The Dynamics of Complex Systems

Warning Signals of Imminent Phase Transitions

Cross-Recurrence Quantification Analysis and other flavours of RP's

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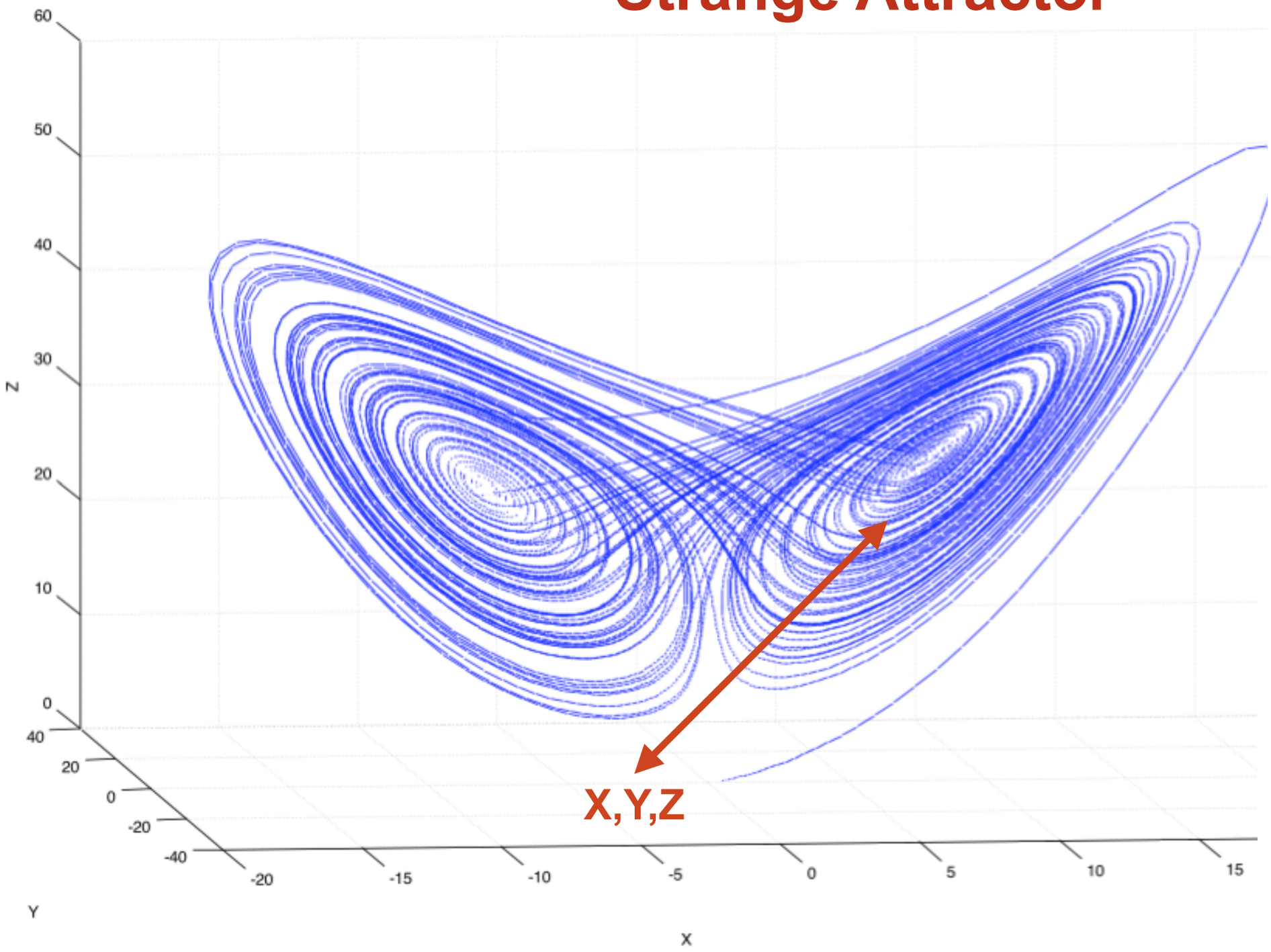
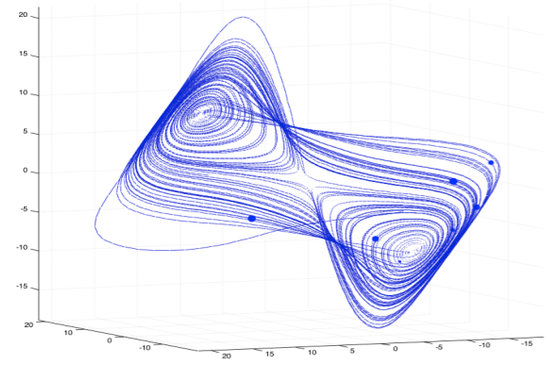
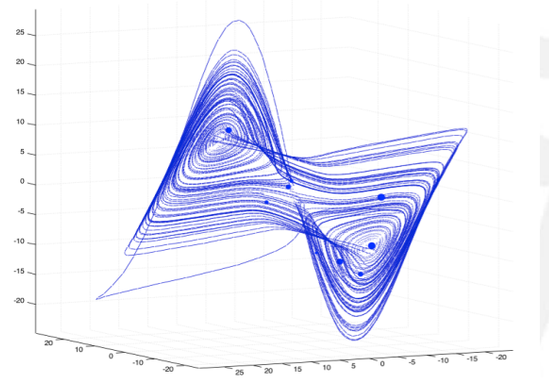
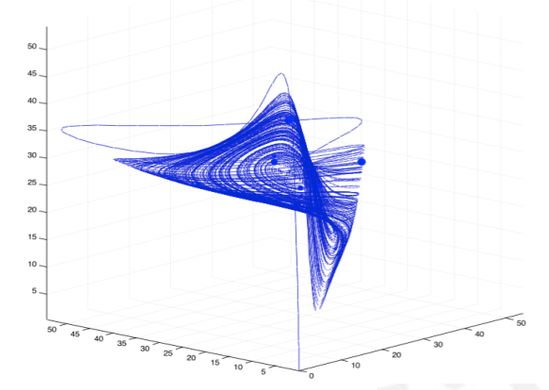
Lorenz system – X,Y,Z State space
Strange Attractor



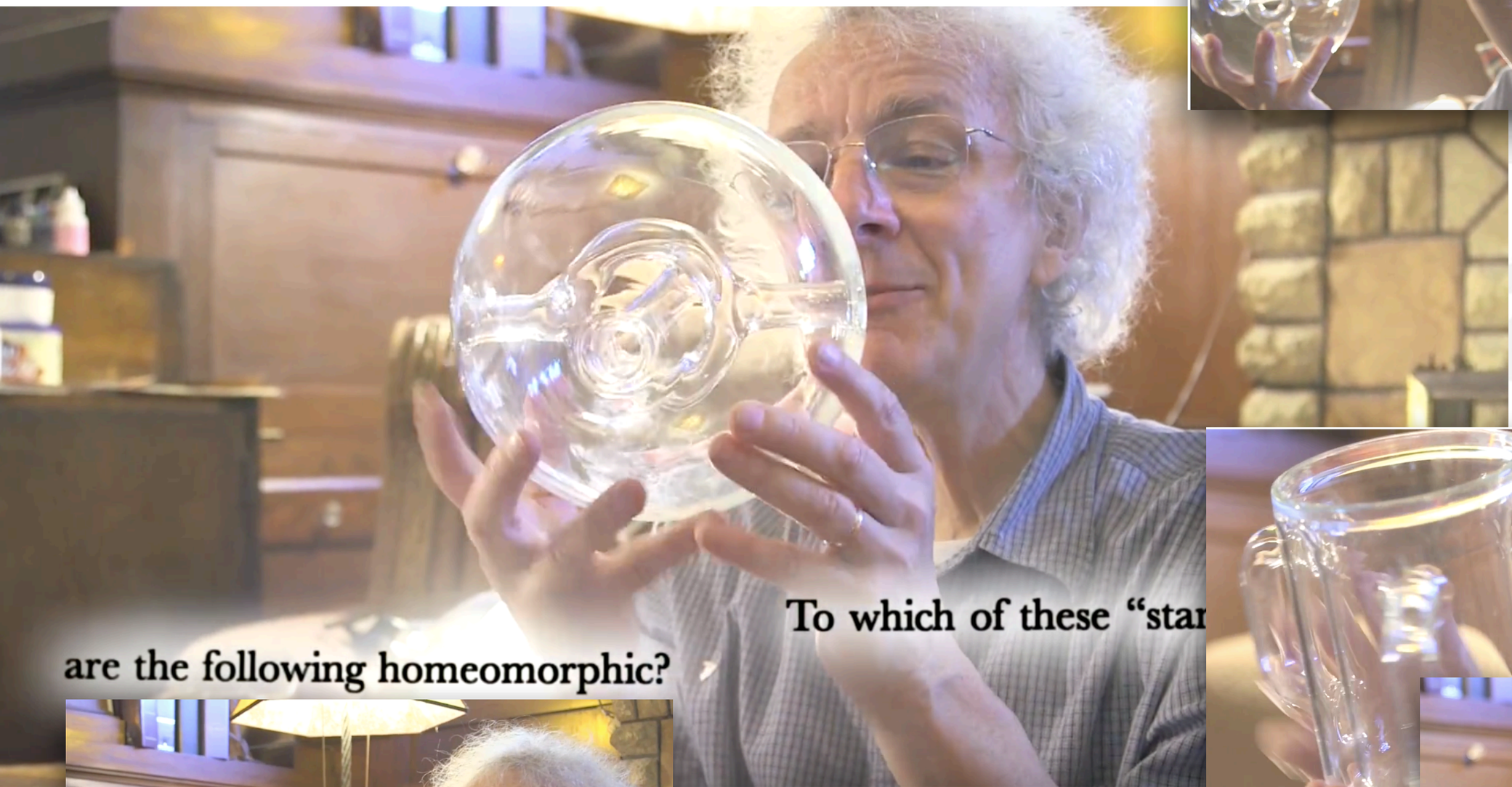




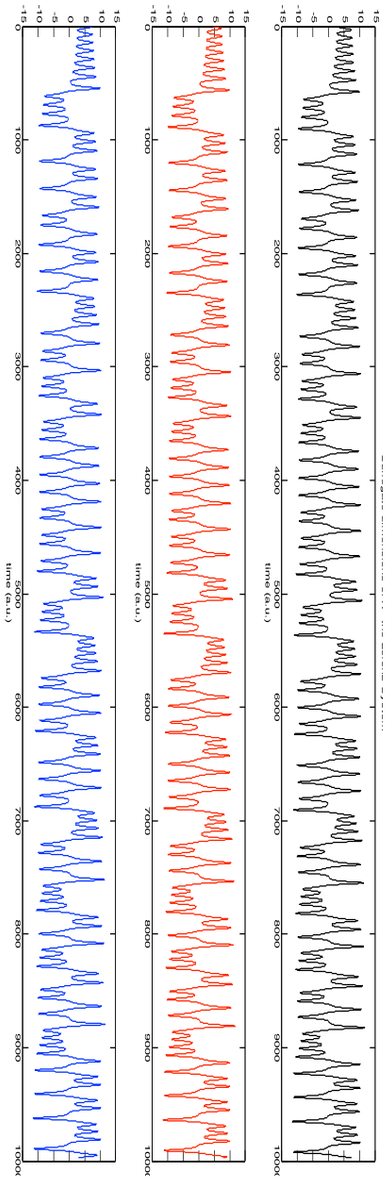


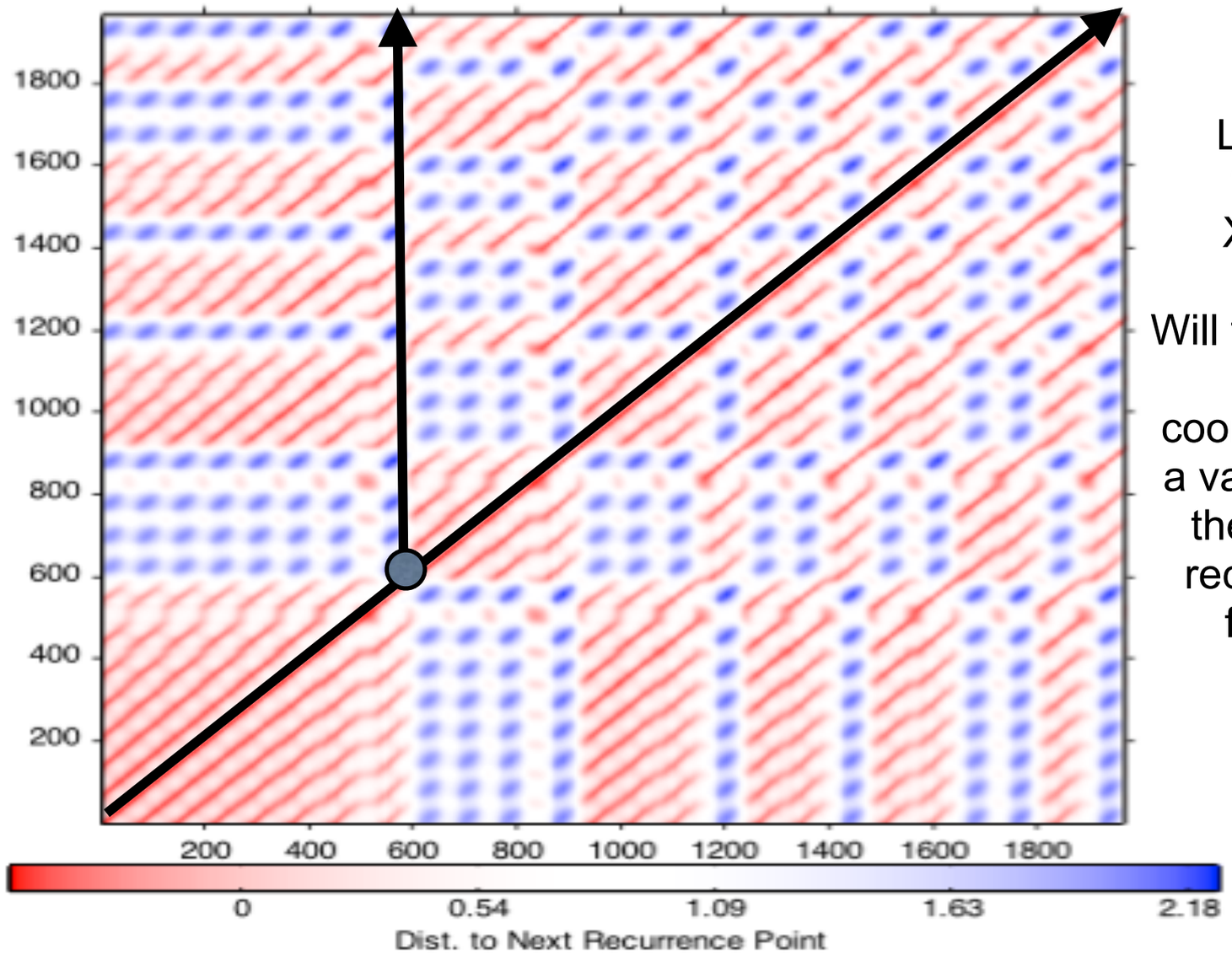
https://www.youtube.com/watch?v=k8Rxep2Mkp8

Topological Equivalence (~Homeomorphic)



Recurrence Quantification





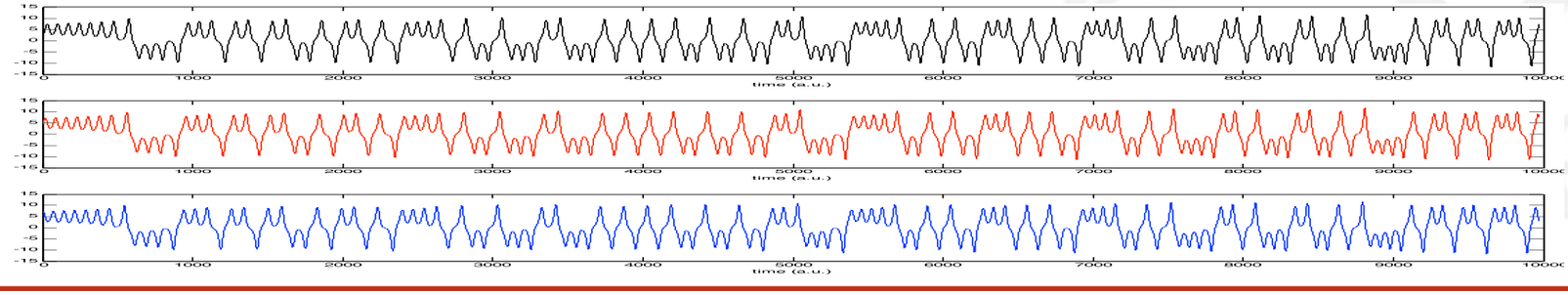
Looking "up" at X(600):

Will the current X,Y,Z coordinate (or a value within the radius) recur in the future?

