Calculation of music note frequencies and bank interest

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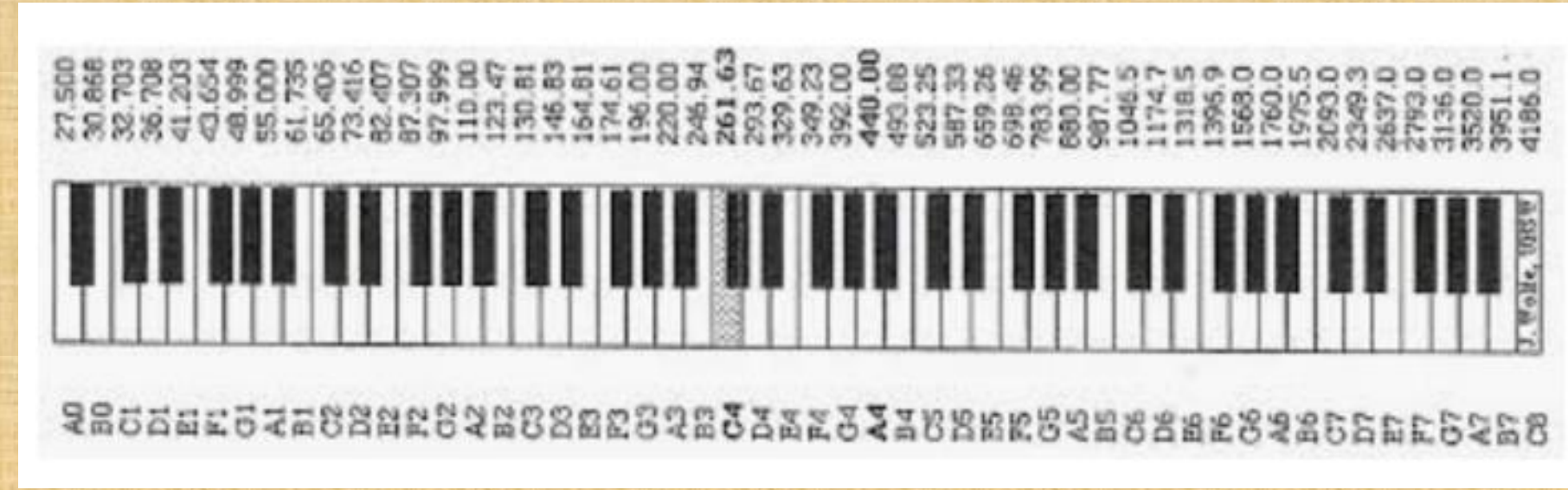
(home stay during the Coronavirus pandemic crisis)

Music note frequencies
how do you get these numbers?



C4 = Piano Central C

Music note frequencies
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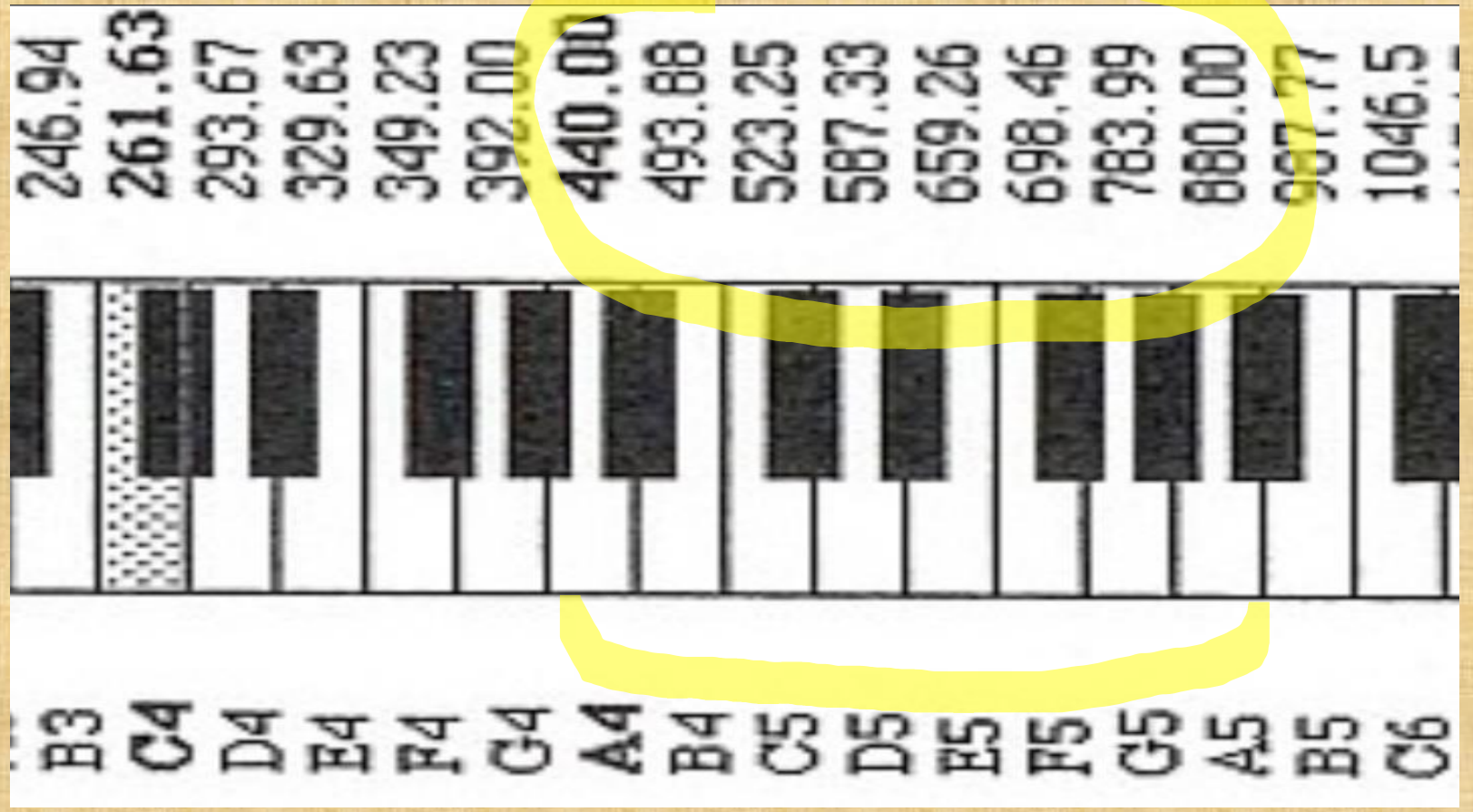


