**Computability Theory** **Fall 2020**

# Problem Set 1

*Due Date: 11/15/2020 Before 11:59pm*

**1**. Let $A_n = \{a^k \mid \text{where } k \text{ is a multiple of } n\}$. Show that for each $n \geqslant 1$, the language $A_n$ is regular.

**2**. Read Spiser section 1.4 (page 77 - 82), show that $L = \{0^m 1^n \mid m \neq n\}$ is not regular.

**3**. Give a context-free grammar that generates the language

$$B = \{a^i b^j c^k \mid i, j, k \geqslant 0 \text{ and either } i = j \text{ or } j = k\}.$$

**4**. Let $C$ be a context-free language and $R$ be a regular language. Show that the language $R \cup C$ is context free.

**5**. Show that every regular language is Turing decidable.

**6**. Design a Turing machine that computes $f(n) = n!$.

**7**. Let $g_i(x)$ and $P_i(x)$ be primitive recursive for $i = 0, 1, \dots k$. Show that

$$f(x) = \begin{cases} g_1(x), & \text{if } P_1(x) \\ g_2(x), & \text{if } P_2(x) \\ \cdots & \\ g_k(x), & \text{otherwise} \end{cases}$$

is primitive recursive.

**8**. Show that every primitive recursive function is total.

**9**. Show that for each primitive recursive function there is a monotone primitive recursive function that is everywhere greater.

**10**. Show that not all recursive functions are primitive recursive.