

# Lecture 4: mmWave Sensing

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# Contents

- Basics of FMCW Radar Signals
- Range Estimation
- Doppler/Velocity Estimation
- Angle Estimation
- Periodicity Estimation
- FMCW Sensing Applications

# Why mmWave/RF Sensing?

**What information is missing?**



# Why RF Sensing?

## Any other concerns for Vision?

### Tesla Vision Update: Replacing Ultrasonic Sensors with Tesla Vision

Safety is at the core of our design and engineering decisions. In 2021, we began our transition to Tesla Vision by removing radar from Model 3 and Model Y, followed by Model S and Model X in 2022. Today, in most regions around the globe, these vehicles now rely on Tesla Vision, our camera-based Autopilot system.

Now it appears **radar** is back. It's not yet clear which models will get the new **radar**. The type of **radar** Tesla intends to market next year is of a frequency that's allocated by the FCC for ADAS use cases, according to Ram Machness, chief business officer at Arbe Robotics, which produces ultra-high-resolution 4D imaging **radar**.

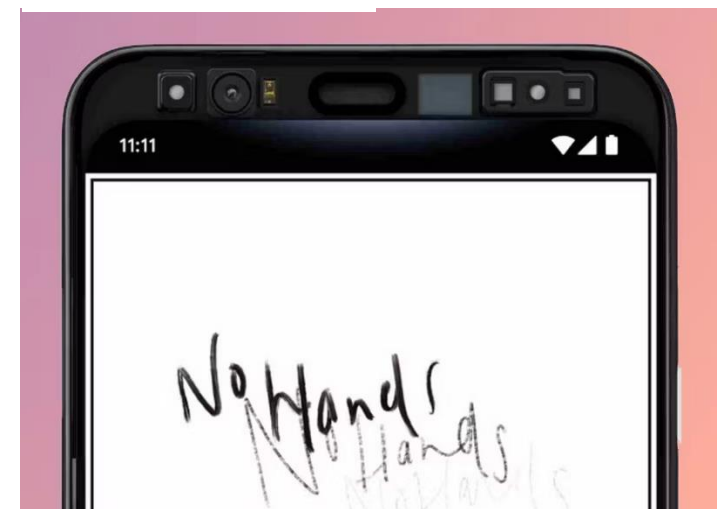
Tesla had originally filed with the FCC to use the new **radar** — which is described in filings as “76-77 GHz Automotive **Radar**” — in its vehicles back in June.

“From the frequency of operation (76-77GHz) as well as the mechanical design of the sensor from Tesla’s FCC filing, it

### Tesla Adding Radar Back

Steven Hong, VP and general manager of **radar** ch.

### Google Pixel 4



### Amazon + Vayyar

Constant protection.  
Touchless fall detection.

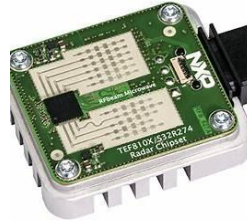
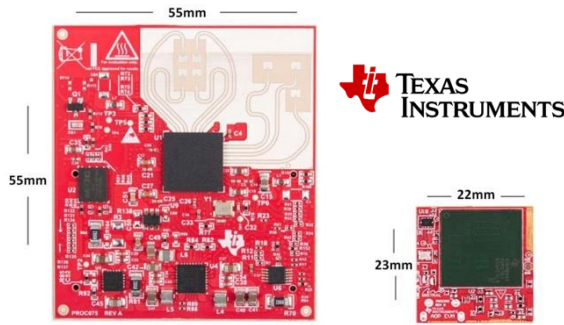
[BUY NOW FROM AMAZON](#)

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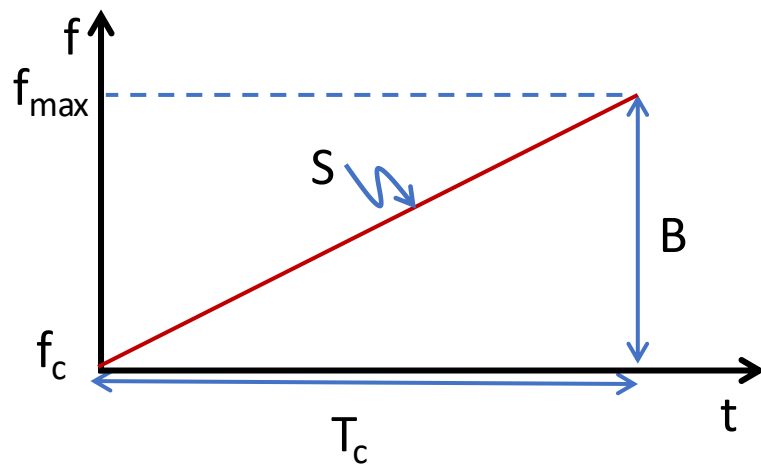
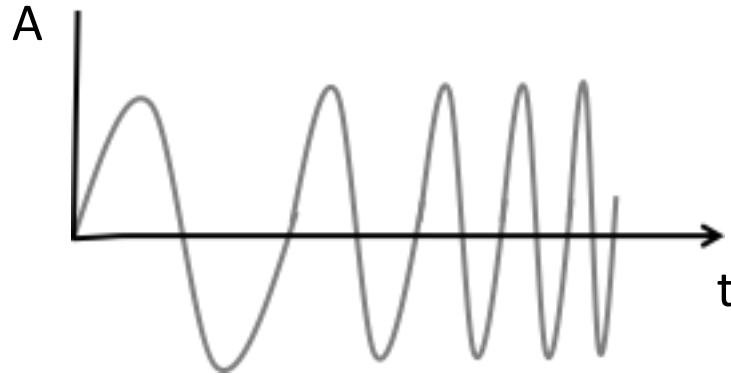
# Commodity mmWave Radars



Attribute	Texas Instruments (TI)	Infineon Technologies	NXP Semiconductors	Analog Devices	Vayyar Imaging	Calterah Semiconductor	ICLEGEND
Model	IWR6843 / AWR1843	BGT24MTR12	TEF810X	ADF5904	VTRIG-74	Alps-Pro	ICL1122
Frequency Band	60 GHz / 78 GHz	24 GHz	77 GHz	24 GHz	79 GHz	77-79 GHz	24 GHz
Bandwidth	4 GHz	250 MHz	1 GHz	250 MHz	4 GHz	Up to 5 GHz	250 MHz
Number of Antennas	3 TX, 4 RX	1 TX, 1 RX	3 TX, 4 RX	4 RX	12 TX, 12 RX	4 TX, 4 RX	1 TX, 2 RX

\*Price: <US\$20 for TI radar; price information is usually not publicly available.

# Revisit Chirp Signals



- FMCW Radar

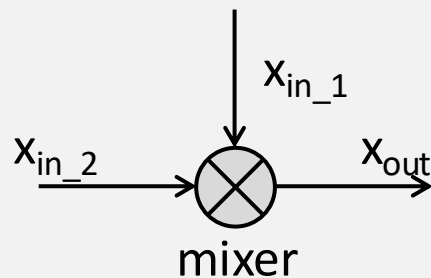
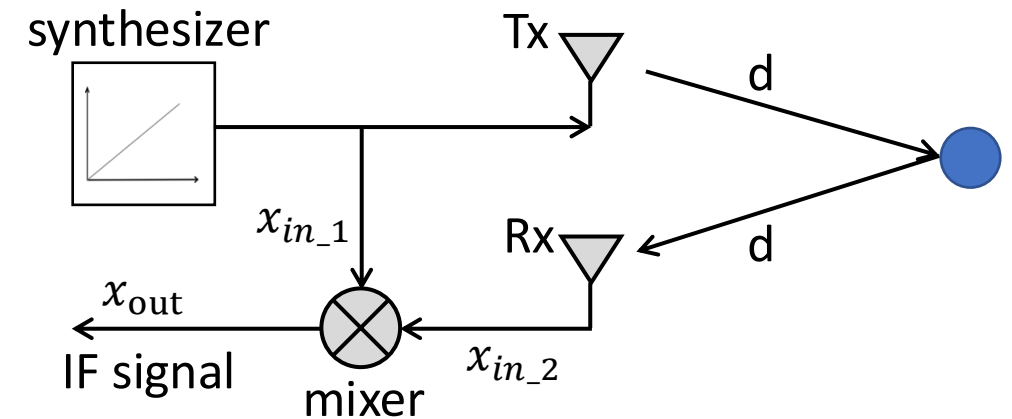
- A Frequency Modulated Continuous Wave radar using chirp signals

- Chirp: sinusoid with  $f$  increasing linearly with time

- Amplitude vs. time plot
- Frequency vs. time plot
  - A slope  $S=B/T_c$  defines the rate at which the chirp ramps up
  - $f_c$ : carrier frequency (mmWave frequency band)
- Recall CSS in LoRa communication

# 1Tx-1Rx FMCW Radar

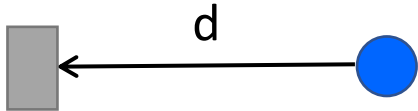
- IF (Intermediate Frequency) signal
  - Rx signal and Tx signal are “mixed” together as the IF signal



- Mixer: 3-port device with 2 inputs & 1 output
  - $x_{in\_1} = \sin(2\pi f_1 t + \phi_1)$
  - $x_{in\_2} = \sin(2\pi f_2 t + \phi_2)$
  - $x_{out} = \sin[2\pi(f_1 - f_2)t + (\phi_1 - \phi_2)]$
  - Eliminate the carrier frequency  $f_c$



# FMCW Radar Sensing



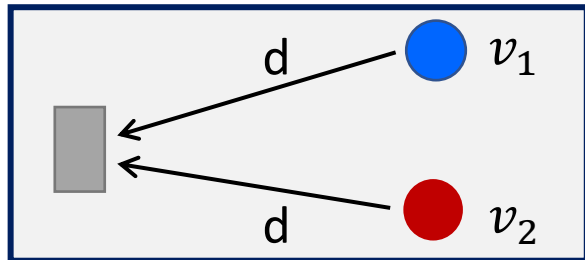
How to estimate the range of an object?

What if there are multiple objects?

How close can two objects be while still being separatable?

How to measure the velocity of the objects?

How to find the direction of the objects?



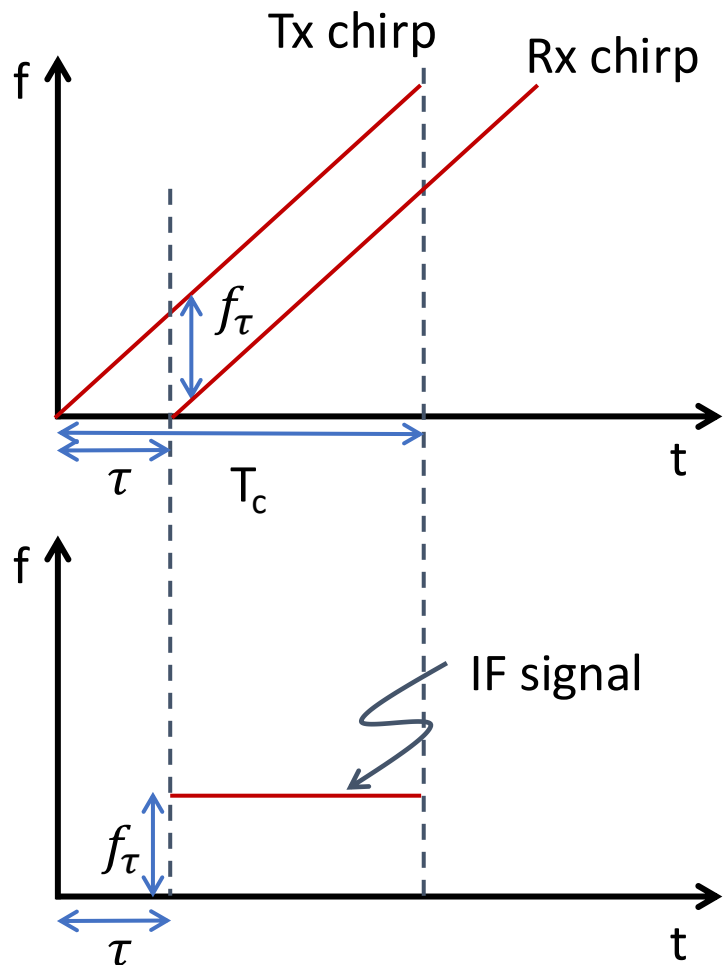
- Range
- Doppler
- Angle (Angle of Arrival, AoA)

Can we also monitor periodic motions such as vibrations, heartbeats?

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# IF Signal – Single Object



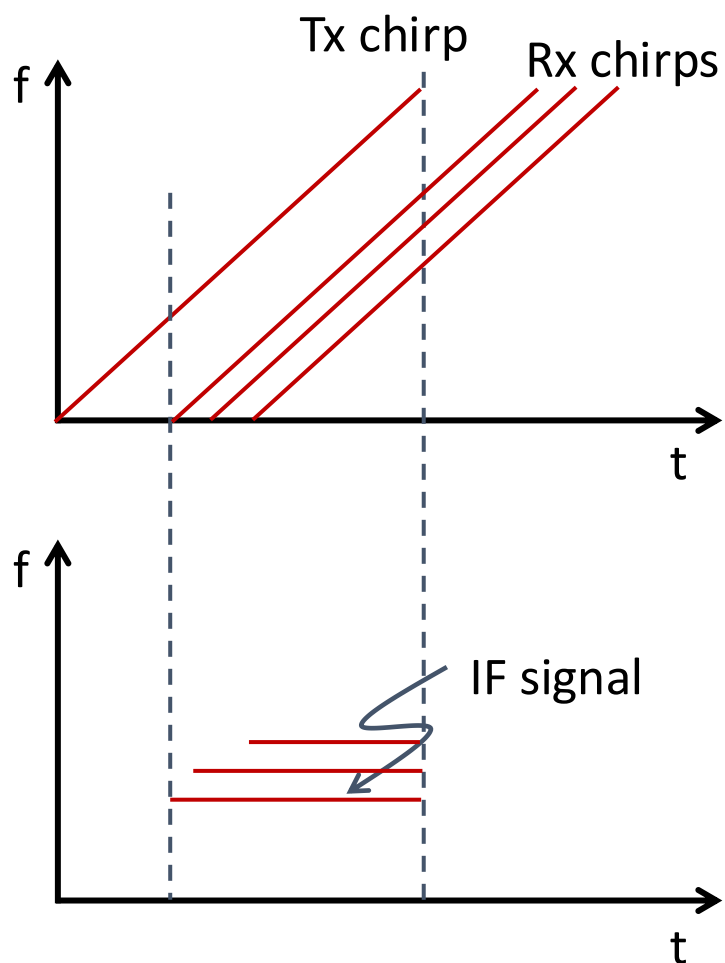
- Rx signal is a delayed version of the Tx signal, with delay  $\tau$ .
- A single object in front of the radar produces an IF signal with a constant frequency of  $f_\tau = S\tau$

$\tau = \frac{2d}{c}$ : round-trip time (RTT) between the radar and the object

$$f_\tau = S\tau = \frac{2dS}{c} \rightarrow d = \frac{cf_\tau}{2S}$$

- $\tau$  (and thus the non-overlapping part of Tx and Rx signals) is typically very small and negligible compared to  $T_c$ .
- E.g., for a max  $d$  of 100m and  $T_c = 40\mu s$ ,  $\frac{\tau}{T_c} = \frac{1}{60} = 1.7\%$ .