C4 = Piano Central C

Who defined these numbers?

a musician?
a physicist?
an engineer?

or a banker?

Bank interest calculation

P = Principal or initial deposit amount

r = annual interest rate in %, compounded annually, e.g., r = .03 for 3%

t = time or number of years

A = final balance at the end of t years

$$\mathbf{A = P(1 + r)^t}$$

Exponential model: $A = Pe^{rt}$

Q: how long does it take to double my money?

Exact: $2P=P(1+r)^t$, solve for t: $t=\frac{ln\ 2}{ln\ (1+r)}$

$2P = Pe^{rt}$, solve for t

t = $\frac{ln\ 2}{r} = \frac{.693147180\ldots\ldots}{r}$

Approximation: (rule of 70):

t= $\frac{70}{100r}$ e.g., t = $\frac{70}{100\ x\ .03} = \frac{70}{3}$ ≈ 23 years

Q: Deposit of $440, and want to double in 12 years. What is the interest rate needed?

$P = 440, A = 880$, t=12 and solve for r.

Exact: $2P=P(1+r)^t$, $1+r = e^{\frac{ln\,2}{t}} = e^{\frac{ln\,2}{12}}$ =1.059463094359....

$2P = Pe^{rt}$, $1+r=1+\frac{ln\,2}{12}$ =1.057762265047.....

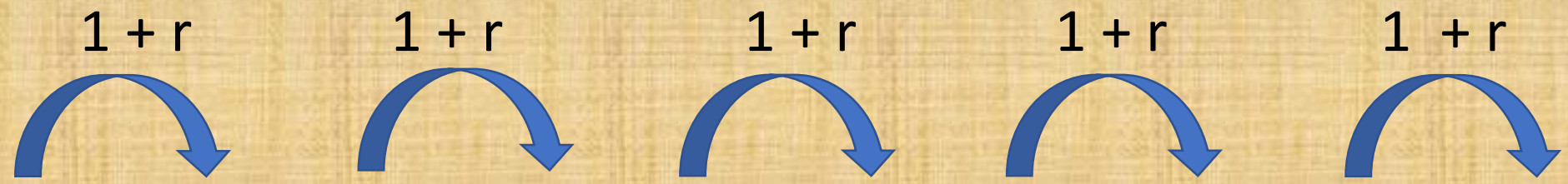
Approximation (Rule of 70):

t= $\frac{70}{100r}$, r = $\frac{70}{100t}$= $\frac{70}{1200}$ ≈ .058, 1+r ≈ 1.058

Q:Deposit of $440, and want to double in 12 years. What is the interest rate needed? r = .059463094359….

Show the balance at the beginning for each of the 12 years.

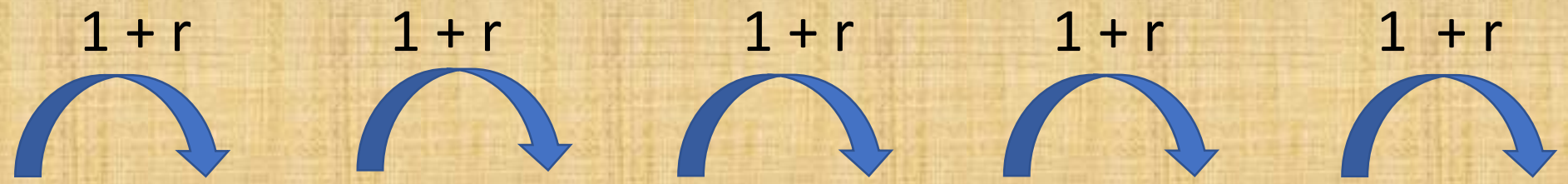
Deposit of $440, and want to double in 12 years. What is the interest rate? r = .059463094359....



