The Idea of Proof: Part I

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Outline

Knowledge and Reasoning

- Part of the process by which we acquire knowledge is the process of reasoning.
- Example: scientific knowledge
  - Scientists gather data, and use data to show that some theories are right while other theories are wrong.
  - To understand science, we need to understand when it is legitimate or not legitimate to draw a conclusion from what we have already known.

Knowledge and Reasoning

- The process of reasoning for a conclusion is also called **argument**.
- In order to know how we obtain new knowledge from what we have known, we need to understand the difference between good and bad arguments.
- Logic is the study of argumentation.

The Structure of Arguments

- **Premises**: what we presuppose.
- **Conclusion**: what we conclude from the premises.

An Example of Arguments

- No medieval king had absolute power over his subjects.
- Louis VII of France was a medieval king.

- So, Louis VII of France did not have absolute power over his subjects.

Validity of Arguments

- **Valid argument**: an argument where the conclusion really follows from the premises.
- Invalid argument: an argument that is not valid.

An Example of Invalid Argument

- No medieval king had absolute power over his subjects.
- Louis VII of France was a great horseman.

- So, Louis VII of France did not have absolute power over his subjects.

Valid Argument with False Premises

- No medieval king had absolute power over his subjects.
- Renjie Yang was a medieval king.

- So, Renjie Yang did not have absolute power over his subjects.

Deductive and Inductive Arguments

- **Deductive Argument**: an argument in which the truth of the premises absolutely guarantees the truth of the conclusion.
- **Inductive Argument**: an argument where the truth of the premises gives good reason to believe the conclusion, but does not absolutely guarantee the truth of the conclusion.

An Example of Deductive Arguments

- No medieval king had absolute power over his subjects.
- Louis VII of France was a medieval king.

- So, Louis VII of France did not have absolute power over his subjects.

An Example of Inductive Arguments

- None of the medieval texts we have studied argues against the existence of God.

- So, nobody in the Middle Ages argued against the existence of God.

The Philosophical Problem of Deductive Reasoning

Deductive reasoning has historically seemed the most fundamental, the very first thing a philosopher should try to understand.

- First, unlike other forms of reasoning, valid deductive reasoning provides a guarantee: we can be certain that if the premises of such an argument are true, the conclusion is also true.
- Second, the very possibility of deductive reasoning must be somehow connected both with language and with the structure of the world. How can the world and language be so structured that some claims make others necessary?

Problems for A Theory of Deductive Reasoning

- How can we determine whether or not a piece of reasoning from premises to a conclusion is a valid deductive argument?
- How can we determine whether or not a conclusion is necessitated by a set of premises? If a conclusion is necessitated by a set of premises, how can we find a valid deductive argument that demonstrates that necessary connection?
- What features of the structure of the world, the structure of language, and the relation between words and thoughts and things make deductive reasoning possible?

Outline

Euclid's Geometry

Euclid developed geometry as an *axiomatic system*.

- Definitions
- Common notions
- Postulates
- Theorems

Euclid's Geometry

Definitions:

1. A point is that which has no part.
2. A line is breadthless length.
3. The extremities of a line are points.
4. A straight line is a line that lies evenly with the points on itself.
5. A surface is that which has length and breadth only.

Euclid's Geometry

Definitions:

6. The extremities of a surface are lines.
7. A plane surface is a surface that lies evenly with the straight lines on itself.
8. A plane angle is the inclination to one another of two lines in a plane that meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called rectilinear.

Euclid's Geometry

Definitions:

10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right, and the straight line standing on the other is called a perpendicular to that on which it stands.
11. An obtuse angle is an angle greater than a right angle.
12. An acute angle is an angle less than a right angle. one another and do not lie in a straight line.
13. A boundary is that which is an extremity of anything.

Euclid's Geometry

Definitions:

14. A figure is that which is contained by any boundary or boundaries.
15. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another.
16. And the point is called the center of the circle.

Euclid's Geometry

Definitions:

19. Rectilinear figures are those contained by straight lines, trilateral figures being those contained by three.
20. Of trilateral figures, an equilateral triangle is that which has its three sides equal.

   …

23. Parallel straight lines are straight lines that, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

Euclid's Geometry

Common Notions:

1. Things that are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Euclid's Geometry

Postulates:

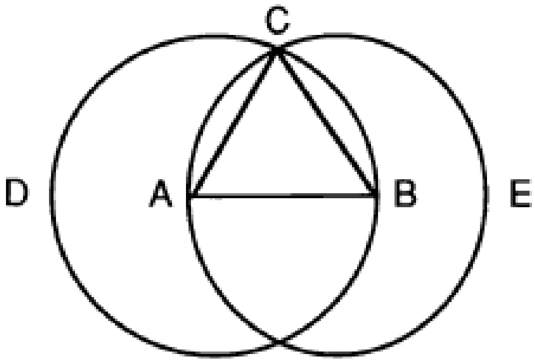
1. It is possible to draw a straight line from any point to any point.
2. It is possible to produce a finite straight line continuously in a straight line.
3. It is possible to describe a circle with any center and distance.
4. All right angles are equal to one another.
5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on the side of the angles less than the two right angles.

