Curso Superior de Tecnologia em Sistemas de Computação
Disciplina: Matemática para Computação
AD1 - 2^o semestre de 2014 — Gabarito

Questões

1. (1,0 ponto)

   Diga qual o domínio e a imagem das seguintes funções:

   (a) $$f(x) = \begin{cases} x^2 & \text{se} \quad 2 \leq x \leq 4 \\ x+1 & \text{se} \quad 1 \leq x < 2 \end{cases}$$

   (b) $f(x) = x^2 + 4$

   (c) $f(x) = \sqrt{x^2 + 4}$

   (d) $f(x) = \sqrt{x^2 - 4}$

   (e) $f(x) = \dfrac{x}{x+3}$

   (f) $f(x) = \dfrac{2x}{(x-2)(x+1)}$

   (g) $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$

   (h) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

   (i) $f(x) = \sqrt{\dfrac{x}{2-x}}$

Solução:

(a) $f(x) = \begin{cases} x^2 & \text{se} \quad 2 \le x \le 4 \\ x+1 & \text{se} \quad 1 \le x < 2 \end{cases}$

Dom $f = \{x \in \mathbb{R} \text{ tais que } 1 \le x \le 4\}$

Im $f = \{x \in \mathbb{R} \text{ tais que } 2 \le x < 3 \text{ e } 4 \le x \le 16\}$

(b) $f(x) = x^2 + 4$

Dom $f = \{x \in \mathbb{R}\}$

Im $f = \{x \in \mathbb{R}\}$

(c) $f(x) = \sqrt{x^2 + 4}$

Dom $f = \{x \in \mathbb{R}\}$

Im $f = \left\{x \in \mathbb{R} \text{ tais que } \sqrt{\ 4\ } \le x < \infty\right\}$

(d) $f(x) = \sqrt{x^2 - 4}$

Para a função ser definida $x^2 - 4 \ge 0$ ou $x^2 \ge 4$, isto é $x \ge 4$ ou $x \le -4$

logo

Dom $f = \{x \in \mathbb{R} \text{ tais que } x \le -4 \text{ ou } 4 \le x\}$

Im $f = \{x \in \mathbb{R} \text{ tais que } 0 \le x < \infty\}$

(e) $f(x) = \dfrac{x}{x+3}$

Dom $f = \{x \in \mathbb{R} \text{ tais que } x \ne -3\}$

Im $f = \{x \in \mathbb{R}\}$

(f) $f(x) = \dfrac{2x}{(x-2)(x+1)}$

Dom $f = \{x \in \mathbb{R} \text{ tais que } x \ne -1 \text{ e } x \ne 2\}$

Im $f = \{x \in \mathbb{R}\}$

(g) $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$

Para a função ser definida $9 - x^2 > 0$ ou $9 > x^2$, isto é $-3 < x < 3$

logo

Dom $f = \{x \in \mathbb{R} \text{ tais que } -3 < x < 3\}$

Im $f = \left\{x \in \mathbb{R} \text{ tais que } \dfrac{1}{9} \le x < \infty\right\}$

(h) $$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

$$\text{Dom } f = \{x \in \mathbb{R}\}$$

$$\text{Im } f = \{x \in \mathbb{R} \text{ tais que } -\infty < x < 1\}$$

(i) $$f(x) = \sqrt{\frac{x}{2 - x}}$$

$$\text{Dom } f = \{x \in \mathbb{R} \text{ tais que } x \neq 2\}$$

$$\text{Im } f = \{x \in \mathbb{R} \text{ tais que } 0 \leq x < \infty\}$$

2. (1,0 ponto) ———

Encontre os seguintes limites:

(a) $$\lim_{x \to 0} \frac{1}{x^2}$$

(b) $$\lim_{x \to 1} \frac{-1}{(x-1)^2}$$

(c) $$\lim_{x \to +\infty} \frac{1}{x}$$

(d) $$\lim_{x \to +\infty} \left(2 + \frac{1}{x^2}\right)$$

Solução:

(a) $$\lim_{x \to 0} \frac{1}{x^2} = +\infty$$

(b) $$\lim_{x \to 1} \frac{-1}{(x-1)^2} = -\infty$$

(c) $$\lim_{x \to +\infty} \frac{1}{x} = 0$$

(d) $$\lim_{x \to +\infty} \left(2 + \frac{1}{x^2}\right) = \left(\lim_{x \to +\infty} 2 + \lim_{x \to +\infty} \frac{1}{x^2}\right) = (2 + 0) = 2$$

