Lab 3 Appendix A: Max Distance

Parameters

```
Remove["`*"]; (* Remove all global symbols *)
```

Physical Constants

```
g := {0, -9.81}; (* Acceleration of gravity, m/s^2 *)
```

```
γ := 1; (* Type of flow, 1 = laminar *)
```

```
ρ := 1.3; (* Fluid mass density, kg/m^3 *)
```

```
CD := 0.3; (* Drag coefficient *)
```

Object Properties

```
m := 0.045; (* Particle mass, kg *)
```

```
r := 0.02; (* Particle radius, m *)
```

Initial Conditions

```
t0 = 0; (* Initial time *)
```

```
tf = 12; (* Final time *)
```

```
x0 := {0, 0}; (* Initial position, m *)
```

```
v0 := 80; (* Initial velocity, m/s *)
```

Forces and Energy

Total Force

$$F[x\_?VectorQ, v\_?VectorQ] := m\, g - kD\, Norm[v]^{\gamma} \frac{v}{Norm[v]};$$

where

$$kD := \frac{1}{2} CD\, \rho\, A; \text{ (* Viscous damping coefficient *)}$$

$$A := r^2 \pi; \text{ (* Cross section area, } m^2 \text{ *)}$$

Potential and Kinetic Energy

```
Ep[x_?VectorQ] := -m g.x;
```

$$Ek[v\_?VectorQ] := \frac{m\, v.v}{2};$$

Equations of Motion

```
SolveODE[
   t0_, tf_,
   x0_?VectorQ, v0_?VectorQ,
   method_: Automatic, h_: Automatic
  ] := Module[
   {X, V, sol},
   sol = NDSolve[
     {
      (* Differential Equations: *)
          X'[t] == V[t],
          m V'[t] == F[X[t], V[t] ],
      (* Initial conditions: *)
          X[t0] == x0,
          V[t0] == v0
     },
     {X, V}, (* Dependent variables *)
     {t, t0, tf} (* Range of the independent variable *)
     , Method → method
     , StartingStepSize → h
     , MaxStepSize → h
     , MaxSteps → Infinity
    ];
   ( {X, V} /. sol )[[1]]
  ];
```

Solve Equations

Subroutine that solves ode using Runge-Kutta 4th order with $h = 0.01$ for a given angle

```
FindMax[θ_] := Module[
   {X, V, tmax},
   {X, V} = SolveODE[t0, tf, x0,
     {v0 Cos[θ π / 180], v0 Sin[θ π / 180]}, "ExplicitRungeKutta", 0.01];
   tmax = t /. FindRoot[X[t][[2]] == 0, {t, 10, tf}] ;
   X[tmax][[1]]
  ];
```

Find max distance without viscous damping (reference solution)

```
CD = 0.0;
MaxXref = Quiet[Table[{θ, FindMax[θ]}, {θ, 40, 50, 0.2}]];
```

Find max distance with viscous damping enabled, laminar flow

```
CD = 0.3; γ := 1;
MaxXvisc = Quiet[Table[{θ, FindMax[θ]}, {θ, 40, 50, 0.2}]];
```

Find max distance with viscous damping enabled, turbulent flow

```
CD = 0.3; γ := 2;
MaxXvisc2 = Quiet[Table[{θ, FindMax[θ]}, {θ, 32, 42, 0.2}]];
```

