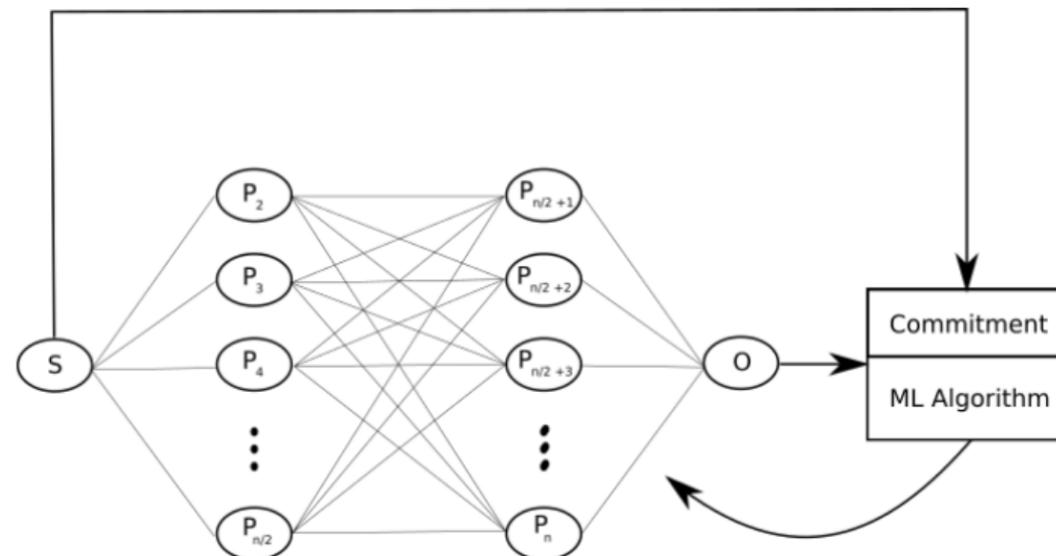
Hybrid Quantum / Classical Networks

Status: Basic Physics Research

Fully Quantum Networks

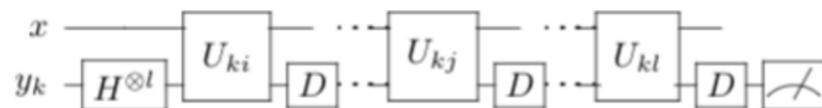
Switching Quantum Networks

# Augmented QHBA



# Associative Measuring Neurons

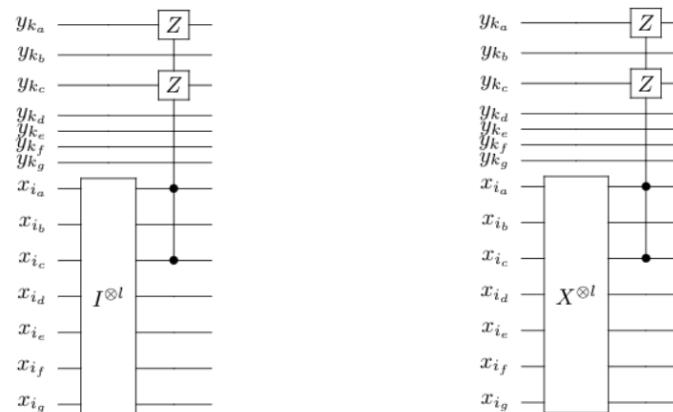
- High-dimensional threshold classifier



- Uses competing Grover's searches with diffusion op D

$$H^{\otimes l} \xrightarrow{2|0^l\rangle\langle 0^l| - I_l} H^{\otimes l}$$

- Uses classically parameterized oracles  $U_{ki}$

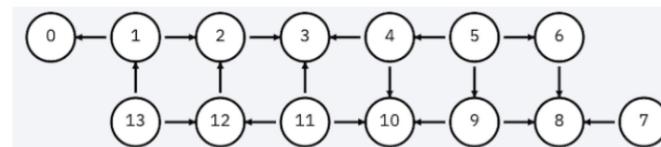


Lov K. Grover. "A fast quantum mechanical algorithm for database search". In: Proceedings of the twenty-eighth annual ACM symposium on Theory of computing - STOC 96 (1996). doi: 10.1145/237814.237866.

