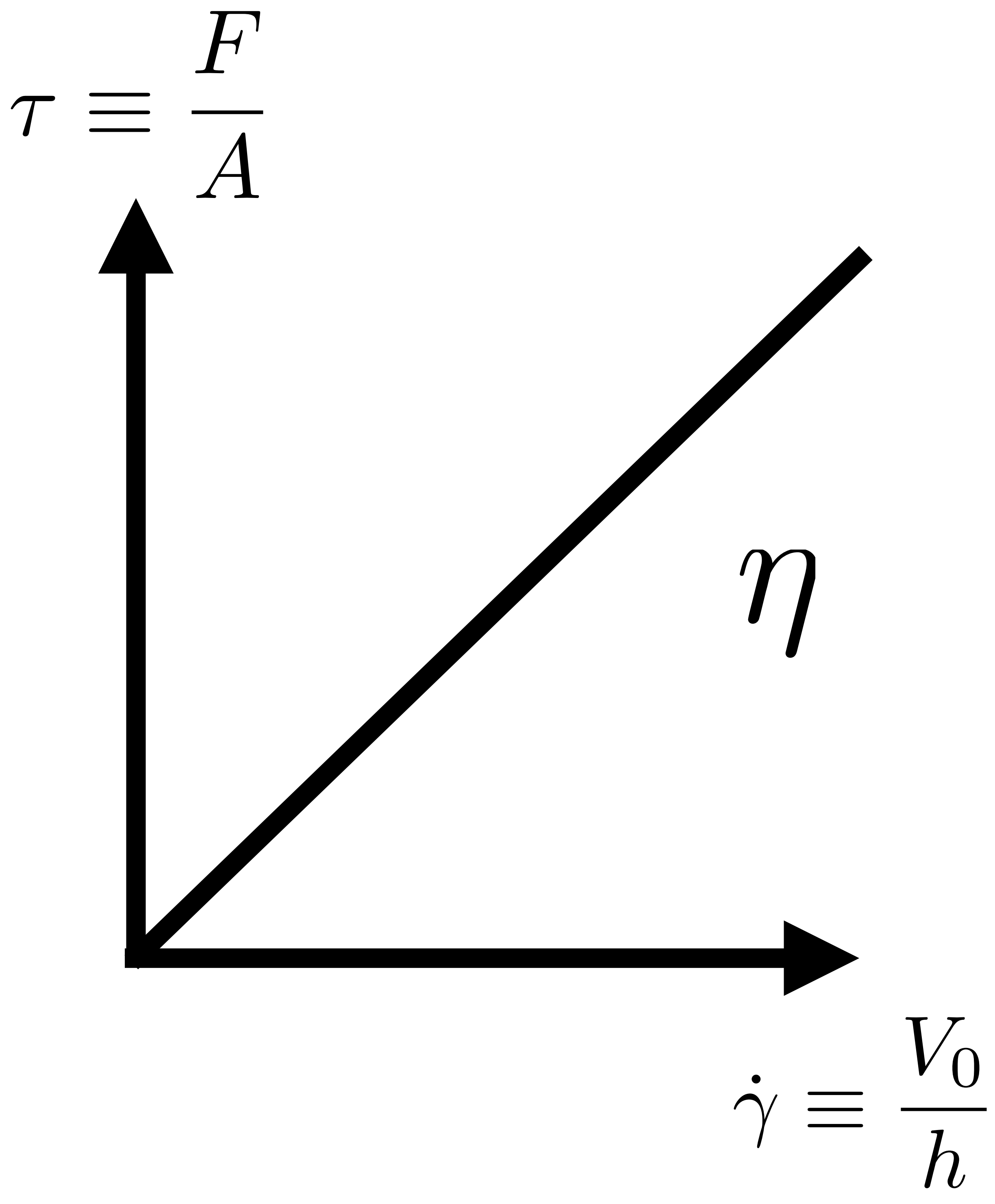
