Computability Theory **Fall 2020**

Problem Set 1

Due Date: 11/15/2020 Before 11:59pm

1. Let $A_n = \{a^k |$ where k is a multiple of $n\}$. Show that for each $n \geqslant 1$, the language A_n is regular.

2. Read Spiser section 1.4 (page 77 - 82), show that $L = \{0^m 1^n |\ m \neq n\}$ is not regular.

3. Give a context-free grammar that generates the language

$$B = \{a^i b^j c^k |\ i, j, k \geqslant 0 \text{ and either } i = j \text{ or } j = k\}.$$

4. Let C be a context-free language and R be a regular language. Show that the language $R \cup C$ is context free.

5. Show that every regular language is Turing decidable.

6. Design a Turing machine that computes $f(n) = n!$.

7. Let $g_i(x)$ and $P_i(x)$ are primitive recursive for $i = 0, 1, \ldots k$. Show that

$$f(x) = \begin{cases} g_1(x), & \text{if } P_1(x) \\ g_2(x), & \text{if } P_2(x) \\ \cdots & \\ g_k(x), & \text{otherwise} \end{cases}$$

is primitive recursive.

8. Show that every primitive recursive function is total.

9. Show that for each primitive recursive function there is a monotone primitive recursive function that is everywhere greater.

10. Show that not all recursive functions are primitive recursive.