Experience, Knowledge, and Belief: Part I

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Three Principal Questions about Deductive Reasoning

1. How can we determine whether or not a piece of reasoning from premises to a conclusion is a valid deductive argument?
2. How can we determine whether or not a conclusion is necessitated by a set of premises? If a conclusion is necessitated by a set of premises, how can we find a valid deductive argument that demonstrates that necessary connection?
3. What features of the structure of the world, the structure of language, and the relations among words, thoughts, and things make deductive reasoning possible?

Three Principal Questions about Deductive Reasoning

Problem 1: How can we determine whether or not a piece of reasoning from premises to a conclusion is a valid deductive argument?

- Given a deductive argument, we put the premises, the conclusion, and the intermediate steps of the argument into formal notation and then determine whether or not each step of the formal argument follows from preceding steps or from the premises by rules of proof or by rules that can be derived from those rules.
- Using Frege's theory, we can represent proofs in number theory, algebra, geometry, set theory, and those empirical sciences that use mathematical reasoning.
- David Hilbert (1862 – 1943), Euclid's geometry

Three Principal Questions about Deductive Reasoning

Problem 3: How can we determine whether or not a conclusion is necessitated by a set of premises?

- Leibniz hoped that by formulating all of science in a formal language, an "alphabet of thought," we would obtain an algorithm, a mechanical means, to derive all of the consequences of any proposition.
- No algorithm exists that will determine for every first-order formula whether or not that formula is logically true. Hence no algorithm exists that will determine for every set of premises and every possible conclusion whether or not the premises entail the conclusion. Leibniz dreamed an impossible dream.
- Theory of computation → theoretical computer science.

Three Principal Questions about Deductive Reasoning

Problem 2: What features of the structure of the world, the structure of language, and the relations among words, thoughts, and things make deductive reasoning possible?

- The names and predicates of a language denote objects, properties, and relations, and in the actual world some objects may happen to exemplify any particular property or relation, and some may not.
- The denotations and the facts of the world determine the truth-values of sentences.
- What constitutes the relation of denotation between words on the one hand and things, properties, and relations on the other remains mysterious.

An Alternative Type of Science

- Euclid's geometry was the paradigm of science shared by Aristotle, Descartes, and many people in between.
- Revolutionary developments in science were replacing the Cartesian view of how knowledge is acquired with a quite different conception.
- Andreas Vesalius (1514 – 1564)
- Galileo Galilei (1564 - 1642)
- William Gilbert (1544 – 1603)
- William Harvey (1578 – 1657)

An Alternative Type of Science

- These and other discoveries did not seem to have the form expected by either Aristotelian or Cartesian science.
- There were no intuitions of general principles about lodestones or human physiology from which everything else in these subjects was deduced.
- Instead, examples of particular phenomena were observed, found to be repeatedly and regularly produced, and thus taken to hold generally.
- Inferences from particular instances to general conclusions were called **inductive**.

A More Ambitious Goal

- The goal of the new science was to go beyond simple generalizations of observed regularities to find their **causes** and the **laws** governing those causes.
- Scientists of the time searched for the hidden powers, causes, and structures that produce appearances, and they searched for the laws that govern such powers, causes, and structures.

A More Ambitious Goal

Copernicus and Harvey and Galileo seemed to be penetrating the "hidden springs" of nature.

- Copernicus (1473 – 1543): The movements of the sun and the star are only appearance.
- Johannes Kepler (1571 – 1630) used Copernican theory to establish three laws of motion of the planets.
- Harvey explained the pulse by postulating that the heart acts as a pump.

The Epistemology of Empirical Inquiry

How could scientists find the hidden structures and mechanism of the reality? What could the method be?

Outline

Bacon's Inductive Method

- Bacon's *Novum Organum*, 1620
- The content of scientific laws as conjunctions of conditions necessary and sufficient for observed effects.
- Scientific laws are to be discovered by applying the appropriate kind of reasoning to observations of particular cases.
- The important questions for scientific method concern what observations to make and how to reason from the observations to conclusions about causes.

Bacon's Inductive Method

To discover the cause of a phenomenon such as heat or light or gravity, one should collect three different kinds of instances.

- A list of positive instances;
- A list of negative instances;
- a list of degrees.