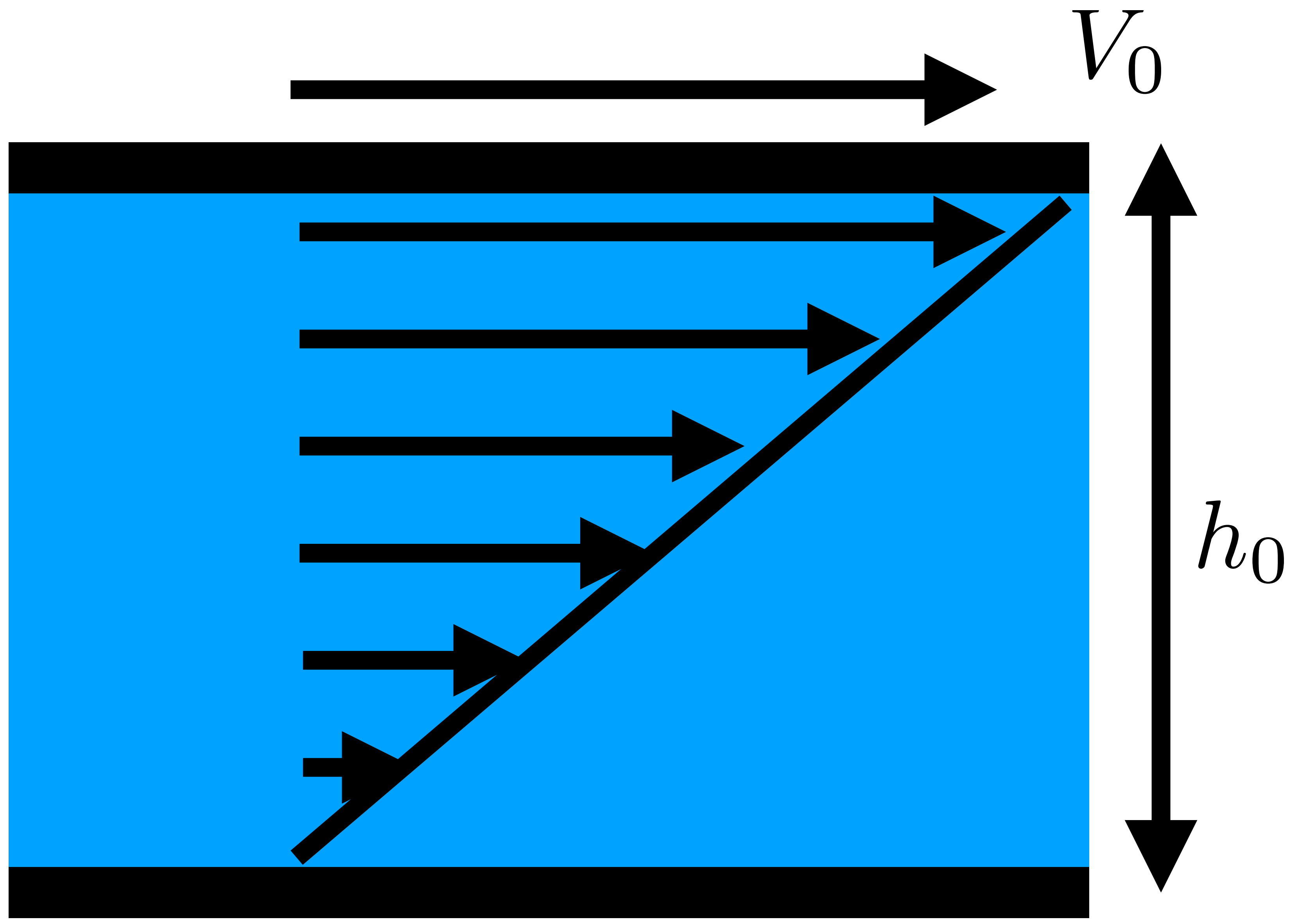
What is viscosity?

