Curso Superior de Tecnologia em Sistemas de Computação
Disciplina: Matemática para Computação
AP1 - 1^o semestre de 2011 - Gabarito

Questões

1. (1,00 ponto)

   Determine as inversas das seguintes funções:

   (a) $f(x) = 2x^4 - 3$

   (b) $f(x) = \sqrt[4]{x-1}$

   Solução:

   (a) $f(x) = 2x^4 - 3$

   $$y = 2x^4 - 3 \implies y + 3 = 2x^4 \implies \frac{y+3}{2} = x^4 \implies \sqrt[4]{\frac{y+3}{2}} = x$$

   $$f^{-1}(x) = \sqrt[4]{\frac{x+3}{2}} \quad x \geq -3$$

   (b) $f(x) = \sqrt[4]{x-1}$

   $$y = \sqrt[4]{x-1} \implies y^4 = x - 1 \implies x = y^4 + 1$$

   $$f^{-1}(x) = x^4 + 1$$

2. (1,50 pontos)

   Calcule os limites abaixo:

(a) $$\lim_{x\to 1} \frac{(3x-1)^2}{(x+1)^3}$$

(b) $$\lim_{x\to 4} \frac{x-4}{x^2-x-12}$$

Solução:

(a) $$\lim_{x\to 1} \frac{(3x-1)^2}{(x+1)^3} = \frac{(3\cdot 1-1)^2}{(1+1)^3} = \frac{(2)^2}{(2)^3} = \frac{1}{2}$$

(b) $$\lim_{x\to 4} \frac{x-4}{x^2-x-12} = \lim_{x\to 4} \frac{x-4}{(x+3)(x-4)} = \lim_{x\to 4} \frac{1}{(x+3)} = \frac{1}{7}$$

3. (1,50 pontos)

Calcule os seguintes limites infinitos,

(a) $$\lim_{x\to +\infty} \frac{2x+3}{4x-5}$$

(b) $$\lim_{x\to +\infty} \frac{x^2+5x+6}{x+1}$$

Solução:

(a) $$\lim_{x\to +\infty} \frac{2x+3}{4x-5} = \lim_{x\to +\infty} \frac{2x+3}{4x-5}\cdot\frac{x}{x} = \lim_{x\to +\infty} \frac{2x/x+3/x}{4x/x-5/x} = \lim_{x\to +\infty} \frac{2+3/x}{4-5/x} =$$

$$\frac{2+0}{4-0} = \frac{1}{2}$$

(b) $$\lim_{x\to +\infty} \frac{x^2+5x+6}{x+1} = \lim_{x\to +\infty} \frac{x^2+5x+6}{x+1}\cdot\frac{x^2}{x^2} = \lim_{x\to +\infty} \frac{x^2/x^2+5x/x^2+6/x^2}{x/x^2+1/x^2} =$$

$$\lim_{x\to +\infty} \frac{1+5/x+6/x^2}{1/x+1/x^2} = \frac{1+5/x+6/x^2}{1/x+1/x^2} = +\infty$$

4. (2,00 pontos)

Ache as descontinuidades das seguintes funções (se existirem), justifique sua resposta.

(a) $$f(x) = \frac{x-1}{(x+3)(x-2)}$$

(b) $$f(x) = \mid x \mid - x$$

Solução:

(a) $$f(x) = \frac{x-1}{(x+3)(x-2)}$$

$f(x)$ é descontínua em $x=-3$ e $x=2$, já que sequer é definida nestes pontos.

(b) $f(x) = \mid x \mid - x$

$f(x)$ é sempre contínua, posto que é a soma de duas funções contínuas.

5. (2,00 pontos) ---

Calcule as derivadas das seguintes funções.

(a) $f(x) = \dfrac{x^2}{\sqrt{4-x^2}}$

(b) $f(t) = (t^2-3)^4$

Solução:

(a) $f(x) = \dfrac{x^2}{\sqrt{4-x^2}}$

$$f'(x) = \frac{(x^2)'(4-x^2)^{1/2} - (x^2)[(4-x^2)^{1/2}]'}{(\sqrt{4-x^2})^2}$$

$$f'(x) = \frac{2x(4-x^2)^{1/2} - (x^2)(1/2)(4-x^2)^{-1/2}(-2x)}{4-x^2}$$

$$f'(x) = \frac{2x(4-x^2)^{1/2} + x^3(4-x^2)^{-1/2}}{4-x^2} \cdot \frac{(4-x^2)^{1/2}}{(4-x^2)^{1/2}}$$

$$f'(x) = \frac{2x(4-x^2) + x^3}{(4-x^2)^{3/2}} = \frac{8x - x^3}{(4-x^2)^{3/2}}$$