The Investigation of the Cause of Heat

Positive instances of heat:

- Everyday inanimate substances, such as rocks and metals
- Substances formerly hot, such as horse dung, lime, and ashes from fires
- The heat of animals
- Animal heat is increased by motion and exercise, wine, feasting, sex, fevers, and pains
- The heat of the sun in warm climates
- The heat of a flame, which depends on the body ignited ...

The Investigation of the Cause of Heat

Negative instances of heat:

- Rays of the Moon, stars, and comets
- The rays of the Sun on the tops of mountains where the air is thin
- The rays of the Sun in the polar regions
- The rays that emerge from a reverse burning glass
- Rotten wood that shines by night but is not hot
- The scales of fish and the body of the glowworm, which give light but are not warm

...

The Investigation of the Cause of Heat

Degrees of heat:

- Everyday inanimate substances, such as rocks and metals
- Substances formerly hot, such as horse dung, lime, and ashes from fires
- The heat of animals
- Animal heat is increased by motion and exercise, wine, feasting, sex, fevers, and pains
- The heat of the sun in warm climates
- The heat of a flame, which depends on the body ignited

The Investigation of the Cause of Heat

Bacon proposed that the cause or "form" be found by seeking a conjunction of properties such that

1. each property in the conjunction occurs in every positive instance;
2. in every negative instance, some property in the conjunction is absent;
3. the combination of properties increases in intensity as the phenomenon increases in intensity.

Conclusion: heat is the chaotic motion of the very small parts of a thing.

Outline

Issac Newton

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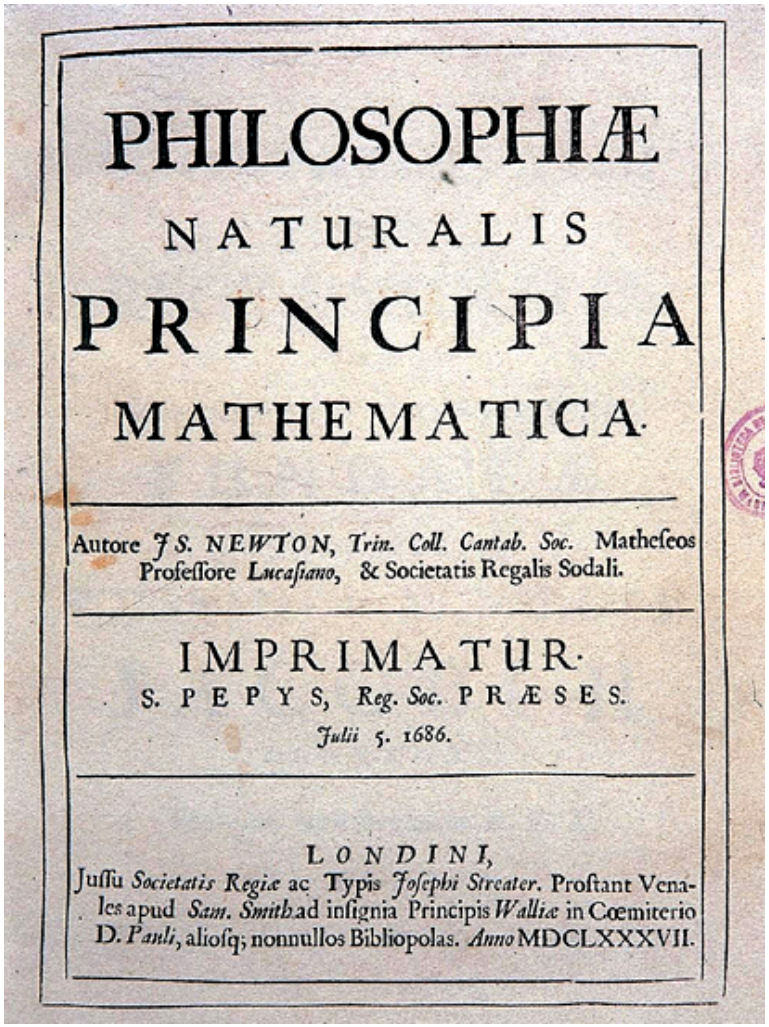
Issac Newton's *Principia*



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J S. NEWTON, Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtant Venales apud *Sam. Smith* ad inſignia Principis *Walliæ* in Cœmiterio D. *Pauli*, alioſq; nonnullos Bibliopolas. *Anno* MDCLXXXVII.

