Limits on B_s Mixing

Study on Setting Limits

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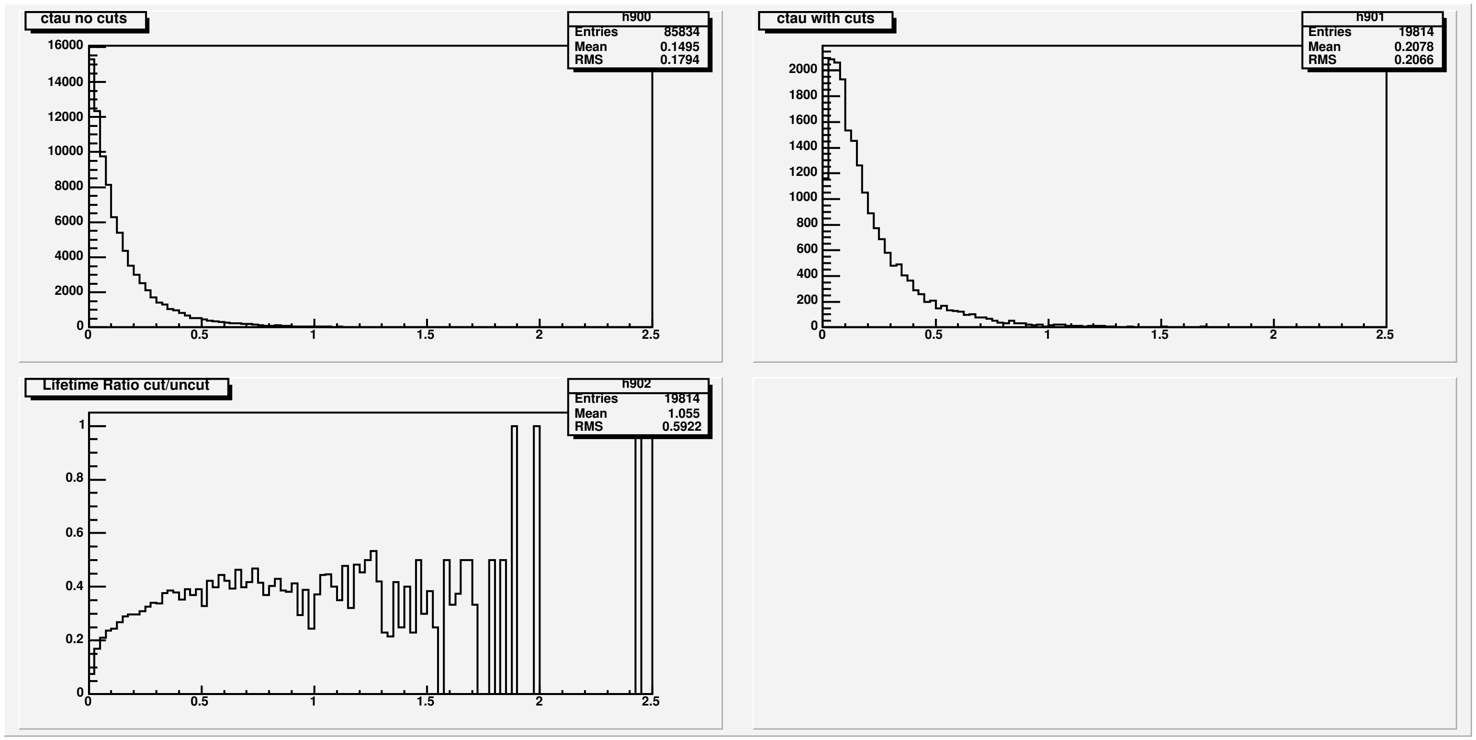
Approaches

- Two basic approaches
  - Fit mixed and unmixed time distributions
    - This can be binned or unbinned
  - Amplitude method
    - This can also be binned or unbinned to mixed and unmixed time distributions
    - Can be applied to binned asymmetry distribution
- The questions we wish to answer is how best to set limit on Δm_s given our current data set.
- **This study concentrates on the Amplitude Method**