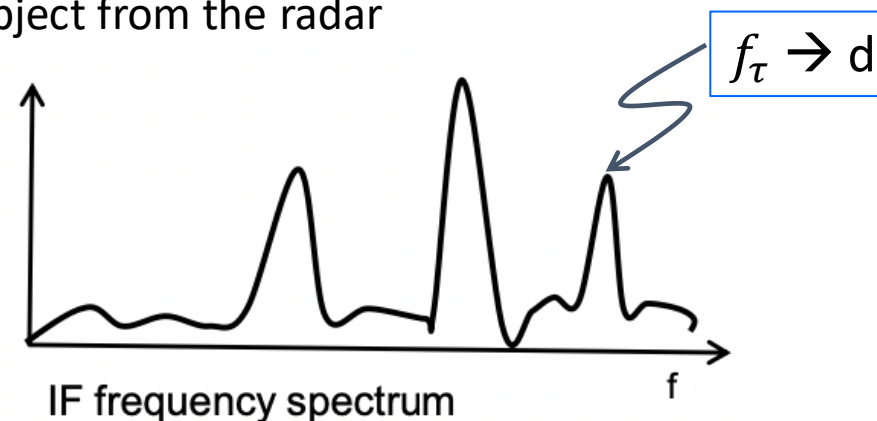
# Multiple objects



- Multiple objects  $\rightarrow$  multiple reflected copies of chirps

A frequency spectrum of the IF signal will reveal multiple tones, the frequency of each being proportional to the range of each object from the radar

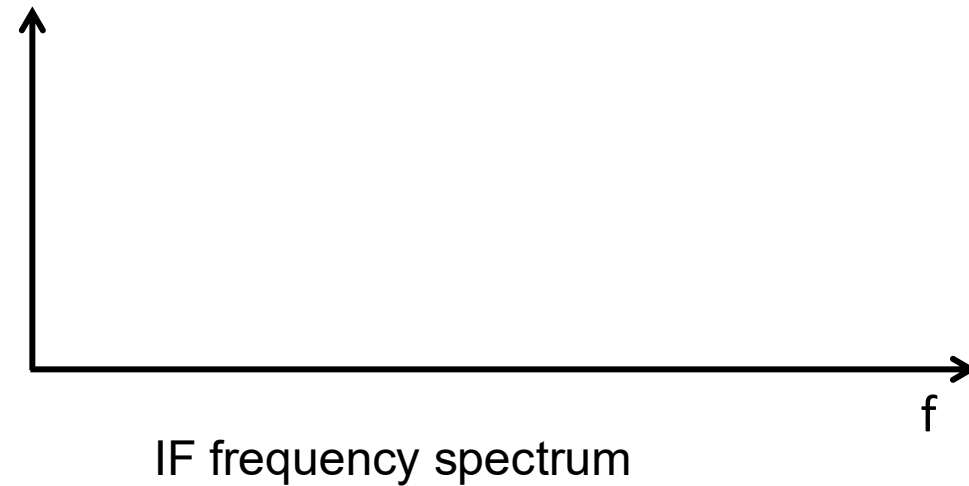
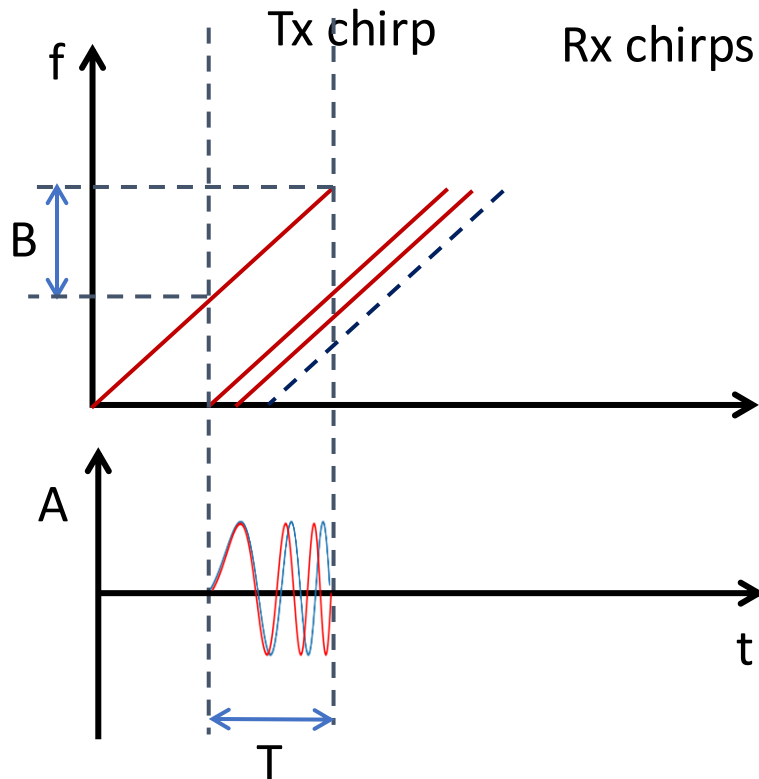
Range-FFT



- If two objects are very close to each other, can they be separated?
- What's the minimum separation of two objects that can be resolved?

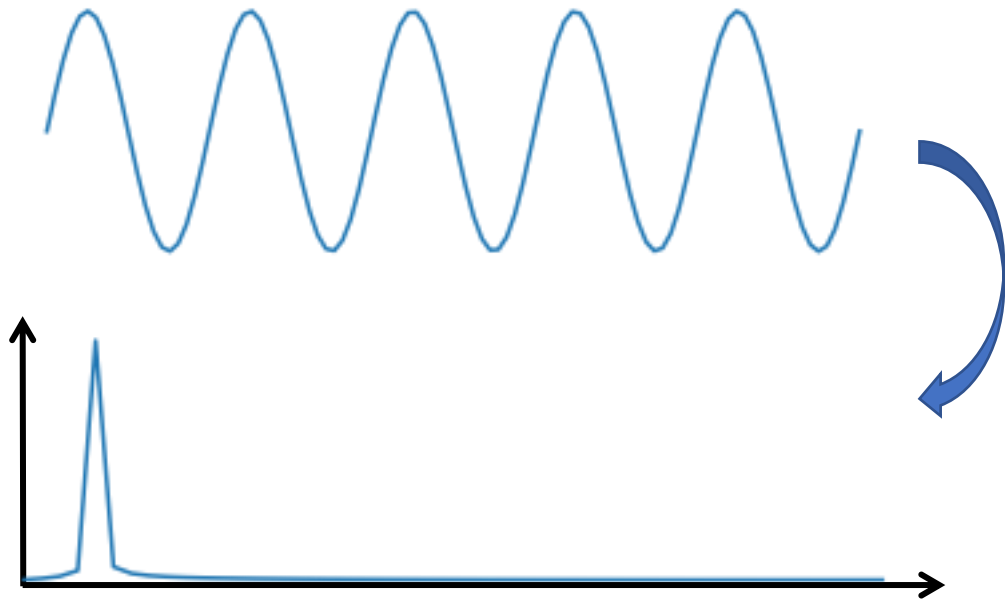
# Range Resolution

- The ability to resolve/separate two closely spaced (in range dimension) objects



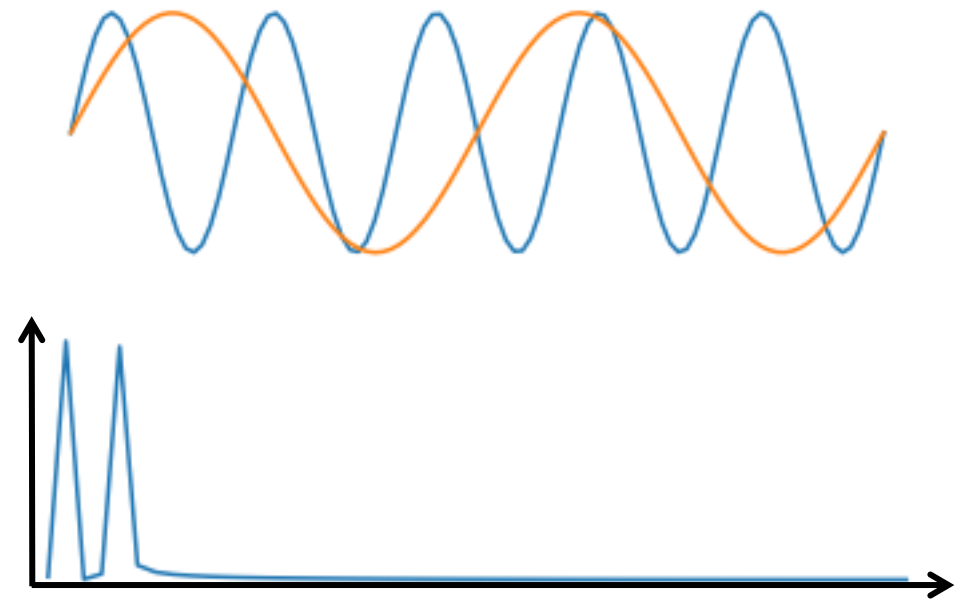
# Fourier Transforms

One single frequency



Two sufficiently different frequencies

FFT

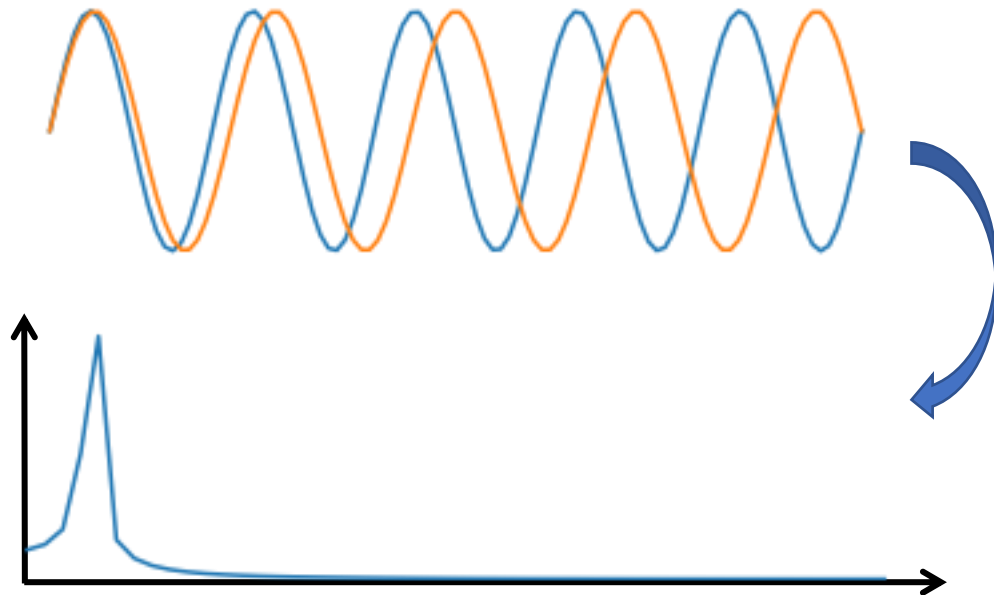


A visual intro to FFT: <https://www.youtube.com/watch?v=spUNpyF58BY>

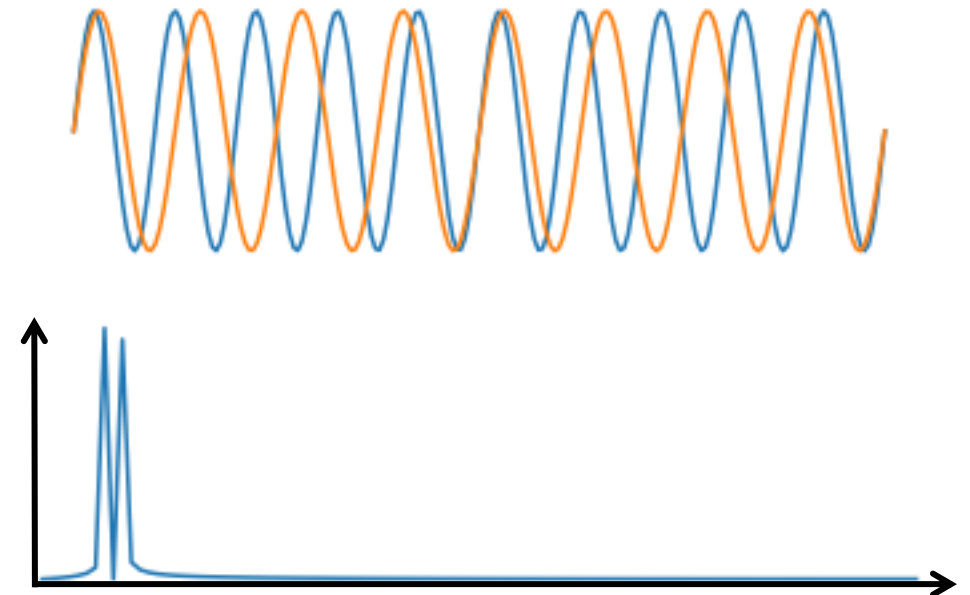


# Fourier Transforms

Two close frequencies



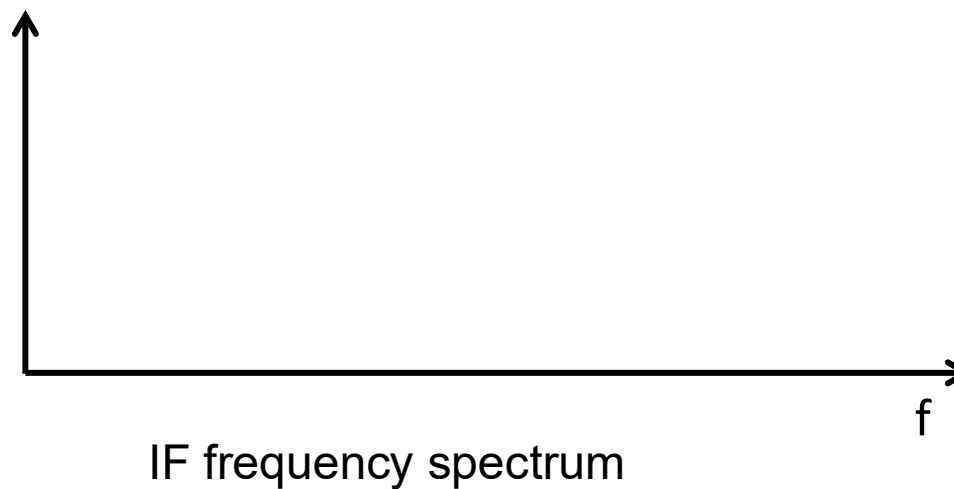
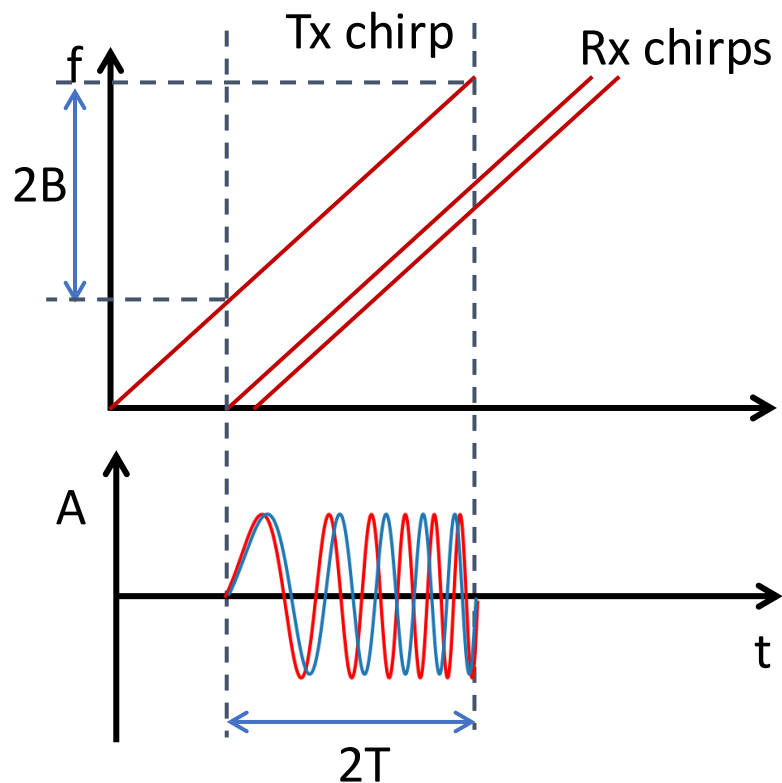
Two close frequencies with a longer observation window



Longer observation period  $\rightarrow$  higher frequency resolution.  
In general, frequency resolution =  $1/T$  for an observation window of  $T$ .

# Range resolution vs. Bandwidth

- Range resolution improves with increased observation period  $T$ , i.e., the length of the IF signal
- Proportionally increases the bandwidth
- Thus: Larger bandwidth  $\rightarrow$  Better resolution



# Range resolution vs. Bandwidth

- Recall

- An object at a distance  $d$  results in an IF tone of frequency  $f_t = S2d/c$
- Two tones can be resolved in frequency if  $\Delta f > \frac{1}{T}$

- Range resolution of a radar?

Two objects at  $d_1$  and  $d_2$  with a distance difference  $\Delta d$ , their IF frequency difference is  $\Delta f = \frac{S2\Delta d}{c}$ . The observation period is  $T_c$ , and thus  $B = ST_c$ .

To resolve the two frequencies/objects, we need to have

$$\Delta f > \frac{1}{T_c} \Rightarrow ?$$

The Range Resolution depends on only the bandwidth:

$$d_{res} = \frac{c}{2B}$$


# Range resolution vs. Bandwidth

- Typical values for Bandwidth and Range resolution

Bandwidth	Range Resolution	
4GHz	3.75 cm	TI FMCW Radar 3.56 GHz
1GHz	15 cm	
600 MHz	25 cm	
160 MHz	0.9375 m	WiFi 802.11ax / WiFi 6
80 MHz	1.875 m	WiFi 5GHz
40 MHz	3.75 m	WiFi 5GHz
20 MHz	7.5 m	WiFi 2.4GHz

The Range Resolution depends on only the bandwidth:

$$d_{res} = \frac{c}{2B}$$

- 
1. What is the range resolution between separated Tx and Rx?
  2. What about non-RF signals, e.g., acoustic signals?

# Ranging summary

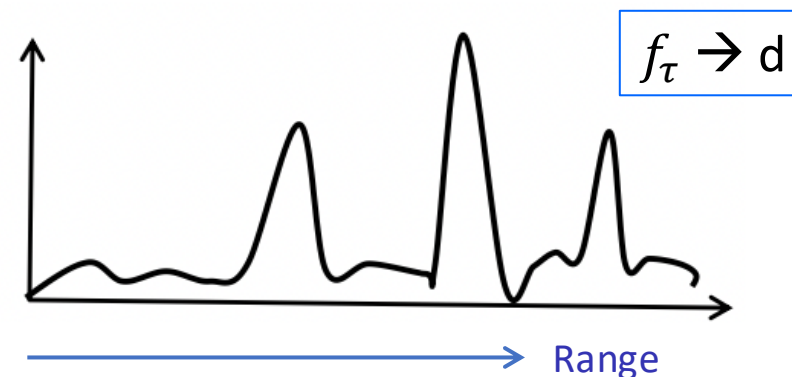
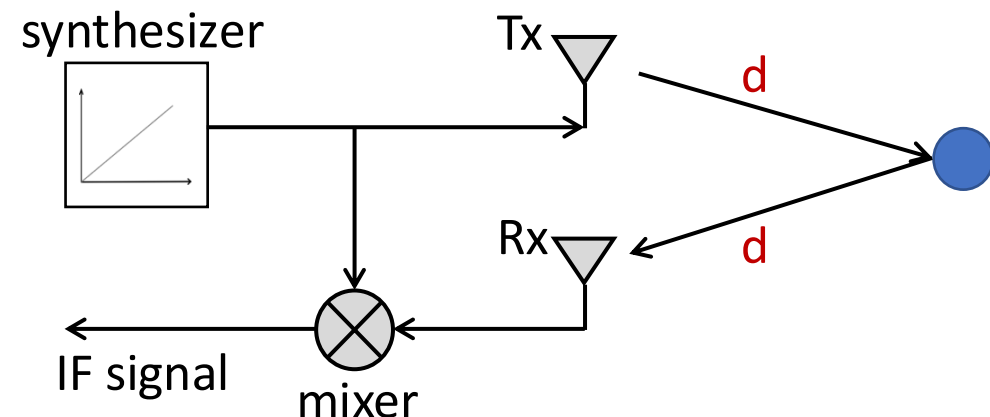
- IF signal consists of one or multiple tones.
- The frequency ( $f$ ) of each tone is proportional to the distance ( $d$ ) of the corresponding object

$$f_{\tau} = S2d/c$$

- Range resolution:

$$d_{res} = c/2B$$

- Range-FFT can be performed on the IF signal to estimate range.
  - Measure the Frequency, not the Time!



