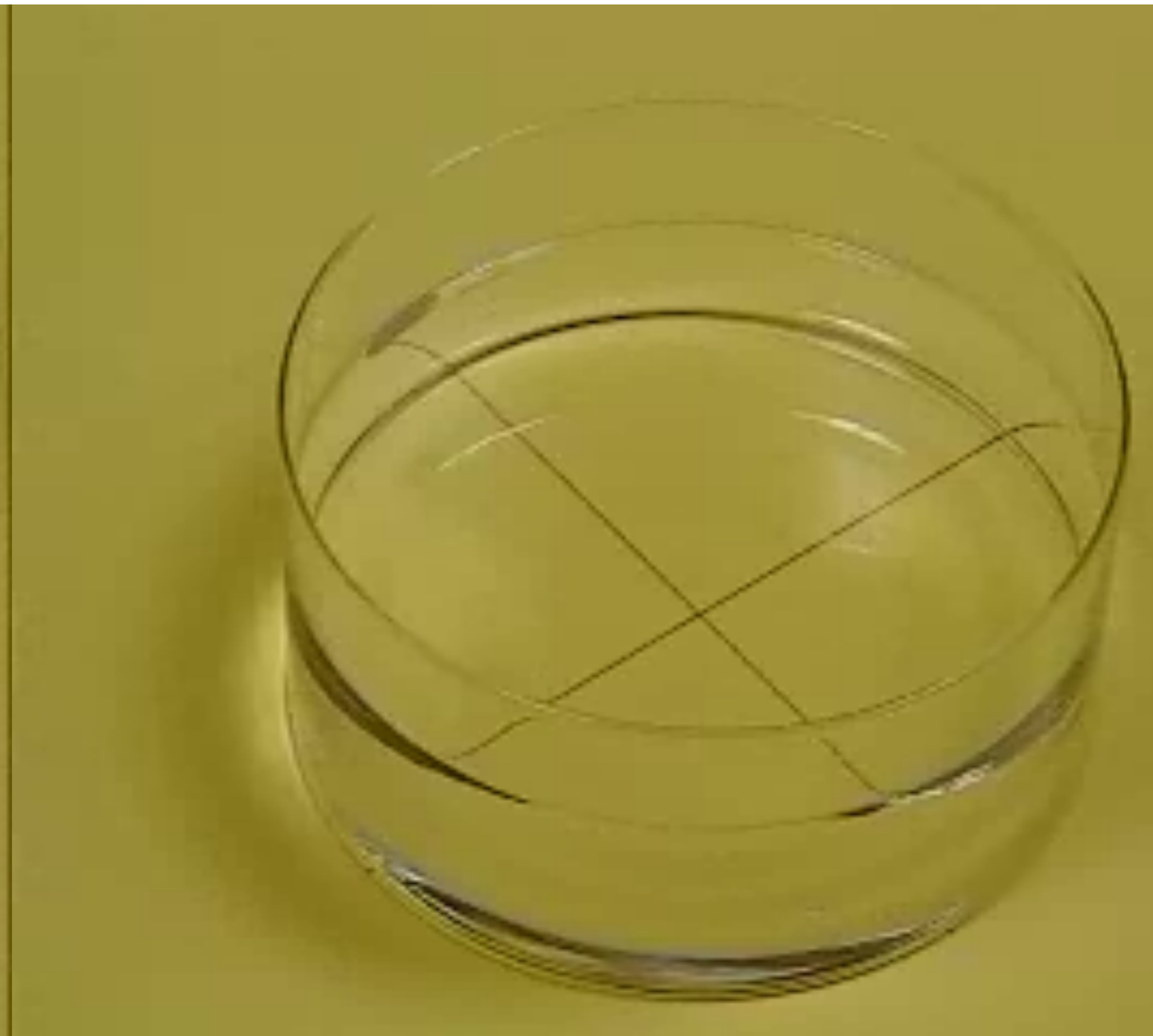