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The Newtonian Revolution

- Philosophiae Naturalis Principia Mathematica, 1686
- Part I: a system of definitions and axioms
- Part II, book 1: theorems on the motions of bodies subject to forces of various mathematical forms
- Part II, book 2: hydrostatics and wave motion
- Part II, book 3: the law of universal gravitation and implications

The Newtonian Revolution

- In Newton's conception, what we see and observe are apparent motions
- There are real motions with respect to absolute space.
- There are real causes as well - those fundamental forces that apparent causes are composed of.

The Rotating-Bucket Experiment.

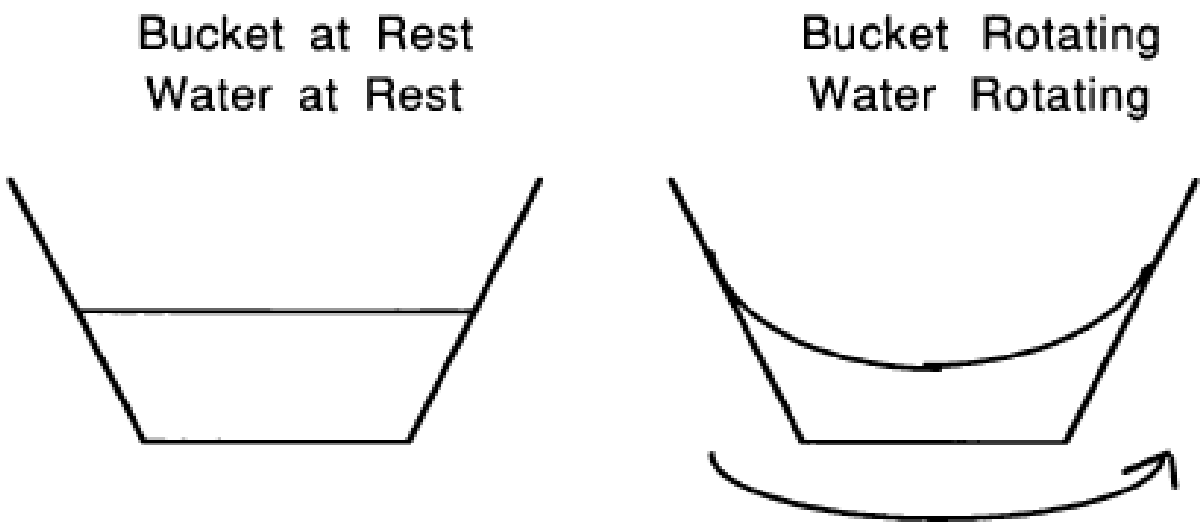

Figure 7.1 in *Thinking Things Through*

How to Infer True Motions and Causes

"It is indeed a matter of great difficulty to discover, and effectually to distinguish, the true motions of particular bodies from the apparent; because the parts of that immovable space, in which those motions are performed, do by no means come under the observation of our senses. Yet the thing is not altogether desperate; for we have some arguments to guide us, partly from the apparent motions, which are the differences of the true motions partly from the forces which are the causes and effects of the true motions. ... But how we are to obtain the true motions from their causes, effects, and apparent differences, and the converse, shall be explained more at large in the following treatise. For to this end it was that I composed it." - *Principia*

Newton's *Principia*

Basic concepts: Definitions

- Definition I: The quantity of matter is the measure of the same arising from its density and bulk conjointly.
- Definition II: The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.
- Definition III: The *vis insita*, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forwards in a right line.
- Definition IV: An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of uniform motion in a right line.

Newton's *Principia*

Phenomena or Appearances:

- Phenomenon I: That the circumjovial planets, by radii drawn to Jupiter's centre, describe areas proportional to the times of description, and that their periodic times, the fixed stars being at rest, are as the 3/2th power of their distances from its centre.
- Phenomenon II: That the circumsaturnal planets, by radii drawn to Saturn's centre, describe areas proportional to the times of description; and that their periodic times, the fixed stars being at rest, are as the 3/2th power of their distances from its centre.

