1 Introduction

D0 and CDF measure the instantaneous luminosity by counting the number of observed $p\overline{p}$ interactions per beam crossing. The mean number of observed collisions per crossing μ can be related to the scattering cross section via:

$$f\mu = \sigma \pounds\epsilon(\eta_{min}, \eta_{max})$$

where f is the crossing frequency, σ is the cross section £is the luminosity and $\epsilon(\eta_m in, \eta_m ax)$ is the acceptance of the detector, which subtends a pseudo-rapidity range from η_{min} to η_{max}. If σ and $\epsilon(\eta_{min}, \eta_{max})$ are known, £can be measured.

It is important to note that both D0 and CDF luminosity detectors do not subtend the full solid angle and are sensitive to only a subset of the total cross section, in particular, the inelastic scattering cross section over a range $3 < |\eta| < 7$, it is this quantity which is need to relate measurements to luminosity.

The σ to input to the D0 and CDF luminosity comes from measurements performed with full acceptance and the ability to measure luminosity and event rates independently. Three experiments have performed this measurement at the Tevatron, E711**??**, E810**??** and CDF**??**.

2 Inputs

The experimental measurements of the scattering cross section use the the optical theorem and elastic scatters to obtain two relations between counting rate and luminosity.

The optical theorem relates the $t = 0$ intercept of the elastic scattering cross section to the total cross section:

$$(1+\rho^2)\sigma_T^2 = 16\pi(\hbar c)^2 \frac{d\sigma_{el}}{dt}|_{t=0}$$

$$\sigma_T = \sigma_{inel} + \sigma_{el}$$

$$\sigma_T = \frac{1}{\pounds}\frac{N_{inel}}{\epsilon_{inel}} + \frac{N_{el}}{\epsilon_{el}}$$

$$\frac{d\sigma_{el}}{dt}|_{t=0} = \frac{1}{\pounds}\frac{dN_{el}}{\epsilon_{el}(t)dt}|_{t=0}$$

There experiments were able to measure both $N_{el}(t)$ and $N_{el} + N_{inel}$, determine the luminosity and from that the cross sections. To the authors of these publications, the determination of the total cross section was most important. For D0 and CDF luminosity determinations the important quantity is the inelastic cross section measured over the pseudo-rapidity ranges covered by the D0 and CDF luminosity monitors.

The D0 luminosity constant is the acceptance weighted inelastic cross section. In the next section a naive study of its sensitivity to experimental inputs is done. A more precise determination, in which the model used for the D0 acceptance is the same as that used in the luminosity/cross section measurements still needs to be done.

3 Combination of Experiments

The table below shows the results of a simple combination of results from CDF and E811. The numbers were adjusted to have a common ρ value (from E811 world average**??**) of 0.132 ± 0.056 and then averaged. The PDG error quoted is the error consistent with a χ^2/d.o.f. of 1.

The hard cross section is the inelastic cross section with the diffractive components removed. The D0 accepted cross section or luminosity constant is the sum of the diffractive and hard cross sections weighted by the D0 acceptances of 0.97, 0.14 and 0.72 for hard, single and double diffractive scattering respectively. There is an additional common factor of 0.907 ± 0.025 from halo and detector efficiency corrections.

Cross Section, mb	Value	error	PDG Error	Chi2/Dof	expt expt expt	Used?
Total	75.8	1.92	4.64	8.25/ 1.		
	72.1	3.3			E710	no
	71.4	2.4			E811	yes
	80.0	2.2			CDF	yes
Elastic	17.8	0.662	2.02	10.9/1.		
	16.6	1.6			E710	no
	15.8	0.9			E811	yes
CDF	19.7	0.9			CDF	yes
Inelastic	57.9	1.24	2.48	6.62/ 1.		
E710	55.5	2.2				no
E811	55.9	1.2				yes
CDF	60.3	1.4				yes
Single Diffractive	9.54	0.432	0.432	0.91/1.		
E711	11.7	2.3				yes
						no
CDF	9.5	0.5				yes
Double Diffractive	7.48	1.99	1.99	0	0	
E711	(2.06)	(0.83)				no
						no
CDF	7.5	2.0				yes
Derived Hard	40.8	2.38	3.21			
D0 Inelastic accpt.	42.1	1.94	2.71			

3.1 Sensitivity to double diffractive rate

If the double diffractive cross section estimated by E710 from the elastic and inelastic cross sections (2.06 ± 0.84) mb is used instead of the recent CDF measurement of 7.5 ± 2.0 mb, the hard cross section rises substantially to $46.3 \pm 1.6 (\pm 2.7)$ but the D0 luminosity constant only rises to $43.4 \pm 1.9 \pm 2.7$ (3.5%). This is because the acceptance for the D0 luminosity detector is not radically different for the two processes.

3.2 Sensitivity to choice of input experiments

3.2.1 add E710

If E710 measurements are all included, the D0 luminosity constant drops to 41.8 mb or by less than 1%.

3.2.2 CDF only

If CDF data only are used, (except for $\rho0$ the D0 luminosity constant rises to 44.6 mb (+6%)

3.2.3 E811 only

If E811 data only are used(except for the single and double diffraction where E811 has no information), the D0 luminosity constant falls to 39.8 mb (-6%)