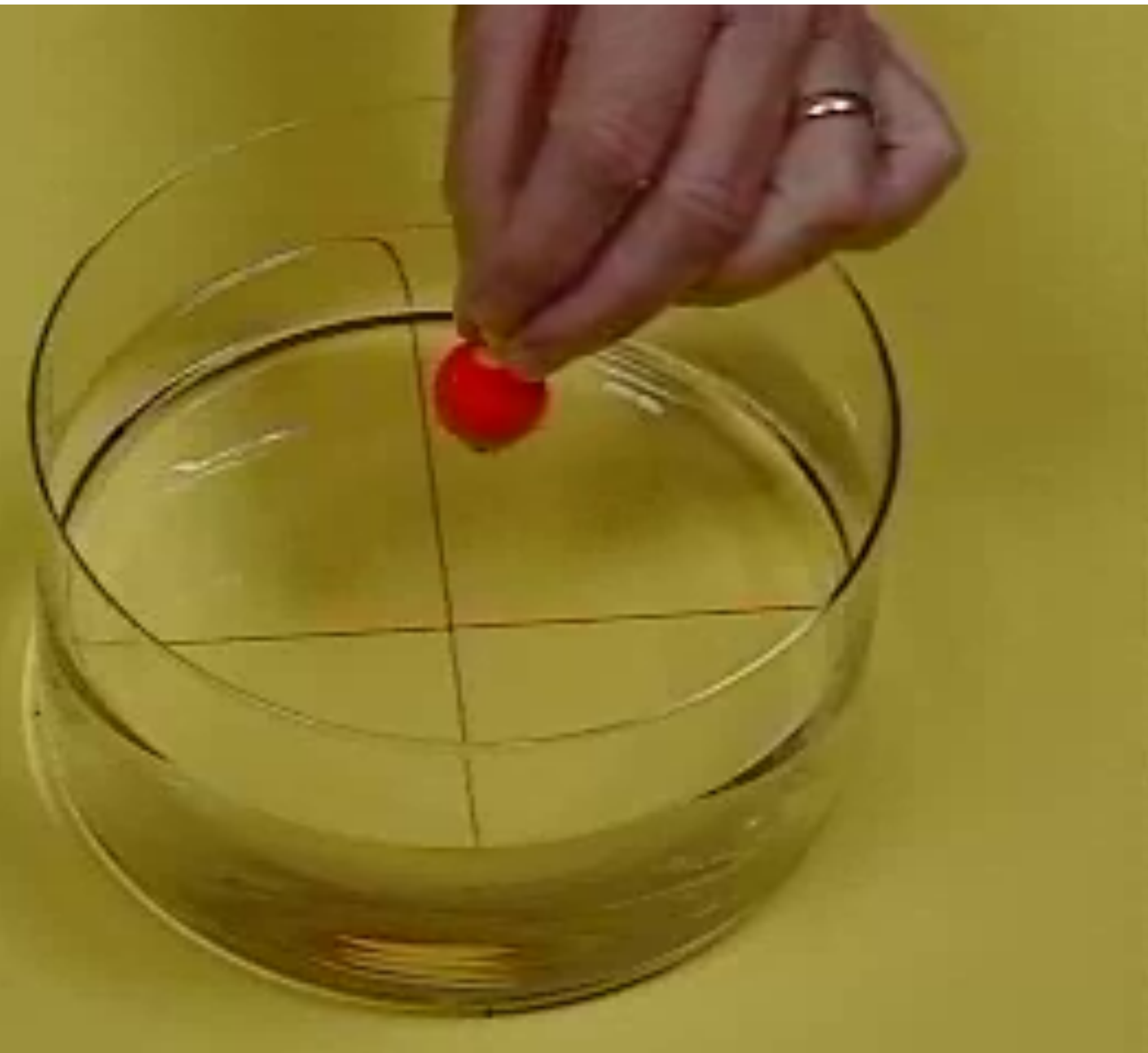