Newton's *Principia*

Phenomena or Appearances:

- Phenomenon III: That the five primary planets, Mercury, Venus, Mars, Jupiter, and Saturn, with their several orbits, encompass the Sun.
- Phenomenon IV: That the circumsaturnal planets, by radii drawn to Saturn's centre, describe areas proportional to the times of description; and that their periodic times, the fixed stars being at rest, are as the 3/2th power of their distances from its centre.

Newton's *Principia*

3 Laws of Motions as axioms:

- Every body continues in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed upon it.
- The change of motion is proportional to the motive force impressed and is made in the direction of the straight line in which that force is impressed.
- To every action there is always opposed an equal reaction; that is, the mutual actions of two bodies upon each other are always equal and directed to contrary parts.

Prop. LXIX, Book I

Proposition

In a system of several bodies A, B, C, D, etc., if any one of those bodies, as A, attract all the rest, B, C, D, etc.,with accelerative forces that are reciprocally as the squares of the distances from the attracting body; and another body, as B, attracts also the rest, A, C, D, etc., with forces that are reciprocally as the squares of the distances from the attracting body; the absolute forces of the attracting bodies A and B will be to each other as those very bodies A and B to which those forces belong.

The Universal Law of Gravitation

Theorem VII

That there is a power of gravity pertaining to all bodies, proportional to the several quantities of matter which they contain.

Proof.

That all the planets gravitate one towards another, we have proved before; as well as that the force of gravity towards every one of them, considered apart, is inversely as the square of the distance of places from the centre of the planet. and thence (by Prop. LXIX, Book I, and its Corollaries) it follows, that the gravity tending towards all the planets is proportional to the matter which they contain.

The Universal Law of Gravitation

Proof.

Moreover, since all the parts of any planet A gravitate towards any other planet B; and the gravity of every part is to the gravity of the whole as the matter of the part to the matter of the whole; and (by Law III) to every action corresponds an equal raction; therefore the planet B will, on the other hand, gravitate towards all the parts of the planet A; and its gravity towards any one part will be to the gravity towards the whole as the matter of the part to the matter of the whole. Q.E.D. □

Rules of Reasoning in Philosophy

- Rule I: We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.
- Rule II: Therefore to the same natural effects we must, as far as possible, assign the same causes.
- Rule III: The qualities of bodies, which admit neither intension nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatsoever.

Rules of Reasoning in Philosophy

- Rule IV: In experimental philosophy we are to look upon propositions collected by general induction from, phenomena as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may either be made more accurate, or liable to exceptions.

Argument for the Laws of Motion

"When a body is falling, the uniform force of its gravity acting equally, impresses, in equal intervals of time, equal forces upon that body, and therefore generates equal velocities; and in the whole time impresses a whole force, and generates a whole velocity proportional to the time. And the spaces described in proportional times are as the product of the velocities and the times; that is, as the squares of the times." - *Principia*, Book I, The Scholium to the Laws of Motion

Argument for the Laws of Motion

- $P \wedge S \rightarrow Q$
- Q

- P

Bootstrap Argument for the Laws of Motion

- Observe that for one class of bodies the distance traveled in free fall is proportional to the square of the elapsed time
- Use Newton ' s second law to infer that for this class of bodies the force of gravity is constant for each body.
- By induction, for any body in free fall near the surface of the Earth, the force of gravity is constant.
- Apply this conclusion to a new class of bodies in free fall and measuring the time elapsed to cover a fixed distance of fall
- The second law of motion is satisfied

Outline

Inductive Skepticism

- Whether in Newton's "general induction" or in Bacon's simpler framework, inductive inference is subject to an important objection.
- "All virtuous people are just"
- "All pure water boils at 100 degrees centigrade at 1 atmosphere of pressure"
- Suppose that after looking at a sufficiently large sample, we make the right conclusion.
- Plato has Meno ask how we will know the conclusions are true.

Ancient Inductive Skepticism

"Meno: How will you look for it, Socrates, when you do not know at all what it is? How will you aim to search for something you do not know at all? If you should meet with it, how will you know that this is the thing that you did not know?"

