COMP3516: Data Analytics for IoT

# Lecture 3: A Primer on Signals

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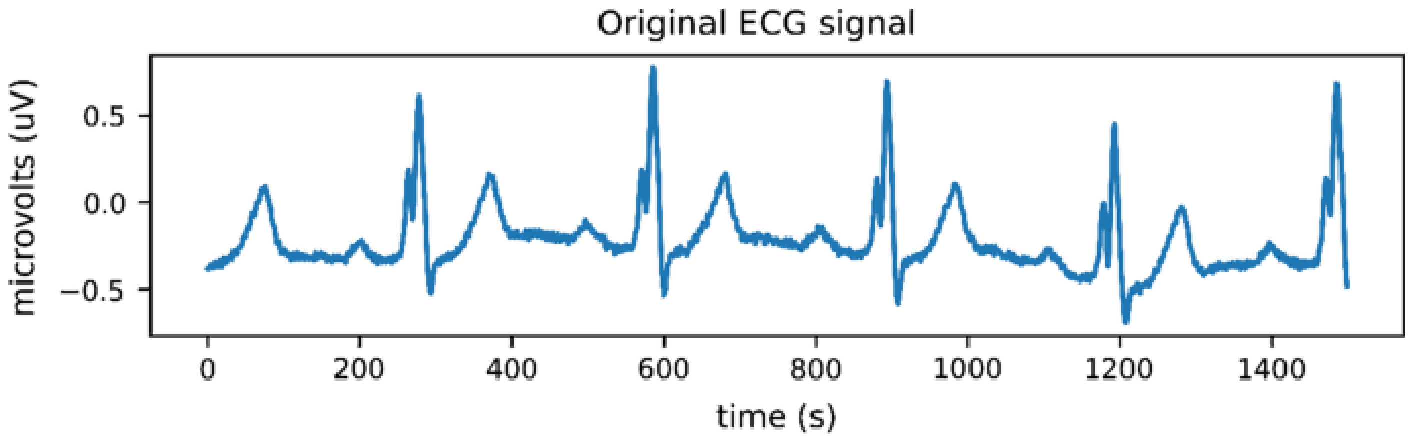
# Contents

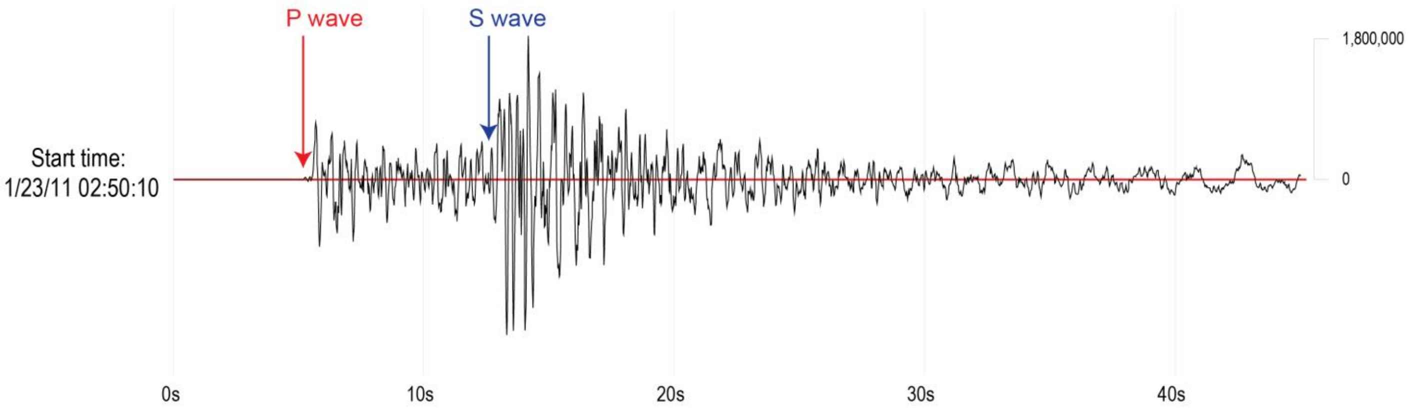
- What are Signals/Data?
- Analog/Digital Signals
- Common Signals: Sinusoids
- Complex Signals
- Sampling
- Time-Frequency Domains

# What Are Signals?

- Certain physical variables
  - Voltage, current…
  - Light, temperature, humidity…
  - ECG, blood pressure…
  - Position, velocity, mass, …
  - Price of stocks, interest/exchange rate, …
  - Sound, water ripples, WiFi, infrared, …
  - …
- Time-series data: signals as a function of time (or other variables)
  - The physical variable at a set of times
  - Data!
- Signal Processing / Data Analytics
  - Extract information/contexts from data/signals

# Examples


Original ECG signal
microvolts (uV)
0.5
0.0
−0.5
0
200
400
600
800
1000
1200
1400
time (s)

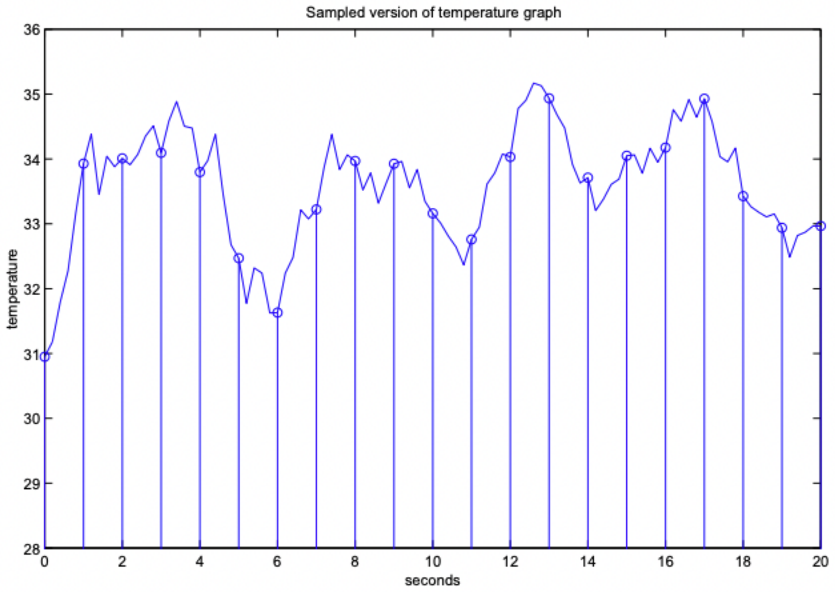

P wave
S wave
Start time:
1/23/11 02:50:10
1,800,000
0
0s
10s
20s
30s
40s

# Analog/Digital Signals

- Analog signals: <u>Continuous</u> domain and range
  - Continuous-time signals, x(t)
- Digital signals: <u>Discrete</u> (and often finite) domain and range
  - Discrete-time signals, x[n]

- Digitalize an analog signal (Why?)
  - <u>**Sampling**</u>: Digitalize the (time) domain
  - <u>**Quantization**</u>: Digitalize the range

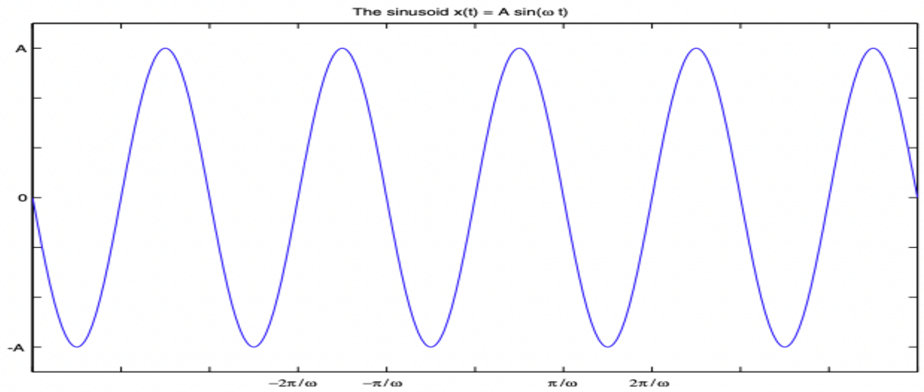
  - Analog-to-Digital Conversion/DAC

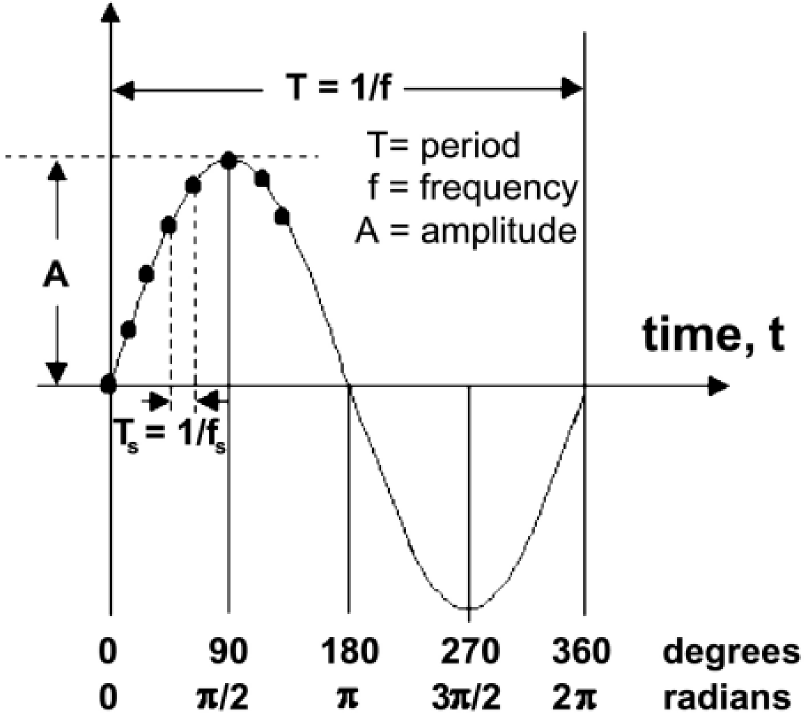


# Common Signals: Sinusoids

- $x(t) = A\sin(\omega t) = A\sin(2\pi f t)$
  - A: amplitude
  - ω: radian frequency in radians/s
  - f: frequency in Hertz (Hz) / cycles per sec

- $x(t) = A\cos(2\pi f t - \pi/2)$
  - phase: the position on a waveform cycle
  - angle-like quantity representing the fraction of the cycle covered up to t