# IBM Q Melbourne

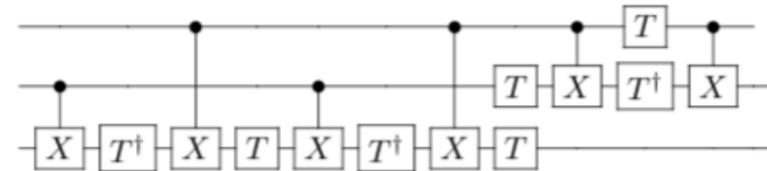
qubit	multi_qb_gate_error	T1 (us)	T2 (us)	Frequency (GHz)	readout_error	gate_error
Q0		73.32348273	23.48828043	5.100090141	0.0215	0.004031062
Q1	CX1_0: 0.03, CX1_2: 0.04	63.23181621	116.7289054	5.238609742	0.054	0.012242205
Q2	CX2_3: 0.04	46.13953307	74.56571753	5.032644087	0.1864	0.010450744
Q3		81.05055849	74.78940464	4.896205701	0.047	0.002494886
Q4	CX4_3: 0.03, CX4_10: 0.04	55.43102145	27.63898146	5.028667392	0.1226	0.002551687
Q5	CX5_4: 0.05, CX5_6: 0.05, CX5_9: 0.07	27.79450766	50.71953989	5.06718735	0.0568	0.004714312
Q6	CX6_8: 0.04	56.16840169	56.0630866	4.923906934	0.0478	0.004816689
Q7	CX7_8: 0.03	32.50641909	45.28966051	4.974534967	0.0598	0.004438222
Q8		47.68062524	71.45643335	4.739563654	0.0389	0.004361702
Q9	CX9_8: 0.04, CX9_10: 0.05	38.43726664	79.71232612	4.963421912	0.0443	0.006372041
Q10		56.99362705	69.83941723	4.945065458	0.037	0.003278348
Q11	CX11_3: 0.05, CX11_10: 0.05, CX11_12: 0.06	57.53451171	71.43323367	5.004981691	0.0357	0.0044898
Q12	CX12_2: 0.06	78.13277541	117.4664528	4.760047973	0.0918	0.007732648
Q13	CX13_1: 0.12, CX13_12: 0.1	21.39891833	41.28178002	4.968495889	0.0498	0.011006778

CX Gate	GF Gate Time (ns)
CX1_0	239
CX1_2	174
CX2_3	261
CX4_3	266
CX5_4	300
CX5_6	300
CX7_8	220
CX9_8	434
CX9_10	300
CX11_10	261
CX11_12	261
CX13_12	300
CX13_1	652
CX12_2	1043
CX11_3	286
CX4_10	261
CX5_9	348
CX6_8	348



# Circuit Compilation for IBM Q

- CZ gate compilation
- Available single- and double-param 1-qubit gates
- T and  $T^\dagger$  compilation
- Hadamard compilation



$$u1(\lambda) = \begin{bmatrix} 1 & 0 \\ 0 & e^{\lambda i} \end{bmatrix} \quad u2(\phi, \lambda) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -e^{\lambda i} \\ e^{\phi i} & e^{(\phi i + \lambda i)} \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{\pi}{4}i} \end{bmatrix} = u1\left(\frac{\pi}{4}\right) \quad T^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{\pi}{4}i} \end{bmatrix} = u1\left(-\frac{\pi}{4}\right)$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = u2(2\pi, 3\pi)$$

Vivek V. Shende and Igor L. Markov. "On the CNOT-cost of TOFFOLI gates". In: arXiv e-prints, arXiv:0803.2316 (Mar. 2008), arXiv:0803.2316. arXiv: 0803. 2316 [quant-ph].