3. (1,0 ponto) ———

Encontre os seguintes limites:

(a) $$\lim_{x \to 4} \sqrt{25 - x^2}$$

(b) $$\lim_{x \to -5} \frac{x^2 - 25}{x + 5}$$

(c) $$\lim_{x \to 4} \frac{x - 4}{x^2 - x - 12}$$

(d) $$\lim_{x\to 2} \frac{x^2+x-2}{(x-1)^2}$$

Solução:

(a) $$\lim_{x\to 4} \sqrt{25-x^2} = \sqrt{\lim_{x\to 4}(25-x^2)}$$
$$= \sqrt{\left(\lim_{x\to 4} 25 - \lim_{x\to 4} x^2\right)}$$
$$= \sqrt{(25-16)}$$
$$= \sqrt{9} = 3$$

(b) $$\lim_{x\to -5} \frac{x^2-25}{x+5} = \lim_{x\to -5} \frac{(x+5)(x-5)}{x+5}$$
$$= \lim_{x\to -5}(x-5)$$
$$= (\lim_{x\to -5} x - \lim_{x\to -5} 5)$$
$$= (-5-5) = -10$$

(c) $$\lim_{x\to 4} \frac{x-4}{x^2-x-12} = \lim_{x\to 4} \frac{x-4}{(x+3)(x-4)}$$
$$= \lim_{x\to 4} \frac{1}{(x+3)} = \frac{1}{7}$$

(d) $$\lim_{x\to 2} \frac{x^2+x-2}{(x-1)^2} = \lim_{x\to 2} \frac{(x-1)(x+2)}{(x-1)^2}$$
$$= \lim_{x\to 2} \frac{(x+2)}{(x-1)}$$
$$= \frac{(2+2)}{(2-1)} = \frac{4}{1} = 4$$

4. (1,0 ponto) ____

Nos itens a seguir $\lim\limits_{x\to\pm\infty}$ pode ser interpretado como $\lim\limits_{x\to+\infty}$ ou $\lim\limits_{x\to-\infty}$. Calcule então os limites:

(a) $$\lim_{x\to\pm\infty} \frac{x^2+x-2}{4x^3-1}$$

(b) $$\lim_{x\to\pm\infty} \frac{2x^3}{x^2+1}$$

(c) $$\lim_{x\to\pm\infty} (x^5-7x^4-2x+5)$$

Solução:

(a)

$$\begin{aligned}
\lim_{x\to\pm\infty}\frac{x^2+x-2}{4x^3-1} &= \lim_{x\to\pm\infty}\frac{x^2/x^3+x/x^3-2/x^3}{4x^3/x^3-1/x^3}\\
&= \lim_{x\to\pm\infty}\frac{1/x+1/x^2-2/x^3}{4-1/x^3}\\
&= \frac{\lim\limits_{x\to\pm\infty}1/x+\lim\limits_{x\to\pm\infty}1/x^2-\lim\limits_{x\to\pm\infty}2/x^3}{\lim\limits_{x\to\pm\infty}4-\lim\limits_{x\to\pm\infty}1/x^3}\\
&= \frac{0+0-2\cdot 0}{4-0}=\frac{0}{4}=0
\end{aligned}$$

(b)

$$\begin{aligned}
\lim_{x\to\pm\infty}\frac{2x^3}{x^2+1} &= \lim_{x\to\pm\infty}\frac{2x^3/x^2}{x^2/x^2+1/x^2}\\
&= \lim_{x\to\pm\infty}\frac{2x}{1+1/x^2}\\
&= \frac{\lim\limits_{x\to\pm\infty}2x}{\lim\limits_{x\to\pm\infty}1+\lim\limits_{x\to\pm\infty}1/x^2}\\
&= \frac{\pm\infty}{1+0}=\pm\infty
\end{aligned}$$