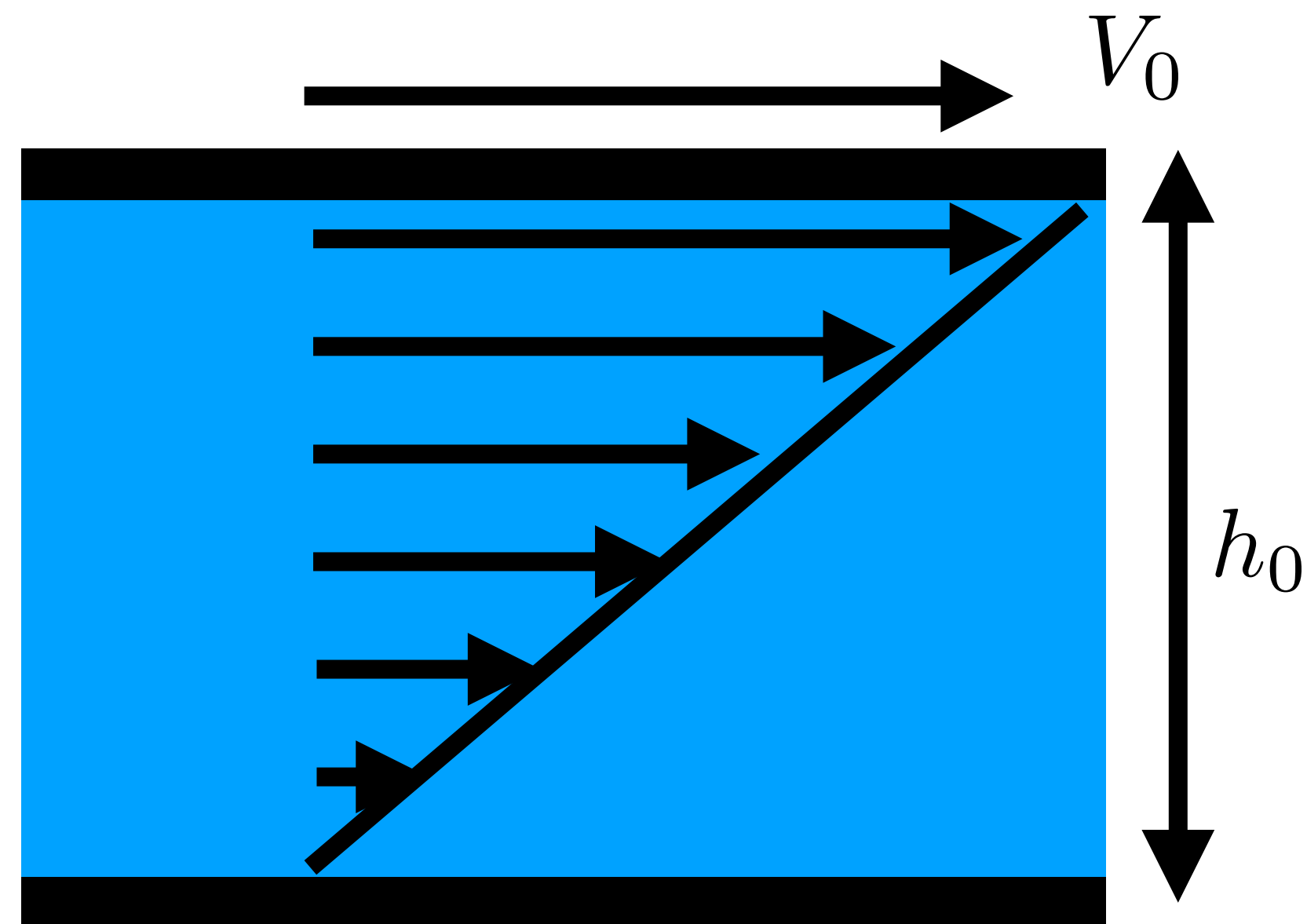