# Time Penalties

- Penalty for Uki

$$T(U_{ki}) \doteq 2 \cdot 0\text{ns} + 6 \cdot 350\text{ns} = 2100\text{ns}$$

- Penalty for D

$$T(D) \doteq 2 \cdot 0\text{ns} + 4 \cdot 70\text{ns} + 1 \cdot 350\text{ns} = 640\text{ns}$$

- Penalty per repetition

$$T_{rep} = T(U_{ki}) + T(D) = 2740\text{ns}$$

- Associative measuring neuron penalty

$$T_{assoc} = \sum_i \lfloor b_k + N_i \rfloor \cdot 2740\text{ns}$$

- Bounded neuron penalty < T2

$$T2 = \max(T_{assoc} | |P|) = \sum_{i=0}^{\frac{|P|}{2}} \lfloor b_k + \max(N_i) \rfloor \cdot 2740\text{ns}$$

# Scaling in Network Nodes

$$22.40\mu s = \sum_{i=0}^{\frac{|P|}{2}} \lfloor b_k + \max(N_i) \rfloor \cdot 2740ns$$

In the worst case,  $\sum_{i=0}^{\frac{|P|}{2}} b_k \rightarrow \frac{|P|}{2}$  since  $0 \leq b_k \leq 1$ .

$$\frac{22.40\mu s}{2740ns} = \sum_{i=0}^{\frac{|P|}{2}} \lfloor b_k + \max(N_i) \rfloor$$

$$\max(N_i | |P|) + 1 \doteq \frac{2}{|P|} \cdot \frac{22.40\mu s}{2740ns}$$

$$\frac{22.40\mu s}{2740ns} - \sum_{i=0}^{\frac{|P|}{2}} b_k \doteq \sum_{i=0}^{\frac{|P|}{2}} \max(N_i)$$

$$\frac{2}{|P|} \cdot \frac{22.40\mu s}{2740ns} \geq 1$$

$$\frac{22.40\mu s}{2740ns} - \sum_{i=0}^{\frac{|P|}{2}} b_k \doteq \frac{|P|}{2} \max(N_i)$$

$$\max(N_i | |P|) \doteq \frac{2}{|P|} \cdot \frac{22.40\mu s}{2740ns} - \frac{2}{|P|} \cdot \sum_{i=0}^{\frac{|P|}{2}} b_k$$

# Recap: Open Questions Addressed

## Hybrid / Quantum Networks

- Multi-party applications (not reduceable to point-to-point)
  - Anonymous channels
  - Multi-party arbitrary function evaluation
- 

## Trusted Node Networks

- Integration of QKD into security architecture
  - Reduction of trust assumptions
  - Authentication or proof of identity
- 

## Point-to-Point Communication

- Tools to evaluate protocols
- Abilities to handle imperfections
- More applications

# Discussion



In each case we demonstrated that different aspects of the quantum internet are useable in a limited sense today, and bounded this usefulness.

We also provided several novel algorithms and technologies that aim to accelerate the internet through a fair, non-wasteful and stable transition into the quantum age.



It remains to be seen if IBM will meet with their projected success of a quantum “Moore’s law”.



There are currently no known degree-2 hash functions that are proven to be non-collapsing.



It remains for someone to develop a true testing framework on top of our WASM transpiler.



Blockchain technology may be yet to stand the test of time.



