Computational Physics / PHYS-UA 210 / Problem Set #7
Due October 27, 2017

You *must* label all axes of all plots, including giving the *units*!!

Here you will solve the heat equation using the same techniques used in class for the vibrating string. This equation describes a heat-conducting rod, with in this case insulated ends.

The equation in one dimension is:

$$\frac{\partial u(x,t)}{\partial t} - \alpha \frac{\partial^2 u(x,t)}{\partial x^2} = 0 \tag{1}$$

and if its ends are insulated it is subject to the boundary condition:

$$\frac{\partial u(x,t)}{\partial x} = 0 \tag{2}$$

Constraints on the derivative at the boundary are known as *Neumann* boundary conditions (constraints on the function value at the boundary are known as *Dirichlet* boundary conditions.

1. Use the method of separation of variables to analytically find the solutions of this equation for constant α.

2. Now use the finite difference method presented in class to solve the same problem using a finite eigensystem. Compare the eigenfunctions to your analytic method and test how its accuracy varies with N. Demonstrate how a central temperature excess (use a Gaussian with a standard deviation of about one pixel) evolves over time. Feel free to use an altered version of the `StringProb` class in the Jupyter notebook for this lecture. However, you will need to account for the boundary conditions differently. To do so, use a central difference approximation to the derivative at the first node, and use this to alter the first and last finite difference equations.

3. Now alter the thermal diffusivity coefficient α to be a function of position. Try putting a "barrier" of low diffusivity somewhere (not right at the location of the Gaussian), and see what happens. Try some other pattern you invent too.