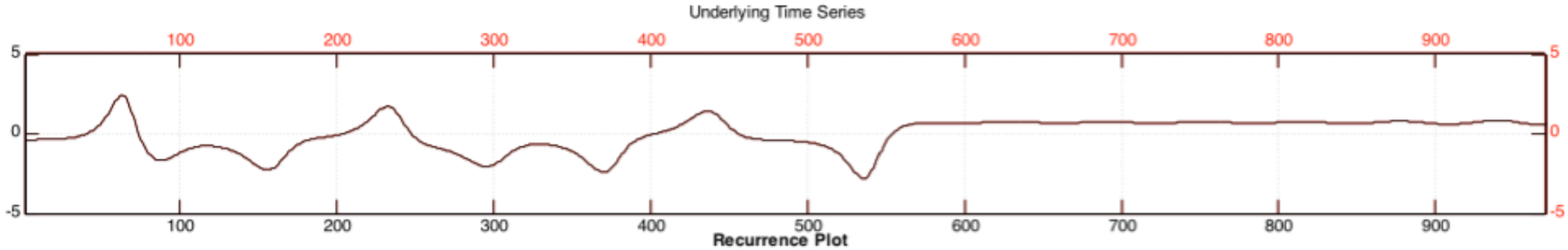
X

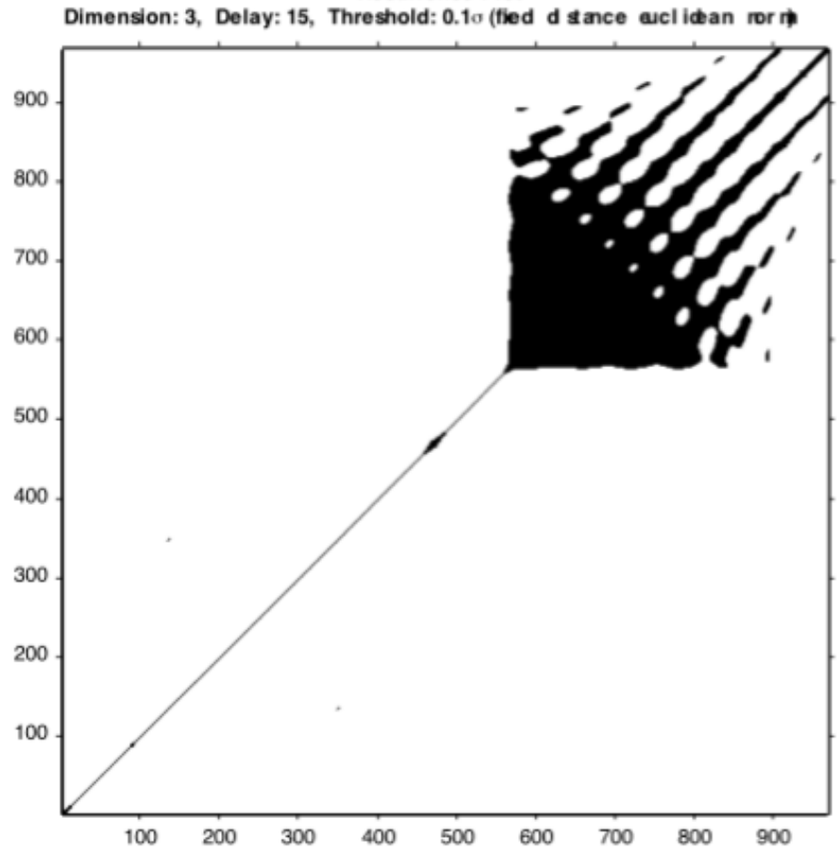
$X_{t+\tau}$

$X_{t+2\tau}$



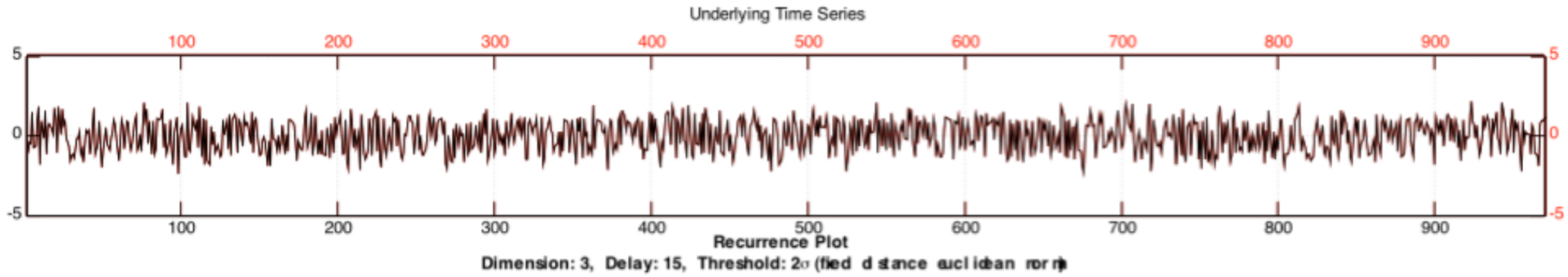
Recurrence Quantification

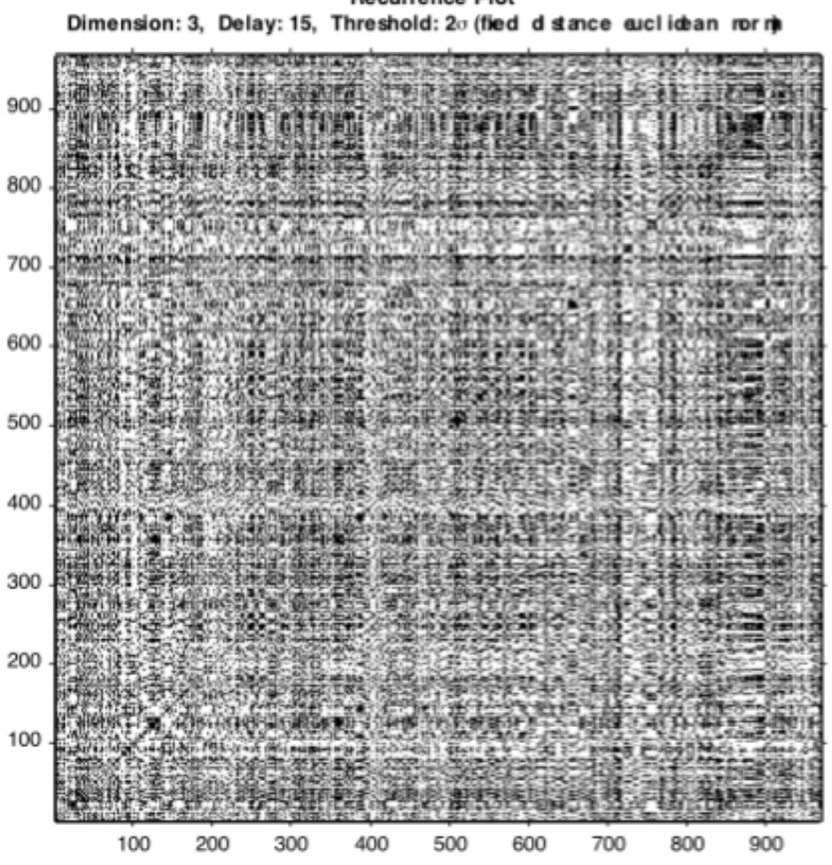




Original:
%REC = 7%
%DET = 100%
Av. LINE = 58
ENTROPY = 4.34

Recurrence Quantification





Original:
%REC = 7%
%DET = 100%
Av. LINE = 58
ENTROPY = 4.34

Shuffled:
%REC = 7%
%DET = 14%
Av. LINE = 2.1
ENTROPY = 0.25

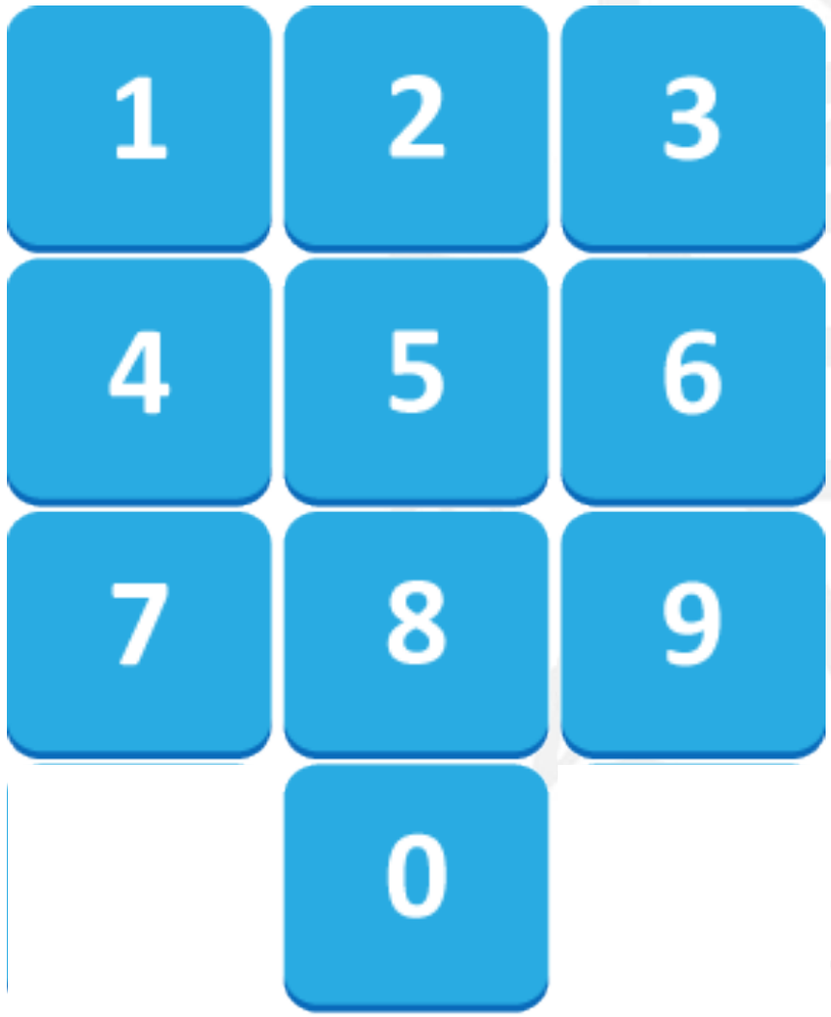
Or use a surrogate

Many Applications of RQA

Oomens, W., Maes, J. H., Hasselman, F., & Egger, J. I. (2015). A time series approach to random number generation: using recurrence quantification analysis to capture executive behavior. *Frontiers in Human Neuroscience*, *9*

Executive control:

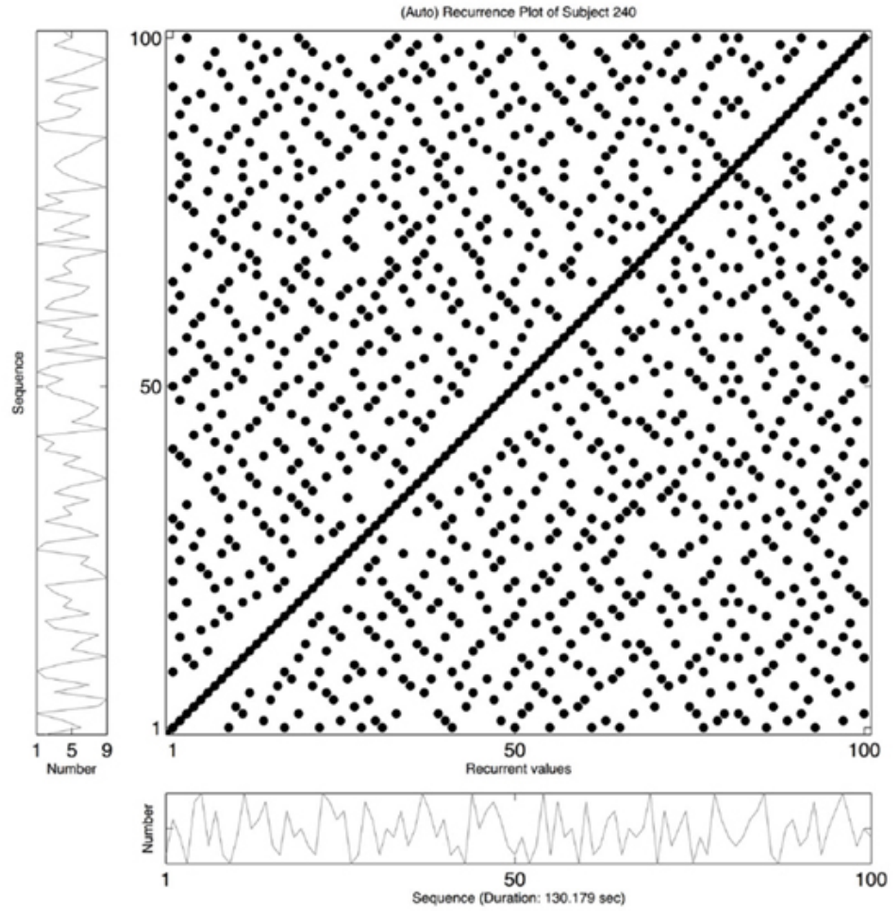
"be as random as you can"



A

RQA measures:

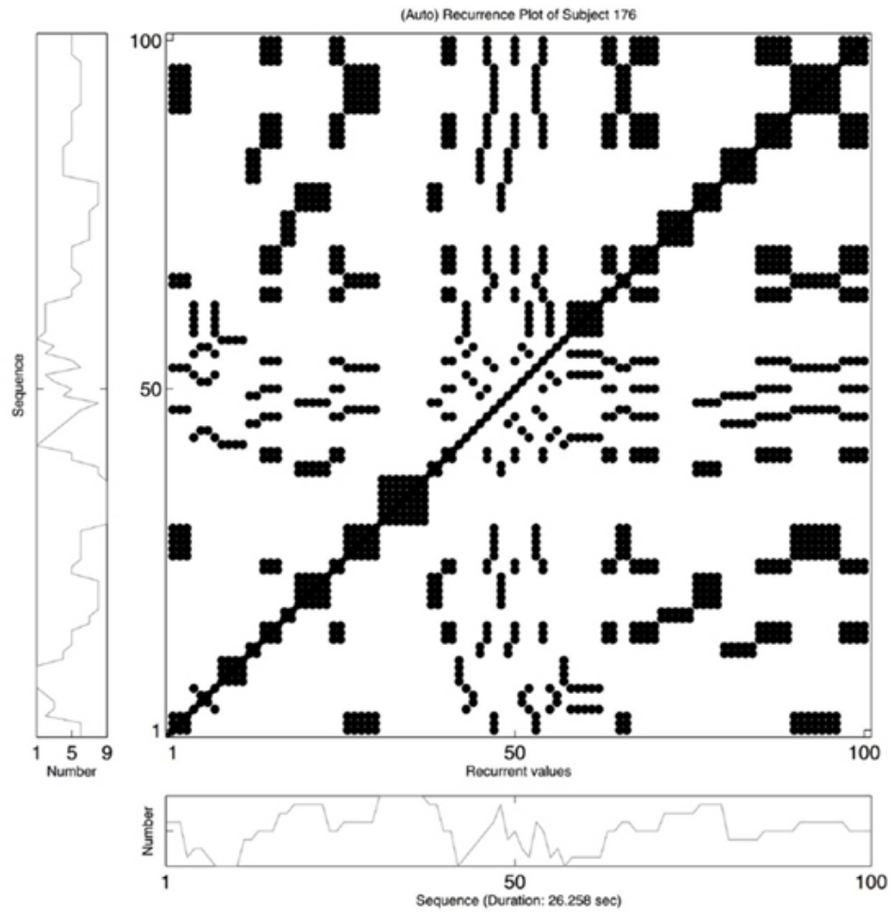
REC = 0.105
DET = 0.185
Lmn = 2.04
Lmx = 3
ENT = 0.176
LAM = 0
Vmn = NaN
Vmx = 0



C

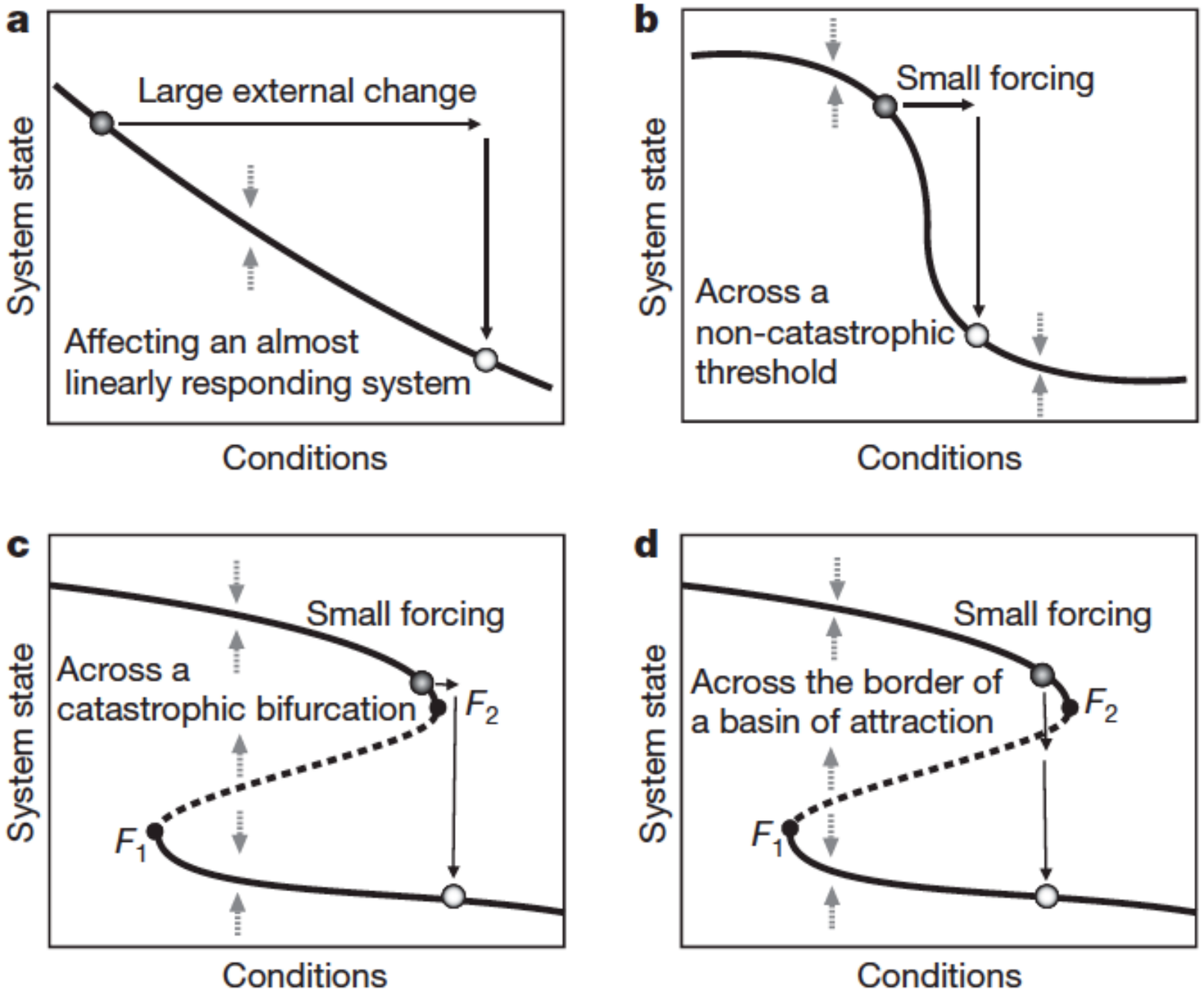
RQA measures:

