Experience, Knowledge, and Belief: Part II

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An Alternative Type of Science

- Euclid's geometry was the paradigm of science shared by Aristotle, Descartes, and many people in between.
- Revolutionary developments in science were replacing the Cartesian view of how knowledge is acquired with a quite different conception.
- Andreas Vesalius (1514 – 1564)
- Galileo Galilei (1564 - 1642)
- William Gilbert (1544 – 1603)
- William Harvey (1578 – 1657)

An Alternative Type of Science

- These and other discoveries did not seem to have the form expected by either Aristotelian or Cartesian science.
- There were no intuitions of general principles about lodestones or human physiology from which everything else in these subjects was deduced.
- Instead, examples of particular phenomena were observed, found to be repeatedly and regularly produced, and thus taken to hold generally.
- Inferences from particular instances to general conclusions were called **inductive**.

A More Ambitious Goal

- The goal of the new science was to go beyond simple generalizations of observed regularities to find their **causes** and the **laws** governing those causes.
- Scientists of the time searched for the hidden powers, causes, and structures that produce appearances, and they searched for the laws that govern such powers, causes, and structures.

The Epistemology of Empirical Inquiry

How could scientists find the hidden structures and mechanism of the reality? What could the method be?

Newton's Rules of Reasoning in Philosophy

- Rule I: We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.
- Rule II: Therefore to the same natural effects we must, as far as possible, assign the same causes.
- Rule III: The qualities of bodies, which admit neither intension nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatsoever.

Newton's Rules of Reasoning in Philosophy

- Rule IV: In experimental philosophy we are to look upon propositions collected by general induction from, phenomena as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may either be made more accurate, or liable to exceptions.

Hume's Inductive Skepticism

- Hume's conclusion is that inductive inference is not founded on reason.
- We have no rational grounds for believing such inferences to be reliable, and so when we do empirical science, we are not engaged in a rational activity.
- Inductive inferences are founded on custom and habit rather than reason.
- We are so constructed psychologically that from observed instances of regularities we come naturally to expect the same regularity in future instances.

Hume's Inductive Skepticism

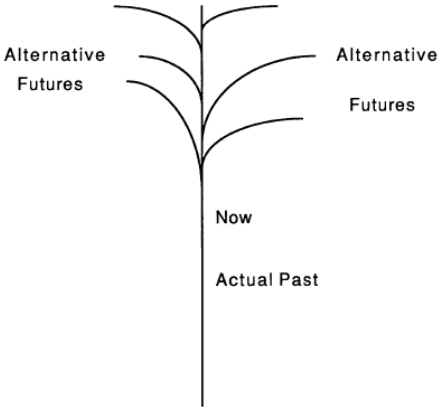

Figure 7.2 in *Thinking Things Through*

Metaphysical Skepticism

Brain in a vat - Wikipedia

Metaphysical Skepticism

Consider the following claims that we typically believe we know:

- Ordinary objects and persons continue to exist when I do not perceive them.
- Other persons have minds.
- Some past events really took place.
- The world I perceive really exists.

The metaphysical skepticism argues that we do not know any of these things!

Outline

1. The Theory of Probability
2. Personal Probability Theory
3. Bayesian Epistemology

Probability theory

- An algebra is a collection of propositions closed under the Boolean operations $\wedge$, $\vee$, $\neg$.
- A probability function P satisfies the following axioms:
  1. If $\vDash T$ then $P(T) = 1$
  2. $P(A) \leq 0$
  3. If $\vDash \neg(A \wedge B)$ then $P(A \vee B) = P(A) + P(B)$ (finite additivity)
- Optional: If for all $i \neq j$, $\vDash \neg(A_i \wedge A_j)$, then $P(A_0 \vee A_1 \vee \cdots \vee A_n \vee \cdots) = \Sigma_i P(A_i)$ (countable additivity).
- Conditional probability: If $P(B) > 0$ then define $P(A|B) = \dfrac{P(A \wedge B)}{P(B)}$

Interpretations of Probability

- Frequentism
- Propensity
- Classical
- Logical

Frequentism

- Consider a repeated experiment (flip a coin)
- Consider a type of outcome of that experiment (heads comes up)
- The frequency of an outcome type is the ratio: $\frac{\#outcomes\ of\ that\ type}{\#of\ trials}$
- The limiting frequency of an outcome type is the limit of the frequency as the number of trials goes to infinity.
- The probability of the outcome type is the limiting frequency of the type. Not countable additive.