Ancient Inductive Skepticism

- Even if we are in fact right about the boiling point of water, it is still logically possible that some as yet unexamined sample of pure water will boil at a different temperature at 1 atmosphere of pressure.
- For Plato, someone can know a proposition only if the proposition is true, the person believes it, and the person also knows why the proposition is true.

Ancient Inductive Skepticism

Proposition

No procedure is mathematically possible that, in every logically possible world or circumstance, will correctly decide the truth or falsity of a universal hypothesis that is not logically necessary (i.e., except for logical truths) from a finite sample of singular facts.

The Difficulty with Inductive Inference

- The difficulty with inductive inference is that it can be unreliable.
- Many of the ancient Greeks thought the heavens were perfect and unalterable, for they exhibited perfectly regular motions that seemed never to change. But then in the sixteenth century there appeared a new star (now thought to be a supernova): the heavens did change.
- Until the eighteenth century, no means had ever been found to synthesize a biological chemical from inorganic chemicals, but then Friedrich Wöhler (1800 – 1882) found a way to synthesize urea.

Hume's Psychological Theory

- Ideas and impressions;
- Ideas combine and separate in thought according to natural law.
- One idea tends to lead to another if the two ideas are *resemblance*, if they have *contiguity*, or ideas of cause and effect.
- Ideas (all the materials of thinking) are derived either from our outward or inward sentiment.
- The mind has traditional faculties or capacities: will, imagination, understanding. Reason is the operation of the understanding, and Hume thought of reasoning as deduction, analysis, and synthesis.

Hume's Inductive Skepticism

If, as Bacon and Newton seem to suggest, inductive inference is the inference to causes from effects (and effects from causes) and to general principles about causes and effects, then the question, as Hume saw it, is how such relations may be discovered (come to know) and what it would be to discover them.

Hume's Inductive Skepticism

"I shall venture to affirm, as a general proposition, which admits of no exception, that the knowledge of this relation is not, in any instance, attained by reasonings a priori , but arises entirely from experience, when we find, that any particular objects are constantly conjoined with each other. Let an object be presented to a man of ever so strong natural reason and abilities; if that object be entirely new to him, he will not be able, by the most accurate examination of its sensible qualities, to discover any of its cause or effects."

Hume's Inductive Skepticism

"Adam, though his rational faculties be supposed at the very first, entirely perfect, could not have inferred from the fluidity, and transparency of water, that it would suffocate him, or from the light and warmth of fire, that it would consume him. No object ever discovers, by the qualities which appear to the senses, either the causes which produced it, or the effects which will arise from it; nor can our reason, unassisted by experience, ever draw any inference concerning real existence and matter of fact."

Hume's Inductive Skepticism

"In a word, then, every effect is a distinct event from its cause. It could not, therefore, be discovered in the cause, and the first invention or conception of it, a priori , must be entirely arbitrary. And even after it is suggested, the conjunction of it with the cause must appear equally arbitrary; since there are always many other effects which, to reason, must seem fully as consistent and natural. In vain, therefore, should we pretend to determine any single event, or infer any cause or effect, without the assistance of observation and experience."

Hume's Inductive Skepticism

"These two propositions are far from being the same, I have found that such an object has always been attended with such an effect, and I foresee, that other objects, which are, in appearance, similar, will be attended with similar effects. I shall allow, if you please, that the one proposition may justly be inferred from the other; I know in fact, that it always is inferred. But if you insist, that the inference is made by a chain of reasoning, I desire you to produce that reasoning. The connexion between these propositions is not intuitive. There is required a medium, which may enable the mind to draw such an inference, if indeed it be drawn by reasoning and argument. What that medium is, I must confess, passes my comprehension; and it is incumbent on those to produce it, who assert, that it really exists, and is the origin of all our conclusions concerning matter of fact."

Hume's Inductive Skepticism

"If we be, therefore, engaged by arguments to put trust in past experience, and make it the standard of our future judgement, these arguments must be probable only, or such as regard matter of fact and real existence, according to the division above mentioned. But that there is no argument of this kind, must appear, if our explications of that species of reasoning be admitted as solid and satisfactory. We have said, that all arguments concerning existence are founded on the relation of cause and effect, that our knowledge of that relation is derived entirely from experience, and that all our experimental conclusions proceed upon the supposition, that the future will be conformable to the past. To endeavour, therefore, the proof of this last supposition by probable arguments, or arguments regarding existence, must be evidently going in a circle, and taking that for granted which is the very point in question."