Euclid's Geometry

Proposition 1

- For every straight line segment, there exists an equilateral triangle having that line segment as one side.

Proposition 1



Thinking Things Through, Figure 1.2

Proof of Proposition 1

Proof: Let AB be the given finite straight line. Thus it is required to construct an equilateral triangle on the straight line AB. Let circle BCD be drawn with center A and distance (i.e., radius) AB (postulate 3). Again, let circle ACE be drawn with center B and distance BA (postulate 3). And from point C, at which the circles cut one another, to points A, B, let the straight lines CA, CB be joined (postulate 1). (See figure 1.2) Now since point A is the center of the circle CDB, AC is equal to AB. Again, since point B is the center of circle ACE, BC is equal to BA. But CA was also proved equal to AB. Therefore, each of the straight lines CA, CB is equal to AB. And things that are equal to the same thing are also equal to one another (common notion 1). Therefore, CA is also equal to CB. Therefore, the three straight lines CA, AB, BC are equal to one another. Therefore, triangle ABC is equilateral, and it has been constructed on the given straight line AB. QED.

Characterizations of Euclid' s proof

- The proof is like a short essay in which one sentence follows another in sequence.
- Each sentence of the proof is justified either by preceding sentences of the proof or by the definitions, postulates, or common notions.
- The conclusion to be proved is stated in the last sentence of the proof.
- The proposition proved is logically quite complex. It asserts that for every line segment L, there exists an object T that is an equilateral triangle, and that T and L stand in a particular relation, namely that the equilateral triangle T has line segment L as one side.

Characterizations of Euclid' s proof

- Euclid actually claims to prove something stronger. What he claims to prove is that if his postulates are understood to guarantee a method for finding a line segment connecting any two points (as with a ruler) and a method for constructing a circle of any specified radius (as with a compass), then there is a procedure that will actually construct an equilateral triangle having any given line segment as one side. Euclid's proof that such triangles exist is constructive. It shows they exist by giving a general procedure, or algorithm, for constructing them.

Characterizations of Euclid' s proof

- The proof comes with a picture (figure 1.2). The picture illustrates the idea of the proof and makes the sentences in the proof easier to understand. Yet the picture itself seems not to be part of the argument for the proposition, but rather a way of making the argument more easily understood. Or does the picture do more than that - is it really a silent part of the proof?

God and Saint Anselm

- God cannot be thought of as nonexistent. And certainly it exists so truly that it cannot be thought of as nonexistent. For something can be thought of as existing, which cannot be thought of as not existing, and this is greater than that which can be thought of as not existing. Thus if that than which a greater cannot be thought can be thought of as not existing, this very thing than which a greater cannot be thought is not that than which a greater cannot be thought. But this is contradictory. So, then, there truly is a being than which a greater cannot be thought - so truly that it cannot even be thought of as not existing

Anselm's Argument

- Premise 1: A being that cannot be thought of as not existing is greater than a being that can be thought of as not existing.
- Therefore, if God can be thought of as not existing, then a greater being that cannot be thought of as not existing can be thought of.
- Premise 2: God is the being than which nothing greater can be thought of.

- Conclusion: God cannot be thought of as not existing.

Anselm's Another Argument

- God is the greatest thing we can think of.
- Things can exist only in our imaginations, or they can also exist in reality.
- Things that exist in reality are always better than things that exist only in our imaginations.
- If God existed only in our imagination, he wouldn't be the greatest thing that we can think of, because God in reality would be better.

- Conclusion: Therefore, God must exist in reality.

Outline

Aristotle's Theory of Demonstration and Proof

- The Platonic Conception of Knowledge
- Aristotle's Conception of Nature
- Aristotle's Conception of Science
- Aristotle's Logic
- The Theory of the Syllogism
- Limitations of Aristotle's Syllogistic Theory of Deductive Argument
- After Aristotle
- Aristotelian Reasoning in Artificial Intelligence

The Platonic Conception of Knowledge

- For Plato, the paradigmatic scientific question is of the form "What is x?"
- Plato believed that any acceptable answer must give a combination of features shared by all things that are x and by no things that are not x.
- Knowledge requires certainty through infallible methods.
- Experience can never provide us with the kind of certainty that he required for knowledge.
- We really don't ever come to know anything. Learning from experience is just a process of recollection.