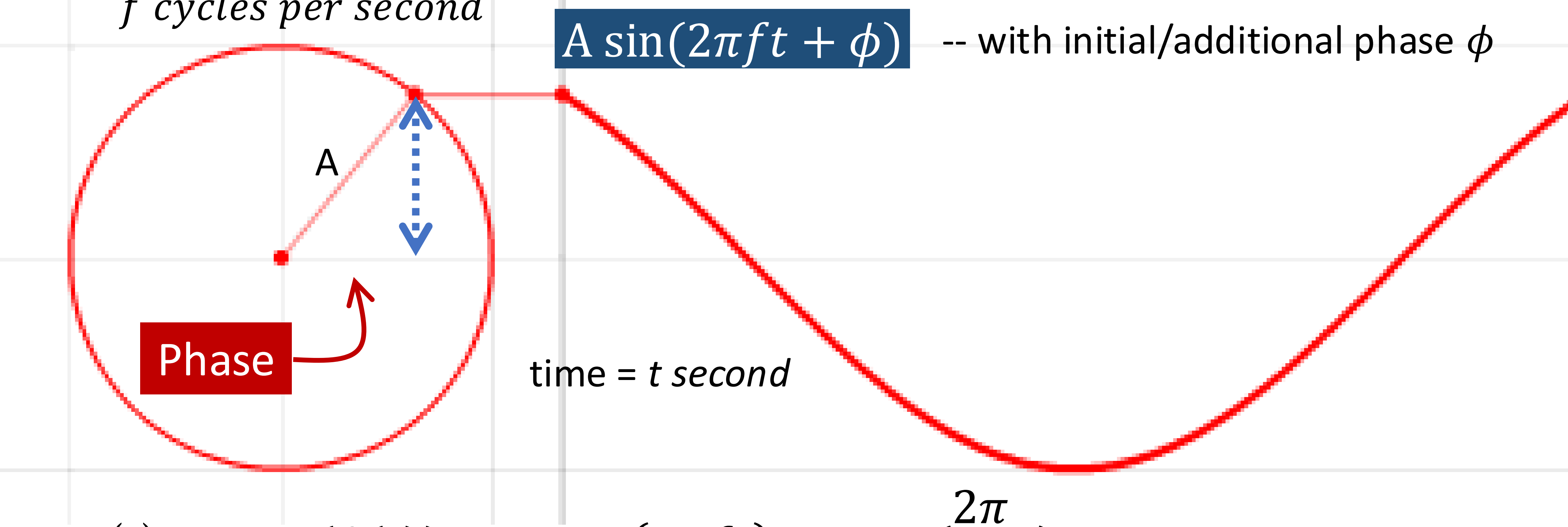



# Frequency, Amplitude, and Phase

# Frequency, Amplitude, and Phase

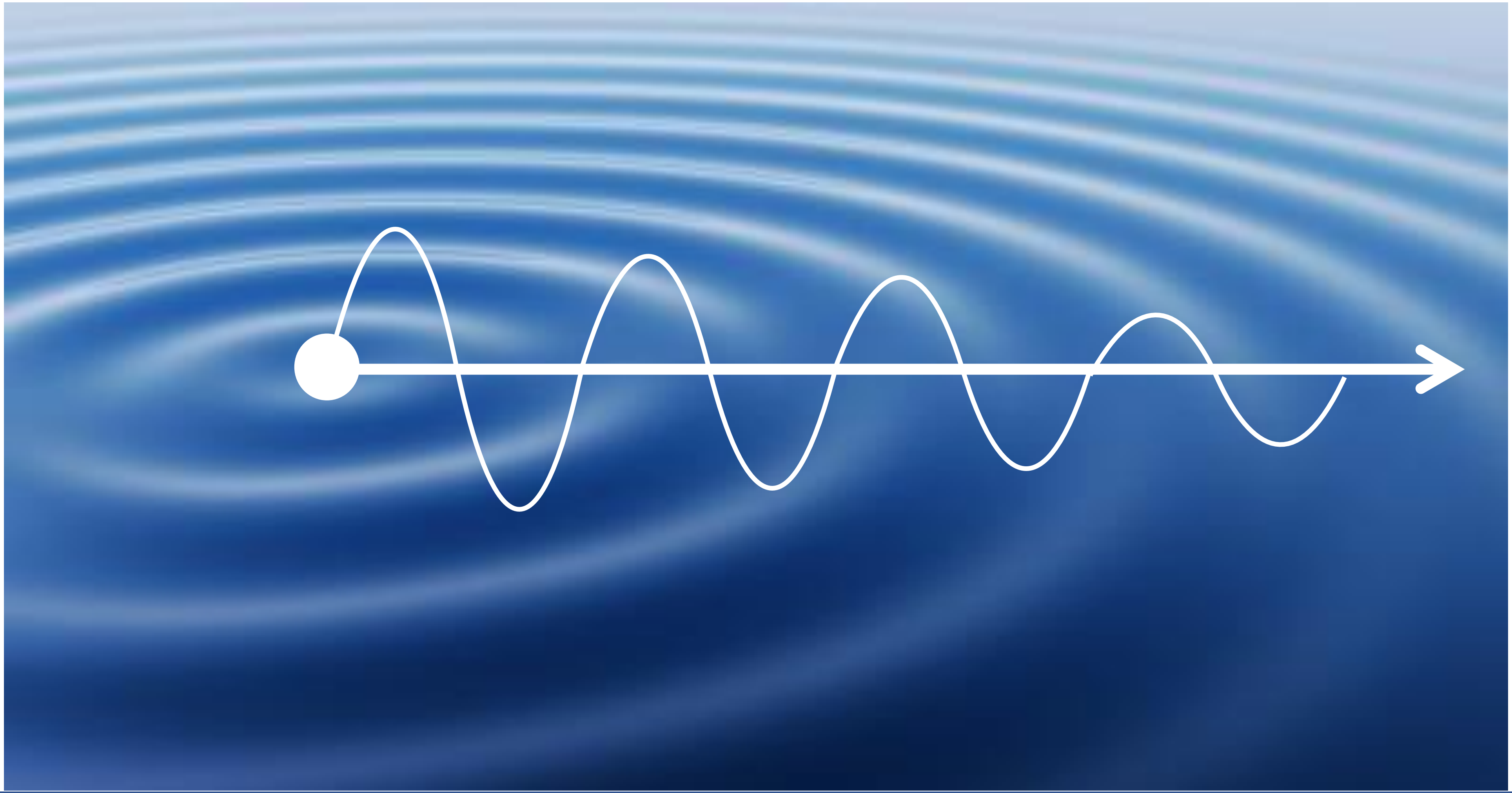
$2\pi$ *angles per cycle*

$f$ *cycles per second*

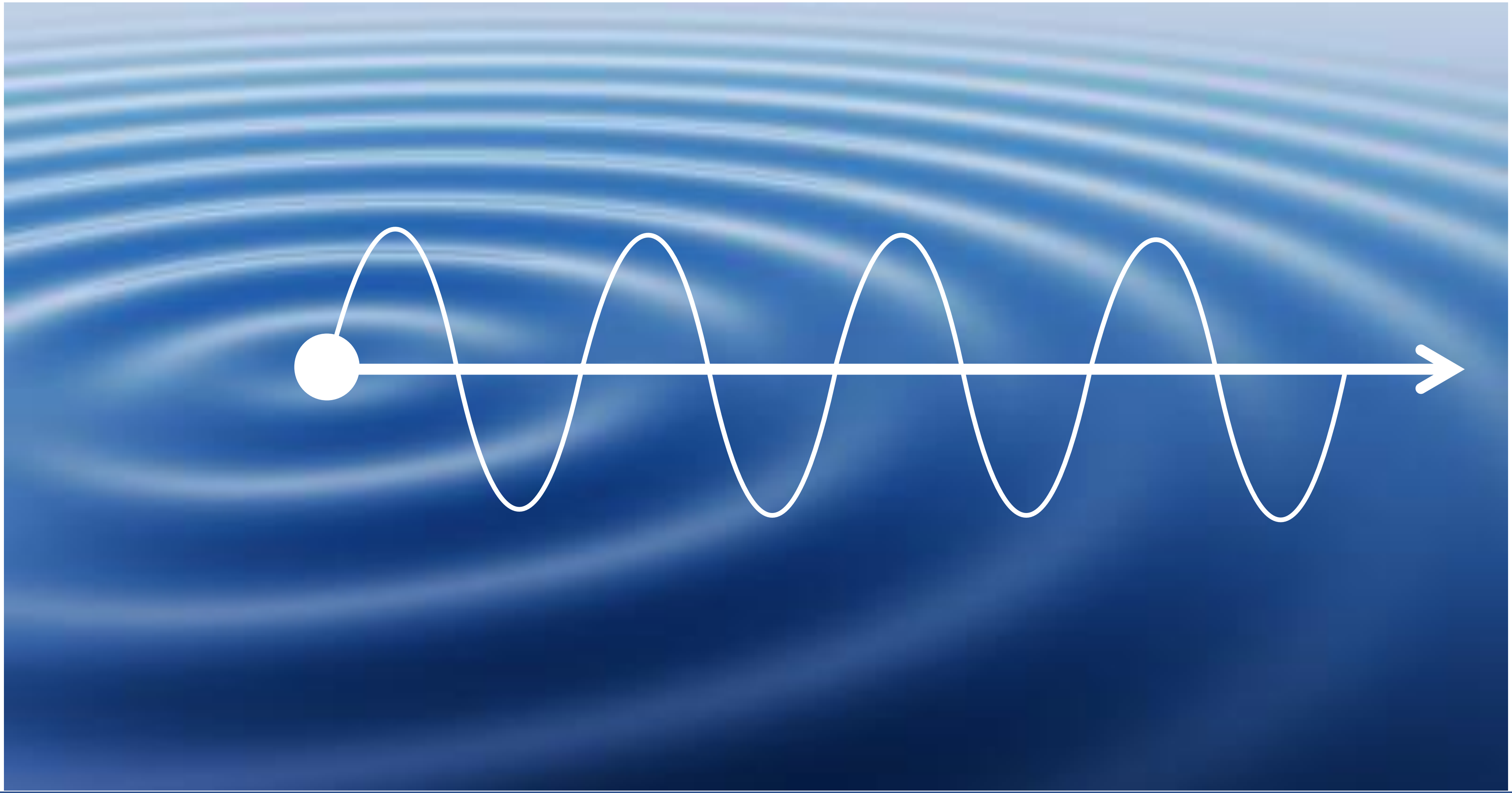


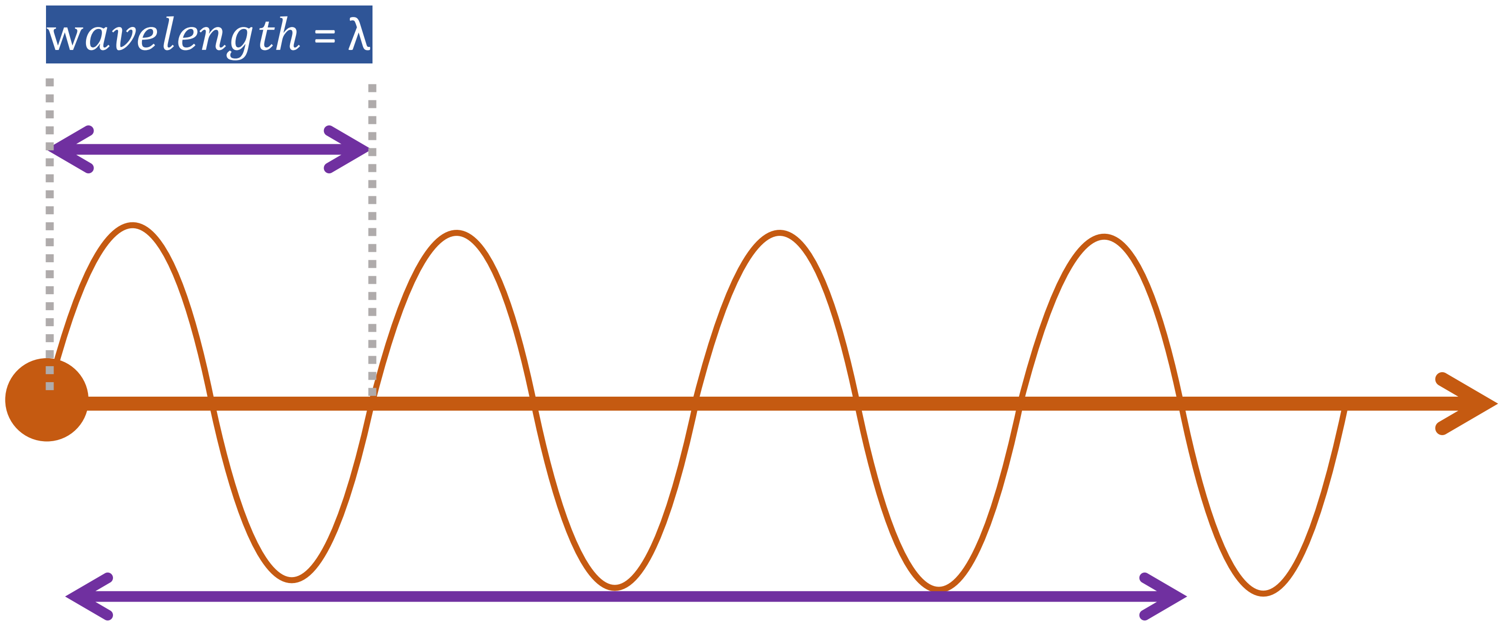

$$x(t) = A\sin(\theta(t)) \;\; = A\sin(2\pi f t) = A\sin(\frac{2\pi}{T}t)$$