Hume's Inductive Skepticism

- Hume's conclusion is that inductive inference is not founded on reason.
- We have no rational grounds for believing such inferences to be reliable, and so when we do empirical science, we are not engaged in a rational activity.
- Inductive inferences are founded on custom and habit rather than reason.
- We are so constructed psychologically that from observed instances of regularities we come naturally to expect the same regularity in future instances.

Hume's Inductive Skepticism

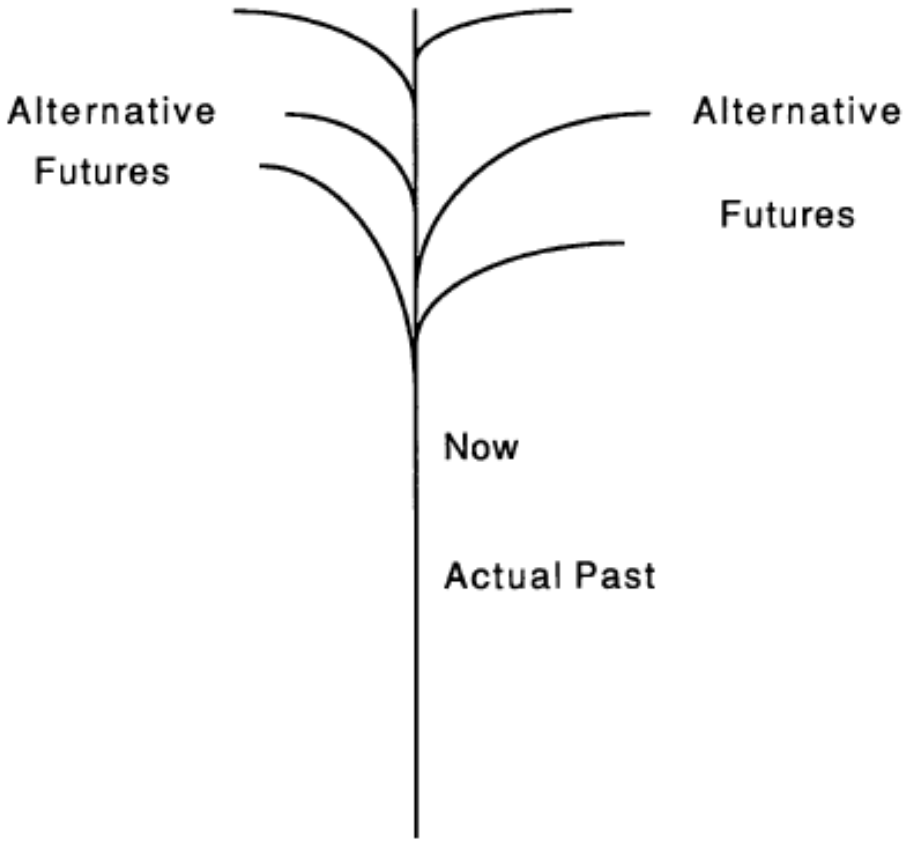

Figure 7.2 in *Thinking Things Through*

Outline

Metaphysical Skepticism

Suppose somehow that we had before us all possible facts of a certain kind; suppose that by magic we could survey an infinity of possible experiences. Then Hume' skepticism would not apply. But one can imagine that skeptical doubt would remain even then.

Descartes' Metaphysical Skepticism

"I will suppose not a supremely good God, the source of truth, but rather an evil genius, as clever and deceitful as he is powerful, who has directed his entire effort to misleading me. I will regard the heavens, the air, the earth, colors, shapes, sounds and all external things as nothing but the deceptive games of my dreams, with which he lays snares for my credulity. I will regard myself as having no hands, no eyes, no flesh, no blood, no senses, but as nevertheless falsely believing that I possess all these things."

Metaphysical Skepticism

Consider the following claims that we typically believe we know:

- Ordinary objects and persons continue to exist when I do not perceive them.
- Other persons have minds.
- Some past events really took place.
- The world I perceive really exists.

