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1 Introduction

D0 and CDF measure the instantaneous luminosity by counting the number of observed $p\overline{p}$ interactions per beam crossing. The mean number of observed collisions per crossing μ can be related to the scattering cross section via:

$$f\mu = \sigma \epsilon \pounds(\eta_{min}, \eta_{max})$$

where f is the crossing frequency, σ is the cross section $\pounds$ is the luminosity and $\epsilon(\eta_{min}, \eta_{max})$ is the acceptance of the detector, which subtends a pseudo-rapidity range from η_{min} to η_{max}. If σ and $\epsilon(\eta_{min}, \eta_{max})$ are known, $\pounds$ can be measured.

It is important to note that both D0 and CDF luminosity detectors do not subtend the full solid angle and are sensitive to only a subset of the total cross section, in particular, the inelastic scattering cross section over a range from $3 < |\eta| < 7$, it is this quantity which is need to relate measurements to luminosity.

The σ to input to the D0 and CDF luminosity comes from measurements performed with full acceptance and the ability to measure luminosity and event rates independently. Three experiments have performed this measurement at the Tevatron, E711[**?**], E810[**?**, **?**] and CDF[**?**].

2 Inputs

The experimental measurements of the scattering cross section use the the optical theorem and elastic scatters to obtain two relations between counting rate and luminosity.

The optical theorem relates the $t = 0$ intercept of the elastic scattering cross section to the total cross section:

$$(1 + \rho^2)\sigma_T^2 = 16\pi(\hbar c)^2 \frac{d\sigma_{el}}{dt}|_{t=0}$$

$$\sigma_T = \sigma_{inel} + \sigma_{el}$$
$$\sigma_T = \frac{1}{\pounds}\frac{N_{inel}}{\epsilon_{inel}} + \frac{N_{el}}{\epsilon_{el}}$$
$$\frac{d\sigma_{el}}{dt}|_{t=0} = \frac{1}{\pounds}\frac{dN_{el}}{\epsilon_{el}(t)dt}|_{t=0}$$

There experiments were able to measure both $N_{el}(t)$ and $N_{el} + N_{inel}$, determine the luminosity and from that the cross sections. To the authors of these publications, the determination of the total cross section was most important. For D0 and CDF luminosity determinations the important quantity is the inelastic cross section measured over the pseudo-rapidity ranges covered by the D0 and CDF luminosity monitors.

In particular, the desired **luminosity constant**for each experiment is the acceptance weighted inelastic cross section.

$$\sigma_{accep} = \sigma_{hard}\epsilon_{hard} + \sigma_{SD}\epsilon_{SD} + \sigma_{DD}\epsilon_{DD}$$

For D0 this is

$$\sigma_{accep} \approx 0.91 \times (0.97\sigma_{hard} + 0.15\sigma_{SD} + 0.72\sigma_{DD}))$$

3 Measurements

Cross Section, mb	Value	error
Total		
E710[1]	72.1	3.3
E811[3]	71.4	2.4
CDF[4]	80.0	2.2
Elastic		
E710[1]	16.6	1.6
E811[2]	15.8	0.9
CDF[4]	19.7	0.9
Inelastic		
E710[1]	55.5	2.2
E811[2]	55.9	1.2
CDF[4]	60.3	1.4
Single Diffractive		
E711[1]	11.7	2.3
–		
CDF[4]	9.5	0.5
Double Diffractive(*)		
E711[1]	(2.06)	(0.83)
–		
CDF[4]	(1.29)	(0.20)
CDF[*]	7.5(*)	2.0
D0 'Luminosity Constant'	41.8	1.9

(∗) indicates an unpublished CDF measurement. The numbers in parentheses are indirect estimates of the double diffractive cross section from single diffractive and elastic cross sections.

References

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