# Waves in time and space



# Waves in time and space






*f number of cycles per second*

*C meters of distance per sec*
*(speed of the wave)*

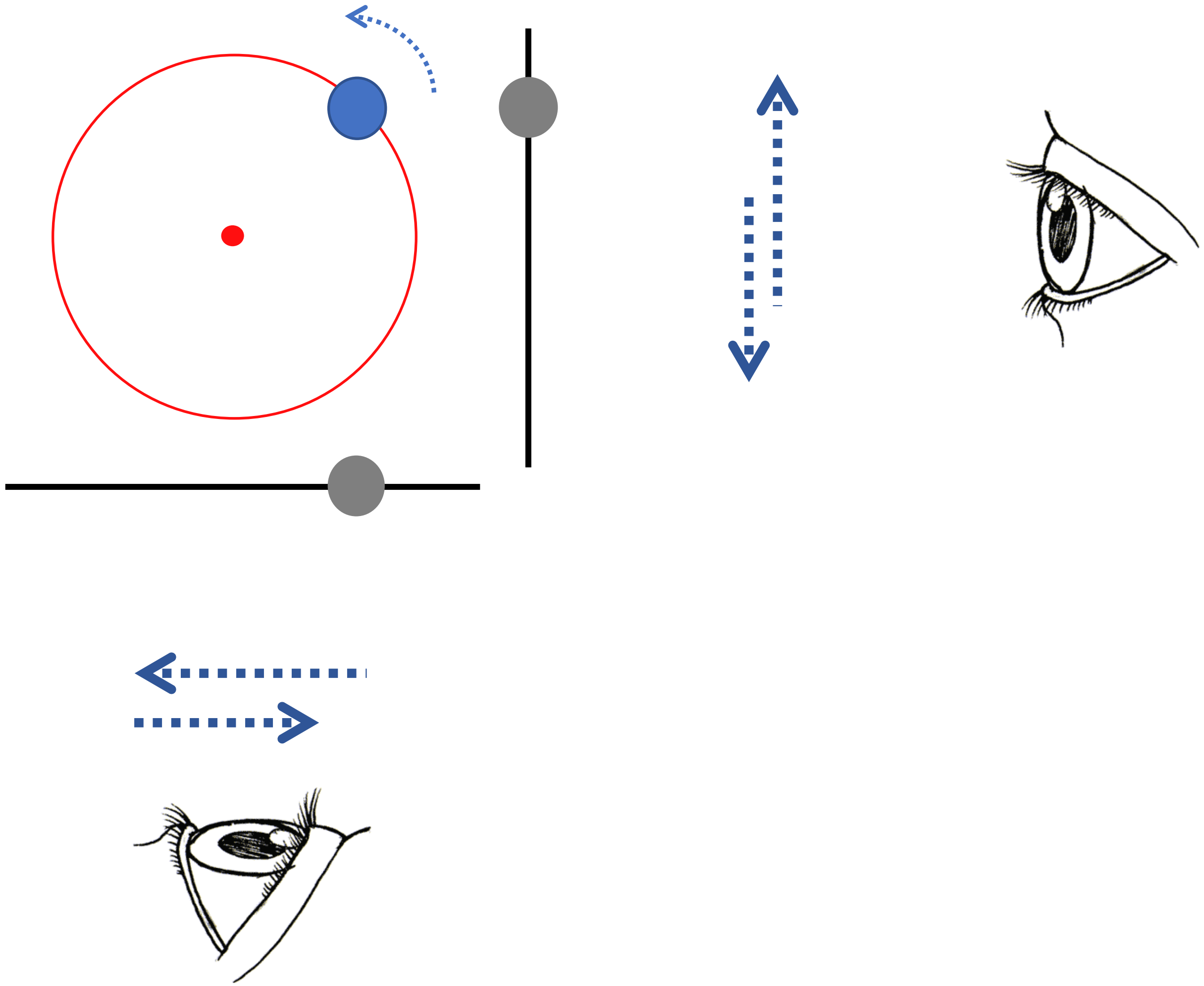
*distance per cycle* $= \lambda = C / f$

*wavenumber* $= 1/\lambda$
*(cycles per unit distance)*

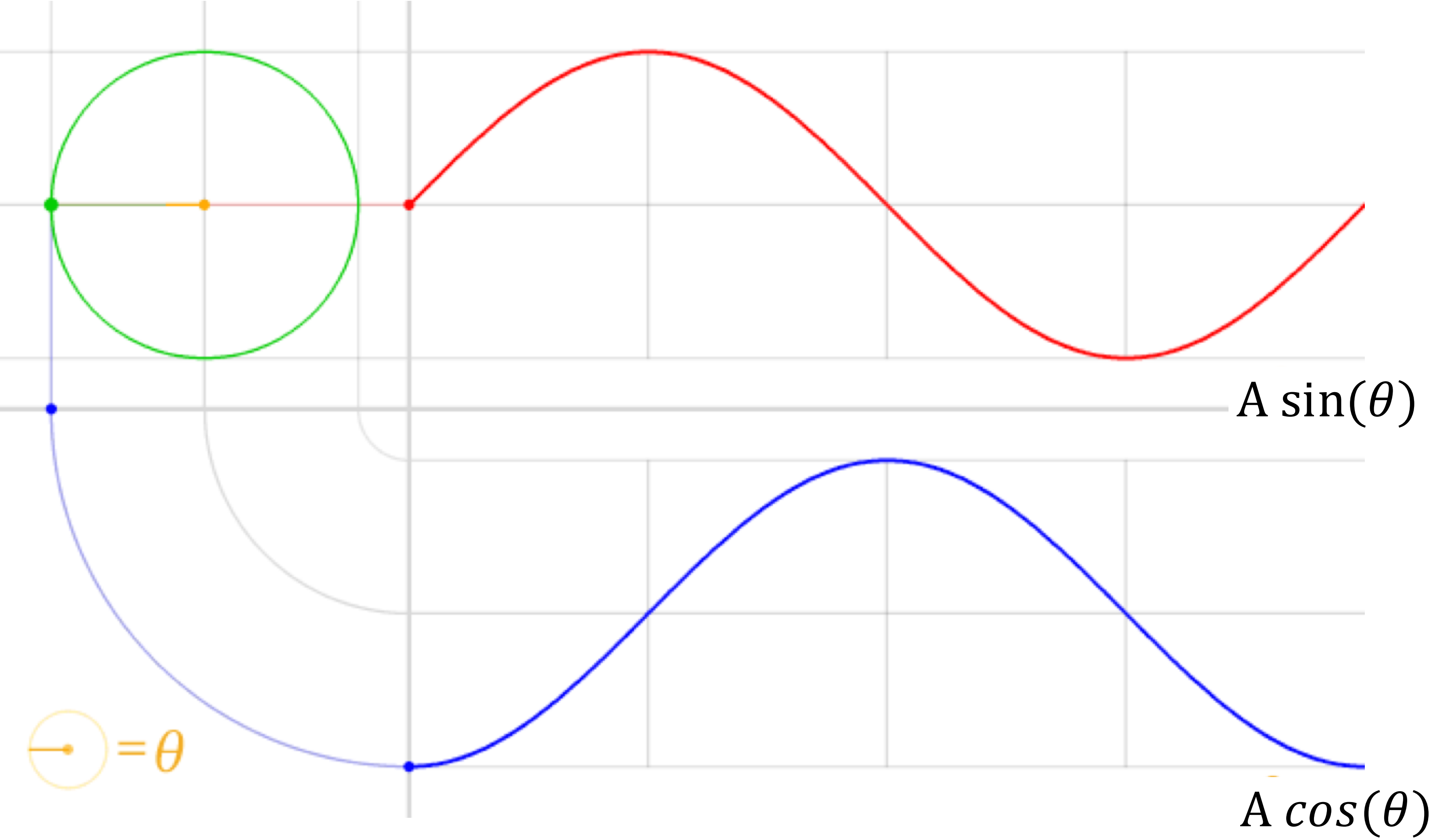
$$x(t) = A\sin(2\pi f t) = A\sin(\frac{2\pi}{T} t)$$