# THANK YOU

[Attributions](#)[Supplementary Slides](#)[References](#)

# Background / Literature Review

## Quantum Physics

- Quantum fundamentals
- Energy Distributions
- Schrodinger's Equation

## Quantum Computing

- The Qubit model
- Quantum circuits
- Useful operators

## Quantum Blockchain

- Blockchain fundamentals
- Quantum blockchains
- Hybrid blockchains

# Quantum Information

- Energy distributions or "complexes"

$$R = \frac{(N + P)^{N+P}}{N^N \cdot P^P}$$

- Entropy of a quantum system

$$S_N = k \log(R) = k \{(N + P) \log(N + P) - N \log N - P \log P\}$$

- Entropy of a particle

$$S = k \left\{ \left(1 + \frac{U}{\epsilon}\right) \log \left(1 + \frac{U}{\epsilon}\right) - \frac{U}{\epsilon} \log \frac{U}{\epsilon} \right\}$$

- Quantum Harmonic Oscillator

$$E_n = \hbar\omega(n + \frac{1}{2}) = (2n + 1)\frac{\hbar}{2}\omega$$

- "Complexes" become "quantum states"  $\psi_r$

$$\rho = \sum_{r=0}^R \alpha_r |\psi_r\rangle \langle \psi_r|$$

- Pure states are complex-weighted sums

$$|\psi\rangle = \sum_{r=0}^R \alpha_r |r\rangle$$

M. Planck. "On the Theory of the Energy Distribution Law of the Normal Spectrum". In: Annalen der Physik (1901), p. 553.

# Quantum Physics

- Einstein's photons
- De Broglie's energy and momentum / frequency and wave number equivalence
- Momentum is proportional to reduced Planck's constant

$$E = hf \text{ for photons}$$

$$c = f\lambda \text{ for waves}$$

$$E^2 = p^2c^2 + (mc^2)^2$$

$$m \rightarrow 0 \text{ for photons}$$

$$\therefore E = pc \rightarrow p = \frac{E}{c}$$

$$\therefore p = \frac{h}{\lambda} = \hbar k$$

D. J. Griffiths and D. F. Schroeter, "Introduction to Quantum Mechanics," 2018.

# Quantum System Energy

- From here we can describe the kinetic energy of a particle

$$KE = \frac{1}{2}mv^2$$

$$p = mv \rightarrow KE = \frac{p^2}{2m}$$

$$= \frac{h^2}{2\lambda^2 m} = \frac{h^2 k^2}{4\pi^2 2m}$$

$$\therefore KE = \frac{\hbar k^2}{2m}$$

- Schrodinger's equation describes system energy

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x, t)}{\partial x^2} + U(x, t)\psi(x, t) = \frac{i\hbar \partial \psi(x, t)}{\partial t}$$

E. Schrödinger. An undulatory theory of the mechanics of atoms and molecules. Physical Review, 28(6):1049-1070, Jan 1926.

# Differential Values and Operators

- Momentum from kinetic energy
- Momentum operator
- Observable value probability
- Expectation values
- i.e.

$$p^2 = \frac{-\hbar^2 \partial^2}{\partial x^2} \text{ since } mv^2 = \frac{p^2}{2m}$$

$$p = \frac{-i\hbar\partial}{\partial x}$$

$$\int_a^b |\psi(x, t)|^2 dx$$

$$\langle Q \rangle = \int_{-\infty}^{\infty} \psi^*(x, t) \hat{Q} \psi(x, t) dU$$

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^*(x, t) \frac{\hbar}{i} \frac{\partial}{\partial x} \psi(x, t) dx$$

D. J. Griffiths and D. F. Schroeter, "Introduction to Quantum Mechanics," 2018.

# Quantum Computing – Qubits

- Quantum state as a vector

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle = \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}$$

- Polar qubit equations

$$\alpha_0 = \cos\left(\frac{\theta}{2}\right)e^{i\phi} \quad \alpha_1 = \sin\left(\frac{\theta}{2}\right)e^{i\phi}$$

- Natural normalization

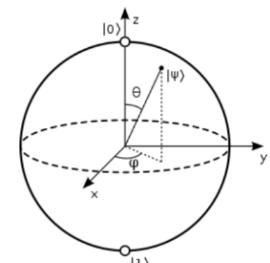
$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

- Pure multi-qubit systems

$$|\Psi\rangle = \sum_{i=0}^N \alpha_i |i\rangle \quad \sum_{i=1}^N |\alpha_i|^2 = 1$$

- Tensor representation

$$|\Phi\rangle = |\phi_0\rangle \otimes |\phi_1\rangle \otimes \cdots \otimes |\phi_n\rangle$$



N. David Mermin. Quantum Computer Science: An Introduction. Cambridge University Press, 2007. doi: 10.1017/CBO9780511813870.