(c)

$$\begin{aligned}
\lim_{x\to\pm\infty}(x^5-7x^4-2x+5) &= \lim_{x\to\pm\infty}x^5(1-\frac{7x^4}{x^5}-\frac{2x}{x^5}+\frac{5}{x^5})\\
&= \lim_{x\to\pm\infty}x^5(1-\frac{7}{x}-\frac{2}{x^4}+\frac{5}{x^5})\\
&= \lim_{x\to\pm\infty}x^5\cdot\lim_{x\to\pm\infty}(1-\frac{7}{x}-\frac{2}{x^4}+\frac{5}{x^5})\\
&= \lim_{x\to\pm\infty}x^5\cdot(\lim_{x\to\pm\infty}1-\lim_{x\to\pm\infty}\frac{7}{x}-\lim_{x\to\pm\infty}\frac{2}{x^4}+\lim_{x\to\pm\infty}\frac{5}{x^5})\\
&= \lim_{x\to\pm\infty}x^5\cdot(1-0-0+0)\\
&= \lim_{x\to\pm\infty}x^5\cdot(1)\\
&= \lim_{x\to\pm\infty}x^5=\pm\infty
\end{aligned}$$

5. (1,0 ponto)

Estude a continuidade das seguintes funções:

(a)

$$f(x)=\begin{cases} x^2 & \text{se} \quad x\neq 2\\ 0 & \text{se} \quad x=2\end{cases}$$

(b) $$f(x) = \frac{\mid x \mid}{x}$$

Solução:

(a) $$f(x) = \begin{cases} x^2 & \text{se} \quad x \neq 2 \\ 0 & \text{se} \quad x = 2 \end{cases}$$

No ponto $x = 2$

$$\lim_{x \to 2} f(x) = 4$$

já que

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = 4$$

e

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 = 4$$

mas

$$f(2) = 0$$

sendo então o valor do limite diferente do valor da função no ponto. Daí f é descontínua no ponto $x = 2$.

(b) $$f(x) = \frac{\mid x \mid}{x}$$

A função não é definida no ponto $x = 0$. Além disso, como $\mid x \mid$ é definida por

$$\mid x \mid \quad = \begin{cases} -x & \text{se} \quad x < 0 \\ x & \text{se} \quad x \geq 0 \end{cases}$$

teremos

$$\lim_{x \to 0^-} \frac{\mid x \mid}{x} = \lim_{x \to 0^-} \frac{-\,x}{x} = \lim_{x \to 0^-} -1 = -1$$

e

$$\lim_{x \to 0^+} \frac{\mid x \mid}{x} = \lim_{x \to 0^-} \frac{x}{x} = \lim_{x \to 0^-} 1 = 1$$

e

$$\lim_{x \to 0^-} \frac{\mid x \mid}{x} \neq \lim_{x \to 0^+} \frac{\mid x \mid}{x} \Longrightarrow \quad \lim_{x \to 0} \frac{\mid x \mid}{x} \quad \textbf{não existe}$$

Portanto, f é descontínua em $x = 0$.

6. (1,0 ponto) ———

Calcule as derivadas das funções abaixo, usando sua definição por **limite**, isto é:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(a) $$f(x) = \frac{1}{x-2}$$

(b) $$f(x) = \frac{2x-3}{3x+4}$$

Solução:

(a) $$f(x) = \frac{1}{x-2}$$

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\dfrac{1}{(x+h)-2} - \dfrac{1}{x-2}}{h} \\
&= \lim_{h \to 0} \frac{\dfrac{(x-2)-(x+h-2)}{(x+h-2)(x-2)}}{h} \\
&= \lim_{h \to 0} \frac{(x-2)-(x+h-2)}{h(x+h-2)(x-2)} \\
&= \lim_{h \to 0} \frac{(x-2)-(x+h-2)}{h(x+h-2)(x-2)} \\
&= \lim_{h \to 0} \frac{-1}{(x+h-2)(x-2)} \\
&= -\frac{1}{(x-2)(x-2)} \\
&= -\frac{1}{(x-2)^2}
\end{aligned}$$

