Homework #7

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **You do not need to turn in these problems. The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Thievery

There are n houses built in a line, each of which contains some value in it. A thief is going to steal the maximal value in these houses, but he cannot steal in two adjacent houses because the owner of a stolen house will tell his two neighbors on the left and right side.

For example, if there are four houses with values 6, 1, 2, 7, the maximal stolen value is 13 when the first and fourth houses are stolen. All values are nonnegative.

A function F(i) is defined to denote the maximal stolen value from the first house to the ith house, and the value contained in the ith house is denoted as v_i. When the thief reaches the ith house, he has two choices: to steal or not. Define F(i) for the following cases:

(a) Provide F(0) and F(1). Then provide F(i) in terms of previously computed values of F.

Problem 2: Summing Integers

Suppose you are given a collection $A = \{a_1, a_2, \ldots, a_n\}$ of n positive integers that add up to $2Z$. Design an $O(nZ)$ time algorithm to decide if the set can be partitioned into two groups B and $A - B$ such that:

$$\sum_{a_i \in B} a_j = \sum_{a_i \in (A-B)} a_i = Z$$

Problem 3: Longest Paths

Given an undirected graph $G = (V, E)$ with positive edge weights w_e for each edge $e \in E$, give a dynamic programming algorithm to compute the longest path in G from a given source s that contains at most n edges.
(Hint: Let $A[v, k]$ denote the weight of the longest path from s to node v of at most k edges).