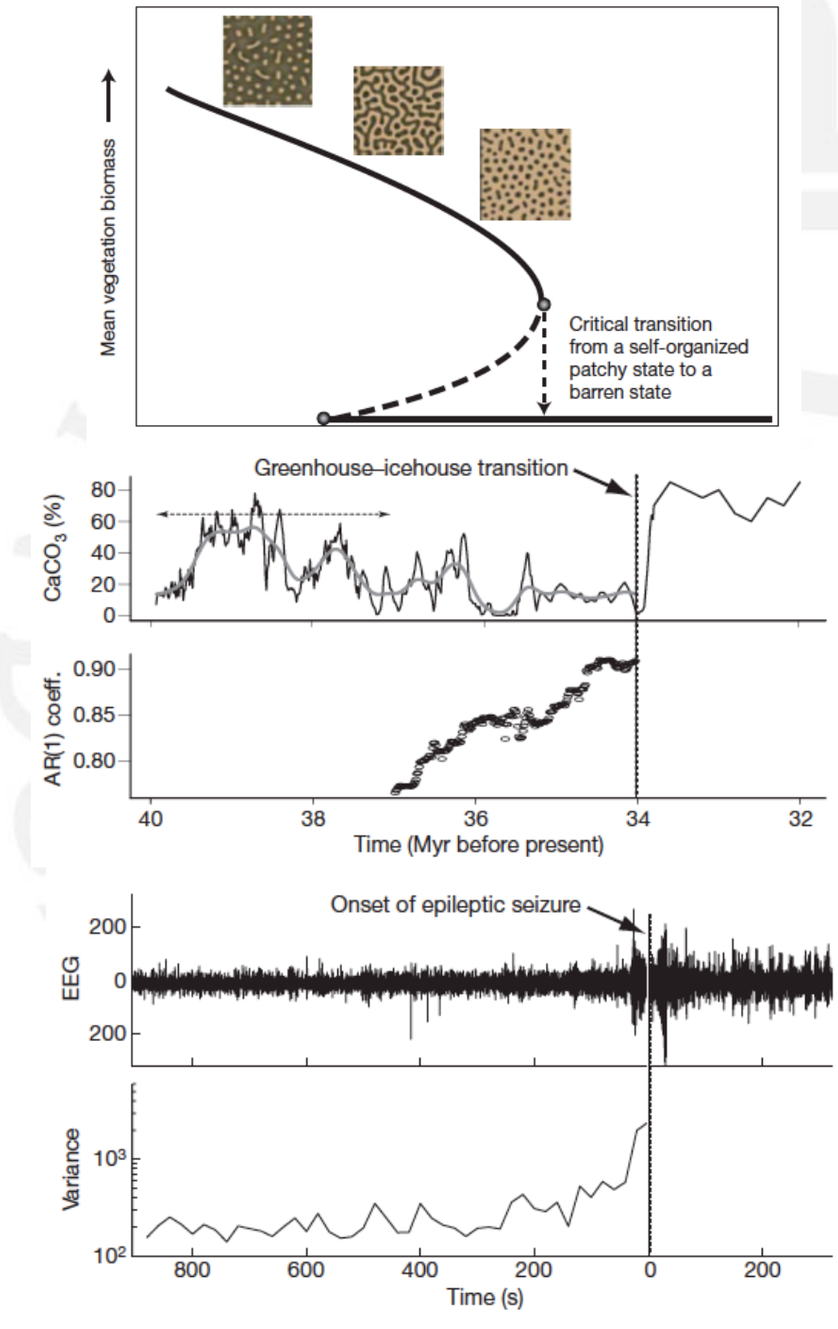
REC = 0.138
DET = 0.654
Lmn = 2.65
Lmx = 7
ENT = 1.1
LAM = 0.82
Vmn = 3.3
Vmx = 7



Warning Signals for Critical transitions

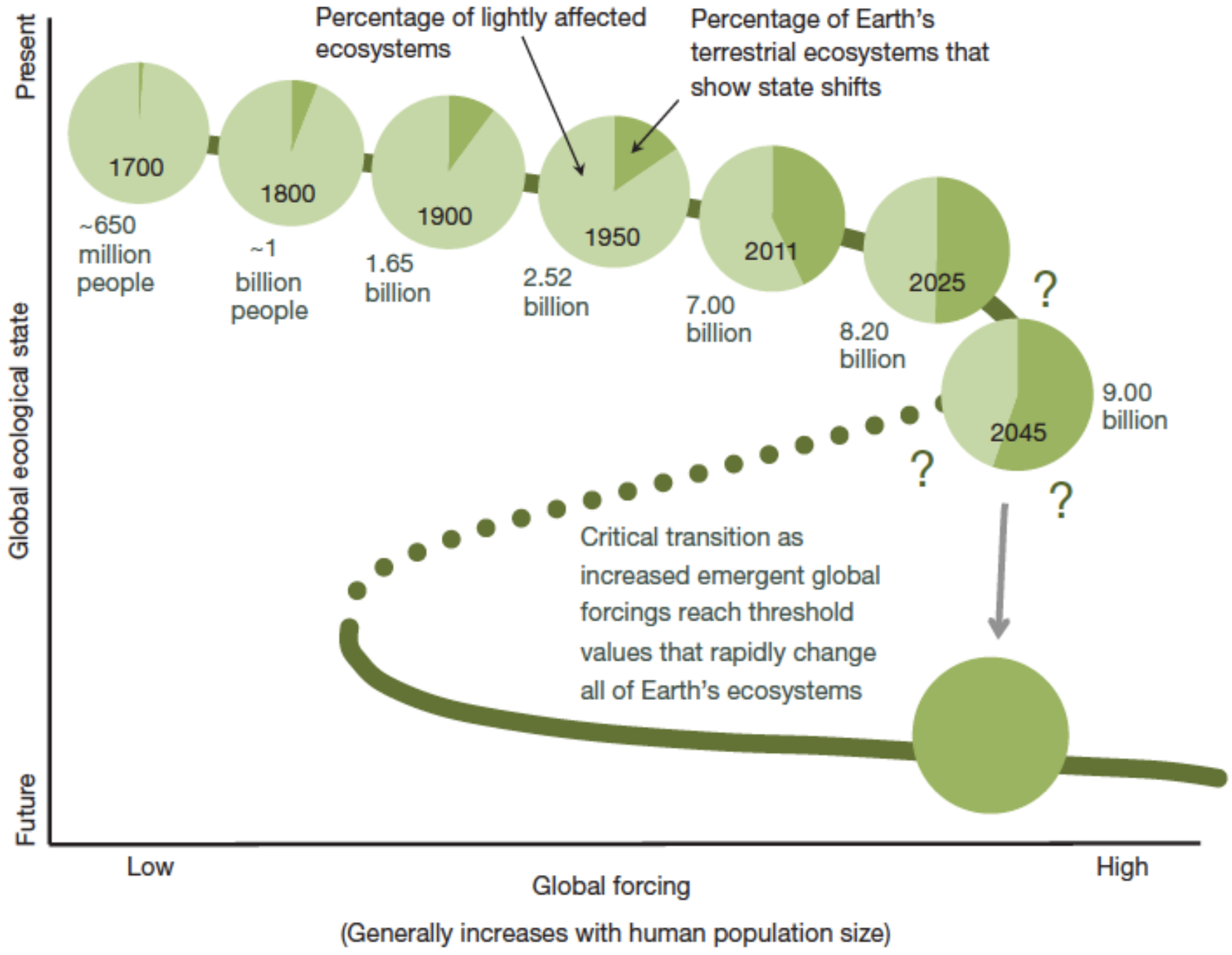
(R package: earlywarnings)





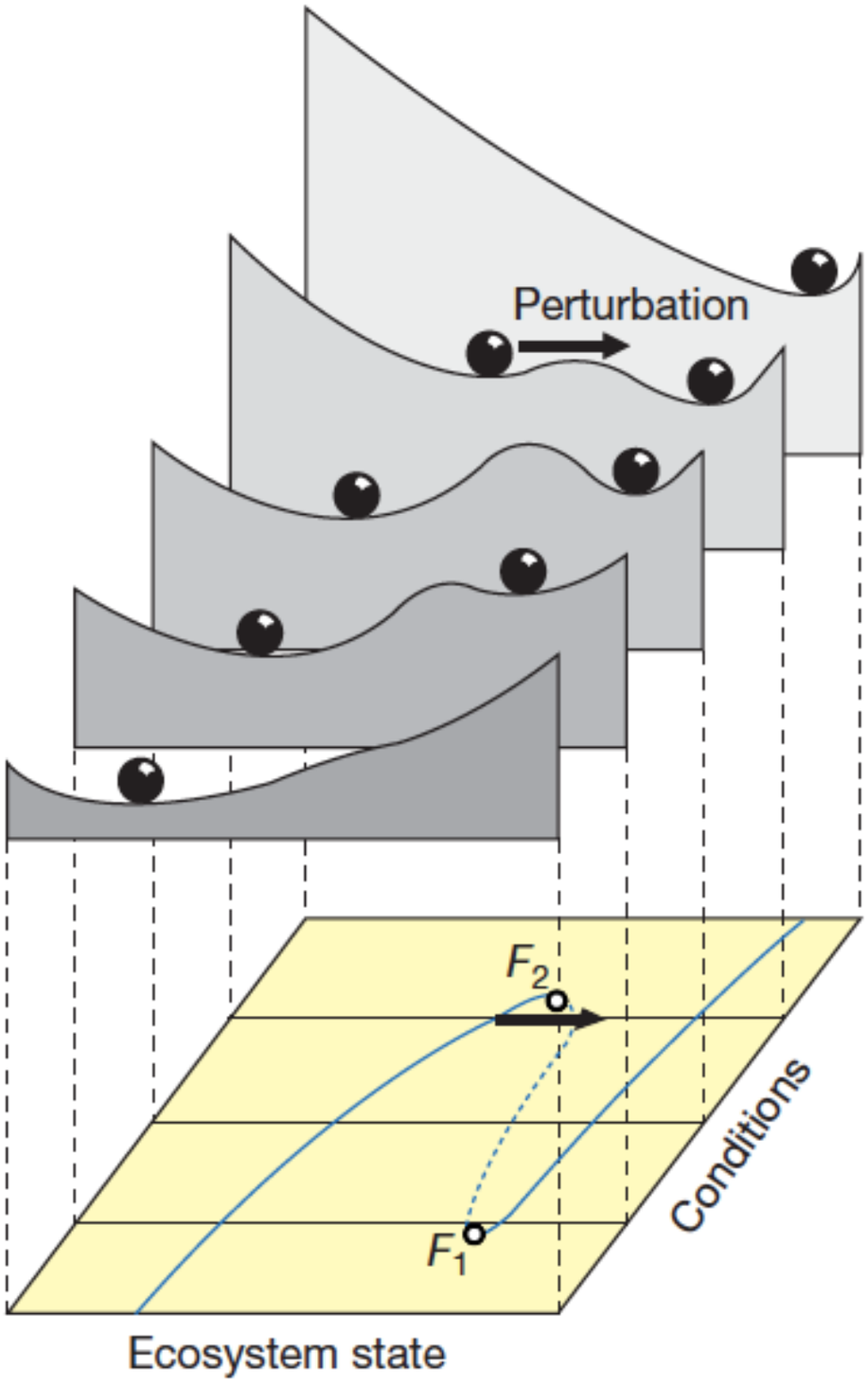
Scheffer, M., Bascompte, J., Brock, W. a, Brovkin, V., Carpenter, S. R., Dakos, V., Held, H., et al. (2009). Early-warning signals for critical transitions. *Nature*, *461*(7260), 53-9.

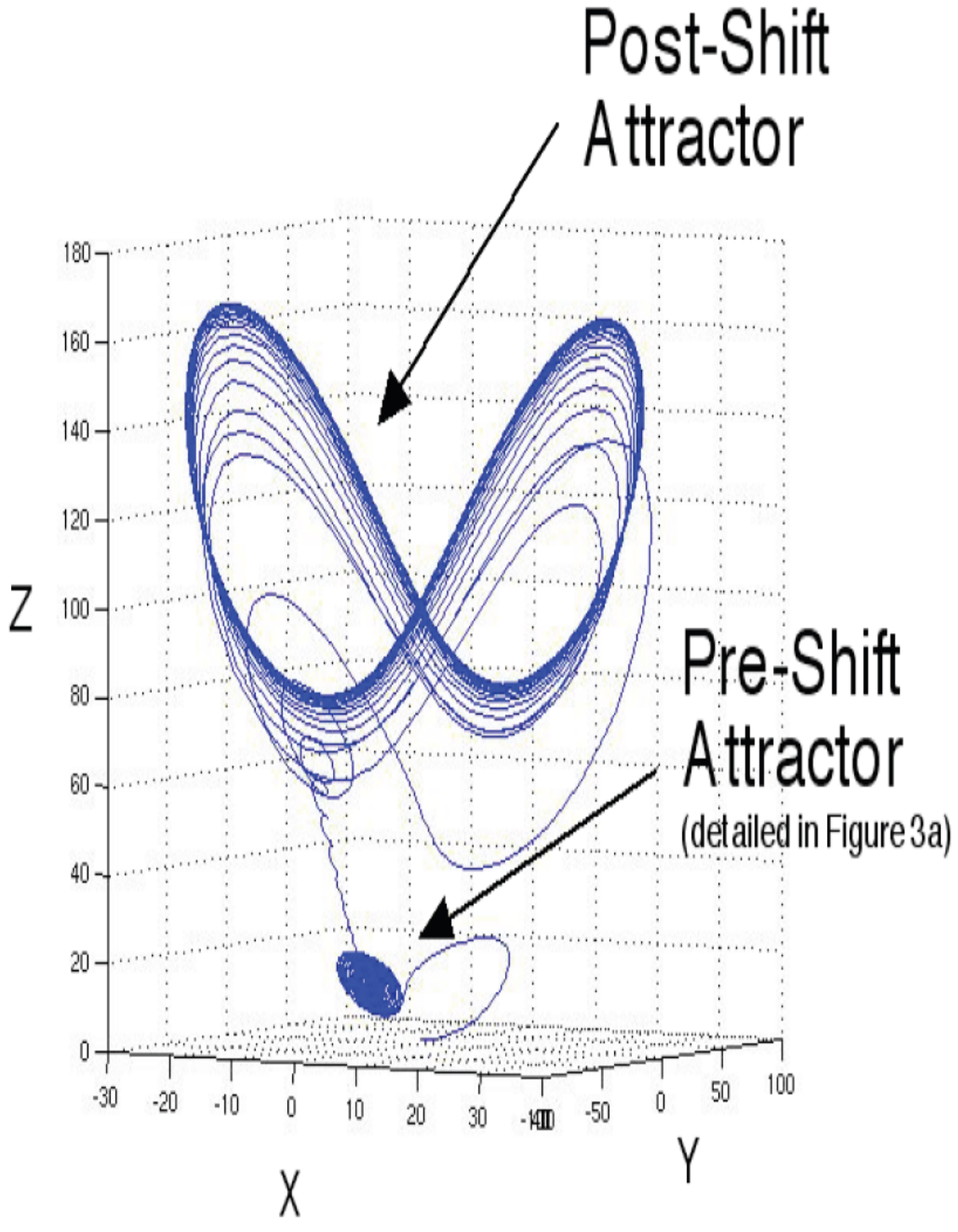
Warning Signals for Critical transitions



Barnosky, A. D., Hadly, E. a., Bascompte, J., Berlow, E. L., Brown, J. H., Fortelius, M., Getz, W. M., et al. (2012). Approaching a state shift in Earth's biosphere. *Nature, 486(7401)*, 52-58.

