Controlled Teleportation on the IBM Quantum Computing Platform

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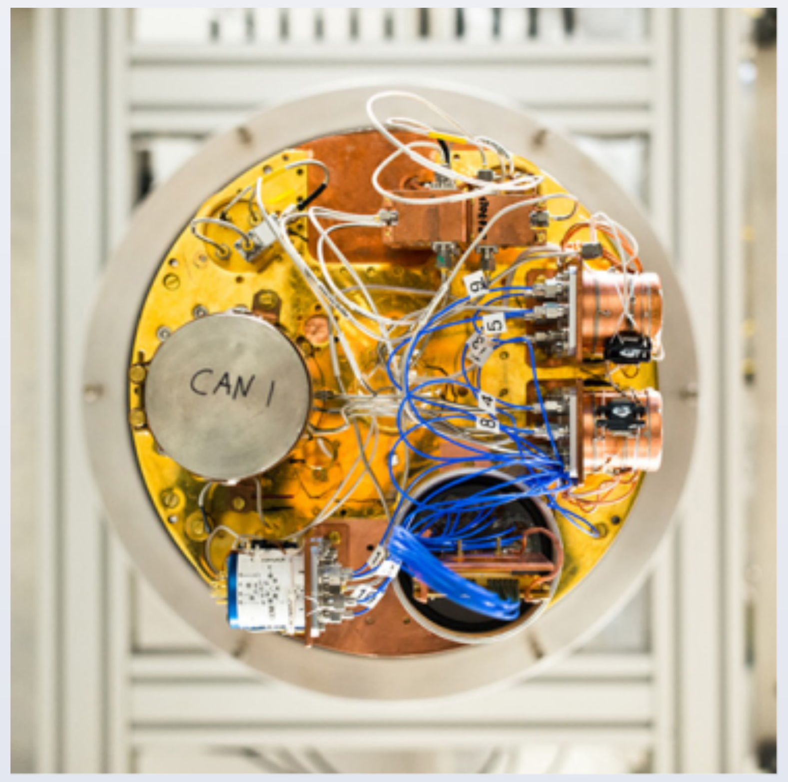
Controlled Teleportation

Quantum teleportation takes advantage of entanglement to achieve the secure sharing of unknown quantum state information between separated sending and receiving parties. An extension of the quantum teleportation protocol, called controlled teleportation, allows a third party to dictate when a teleportation is allowed to be executed successfully. This procedure had not been tested, only theorized, until our experimental implementation using the IBM quantum computer.



Project Goals

The goal in doing this project has been to demonstrate the effectiveness of a controlled teleportation protocol implemented experimentally using the IBM Q. The experiment was designed to provide insight into both the actual utility of the protocol and into the performance of one of today's most promising quantum computing machines.

Theoretical Procedure

The controlled teleportation procedure begins with the preparation of what is known as a Greenberger-Horne-Zelinger state, an entanglement involving three qubits. This three-qubit state can be represented by the following formula.

$$|\psi_{GHZ}>_{ABC} = \frac{|000>_{ABC} + |111>_{ABC}}{\sqrt{2}}$$

The three qubits in this entanglement each represent a piece of information. The owner of each qubit can change or observe its state. In this protocol, we have a sender of information (Alice), a receiver of information (Bob) and a controller (Charlie). The goal is to demonstrate that the controller has the power to decide whether or not a transmission is successful.

To prepare her qubit with the message she wants to teleport to Bob, Alice first changes her qubit's state to the state which she intends Bob to receive. This can be achieved by performing a Control-NOT operation on her entangled qubit, with a qubit representing her intended message used as the controller. We will call this qubit x. Then Alice's entangled qubit will have some state of the following form.

$$|x>_x = \alpha|0>_x + \beta|1>_x$$

Alice will then perform a Bell measurement on her qubits A and x. This type of measurement ensures that when these qubits are measured later, they will be observed to have one of four states. These four particular states are called the "Bell basis" and they are given by the following formulas.

$$|\psi^{\pm}>= \frac{1}{\sqrt{2}}(|0>_x|1>_A \pm |1>_x 0>_A)$$

$$|\phi^{\pm}>= \frac{1}{\sqrt{2}}(|0>_x|0>_A \pm |1>_x|1>_A)$$

Since we now know that qubits A and x will be observed to have one of these states, each with equal probability, we can represent the entire system of 4 qubits in terms of these four possibilities, yielding the state below.