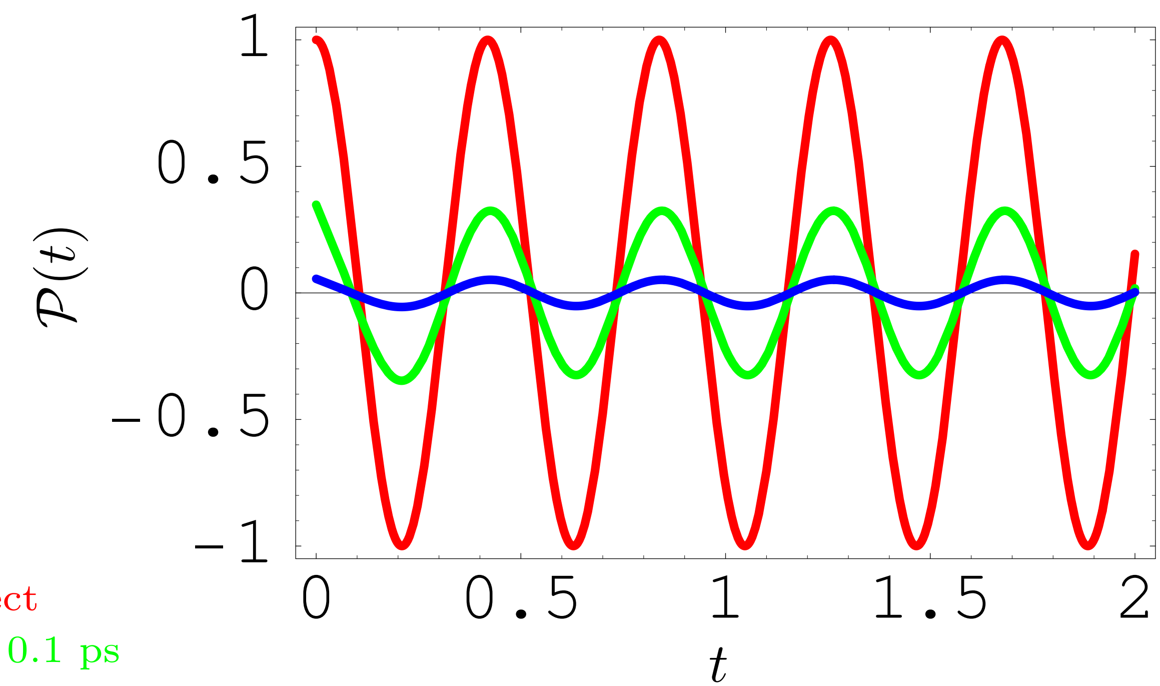
Comments

- To reduce background, apply cuts to lifetime distributions
  - Fit to lifetime plot requires unknown function $f(t)$
  - This is important for likelihood fits



Amplitude Method

- Amplitude of asymmetry distribution decreases as a function of σ_t and D



Perfect

$\sigma_t = 0.1$ ps

$\sigma_t = 0.1$ ps & $D = 0.16$

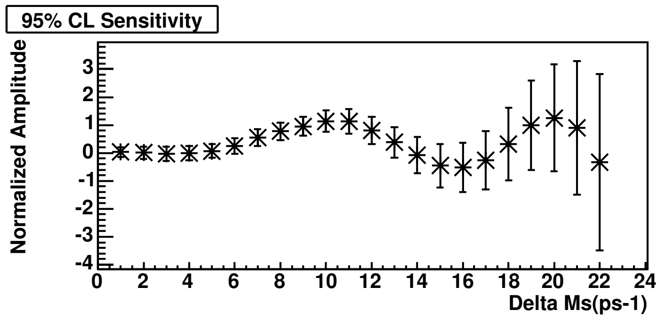
$\Delta m = 15\ \text{ps}^{-1}$

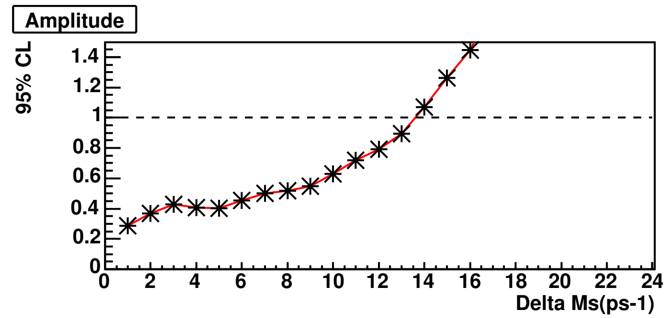
Amplitude Method

- Fit to the oscillation amplitude for different values of Δm_s
  - For asymmetry distribution: $A\cos(\Delta m_s t)$
  - For mixed and unmixed lifetime distributions:
    $\frac{\Gamma}{2}e^{-\Gamma t}\left[1 \pm A\cos(\Delta m_s t)\right]$
- Then compare to expected amplitude for given dilution and proper time resolution: $De^{-(\Delta m_s \sigma_t)^2/2}$
- Δm_s found when $A = 1$
  - Sensitivity assume $A = 0$ what is probability to fluctuate to $A = 1$: $S = 1.65\sigma$ for 95% exclusion
  - Limit given A what is probability to fluctuate to $A = 1$
    $A < 1 - 1.65\sigma$ 95% exclusion