Propensity

- Strong: propensities are conterfactual limiting relative frequencies: what would happen if a given process were continued forever.
- Moderate: propensities entail counterfactual limiting relative frequencies, but are not definable as such frequencies.
- Weak: propensities are modalities in the world that satisfy the probability axioms. Imply limiting relative frequencies only with unit propensity. May be assumed to be countably additive. Hard to say what they practically imply or how we can discover them.

Logical

- Rudolph Carnap's idea of Partial Entailment.
- $E \vDash H$ means E support H completely.
- $P(H|E) = r$ means E confirms H to degree r.
- $P(H|E)$ measures the degree to which H is supported by E.

Goodman's New Riddle of Induction

- x is Grue just in case x is Green prior to 2050 or x is Blue after 2050.
- Consider the cases of x we have collected till now. Say they are all Green. (They are also all Grue)
- This increase the support of Green(2050), and decrease the support of Blue(2050).
- But Grue(2050) is Blue(2050). So the evidence of Grue decrease the support of Grue(2050). Same logical form, different partial entailment.

Outline

1. The Theory of Probability
2. **Personal Probability Theory**
3. Bayesian Epistemology

Personal Probability Theory

- Probability is a measure of rational degree of belief
- $P(A) = r$ says that a particular individual believes A to degree r.
- Different people may have different probabilities for the same proposition.
- Probability measures the strength of a propositional attitude, rather than a causal feature of the world.
- To check whether a radium atom has a ten percent chance of decaying, do psychological tests on yourself, rather than experiments on the atom.

Dutch Book Argument

- Your degrees of belief should satisfy the probability axioms or somebody clever could take advantage of you.

Bets

- The bet $(\$s, A)$ is a contract to pay $\$s$ to the buyer if A is true and to pay nothing if A is false.
- $(\$s, A)$:
  - Pays $\$s$ if A
  - Pays $\$0$ if $\neg A$
- $\$s$ is the prize you win if A turns out true.

Negative Prizes and Selling

- Selling the bet $(\$s, A)$ at the rate r means receiving $\$sr$ prior to the bet and paying out the prize $\$s$ to your customer if A occurs.
- This may be redescribed as paying the negative amount $\$-sr$ for the bet and receiving the unwelcome "prize" of $\$-s$ when the customer "win" (when A occurs).
- Selling the bet $(\$s, A)$ is just the same as buying the bet $(\$-s, A)$.
- So it suffices to speak only of buying bets.

"Put Your Money Where Your Mouth Is" Principle

- If your degree of belief in A is r, then
  - you are willing to contribute fraction r of the stake in a bet for A
  - you are willing to contribute fraction $(1-r)$ of the stake in a bet for $\neg A$
- If $P(A) = r$ then
  - you will trade \$$rs$ for the bet (\$$s, A$) and
  - you will trade \$$(1-r)s$ for the bet (\$$s, not-A$).

Packages of Bets

- The bet $(\$s, A) + (\$s', B)$ is like owning both bets at once:
  - $(\$s, A) + (\$s', B)$:
  - Pays $\$(s + s')$ if A, B.
  - Pays $\$s$ if A, $\neg B$.
  - Pays $\$s'$ if $\neg A$, B
  - Pays $\$0$ if $\neg A$, $\neg B$

"Package" principle

- A package of bets is worth the sum of the amounts you would have paid for each bet in the package individually.
- If you will pay $\$rs$ for the bet $(\$s, A)$ and $\$r's'$ for the bet $(\$s', B)$ then you will pay $\$(rs + r's')$ for the bet $(\$s, A) + (\$s', B)$.

Coherence

- Degrees of belief assignment P is coherent just in case P satisfies the probability axioms.
- Personalism is about coherence, or beliefs fitting together, rather than about reliability or finding the truth.

Dutch Book

- A package of bets that you would buy and lose money on no matter what happens.
- Bad luck can happen to anybody.
- But losing money no matter what is dumb.

Dutch Book

Theorem 1

Dutch Book theorem: One will be willing to buy a package of bets on which one lose money no matter what if and only if one violate the probability axioms.

Dutch Book

Proof.

First axiom: Let $P(T) = r$. Suppose $r > 1$. Then you are willing to pay $\$r > \$1$ for the "Dutch book" bet DB $= (\$1, T)$. You are guaranteed to win DB, but you foolishly paid more for DB than DB pays out, so you lose money no matter what. Suppose $r < 1$. Then the same holds for the bet DB $= (\$1, \neg A)$. (Here we use the assumption that you would take both sides of a bet at a given degree of belief).