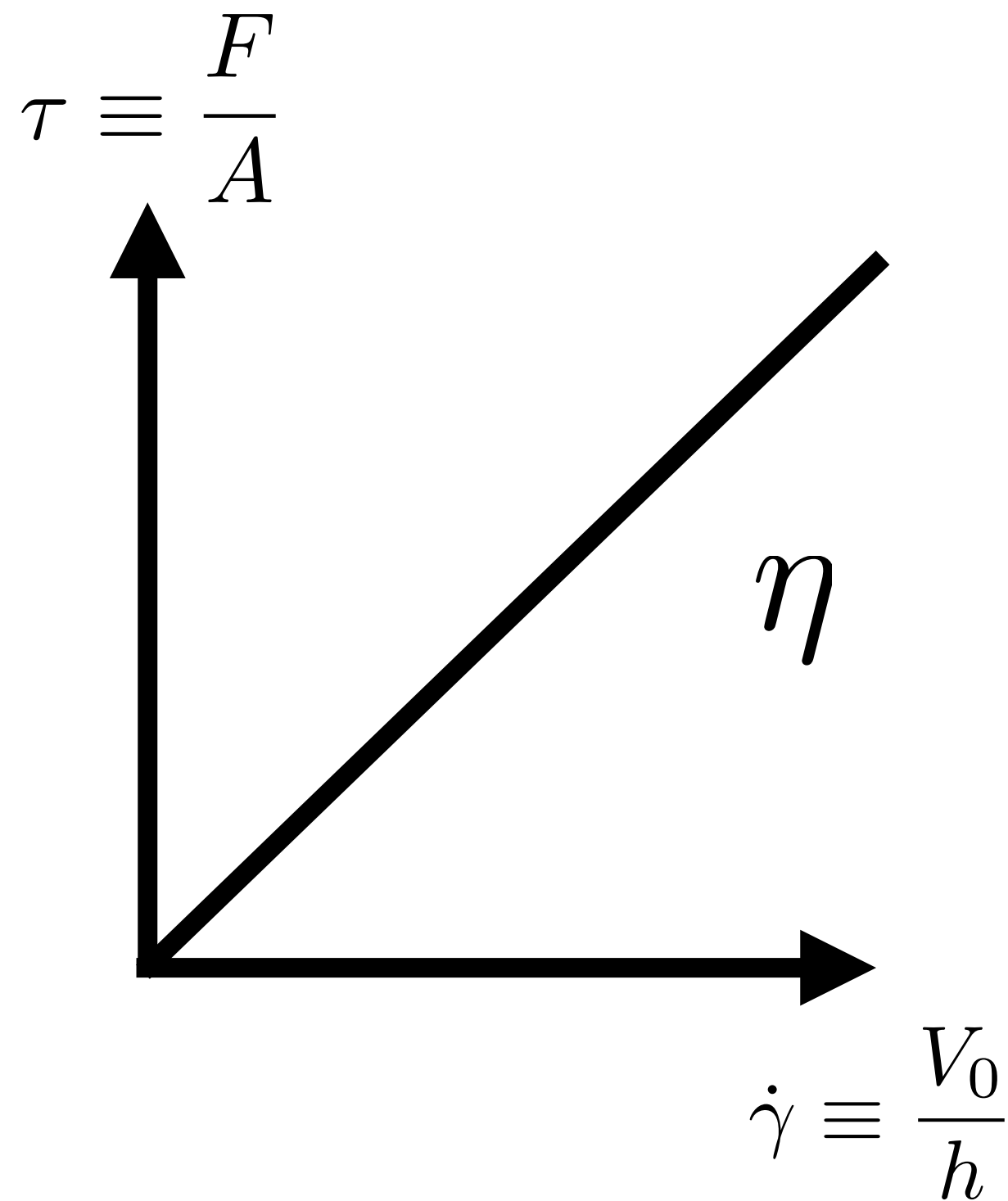
What is viscosity?