$$x(t, d) = A\sin(2\pi f t - \theta(d))$$

# Complex Signals



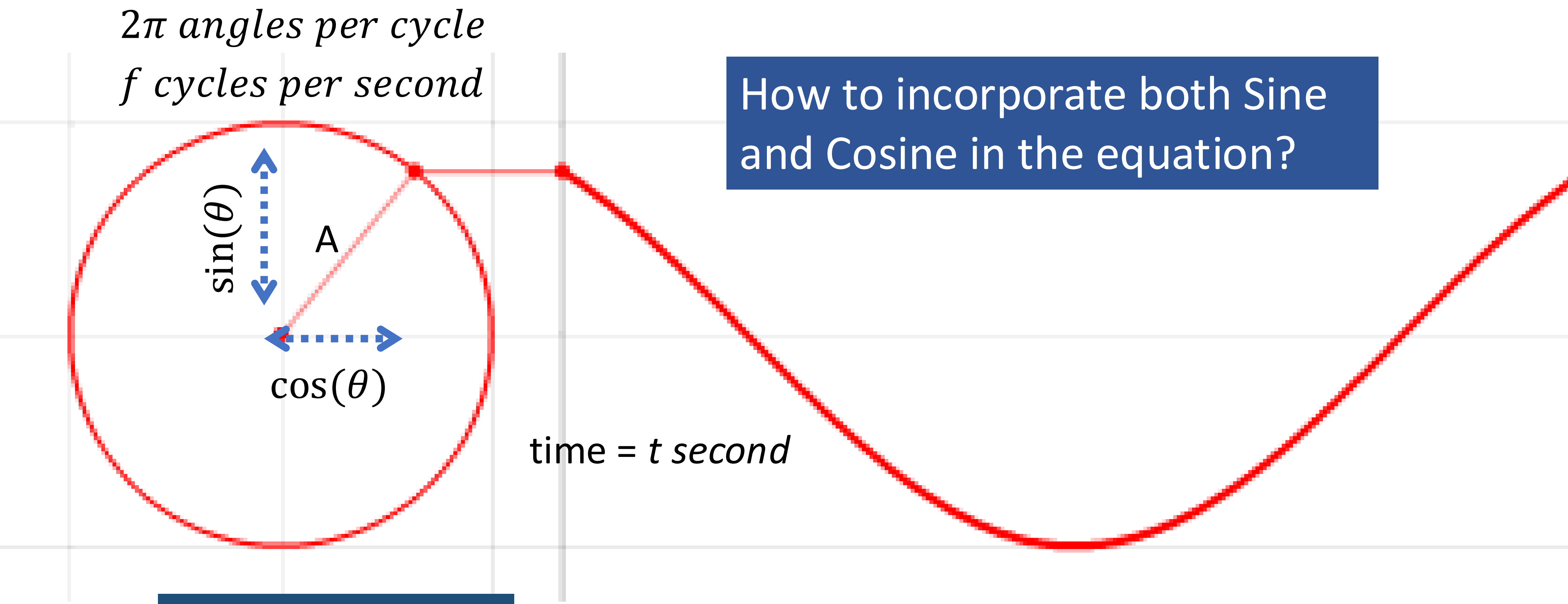
# Complex Signals



# Complex Signals


$A \sin(2\pi f t + \phi)$ -- with initial/additional phase $\phi$

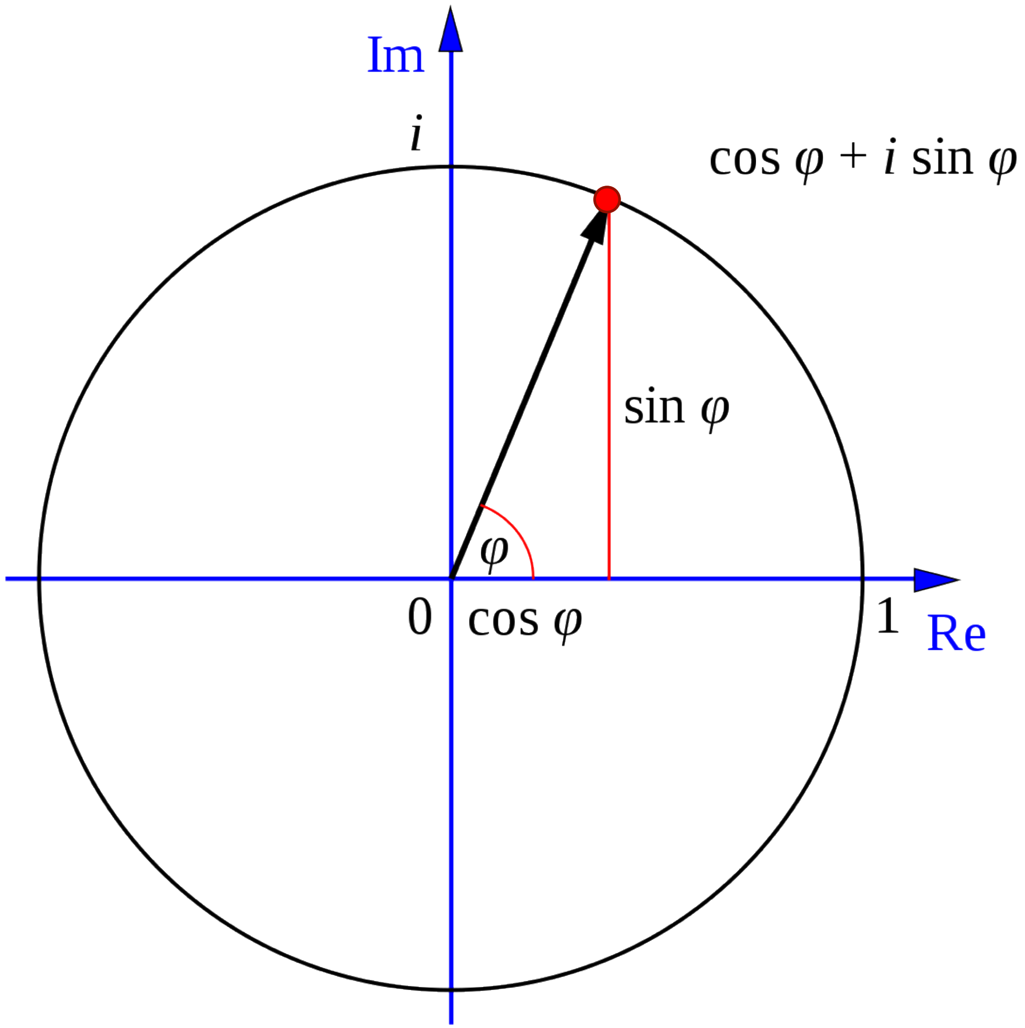
# Complex Signals



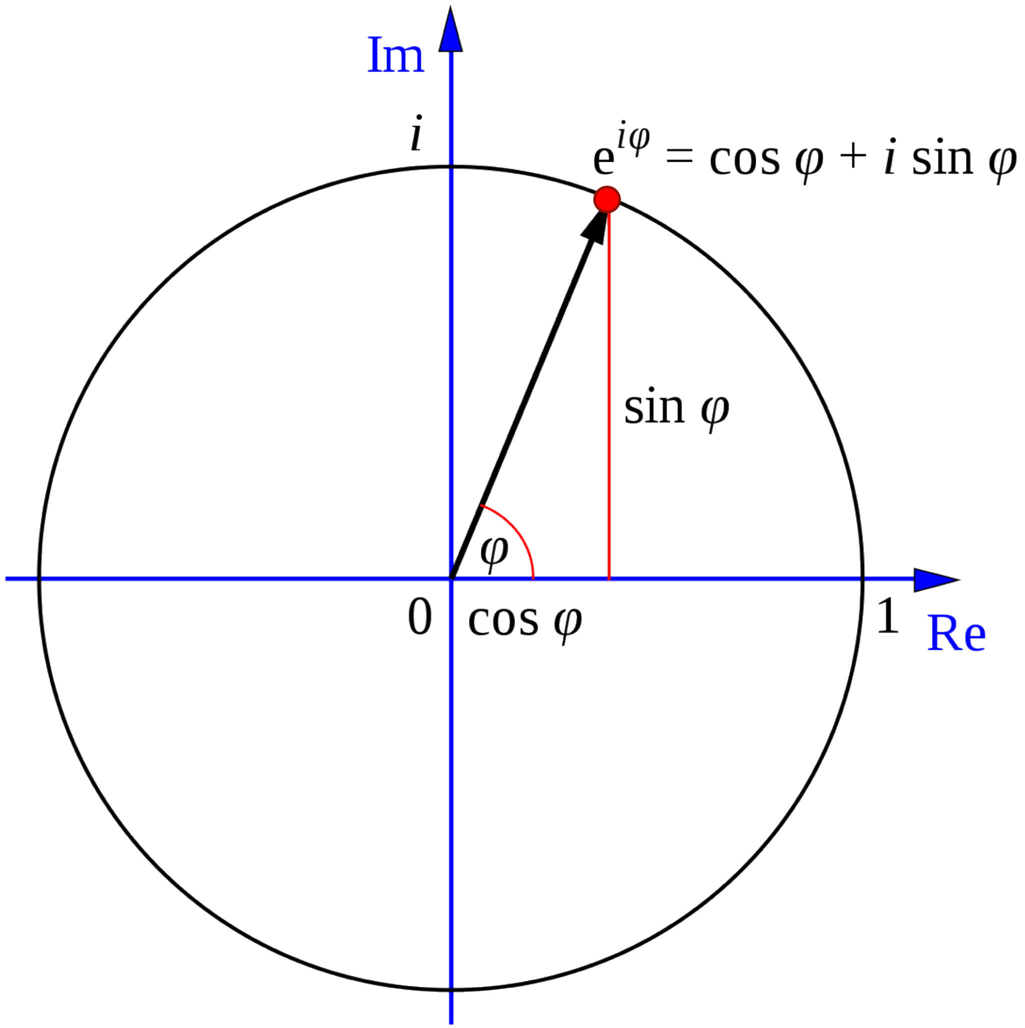

How to incorporate both Sine and Cosine in the equation?

