COMP285: Analysis of Algorithms
North Carolina A&T State University
Dr. Allison Sullivan

Homework #10
Due: November 9, 2018 (*in class quiz*)

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You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **You do not need to turn in these problems. The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Fair Cake Portions

You have a cake sliced into n differently sized pieces. You need to split it among $k \leq n$ students in your class and you want to give each student exactly 1 piece of cake. However, you know that since the pieces are differently sized, the students who end up with smaller pieces will most likely complain about unfairness. You want to minimize the complaints by minimizing unfairness. If you select k cake pieces of size $S_1, \ldots S_k$ then unfairness is defined as:

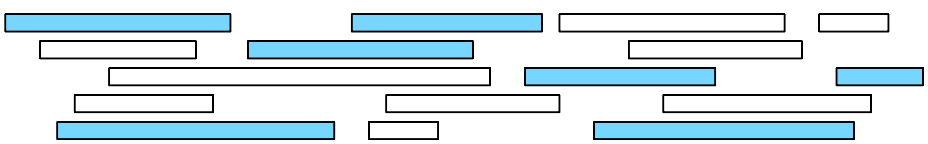
$$max\{S_1, \ldots S_k\} - min\{S_1, \ldots S_k\}$$

Design an algorithm to minimize unfairness.

Problem 2: Another Interval Problem

Let X be a set of n intervals on the real line. A subset of intervals $Y \subseteq X$ is called a *tiling path* if the intervals in Y cover the intervals in X, that is, any real value that is contained in some interval in X is also contained in some interval in Y. The *size* of a tiling cover is just the number of intervals.

Describe an algorithm to compute the smallest tiling path of X. Assume that your input consists of two arrays $X_L[1\,..\,n]$ and $X_R[1\,..\,n]$, representing the left and right endpoints of the intervals in X. Argue that your algorithm is correct (hint: remember that an argument of correctness for any greedy algorithm has two components). The figure below shows an example of a tiling path, though a non-optimal one.



A set of intervals. The seven shaded intervals form a tiling path.

Problem 3: Phone Base Stations around the Lake

Remember the problem with the phone base stations. Here we will see a variation of this problems. This time there are houses scattered sparsely along the perimeter of a cyclic lake with radius far longer than 4 miles. The positions of the houses are given to you. Your goal is to place as few phone base stations as possible, along the perimeter, ensuring that every house is within 4 miles from some base station (you can assume that every phone base station is covering an arc of 8 miles with its position as the middle point of the arc). Give an algorithm with time complexity $O(n^2)$