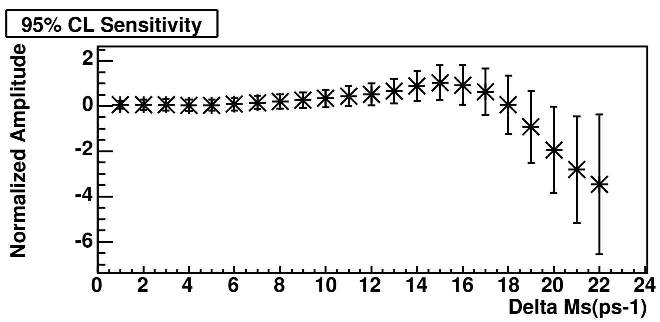
Examples

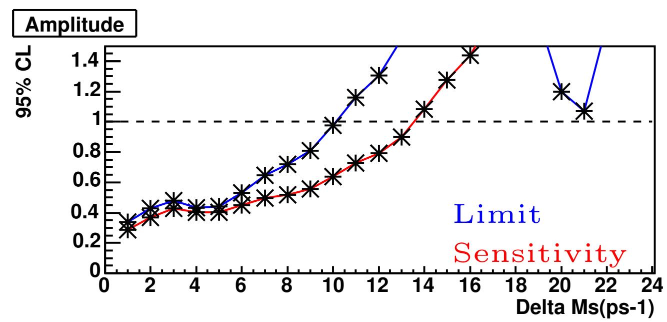
$\Delta m_s = 10\ \mathrm{ps}^{-1}$





$\Delta m_s = 1000\ \mathrm{ps}^{-1}$

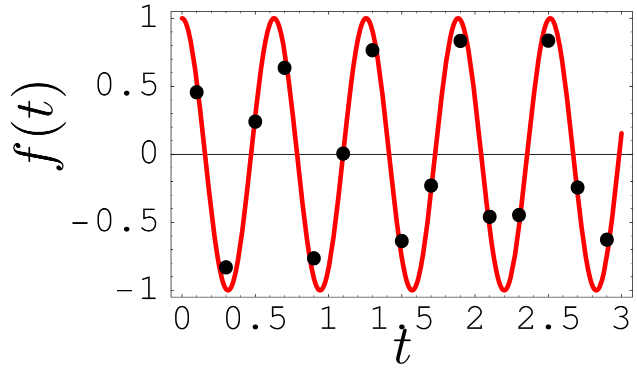




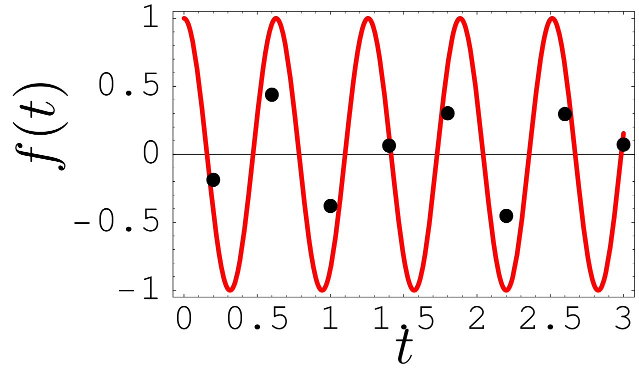
Side Note

- Amplitudes on previous page calculated using from χ^2 fit to asymmetry distribution.
  - The fit is to the integral of $A\cos(\Delta m_s t)$ over each bin
    - Important for large bin size