(b) $$f(x) = \frac{2x-3}{3x+4}$$

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\dfrac{2(x+h)-3}{3(x+h)+4} - \dfrac{2x-3}{3x+4}}{h}
\end{aligned}$$

$$= \lim_{h \to 0} \frac{(2(x+h)-3)(3x+4)-(2x-3)(3(x+h)+4)}{h(3(x+h)+4)(3x+4)}$$

$$= \lim_{h \to 0} \frac{(2x+2h-3)(3x+4)-(2x-3)(3x+3h+4)}{h(3(x+h)+4)(3x+4)}$$

$$= \lim_{h \to 0} \frac{(6x^2+8x+6hx+8h-9x-12)-(6x^2+6hx+8x-9x-9h-12)}{h(3(x+h)+4)(3x+4)}$$

$$= \lim_{h \to 0} \frac{6x^2+8x+6hx+8h-9x-12-6x^2-6hx-8x+9x+9h+12}{h(3(x+h)+4)(3x+4)}$$

$$= \lim_{h \to 0} \frac{8h+9h}{h(3x+3h+4)(3x+4)}$$

$$= \lim_{h \to 0} \frac{17h}{h(3x+3h+4)(3x+4)}$$

$$= \lim_{h \to 0} \frac{17}{(3x+3h+4)(3x+4)}$$

$$= \frac{17}{(3x+4)(3x+4)}$$

$$= \frac{17}{(3x+4)^2}$$

7. (1,0 ponto) ---

Ache a derivada de $f(x) = |\ x\ |$.

Solução:

Sabemos que

$$|\ x\ | = \begin{cases} -x & \text{se} \quad x < 0 \\ 0 & \text{se} \quad x = 0 \\ x & \text{se} \quad x > 0 \end{cases}$$

Se $x < 0$

$$f'(x) = (-x)' = -1$$

e se $x > 0$

$$f'(x) = (x)' = 1$$

ou

se $x < 0$

$$f'_{-}(0) = \lim_{h\to 0-} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0-} \frac{-\,(x+h) - (-x)}{h}$$

$$= \lim_{h\to 0-} \frac{-\,h}{h} = -1$$

e se $x > 0$

$$f'_{+}(0) = \lim_{h\to 0+} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0+} \frac{(x+h) - (x)}{h}$$

$$= \lim_{h\to 0+} \frac{h}{h} = 1$$

Portanto a derivada não existe em $x = 0$. Resumindo

$$x < 0 \implies f'(x) = -1$$

$$x = 0 \implies \textbf{não existe}$$

$$x > 0 \implies f'(x) = 1$$

8. (1,0 ponto) ———

Diferencie as funções:

(a) $f(x) = 2x^{1/2} + 6x^{1/3} - 2x^{3/2}$

(b) $f(x) = \dfrac{2}{x^{1/2}} + \dfrac{6}{x^{1/3}} - \dfrac{2}{x^{3/2}} - \dfrac{4}{x^{3/4}}$

(c) $s(t) = (t^2 - 3)^4$

(d) $w(y) = (y^2 + 4)^2(2y^3 - 1)^3$

Solução:

(a)

$$f(x) = 2x^{1/2} + 6x^{1/3} - 2x^{3/2}$$

$$\begin{aligned}
f'(x) &= \left[2x^{1/2} + 6x^{1/3} - 2x^{3/2}\right]' \\
&= \left[2\left(\frac{1}{2}\right)x^{1/2-1} + 6\left(\frac{1}{3}\right)x^{1/3-1} - 2\left(\frac{3}{2}\right)x^{3/2-1}\right] \\
&= \left(\frac{2}{2}\right)x^{-1/2} + \left(\frac{6}{3}\right)x^{-2/3} - \left(\frac{6}{2}\right)x^{1/2} \\
&= \frac{1}{x^{1/2}} + \frac{2}{x^{2/3}} - 3x^{1/2} \\
&= \frac{1}{\sqrt{x}} + \frac{2}{\sqrt[3]{x^2}} - 3\sqrt{x}
\end{aligned}$$

(b)

$$f(x) = \frac{2}{x^{1/2}} + \frac{6}{x^{1/3}} - \frac{2}{x^{3/2}} - \frac{4}{x^{3/4}}$$

$$\begin{aligned}
f'(x) &= \left[2x^{-1/2} + 6x^{-1/3} - 2x^{-3/2} - 4x^{-3/4}\right]' \\
&= \left[2\left(-\frac{1}{2}\right)x^{-1/2-1} + 6\left(-\frac{1}{3}\right)x^{-1/3-1} - 2\left(-\frac{3}{2}\right)x^{-3/2-1} - 4\left(-\frac{3}{4}\right)x^{-3/4-1}\right] \\
&= -\frac{2}{2}x^{-3/2} - \frac{6}{3}x^{-4/3} + \frac{6}{2}x^{-5/2} + \frac{12}{4}x^{-7/4} \\
&= -\frac{1}{\sqrt[2]{x^3}} - \frac{2}{\sqrt[3]{x^4}} + \frac{3}{\sqrt[2]{x^5}} + \frac{3}{\sqrt[4]{x^7}}
\end{aligned}$$