Dutch Book

Proof.

Second axiom: Let $P(A) = r < 0$. Let $\$s < \$0$. Then DB $= (\$s, A)$ is the awful bet in which you "win" a bill charging you $\$s$ when A occurs. But you are willing to pay the positive amount $\$rs > 0$ for DB. You lose this amount if you lose, and you lose even more when you win and get the bill for $\$s$.

Dutch Book

Proof.

Third axiom: Suppose $P(A) = a$, $P(B) = b$, $P(A \vee B) = c$, and $\vDash \neg(A \wedge B)$. Consider case $a + b > c$. You would pay $\$a$ for $(\$1, A)$, $\$b$ for $(\$1, B)$ and $\$(1 - c)$ for $(\$1, \neg A \wedge \neg B)$ (This uses the assumption that you will take both sides of the bet $(\$1, A \vee B)$). So you would pay $\$(a + b + 1 - c)$ for the package DB $= (\$1, A) + (\$1, B) + (\$1, \neg A \wedge \neg B)$ (by the package principle).

Dutch Book

Proof.

Third axiom(continued):

- Case: A, B: impossible
- Case: A, $\neg B$: Outlay for bets $= \$(-a-b-1+c)$. Winnings from $(\$1, A) = \$1$. You win nothing from the others. Net from bets $= \$1 + \$(-a-b-1+c) = \$(c-a-b) = \$(c-(a+b)) < \$0$
- Case: B, $\neg A$: Switch A and B in the second case.
- Winnings from $(\$1, \neg A \wedge \neg B) = \$1$. You win nothing from the others. Net from bets $= \$1 + \$(-a-b-1+c) < \$0$.

Objections

- Bettable propositions: If P is a proposition that nobody could determine the truth of, nobody could force you to pay when you lose, so you can't lose money. (Response: if you value the truth, you can lose or gain it whether or not you or anyone else knows that you have).
- Package principle: suppose the bets are bought in sequence through time. Your assessment of the value of money could change during this period, so that your total outlay for the bets differs from the sum of your origional degrees of belief.

Personalist Evidence and Learning

- In personalism, conditional probability is used as a procedure for learning.
- Let $P_n(A)$ be the agent's degrees of belief at stage n.
- Suppose that at time $n+1$ the agent learns E. Then the personalist idea for learning is: $P_{n+1}(H) = P_n(H|E)$
- This procedure is called updating by conditionalization or **Bayesian updating**. And yes there is a Dutch book argument for it.

Outline

Bayes' Theorem

Theorem 2

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

Proof.

$$\frac{P(E|H)P(H)}{P(E)} = \frac{\frac{P(E \wedge H)}{P(H)}P(H)}{P(E)} = \frac{P(E \wedge H)}{P(E)} = P(H|E) \qquad \square$$

Bayes' Theorem Terminologies

- $P(H)$ = the prior probability of H
- $P(E|H)$ = the likelihood of E
- $P(H|E)$ = the posterior probability of H
- $P(E)$ = the prior probability of E

Total Probability Theorem

Theorem 3

$$P(E) = P(E|H)P(H) + P(E|\neg H)P(\neg H)$$

Proof.

$$\begin{aligned}
RHS &= P(E|H)P(H) + P(E|\neg H)P(\neg H) \\
&= \frac{P(E \wedge H)}{P(H)}P(H) + \frac{P(E \wedge \neg H)}{P(\neg H)}P(\neg H) \\
&= P(E \wedge H) + P(E \wedge \neg H) \\
&= P((E \wedge H) \vee (E \wedge \neg H)) \quad Axiom3 \\
&= P(E)
\end{aligned}$$

$\square$

Bayesian Confirmation

- Positive relevance provides us with a quantitative concept of confirmation.
- Conditional probability provides an intuitively appealing theory of justification.

Bayesian Confirmation

Refutation is bad

Suppose evidence E refutes hypothesis H. Then $E \vDash \neg H$. Then

$$P(H|E) = \frac{P(E \wedge H)}{P(E)} = \frac{0}{P(E)} = 0.$$

Bayesian Confirmation

Surprising predictions are good

Suppose $P(E)$ is very low and $P(E|H)$ is very high. So H makes a surprising prediction. Now suppose the prediction is observed.

Then $P(H|E) = \dfrac{P(E|H)P(H)}{P(E)}$.