Channel Impulse Response  
Power Delay Profile (Amplitude only)

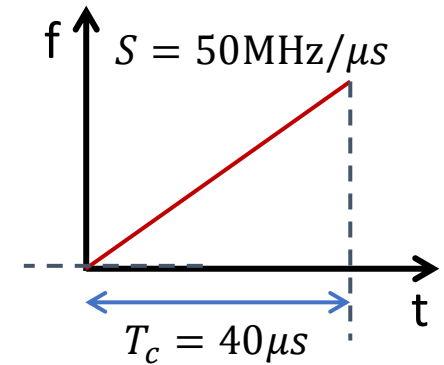
# Quiz



Quiz Time

Let's have  
some fun!

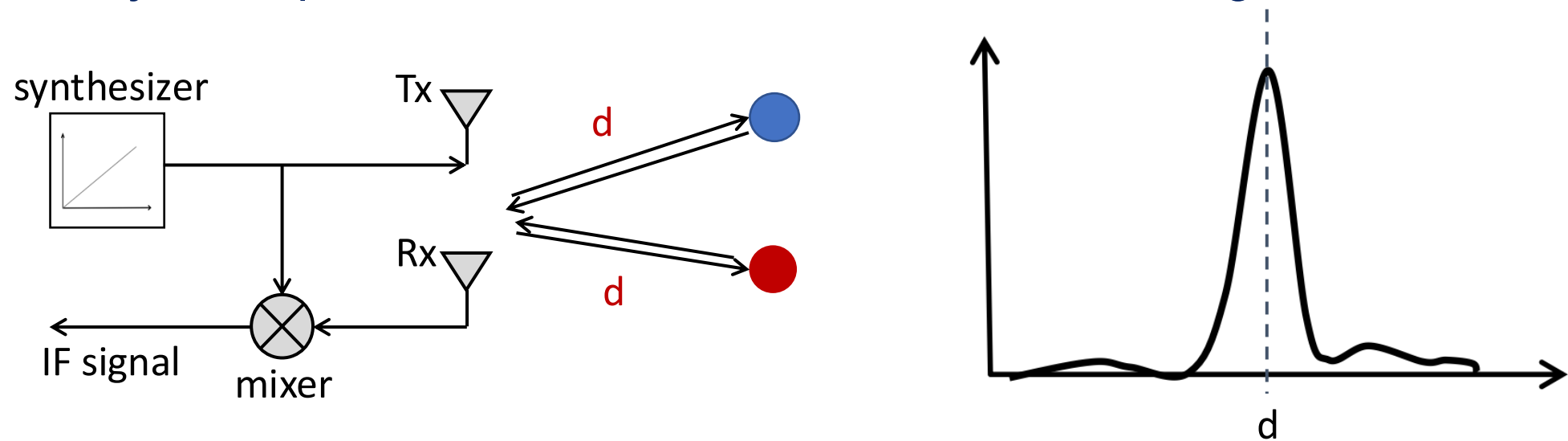
- Given the FMCW radar signal in the diagram
- (1) Assume it works on the 60GHz frequency band, what is the wavelength of the radar signal?
- (2) What is the range resolution of this radar?
- (3) Suppose we observe a frequency of 1MHz in the IF signal, what is the object's range?





# Ranging summary

- Two objects equidistant from the radar. How will the range-FFT look like?



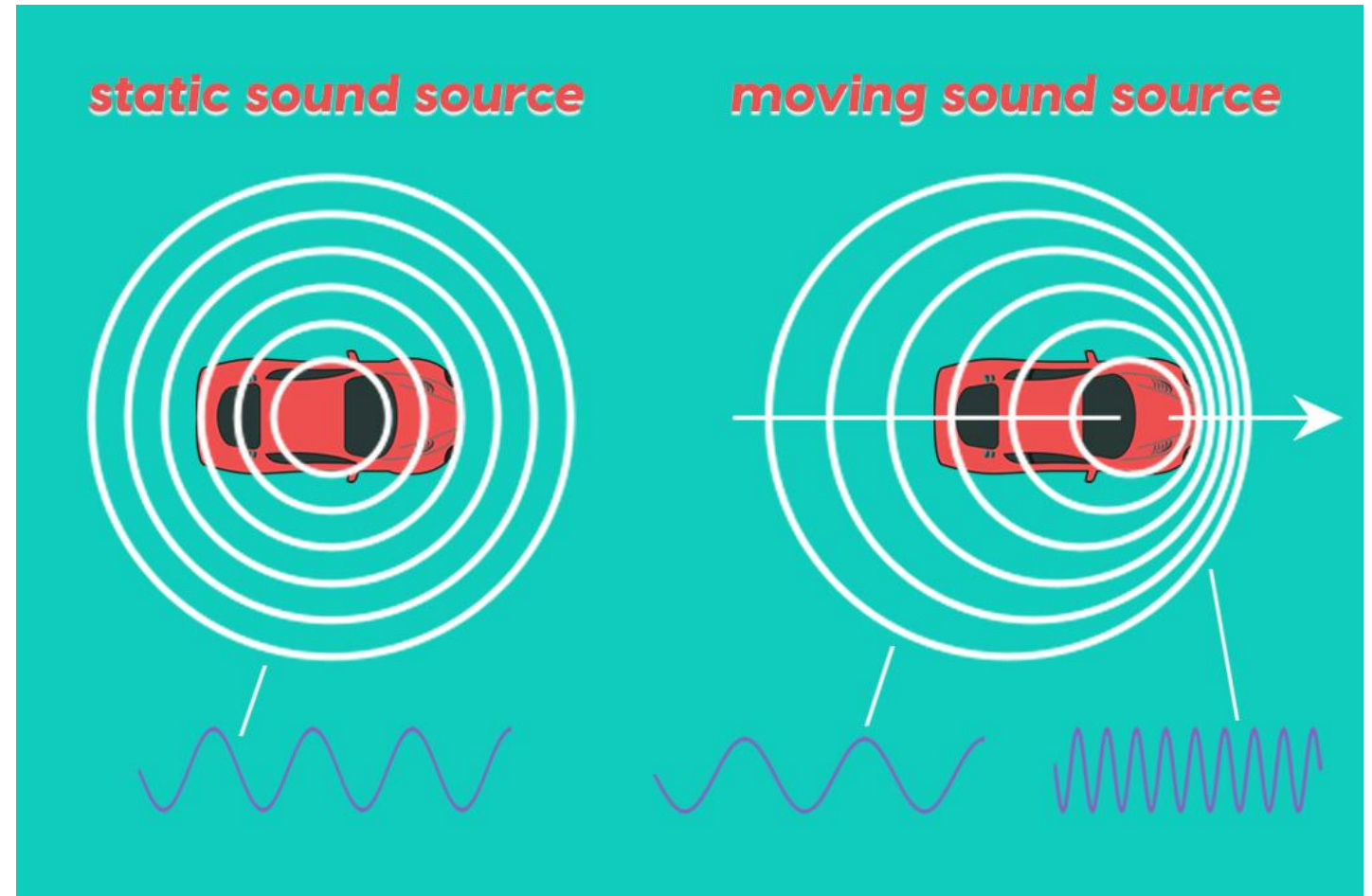
- How do we separate these two objects?
  - Doppler-FFT

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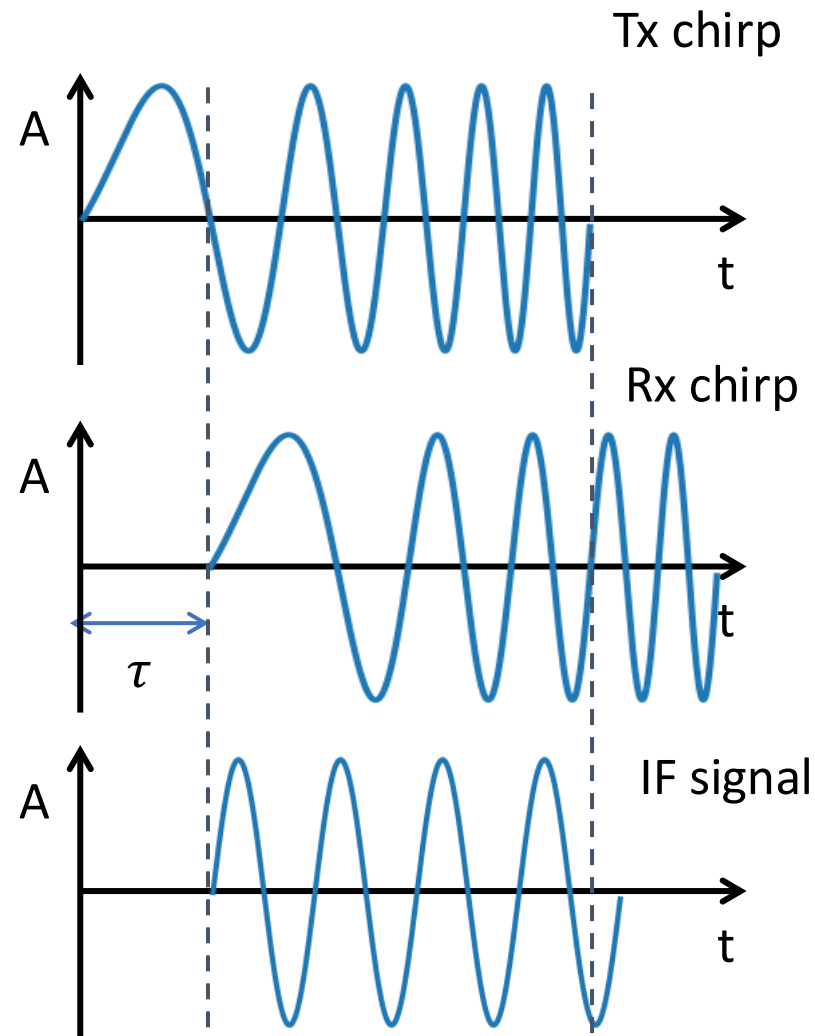
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# Doppler Effect

- The change in frequency of a wave in relation to an observer who is moving relative to the wave source



# The Phase of IF signal

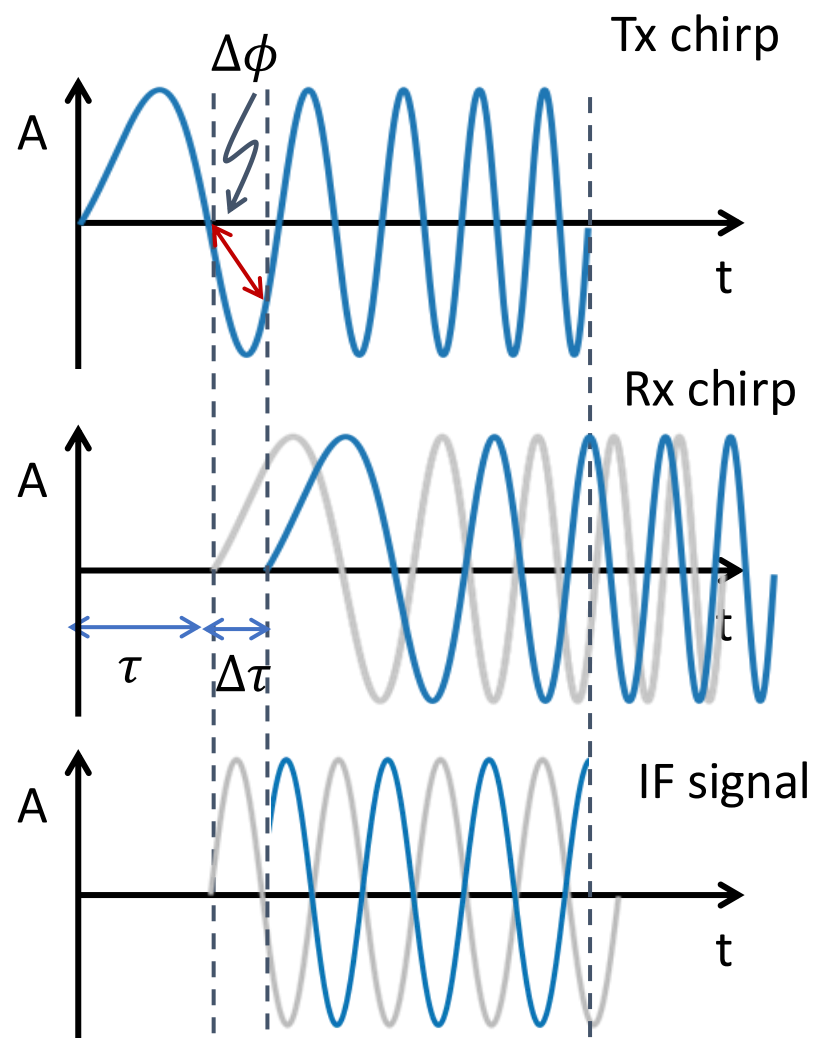


- Assume one object at a distance  $d_0$ , the resulted IF signal:

$$A \sin(2\pi f_0 t + \phi_0)$$

- Frequency:  $f_0 = \frac{S2d_0}{c}$
- Phase  $\phi_0 = -2\pi f \frac{2d_0}{c} = -4\pi \frac{d_0}{\lambda}$ : the difference of the initial phases of the Tx and Rx chirps (Rx chirp delayed by  $\phi_0$ )

# The Phase of IF signal



- What happens if the RTT delay changes a bit by  $\Delta\tau$  (the object moves for a short distance  $\Delta d$ )? The phase of Rx chirps changed by

$$\Delta\phi = 2\pi f_c \Delta\tau = \frac{4\pi\Delta d}{\lambda}$$

- The IF signal experiences the same amount of phase changes

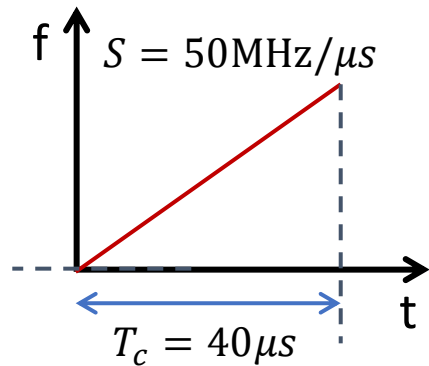
$$A \sin(2\pi f t + \phi_0 + \Delta\phi)$$

Equal to the phase of the peak in FFT response



# Phase changes: An example

- Assuming a center frequency of 77GHz ( $\lambda \approx 4$  mm), what is the phase change if an object moves by 1mm ( $\approx \lambda/4$ )?



$$\Delta\phi = \frac{4\pi\Delta d}{\lambda} = \pi$$

Huge!

- The frequency of the IF signal changes by

$$\Delta f = \frac{S2\Delta d}{c} = 333 \text{ Hz}$$

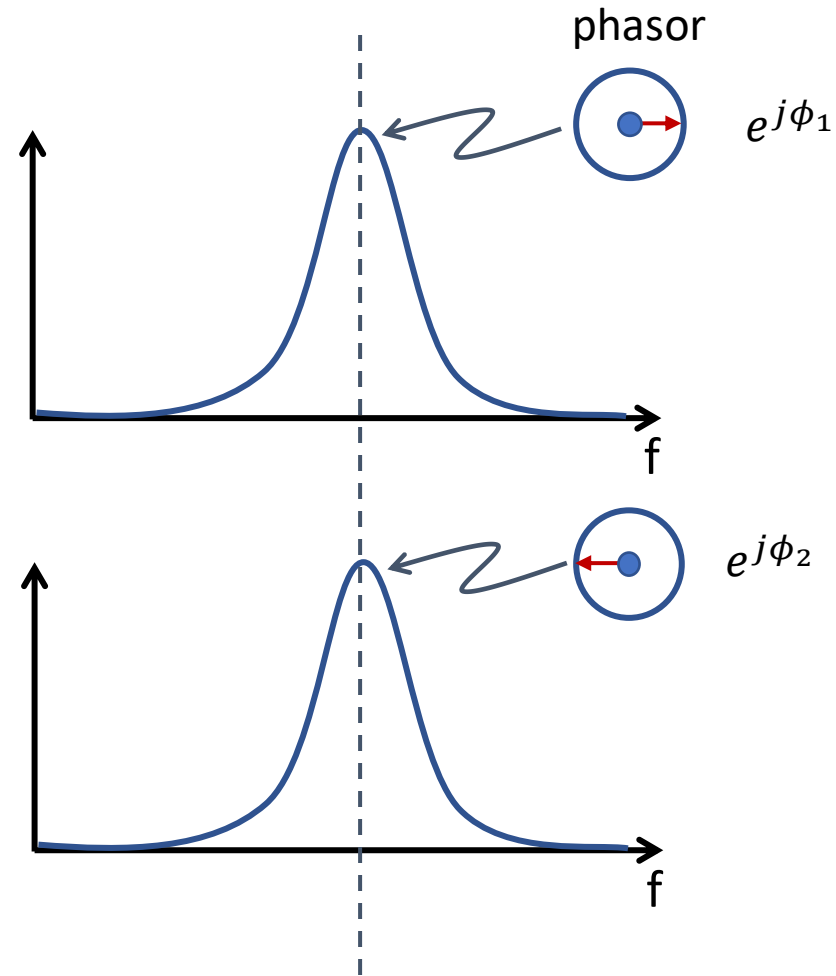
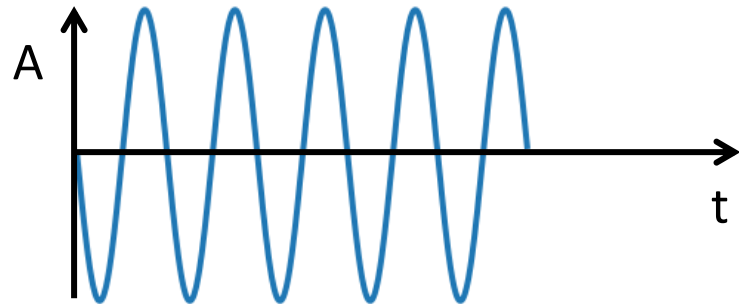
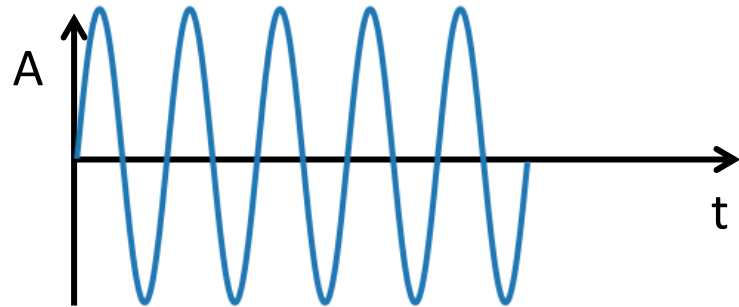
Small!