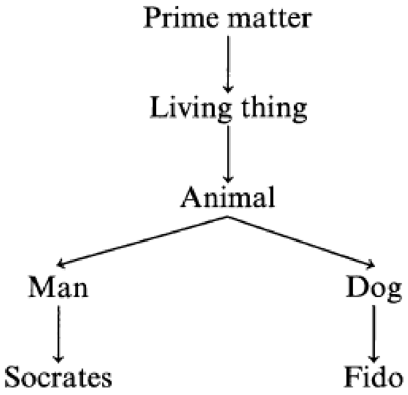
Aristotle's Conception of Nature

- If a thing changes, it acquires some new property or loses some old property. **Substance** is whatever that endures through change.
- **Attributes** or **properties** are features that can attach to a substance at one time and not attach to it at other times.
- Substance that has no properties and is completely unformed is calls **prime matter**.
- Aristotle's conception of the fundamental stuff of the universe can be very roughly pictured as gobs of stuff enduring through time but having various attributes stuck to it at any moment.

Aristotle's Conception of Nature

- The world is put together in the same way that our descriptions of it are assembled.
- Subject-Predicate structure:
  - The cat is black.
  - The animal that is a cat is trained.
  - Aristotle thought of nature in terms of hierarchies.

Aristotle's Conception of Nature



Aristotle's Conception of Nature

- A thing can have **accidental** or **essential** properties.
- For each part of nature, there is a hierarchy that includes only the essential attributes or forms of objects and ignores accidental attributes.
- The goal of science is to find the structure of the appropriate hierarchy for any subject, whether it is astronomy, biology, or cosmology.

Aristotle's Conception of Nature

"Why does the sun give warmth?"

- Formal cause
- Material cause
- Efficient cause
- Final cause

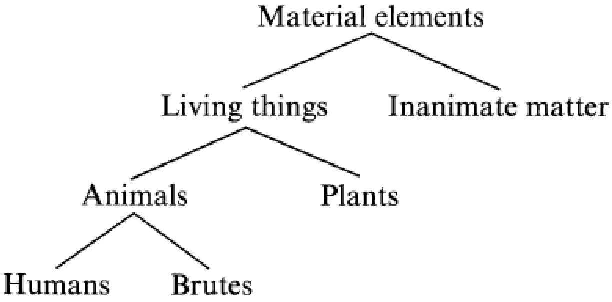
Aristotle's Conception of Science

- the science of any subject should constitute a system of knowledge claims.
- Fundamental claims, or axioms, could be used to deduce less fundamental claims.
- The scientific explanation of a general fact about the world consists in a valid deductive argument that has a description of that general fact as its conclusion and has true, fundamental claims as its premises.

Aristotle's Conception of Science

- Different sciences might have quite different axiomatic systems; there is one theory for biology, another for the constitution of matter, another for astronomy, and so on.
- The axioms of a scientific subject would be divided more or less as Euclid divided his axioms into common notions and postulates of a peculiarly geometric character.

Aristotle's Conception of Science



Aristotle's Conception of Science

- Each link in this picture corresponds to a general truth about the generation of humans: All humans are animals.
- Each of these sentences predicates something essential of its subject.
- Each point in the illustration represents a kind of thing obtained by imposing additional form on the matter that is the kind of thing at the point above it.
- The imposition of a form upon matter is brought about by a characteristic kind of efficient cause.

Aristotle's Conception of Science

An Aristotelian demonstration

1. All humans are animals.
2. All animals are living things.
3. Therefore, all humans are living things.
4. All humans are living things.
5. All living things are composed of matter.

Conclusion: all humans are composed of matter.

Aristotle's Logic

Aristotle's logic concerns sentences that have the following structure:

- a quantifier such as "all" or "some" or 'no"
- a subject term such as "humans" or "Socrates"
- a predicate term such as "are animals"

Aristotle's Logic

The characteristic form of inference in Aristotle's logic is the syllogism:

1. All humans are animals.
2. All animals are mortal.

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3. Therefore, all humans are mortal.

Venn Graph of the Syllogism

An Invalid Syllogism

1. All humans are animals.
2. Some animals are mortal.

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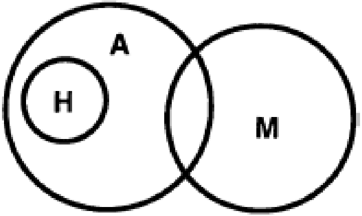
3. Therefore, all humans are mortal.

