Network Flow Practice Problems

Problem 1: Iceapolis VOL.2

Yet another obstacle to Iceapolis receiving supplies! The limited security personnel of each in-between city i must inspect the supplies each train brings in, before they are packed in another train for re-shipping. Thus each city also has a capacity c of supplies it allows to be sent through it in a single day. Again, show how to send the greatest amount of supplies possible from the allies to Iceapolis in a single day, and argue about the complexity of your algorithm.

Problem 2: Hospital Flow

Paramedics have identified n injured people who need to be rushed to hospitals. There are k hospitals, and each of the n people needs to be brought to a hospital that is within a half-hour's driving time of their location (so different people will have different options for hospitals). We want to work out whether we can choose a hospital for each of the injured people in such a way that the load on the hospitals is balanced (in order not to overload the hospitals): Each hospital should receive at most $\lceil n/k \rceil$ people.

Phrasing this problem as a max-flow problem, given information about the people's locations, describe a polynomial-time algorithm to determine whether this is possible. Give the running-time bound of your algorithm in terms of n and k.

Problem 3: Deleting Edges

Consider the following problem. You are given a flow network with unit capacity edges: It consists of a directed graph $G = (V, E)$, a source $s \in V$, and a sink $t \in V$; and $c_e = 1$ for all $e \in E$. You are also given a parameter k.

The goal is to delete k edges so as to reduce the maximum s-t flow in G by as much as possible. In other words, you should find a set of edges $F \subseteq E$ so that $|F| = k$ and the maximum s-t flow in $G' = (V, E - F)$ is as small as possible subject to this.

Give a polynomial time algorithm to solve this problem. Argue (prove) that your algorithm does in fact find the graph with the smallest maximum flow.