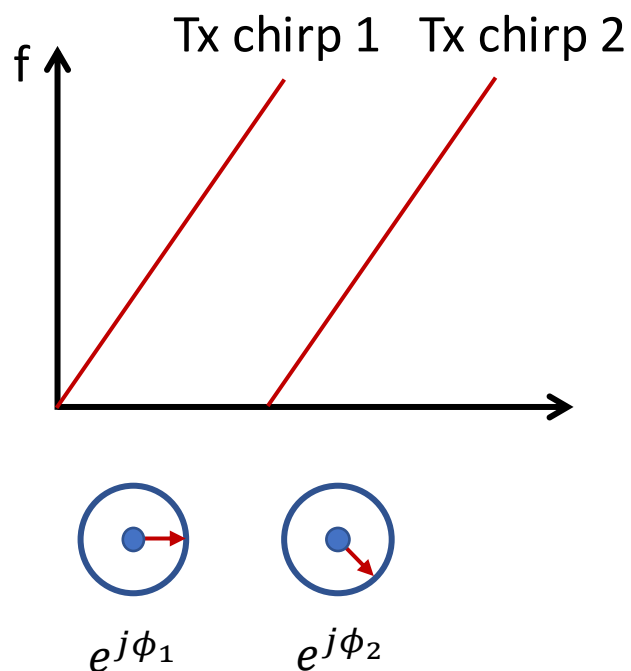
The phase of the IF signal is extremely sensitive to distance/range changes of an object.



# Phase changes due to small motion



# Measure velocity from phase changes



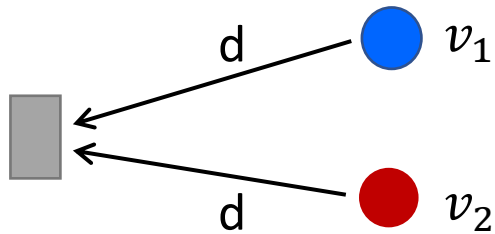
- Transmit continuous chirps separated by  $T_c$
- Consider one moving object
- Range-FFT for each chirp produces the same peak, but with different phase
- The phase difference  $\Delta\phi$  implies the moving speed

$$\Delta\phi = \frac{4\pi\Delta d}{\lambda} = \frac{4\pi v T_c}{\lambda}$$



$$v = \frac{\lambda\Delta\phi}{4\pi T_c}$$

# Doppler-FFT



- Doppler-FFT: FFT on the sequence of phasors corresponding to the range-FFT peaks
- Resolve two objects at the same  $d$  but moving at different speeds

$$e^{jk\omega_1}, \omega_1 = 4\pi v T_c / \lambda$$

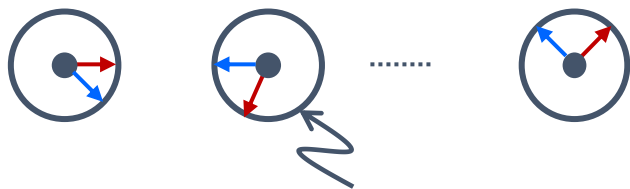
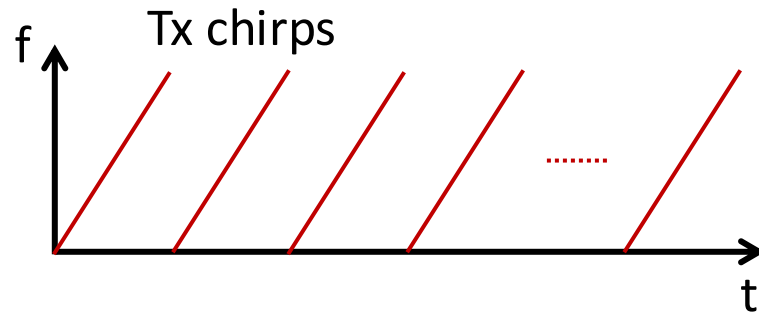
$$\phi_1[0] = \frac{4\pi v T_c}{\lambda} \cdot 1$$

$$\phi_1[1] = \frac{4\pi v T_c}{\lambda} \cdot 2$$

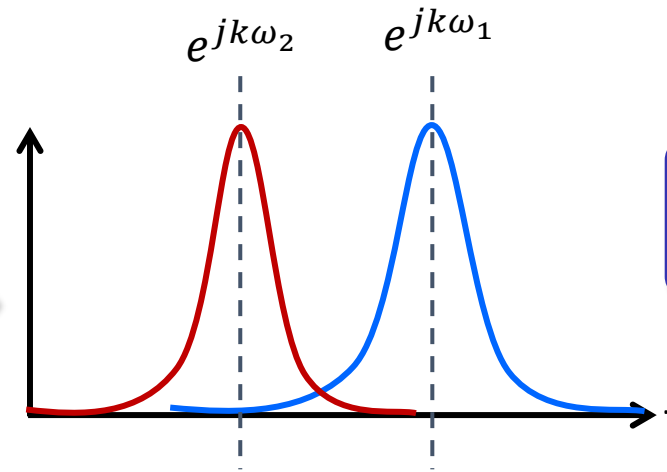
$$\phi_1[2] = \frac{4\pi v T_c}{\lambda} \cdot 3$$

...

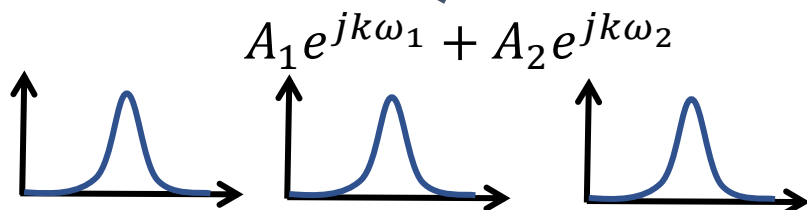
$$\phi_1[N-1] = \frac{4\pi v T_c}{\lambda} \cdot N$$



Doppler-FFT



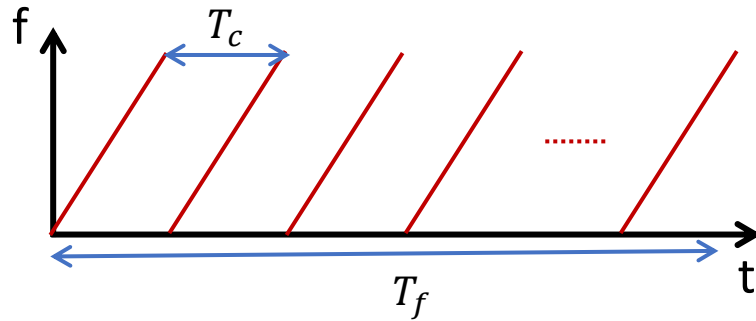
$$v_1 = \frac{\lambda \omega_1}{4\pi T_c}, v_2 = \frac{\lambda \omega_2}{4\pi T_c}$$



Can we still directly calculate phase differences of the IF signal to calculate the speeds?



# Velocity resolution



- FFT Resolution: FFT on  $N$  discrete observations can separate two radian frequencies  $\omega_1$  and  $\omega_2$  if  $|\omega_1 - \omega_2| > 2\pi/N$
- $T_c$ : Chirp duration
- $T_f$ : Frame duration ( $N$  consecutive chirps)

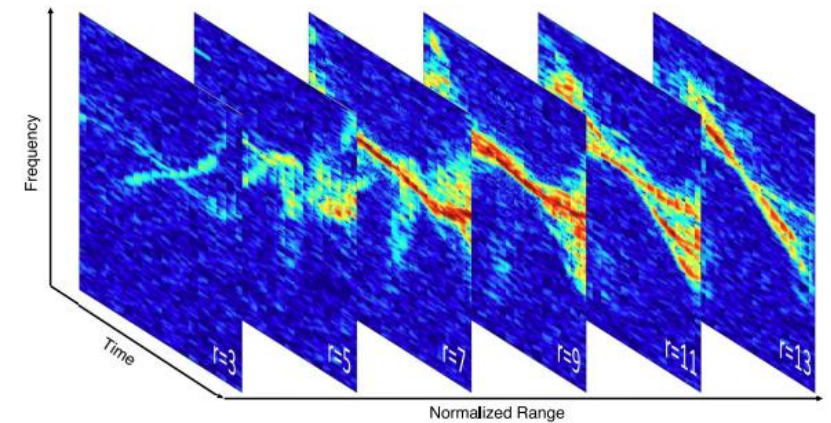
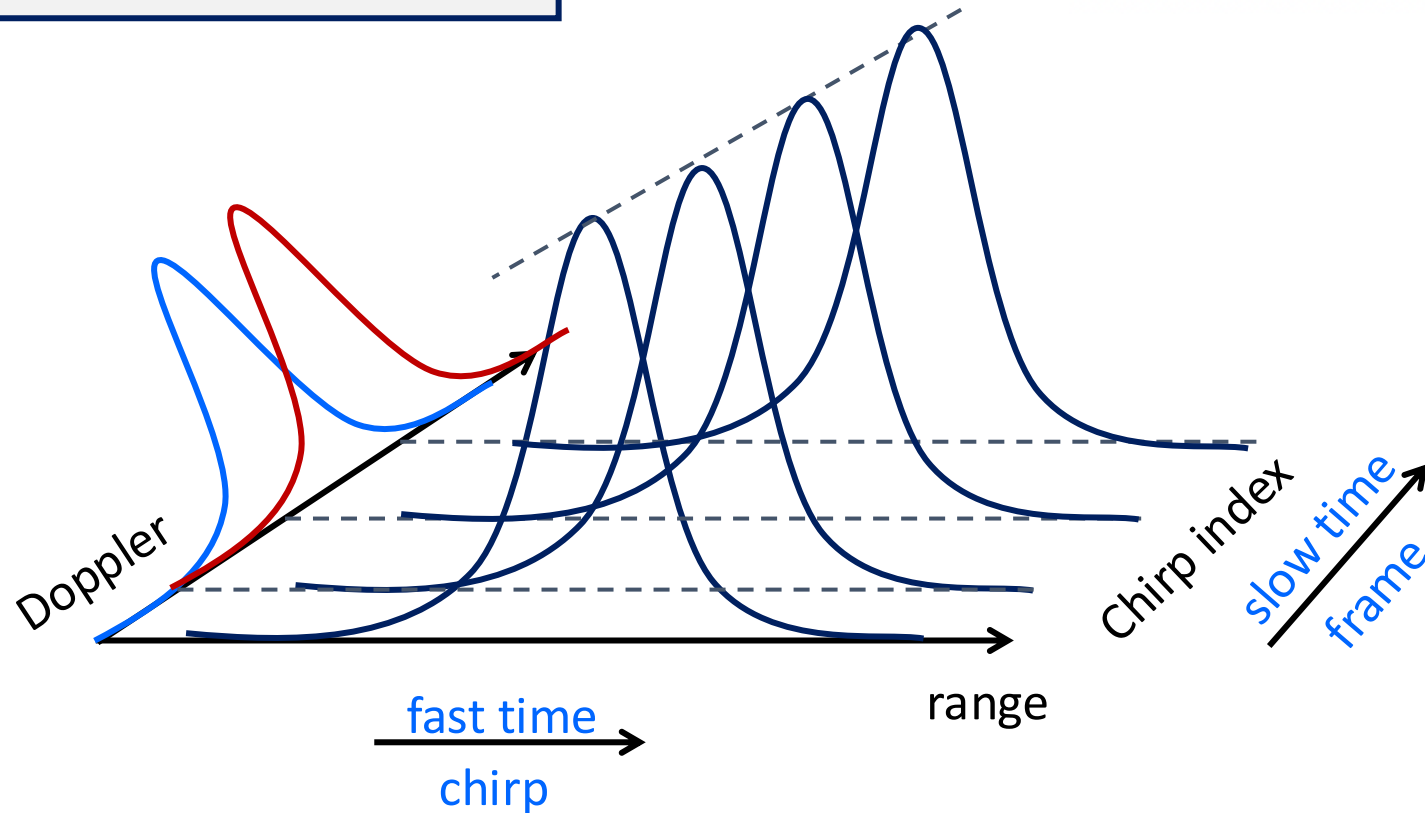
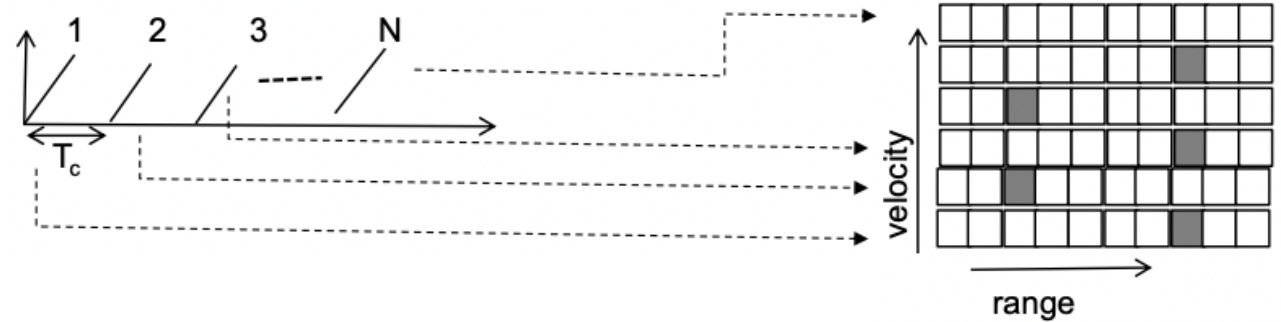
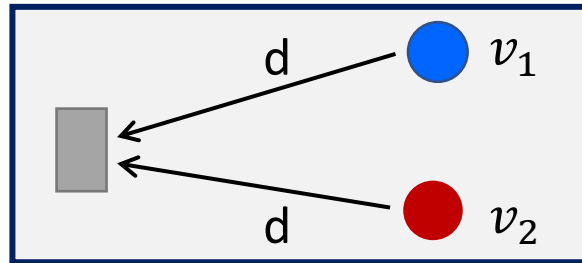
- Velocity resolution of Doppler-FFT?

- Minimum separation between  $v_1$  and  $v_2$  so that they show up as two distinguishable peaks in Doppler-FFT

$$\Delta\omega = \frac{4\pi\Delta v T_c}{\lambda} > \frac{2\pi}{N}$$
$$\rightarrow \Delta v > \frac{\lambda}{2NT_c}$$

$$v_{res} = \frac{\lambda}{2T_f}$$

# Range-Doppler 2D-FFT



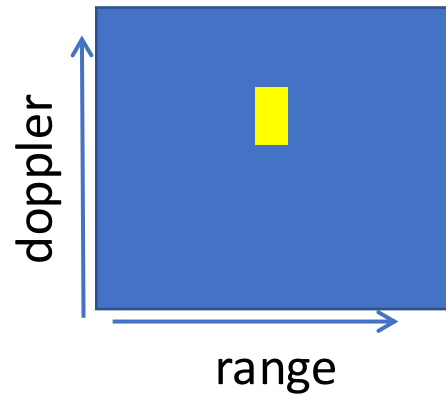
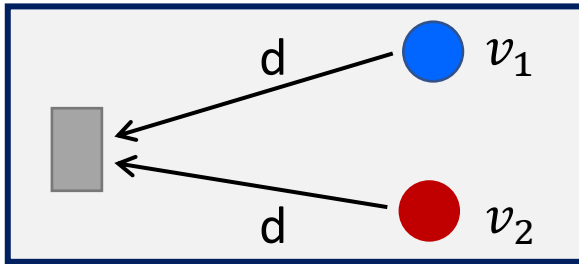
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# Angle Estimation

- What if  $v_1 = v_2$ ?



Single peak in the range-Doppler 2D-FFT plot, as they have the same range and velocity.

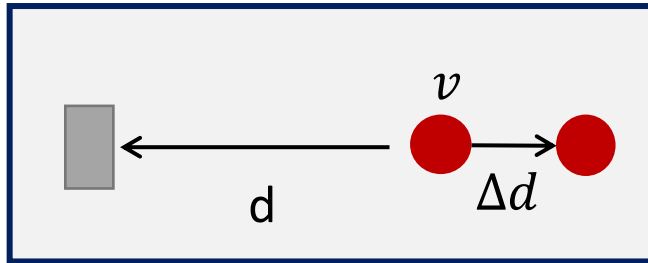
How do we separate the two objects?