Venn Graph of the Syllogism



The Theory of the Syllogism

The four figures of syllogistic arguments:

Figure 1	Figure 2	Figure 3	Figure 4
A are *B*	*A* are *B*	*B* are *A*	*B* are *A*
B are *C*	*C* are *B*	*B* are *C*	*C* are *B*
A are *C*	*A* are *C*	*A* are *C*	*A* are *C*

The Theory of the Syllogism

Valid Aristotelian syllogisms in the first three figures:

1st figure	Barbara	Celarent	Darii	Ferio
A are B B are C A are C	All A are B All B are C All A are C	All A are B No B are C No A are C	Some A are B All B are C Some A are C	Some A are B No B are C Not all A are C
2nd figure	Cesare	Camestres	Festino	Baroco
A are B C are B A are C	All A are B No C are B No A are C	No A are B All C are B No A are C	Some A are B No C are B Not all A are C	Not all A are B All C are B Not all A are C
3rd figure	Darapti	Felapton	Disamis	Datisi
B are A B are C A are C	All B are A All B are C Some A are C	All B are A No B are C Not all A are C	All B are A Some B are C Some A are C	Some B are A All B are C Some A are C
	Bocardo	Ferison		
	All B are A Not all B are C Not all A are C	Some B are A No B are C Not all A are C		

The Theory of the Syllogism

- Aristotle held that the valid syllogisms of the first figure are *perfect*, by which he meant that their validity is obvious and self-evident and requires no proof.
- To show that the other syllogisms are valid, Aristotle assumed certain **rules of conversion**, which are really logical rules for inferring one sentence from another.

The Theory of the Syllogism

Rule 1 From "No X are Y," infer "No Y are X."

Rule 2 From "All X are Y," infer "Some Y are X."

Rule 3 From "Some X are Y," infer "Some Y are X."

The Theory of the Syllogism

Cesare	Celarent
All A are B	All A are B
No C are B	No B are C
No A are C	No A are C

The Theory of the Syllogism

How did Aristotle show that the many syllogistic forms of the second and third figures that do not occur in the table above are not valid?

- A syllogistic argument form is valid provided that, however we substitute real terms for the abstract A, B, and C in the syllogistic form, if the result is a syllogism with true premises, then the resulting conclusion is also true.
- So in order to show that a syllogistic form is not valid, Aristotle needed only to find examples of syllogisms of that form in which the premises are both true and the conclusion is false.

Limitations of Aristotle's Syllogistic Theory of Deductive Argument

- Aristotle developed his theory of the syllogism as part of a theory of scientific demonstration.
- While geometry was the paradigmatic Greek science, the theory of the syllogism cannot account for even the simplest demonstrations in Euclid's Elements.

Limitations of Aristotle's Syllogistic Theory of Deductive Argument

Proposition 1: For every straight line segment, there exists an equilateral triangle having that line segment as one side.

- The propositions of geometry are not all of a simple subject-predicate form. In fact, rather few of them are. Instead, geometrical propositions deal with relations among objects.
- The propositions of geometry do not all have just one quantifier; they may essentially involve repeated uses of "all" and "there exists."
- Proofs require devices for referring to the same object in different ways within the same sentence.

Limitations of Aristotle's Syllogistic Theory of Deductive Argument

Hypothetical syllogism

If Celarent is valid, then Cesare is valid.
Celarent is valid.
___
Therefore, Cesare is valid.

Limitations of Aristotle's Syllogistic Theory of Deductive Argument

- Aristotle meant "Some A are C" to be read as "There exist some things that are A and C."
- Rule 2: From "All X are Y," infer "Some Y are X."
- "All numbers that are both even and odd are divisible by two", therefore "Some numbers that are divisible by two are both even and odd"

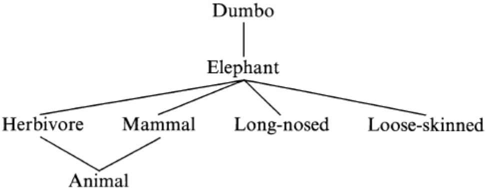
After Aristotle

- No fundamental advances appeared for the next 2400 years
- **Modus ponens** From "P" and "If P then Q," infer " Q."
- **Modus tollens** From 'Not Q" and "If P then Q," infer "Not P."

Aristotelian Reasoning in Artificial Intelligence

- Research in artificial intelligence attempts to devise programs that will enable computers to solve problems that require intelligence in human problem solvers.
- Most AI work is not committed to making computers solve problems in exactly the same way in which humans solve them.
- Knowledge organization: *is-a hierarchy*

Aristotelian Reasoning in Artificial Intelligence



Aristotelian Reasoning in Artificial Intelligence

- “Tweety is a bird.”“Can Tweety fly?”
- “Tweety is an ostrich.”“Can Tweety fly?”
- Monotonic inference: if a conclusion C can be validly inferred from a set of premises, then it can also he validly inferred from any set of premises that include the original premises.
- To make a computer reason as humans do in contexts where knowledge consists of generalized but not universal sentences, the computer must make inferences according to principles of nonmonotonic logic.

Reference

- Clark Glymour. (2015) *Thinking Things Through: An Introduction to Philosophical Issues and Achievements*, 2nd edition. A Bradford Book.