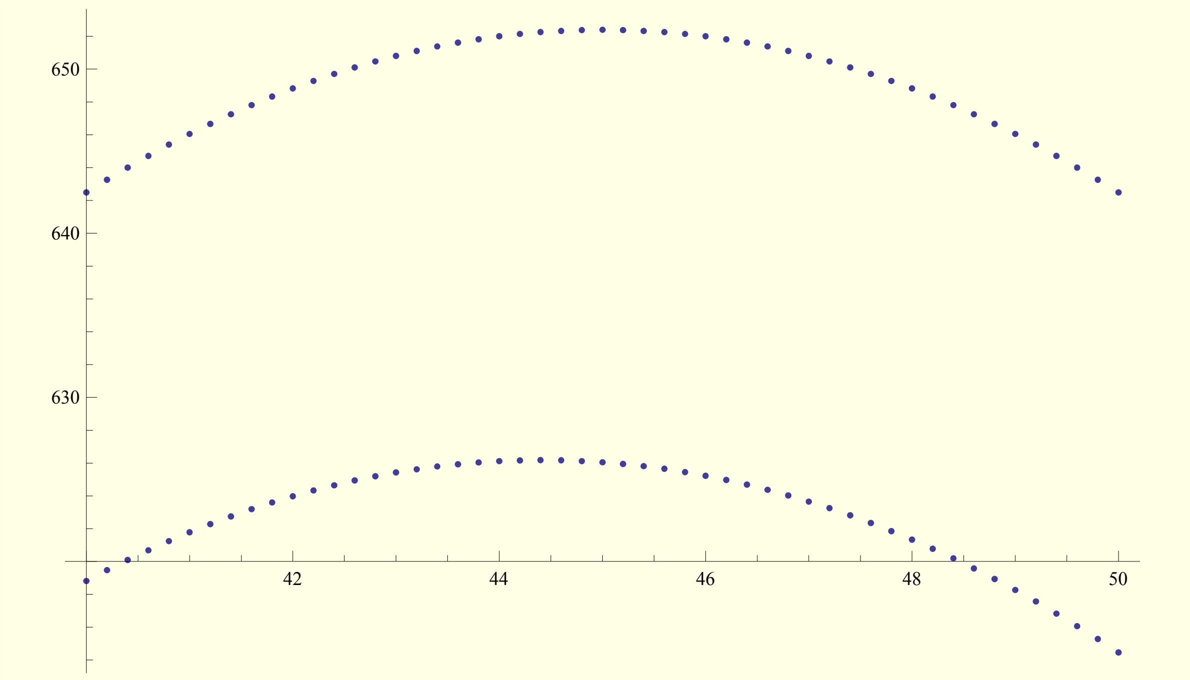
The Results

```
MaxX = Transpose[{
    MaxXvisc[[All, 1]], MaxXvisc[[All, 2]], MaxXref[[All, 2]],
    Table["    ", {i, 0, 10, 0.2}],
    MaxXvisc2[[All, 1]], MaxXvisc2[[All, 2]]
   }] // TableForm
```