In the limiting case, $H \vDash E$ so $P(E|H) = 1$. So $P(H|E) = \dfrac{P(H)}{P(E)}$.

Since $P(E) < 1$, $P(H|E) > P(H)$, and the less plausible E is, the bigger the evidential boost received by H.

Bayesian Confirmation

Unification of apparently independent phenomena is good

- Unifying disparate phemonena reveals that the one really does provide information about the other.
- For example, the wave theory of optics provided a perfect correlation between the swirling pink and blue color of an oily puddle with the bands or fringes around the edge of a shadow. On Newton's particle-based theory of optics, these were entirely uncorrelated phenomena.

Bayesian Confirmation

Unification of apparently independent phenomena is good: Suppose E is irrelevant to E' under theory N, but conditional on W, E is highly relevant to E'. Then

$$P(E|E' \wedge N) = P(E|N) \text{ but } P(E|E' \wedge W) = 1.$$

$$P(E \wedge E'|N) = P(E'|N)P(E|N)$$

$$P(E \wedge E'|W) = P(E|E' \wedge W)P(E'|W) = P(E|W)$$

Then we have

$$\frac{P(N|E \wedge E')}{P(W|E \wedge E')} = \frac{P(N)P(E \wedge E'|N)}{P(W)P(E \wedge E'|W)}$$

$$= \frac{P(N)P(E|N)P(E'|N)}{P(W)P(E|W)}$$

Bayesian Confirmation

If we define the Unifying coefficent of W given $E \wedge E'$ as follows:

$$U(H|E \wedge E') = \frac{P(E \wedge E'|H)}{P(E|H)P(E'|H)}$$

Then the likelihood ratio can be written as:

$$\frac{P(N|E \wedge E')}{P(W|E \wedge E')} = \frac{U(N|E \wedge E')(P(E|N)P(E'|N))}{U(W|E \wedge E')(P(E|W)P(E'|W))}$$

Bayesian Induction

Suppose H is $\forall i E_i$. Then

$$\begin{aligned}\forall i E_i &= \neg(\exists i \neg E_i) \\ &= \neg(\neg E_0 \vee \neg E_0 \vee \cdots \vee \neg E_n \vee \cdots) \\ &= \neg \bigvee_{i=0}^{\infty} \neg E_i \\ &= \neg \bigvee_{i=0}^{\infty} (E_0 \wedge E_1 \wedge \cdots \wedge \neg E_i)\end{aligned}$$

By countable additivity (optional axiom 4) we have:

$$P(\forall i E_i) = 1 - \sum_{i=0}^{\infty} P(E_0 \wedge \cdots \wedge E_{n-1} \wedge \neg E_n).$$

Bayesian Induction

Since
for each n, $\forall i E_i \vDash (E_0 \wedge \cdots \wedge E_{n-1} \wedge E_n)$
We have

$$P(\forall i E_i | (E_0 \wedge \cdots \wedge E_n)) = \frac{P(\forall i E_i)}{P(E_0 \wedge \cdots \wedge E_n)}$$

$$= \frac{1 - \sum_{i=0}^{\infty} P(E_0 \wedge \cdots \wedge E_{i-1} \wedge \neg E_i))}{1 - \sum_{i=0}^{n} P(E_0 \wedge \cdots \wedge E_{i-1} \wedge \neg E_i)}$$

The value approaches 1 as $n \to \infty$

Bayesian Response to Skepticism

- Modern Bayesians abandon the notion of knowledge altogether, focusing instead on the notion of rational degrees of belief and rational changes of degrees of belief.
- Rather than obtaining the truth after a finite amount of evidence has appeared and knowing when one has obtained the truth, Bayesians conceive reliability as at most convergence to the truth in the limit, so that as evidence increases without bound, one's probability distribution becomes concentrated more and more tightly around the true hypothesis.

Bayesian Response to Skepticism

- Rather than requiring that inquiry lead to the truth in all logically possible worlds, Bayesians allow many logically possible worlds to have a zero initial probability; if the truth lies in such a world, it will not be found.
- Bayesians maintain that for deliberation, decision, and action, only expected utilities matter, and these may be defined and computable even in the presence of skeptical hypotheses.
- The Bayesian picture does not refute metaphysical skepticism.

Reference

- Clark Glymour. (2015) *Thinking Things Through: An Introduction to Philosophical Issues and Achievements*, 2nd edition. A Bradford Book.