Homework #6

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Changing the Graph

Consider an undirected graph $G = (V, E)$ with distinct nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of G, and that you have also computed shortest paths to all nodes from a particular node $s \in V$. Now suppose each edge weight is increased by 1: the new weights are $w'_e = w_e + 1$.

1. Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

2. Do the shortest paths change? Give an example where they change or prove they cannot change.

Problem 2: Greedy Ski Distribution

Consider the problem of matching a set of available skis to a set of skiers. The input consists of n skiers with heights $p_1, \ldots p_n$ and n sets of skis with heights $s_1, \ldots s_n$. The problem is to assign each skier a ski to minimize the average difference between the height of a skier and his or her assigned set of skis. That is, if the i^{th} skier is given the $\alpha(i)^{th}$ pair of skis, then you want to minimize:

$$\frac{1}{n}\sum_{i=1}^{n} |p_i - s_{\alpha(i)}|$$

Design a greedy algorithm that solves this problem.

Problem 3: Phone Base Stations

Consider a long, quiet country road with houses scattered sparsely along it. (We can picture the road as a long line segment, with an eastern endpoint and a western endpoint.) Further, let's suppose that despite the bucolic setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road, so that every house is within 4 miles of one of the base stations.

(a) Give an efficient algorithm that achieves this goal, using as few base stations as possible.

(b) Prove that the greedy choice that your algorithm makes is the optimal choice.