Inviscid vs. Viscous liquid

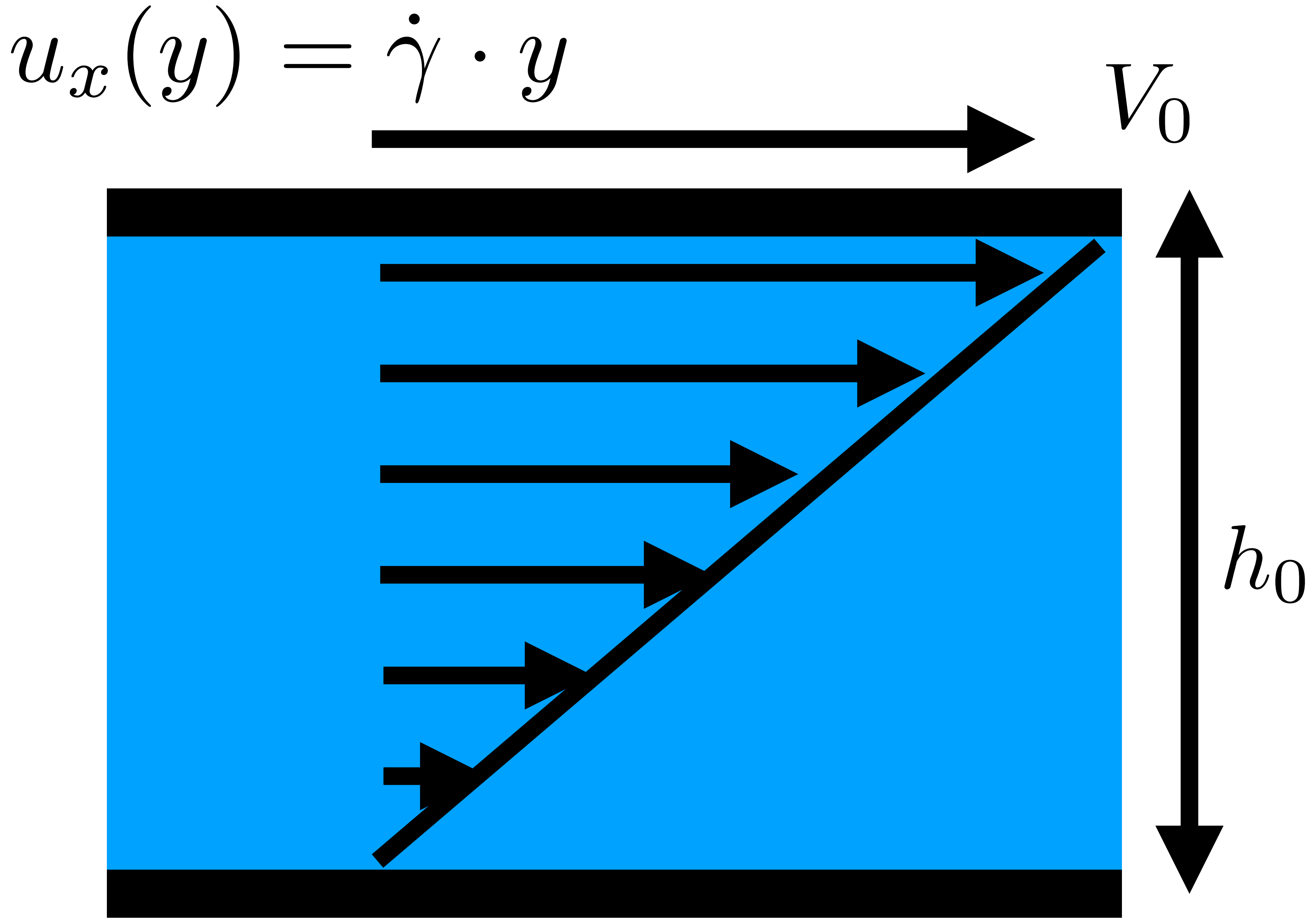
Viscosity and Newton

$$\boxed{\tau_{xy} = \eta\dot{\gamma} = \eta\frac{\partial u_x}{\partial y}}$$





Viscosity from kinetic theory



$\lambda:$ Mean free path of the molecules

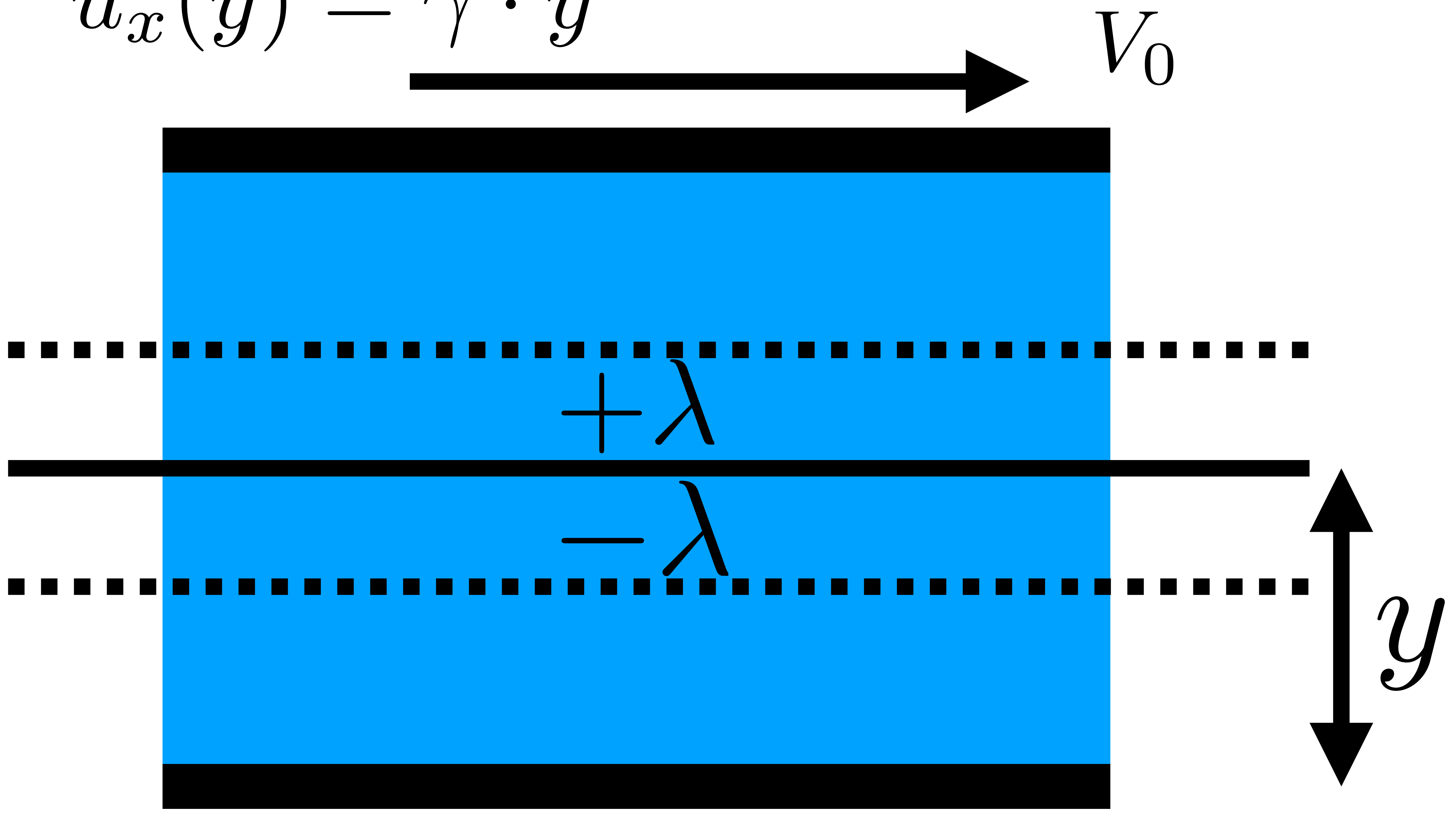
Mean speed of a monoatomic gas:

$$\bar{c} = \sqrt{8k_B T/(\pi m)}$$

Molecules at height y have average x-velocity $u_x(y)$. But they only came to height y after traveling a distance $\sim \lambda$ through the fluid. So the x-momentum they carry reflects the conditions they saw $\sim \lambda$ away. That's where the gradient matters.

How many molecules cross a plane per unit area per unit time?