# Angle of Arrival (AoA)

- How to estimate the angle of arrival of an object?
- How to estimate AoAs of multiple objects at different angles?
- Angle resolution?
- What is the Field of View (FoV)?

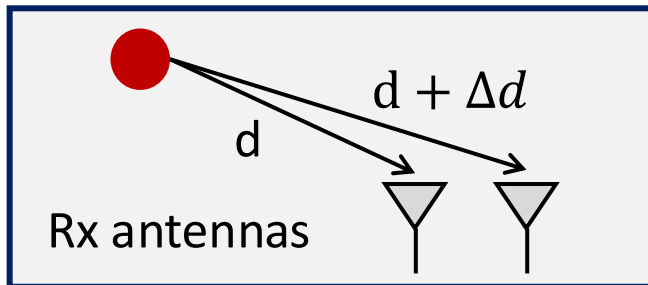
# AoA Estimation

- Recall: A small change  $\Delta d$  in  $d$  results in a phase change  $\Delta\phi$



$$\Delta\phi = 4\pi\Delta d/\lambda$$

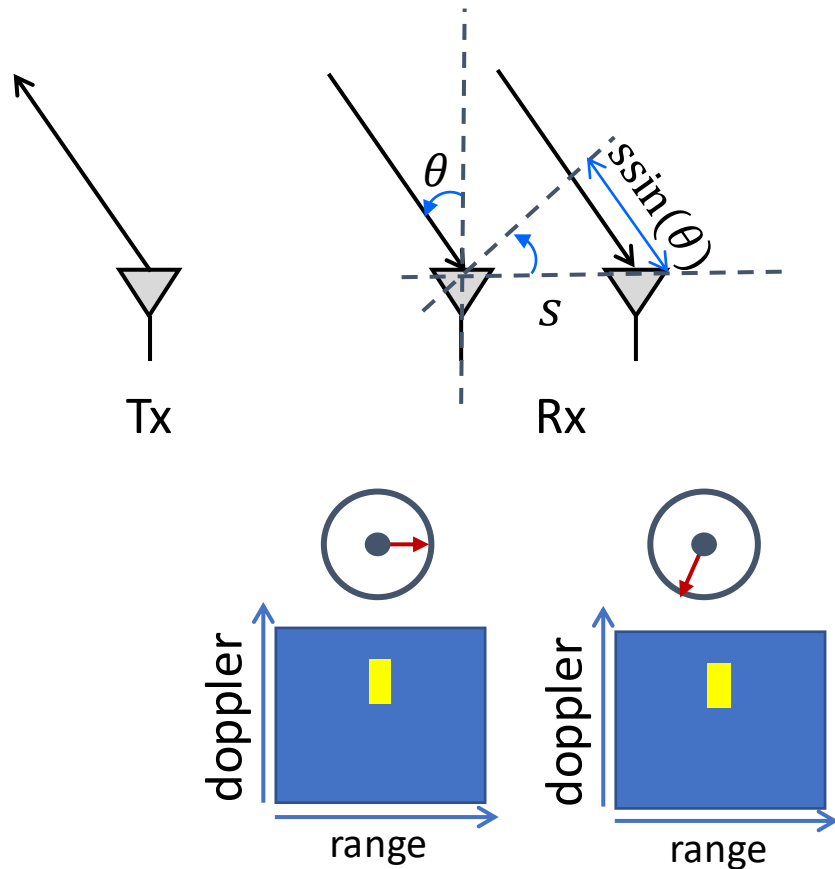
- AoA estimation requires multiple antennas
  - The slight difference in propagation distances from the object to each antennas also lead to phase changes in the 2D-FFT.



$$\Delta\phi = 2\pi\Delta d/\lambda$$

Why  $2\pi$ ? 🤔

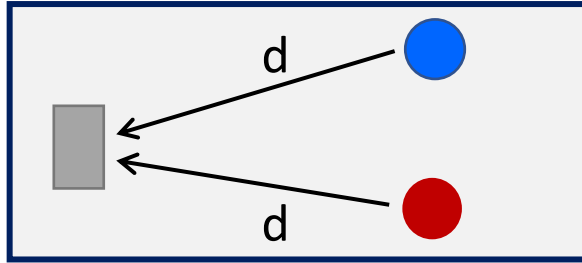
# AoA Estimation



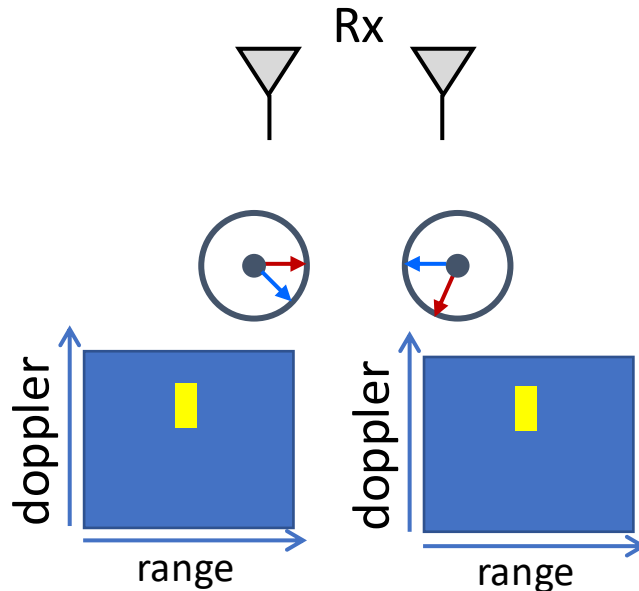
- The measured phase difference can be used to infer AoA

$$\Delta\phi = \frac{2\pi s \sin(\theta)}{\lambda}$$
$$\rightarrow \theta = \sin^{-1}\left(\frac{\lambda \Delta\phi}{2\pi s}\right)$$

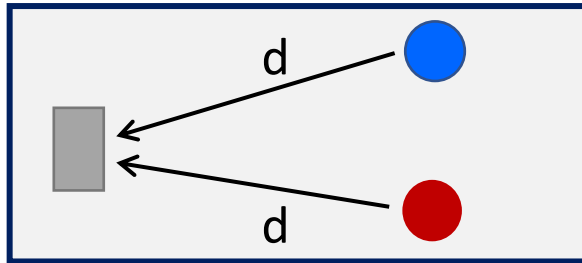
# AoA Estimation



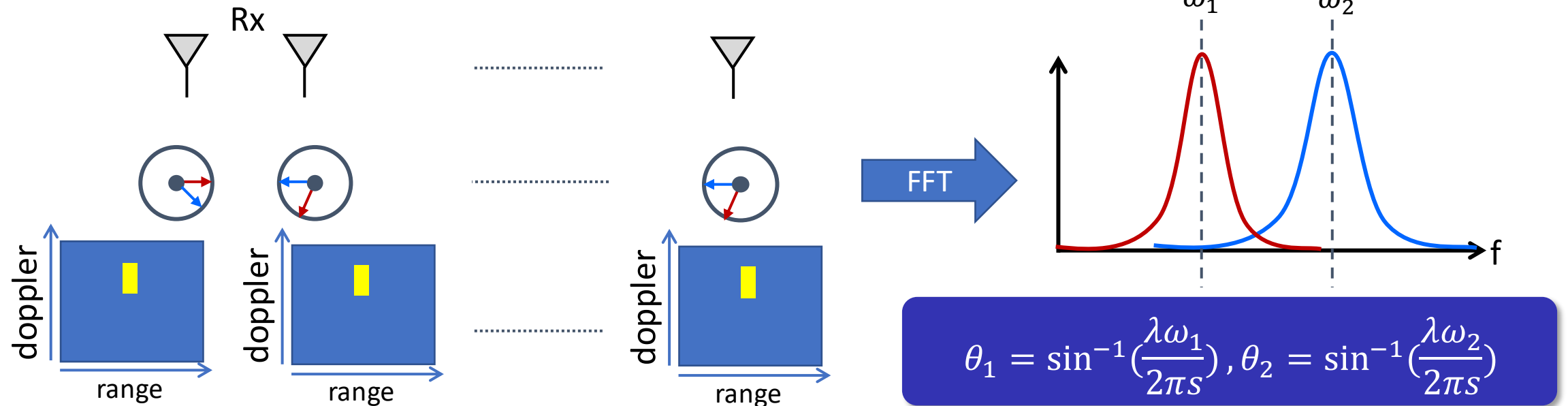
- The method will not work in presence of multiple objects
  - Multiple phase components
- An array of more receive antennas is needed



# AoA Estimation

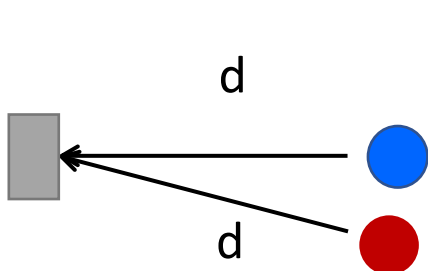


- An array of more receive antennas is needed
- Angle-FFT: FFT on the sequence of phasors corresponding to the Range-Doppler 2D-FFT peaks



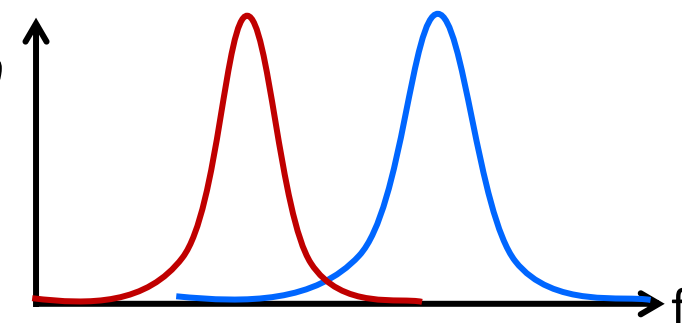
# Angle Resolution

- $\theta_{res}$ : The minimum angle separation that can be differentiated by Angle-FFT.



$$\Delta\omega = \frac{2\pi s}{\lambda} (\sin(\theta + \Delta\theta) - \sin(\theta)) \approx \frac{2\pi s}{\lambda} \cos(\theta) \Delta\theta$$

Recall FFT,  $\Delta\omega > 2\pi/M \leftarrow \# \text{ of samples (antennas)}$   
 $\rightarrow \Delta\theta > \frac{\lambda}{M s \cos(\theta)}$



$$\theta_{res} = \frac{\lambda}{M s \cos(\theta)}$$

Assume  $s = \lambda/2, \theta = 0$

$$\theta_{res} = \frac{2}{M}$$

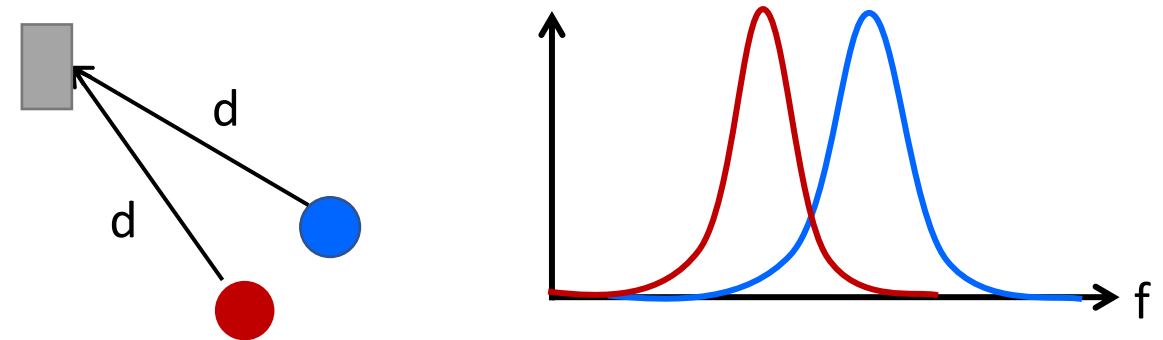
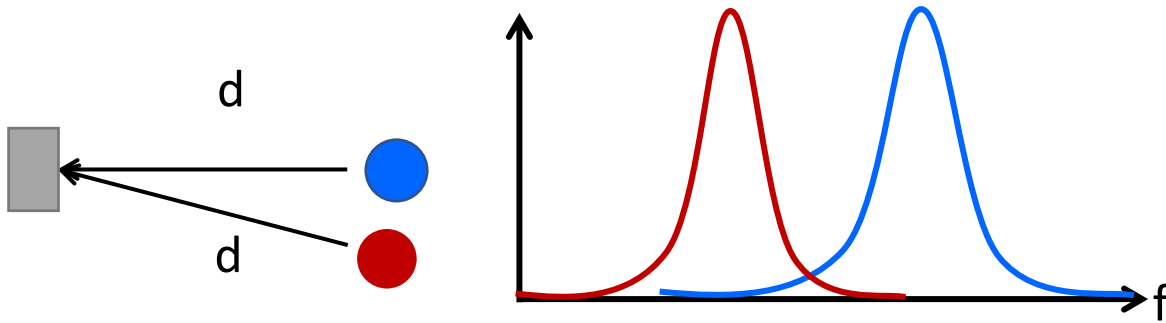
Ms: Aperture

# Angle Resolution

- AoA resolution decreases as AoA increases.

$$\theta_{res} = \frac{\lambda}{M \sin(\theta)}$$

Resolution depends on  $\theta$ . Best at  $\theta = 0$ .





# Beamforming

- Horn antenna
  - Mechanical motor



- Digital Beamforming
  - Classical beamforming
  - Minimum Variance Distortionless Response (MVDR) / Capon
  - MUSIC (MUltiple Signal Classification)

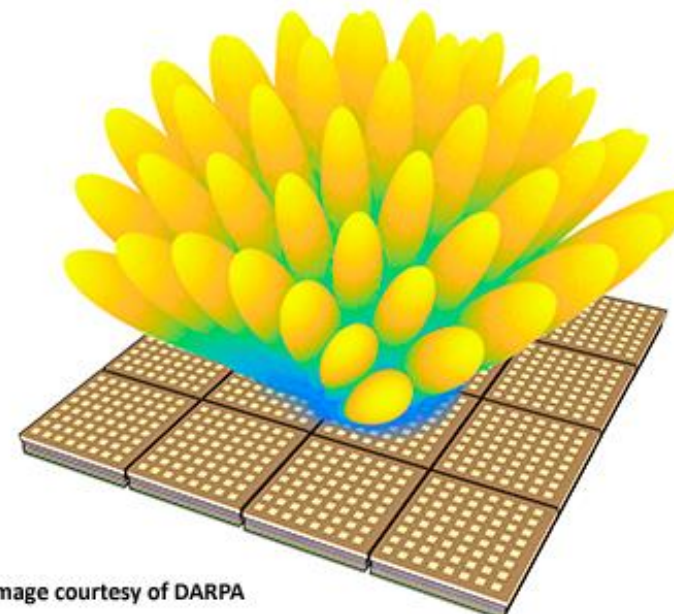


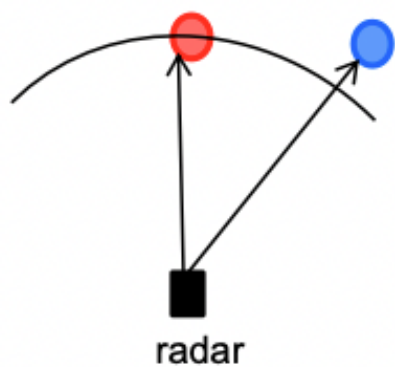
Image courtesy of DARPA

[mmEye: Super-Resolution Millimeter Wave Imaging](#)

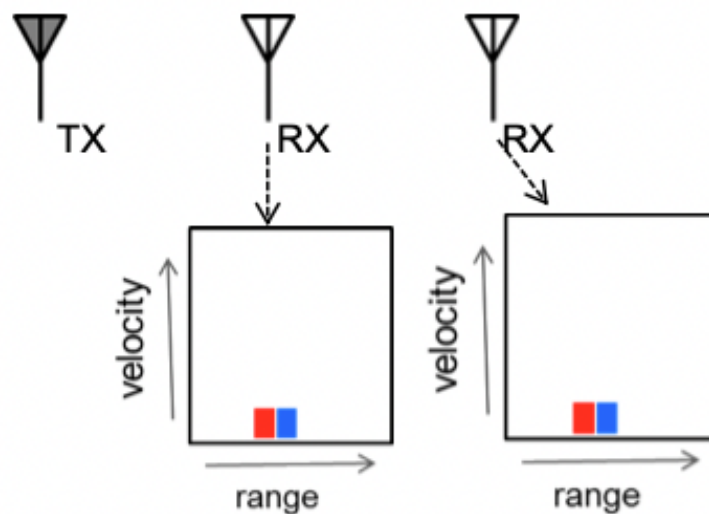
Feng Zhang, Chenshu Wu, Beibei Wang, and K. J. Ray Liu

IEEE Internet of Things Journal, 2021.