$440	$466.163	$493.883	$523.251	$554.365	$587.330	
year 1	2	3	4	5	6	

$622.254	$659.255	$698.456	$739.989	$783.991	$830.609	$880.00
7	8	9	10	11	12	

Does this have anything to do with music?



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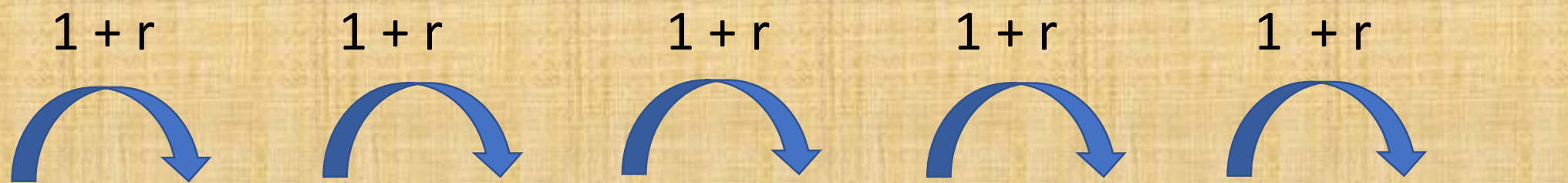
They are exactly the same as music note frequencies from A4 to A5

r = .059463094359....



440	466.163	493.883	523.251	554.365	587.330
A4	A#	B4	C5	C#	D5
La		Ti	Do		Re

622.254	659.255	698.456	739.989	783.991	830.609	880.00
D#	E5	F5	F#	G5	G#	A5
	Mi	Fa		Sol		La

1 + r 1 + r 1 + r 1 + r 1 + r

440 466.163 493.883 523.251 554.365 587.330

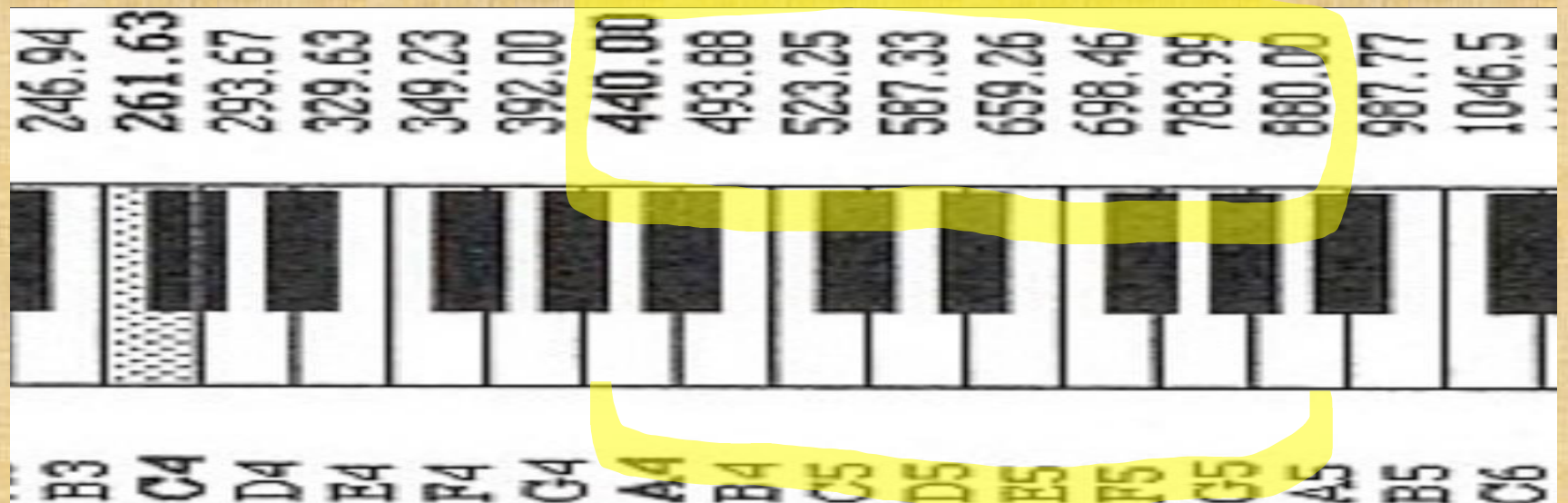
A4 A# B4 C5 C# D5

La Ti Do Re

622.254 659.255 698.456 739.989 783.991 830.609 880.00

D# E5 F5 F# G5 G# A5

Mi Fa Sol La



conclusion

Calculation of music note frequencies is exactly the same as calculation of interest payments.

Hope that makes both subjects more interesting to you.

Thank you