Homework #12

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **You do not need to turn in these problems. The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Updating a Maximum Flow

Suppose you are given a flow network $G = (V, E)$ with source s, sink t, integer capacities, and a maximum flow, f of G.

1. We increase the capacity of a single edge $(u, v) \in E$ by one. Give a $O(m + n)$ time algorithm to update the maximum flow.
2. We decrease the capacity of a single edge $(u, v) \in E$ by one. Give a $O(m + n)$ time algorithm to update the maximum flow.

Problem 2: Efficient Recruiting

Suppose you're helping to organize a summer sports camp, and the following problem comes up. The camp is supposed to have at least one counselor who is skilled at each of the n sports covered by the camp (baseball, volleyball, etc.). They have received job applications from m potential counselors. For each of the n sports, there is some subset of the m applicants qualified in that sport. The question is: For a given number $k < m$, is it possible to hire at most k of the counselors and have at least one counselor qualified in each of the n sports? We'll call this the *Efficient Recruiting* Problem. Show that *Efficient Recruiting* is NP-Complete by reducing from the vertex cover problem.

The Vertex Cover Problem. *Given a graph G and a number k, does G contain a vertex cover of size at most k? (Recall that a vertex cover $V' \subseteq V$ is a set of vertices such that every edge $e \in E$ has at least one of its endpoints in V'.)*

Problem 3: Zero-Weight Cycle

You are give a directed graph $G = (V, E)$ with weights w_e on its edges $e \in E$. The weights can be negative or positive. The *Zero-Weight Cycle* Problem is to decide if there is a simple cycle in G so that that sum of the edge weights on this cycle is exactly 0. Prove that *Zero-Weight Cycle* is NP-Complete by reducing from the subset sum problem.

The Subset Sum Problem. *Given natural numbers $w_1, w_2, \ldots, w_n$ and a target number W, is there a subset of $\{w_1, w_2, \ldots, w_n\}$ that adds up to precisely W?*