Bin Width 0.2



Bin Width 0.4



Dots correspond to integral over bin width

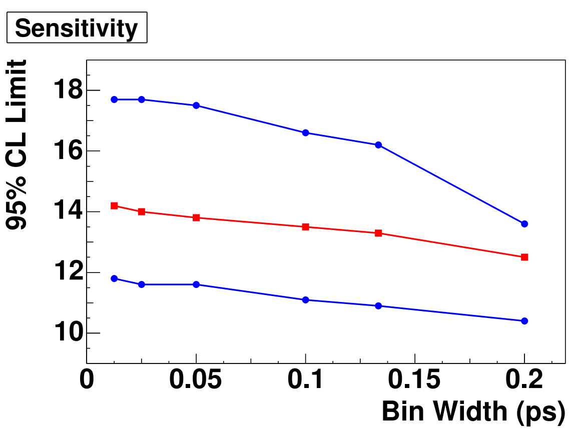
Expected Sensitivity

Using Toy Monte Carlo and the amplitude method, calculate sensitivity from asymmetry distribution for different bin width, resolution, and dilution.

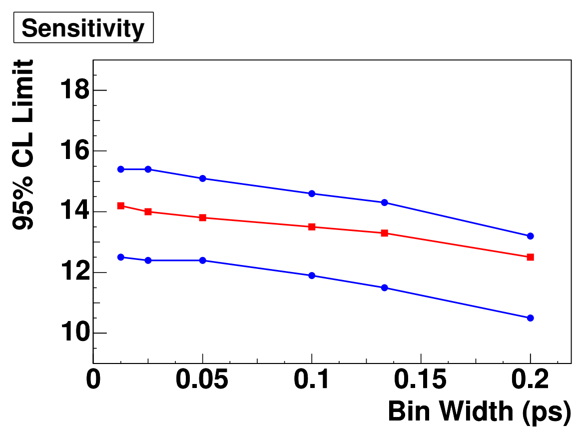
Nominal Values

Range	0 to 0.8 ps	Events in range	≈ 2000
Resolution	0.1 ps	Dilution	0.16

Vary Resolution $\pm 20\%$



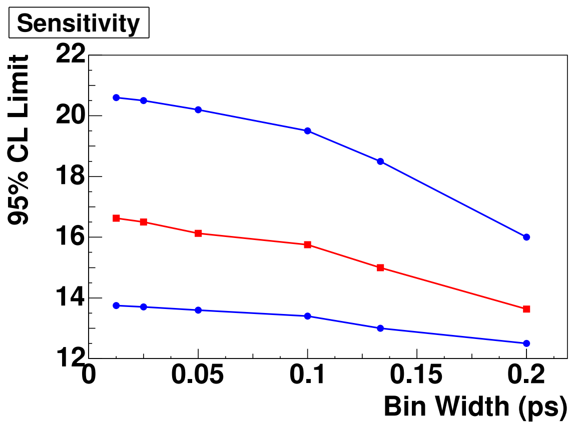
Vary Dilution $\pm 20\%$



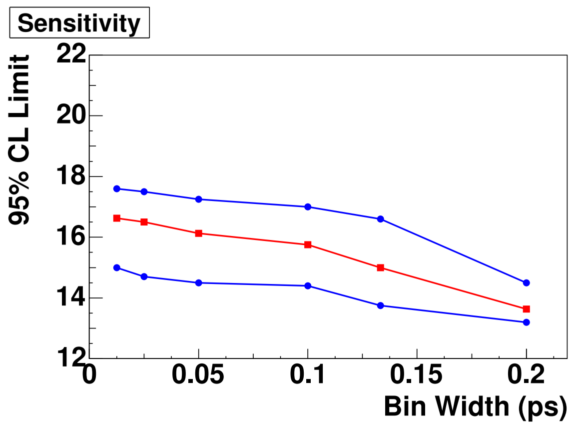
Expected Sensitivity—cont.

To compare with the the unbinned method, we increased the number of events to 4000

Vary Resolution ±20%



Vary Dilution ±20%



- For the nominal values, the sensitivity is
  - 2000 Events 13.5 ps^{-1}
  - 4000 Events 15.75 ps^{-1} (16.0 ps^{-1} if range extended to 3 ps)

Unbinned Amplitude Fit

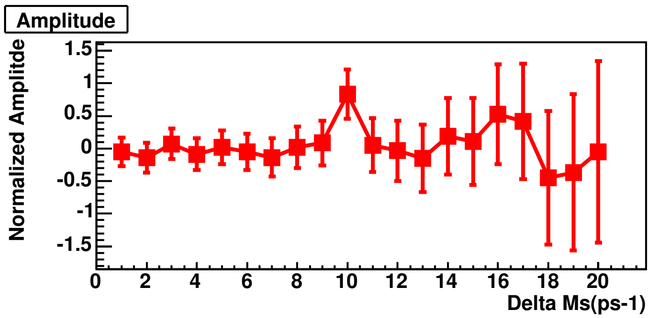
Again we use a toy MC, this time we determine the Amplitude from an unbinned fit.