$A \sin(2\pi f t + \phi)$ -- with initial/additional phase $\phi$

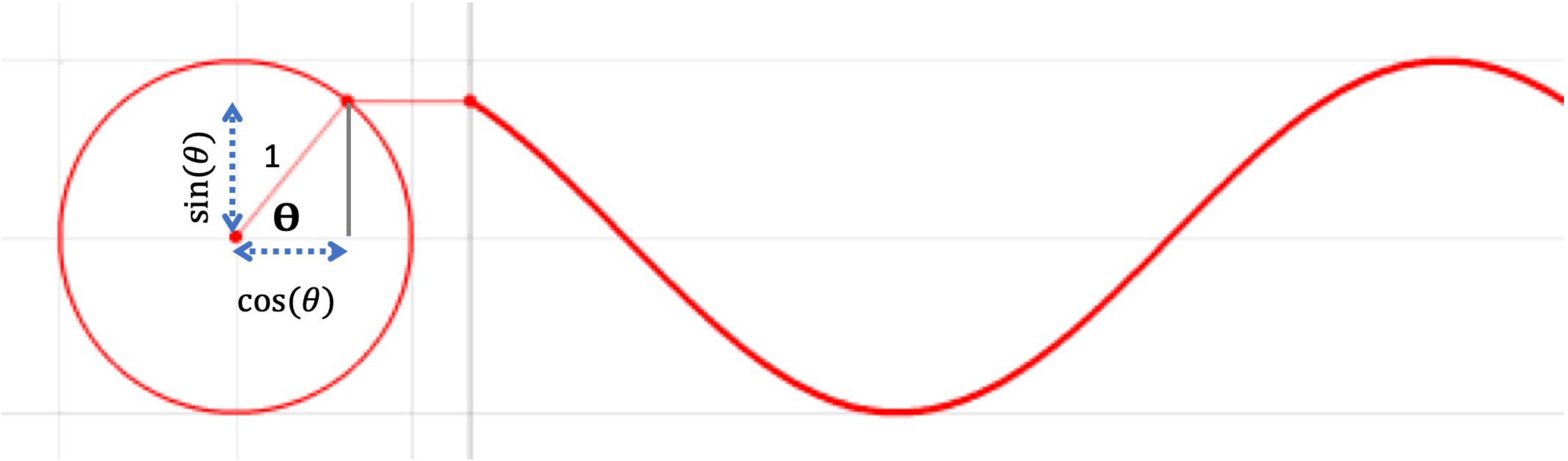
# Complex Signals



# Complex Signals



# Complex Signals




$$cos(\theta) + j\ \sin(\theta) = e^{j\theta} = e^{j\,2\pi f t}$$

# Complex Signals

- The most basic complex-valued signal is the complex exponential $e^{j\omega t}$.

$$\boxed{e^{j2\pi ft} = cos(2\pi ft) + j\sin(2\pi ft)}$$

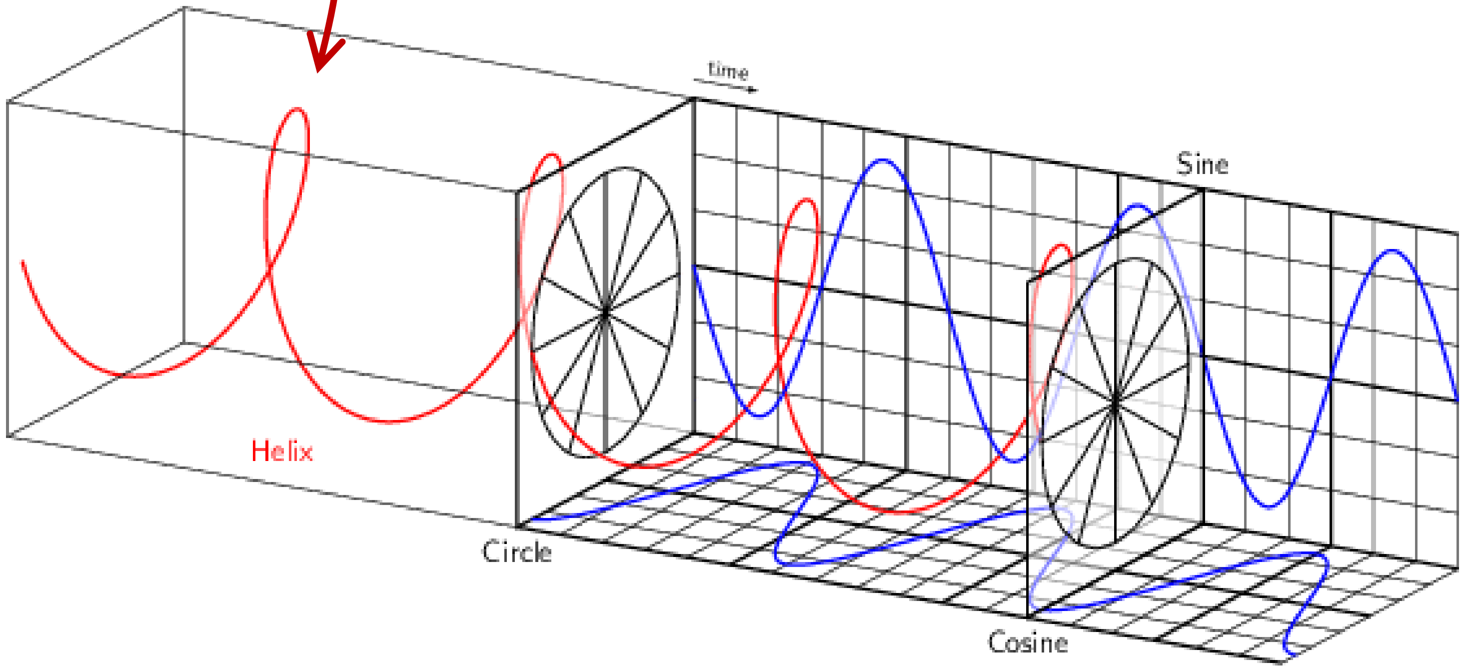
$$\boxed{e^{-j2\pi ft} = cos(2\pi ft) - j\sin(2\pi ft)}$$

How about real sinusoids?

