Computability Theory **Fall 2020**

Lecture 13: Incompleteness II

Lecturer: Renjie Yang

Peano arithmetic PA

$$\varphi(0) \wedge \forall x(\varphi(x) \rightarrow \varphi(x+1)) \rightarrow \forall x \varphi(x)$$

- If $PA \vdash \varphi$, then $PA \vdash Prov_{PA}(\ulcorner \varphi \urcorner)$
- For every formula φ and ψ, $PA \vdash Prov_{PA}(\ulcorner \varphi \rightarrow \psi \urcorner) \rightarrow (Prov_{PA}(\ulcorner \varphi \urcorner) \rightarrow Prov_{PA}(\ulcorner \psi \urcorner))$
- For every formula ϕ, $PA \rightarrow Prov_{PA}(\ulcorner \varphi \urcorner) \rightarrow Prov_{PA}(\ulcorner Prov_{PA}(\ulcorner \varphi \urcorner) \urcorner)$

Theorem 13.1 *Let T be any theory extending Q, and let $\psi(x)$ be any formula with free variable x. Then there is a sentence φ such that T proves $\varphi \leftrightarrow \psi(\ulcorner \varphi \urcorner)$.*

Theorem 13.2 *Let T be any ω-consistent, computably axiomatized theory extending Q. Then T is not complete.*

Theorem 13.3 *Assuming PA is consistent, then PA does not prove $\neg Prov_{PA}(\ulcorner 0 = 1 \urcorner)$.*