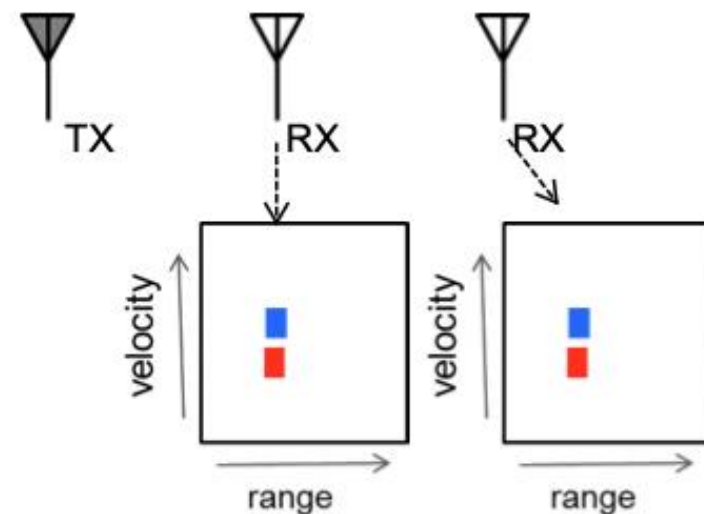
# Radar Resolution



Angle



Range



Doppler

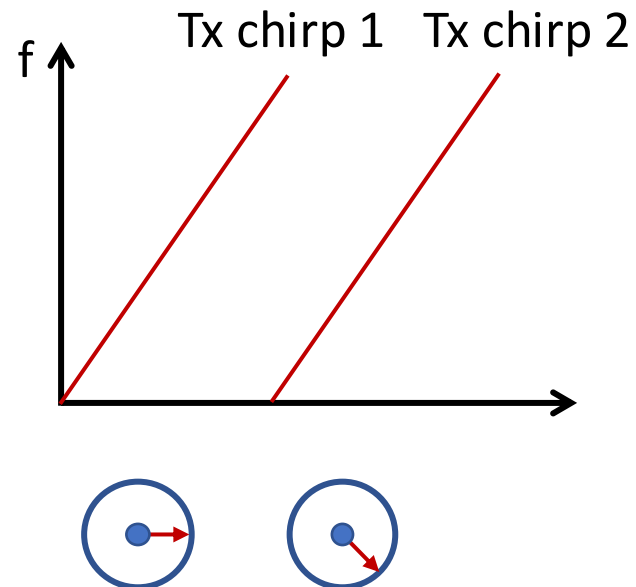
Good range and velocity resolution can allow relatively poor angle/spatial resolution → Fewer antennas, smaller size.

# Contents

- Basics of FMCW Radar Signals
- Range Estimation
- Doppler/Velocity Estimation
- Angle Estimation
- Periodicity Estimation
- FMCW Sensing Applications

# Periodicity

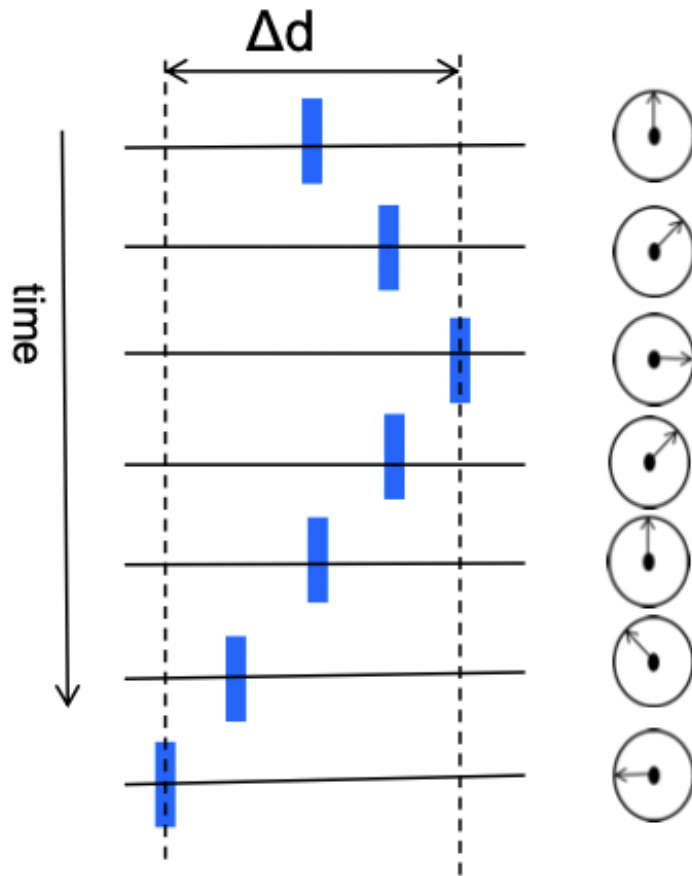
- Measuring the frequency of periodic movements.
- Recall velocity estimation based on phase changes:



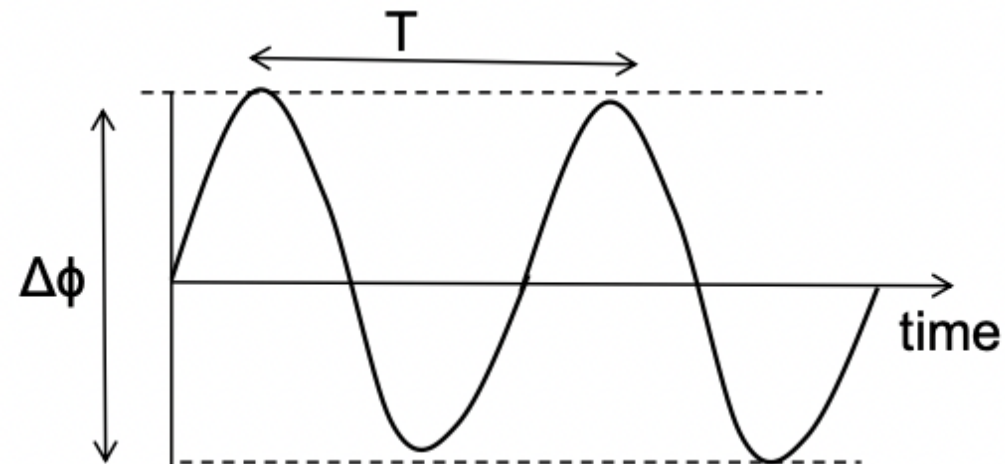
$$\Delta\phi = \frac{4\pi\Delta d}{\lambda} = \frac{4\pi v T_c}{\lambda}$$

What if the movement is periodic?

# Periodicity – Vibration Object



- Vibrating motor/machinery, heartbeat, breathing ...
- Cause small but periodic changes in IF signal phase



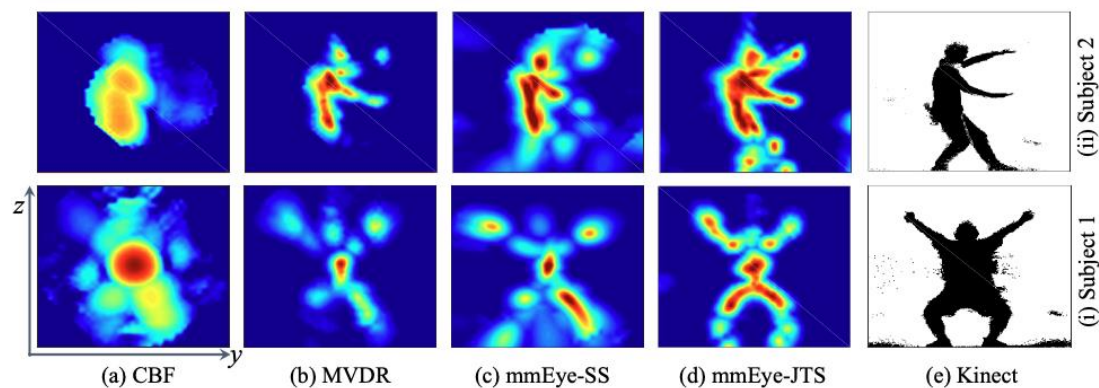
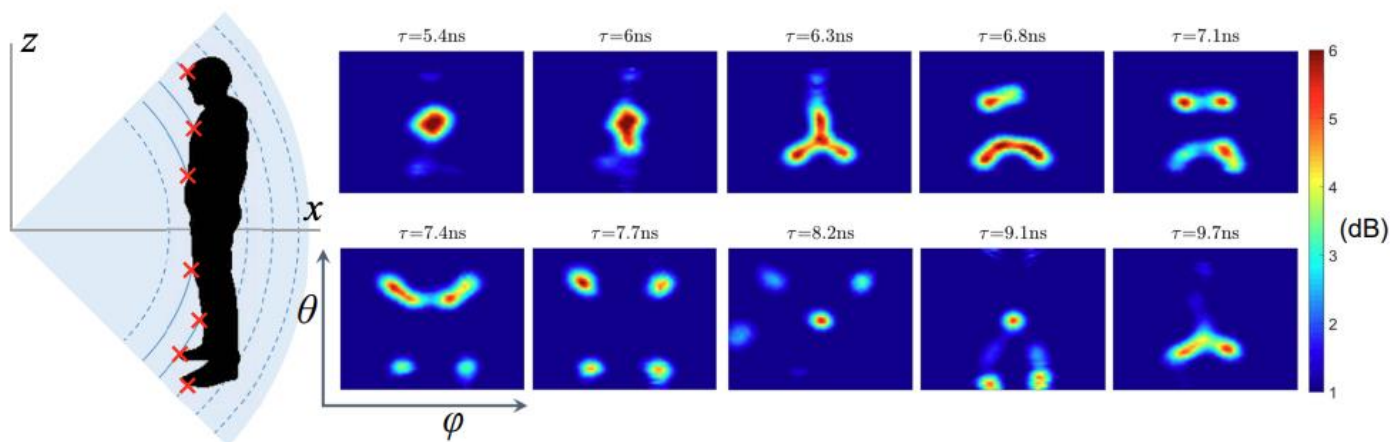
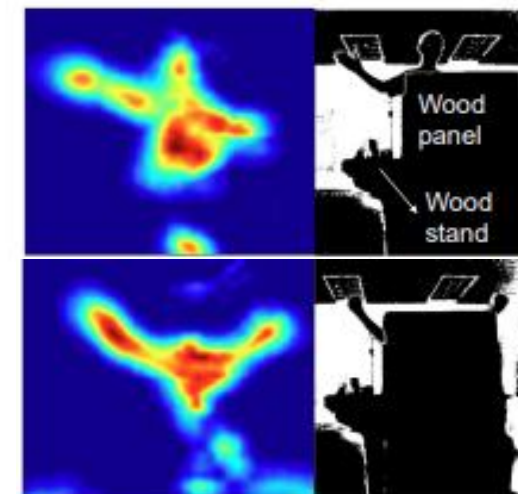
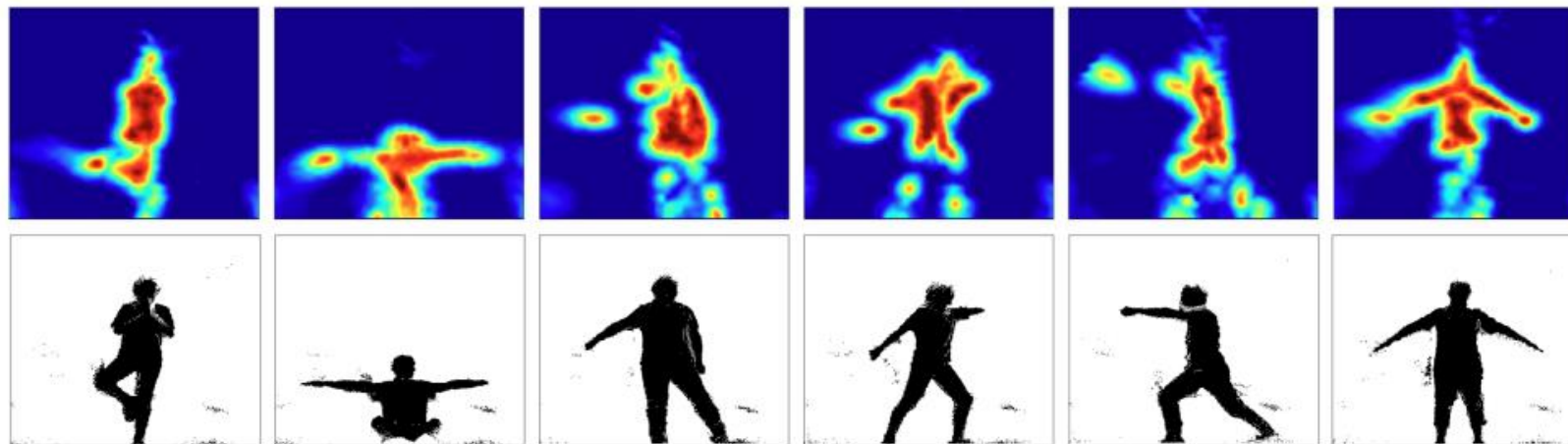
$$\omega = \frac{4\pi\Delta d}{\lambda}$$

Both amplitude and  
periodicity (frequency)

# Contents

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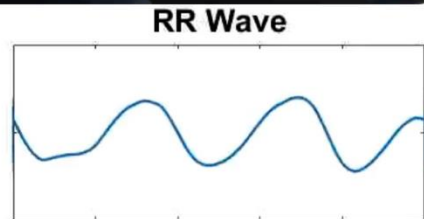
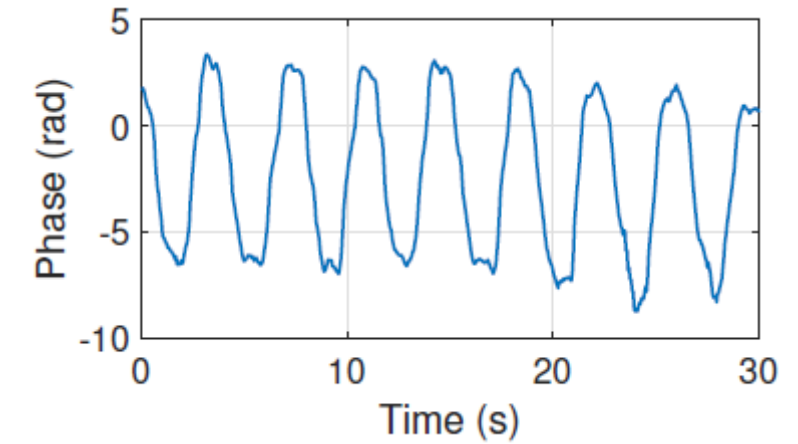
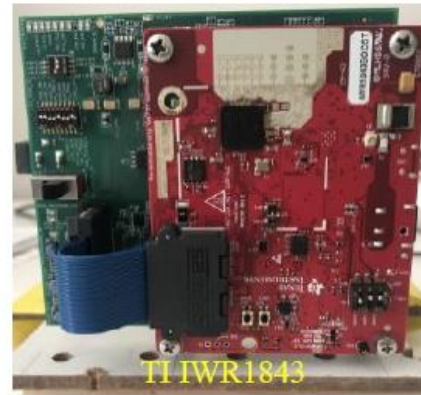
# Imaging



mmEye: Super-Resolution Millimeter Wave Imaging, IEEE IoTJ, 2021



# Vital Sign Monitoring

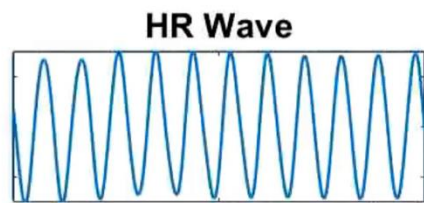


Respiration Rate Truth

16 rpm

Respiration Rate Estimation

16.06 rpm



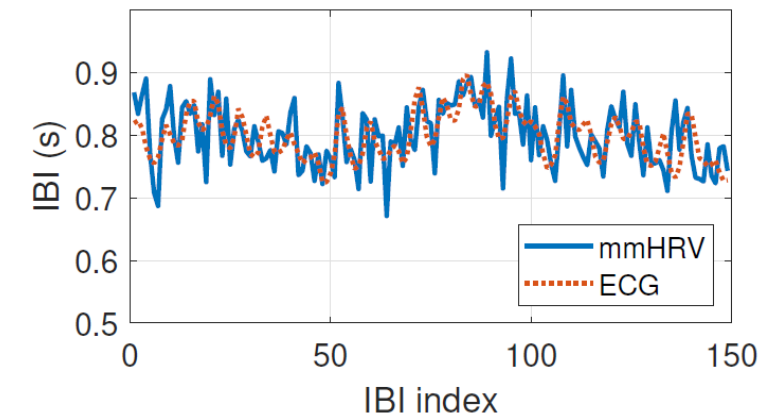
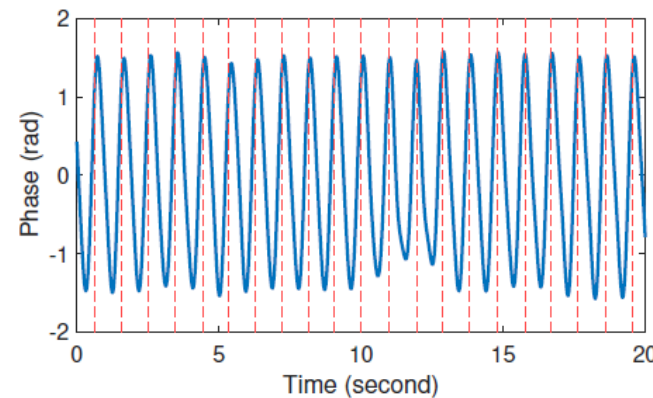
Heart Rate Truth

