Homework #4

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Shortest Path-ish

Suppose that you want to get from vertex s to vertex t in an unweighted graph $G = (V, E)$, but you would like to stop by vertex u if it is possible to do so without increasing the length of your path by more than a factor of α.

Describe an efficient algorithm that would determine an optimal $s-t$ path given your preference for stopping at u along the way if doing so is not prohibitively costly. (It should either return the shortest path from s to t or the shortest path from s to t containing u, depending on the situation.) If it helps, imagine that there are burgers at u.

Problem 2: Discipline in Groups of Children

Your job is to arrange n rambunctious children in a straight line, facing front. You are given a list of m statements of the form "i hates j". If i hates j, then you do not want to put i somewhere behind j because then i is capable of throwing something at j. Give an algorithm that orders the line (or says it's not possible) in $O(m + n)$ time. Justify that you algorithm runs in the required time.

Problem 3: Counting Shortest Paths

Suppose we are given an undirected graph $G = (V, E)$, and we identify two nodes v and w in G. Give an algorithm that computes the number of shortest v-w paths in G. (The algorithm should not list all the paths; just the number suffices.) The running time of your algorithm should be $O(m + n)$ for a graph with n nodes and m edges.