# Inviscid vs. Viscous liquid

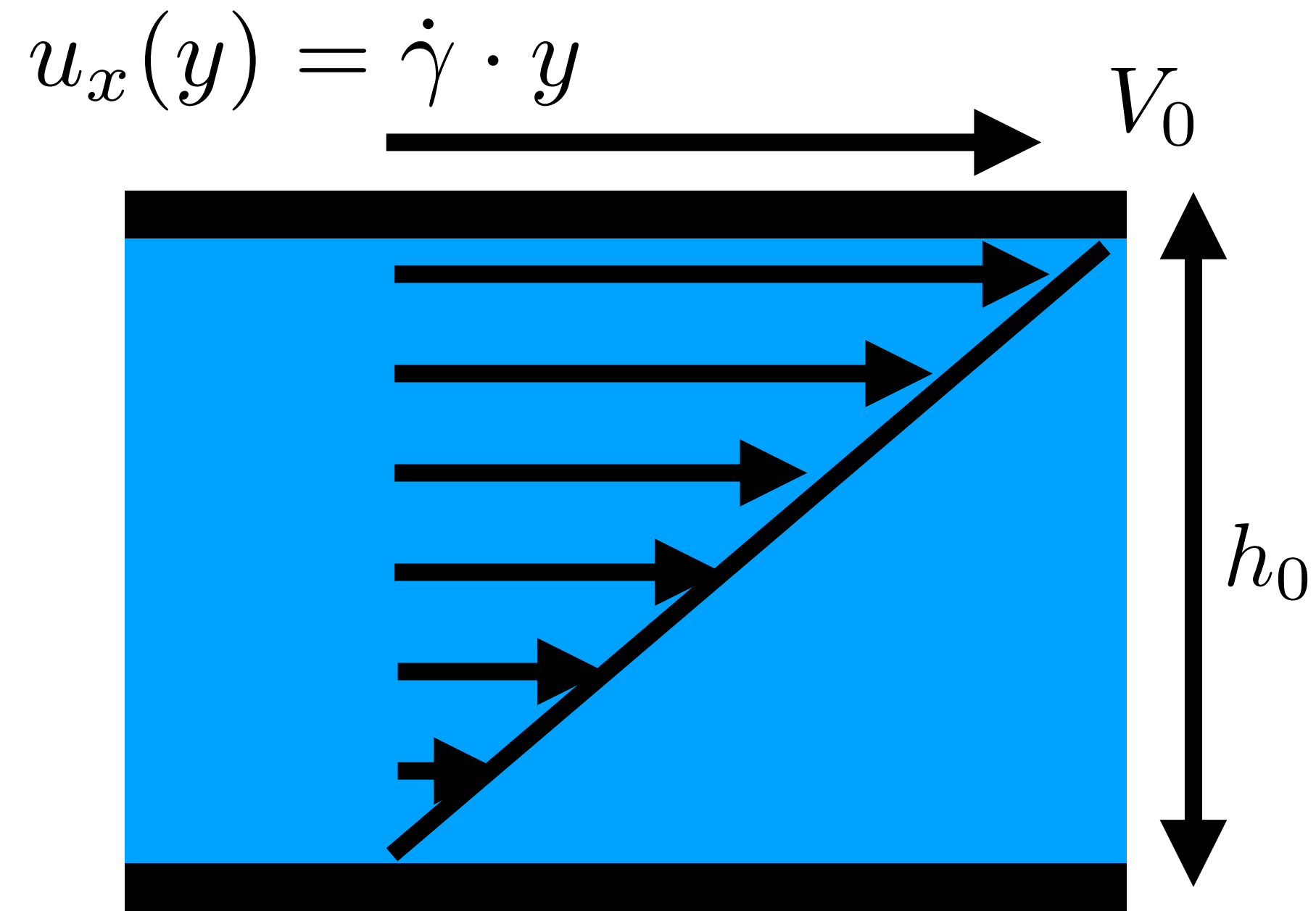


# Viscosity and Newton

$$\tau_{xy} = \eta \dot{\gamma} = \eta \frac{\partial u_x}{\partial y}$$



# Viscosity from kinetic theory



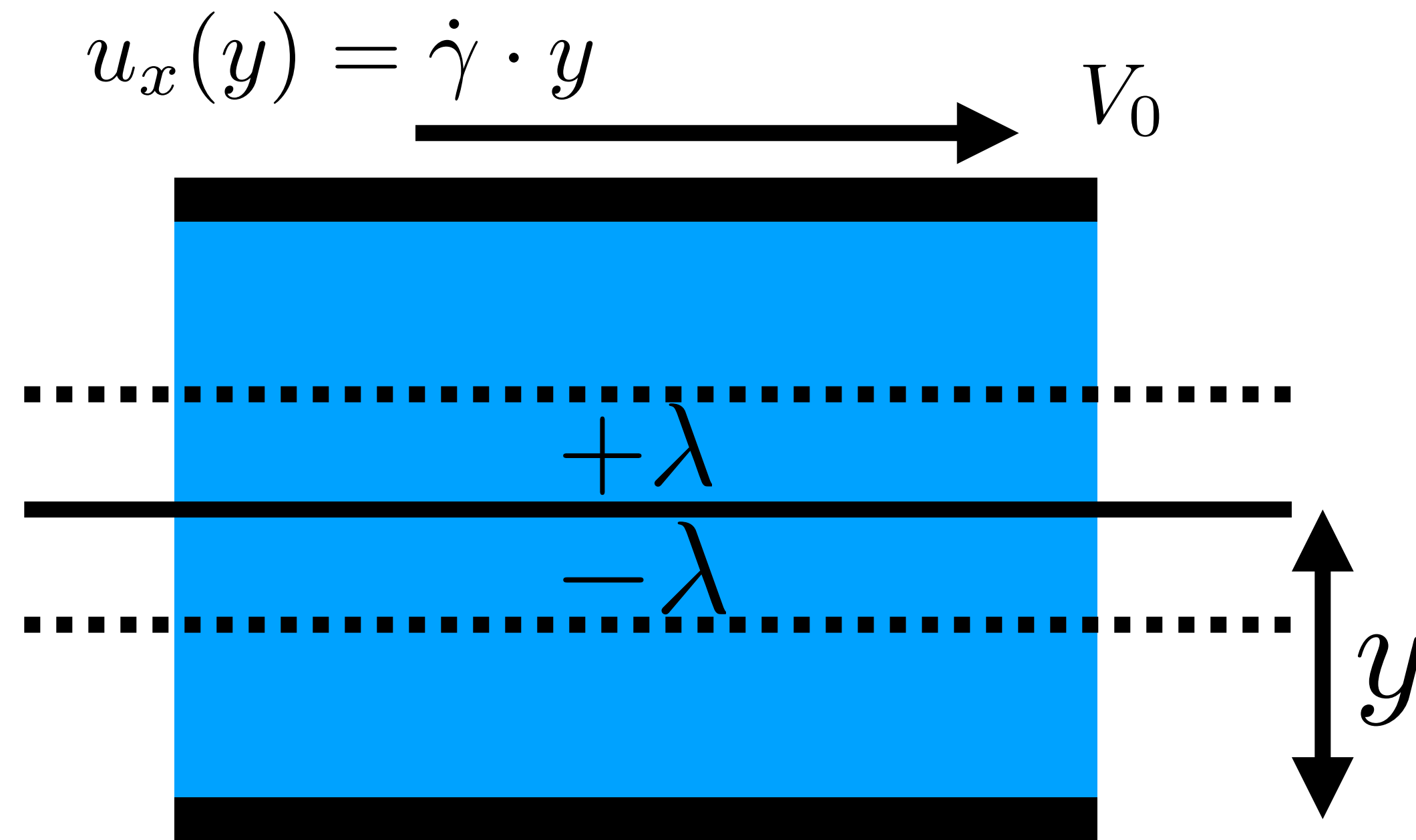
$\lambda$  : Mean free path of the molecules

Mean speed of a monoatomic gas:

$$\bar{c} = \sqrt{8k_B T / (\pi m)}$$

Molecules at height  $y$  have average  $x$ -velocity  $u_x(y)$ . But they only came to height  $y$  after traveling a distance  $\sim \lambda$  through the fluid. So the  $x$ -momentum they carry reflects the conditions they saw  $\sim \lambda$  away. That's where the gradient matters.