Warning Signals for Critical transitions





Scheffer, M., Carpenter, S., Foley, J. a, Folke, C., & Walker, B. (2001). Catastrophic shifts in ecosystems. Nature, 413(6856), 591-6. doi:10.1038/35098000

Warning Signals for Critical transitions

- Increase in variance (**Fluctuation**), autocorrelation (FD), **critical "slowing down"**

- **Increase in Entropy...** in physics entropy is a measure of disorder (2nd law of thermodynamics), or a loss of structure in a system. Everything in the universe is moving towards a maximum entropy state, which means all structure is lost, all energy in the universe is distributed homogeneously

[Entropy is also used in information theory: the amount of information needed to describe a message / data stream. If much information is needed, the message is disorderly, random, not patterned -> High entropy]

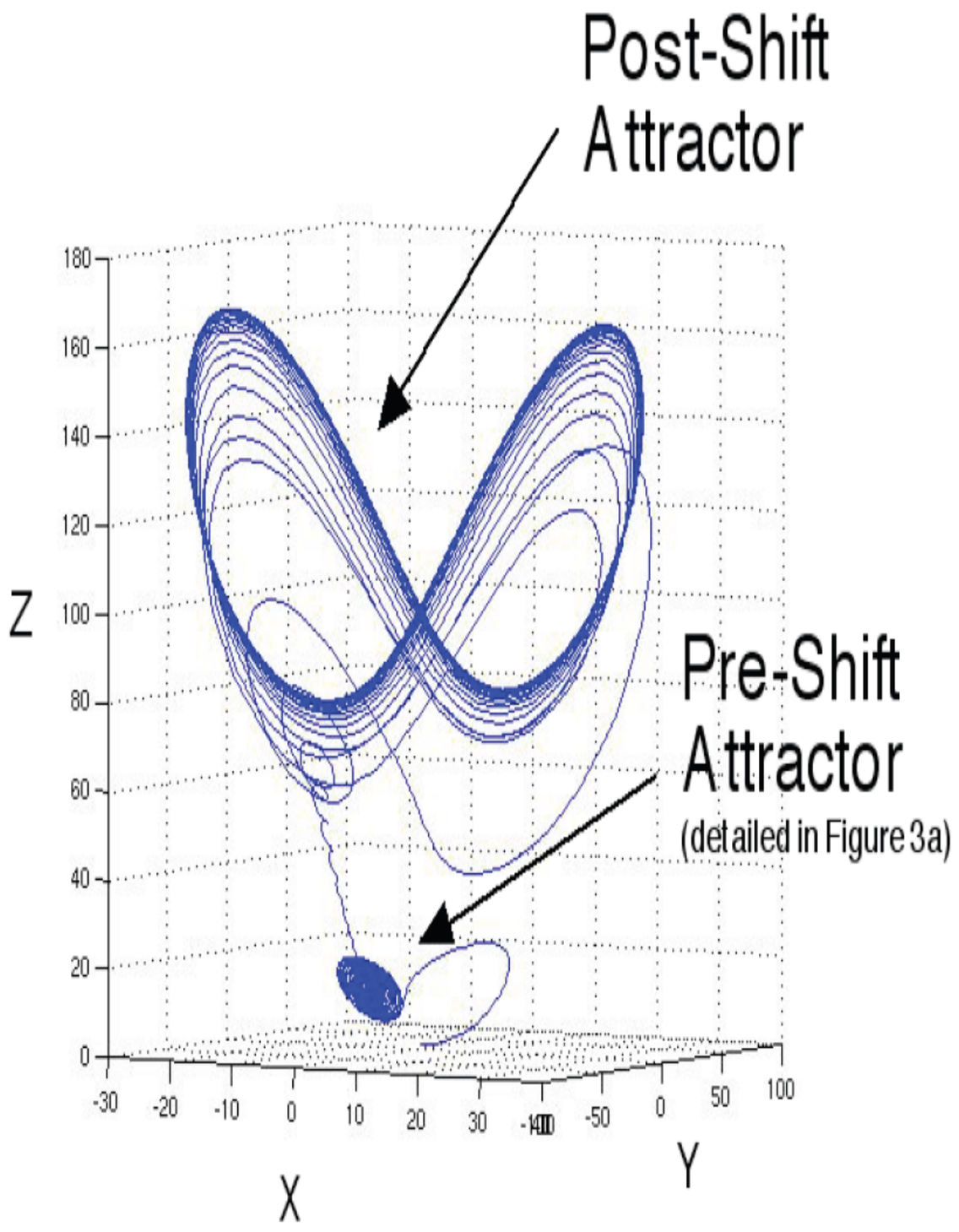
- **Entropy in RQA:** the entropy in the distribution of line lengths in the Recurrence Plot. If the entropy is high there are a lot of different line lengths, pointing towards less stable dynamics in reconstructed phase space. If entropy is low, the system is more deterministic, there are few very stable recurring patterns.

1. If we can reconstruct the state space of a complex dynamical system from one observable dimension....

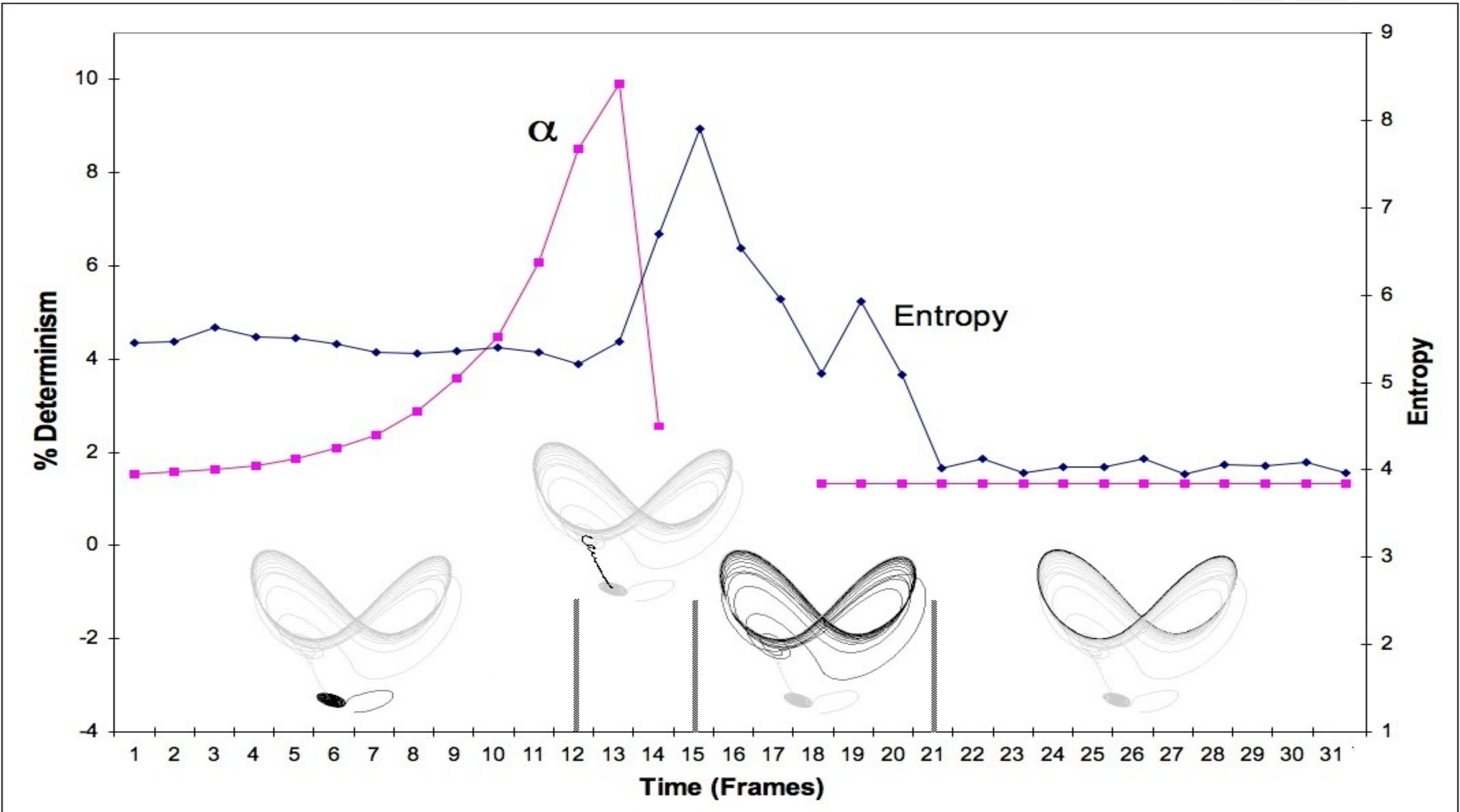
2. If we can quantify the attractor dynamics in this state space...

3. Direct measurements of physical observables in humans should tell us something about the the dynamics of the unobservable cognitive system

4. Could we predict insight in problem solving from a phase transition in phase space reconstructed from hand movements?



Lorenz system – Transitions in phase space



Insight as a phase transition

- Stephen, D.G., Dixon, J.A., & Isenhower, R.W. (2009). Dynamics of representational change: Entropy, action, and cognition. JEP: HPP.

Gear Domain

- Gear systems problems
- Solve problem any way they wish
- Code strategies
  - Force-tracing
- Gear system does not move

- Force-tracing actions create information about the system
- Discovery of Alternation

Insight as a phase transition

Optotrak

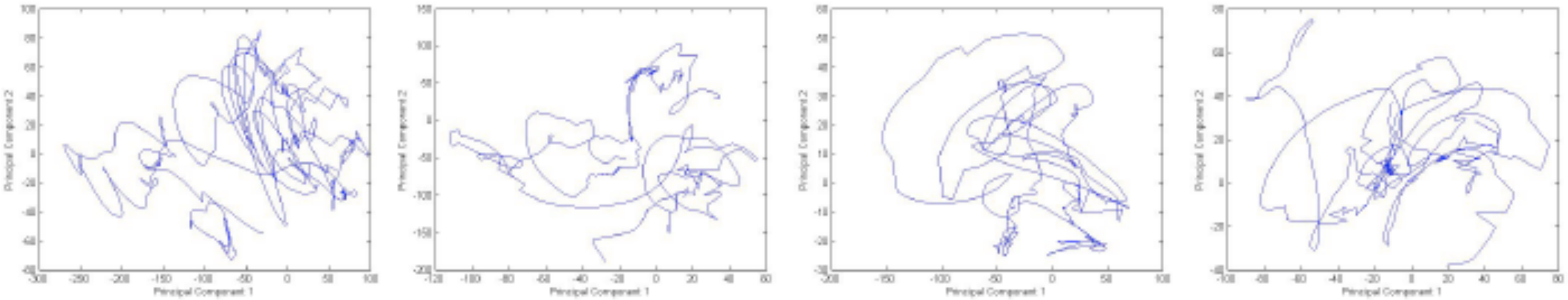
100 Hz sampling rate, 4 markers Velcro-ed to forefinger

