Homework #9

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **You do not need to turn in these problems. The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Greedy Choice

You are consulting for a trucking company that does a large amount of business shipping packages between New York and Boston. The volume is high enough that they have to send several trucks each day between the two locations. Trucks have a fixed limit W on the maximum amount of weight they are allowed to carry. Boxes arrive at the New York station one by one, and each package i has a weight w_i. The trucking station is quite small, so at most one truck can be in the station at any time. Company policy requires that boxes are shipped in the order they arrive; otherwise a customer might get upset. At the moment, the company is using a simple greedy algorithm for packing: they pack boxes in the order they arrive, and whenever the next box does not fit, they send the truck on its way.

But they wonder if they might be using too many trucks, and they want your opinion on whether the situation can be improved. Here is how they are thinking. Maybe one could decrease the number of trucks needed by sometimes sending off a truck that was less full, and in this way allow the next few trucks to be better packed.

Prove that, for a given set of boxes with specified weights, the greedy algorithm currently in use actually minimizes the number of trucks that are needed.

Problem 2: Coin Changing

Consider the problem of making change for n cents using the fewest number of coins. Assume that each coin's value is an integer.

(a) Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Prove that your algorithm yields an optimal solution.

(b) Give a set of coin denominations for which your greedy algorithm does not yield an optimal solution. Your set should include a penny to ensure that you can always successfully make change. (Note: you can assume a coin can take any positive integer value).

Problem 3: Phone Base Stations

Consider a long, quiet country road with houses scattered sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road, so that every house is within 4 miles of one of the base stations.

(a) Give an efficient algorithm that achieves this goal, using as few base stations as possible.

(b) Prove that the greedy choice that your algorithm makes is the optimal choice.