

CSE 5559: Computational Topology and Data Analysis

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Home Research pages(TBA) <u>List of Papers</u> (partially constructed) Class Schedule

> Time: SP 2017, Mon 10:20--12:25pm, DL 480

> > **Final Presentations TBA**

Computational topology has played a synergistic role in bringing together research work from computational geometry, algebraic topology, data analysis, and many other related scientific areas. In recent years, the field has undergone particular growth in the area of data analysis. The application of topological techniques to traditional data analysis, which before has mostly developed on a statistical setting, has opened up new opportunities. This course is intended to cover this aspect of computational topology along with the developments of generic techniques for various topology-centered problems.

Objectives

- Be familiar with basics in topology that are useful for computing with data
- Master a subset of algorithms for computing: Betti number, topological persistence, homology cycles, Reeb graphs, Laplace spectra from data
- Be familiar with how to design algorithms for problems in applications dealing with data
- Be familiar with how to research the background of a topic in data analysis, machine learning
- **Prerequisites:** CSE6331; or grad standing and permission of instructor
- **Materials (Transparencies)**

material

Text and class 1. Computational topology, Herbert Edelsbrunner and John L. Harer,

AMS

2. Curve and surface reconstruction: Algorithms with mathematical analysis, Tamal K. Dey, Cambridge U. Press

3. Elements of Algebraic Topology, James R. Munkres, Addison-

Wesley

4. Algebraic Topology, Allen Hatcher, Cambridge U. Press

5. Class materials and notes posted on this web-site

Topics

1. Basics of Topology;

a. Topological spaces, metric space topology [Chapter 1 & 12 of this book Notes

b. Maps: homeomorphisms, homotopy equivalence, isotopy [Notes]

c. Manifolds [Notes]

2. Complexes on data

a. Simplicial complexes [Munkres] [Notes]

b. Chech complexes, Vietoris-Rips

complexes [Notes]

c. Witness complexes [deSilva-<u>Carlsson04</u> paper][<u>Notes</u>] d. Graph induced complexes [DeyFanWang13 paper][Notes]

3. Homology

a. Chains, boundaries, homology groups, betti numbers [Notes, Munkres book]

c. Induced maps among homology groups [Notes, Munkres book] d. Relative homology groups [Notes, Munkres book]

e. Local homology groups [Notes,

Munkres book]

f. Cohomology groups [Notes,

Hatcher book]

4. Topological persistence

a. Filtrations, Persistent homology Notes, C-VII Edelsbrunner-Harer book]

b. Persistence algorithm [Notes, C-VII Edelsbrunner-Harer book, EdelsbrunneLetscherZomorodian02 paper introduced topological

persistence, ZomorodianCarlsson04 paper brings algebra into

persistence]

c. Persistence diagram [Notes, C-VIII Edelsbrunner-Harer H book, Cohen-SteinerEdelsbrunnerHarer07 paper proves the stabilty of

persistence]

d. Variations on persistence

algorithm [Notes,

CarlssonSilvaMorozov09 paper on

zigzag persistence,

DeyFanWang13SM paper on cohomology persistence]

Betti numbers in three dimensions [Notes]

5. Computing a. Data from surfaces and volumes

b. Data from manifolds in higher dimesnions [Notes]

6. from data

a. Basics of reconstruction [Notes, **Reconstruction** Chapter 1, Dey-book] b.Curve reconstruction from data [Notes, Chapter 2, Dey-book, AmentaBernEppstein98 paper introduces LFS, epsilon-sampling, DeyKumar99 paper on a very simple algorithm for curve reconstruction] c. Surface reconstruction in three dimensions from data [Notes, Chapter 3-4, Dey-book, Amenta-Bern99 paper pioneering provable surface reconstruction, AmentaChoiDeyLeekha01 paper proving the homeomorphism for Cocone algorithm] d. Manifold reconstruction in high dimensions from data [Notes, NiyogiSmaleWeinberger06 paper on probabilistic setting, ChazalCohen-SteinerLieutier09 paper on theory of compacts, ChengDeyRamos05 paper on anomalies of restricted Delaunay and its rectification by weighted Delaunay]

7. Topology data

a. Computing homology from data inference from [Notes, ChazalOudot08 paper on homology inference, CCGGO09 paper on interleaving of persistence modules] b. Sparsification to handle big data [Presentation slides], [Sheehy12] paper on sparsified Rips complex, DeyFanWang13 paper on subsampling]

8. Computing optimized homology cycles

a. Computing shortest basis cycles on surfaces [EricksonWhittlesey05 paper on greedy basis construction] b. Computing shortest basis cycles from data points [Presentation slides, DeySunWang09 paper on shortest basis from point data] c. Localizing a cycle class [Presentation slides, DeyHiraniKrishnamoorthy10 paper on LP algorithm for shortest homologous cycle]

9. Reeb graphs a. Reeb graphs [Notes, C-MEHNP]

from data paper on loops in Reeb graphs]

b. Approximating Reeb graphs from data [Notes, <u>DeyWang11</u> paper on approximation of Reeb

graphs from data]

10. Topology of a. Laplace operator [Notes]

Laplace operators,

b. Approximating Laplace from data [Notes, BelkinNiyogi03 paper

spectra on approximating Laplace, **approximation** BelkinSunWang09 paper on PCD

Laplace]

c. Stabilty of Laplace spectra [Notes, <u>DeyWang10</u> paper on

spectra stability]

• Grading

A presentation on a 50%

TDA topic

Midterm 40% Participation 10%