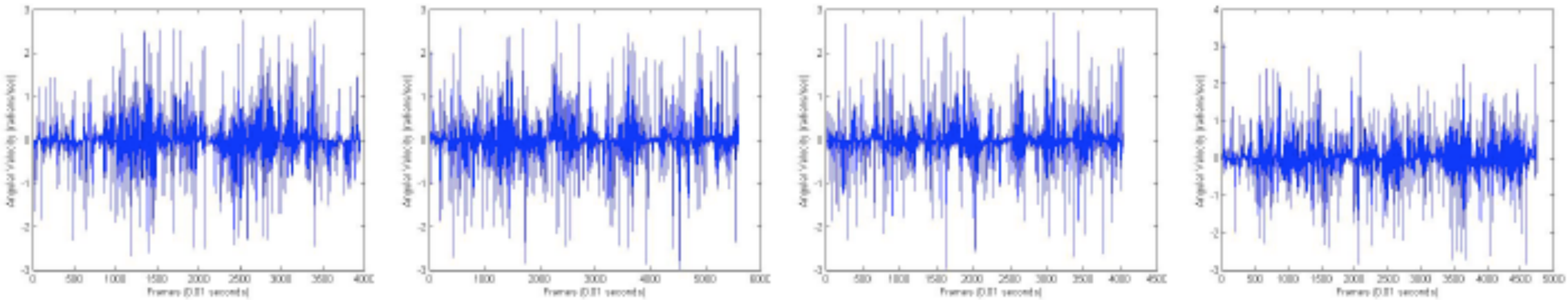
Markers emit infrared light

2 markers for left camera

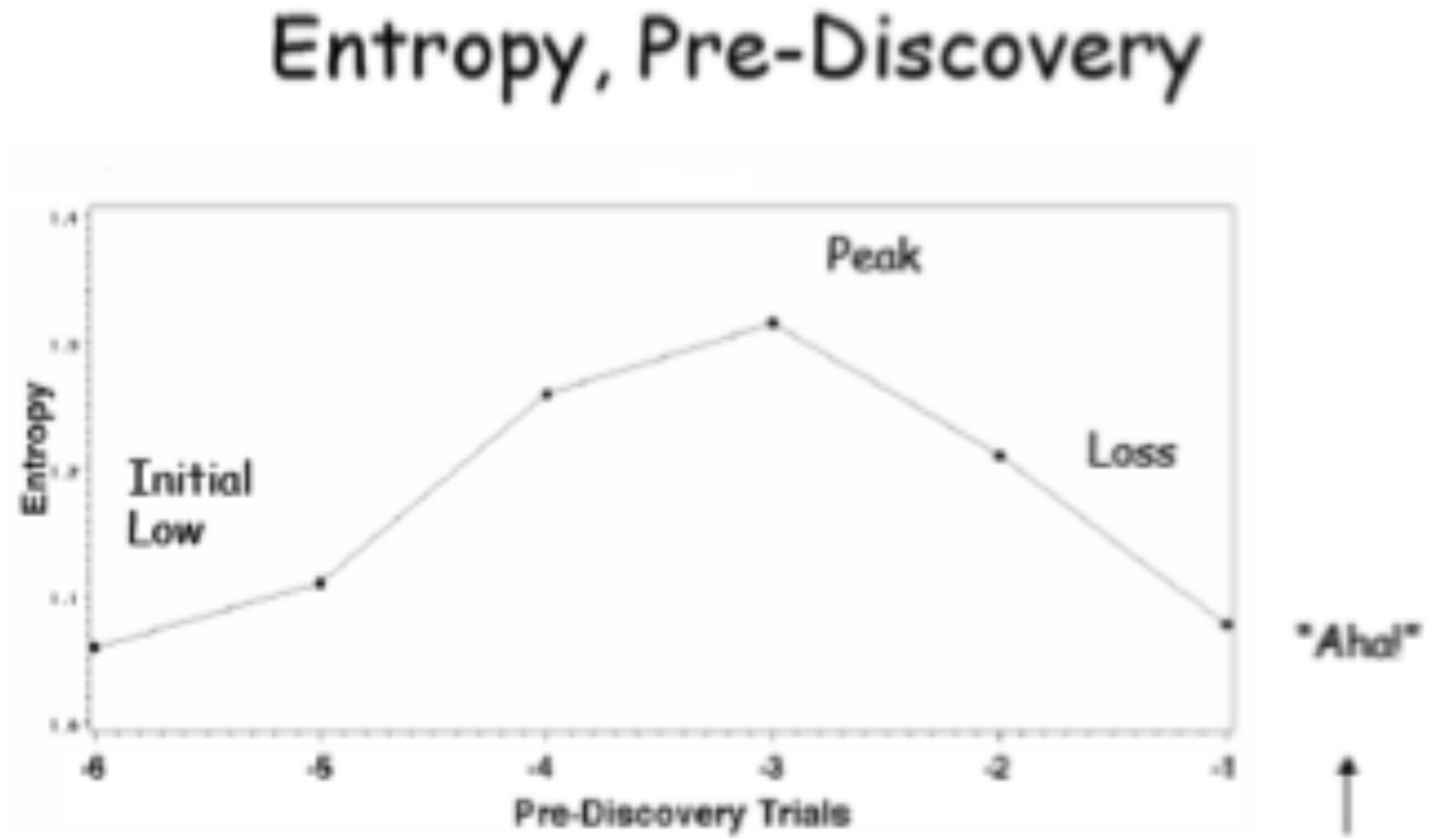
2 markers for right camera

Angular velocity of finger movements

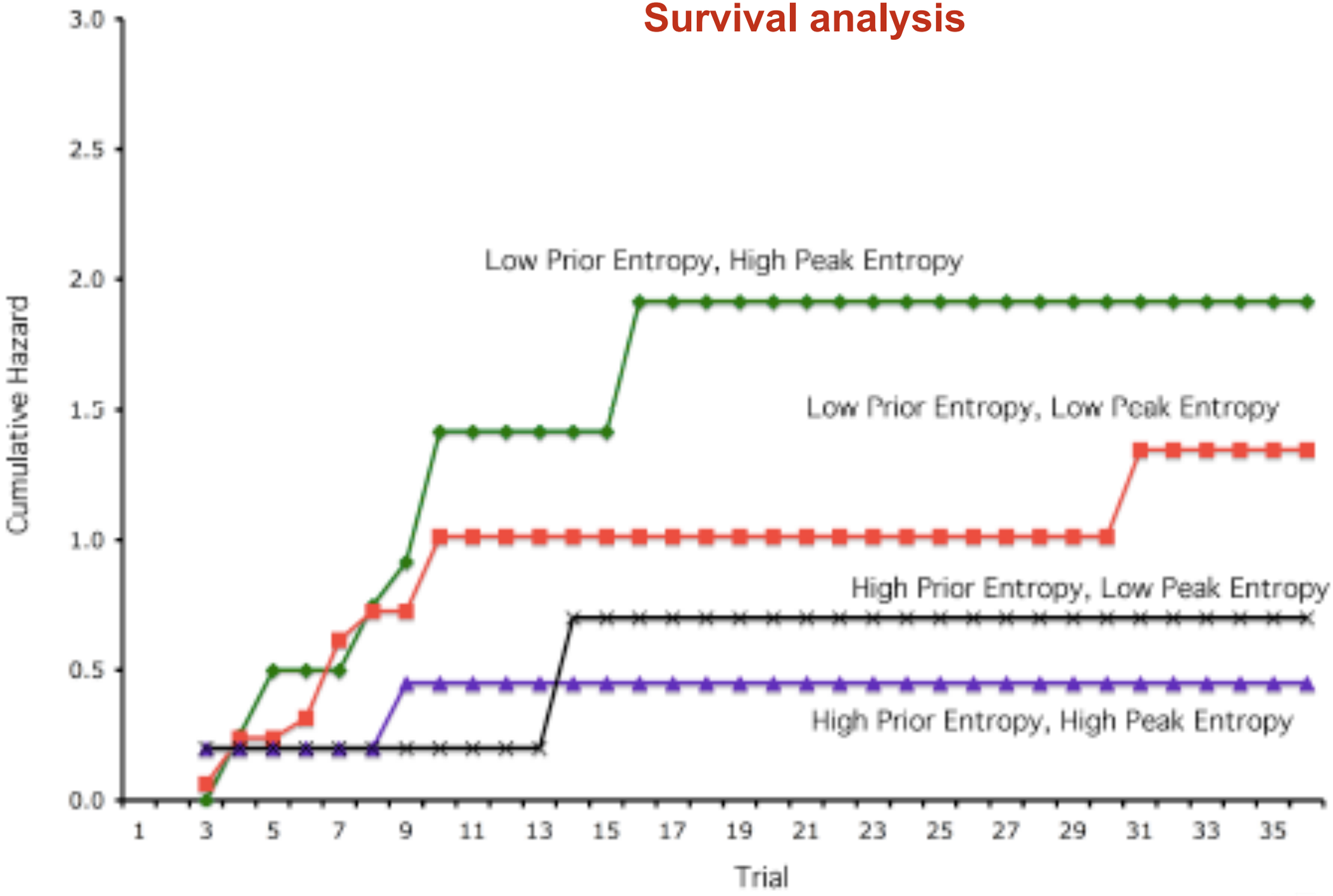




Insight as a phase transition



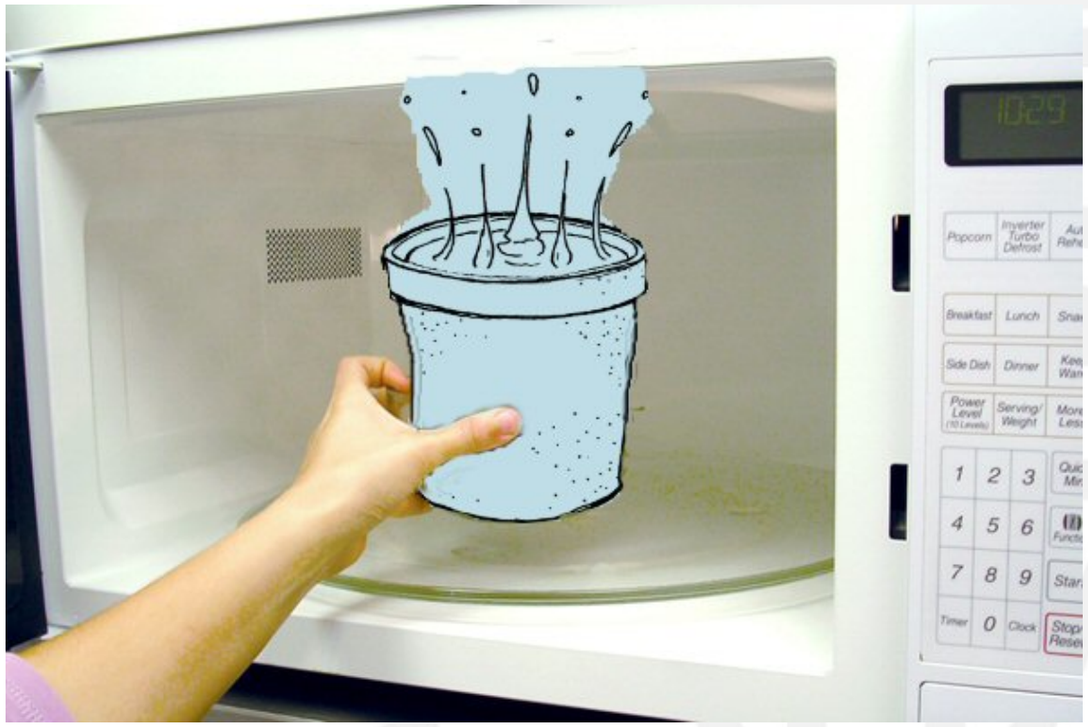
Survival analysis

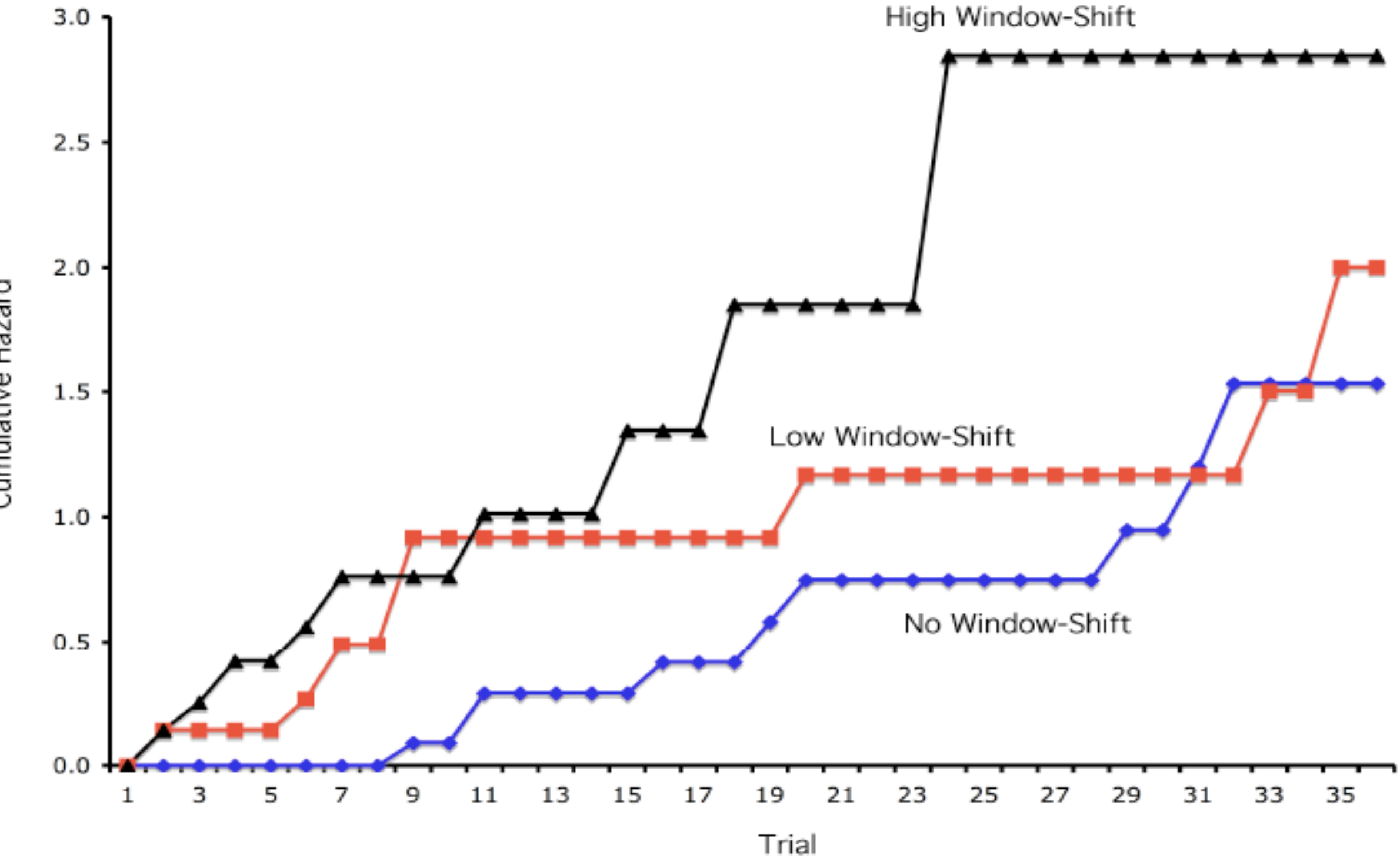


1. Assumption: Noise / Entropy drives the structural change

1. Hypothesis: Increase noise, this will lead to an earlier discovery of the rule

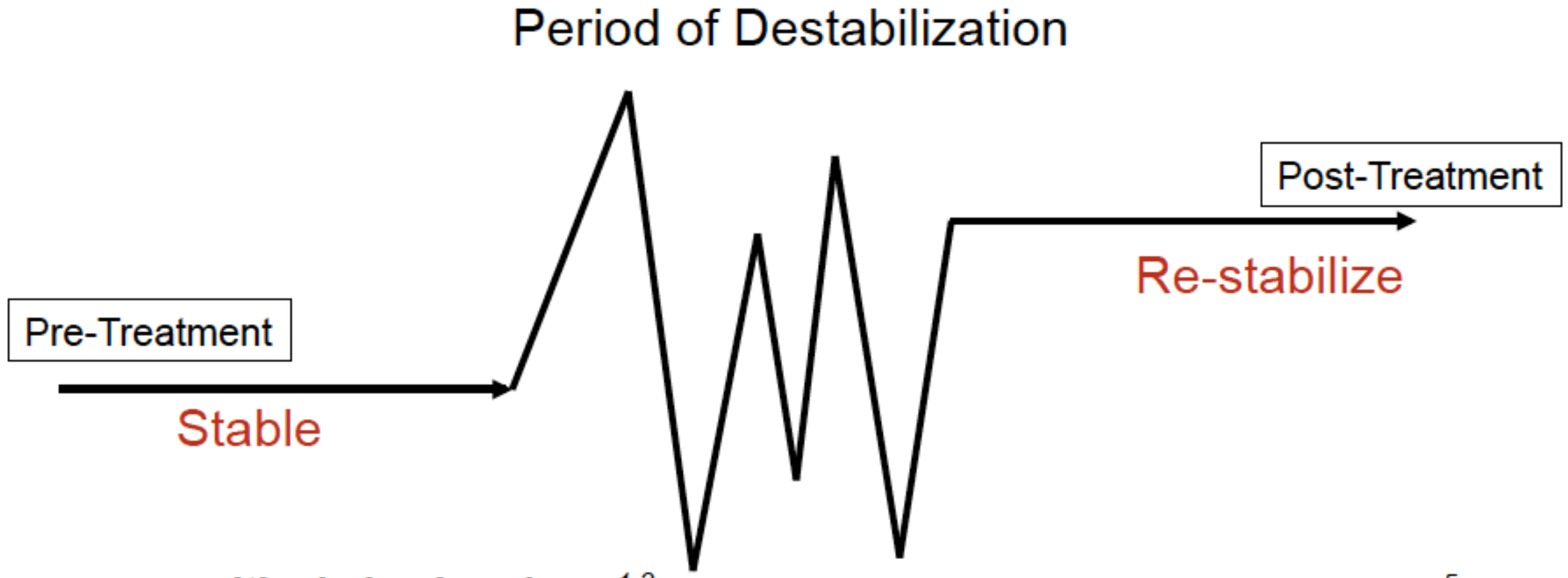
1. Additional condition: increase noise by making the gear problems shift position on the screen




High Window-Shift
Low Window-Shift
No Window-Shift
Cumulative Hazard
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
Trial

Lichtwarck-Asschoff et al.

Main Hypothesis: Improvement preceded by destabilisation

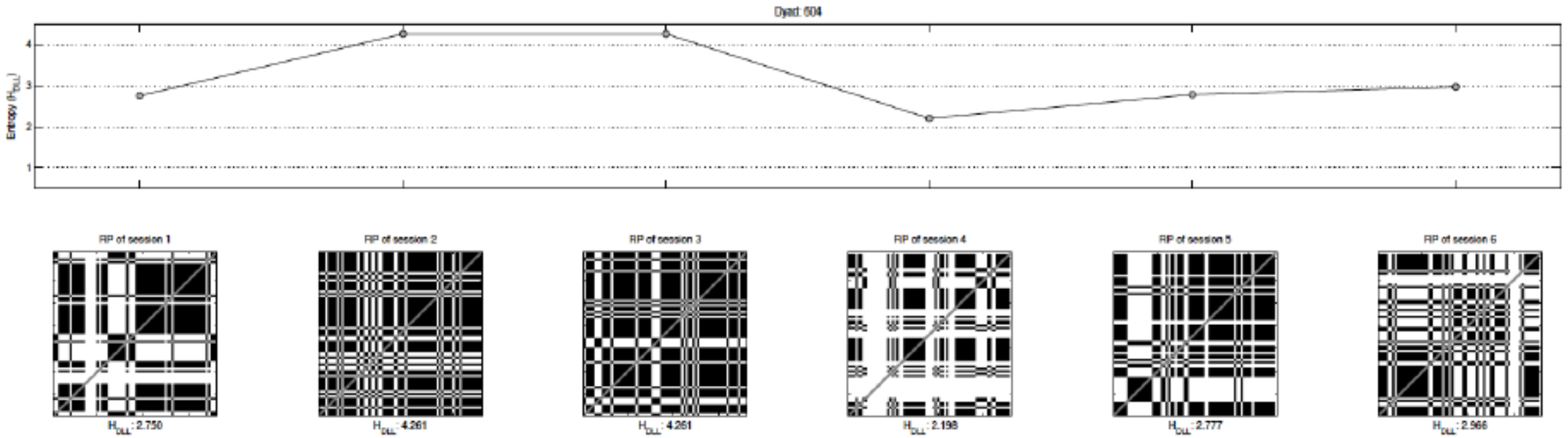


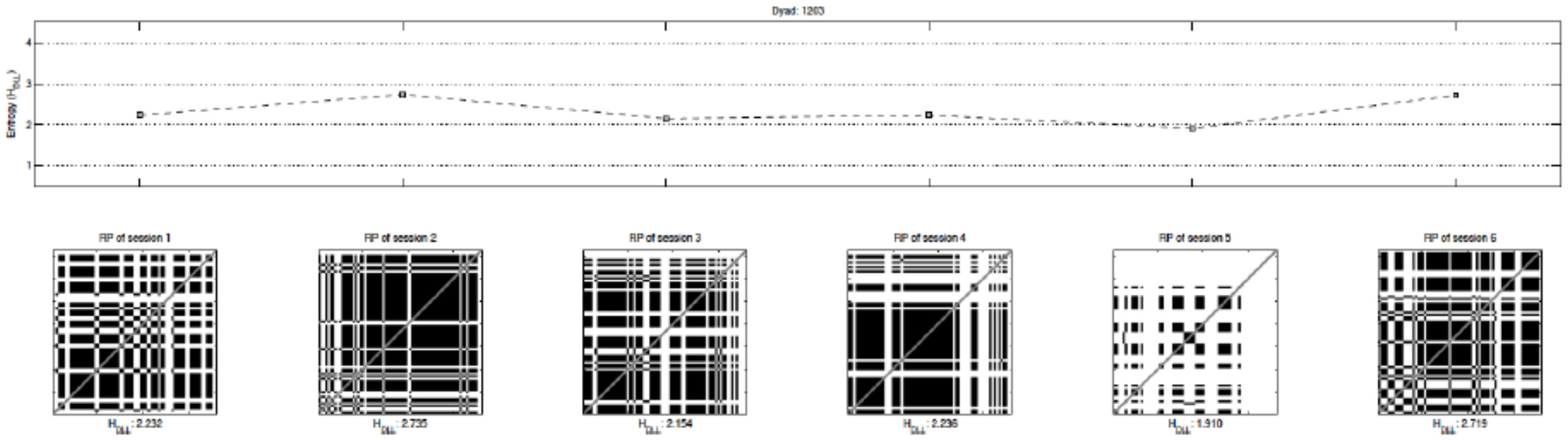
critical slowing down[1,3] ***resilience to perturbation***[5]

critical fluctuations[3,4]

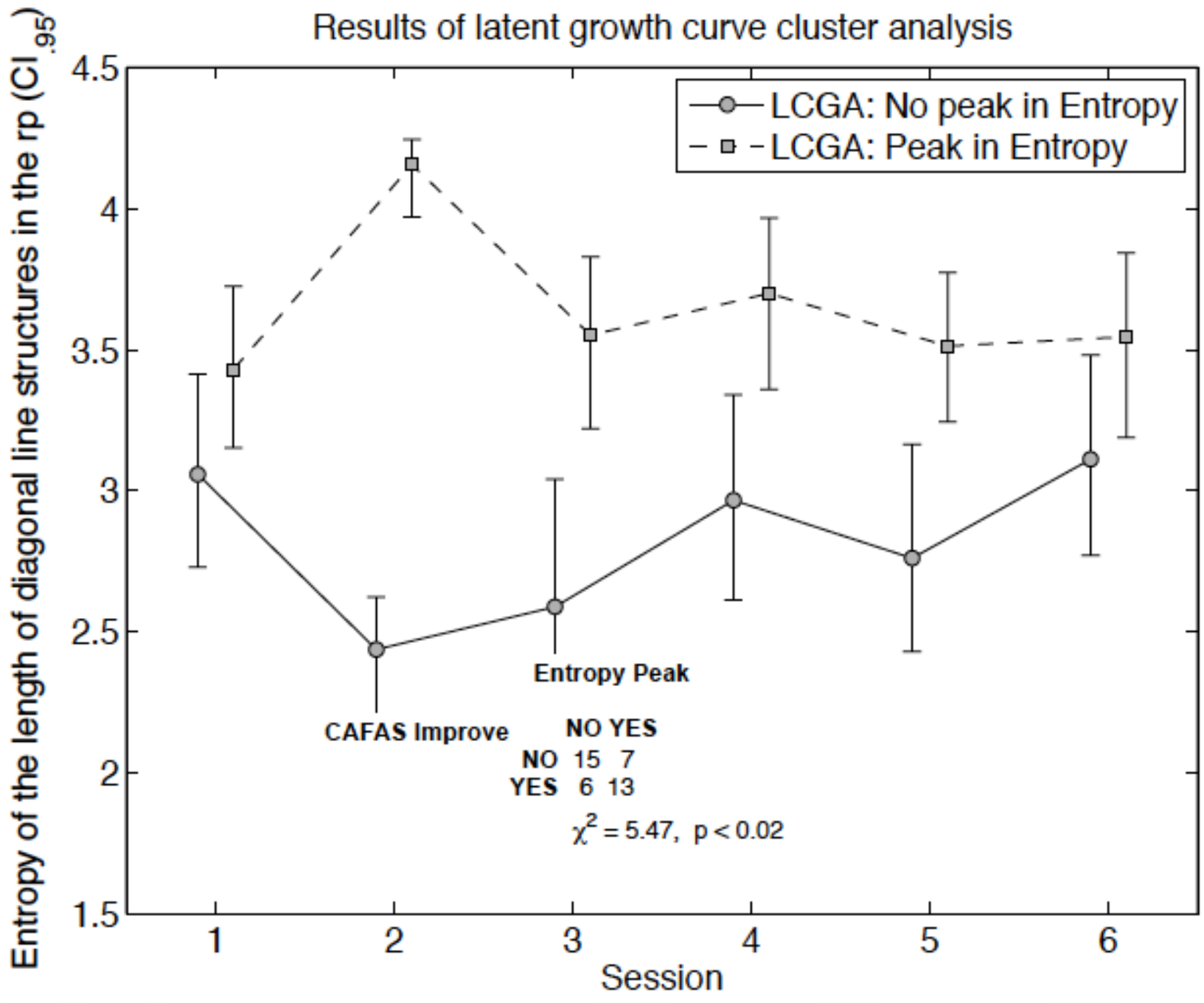
- increase in recovery and switching time after perturbation
- increase in variance, autocorrelation, long-range dependence
- increase in occurrence and diversity of unstable states
- increase in the entropy of the distribution of state occurrences