66bpm

Heart Rate Estimation

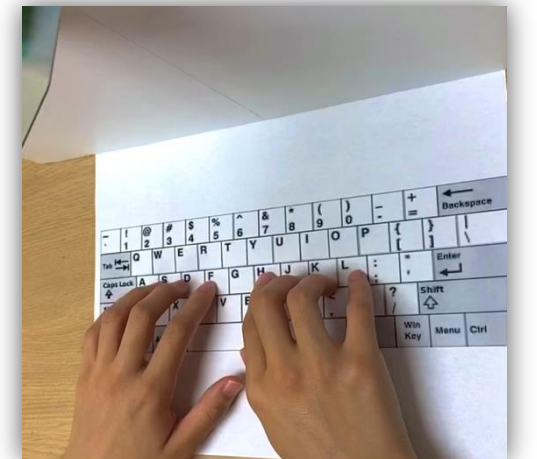
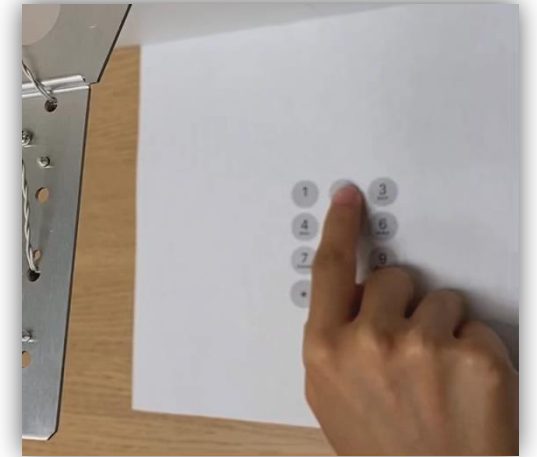
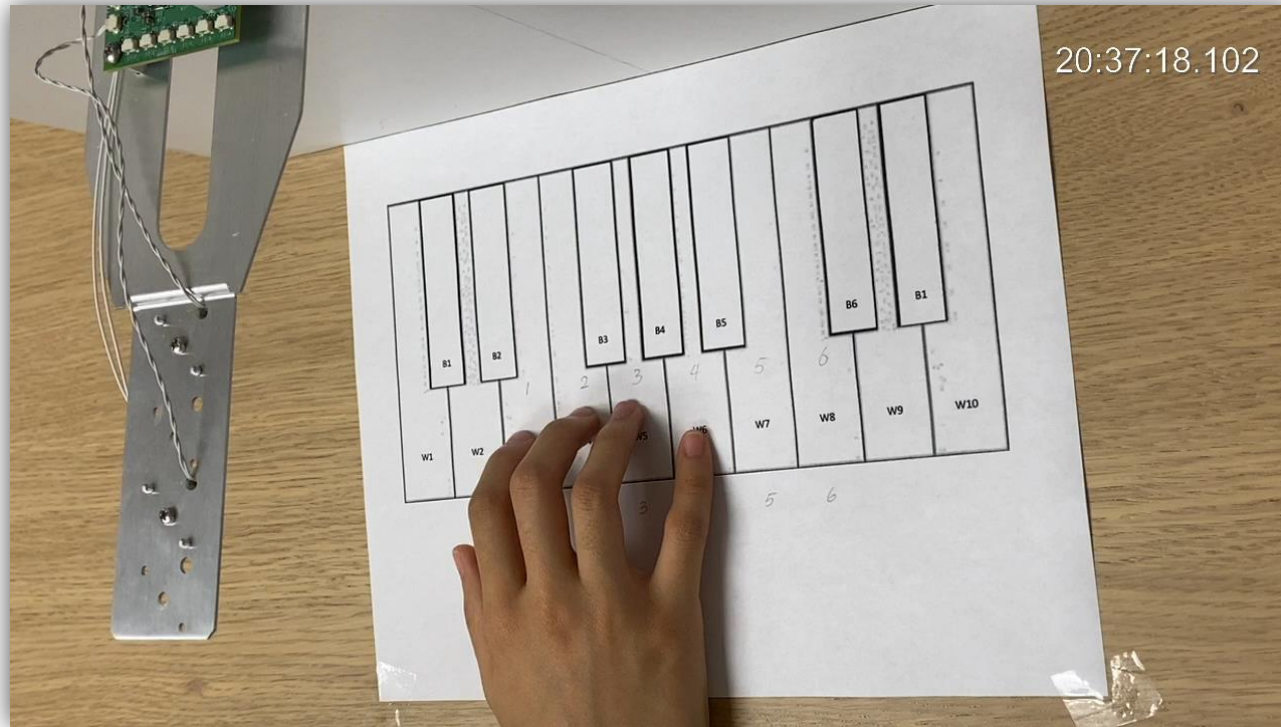
66bpm

Time (second)





# Gesture/Virtual Keyboard



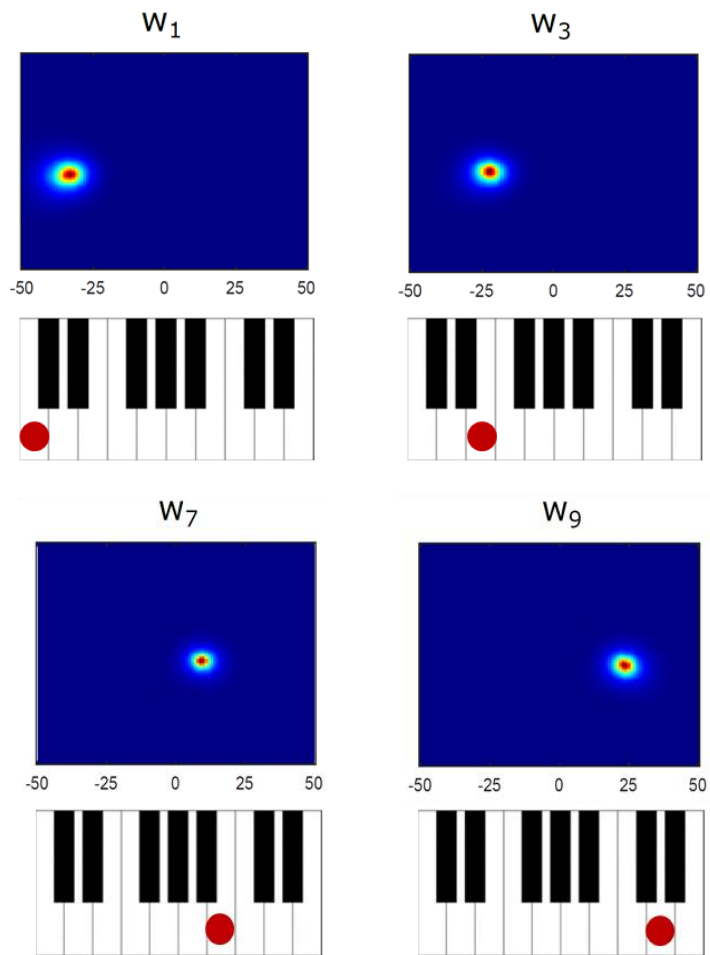
Ground truth:  $w_3 w_3 w_7 w_7 w_8 w_8 w_7 w_6 w_6 w_5 w_5 w_4 w_4 w_3$

Estimation:  $w_3 w_3 w_7 w_7 w_8 w_8 w_7 w_6 w_6 w_5 w_5 w_4 w_4 w_3$

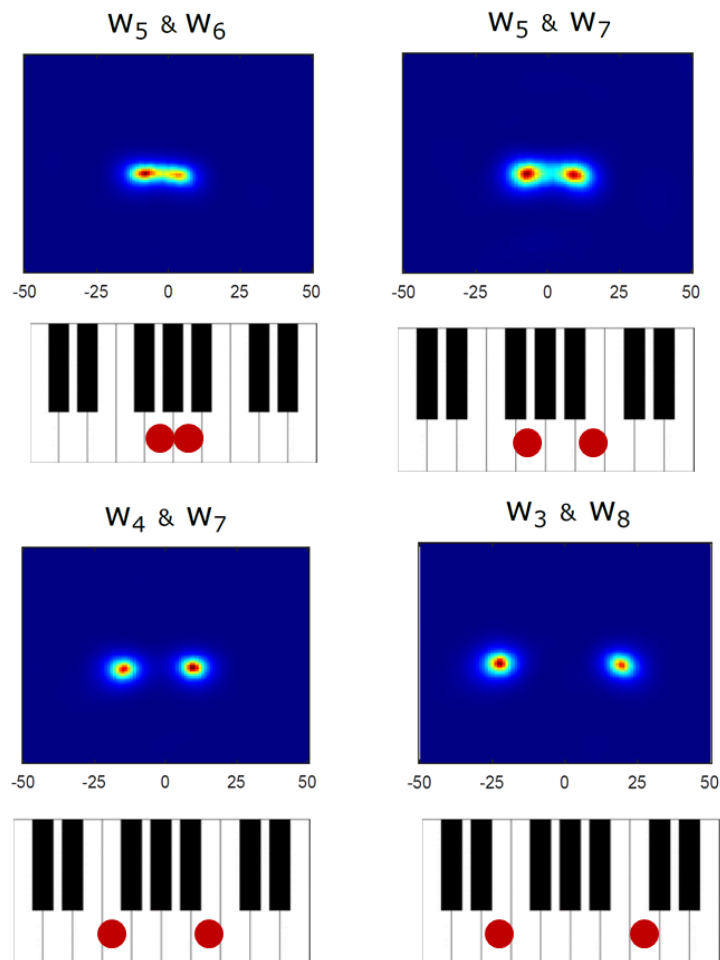
**Estimation = Ground truth**

# Gesture/Virtual Keyboard

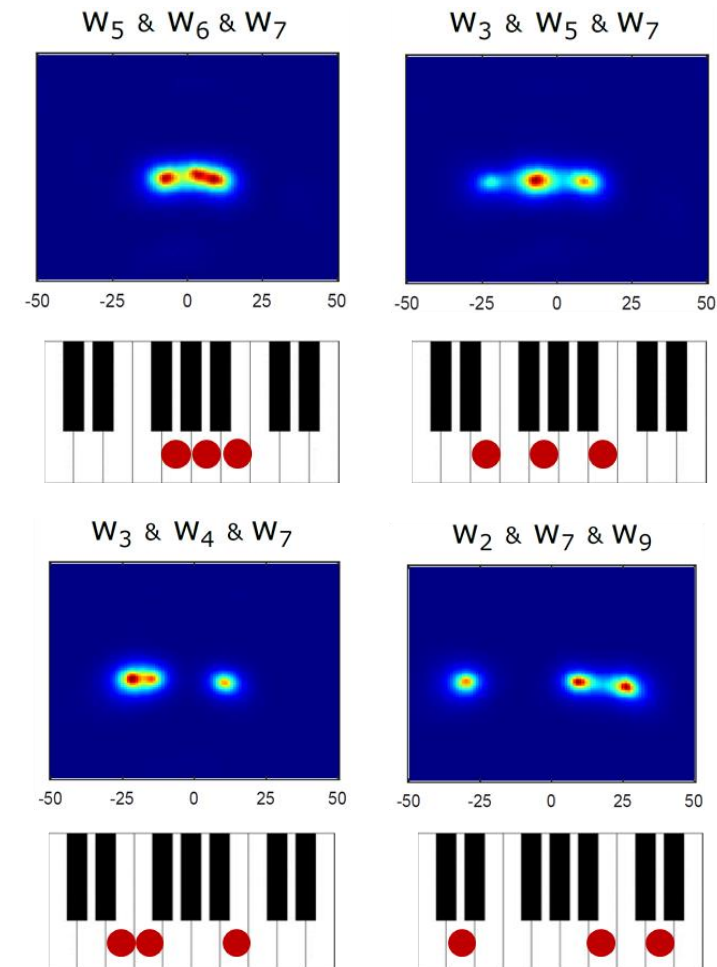
Single key



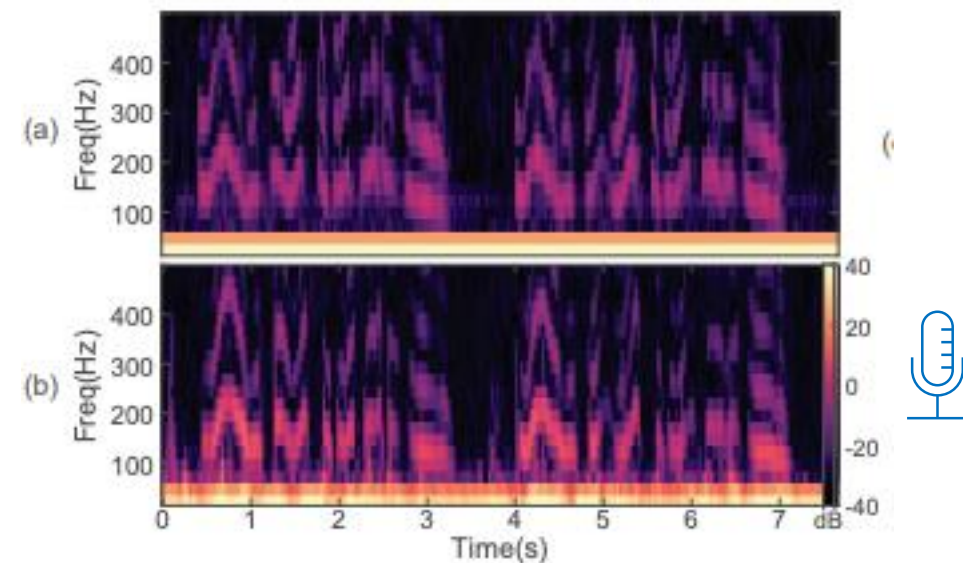
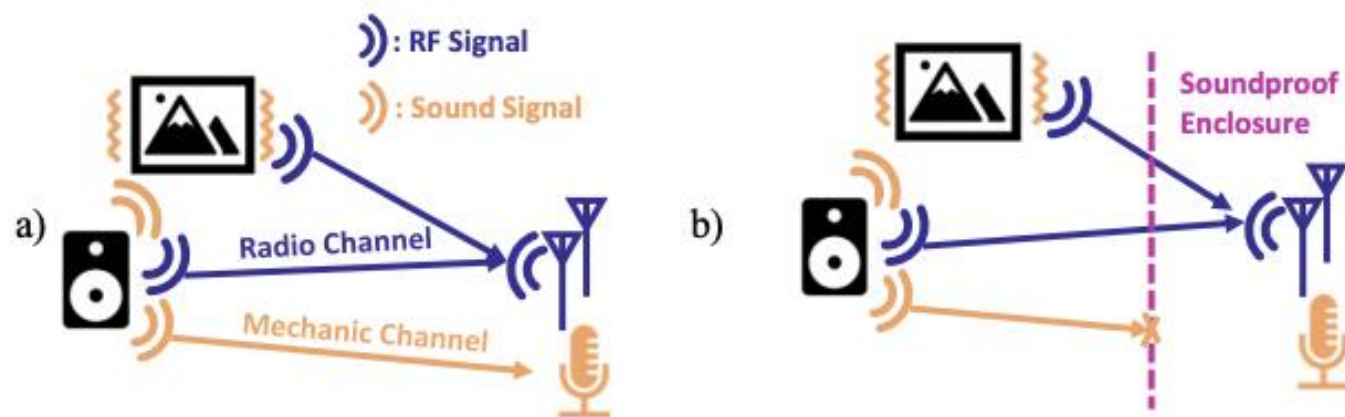
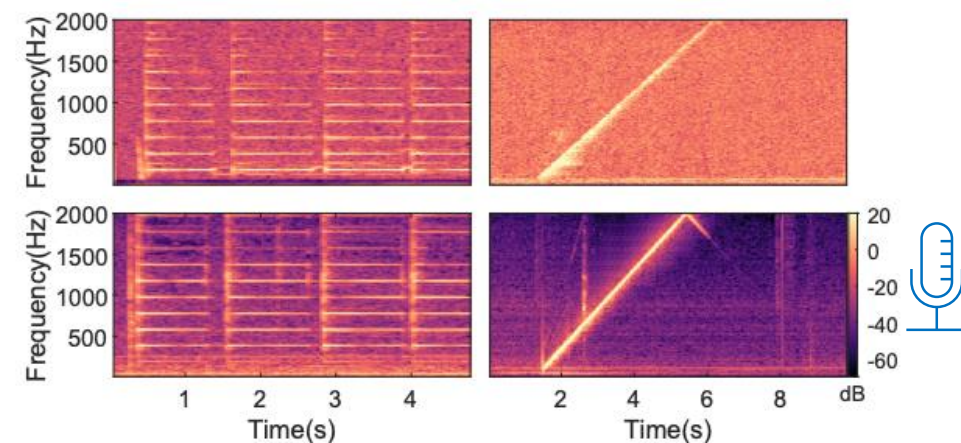
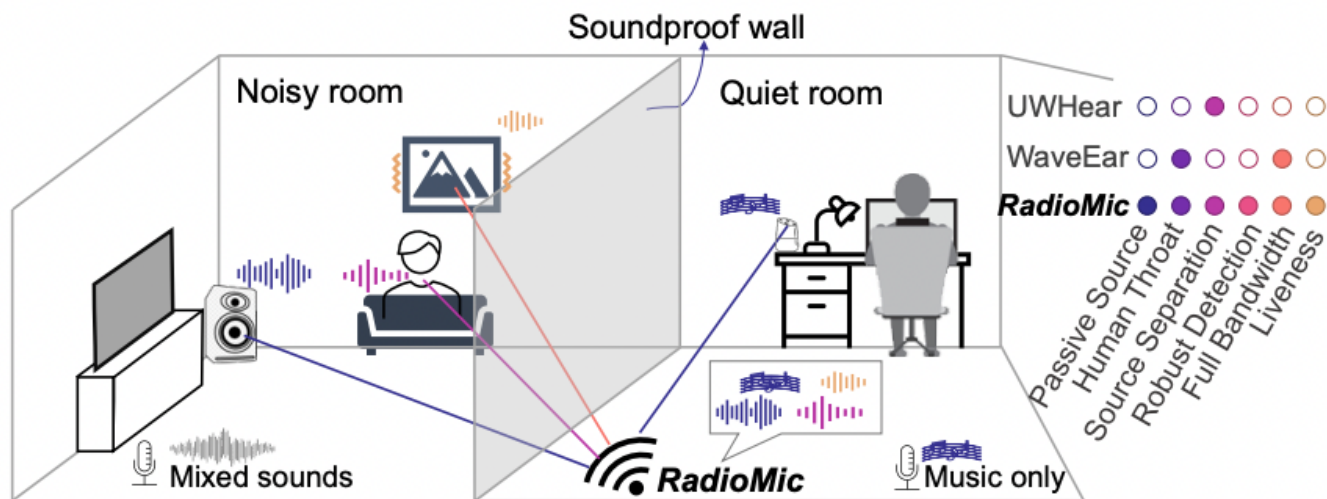
Double keys



Three keys



# Sound Recovery from mmWave Sensing



Ozturk, M. Z., Wu, C., Wang, B., & Liu, K. J. (2021). RadioMic: Sound sensing via mmWave signals. *IEEE IOTJ*, 2022.



