Curso Superior de Tecnologia em Sistemas de Computação
Disciplina: Matemática para Computação
AP1 - 1^o semestre de 2012 - Gabarito

Questões

1. (2,50 pontos)

Determine as inversas das seguintes funções:

(a) $f(x) = 2x^4 - 3$

(b) $f(x) = \sqrt[4]{x-1}$

Solução:

(a) $f(x) = 2x^4 - 3$

$$y = 2x^4 - 3 \implies y + 3 = 2x^4 \implies \frac{y+3}{2} = x^4 \implies \sqrt[4]{\frac{y+3}{2}} = x$$

$$f^{-1}(x) = \sqrt[4]{\frac{x+3}{2}} \quad x \geq -3$$

(b) $f(x) = \sqrt[4]{x-1}$

$$y = \sqrt[4]{x-1} \implies y^4 = x - 1 \implies x = y^4 + 1$$

$$f^{-1}(x) = x^4 + 1$$

2. (2,50 pontos)

Calcule os limites abaixo.

(a) $$\lim_{x\to 3} \frac{x^3-27}{x^2-9}$$

(b) $$\lim_{x\to 1} \frac{x^2+x-2}{(x-1)^2}$$

Solução:

(a) $$\lim_{x\to 3} \frac{x^3-27}{x^2-9} = \lim_{x\to 3} \frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)} = \lim_{x\to 3} \frac{x^2+3x+9}{x+3} = \frac{3^2+3\cdot 3+9}{3+3} = \frac{9}{2}$$

(b) $$\lim_{x\to 1} \frac{x^2+x-2}{(x-1)^2} = \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)^2} = \lim_{x\to 1} \frac{(x+2)}{(x-1)} = \infty$$

3. (2,50 pontos)

Discuta a continuidade da função usando a definição de continuidade:

$$f(x) = \frac{\sqrt{x^2-9}}{x-4}$$

Solução:

$f(x)$ não é definida se o denomindador $x-4$ se anular ou se o radicando x^2-9 for negativo, isto , se $x=4$ ou se $-3<x<3$. Qualquer outro número real está em um dos intervalos $(-\infty,-3]$, $[3,4)$ ou $(4,\infty)$. Temos então que provar a continuidade de $f(x)$ em todo o domínio. Ou seja, no interior dos intervalos (*usando limites*) e nos extremos (*usando limites laterais*).

Intervalo $(-\infty,-3]$:

nos pontos interiores

$$\lim_{x\to c} \frac{\sqrt{x^2-9}}{x-4} = \frac{\sqrt{c^2-9}}{c-4} = f(c) \qquad \forall c \in (-\infty,-3] \cup [3,4) \cup (4,\infty)$$

nos extremos dos intervalos

$$
\begin{aligned}
\lim_{x\to-\infty}\frac{\sqrt{x^2-9}}{x-4} &= \lim_{x\to-\infty}\frac{\sqrt{x^2-9}}{x-4}\frac{\sqrt{x-4}}{\sqrt{x-4}} \\
&= \lim_{x\to-\infty}\frac{\sqrt{(x^2-9)(x-4)}}{\sqrt{(x-4)^3}} \\
&= \lim_{x\to-\infty}\frac{\sqrt{(x^3-4x^2-9x+36)}}{\sqrt{(x-4)(x^2-8x+16)}} \\
&= \lim_{x\to-\infty}\frac{\sqrt{(x^3-4x^2-9x+36)}}{\sqrt{(x^3-12x^2+48x-64)}} \\
&= \sqrt{\lim_{x\to-\infty}\frac{(x^3-4x^2-9x+36)}{(x^3-12x^2+48x-64)}} \\
&= \sqrt{\lim_{x\to-\infty}\frac{\frac{(x^3-4x^2-9x+36)}{x^3}}{\frac{(x^3-12x^2+48x-64)}{x^3}}} \\
&= \sqrt{\lim_{x\to-\infty}\frac{\frac{x^3}{x^3}-\frac{4x^2}{x^3}-\frac{9x}{x^3}+\frac{36}{x^3}}{\frac{x^3}{x^3}-\frac{12x^2}{x^3}+\frac{48x}{x^3}-\frac{64}{x^3}}} \\
&= \sqrt{\lim_{x\to-\infty}\frac{1-\frac{4}{x}-\frac{9}{x^2}+\frac{36}{x^3}}{1-\frac{12}{x}+\frac{48}{x^2}-\frac{64}{x^3}}} \\
&= \sqrt{\frac{\lim_{x\to-\infty}\left[1-\frac{4}{x}-\frac{9}{x^2}+\frac{36}{x^3}\right]}{\lim_{x\to-\infty}\left[1-\frac{12}{x}+\frac{48}{x^2}-\frac{64}{x^3}\right]}} \\
&= \sqrt{\frac{\lim_{x\to-\infty}1-\lim_{x\to-\infty}\frac{4}{x}-\lim_{x\to-\infty}\frac{9}{x^2}+\lim_{x\to-\infty}\frac{36}{x^3}}{\lim_{x\to-\infty}1-\lim_{x\to-\infty}\frac{12}{x}+\lim_{x\to-\infty}\frac{48}{x^2}-\lim_{x\to-\infty}\frac{64}{x^3}}} \\
&= \sqrt{\frac{\lim_{x\to-\infty}1-4\lim_{x\to-\infty}\frac{1}{x}-9\lim_{x\to-\infty}\frac{1}{x^2}+36\lim_{x\to-\infty}\frac{1}{x^3}}{\lim_{x\to-\infty}1-12\lim_{x\to-\infty}\frac{1}{x}+48\lim_{x\to-\infty}\frac{1}{x^2}-64\lim_{x\to-\infty}\frac{1}{x^3}}} \\
&= \sqrt{\frac{1-4\cdot0-9\cdot0+36\cdot0}{1-12\cdot0+48\cdot0-64\cdot0}}=\sqrt{\frac{1-0-0+0}{1-0+0-0}}=\sqrt{\frac{1}{1}}=\sqrt{1}=1
\end{aligned}
$$

$$\lim_{x\to -3} \frac{\sqrt{x^2-9}}{x-4} = \frac{\sqrt{(-3)^2-9}}{(-3)-4} = \frac{\sqrt{9-9}}{-7} = 0$$