$$\begin{aligned}|x>_x|\psi_{GHZ}>_{ABC} = \\ \frac{1}{2}[|\phi^+>_{xA}\otimes(\alpha|00>+\beta|11>)_{BC} \\ +|\phi^->_{xA}\otimes(\alpha|00>-\beta|11>)_{BC} \\ +|\psi^+>_{xA}\otimes(\alpha|11>+\beta|00>)_{BC} \\ +|\psi^->_{xA}\otimes(\alpha|11>-\beta|00>)_{BC}]\end{aligned}$$

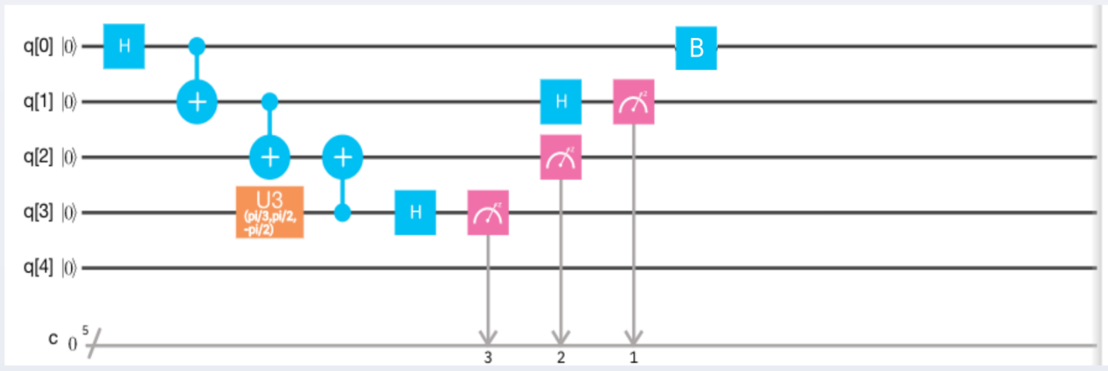
This is useful because when observed, x and A will assume one of the four Bell states and qubits B and C will collapse from this general form to a corresponding state with two outcomes of equal likelihood. For example, in the case that the Bell State $|\phi^+>_{xA}$ is observed, qubits B and C would collapse into the following state.

$$\begin{aligned}|\psi>_{BC} = (\alpha|00>+\beta|11>)_{BC} = \\ \frac{1}{\sqrt{2}}[(\alpha|0>+\beta|1>)_B|+x>_C \\ +(\alpha|0>-\beta|1>)_B|-x>_C]\end{aligned}$$

In this collapsed state, $|\pm x>$ are the eigenvectors of the x basis. The $|\pm x>$ components make it possible for Charlie to determine which of the two outcomes of equal likelihood has taken place by performing a measurement in the x basis of the C qubit. Alice can also measure x and A in order to determine which Bell state has been assumed. If both Charlie and Alice classically transmit the results of these measurements to Bob, it is then possible for Bob to determine what gates need to be applied to qubit B in order for its state to be equal to the transmitted state. Otherwise, Bob has no hope of decoding the message! Therein lies Charlie's control over the procedure.

Experimental Implementation

The implementation of this protocol could theoretically be represented by a relatively simple quantum circuit, with the gate labelled B representing the relevant of the four possible operations for Bob to apply, $q[0]$ as qubit B, $q[1]$ as C, $q[2]$ as A and $q[3]$ as x.



However, there were a few obstacles that made this implementation impossible.

The first obstacle to implementing the protocol was the fact that quantum circuits to be run on the IBM platform must be completely predefined in IBM's own definitional language, Open QASM. This means that measurements taken during a circuit execution cannot affect the gates that are applied as a part of the circuit. This clearly conflicts with Bob's need to use the measurement results from Alice and Charlie to inform his final gate operations.

Secondly, the platform API returns a probability distribution of measurement results over a number of executions. This introduces complexity because in the controlled teleportation protocol, each run will involve a different Bell state and collapsed state that define the context for Bob's final operation. Since the result of a run on the IBMQX4 or IBM simulator provides no insight into these contexts and cannot discriminate between them, the overall probability of measuring qubit B as either 1 or 0 over a number of executions is meaningless.