# How many molecules cross a plane per unit area per unit time?



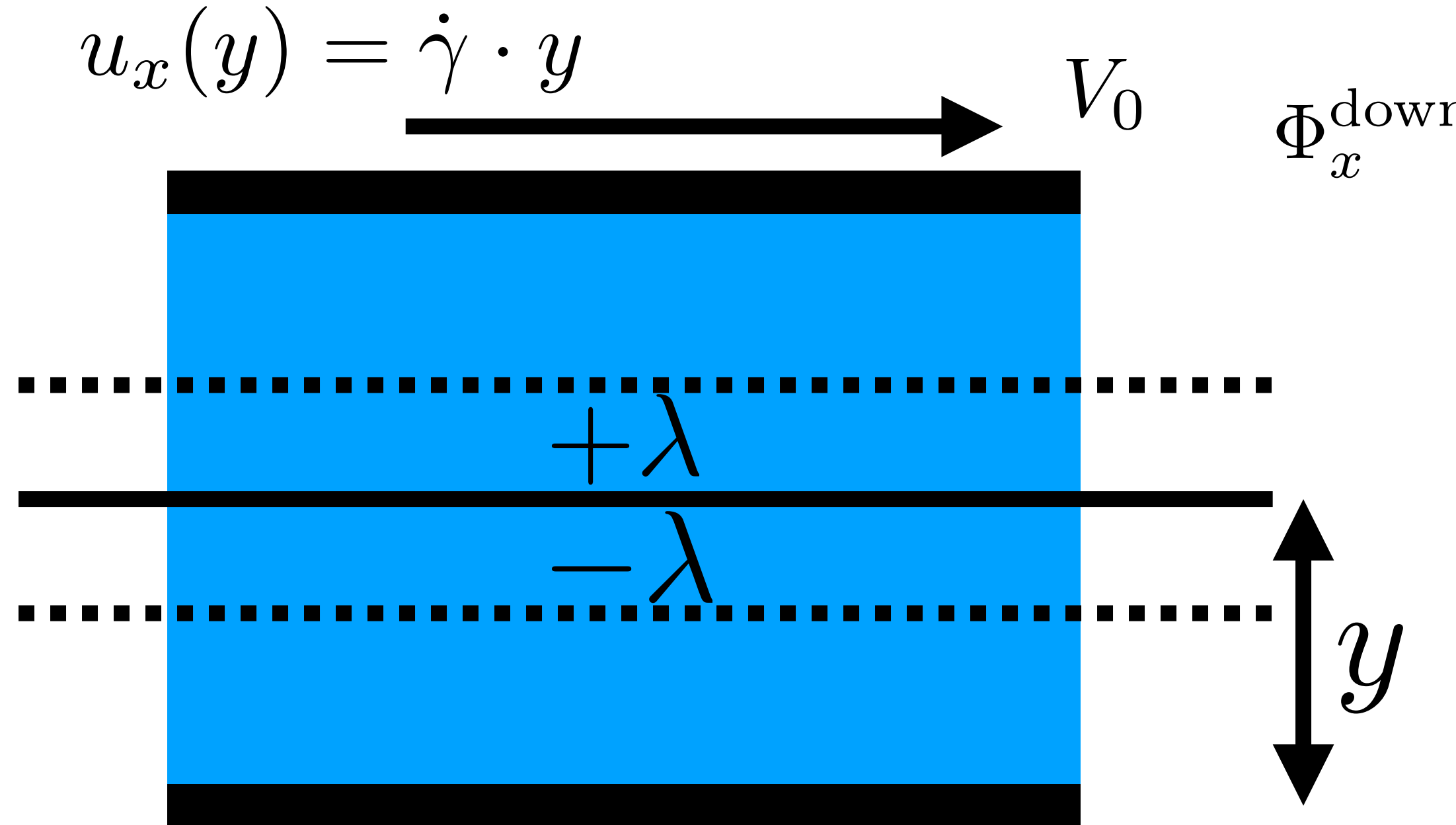
Number flux  $J = \frac{1}{4}n\bar{c}$

A molecule crossing the plane upward was last hit  $\sim \lambda$  below, so it carries the mean x-momentum from around  $y - \lambda$ . A molecule crossing downward carries momentum from around  $y + \lambda$ .

$$\Phi_x^{\text{down}} \approx \frac{n\bar{c}}{4} \cdot m \cdot u_x(y + \lambda)$$

$$\Phi_x^{\text{up}} \approx \frac{n\bar{c}}{4} \cdot m \cdot u_x(y - \lambda)$$

# Net downward momentum flux



The diagram illustrates a fluid layer of thickness  $2\lambda$  between two parallel plates. The top plate is at  $y = +\lambda$  and the bottom plate is at  $y = -\lambda$ . A shear flow is applied, with the velocity profile  $u_x(y) = \dot{\gamma} \cdot y$ . The shear rate  $\dot{\gamma}$  is represented by a horizontal arrow pointing right, labeled  $V_0$ . The vertical coordinate  $y$  is indicated by a double-headed arrow on the right. The fluid is represented by a blue rectangle. Dashed horizontal lines mark the boundaries at  $y = \pm\lambda$ . The momentum fluxes are given by:

$$\Phi_x^{\text{down}} \approx \frac{n\bar{c}}{4} \cdot m \cdot u_x(y + \lambda) \quad \Phi_x^{\text{up}} \approx \frac{n\bar{c}}{4} \cdot m \cdot u_x(y - \lambda)$$

$$\Phi_x^{\text{net}} \approx \frac{nm\bar{c}}{4} \cdot 2\lambda \frac{\partial u_x}{\partial y} = \frac{1}{2} \rho \bar{c} \lambda \frac{\partial u_x}{\partial y}$$

$$\tau_{xy} = -\Phi_x^{\text{net}} = -\frac{1}{2} \rho \bar{c} \lambda \frac{\partial u_x}{\partial y}$$

$$\eta \sim C \rho \bar{c} \lambda, \quad \text{where } C = \mathcal{O}(1)$$

Viscosity is an emergent property

$$\frac{\eta}{\rho} \sim \bar{c} \lambda$$