Main Prediction: Entropy-peak in distribution of recurrent patterns = improve





Main Result: Improvement is associated with destabilization profile



Cross Recurrence Analysis

- Instead of analysing if a system re-visits locations in reconstructed phase space: Analyse if two systems share locations in phase space

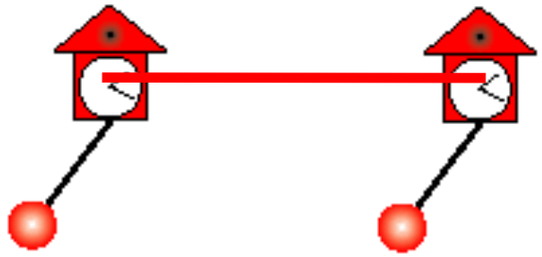
- Cross recurrence analysis tells you something about synchronization or coupling of systems in time.

- Same strategy as autorecurrence: Reconstruct phase space and see if points between two trajectories are adjecent

Synchronisation: Huygens' Clocks

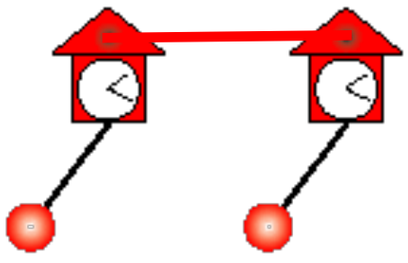
Synchronisation is a science in itself…

For now just note that synchronisation occurs in systems which are coupled in some way

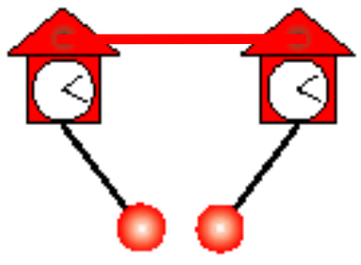


1629-1695

In-Phase



Anti-Phase



CRQA plots
What do these guys have in common?

CRQA plots
Blake vs. Deelder

Auguries of Innocence

To see a World

in a Grain of Sand

And a Heaven

in a Wild Flower,

Hold Infinity

in the palm of your hand

And Eternity

in an hour.

A = 1
B = 2
D = 3
E = 4
F = 5
G = 6
H = 7
I = 8
J = 9
K = 10
L = 11
M = 12
N = 13
O = 14
P = 15
R = 16
S = 17
T = 18
U = 19
V = 20
W = 21
Y = 22

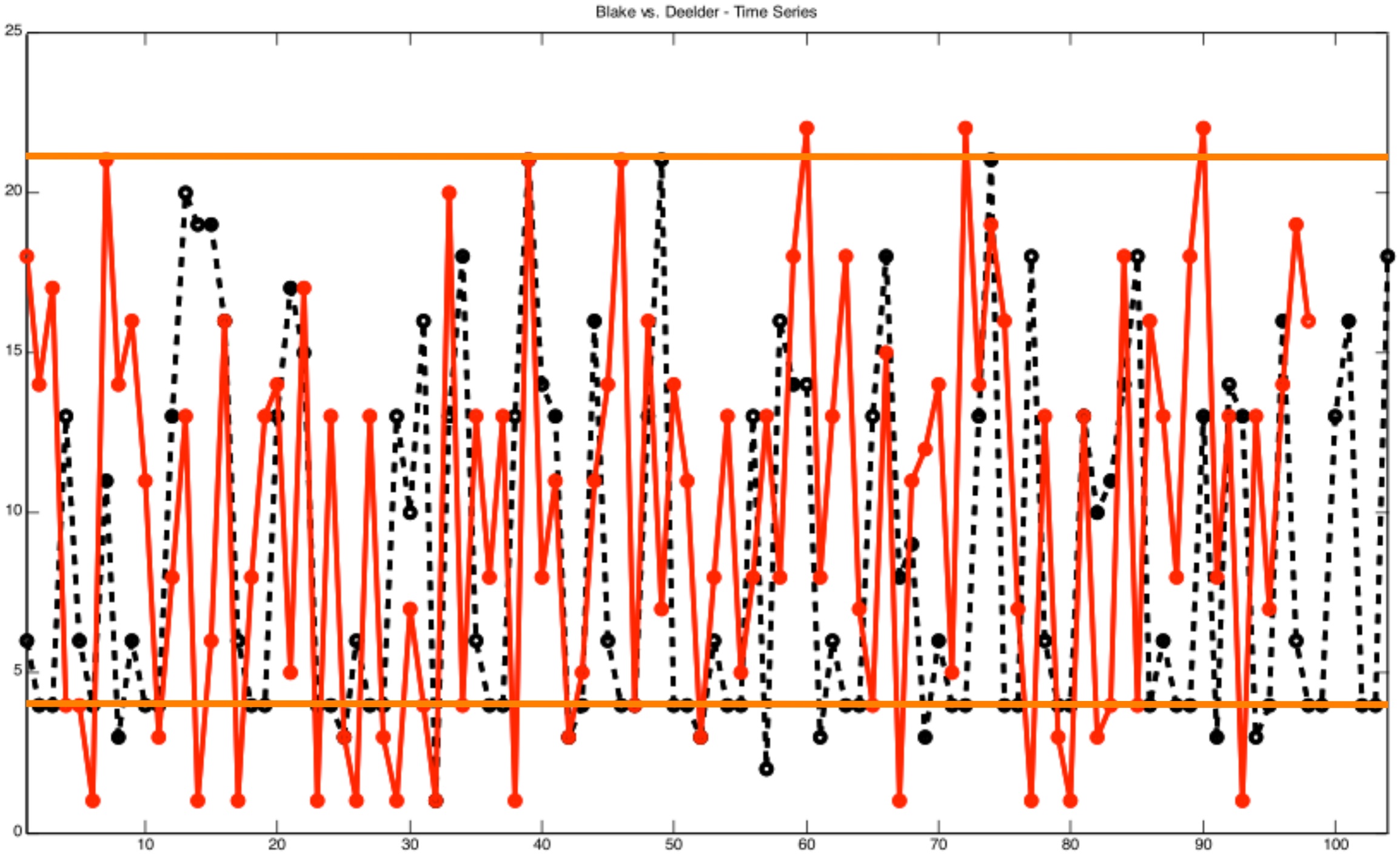
Blues on tuesday

Geen geld.
Geen vuur.
Geen speed.

Geen krant.
Geen wonder.
Geen weed.

Geen brood.
Geen tijd.
Geen weet.

Geen klote.
Geen donder.
Geen reet.


Blake vs. Deelder - Time Series
25
20
15
10
5
0
10
20
30
40
50
60
70
80
90
100

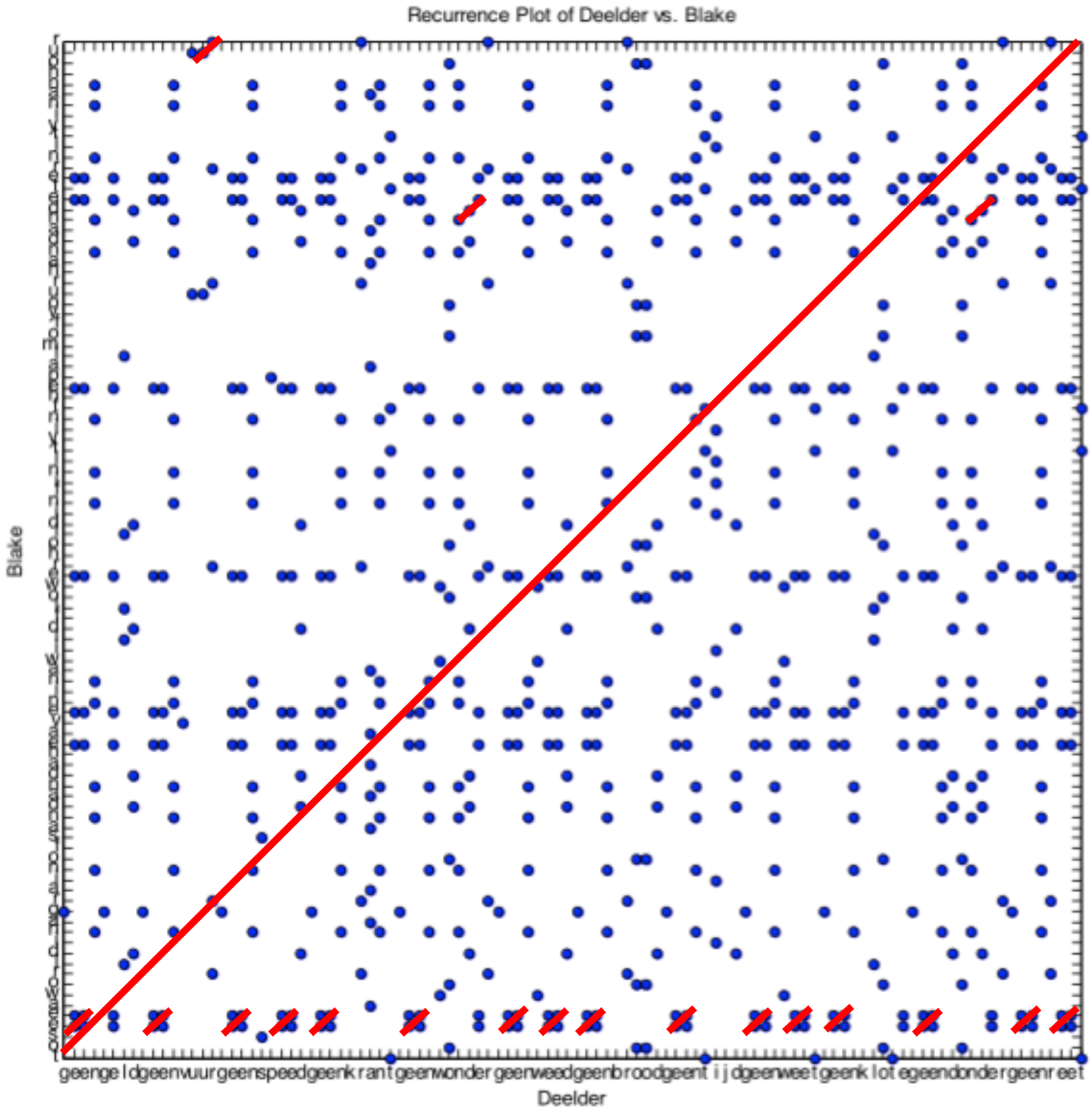
No symmetry in the plot!

Line of Incidence (LOI) =
Line of Synchronisation (LOS),
X(t) = Y(t)

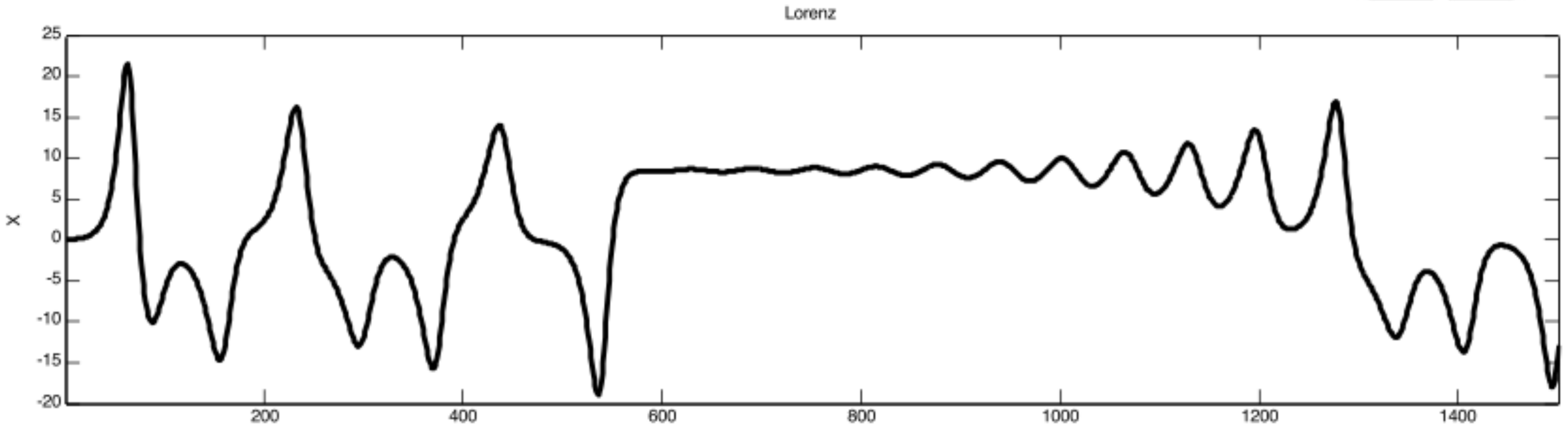
Blake and Deelder
don't synchronise very often
just a few bigrams: **ee - ur**