(c)

$$s(t) = (t^2 - 3)^4$$

$$\begin{aligned}
s'(t) &= 4(t^2 - 3)^{4-1}(2t) \\
&= 8t(t^2 - 3)^3
\end{aligned}$$

(d)

$$w(y) = (y^2 + 4)^2(2y^3 - 1)^3$$

$$\begin{aligned}
w'(y) &= \left[(y^2 + 4)^2(2y^3 - 1)^3\right]' \\
&= \left[(y^2 + 4)^2\right]'(2y^3 - 1)^3 + (y^2 + 4)^2\left[(2y^3 - 1)^3\right]' \\
&= 2\left[(y^2 + 4)^{(2-1)}\right](2y)(2y^3 - 1)^3 + (y^2 + 4)^2(3)\left[(2y^3 - 1)^{(3-1)}\right](6y^2) \\
&= 2(y^2 + 4)(2y)(2y^3 - 1)^3 + (y^2 + 4)^2(3)(2y^3 - 1)^2(6y^2) \\
&= 4y(y^2 + 4)(2y^3 - 1)^3 + 18y^2(y^2 + 4)^2(2y^3 - 1)^2 \\
&= 2y(y^2 + 4)(2y^3 - 1)^2\left[2(2y^3 - 1) + 9y(y^2 + 4)\right] \\
&= 2y(y^2 + 4)(2y^3 - 1)^2\left[4y^3 - 2 + 9y^3 + 36y\right] \\
&= 2y(y^2 + 4)(2y^3 - 1)^2(13y^3 + 36y - 2)
\end{aligned}$$

9. (1,0 ponto)

Se $y = x^2 - 4x$ e $x = \sqrt{2t^2+1}$, ache dy/dt quando $t = \sqrt{2}$.

Solução:

$$y = x^2 - 4x \quad \longrightarrow \quad y' = \frac{dy}{dx} = 2x - 4$$

e

$$x = \sqrt{2t^2+1} \quad \longrightarrow \quad x' = \frac{dx}{dt} = \frac{1}{2} \cdot \frac{1}{\sqrt{2t^2+1}} \cdot (4t) = \frac{2t}{\sqrt{2t^2+1}}$$

Pela regra da cadeia

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2x-4)\frac{2t}{\sqrt{2t^2+1}} = (2(\sqrt{2t^2+1}) - 4)\frac{2t}{\sqrt{2t^2+1}}$$

$$= \frac{(4t\sqrt{2t^2+1} - 8t)}{\sqrt{2t^2+1}}$$

avaliando em $t = \sqrt{2}$

$$\frac{dy}{dt}\left(\sqrt{2}\right) = \frac{\left(4\sqrt{2}\sqrt{2(\sqrt{2})^2+1} - 8\sqrt{2}\right)}{\sqrt{2(\sqrt{2})^2+1}} = \frac{\left(4\sqrt{2}\sqrt{5} - 8\sqrt{2}\right)}{\sqrt{5}} = \frac{4\sqrt{2}\left(\sqrt{5}-2\right)}{\sqrt{5}}$$

10. (1,0 ponto)

Calcule as primeiras e segundas derivadas das seguintes funções:

(a) $f(x) = 3x^{1/2} - x^{3/2} + 2x^{-1/2}$

(b) $f(x) = 2x^2\sqrt{2-x}$

(c) $f(x) = \left(\dfrac{x^2-1}{2x^3+1}\right)^4$

Solução:

(a) $f(x) = 3x^{1/2} - x^{3/2} + 2x^{-1/2}$

$$f'(x) = 3 \cdot \frac{1}{2} \cdot x^{1/2-1} - \frac{3}{2} \cdot x^{3/2-1} + 2 \cdot \frac{-1}{2} \cdot x^{-1/2-1}$$

$$= \frac{3}{2} \cdot x^{-1/2} - \frac{3}{2} \cdot x^{1/2} - \frac{2}{2} \cdot x^{-3/2}$$

$$= \frac{3}{2x^{1/2}} - \frac{3x^{1/2}}{2} - \frac{1}{x^{3/2}}$$

$$f''(x) = \frac{3}{2} \cdot \frac{-1}{2} \cdot x^{-3/2} - \frac{3}{2} \cdot \frac{1}{2} \cdot x^{-1/2} - \frac{-3}{2} \cdot x^{-5/2}$$

