Homework #7

You should try to solve these problems by yourself. I recommend that you start early and get help in office hours if needed. If you find it helpful to discuss problems with other students, go for it. **The goal is to be ready for the in class quiz that will cover the same or similar problems.**

Problem 1: Greedy Choice Justification

You are consulting for a trucking company that does a large amount of business shipping packages between New York and Boston. The volume is high enough that they have to send several trucks each day between the two locations. Trucks have a fixed limit W on the maximum amount of weight they are allowed to carry. Boxes arrive at the New York station one by one, and each package i has a weight w_i. The trucking station is quite small, so at most one truck can be in the station at any time. Company policy requires that boxes are shipped in the order they arrive; otherwise a customer might get upset. At the moment, the company is using a simple greedy algorithm for packing: they pack boxes in the order they arrive, and whenever the next box does not fit, they send the truck on its way.

But they wonder if they might be using too many trucks, and they want your opinion on whether the situation can be improved. Here is how they are thinking. Maybe one could decrease the number of trucks needed by sometimes sending off a truck that was less full, and in this way allow the next few trucks to be better packed.

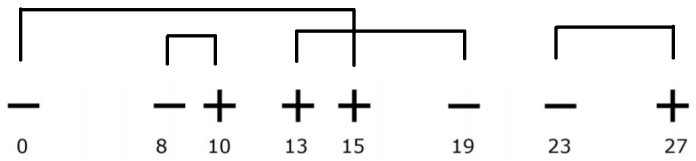
Justify that, for a given set of boxes with specified weights, the greedy algorithm currently in use actually minimizes the number of trucks that are needed.

Problem 2: Phone Base Stations around the Lake

Remember the problem with the phone base stations. Here we will see a variation of this problems. This time there are houses scattered sparsely along the perimeter of a cyclic lake with radius far longer than 4 miles. The positions of the houses are given to you. Your goal is to place as few phone base stations as possible, along the perimeter, ensuring that every house is within 4 miles from some base station (you can assume that every phone base station is covering an arc of 8 miles with its position as the middle point of the arc). Give an algorithm with time complexity $O(n^2)$

Problem 3: Connecting Wires

There are n positive terminals and n negative terminals. They are all at distinct locations along the positive x-axis, i.e., the location of some terminal t can be given by *x(t)* where $\{x(t) \in R : x \geq 0\}$. You must place n wires, each connecting a positive terminal to a negative terminal, so that at the end each terminal is connected to exactly one wire. Design a greedy algorithm to place the wires to minimize the total length of wire used. State and briefly justify the runtime of your algorithm. (**Hint**: You'll need two sorts to solve this problem.)



The above picture shows a possible layout of terminals for n = 4. The location value *x(t)* for each terminal t is written underneath the terminal. The total length of the wire for this example is 15 + 2 + 6 + 4 = 27 (Note : 27 is not the optimal solution here).