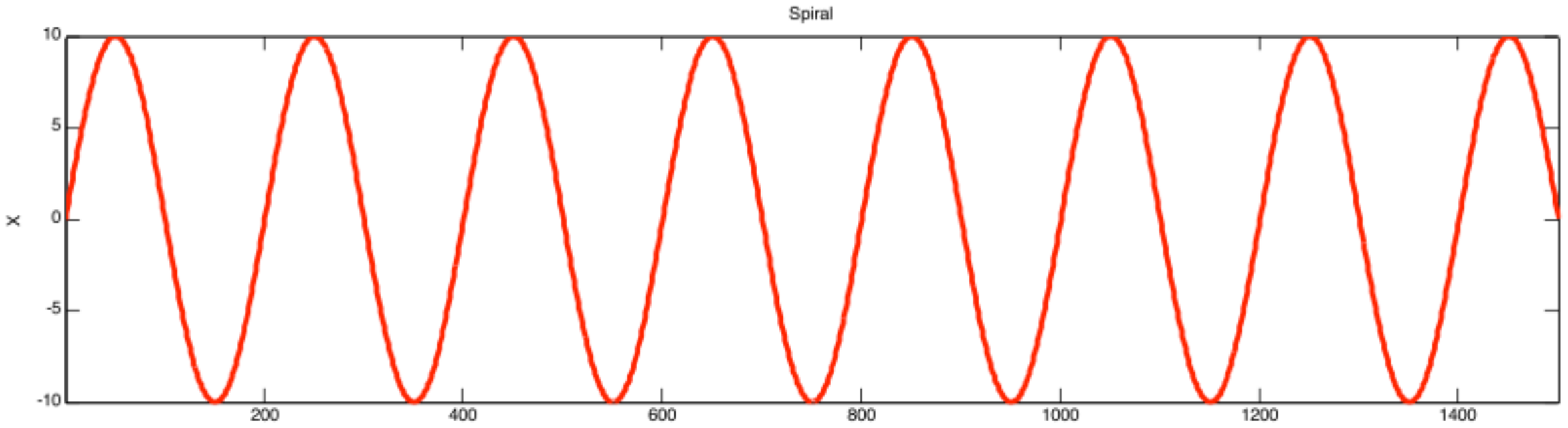
And the odd trigram:
nde

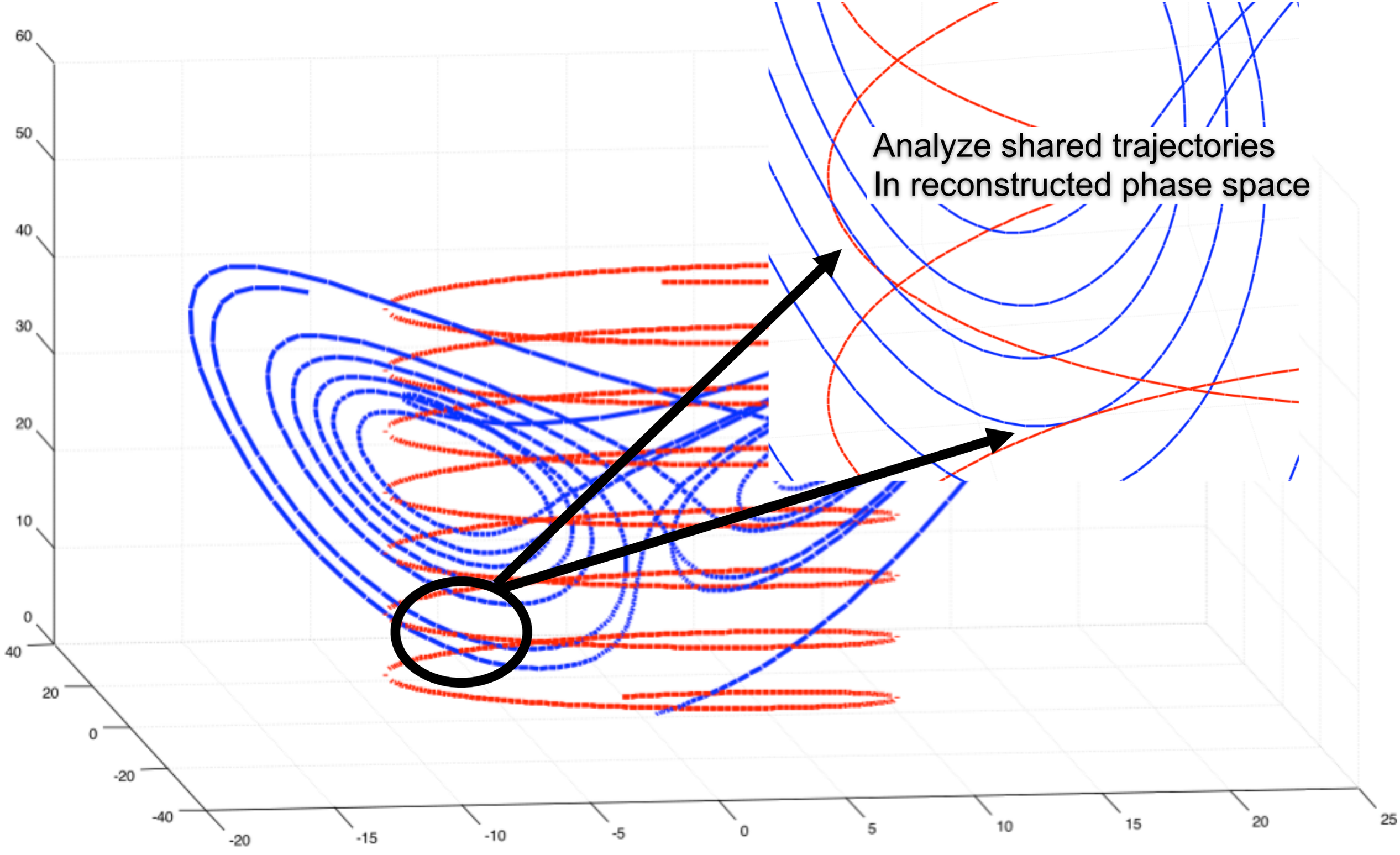
They are'nt really coupled
of course



Time Series of X – Lorenz and Spiral






Analyze shared trajectories
In reconstructed phase space
60
50
40
30
20
10
0
40
20
0
-20
-40
-20
-15
-10
-5
0
5
10
15
20
25

Rescaling *after* Reconstruction

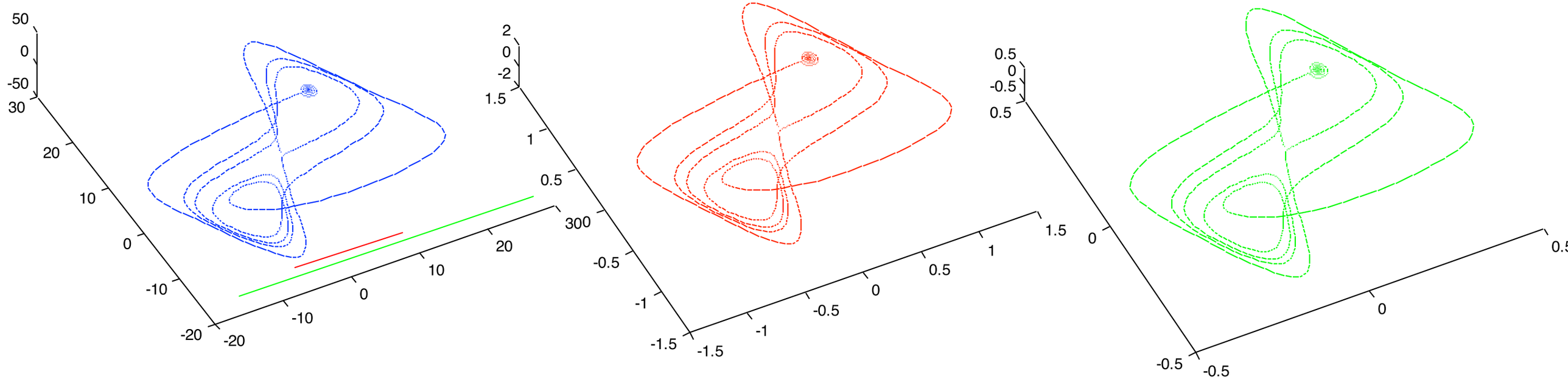
Generally it is a good idea to re-scale your data relative to either the mean or maximum distance separating points in reconstructed phase space before you search for a radius and proceed with RQA analysis.

This also allows relative comparisons across data sets.

No Rescale

Mean Distance Rescale

Maximum Distance Rescale



Maximum distance re-scaling recommended

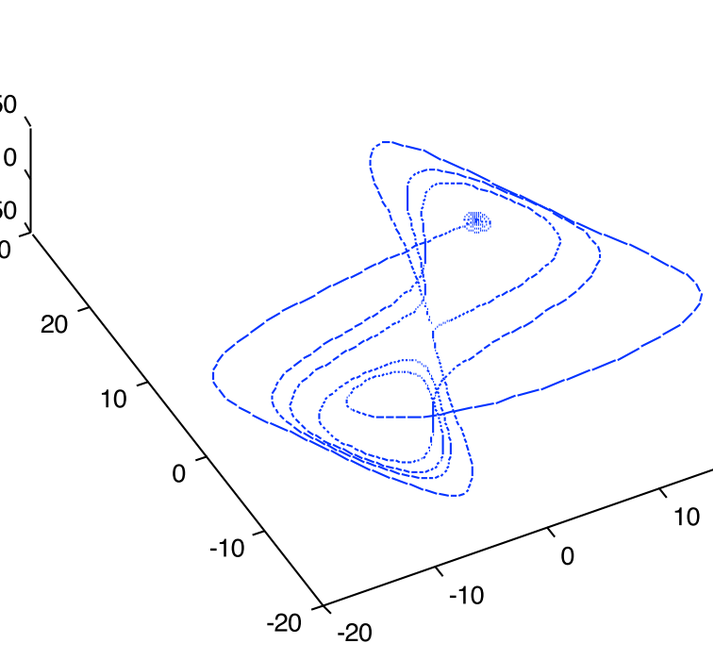
Webber, C.L., Jr., & Zbilut, J.P. (2005). Recurrence quantification analysis of nonlinear dynamical systems. In: *Tutorials in contemporary nonlinear methods for the behavioral sciences*, (Chapter 2, pp. 26-94), M.A. Riley, G. Van Orden, eds. Retrieved June 5, 2007 http://www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.pdf

Rescaling after Reconstruction

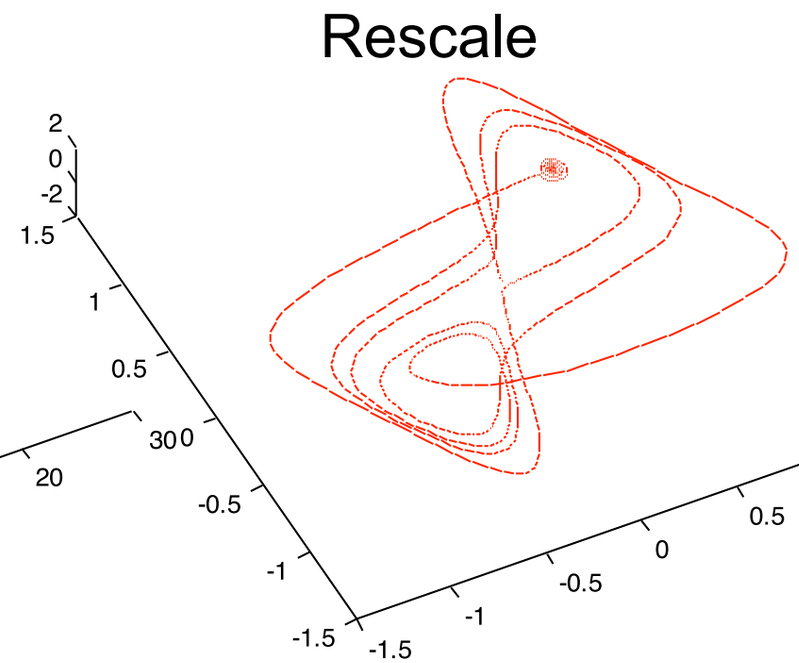
- **MAX scale:** If you have an unthresholded distance matrix (e.g. 'RM') as an R-object, you can run: **RM/max(RM)**

- **Mean scale**: If you have an unthresholded distance matrix (e.g. 'RM') as an R-object, you can run: (**RM-mean(RM)) / sd(RM)**

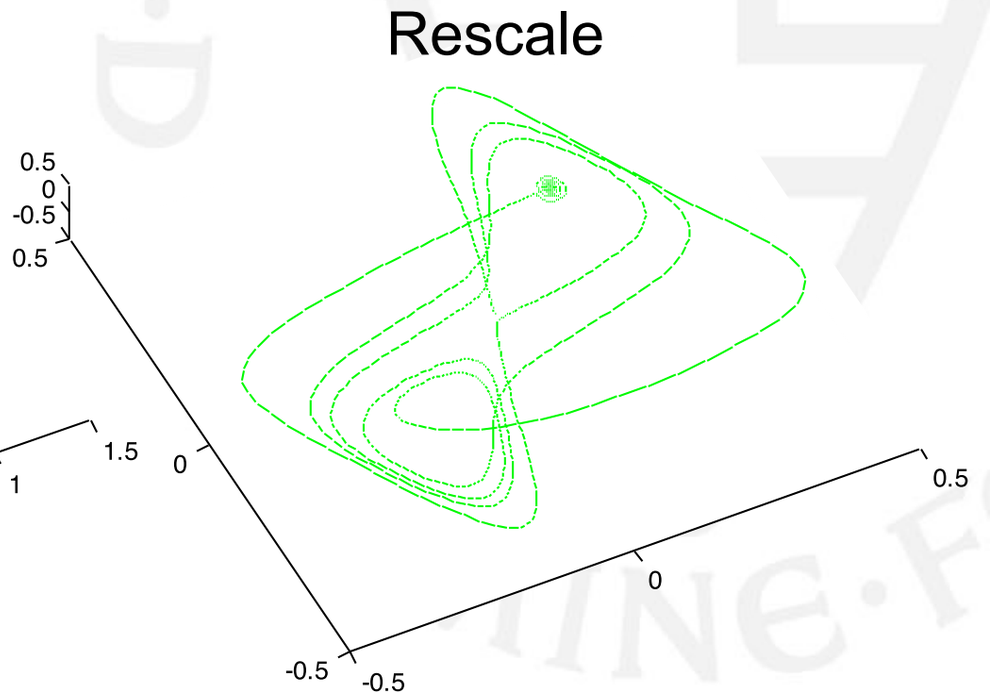
No Rescale



Mean Distance Rescale



Maximum Distance Rescale



Rescaling *before* Reconstruction

- You could also rescale the time series *before* you do the reconstruction:

- **Max distance** -> unit scale $\mathbf{X}_{\mathbf{unit}} = (\mathbf{X}\text{-}\min(\mathbf{X})) / (\max(\mathbf{X})\text{-}\min(\mathbf{X}))$

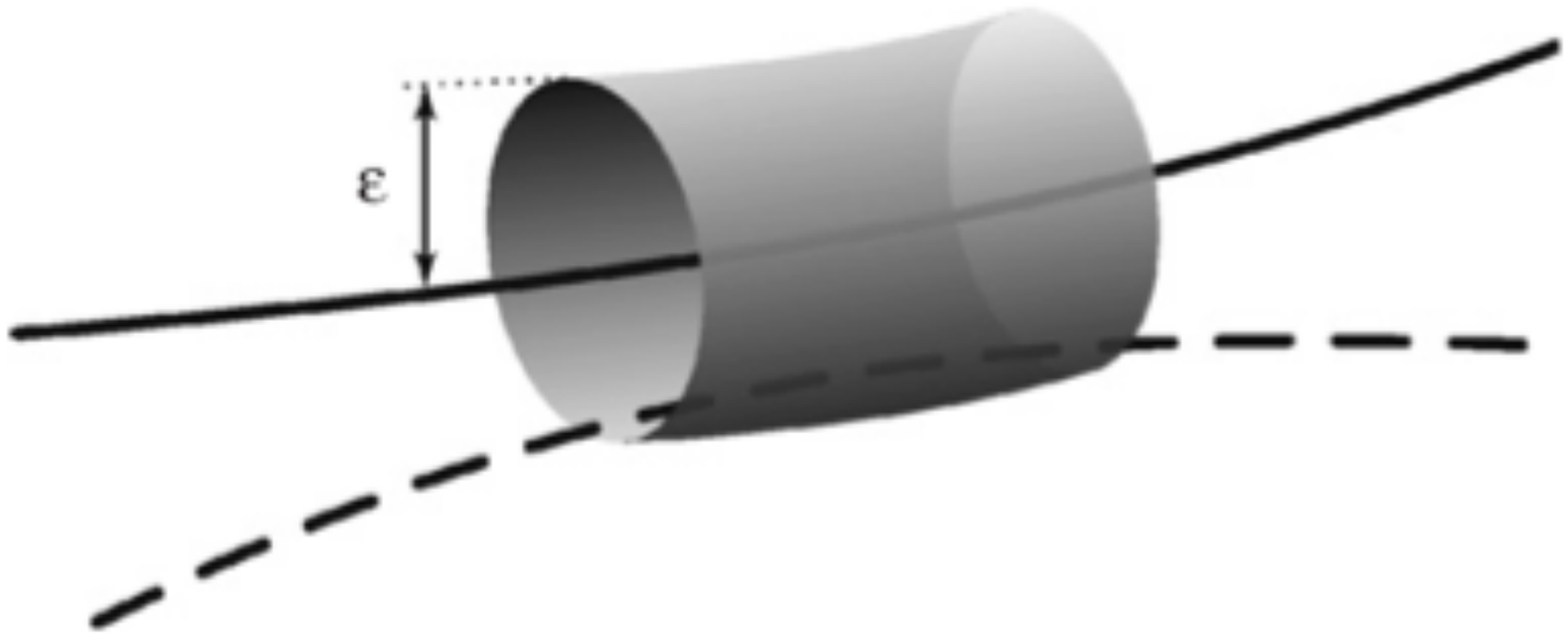
  Scale of 0-1 (in package *casnet* you can use the **elascer** function)

- **Mean distance** -> z-score $\mathbf{X}_{\mathbf{z}} = (\mathbf{X}\text{-}\text{mean}(\mathbf{X})) / \text{std}(\mathbf{X})$

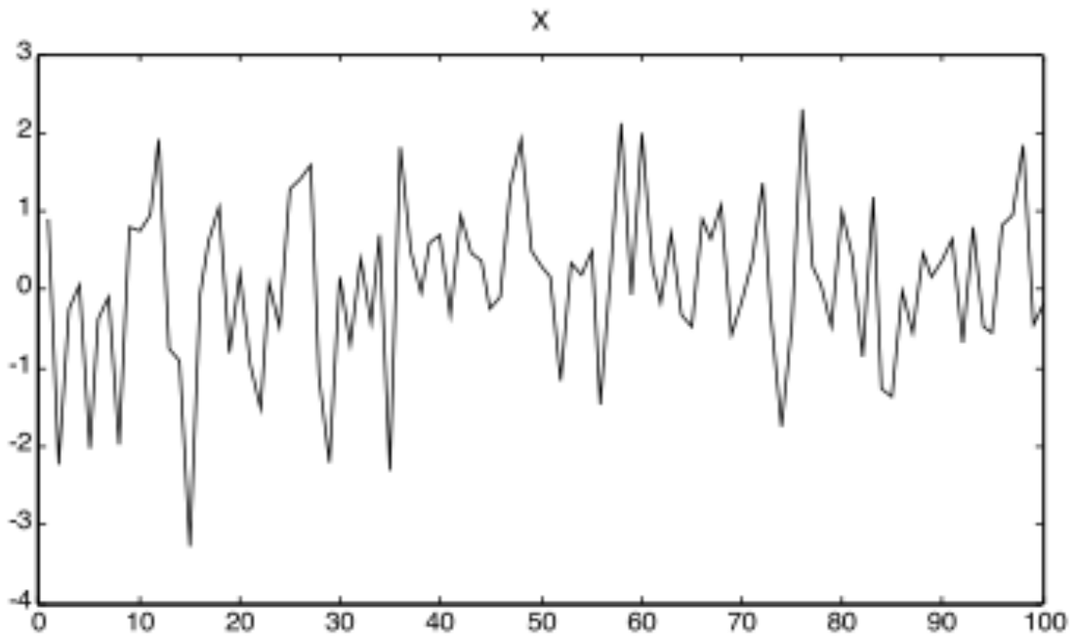
  Z-score scale (in package *casnet* you can use the **ts_standardise** function with: **adjustN = FALSE**)

