Homework #8

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Thievery

There are n houses built in a line, each of which contains some value in it. A thief is going to steal the maximal value in these houses, but he cannot steal in two adjacent houses because the owner of a stolen house will tell his two neighbors on the left and right side. For example, if there are four houses with values 6, 1, 2, 7, the maximal stolen value is 13 when the first and fourth houses are stolen. All values are nonnegative.

A function F(i) is defined to denote the maximal stolen value from the first house to the ith house, and the value contained in the ith house is denoted as v_i. When the thief reaches the ith house, he has two choices: to steal or not. Define F(i) for the following cases:

(a) Provide F(0) and F(1). Then provide F(i) in terms of previously computed values of F.

Problem 2: Dynamic Programming

On a staircase, the i-th step has some non-negative cost cost[i] assigned (0 indexed). Once you pay the cost, you can either climb one or two steps. You need to find minimum cost to reach the top of the floor, and you can either start from the step with index 0, or the step with index 1.
Example: Input: cost = [10, 15, 20] Output: 15

(a) A function C(i) is defined to denote the minimum cost from the first step to the ith step. Provide C(1) (i.e., only the first step is considered) and C(2) (i.e., only the two steps are considered). Then provide C(i) in terms of previously computed values of C.

Problem 3: Minimum Steps to One

On a positive integer, you can perform any of the following three *steps*:

- subtract 1 from it ($n = n - 1$)
- if it is divisible by 2, divide by 2 (if $n \,\%\, 2 == 0$ then $n = n/2$)
- if it is divisible by 3, divide by 3 (if $n \,\%\, 3 == 0$ then $n = n/3$)

Your goal is, for a given positive integer n, find the minimum number of steps to take n to 1.

(a) Consider the following greedy algorithm: of the three choices, choose the one that decreases n the most. Repeat until n is 1. This algorithm does not work. Give a counter-example for this algorithm.

(b) Define the recursive formula for this problem.