# Quantum Computing – Representations

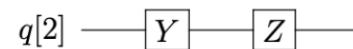
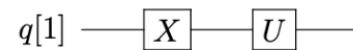
- Pauli operators

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- Bloch vectors

$$U = x\sigma_x + y\sigma_y + z\sigma_z \qquad v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Quantum circuits

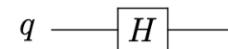


N. David Mermin. Quantum Computer Science: An Introduction. Cambridge University Press, 2007. doi: 10.1017/CBO9780511813870.

# Quantum Computing – Operations

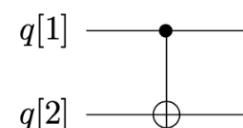
- Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



- Controlled-Not gate

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



- T-Gate

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix}$$



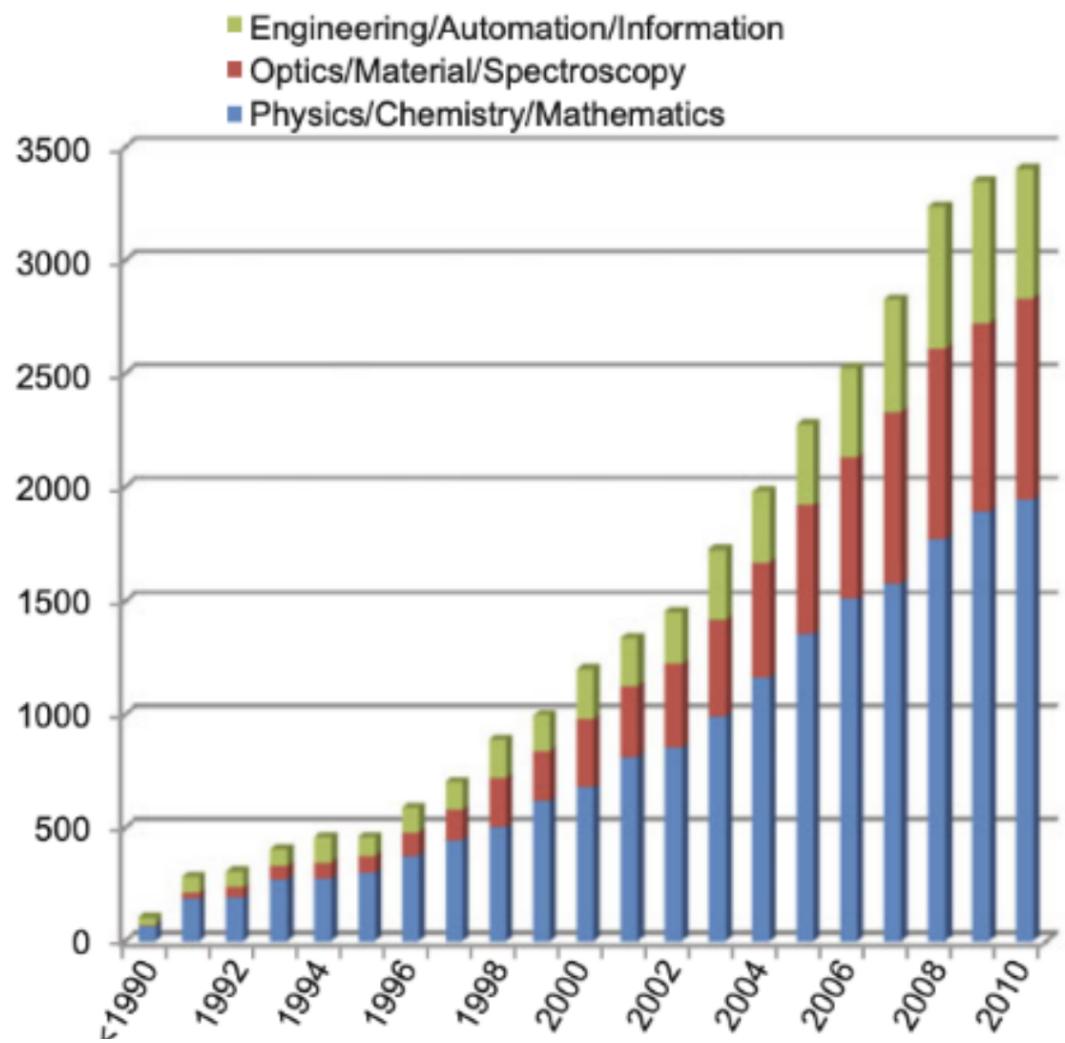
- General form

$$U_3(\theta, \phi, \lambda) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -e^{\lambda i} \sin(\frac{\theta}{2}) \\ e^{\phi i} \sin(\frac{\theta}{2}) & e^{\lambda i + \phi i} \cos(\frac{\theta}{2}) \end{bmatrix}$$

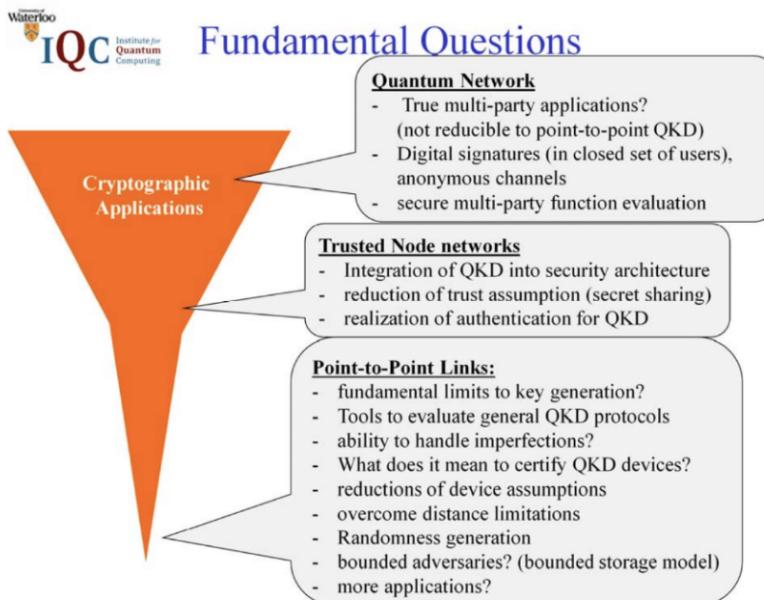
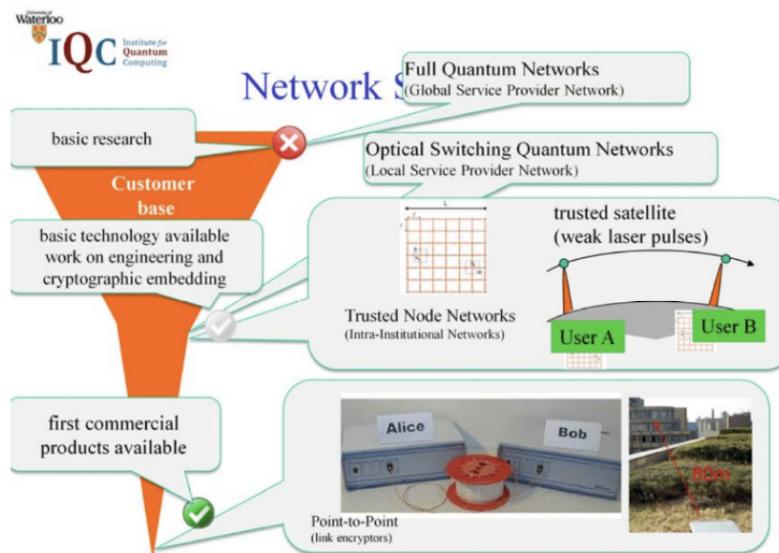
N. David Mermin. Quantum Computer Science: An Introduction. Cambridge University Press, 2007. doi: 10.1017/CBO9780511813870.