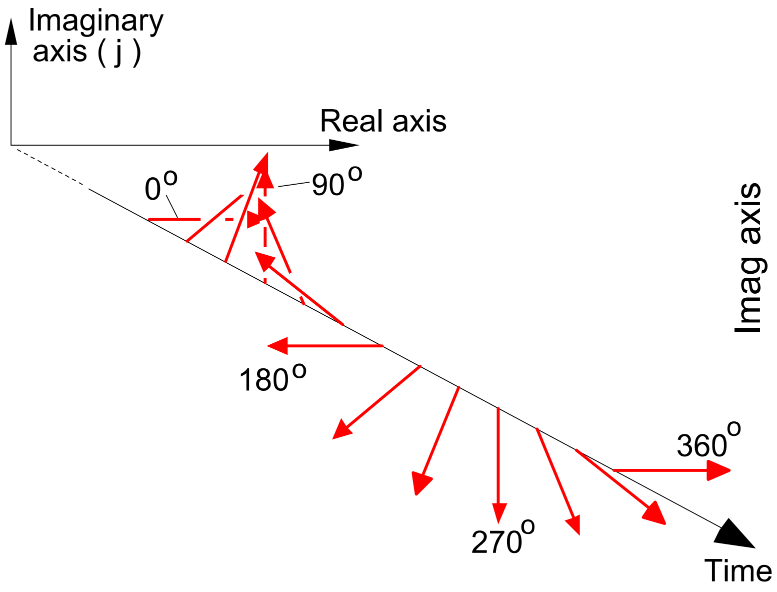
$$cos(\theta) = ?$$
$$sin(\theta) = ?$$

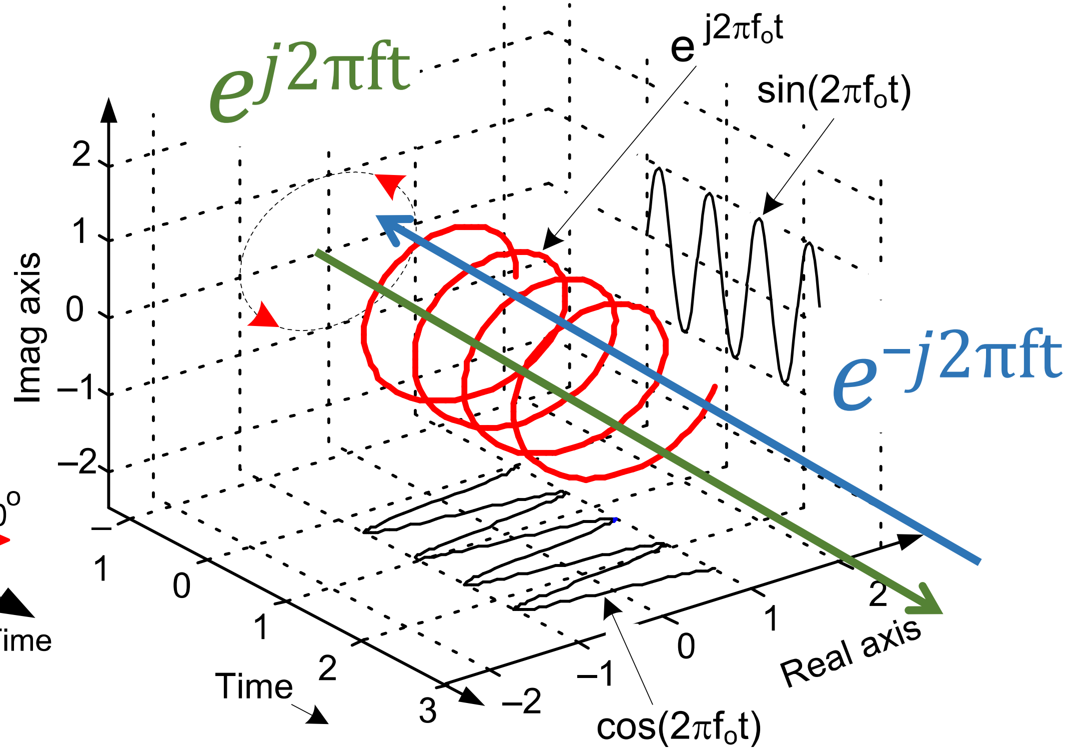
# Complex Signals

$e^{j2\pi ft}$

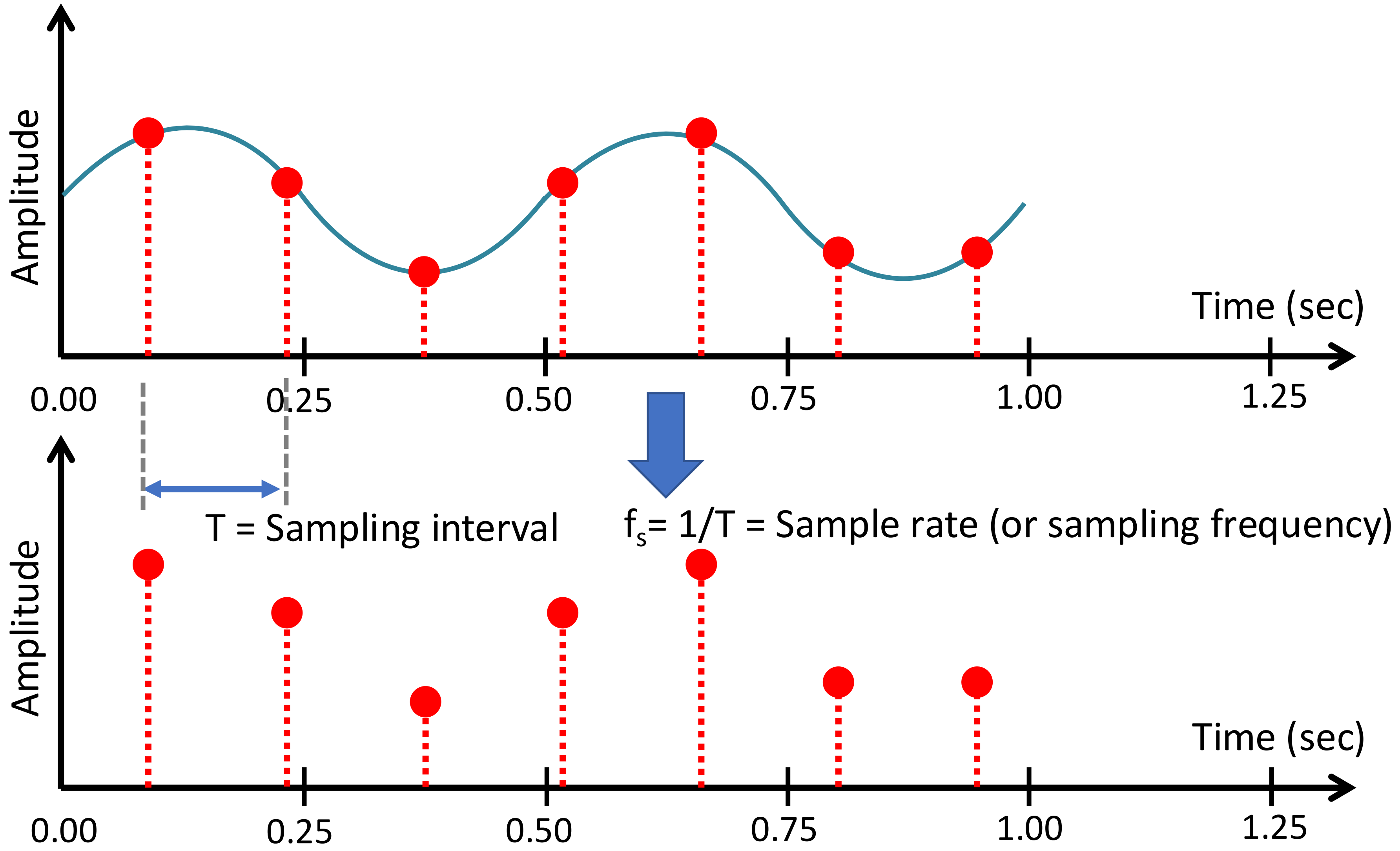


# Complex Signals

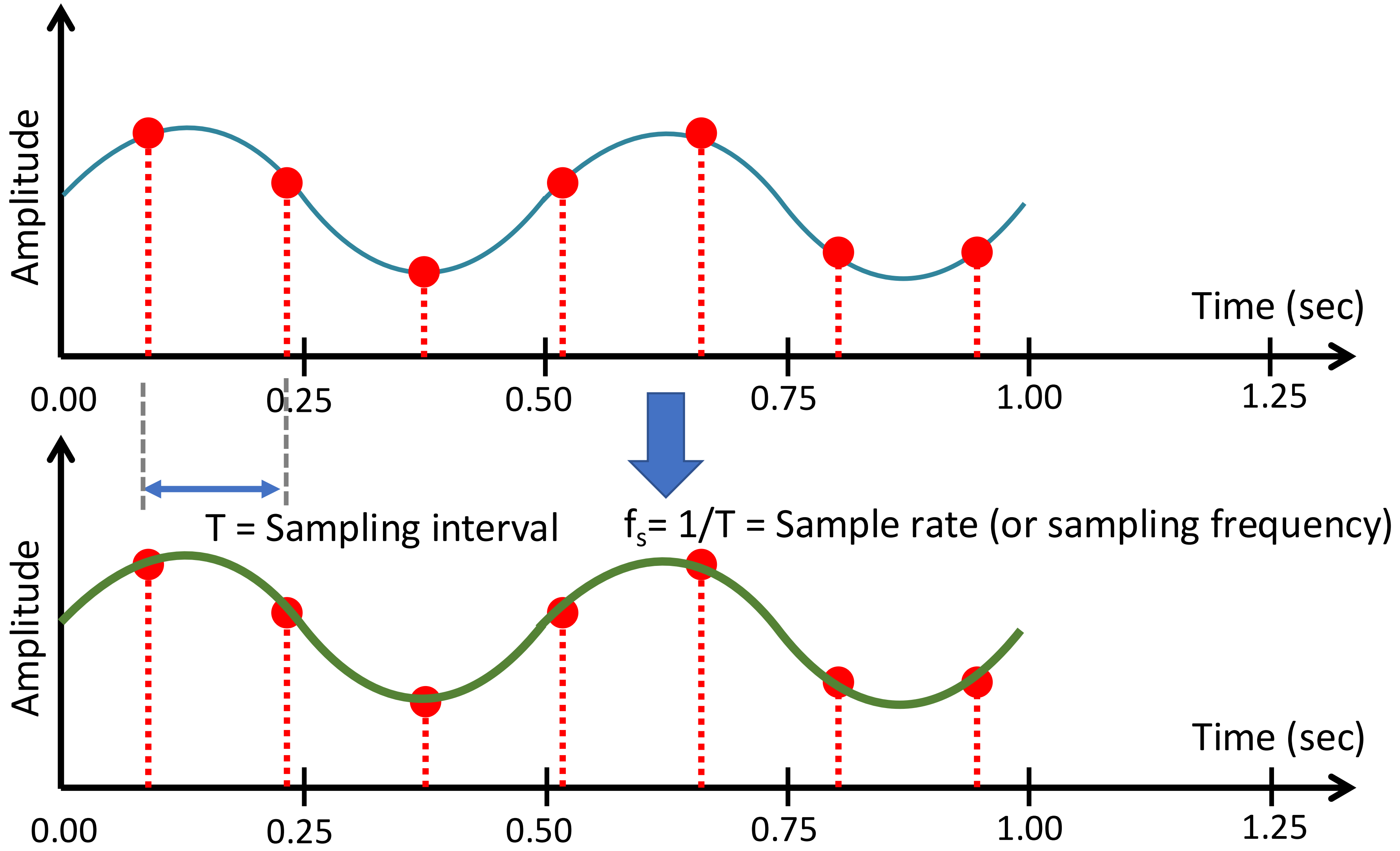


# Sampling


Amplitude
Time (sec)
0.00
0.25
0.50
0.75
1.00
1.25
T = Sampling interval
$f_s$= 1/T = Sample rate (or sampling frequency)
Amplitude
Time (sec)
0.00
0.25
0.50
0.75
1.00
1.25

