Computability Theory **Fall 2020**

Lecture 9: Recursive Enumerable Sets(Unfinished)

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9.1 Semicomputable Predicates

Definition 9.1 *A predicate $P(x^{(n)})$ is called **semicomputable** if there exists a partially computable function whose domain is the set $\{x^{(n)}|\ P(x^{(n)})\}$*

Theorem 9.2 *Every computable predicate is semicomputable.*

Proof: Let $R(x^{(n)})$ be computable. Then, $\{x^{(n)}|\ R(x^{(n)})\}$ is the domain of the partial computable function $min_y[C_R(x^{(n)}) + y = 0]$ ■

Theorem 9.3 *Let $R(x^{(n)}) \leftrightarrow \exists y P(y, x^{(n)})$, where $P(y, x^{(n)})$ is computable. Then $R(y, x^{(n)})$ is semicomputable.*

Proof: $\{x^{(n)}|\ R(x^{(n)})\}$ is the domain of the partial computable function $min_y[C_P(x^{(n)}) = 0]$ ■

Theorem 9.4 *Let $R(x^{(n)})$ be a semicomputable predicate. Then there exists a computable predicate $P(y, x^{(n)})$ such that $R(x^{(n)}) \leftrightarrow \exists y P(y, x^{(n)})$.*

9.2 Recursive Enumerable Sets