The metaphysical skepticism argues that we do not know any of these things!

Outline

Immanuel Kant

Kant's Work

- *The Critique of Pure Reason*
- *Prolegomena to Any Future Metaphysics*
- *Metaphysical Foundations of Natural Science*
- *The Critique of Judgment*

Kant's Claim

Kant claimed to:

- explain why arithmetic and geometry can be known a priori with certainty
- provide a demonstration of the a priori certainty of the most fundamental laws of physics
- provide a demonstration that every event must have a cause
- provide a demonstration that every sequence of causes and effects must follow some rule
- provide a refutation of Hume's skepticism about induction.

Kant's Two Distinctions

- Kant distinguished between propositions (judgments) that are analytic and those that are synthetic.
  - Kant thought that judgments always have a subject-predicate form.
  - In analytic judgments, but not in synthetic judgments, the concept of the predicate contains the concept of the subject.
  - Example: the concept of body includes the concept of extended thing.
- A priori judgments can be known by reason alone; a posteriori judgments cannot be known by reason alone but require the evidence provided by experience.

Kant's Formulation of the Problem

- Kant took the foremost question of epistemology and metaphysics to be this: How are synthetic a priori judgments possible?
- That was Kant's technical way of asking how, for example, the truths of mathematics or physics can be known with certainty by reason alone.

The Heider-Simmel Illusion

Kant's Solution

- Kant's ingenious answer is that we are so constituted that no experience can violate the laws of geometry or arithmetic or part of physics.
- The world we experience is a world constructed by our minds.
- For example, in each instance in which we think of or see a line segment, our constitution provides (in imagination or in sensation) the circles and points that Euclidean geometry says exist.