Intervalo $[3,4)$:

$$\lim_{x\to 3} \frac{\sqrt{x^2-9}}{x-4} = \frac{\sqrt{(3)^2-9}}{(3)-4} = \frac{\sqrt{9-9}}{-7} = 0$$

$$\lim_{x\to 4^-} \frac{\sqrt{x^2-9}}{x-4}$$

4. (2,50 pontos)

Calcule as derivadas de primeira ($f'(x)$), segunda ($f''(x)$) e terceira ($f'''(x)$) ordens das funções

(a)

$$f(x) = \frac{2}{x^{1/2}} + \frac{6}{x^{1/3}} - \frac{2}{x^{3/2}} - \frac{4}{x^{3/4}}$$

(b)

$$f(x) = \frac{3-2x}{3+2x}$$

Solução:

(a) $$f(x) = \frac{2}{x^{1/2}} + \frac{6}{x^{1/3}} - \frac{2}{x^{3/2}} - \frac{4}{x^{3/4}}$$

$$f(x) = 2x^{-1/2} + 6x^{-1/3} - 2x^{-3/2} - 4x^{-3/4}$$

Primeira Derivada:

$$f'(x) = -\frac{1}{2}2x^{-1/2-1} - \frac{1}{3}6x^{-1/3-1} + \frac{3}{2}2x^{-3/2-1} + \frac{3}{4}4x^{-3/4-1}$$

$$f'(x) = -x^{-3/2} - 2x^{-4/3} + 3x^{-5/2} + 3x^{-7/4}$$

$$f'(x) = -\frac{1}{x^{3/2}} - \frac{2}{x^{4/3}} + \frac{3}{x^{5/2}} + \frac{3}{x^{7/4}}$$

Segunda Derivada:

$$f''(x) = -\left(-\frac{3}{2}x^{-3/2-1}\right) - 2\left(-\frac{4}{3}x^{-4/3-1}\right) + 3\left(-\frac{5}{2}x^{-5/2-1}\right) + 3\left(-\frac{7}{4}x^{-7/4-1}\right)$$

$$f''(x) = \frac{3}{2}x^{-5/2} + \frac{8}{3}x^{-7/3} - \frac{15}{2}x^{-7/2} - \frac{21}{4}x^{-11/4}$$

$$f''(x) = \frac{3}{2x^{5/2}} + \frac{8}{3x^{7/3}} - \frac{15}{2x^{7/2}} - \frac{21}{4x^{11/4}}$$

Terceira Derivada:

$$f'''(x) = \frac{3}{2}\left(-\frac{5}{2}x^{-5/2-1}\right) + \frac{8}{3}\left(-\frac{7}{3}x^{-7/3-1}\right) - \frac{15}{2}\left(-\frac{7}{2}x^{-7/2-1}\right) - \frac{21}{4}\left(-\frac{11}{4}x^{-11/4-1}\right)$$

$$f'''(x) = -\frac{15}{4}x^{-7/2} - \frac{56}{9}x^{-10/3} + \frac{105}{4}x^{-9/2} + \frac{231}{16}x^{-15/4}$$

$$f'''(x) = -\frac{15}{4x^{7/2}} - \frac{56}{9x^{10/3}} + \frac{105}{4x^{9/2}} + \frac{231}{16x^{15/4}}$$

(b) $$f(x) = \frac{3-2x}{3+2x}$$

Da regra do quociente:

$$\left(\frac{f(x)}{g(x)}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Primeira Derivada:

$$f'(x) = \frac{(3-2x)'(3+2x) - (3-2x)(3+2x)'}{(3+2x)^2}$$

$$f'(x) = \frac{-6-4x-6+4x}{(3+2x)^2} = \frac{-12}{(3+2x)^2}$$

Segunda Derivada:

$$f''(x) = \frac{(-12)'(3+2x)^2 - (-12)\left[(3+2x)^2\right]'}{(3+2x)^4} = \frac{0\cdot(3+2x)^2 + 12\left[9+12x+4x^2\right]'}{(3+2x)^4}$$

$$f''(x) = \frac{0 + 12\left[12 + 4(2x)\right]}{(3+2x)^4} = \frac{144+96x}{(3+2x)^4}$$

Terceira Derivada:

$$f'''(x) = \frac{(144+96x)'(3+2x)^4 - (144+96x)\left[(3+2x)^4\right]'}{(3+2x)^8}$$

$$f'''(x) = \frac{96(3+2x)^4 - (144+96x)\left[4\cdot(3+2x)^3\cdot(3+2x)'\right]}{(3+2x)^8}$$

$$f'''(x) = \frac{96(3+2x)^4 - 48(3+2x)\left[8(3+2x)^3\right]}{(3+2x)^8}$$

$$f'''(x) = \frac{96(3+2x)^4 - 48\cdot 8(3+2x)^4}{(3+2x)^8}$$

$$f'''(x) = \frac{-288(3+2x)^4}{(3+2x)^8}$$

$$f'''(x) = \frac{-288}{(3+2x)^4}$$