# RadioMic: Examples

Frequency Sweep

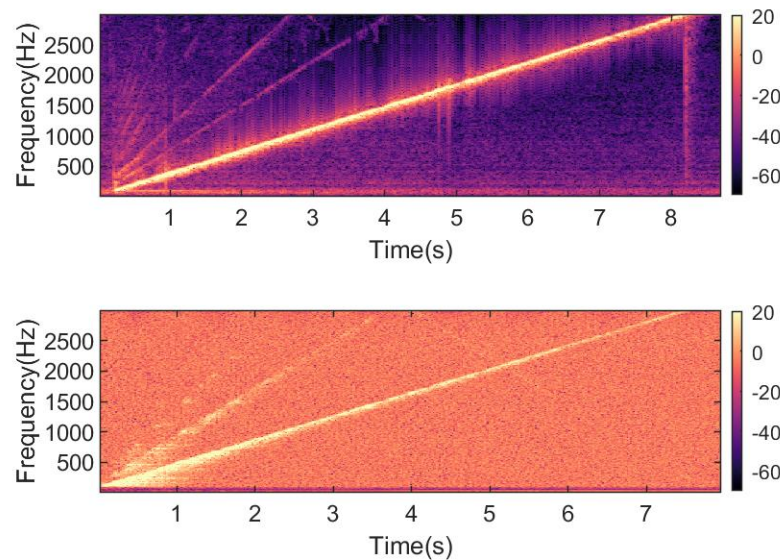


Fig 1. Frequency sweep

Music

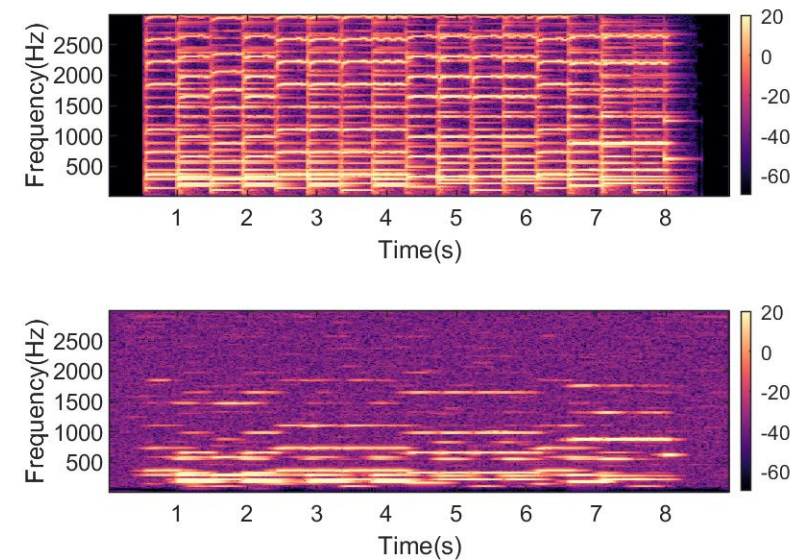


Fig 2. Instrumental music

Warning: Frequency sweep may be annoying to listeners

# RadioMic: Examples

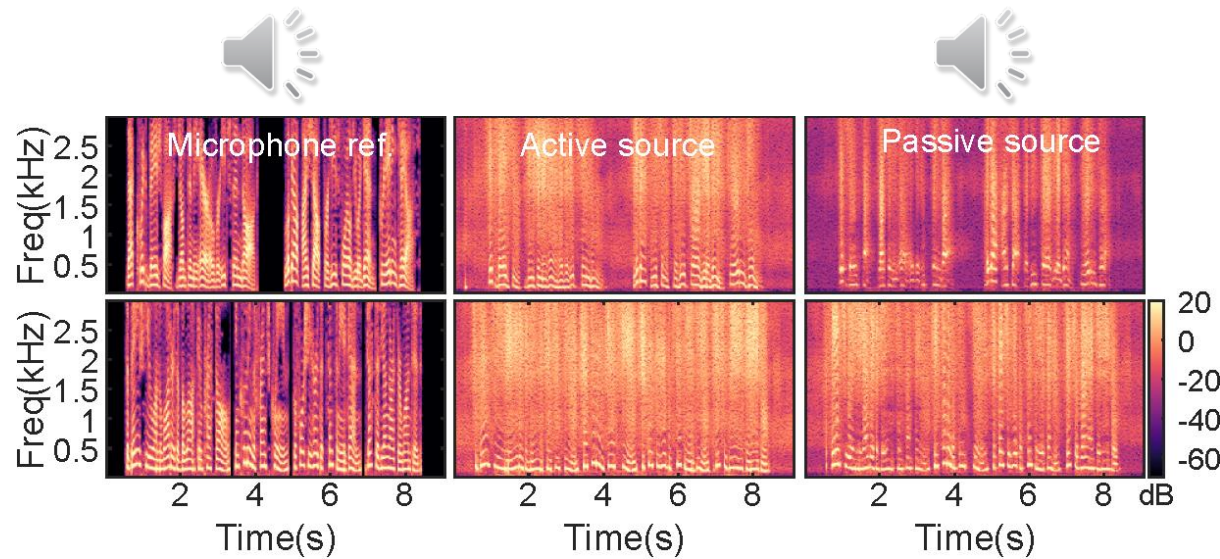


Fig 1. Example spectrograms with RadioMic

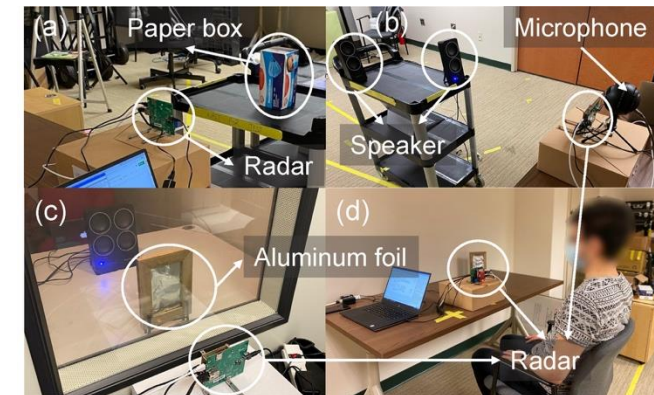
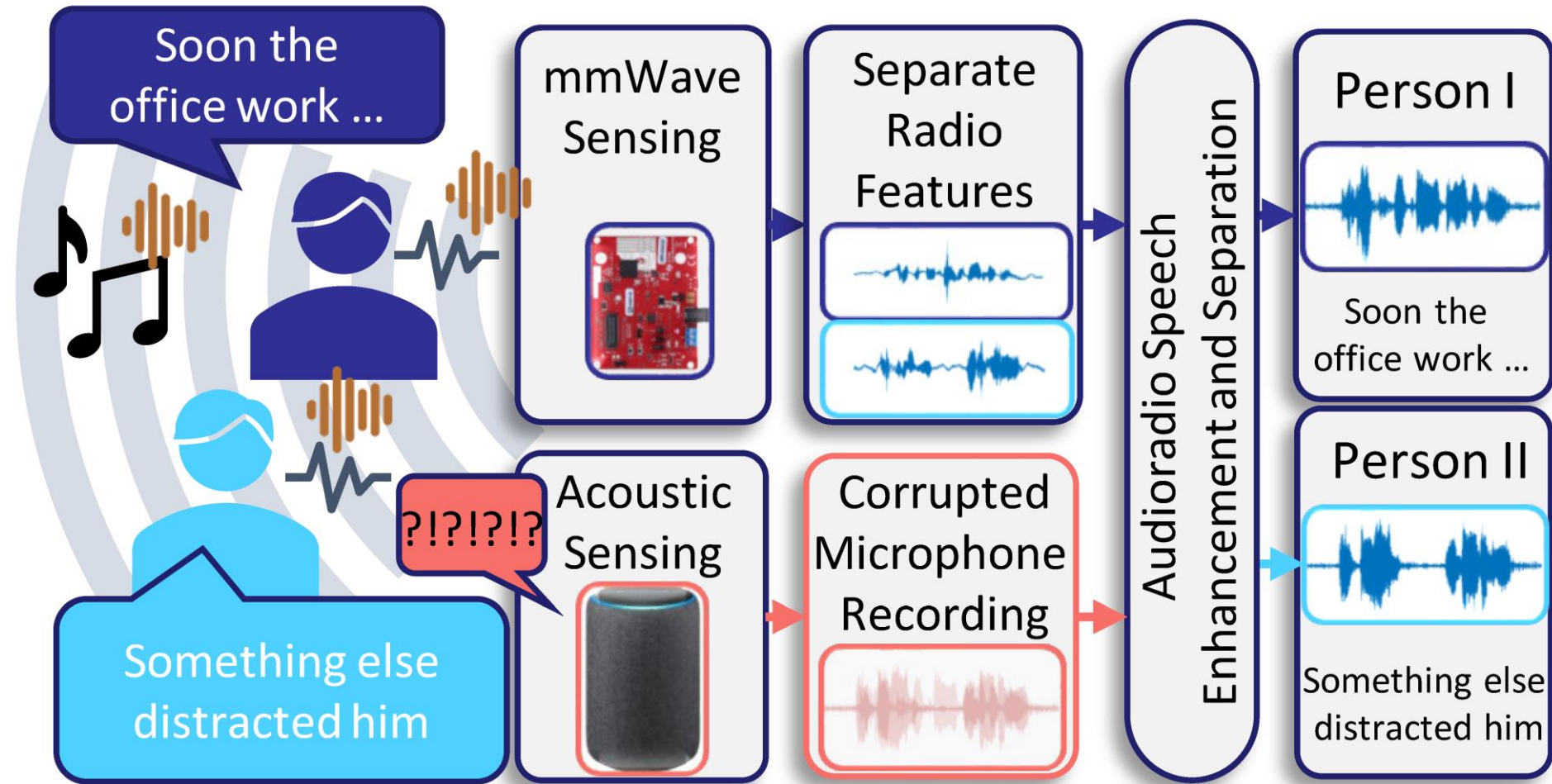


Fig 2. Data collection setups with RadioMic

That quick beige fox jumped in the air over each thin dog. Look out, I shout, for he's foiled you again, creating chaos

# RadioSES: Speech Enhancement and Separation



Ozturk, M. Z., Wu, C., Wang, B., Wu, M., & Liu, K. J. (2022). RadioSES: mmWave-Based Audioradio Speech Enhancement and Separation System. IEEE/ACM TASLP, 2023.



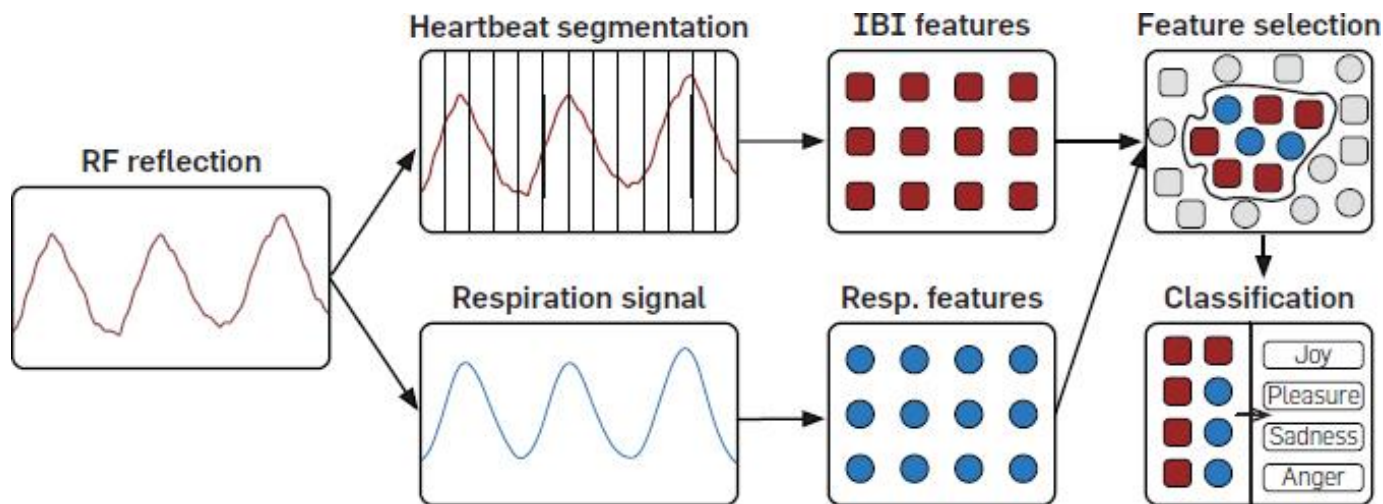
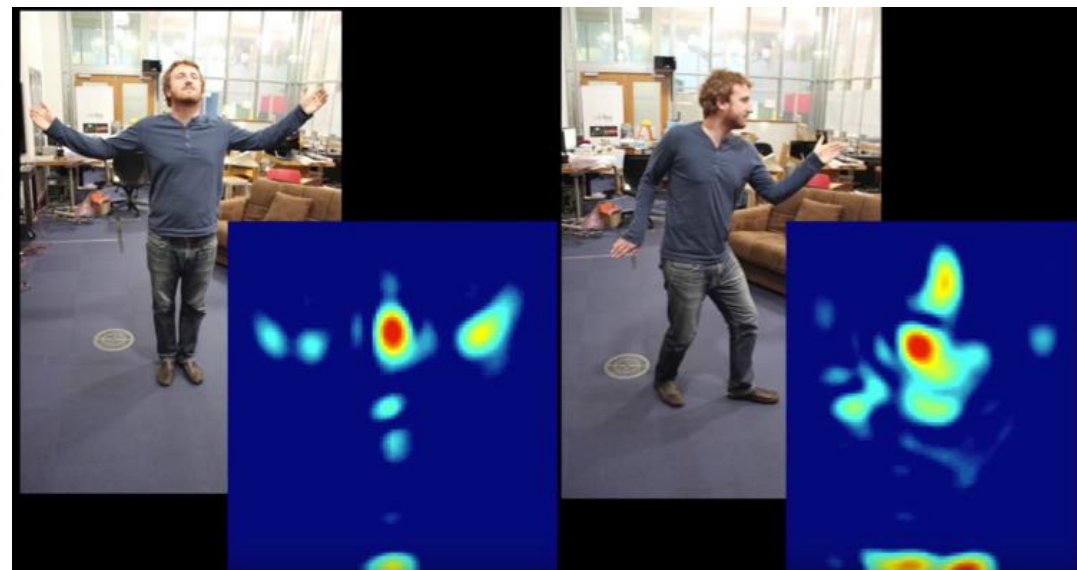
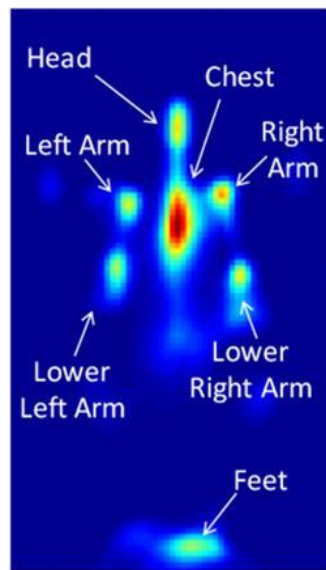
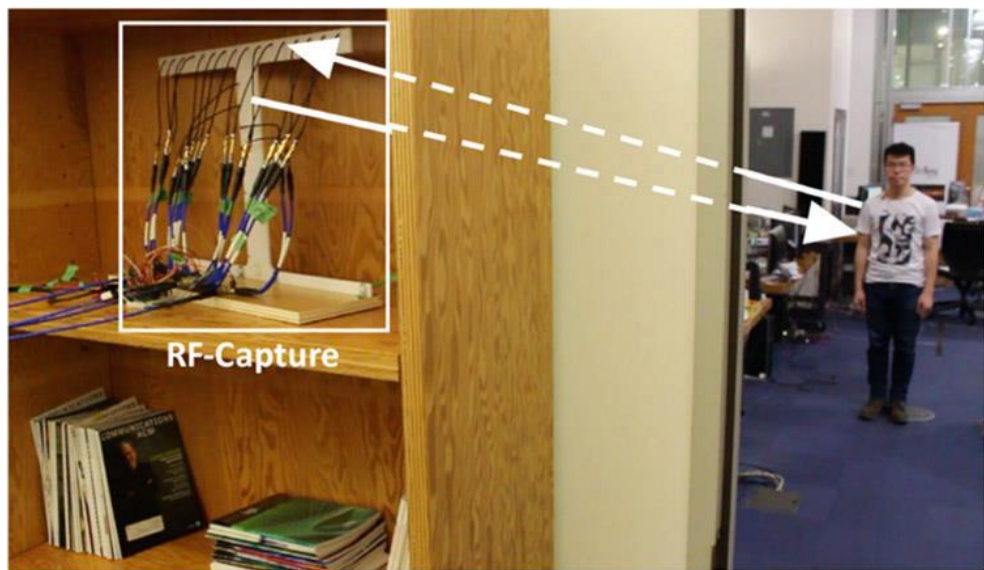
# RadioSES: Audioradio approach



Ozturk, M. Z., Wu, C., Wang, B., Wu, M., & Liu, K. J. (2022). RadioSES: mmWave-Based Audioradio Speech Enhancement and Separation System. IEEE/ACM TASLP, 2023.



# FMCW Radar on WiFi Frequency



A line of awesome work by Dina Katabi's team@MIT



# Questions?

- Thank you!
- Next: Can we transform ubiquitous WiFi into a “Radar”?