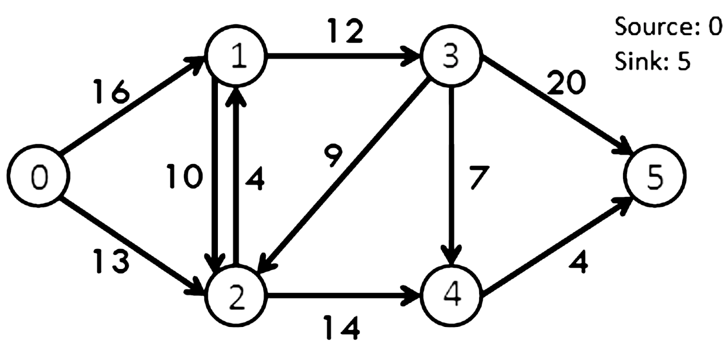
Homework #10

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Computing the maximum flow

Consider the graph $G = (V, E)$ with edges $e = (u, v) \in E$ and capacities $c(e)$ shown in the figure below. The capacity $c(e)$ is annotated for each edge $e \in E$, v_0 is the source, and v_5 is the sink.

Compute the maximum flow for G.



Problem 2: Iceapolis

Iceapolis is facing a winter longer than expected and is in dire need for food! Fortunately, it has k train stations, all of which can receive supplies, and n allies, all of which are willing to send them as many supplies as possible. Unfortunately, there are some constraints. In a given day, there are several routes between the allies and Iceapolis's train stations, that can be described as follows: Let V be the set including the n sources of supplies, k train stations of Iceapolis, and b in-between cities. We can define a directed graph $G = (V, E)$, and capacity function $c(e)$ is s.t. if there is a train connecting cities i and j, then $(i, j) \in E$, and $c((i, j))$ denotes the maximum capacity of that train. Each train runs only once per day (i.e. the trip $i \longrightarrow j$ will only occur once). Show how to send the greatest amount of supplies possible from the allies to Iceapolis in a single day. Argue about the complexity of the algorithm you used.

Problem 3: Double Checking a Max Flow

Suppose someone presents you with a solution to a max-flow problem on some network. Give a linear time algorithm to determine whether the solution does indeed give a maximum flow.