

Review Assignment

You should try to solve these problems by yourself. Consult the answer sheet only after you have given it a real attempt. If you do check the answer sheet along the way, set it aside and try to write up a solution without it.

This is not intended to be a comprehensive review.

Problem 1: Algorithm Analysis

Consider the following algorithm.

Algorithm 1: An Algorithm

```
i = 1
while i ≤ n do
    j = 2
    while j ≤ m do
        mystery()
        j = 2 * j
    i = i + 0.1
```

- (a) Assume the `mystery()` function executes in constant time. Give an asymptotically tight bound, $\Theta(\cdot)$, of the runtime for the algorithm.
- (b) Give an asymptotically tight bound, $\Theta(\cdot)$, of the runtime for the algorithm if `mystery` runs in $O(n)$ time.

Problem 2: Coffee Shop Scheduling

You just started working at a local coffee shop, and they've been doing a terrible job of managing their staff, specifically with respect to having the right number of staff on hand at the busiest times. After just a few days on the job, you notice that the crowds follow a very specific pattern: the number of customers strictly increases up to a point and then strictly decreases. Your registers give you a print out of the average number of customers in every ten minute interval from open to close.

Given this array of the customer counts over a given day, write an efficient algorithm to find the specific 10 minute interval that is the “peak” traffic for your coffee shop. Write and solve the recurrence for your algorithm. Give as efficient an algorithm as possible.

Problem 3: Finding Bridges

Give an $O(m * (n + m))$ time algorithm to find all bridge edges in a connected graph. A bridge of a connected graph is an edge whose removal disconnects the graph. Briefly justify the running time of your algorithm.

Problem 4: Wrestlers

Suppose there are only two types of professional wrestlers: “good guys” and “bad guys.” Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n + r)$ algorithm that determines whether it is possible to designate some of the wrestlers as good guys and the remainder as bad guys such that each rivalry is between a good guy and a bad guy. If it is possible to perform such a designation, your algorithm should produce it.