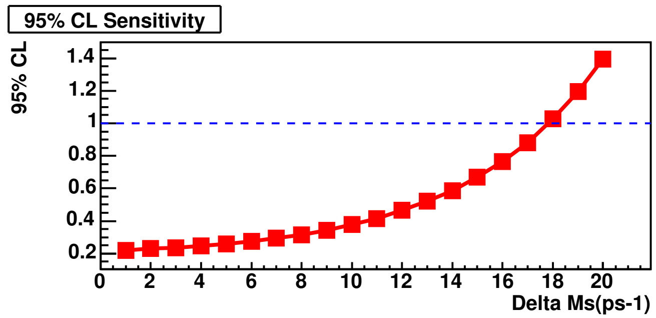
$$\mathcal{P}^{\text{unmixed}}_{\text{mixed}}(t) = \frac{(1-\alpha)}{2\sqrt{2\pi}\sigma\tau}\int_0^\infty e^{-(t-t')^2/2\sigma} e^{-t'/\tau}\left[1 \pm \cos(\Delta m_s t')\right] + \frac{\alpha}{2\sqrt{2\pi}\sigma\tau}\int_0^\infty e^{-(t-t')^2/2\sigma} e^{-t'/\tau}\left[1 \mp \cos(\Delta m_s t')\right]$$

- Fit for α(mistag rate) for different values of Δm_s, σ and τ assumed to be known
- Limits and sensitivity determined as before.

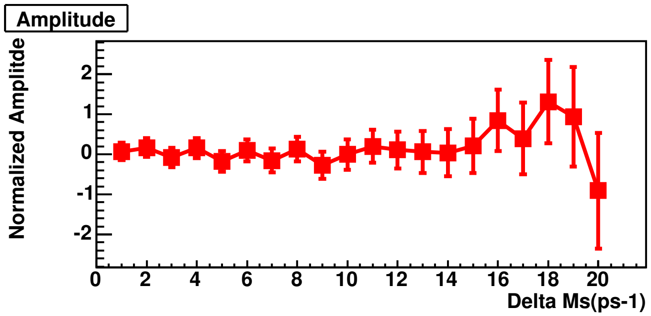
Examples

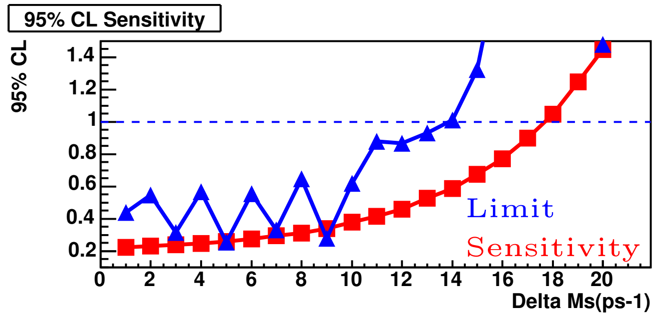
$\Delta m_s = 10\ \text{ps}^{-1}$





$\Delta m_s = 1000\ \text{ps}^{-1}$





Expected Sensitivity—*Likelihood*

Using toy MC, we generate data with different σ_t and D and fit for the amplitude assuming values of σ_t and D (*not necessarily the same as generated*) in the fit.

Sensitivity for fixed $D = 0.16$ vary σ_t

MC/Fit	0.08 ps	0.10 ps	0.12 ps
0.08/ps	22.07/ps	23.56/ps	
0.10/ps	17.66/ps	**17.74/ps**	17.88/ps
0.12/ps		15.74/ps	15.00/ps

Sensitivity for fixed $\sigma_t = 0.1$ ps vary D

MC/Fit	.128	0.160	0.192
0.128	16.35/ps	16.35/ps	
0.160	17.75/ps	**17.74/ps**	17.75/ps
0.192		18.90/ps	18.91/ps

- Significance is given by

$$S(\Delta m, \sigma_t) = \sqrt{\frac{\epsilon N}{2}}\sqrt{\frac{S}{S+B}}De^{-(\Delta m \sigma_t)^2/2}$$