Kant's Solution

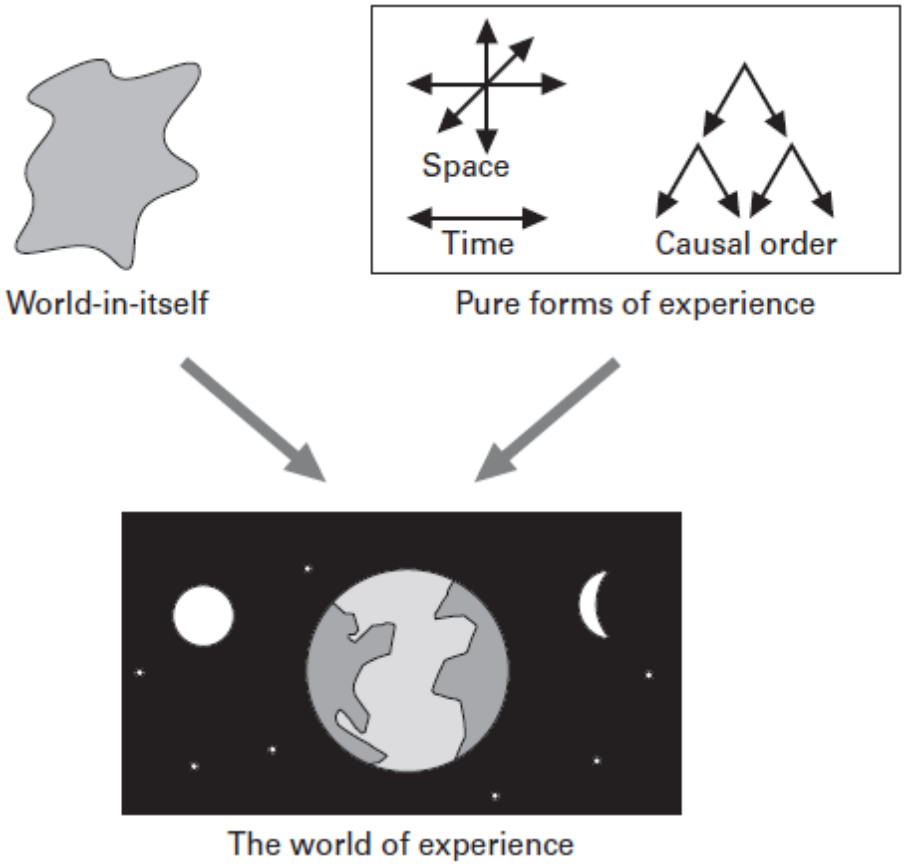

Figure 9.1 in *Thinking Things Through*

Kant's "Transcendental" Argument

- Premises: that the mind produces experience of reidentifiable objects undergoing regular changes in time and in threedimensional space.
- Conclusion:
  - Euclidean geometry is true of the space we experience
  - every event has a cause, that every sequence of causes and effects follows a general rule
  - the most general fundamental laws of natural science necessarily hold in the world we experience
- Hume's inductive skepticism does not apply, but metaphysical skepticism does.

Kant's "Transcendental" Argument

- Kant in fact gives no direct argument that Euclidean geometry is true or is imposed by the mind on the data of sensation.
- Instead, he assumed that Euclidean geometry is true of the space of experience, and he further assumed that it is necessarily true (no possible experience could contradict it), and that we can know a priori that it is true.

Kant's "Transcendental" Argument

- Kant's argument is that his theory *explains* how this could be so: if Euclidean geometrical relations are an artifact of how we synthesize the manifold of experience, then all possible experience must satisfy Euclidean geometry, and we can be certain of its truth, so we do not have to resort to inductive inference.
- Euclidean geometry is the **condition of possibility of** (COPO) our experience of spacial relations. Since we do have spacial experience, therefore Euclidean geometry must be true to our experience.

Kant's Arguments for An a priori Physics

Proposition

Second law of mechanics Every change of matter has an external cause. (Every body remains in its state of rest or motion in the same direction and with the same velocity unless it is compelled by an external cause to forsake this state.)

Kant's Arguments for An a priori Physics

Proof.

(In universal metaphysics there is laid down the proposition that every change has a cause; here [in mechanics] there is only to be proved of matter that its change must always have an external cause.) Matter as mere object of the external senses has no other determinations than those of external relations in space and hence undergoes no changes except by motion. With regard to such change, insofar as it is an exchange of one motion with another, or of motion with rest, and vice-versa, a cause of such change must be found (according to the principle of metaphysics). But this cause cannot be internal, for matter has no absolutely internal determinations and grounds of determination. Hence all change of matter is based upon an external cause. □

Kant's Argument for the Reliability of Inductive Inference

- In synthesizing the manifold of experience, the understanding makes judgments.
- Judgments all have a logical structure.
- There are only a few possible logical structures for judgments, and they can be completely described (according to Kant) in a simple table.
- There are 81 distinct logical forms. Each logical form is determined by a value for quantity, quality, relation, and modality, and each of these four has three possible values.

Kant's Logical Table of Judgments

Example: necessarily, all Manx cats are tailless.

Logical table of judgments

Quantity	Quality	Relation	Modality
Universal	Affirmative	Categorical	Problematic
Particular	Negative	Hypothetical	Assertoric
Singular	Infinite	Disjunctive	Apodeictic

Kant's Argument for the Reliability of Inductive Inference

- When we form the manifold of experience from the matter of experience, we apply the concepts of the understanding to make judgments, and some of these judgments are hypothetical.
- Hypothetical judgments incorporate the concept of causality, and so we form the world of experience so that sequences of events (in the world of experience) make appropriate sentences of the form "If A then B" always true

Kant's Argument for the Reliability of Inductive Inference

- Thus we have an a priori concept of causality and of necessary connection, which we do not need to form from experience but rather use to form experience.
- And the construction of the world of experience guarantees that the causal relations that experience presents to us in particular sequences of events can be generalized and will always be true.

Kant's Responses to Skepticism

- Kant took Hume's skepticism to be refuted by the demonstration of the principles that every event has a cause and that a sequence of causes and effects must satisfy general laws.
- Kant doubted that we could have any knowledge of how the world is in itself, as distinct from our knowledge of things in the world of experience constructed by our minds from sensations.
- What we have is a sequence of claims that together form a general picture of how the world of experience is constituted, why geometry and parts of physics are irrefutable by any possible experience, and why induction is reliable.

Reference

- Clark Glymour. (2015) *Thinking Things Through: An Introduction to Philosophical Issues and Achievements*, 2nd edition. A Bradford Book.