(b) $f(t) = (t^2-3)^4$

$$f'(t) = 4(t^2-3)^3(2t) = 8t(t^2-3)^3$$

6. (2,00 pontos) ---

Encontre as derivadas de primeira e segunda ordens das seguintes funções:

(a) $f(x) = 5x^6 - 2x^3 + x^{-5}$

(b) $f(x) = \left(\dfrac{x}{x+1}\right)^5$

(c) $f(w) = \dfrac{w}{\sqrt[2]{1-4w^2}}$

Solução:

(a) $f(x) = 5x^6 - 2x^3 + x^{-5}$

$$f'(x) = 30x^5 - 6x^2 - 5x^{-6}$$

$$f''(x) = 150x^4 - 12x + 30x^{-7} = 150x^4 - 12x + \frac{30}{x^7}$$

(b)
$$f(x) = \left(\frac{x}{x+1}\right)^5$$

$$f'(x) = 5\left(\frac{x}{x+1}\right)^4\left(\frac{x}{x+1}\right)' = 5\left(\frac{x}{x+1}\right)^4\left(\frac{(x)'(x+1)-(x)(x+1)'}{(x+1)^2}\right)$$

$$f'(x) = 5\left(\frac{x}{x+1}\right)^4\left(\frac{1(x+1)-(x)1}{(x+1)^2}\right) = 5\left(\frac{x^4}{(x+1)^4}\right)\left(\frac{1}{(x+1)^2}\right)$$

$$f'(x) = 5\frac{x^4}{(x+1)^6}$$

$$f''(x) = 5\frac{(x^4)'((x+1)^6)-(x^4)((x+1)^6)'}{((x+1)^6)^2}$$

$$f''(x) = 5\frac{(4x^3)((x+1)^6)-(x^4)(6(x+1)^5(1))}{(x+1)^{12}}$$

$$f''(x) = 5\frac{4x^3(x+1)^6-6x^4(x+1)^5}{(x+1)^{12}}$$

$$f''(x) = 5\frac{4x^3(x+1)-6x^4}{(x+1)^7} = 5\frac{4x^4+4x^3-6x^4}{(x+1)^7} = 5\frac{4x^3-2x^4}{(x+1)^7}$$

$$f''(x) = 10\frac{2x^3-x^4}{(x+1)^7}$$

(c)
$$f(w) = \frac{w}{\sqrt[2]{1-4w^2}}$$

$$f'(w) = \frac{(w)'(\sqrt[2]{1-4w^2})-(w)(\sqrt[2]{1-4w^2})'}{[\sqrt[2]{1-4w^2}]^2}$$

$$f'(w) = \frac{1(\sqrt[2]{1-4w^2})-(w)(1/2)((1-4w^2)^{-1/2}(-8w))}{1-4w^2}$$

$$f'(w) = \frac{(1-4w^2)^{1/2}+4w^2(1-4w^2)^{-1/2}}{1-4w^2}$$

$$f'(w) = \frac{(1-4w^2)^{1/2}+4w^2(1-4w^2)^{-1/2}}{1-4w^2}\cdot\frac{(1-4w^2)^{1/2}}{(1-4w^2)^{1/2}}$$

$$f'(w) = \frac{(1-4w^2)+4w^2}{(1-4w^2)^{3/2}} = \frac{1}{(1-4w^2)^{3/2}}$$

$$f''(w) = \left[\frac{1}{(1-4w^2)^{3/2}}\right]' = \frac{(1)'((1-4w^2)^{3/2})-(1)((1-4w^2)^{3/2})'}{[(1-4w^2)^{3/2}]^2}$$

$$f''(w) = \frac{-1((1-4w^2)^{3/2})'}{(1-4w^2)^3} = \frac{-1((3/2)(1-4w^2)^{1/2}(-8w))}{(1-4w^2)^3}$$

$$f''(w) = \frac{12w(1-4w^2)^{1/2}}{(1-4w^2)^3} = \frac{12w(1-4w^2)^{1/2}}{(1-4w^2)^3} \cdot \frac{(1-4w^2)^{-1/2}}{(1-4w^2)^{-1/2}}$$

$$f''(w) = \frac{12w}{(1-4w^2)^{5/2}} = \frac{12w}{\sqrt{(1-4w^2)^5}}$$