# Sampling



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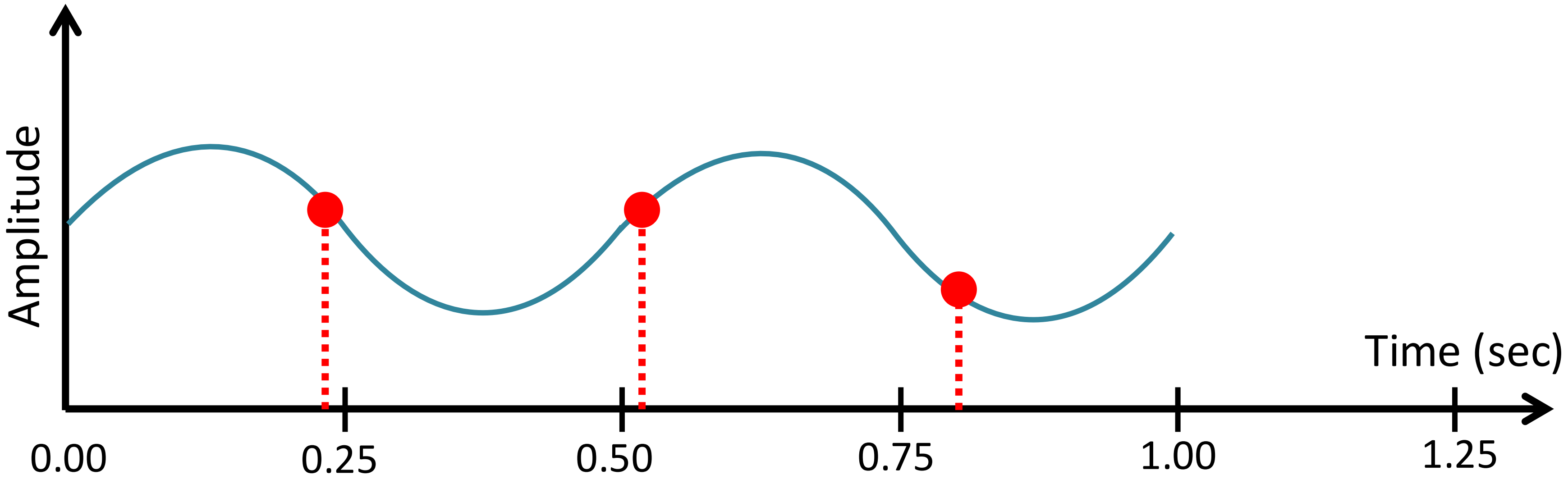
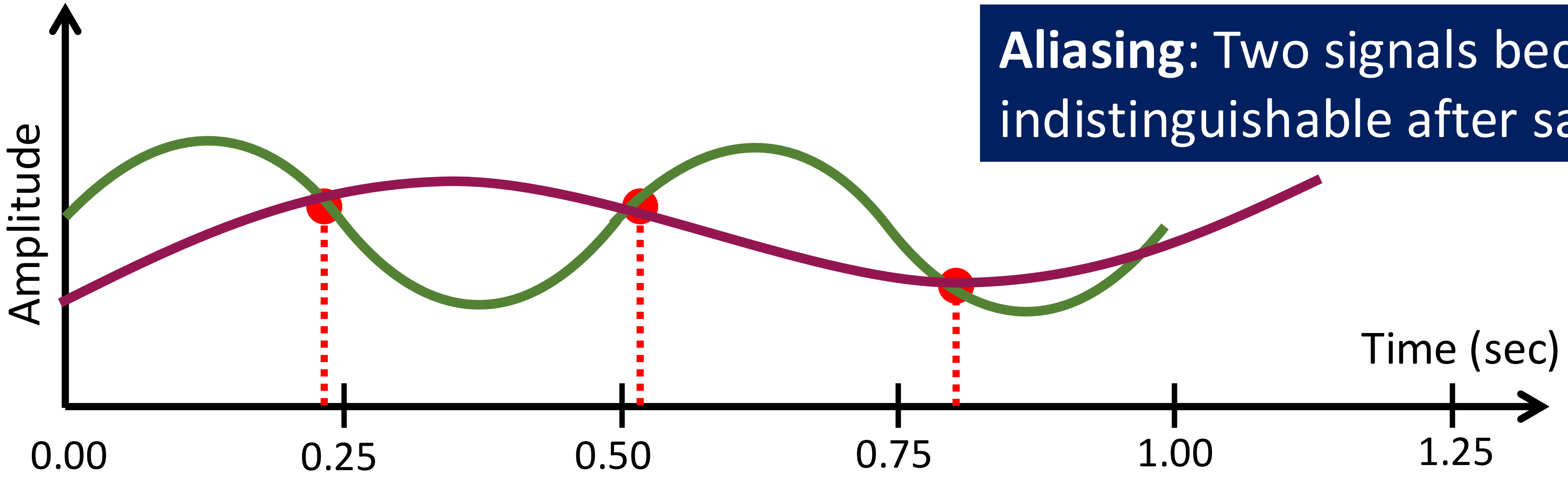
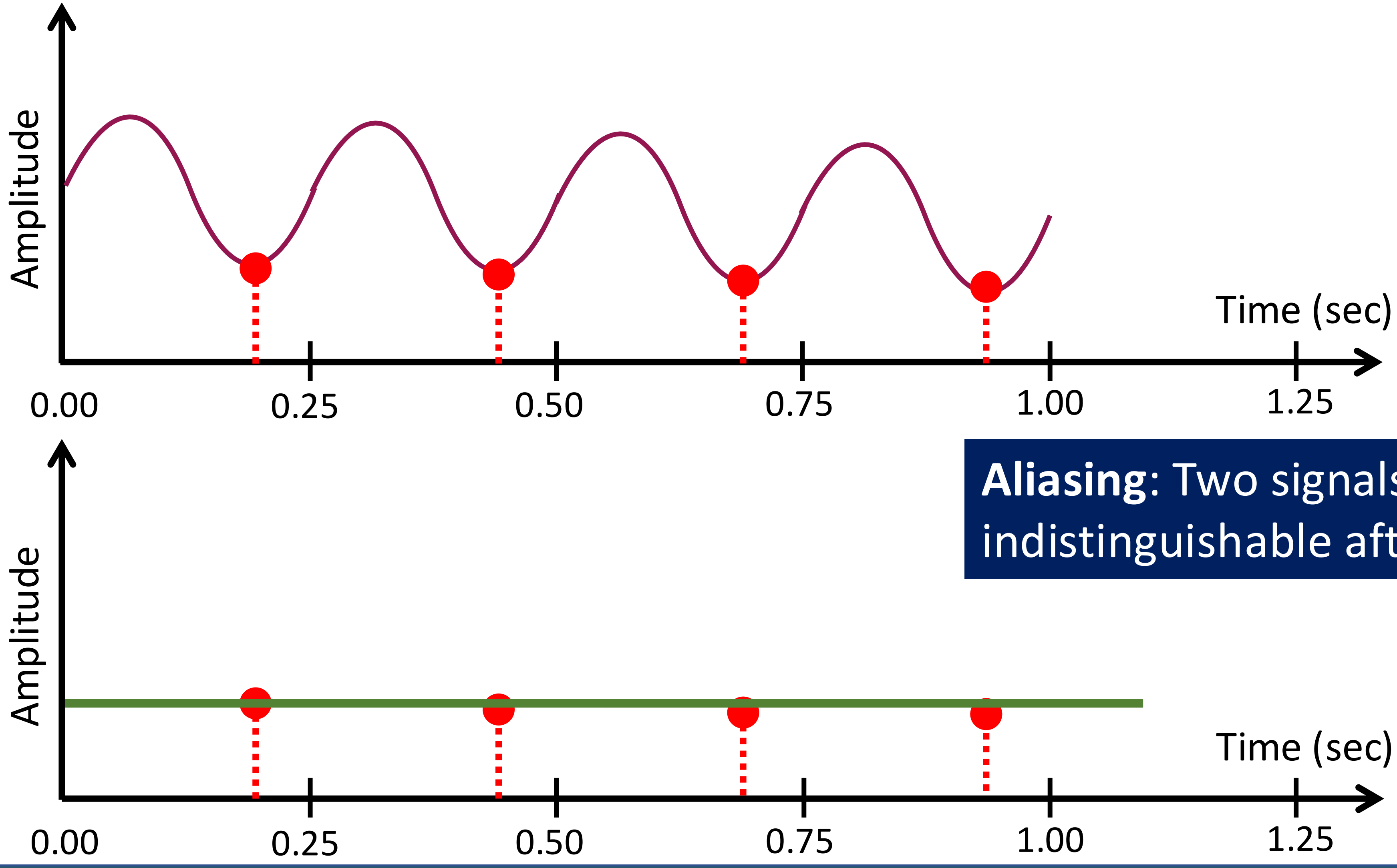
# Sampling




**Aliasing**: Two signals become indistinguishable after sampling

# Sampling

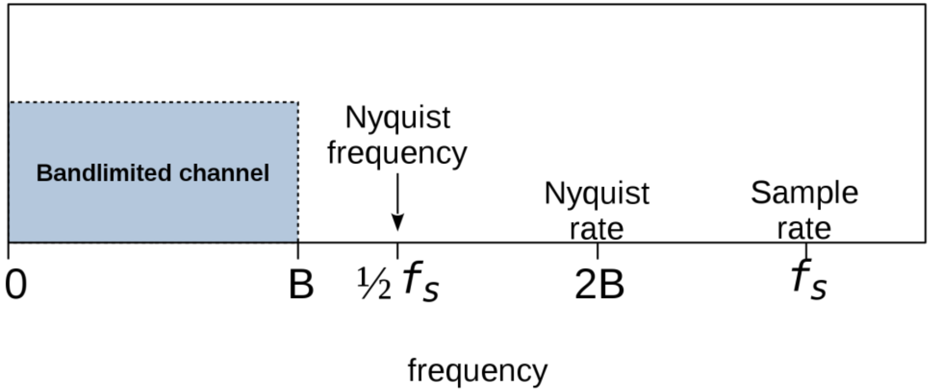



**Aliasing**: Two signals become indistinguishable after sampling

# Sampling

- Nyquist sampling theorem:
  - In order to uniquely represent a signal x(t) by a set of samples, the sampling rate must be more than twice the highest frequency component present in x(t).
  - Sample twice per period!
- If sample rate is $f_s$ and the maximum frequency of interest is $f_{max}$, then $f_s > 2f_{max}$

Relationship of Nyquist frequency & rate (example)




<u>Nyquist frequency</u>: Maximum alias-free frequency for a given $f_s$

<u>Nyquist rate</u>: Minimum sample rate for a given signal = twice the highest frequency