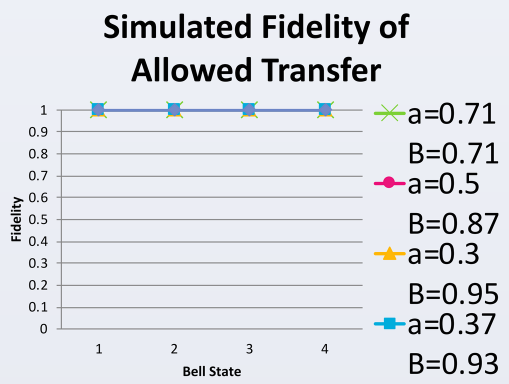
So, instead we predefined four separate circuits, one for each of the possible Bell basis measurement outcomes and tested them each individually. We then implemented a post-selection algorithm in Python which ensured that only the outcomes which matched the circuit under test were considered.

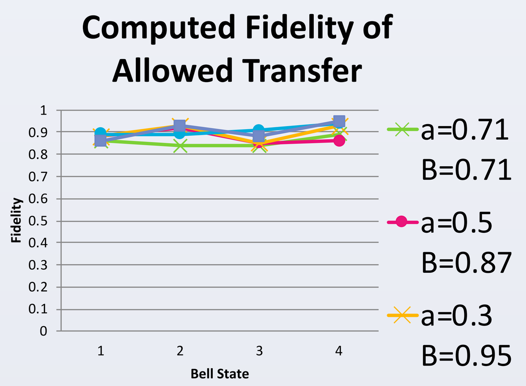
Each of the four circuits contained one of the possible set of decoding operations Bob could need to perform in order to get Alice's message. The possibilities in terms of Charlie and Alice's measurement results are given in the table below.

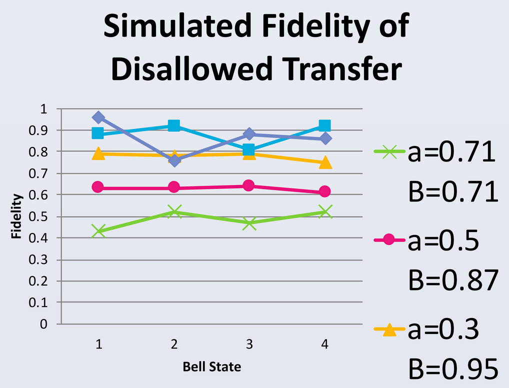
Bell state	Charlie's result	Bob's operation
$\|\phi^+>_{xA}$	$\|+x>$	I
$\|\phi^+>_{xA}$	$\|-x>$	Z
$\|\phi^->_{xA}$	$\|+x>$	Z
$\|\phi^->_{xA}$	$\|-x>$	I
$\|\psi^+>_{xA}$	$\|+x>$	X
$\|\psi^+>_{xA}$	$\|-x>$	XZ
$\|\psi^->_{xA}$	$\|+x>$	XZ
$\|\psi^->_{xA}$	$\|-x>$	X

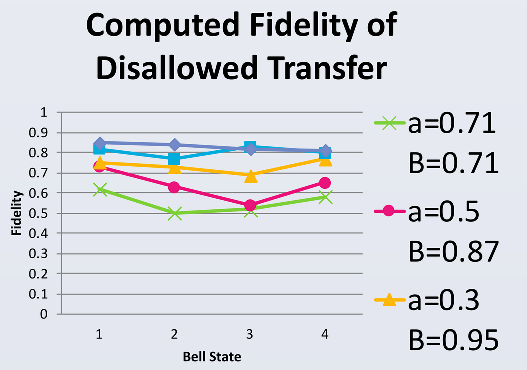
Results

To determine the significance of the results we ran the experiment using a number of teleported messages. We also simulated the procedure to give us a data set to compare our experimental results to. This provides insight into the performance of the IBM QX4. The most interesting comparison is between the fidelities of allowed versus disallowed teleportations.









It is easy to see that there is a significant difference between the allowed and disallowed teleportation fidelity. This demonstrates Charlie's control. There is also a clear difference between the simulated and computed results, especially in the allowed teleportations. The computed results do not have perfect fidelity like the simulated results, due to error in the IBM QX4.

Acknowledgements

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