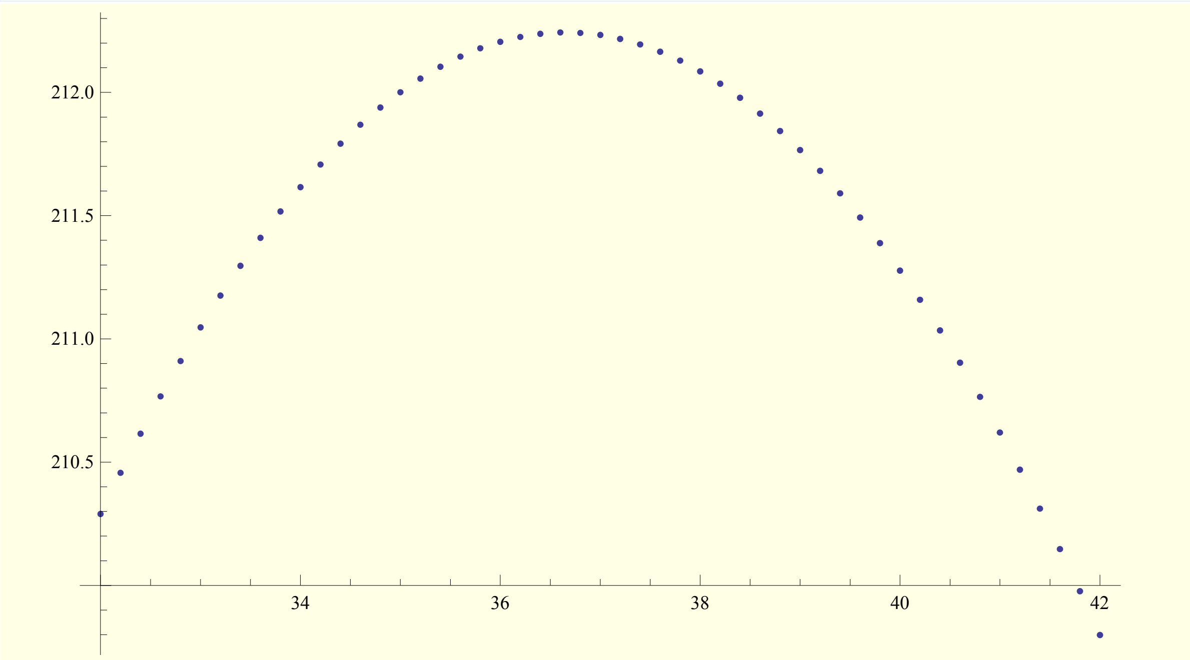
40.	618.821	642.484		32.	210.29
40.2	619.473	643.259		32.2	210.457
40.4	620.094	644.003		32.4	210.616
40.6	620.685	644.716		32.6	210.767
40.8	621.246	645.397		32.8	210.911
41.	621.777	646.046		33.	211.047
41.2	622.278	646.665		33.2	211.176
41.4	622.748	647.251		33.4	211.297
41.6	623.189	647.806		33.6	211.41
41.8	623.599	648.33		33.8	211.517
42.	623.979	648.822		34.	211.616
42.2	624.328	649.282		34.2	211.707
42.4	624.648	649.711		34.4	211.792
42.6	624.937	650.107		34.6	211.869
42.8	625.196	650.473		34.8	211.938
43.	625.425	650.806		35.	212.001
43.2	625.623	651.108		35.2	212.056
43.4	625.791	651.378		35.4	212.104
43.6	625.929	651.617		35.6	212.145
43.8	626.037	651.823		35.8	212.179
44.	626.114	651.998		36.	212.206
44.2	626.162	652.141		36.2	212.225
44.4	626.179	652.252		36.4	212.238
44.6	626.165	652.332		36.6	212.243
44.8	626.122	652.38		36.8	212.241
45.	626.048	652.396		37.	212.233
45.2	625.945	652.38		37.2	212.217
45.4	625.811	652.332		37.4	212.195
45.6	625.647	652.252		37.6	212.165
45.8	625.452	652.141		37.8	212.129
46.	625.228	651.998		38.	212.085
46.2	624.973	651.823		38.2	212.035
46.4	624.689	651.617		38.4	211.978
46.6	624.374	651.378		38.6	211.914
46.8	624.03	651.108		38.8	211.844
47.	623.655	650.806		39.	211.766
47.2	623.251	650.473		39.2	211.682
47.4	622.816	650.107		39.4	211.591
47.6	622.352	649.711		39.6	211.493
47.8	621.857	649.282		39.8	211.388
48.	621.333	648.822		40.	211.277
48.2	620.779	648.33		40.2	211.159
48.4	620.195	647.806		40.4	211.035
48.6	619.582	647.251		40.6	210.903
48.8	618.939	646.665		40.8	210.765
49.	618.266	646.046		41.	210.621
49.2	617.563	645.397		41.2	210.47
49.4	616.831	644.716		41.4	210.312
49.6	616.069	644.003		41.6	210.147
49.8	615.278	643.259		41.8	209.976
50.	614.458	642.484		42.	209.799

Maximimum length (in meters) as a function of initial velocity angle (in degrees):

```
Show[ListPlot[Table[ MaxXref[[i, All]], {i, 1, 51}]],
 ListPlot[Table[ MaxXvisc[[i, All]], {i, 1, 51}]]]
```




```
ListPlot[Table[MaxXvisc2[[i, All]], {i, 1, 51}]]
```



Reference solution without viscous forces:

```
Max[MaxXref[[All, 2]]]
```

```
652.396
```

```
MaxXref[[Position[ MaxXref[[All, 2]], Max[MaxXref[[All, 2]]]][[1]][[1]], 1]]
```

```
45.
```

With viscous damping, laminar flow:

```
Max[MaxXvisc[[All, 2]]]
```

```
626.179
```

```
MaxXvisc[[Position[ MaxXvisc[[All, 2]], Max[MaxXvisc[[All, 2]]]][[1]][[1]], 1]]
```

```
44.4
```

With viscous damping, turbulent flow:

```
Max[MaxXvisc2[[All, 2]]]
```

```
212.243
```

```
MaxXvisc2[[Position[ MaxXvisc2[[All, 2]], Max[MaxXvisc2[[All, 2]]]][[1]][[1]], 1]]
```

```
36.6
```