$$u_x(y) = \dot{\gamma} \cdot y$$



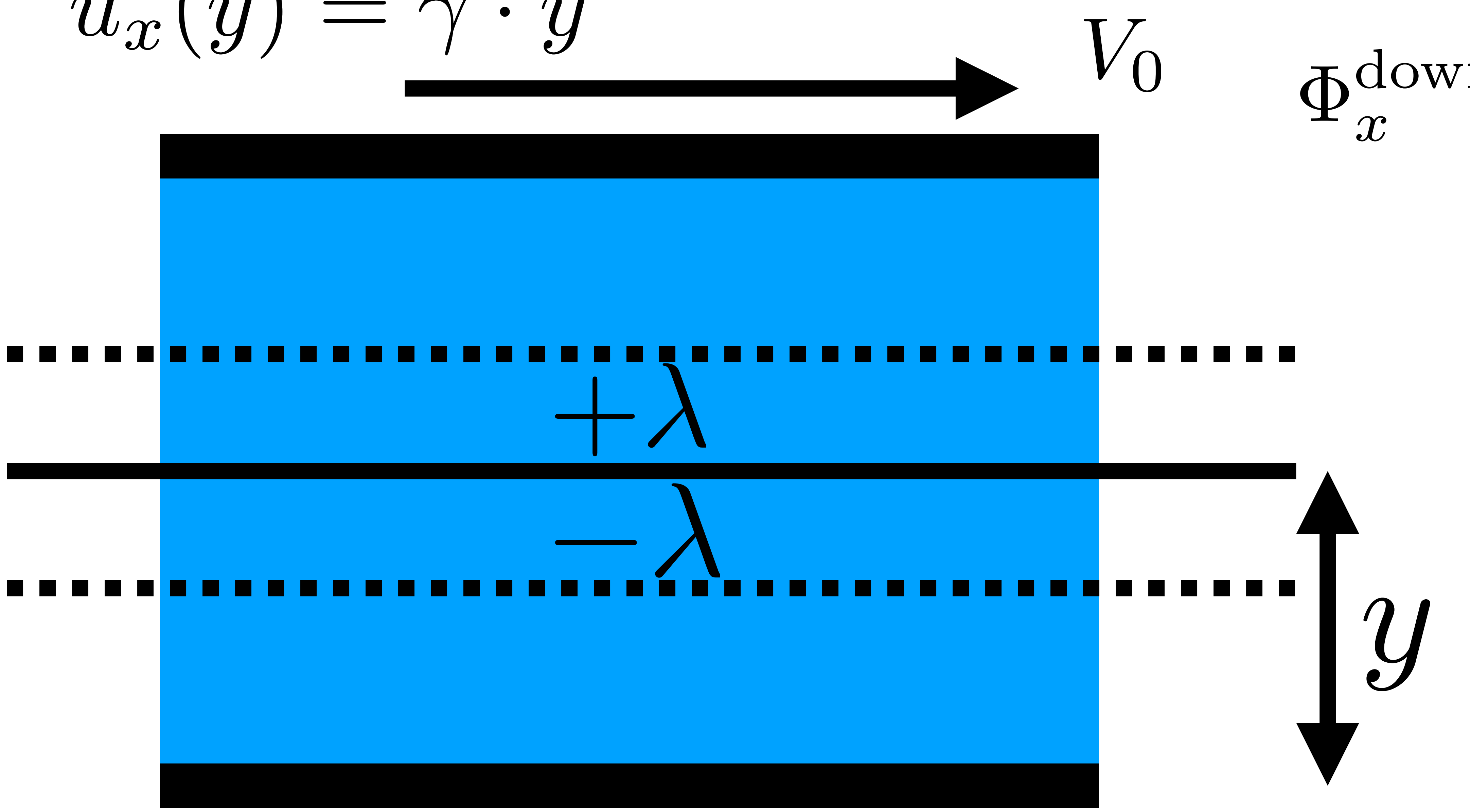
Number flux $\quad J = \dfrac{1}{4} n\bar{c}$

A molecule crossing the plane upward was last hit $\sim \lambda$ below, so it carries the mean x-momentum from around $y - \lambda$. A molecule crossing downward carries momentum from around $y + \lambda$.

$$\Phi_x^{\mathrm{down}} \approx \frac{n\bar{c}}{4} \cdot m \cdot u_x(y+\lambda) \qquad \Phi_x^{\mathrm{up}} \approx \frac{n\bar{c}}{4} \cdot m \cdot u_x(y-\lambda)$$

Net downward momentum flux

$$u_x(y) = \dot{\gamma} \cdot y$$



$$\Phi_x^{\text{down}} \approx \frac{n\bar{c}}{4} \cdot m \cdot u_x(y+\lambda) \qquad \Phi_x^{\text{up}} \approx \frac{n\bar{c}}{4} \cdot m \cdot u_x(y-\lambda)$$

$$\boxed{\Phi_x^{\text{net}} \approx \frac{nm\bar{c}}{4} \cdot 2\lambda \frac{\partial u_x}{\partial y} = \frac{1}{2}\rho\bar{c}\lambda\frac{\partial u_x}{\partial y}}$$

$$\tau_{xy} = -\Phi_x^{\text{net}} = -\frac{1}{2}\rho\bar{c}\lambda\frac{\partial u_x}{\partial y}$$

$$\boxed{\eta \sim C\,\rho\,\bar{c}\,\lambda, \quad \text{where } C = \mathcal{O}(1)}$$

Viscosity is an emergent property

$$\boxed{\frac{\eta}{\rho} \sim \bar{c}\lambda}$$