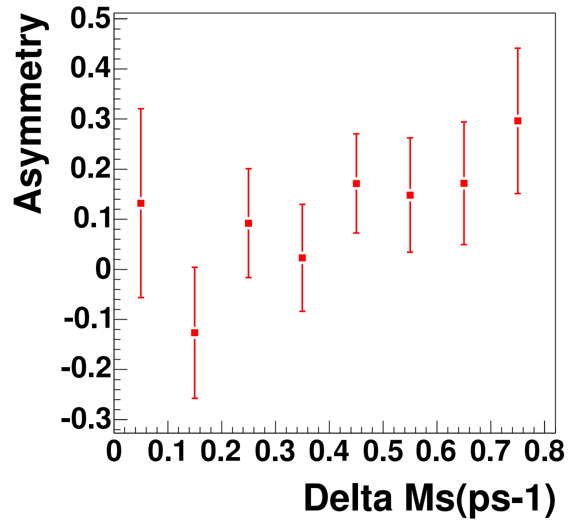
- For the nominal parameters this gives a sensitivity of $17.13\ \text{ps}^{-1}$ which is consistent with the results given above

B_s Mixing Limit

- Taking $B_s \rightarrow D_s(\phi\pi)\mu X$ data (From Vivek 200-220 pb^{-1})
  - Calculate number of mixed and unmixed events from mass fits
  - Calculate asymmetry in bins of 0.1 ps
  - To keep resolution reasonably small, limit proper time < 0.8 ps
    - $\sigma_t = 0.11$ ps $t_p < 0.4$ ps, $\sigma_t = 0.138$ $0.4 < t_p \leq 0.8$ ps (According to Wendy)
  - For amplitude fit use $D = 0.16$ and $\sigma_t = 0.122$ ps (weighted average)

The Limit

Asymmetry Distribution

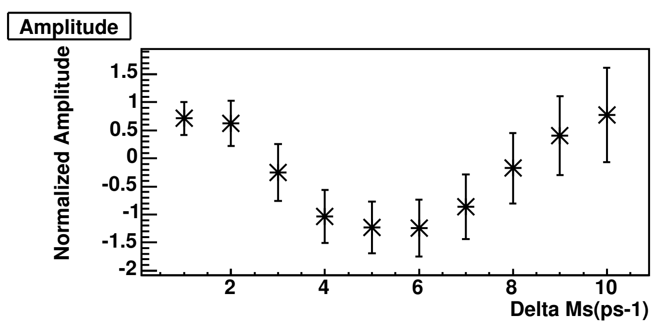


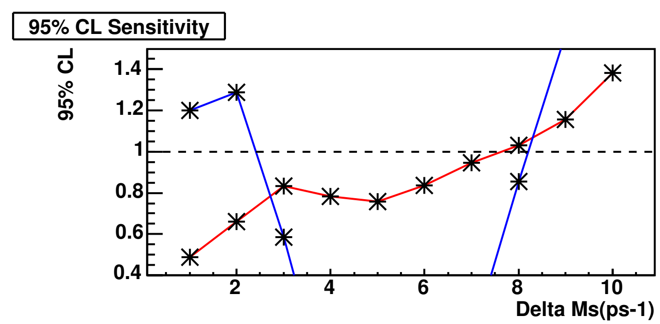
Results

The dilution is taken to be $D = 0.16$

σ_t (ps)	Sensitivity/ps	Limit/ps
0.110	8.63	—
0.122	**7.62**	—
0.138	6.67	—

Using correct σ_t for each bin gives a sensitivity of 6.67 ps^{-1}





Summary

- Likelihood amplitude method gives a better result
  - For same statistics & same fit range $(0 \rightarrow \infty) \approx 2$ ps^{-1} better sensitivity $(17.7 \text{ vs. } 16.0 \text{ ps}^{-1})$
  - For same statistics $t = 0 \rightarrow \infty$, fit range limited for asymmetry fit, ≈ 4 ps^{-1} better sensitivity $(17.7 \text{ vs. } 13.5 \text{ ps}^{-1})$
  - Likelihood gives slightly better than analytic calculation $(17.7 \text{ vs. } 17.1 \text{ ps}^{-1})$
- Likelihood method requires additional work in finding unknown function since we want to reduce backgrounds
- MC needs to be made more realistic
  - Resolution function must depend on proper time
  - Add backgrounds
  - ...