Computational Physics / PHYS-UA 210 / Problem Set #5
Due October 13, 2017

You *must* label all axes of all plots, including giving the *units*!!

1. Exercise 3 in S5.16 of Landau.
2. Exercise 4 in S5.16 of Landau.
3. A common type of multidimensional integral I need to do is as follows. We observe galaxy images focused on a 2D telescope focal plane, using optical fibers in the focal plane that carry the light to a bank of spectrographs. Each fiber has a circular aperture in the focal plane, so the light detected from the galaxy is that within some radius of its center. Two pretty good models for some galaxies are the following (the exponential, and de Vaucouleurs models):

$$\begin{aligned} I_{\mathrm{exp}}(r) &= A \exp\left[-\left(1.678 r/r_{50}\right)\right] \\ I_{\mathrm{deV}}(r) &= A \exp\left[-\left(3466 r/r_{50}\right)^{1/4}\right] \end{aligned} \tag{1}$$

The galaxy is not necessarily azimuthally symmetric, it can appear "squashed" on the sky, due to our viewing it at an angle. This can be accounted for by saying:

$$r = \sqrt{x^2 + \left(\frac{a}{b}\right)^2 y^2} \tag{2}$$

where $b/a < 1$ is called the *axis ratio*. Alternatively, one can express this as:

$$\begin{aligned} r &= r' \sqrt{\cos^2\theta + \left(\frac{a}{b}\right)^2 \sin^2\theta} \\ r' &= \sqrt{x^2 + y^2} \end{aligned} \tag{3}$$

where $\tan\theta = y/x$.

Using $r_{50} = 2$ arcseconds, and assuming a fiber radius of 1 arcsecond, calculate the fraction of the light that goes into the fiber as a function of b/a for each model, within the range 0.1 to 1. First, think about whether you want to integrate over x and y or over r' and θ. Check your convergence carefully for $n = 4$!