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# Divide and Conquer Practice Problems

### Problem 1: Finding the Minimum in a Shifted Array

Suppose you are given an array $A$ of $n$ sorted numbers that has been *circularly shifted* to the right by $k$ positions. For example $\{35, 42, 5, 15, 27, 29\}$ is a sorted array that has been circularly shifted $k = 2$ positions, while $\{27, 29, 35, 42, 4, 15\}$ has been shifted $k = 4$ positions. Give an $O(\log n)$ algorithm to find the smallest number in $A$. You may assume the elements of $A$ are distinct. Write the recurrence for your algorithm and show that its recurrence solves to $O(\log n)$ (e.g., using the Master Method, a recursion tree, or an inductive proof).

### Problem 2: Finding the Maximum

Suppose you are given an array $A$ of $n$ numbers. $A$ has the following property: initially the values in $A$ are increasing and then decreasing. Give an $O(\log n)$ algorithm that finds the largest element in the array. For example, given $\{1, 2, 4, 8, 16, 9, 5\}$, the algorithm should return 16. Write the recurrence relation for your algorithm and show that its recurrence solves to $O(\log n)$ (e.g., using the Master Method, a recursion tree, or the iteration method).

**Name:** **Collaborators:**

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### Problem 3: Finding the Duplicate - Take Two

Suppose you are given an array $A$ of $n$ sorted numbers that contains the values from 1 to $n-1$. However, one value in the array is repeated twice. For example $\{1, 2, 3, 3, 4, 5, 6\}$ is a sorted array of size $n = 7$ with values ranging from 1 to 6 and the value 3 is repeated. Give an $O(\log n)$ algorithm to find the duplicated value in $A$. Write the recurrence relation for your algorithm and show that its recurrence solves to $O(\log n)$ (e.g., using the Master Method, a recursion tree, or the iteration method).