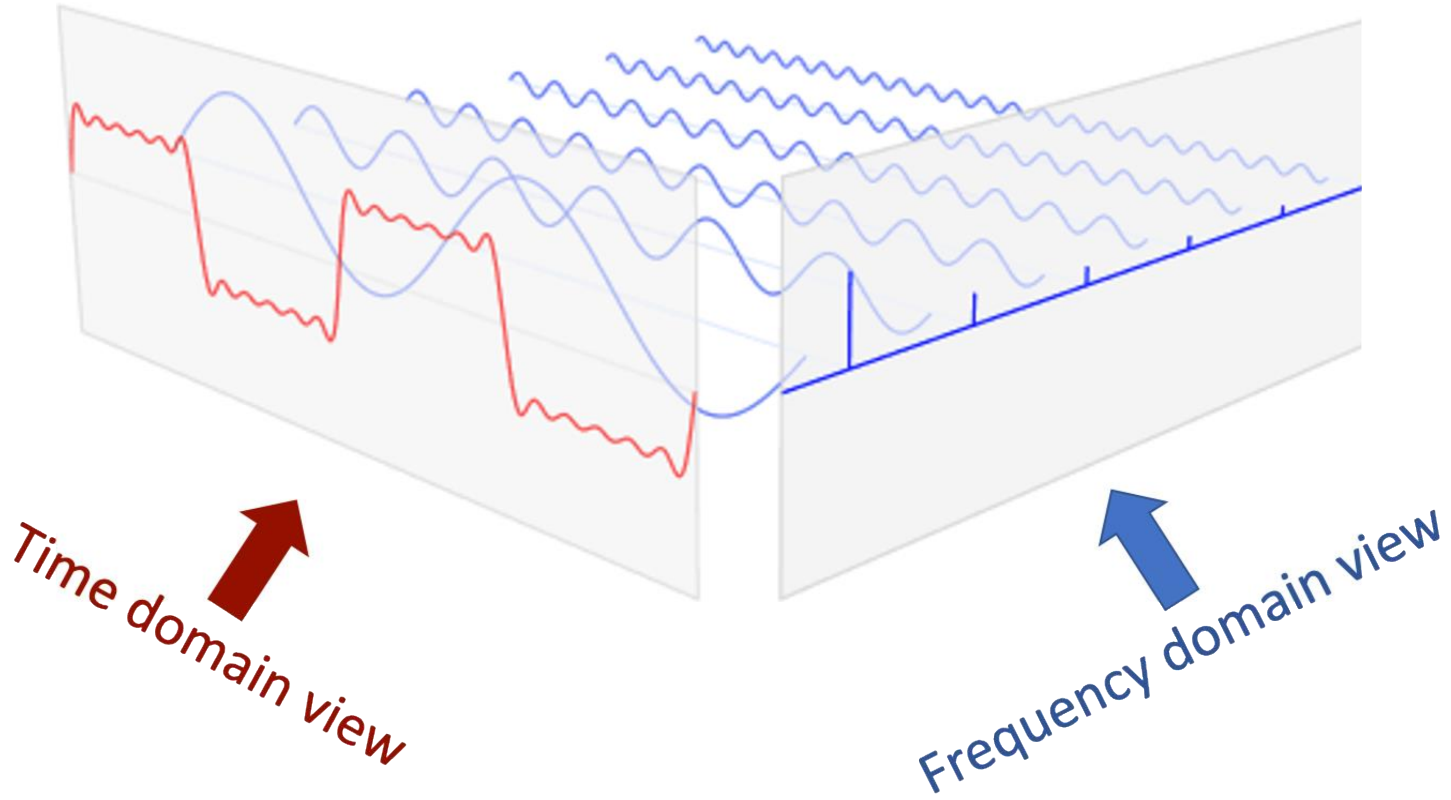
# Aliasing in Real Life

https://www.youtube.com/watch?v=B8EMI3_0TO0

# Aliasing in Real Life


https://www.youtube.com/watch?v=QOwzkND_ooU

# Time and Frequency Domain



# Why Frequency Domain?

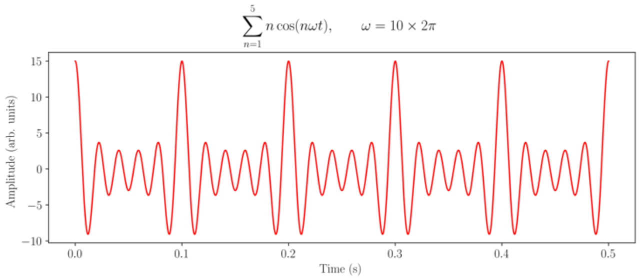
# Why Frequency Domain?

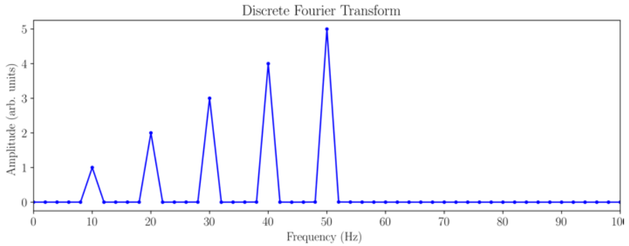

# How to Get Frequency Domain?

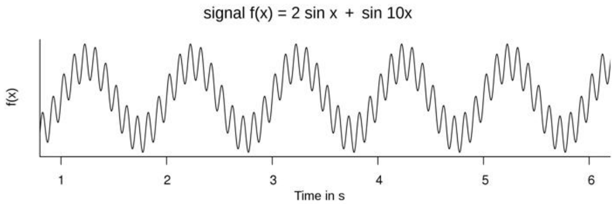
- Discrete Fourier Transform (DFTs)
  - Convert time-domain signals $f(t)$ into the frequency-domain responses $F(\omega)$
  - Time-domain: $x[n], n = 0, 1, \dots, N-1$
  - Frequency-domain: $X[k], k = 0,1, \dots, N-1$
  - $\mathbf{X} = \mathcal{F}\{\mathbf{x}\}$

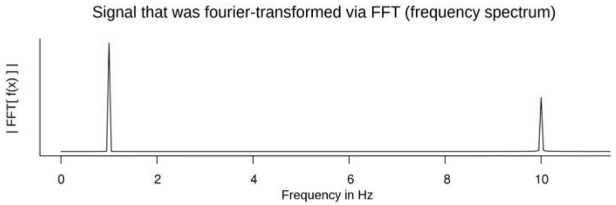
$$\boxed{X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{k}{N} n}}$$

- Fast Fourier transform (FFT)
  - Fast calculation of DFT: $O(N^2)$ to $O(NlogN)$



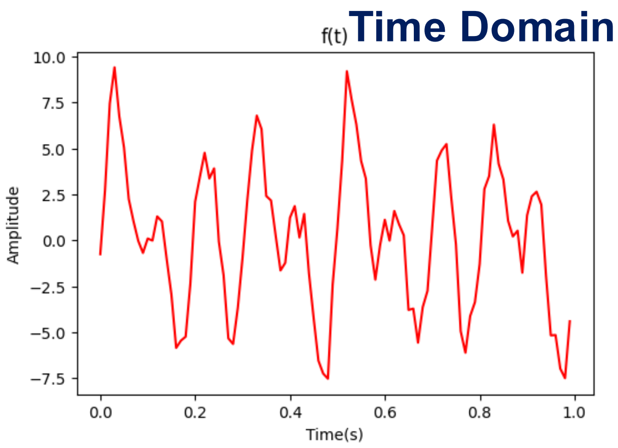



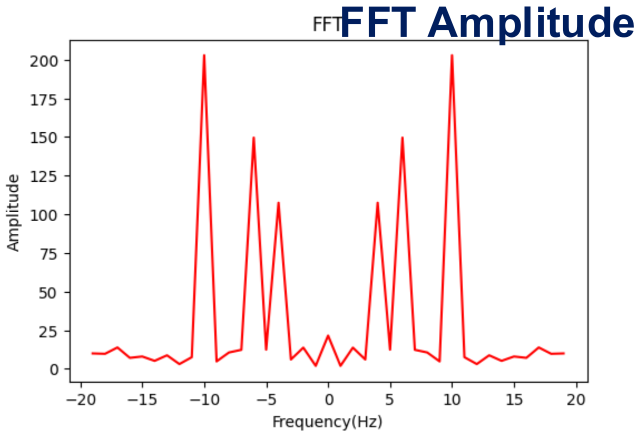
# Understanding FFT (a bit)

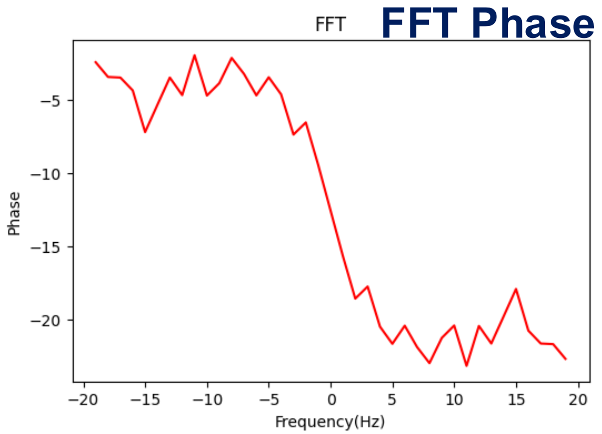
- What
  - Power spectrum of the signal
  - (<u>Single-sided</u>) Range: DC to $\frac{F_s}{2}$
- Why
  - Correlate x[n] with sin() and cos() signals at different frequencies

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi\frac{k}{N}n} = \sum_{n=0}^{N-1} x[n] \sin\left(\frac{2\pi kn}{N}\right) - j\sum_{n=0}^{N-1} x[n] \cos\left(\frac{2\pi kn}{N}\right)$$

- How
  - FFT Resolution of frequency bins: $\Delta f = \frac{F_s}{N} = \frac{1}{N\Delta t} = \frac{1}{T}$
  - The more observations/longer period, the better $\Delta f$
  - Frequency resolution vs. FFT resolution/bin width

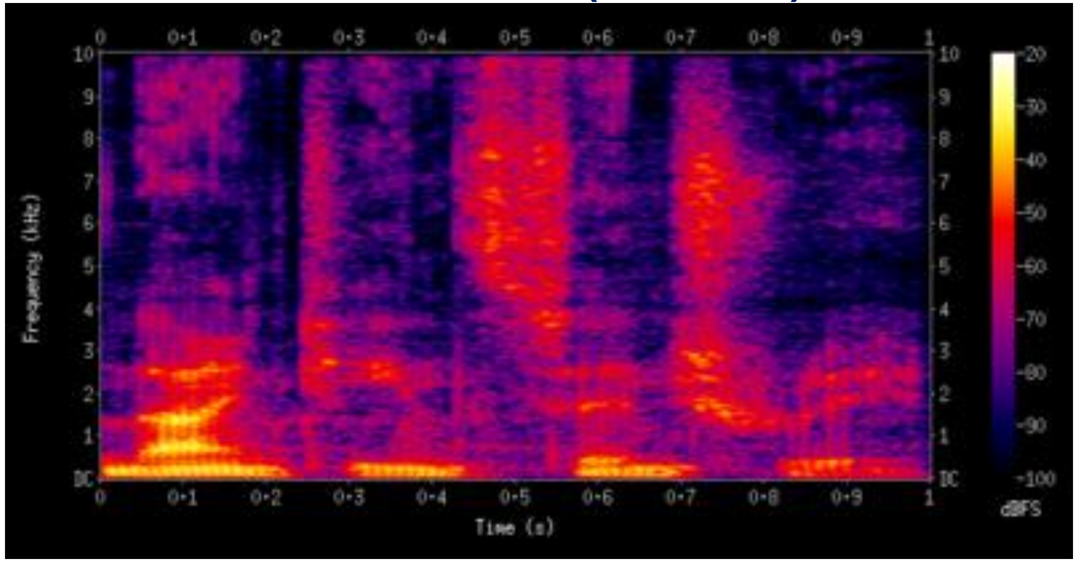


# Time-Frequency Spectrogram

- For each time window a DFT is calculated and frequency intensity (amplitude) is represented with ”colors”.
- Repeat the calculation with a sliding window
- Short-Time Fourier Transform (STFT)



# Questions?

- Thank you!