# Quantum Control Publications

Quantum Networks ∈ Quantum Control



# QIC890 Slides - Open Questions



Norbert Lütkenhaus. "Lecture 22 - QKD Networks." QIC 890 - Applied Quantum Cryptography (Nov. 2020), Waterloo ON, University of Waterloo.

# Controlled Teleportation Equations

- Initial state

$$|\psi_{GHZ}\rangle_{ABC} = \frac{|000\rangle_{ABC} + |111\rangle_{ABC}}{\sqrt{2}}$$

- Alice's preparation

$$|x\rangle_x |\psi_{GHZ}\rangle_{ABC} =$$

$$\frac{1}{2}[|\phi^+\rangle_{xA} \otimes (\alpha|00\rangle + \beta|11\rangle)_{BC}$$

$$+ |\phi^-\rangle_{xA} \otimes (\alpha|00\rangle - \beta|11\rangle)_{BC}$$

$$+ |\psi^+\rangle_{xA} \otimes (\alpha|11\rangle + \beta|00\rangle)_{BC}$$

$$+ |\psi^-\rangle_{xA} \otimes (\alpha|11\rangle - \beta|00\rangle)_{BC}]$$

- Intermediate state example (after Alice's Bell measurement)

$$|\psi\rangle_{BC} = (\alpha|00\rangle + \beta|11\rangle)_{BC} =$$

$$\frac{1}{\sqrt{2}}[(\alpha|0\rangle + \beta|1\rangle)_B | + x \rangle_C$$

$$+ (\alpha|0\rangle - \beta|1\rangle)_B | - x \rangle_C]$$

# Controlled Teleportation Inputs

$\alpha$	$\beta$	Input State
0.71	0.71	$ \psi_1\rangle$
0.5	0.87	$ \psi_2\rangle$
0.3	0.95	$ \psi_3\rangle$
0.37	0.93	$ \psi_4\rangle$
0.17	0.98	$ \psi_5\rangle$

# IBM Q Physical Operators

U1  
( $\lambda$ )

$= \omega_q - FC_{(-\lambda)}$

U2  
( $\varphi, \lambda$ )

$= \omega_q - FC_{(-\lambda)} - GD_{(\pi/2, \pi/2)} - FC_{(-\varphi)}$

FIG. 11. u1 Frame Change Physical Gate

FIG. 12. u2 Frame Change Physical Gate

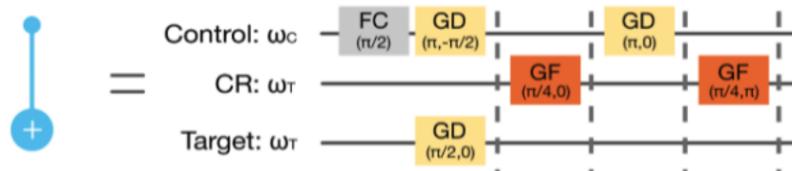


FIG. 13. CX Physical Gate

# Quantum Binding Commitments

- Quantum binding commitments

**Definition 1 (Classical-style binding)** *No algorithm A can output a commitment c and two signatures s, s' that open c to two different messages m and m'.*

**Definition 2 (Collapsing hash function - informal)** *H is a collapsing hash function iff no quantum polynomial time algorithm B can distinguish between Game<sub>1</sub> and Game<sub>2</sub>. An adversary is valid if A outputs a classical value c and a register M where H(m) = c.*