$$= -\frac{3}{4} \cdot x^{-3/2} - \frac{3}{4} \cdot x^{-1/2} + \frac{3}{2} \cdot x^{-5/2}$$

$$= -\frac{3}{4x^{3/2}} - \frac{3}{4x^{1/2}} + \frac{3}{2x^{5/2}}$$

$$= -\frac{3}{4\sqrt[2]{x^3}} - \frac{3}{4\sqrt{x}} + \frac{3}{2\sqrt[2]{x^5}}$$

(b) $f(x) = 2x^2\sqrt{2-x}$

$$f'(x) = \left[2x^2\sqrt{2-x}\right]'$$

$$= 2\left[\left(x^2\right)'\sqrt{2-x} + x^2\left(\sqrt{2-x}\right)'\right]$$

$$= 2\left[2x\sqrt{2-x} + x^2\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2-x}}\right)(-1)\right]$$

$$= 2\left[2x\sqrt{2-x} - \frac{x^2}{2\sqrt{2-x}}\right]$$

$$= 2\left[\frac{4x(2-x) - x^2}{2\sqrt{2-x}}\right]$$

$$= \frac{8x - 5x^2}{\sqrt{2-x}}$$

$$f''(x) = \left[\frac{8x - 5x^2}{\sqrt{2-x}}\right]'$$

$$= \left[\frac{[8x - 5x^2]' \cdot \left[\sqrt{2-x}\right] - [8x - 5x^2] \cdot \left[\sqrt{2-x}\right]'}{\left[\sqrt{2-x}\right]^2}\right]$$

$$= \frac{[8 - 10x] \cdot \left[\sqrt{2-x}\right] - [8x - 5x^2] \cdot \left[\frac{1}{2}\frac{1}{\sqrt{2-x}}(-1)\right]}{\left[\sqrt{2-x}\right]^2}$$

$$= \frac{[8 - 10x] \cdot \left[\sqrt{2-x}\right] + [8x - 5x^2] \cdot \left[\frac{1}{2\sqrt{2-x}}\right]}{\left[\sqrt{2-x}\right]^2}$$

$$= \frac{2\sqrt{2-x}\,[8 - 10x] \cdot \left[\sqrt{2-x}\right] + [8x - 5x^2]}{2\sqrt{2-x}\left[\sqrt{2-x}\right]^2}$$

$$= \frac{2(2-x)\,[8-10x] + [8x-5x^2]}{2\sqrt{2-x}\left[\sqrt{2-x}\right]^2}$$

$$= \frac{2(16-20x-8x+10x^2) + 8x - 5x^2}{2(2-x)\sqrt{2-x}}$$

$$= \frac{32-40x-16x+20x^2+8x-5x^2}{2(2-x)\sqrt{2-x}}$$

$$= \frac{32-48x+15x^2}{2(2-x)\sqrt{2-x}}$$

(c) $$f(x) = \left(\frac{x^2-1}{2x^3+1}\right)^4$$

$$f'(x) = 4\left(\frac{x^2-1}{2x^3+1}\right)^3 \left[\frac{x^2-1}{2x^3+1}\right]'$$

$$= 4\left(\frac{x^2-1}{2x^3+1}\right)^3 \left[\frac{(x^2-1)'\cdot(2x^3+1) - (x^2-1)\cdot(2x^3+1)'}{(2x^3+1)^2}\right]$$

$$= 4\left(\frac{x^2-1}{2x^3+1}\right)^3 \left[\frac{(2x)\cdot(2x^3+1) - (x^2-1)\cdot(6x^2)}{(2x^3+1)^2}\right]$$

$$= 4\left(\frac{x^2-1}{2x^3+1}\right)^3 \left[\frac{(4x^4+2x) - (6x^4-6x^2)}{(2x^3+1)^2}\right]$$

$$= 4\left(\frac{x^2-1}{2x^3+1}\right)^3 \left[\frac{2x-2x^4+6x^2}{(2x^3+1)^2}\right]$$

$$= \frac{8(x^2-1)^3(x-x^4+3x^2)}{(2x^3+1)^5}$$

$f''(x) =$ Item dispensado. Não é necessário fazer.