Lecture 2 - Complementary Questions Answers

Question 1: Coarse-Graining in Practice

Question: Explain what "coarse-graining" means and why it lets us treat soft matter as a continuum. Pick one system (foam, polymer solution, emulsion) and say when this breaks down (give a specific length scale or condition).

Answer:

Coarse-graining means averaging over many microscopic constituents (molecules, droplets, bubbles, chains) to define smooth, space- and time-dependent fields like density, velocity, and stress. When the averaging volume is large compared to the microstructure, local fluctuations cancel and the material can be modeled as a continuous medium obeying conservation laws.

Example (emulsion): If droplets have diameter $d \sim 10\,\mu\mathrm{m}$, a pipe flow with gradients varying over millimeters contains thousands of droplets per "fluid element," so a single-phase continuum with effective properties is accurate.

The approximation **breaks down** when:

1. The observation/gradient length ℓ becomes $O(d)$ or smaller (e.g., microfluidic channels, thin gaps, near interfaces)
2. Time scales approach droplet rearrangement or coalescence times
3. In these regimes, discrete structure, interfacial forces, and nonlocal effects dominate and cannot be neglected

Key insight: Continuum descriptions require separation of scales—the microstructural length must be much smaller than the macroscopic gradients of interest.

Question 2: Continuity in a Nozzle

Question: You squeeze shampoo through a pump and narrow outlet at steady rate. Using only words, explain why the fluid speeds up in the narrow section and name the conservation law you're invoking. Give one case where this reasoning would fail.

Answer:

In steady flow, what enters must leave: the same **mass per unit time** passes every cross-section. If the passage narrows, the fluid must move faster there so that the same mass gets through the smaller area in the same time—this is **conservation of mass** (the continuity principle) in words.

Mathematical statement: For incompressible flow, $\rho A v = \text{constant}$, so if area A decreases, velocity v must increase proportionally.

This reasoning **fails** when:

1. **Density changes significantly**: A compressible or bubbly shampoo (entrained air) undergoing strong pressure drop, where the mass rate stays fixed but the *volume* rate and speeds no longer scale simply with area
2. **Unsteady flow**: "Squeeze-and-release" strokes where fluid is temporarily stored in a compliant pump chamber violate the steady-state assumption

Additional failure mode: Very high shear rates can cause shear-induced migration in suspensions, changing local density and invalidating simple area-velocity scaling.

Question 3: Momentum Balance Around a Bend

Question: Water flows steadily through a pipe that turns 90°. Describe which forces turn the flow (pressure differences versus wall shear) and how the picture changes for very slow flows (creeping) versus very fast flows (inertial).

Answer:

Along a curved streamline, a cross-stream **pressure difference** provides the centripetal "push" that redirects the fluid; **wall shear** transmits and resists motion, setting the velocity profile and causing energy loss.

Very slow (creeping, low Reynolds number):

$$\mathrm{Re} = \frac{\rho v D}{\eta} \ll 1$$

1. Inertia is negligible, so the momentum equation reduces to a balance between pressure gradients and viscous stresses: $\nabla p \approx \eta \nabla^2 \boldsymbol{v}$
2. The flow hugs the bend smoothly with no separation
3. Shear stresses dominate the force budget
4. Velocity profile is nearly parabolic (Poiseuille-like) throughout

Very fast (inertial, high Reynolds number):

$$\mathrm{Re} = \frac{\rho v D}{\eta} \gg 1$$

1. Fluid parcels "want" to go straight due to inertia
2. A larger pressure difference is required on the *outer* wall to provide centripetal acceleration
3. Viscous effects are confined to thin boundary layers
4. Secondary motions (Dean-type vortices) and even flow separation can appear
5. Pressure forces do most of the turning; viscous shear mainly dissipates energy in boundary layers

Physical insight: In both regimes, this description comes from momentum conservation in continuum form ($\rho \frac{D\boldsymbol{v}}{Dt} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{b}$), but the dominant balance shifts from viscous-dominated to inertia-dominated.

Question 4: Where Does the Work Go?

Question: You slide two plates with glycerol between them at constant speed. Explain where the mechanical power you input ends up inside the fluid, name the phenomenon responsible (in words), and give a one-sentence microscopic picture for it.

Answer:

The mechanical power you supply is converted into **heat inside the liquid** via **viscous dissipation** (internal friction): shear stresses acting across velocity gradients irreversibly transform organized motion into thermal energy.

Energy transformation:

Power input:

$$P_{\text{in}} = F \cdot v = \tau A \cdot v$$

where τ is the shear stress, A is the plate area, and v is the sliding velocity.

For Newtonian fluid with viscosity η and gap h:

$$\tau = \eta \frac{v}{h}$$

So:

$$P_{\text{in}} = \eta \frac{v^2}{h} A$$

This power is entirely dissipated as heat (assuming isothermal boundaries that remove heat).

Microscopic picture:

Molecules in faster layers transfer momentum to slower layers through incessant collisions and interactions, randomizing directed motion into molecular agitation—manifested macroscopically as a temperature rise if heat is not removed.

Alternative description: The velocity gradient creates a non-equilibrium state where molecular collisions preferentially transfer momentum down-gradient, doing work against the imposed flow and converting mechanical energy into thermal fluctuations.

Connection to lecture: This is the continuum manifestation of momentum diffusion we derived from kinetic theory—the stress tensor component $\tau = \eta \frac{\partial v}{\partial y}$ represents the momentum flux that ultimately dissipates energy.

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