Game<sub>1</sub>

$$(S, M, U, c) \leftarrow A(1^\gamma)$$

$$ok \leftarrow V_c(M, U)$$

$$m \leftarrow M_{ok}(M)$$

$$b \leftarrow B(1^\gamma, S, M, U)$$

Game<sub>2</sub>

$$(S, M, U, c) \leftarrow A(1^\gamma)$$

$$ok \leftarrow V_c(M, U)$$

$$b \leftarrow B(1^\gamma, S, M, U)$$

**Definition 3 (Collapsing hash function - formal)** *A function H : X → Y is  $\in(q)$ -collapsing if  $cAdv[H](q) := \sup_{SMCU} \delta_q(M, \overline{M} | \overline{C}U) \leq \in(q)$*

*for all q. The supremum is over all states SMCU = S H(M) CU with complexity  $\leq q$ .*

# Quantum Lightning

- “Bolt” generation

1. Randomly choose  $n$  random upper-triangular matrices  $A_i \in \{0, 1\}^{m \times m}$ , and set

$\mathbb{A} = \{A_i\}_i$ .  $\mathbb{A}$  is the public key. Let the hash function  $f_{\mathbb{A}} : \{0, 1\}^m \rightarrow \{0, 1\}^n$  be  $f_{\mathbb{A}}(x) = (x^T \cdot A_i \cdot x)_i$ . If we let operations be taken mod 2, this captures general degree 2 functions over  $\mathbb{F}_2$ .

2. Begin with a state  $|\phi_0\rangle = \sum_{\Delta} \sum_{x \in S_{\Delta}} \frac{1}{2^{kn/2} \sqrt{|S_{\Delta}|}} |\Delta, x\rangle$

3.  $\Delta$  is defined such that we can run a computation which maps  $\Delta = (\Delta_1, \dots, \Delta_k)$  to an affine space  $S_{\Delta}$  s.t.  $\forall x \in S, f_{\mathbb{A}}(x) = f_{\mathbb{A}}(x + \Delta_j) \forall j$ . Then we construct a uniform superposition of elements in  $S_{\Delta}$  to yield:

$$\sum_{\Delta} \sum_{x \in S_{\Delta}} \frac{1}{2^{kn/2} \sqrt{|S_{\Delta}|}} |\Delta, x\rangle$$

5. Compute the maps  $(x, \Delta_1, \dots, \Delta_k)$  to  $(x, x - \Delta_1, \dots, x - \Delta_k)$  in superposition. The final state is a bolt

# A Quantum Voting Protocol

- Ballot commitment
  1. For each  $i \in \{1, \dots, n\}$  voter  $V_i$  generates the  $i^{th}$  row of an  $n \times n$  matrix of integers  $r_{i,1}, \dots, r_{i,n}$  such that  $\sum_j r_{io,j} = 0 \pmod{n+1}$ .
  2. For each  $i, j$  voter  $V_i$  sends  $r_{i,j}$  to voter  $V_j$  via a quantum secure communication.
  3. Then each voter  $V_i$  knows the  $i^{th}$  column  $r_{1,i}, \dots, r_{n,i}$ .  $V_i$  computes his/her masked ballot  $\hat{v}_i = v_i + \sum_j r_{j,i} \pmod{n+1}$ .  $V_i$  commits  $\hat{v}_i$  to every tallier of the blockchain via a quantum commitment protocol.
- Ballot tallying
  1. Each voter  $V_i$  reveals  $\hat{v}_i$  to every tallier of the blockchain by opening his/her commitment.
  2. The talliers each run the Quantum Honest Success Byzantine Agreement Protocol to reach a consensus on the value of the masked ballot  $\hat{v}_1, \dots, \hat{v}_n$ .
  3. The result of the vote is  $\sum_i \hat{v}_i = \sum_i v_i \pmod{n+1}$ .

# Attributions

Icons were made by Pixel Perfect, Freepik, Icongeek26, Kiranshastry, Good Ware, surang, Flat Icons, Pixel perfect from [www.flaticon.com](http://www.flaticon.com).

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