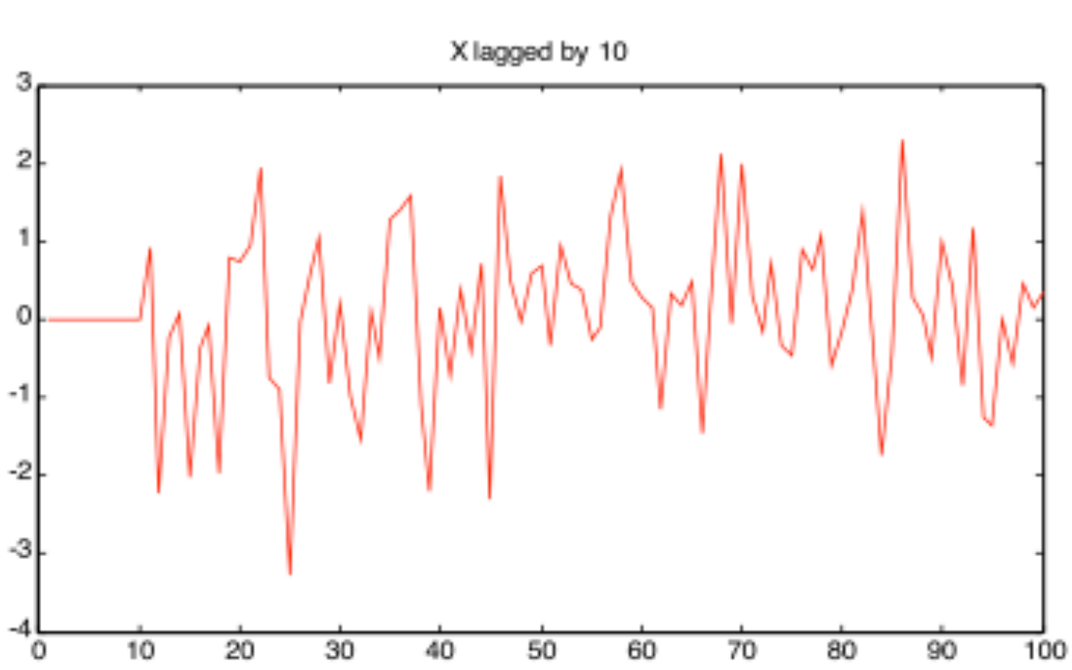
Within radius / threshold = shared trajectory

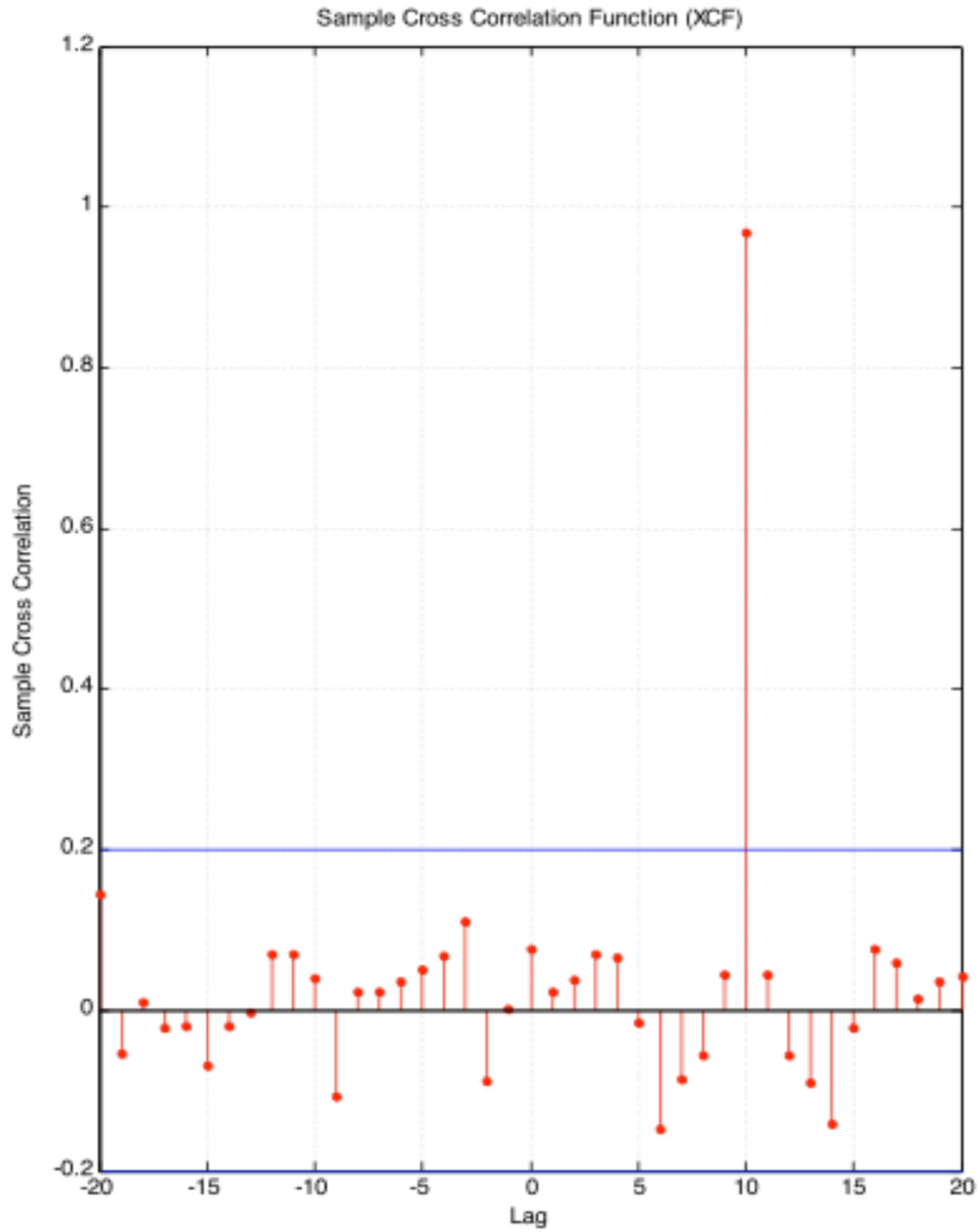
N. Marwan et al. / Physics Reports 438 (2007) 237–329



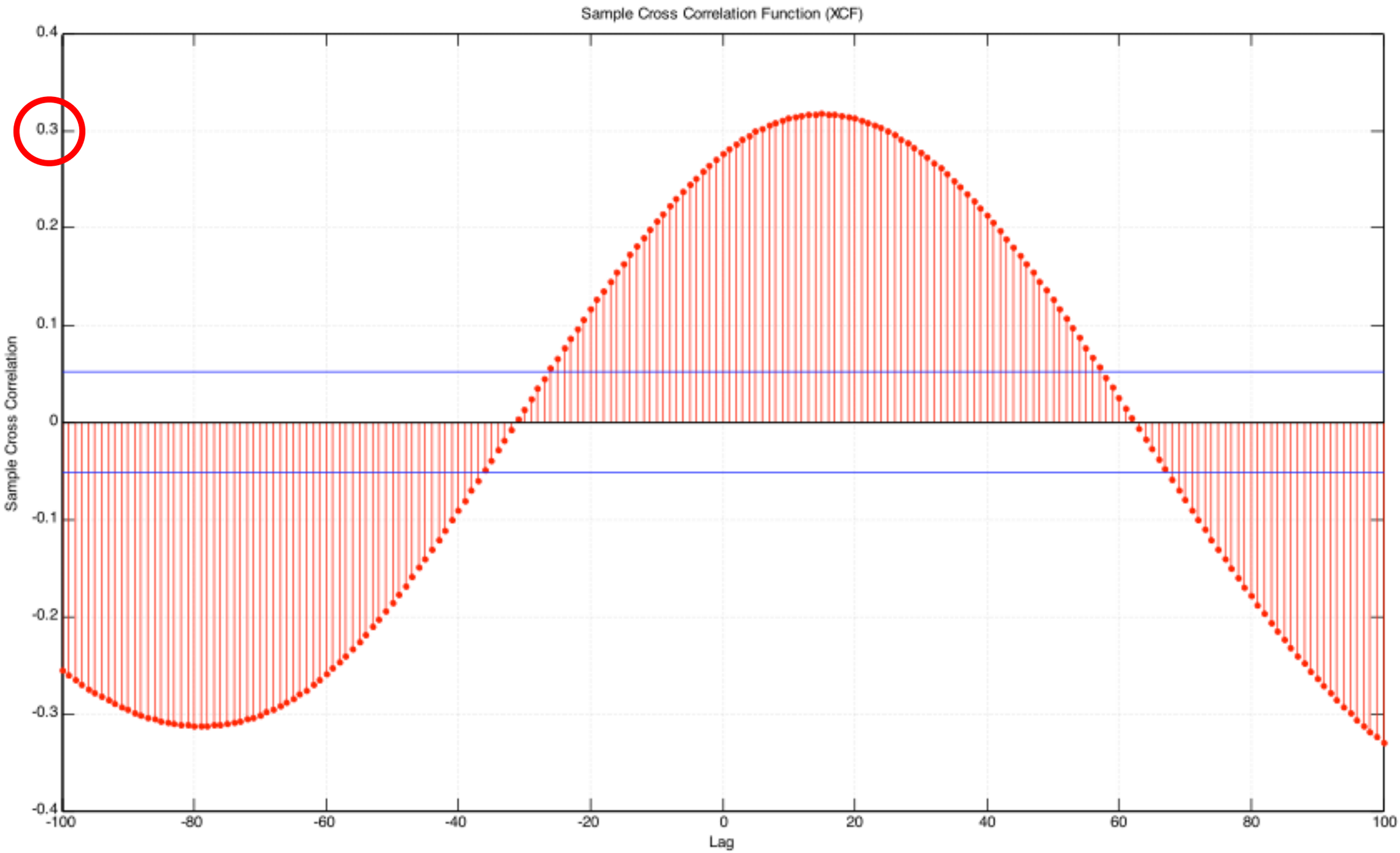
Intuitive notion of synchronisation – Cross Correlation



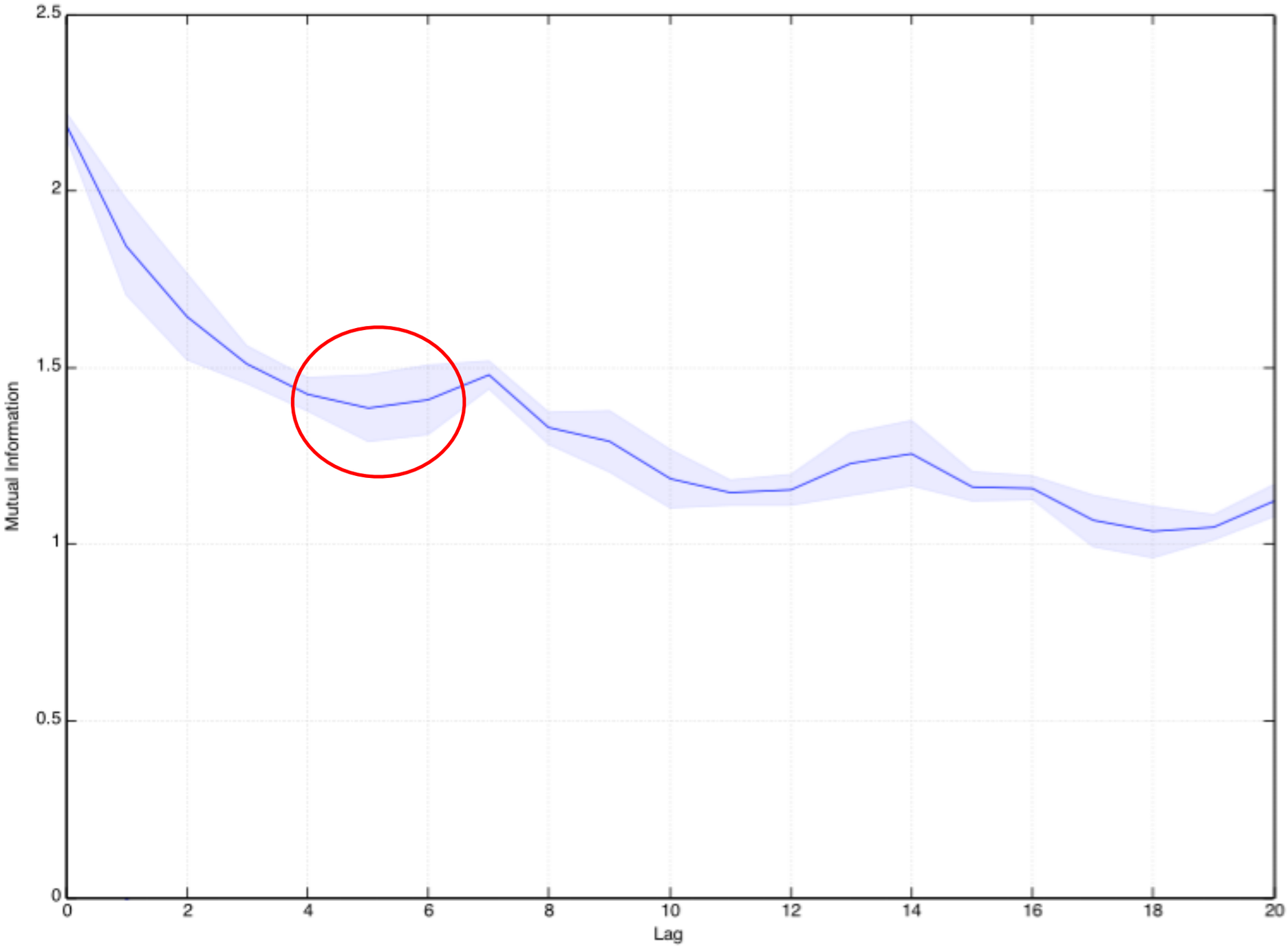


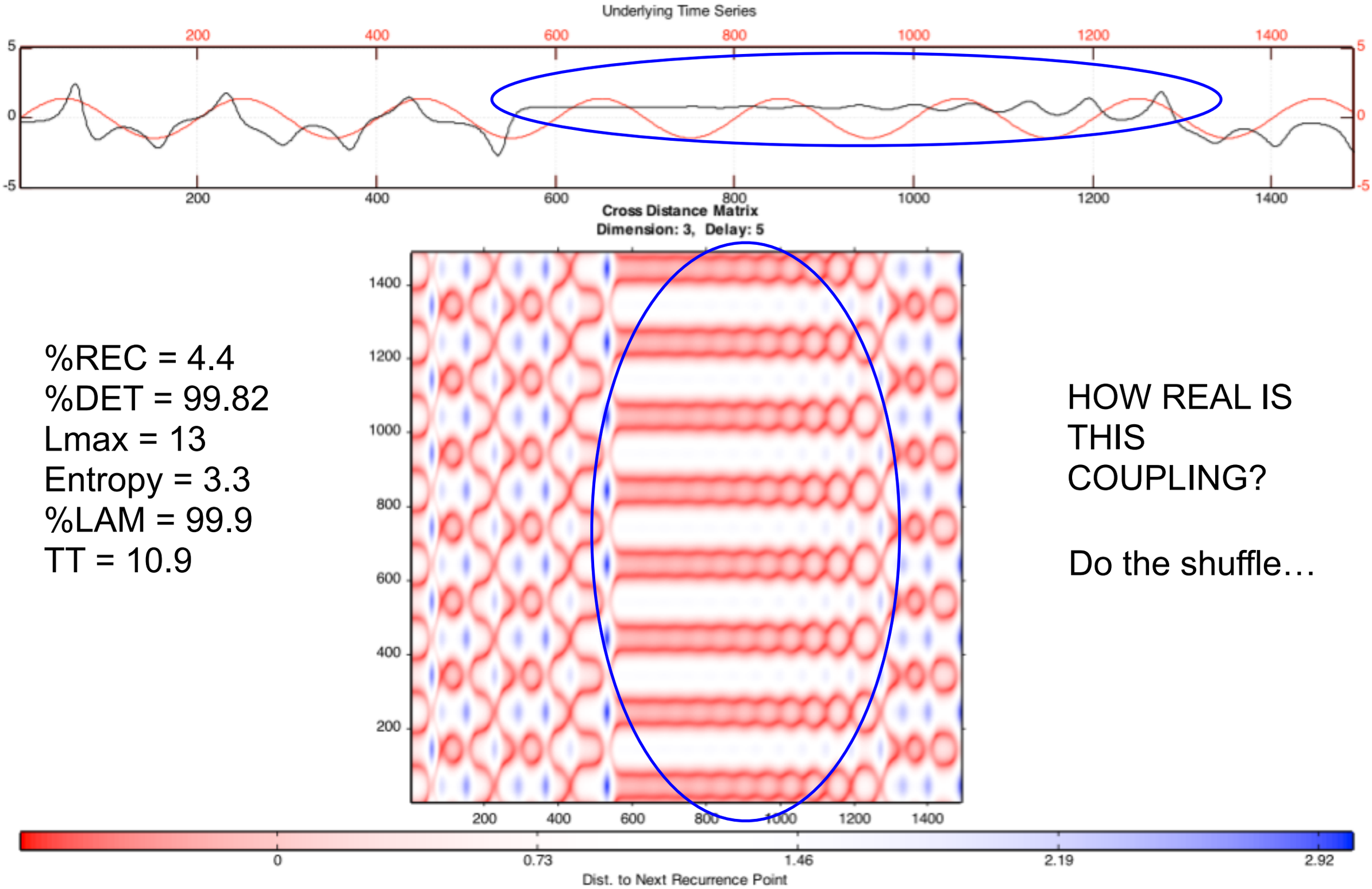


Intuitive notion of synchronisation – Cross Correlation



Lorenz and Spiral – Mutual Information

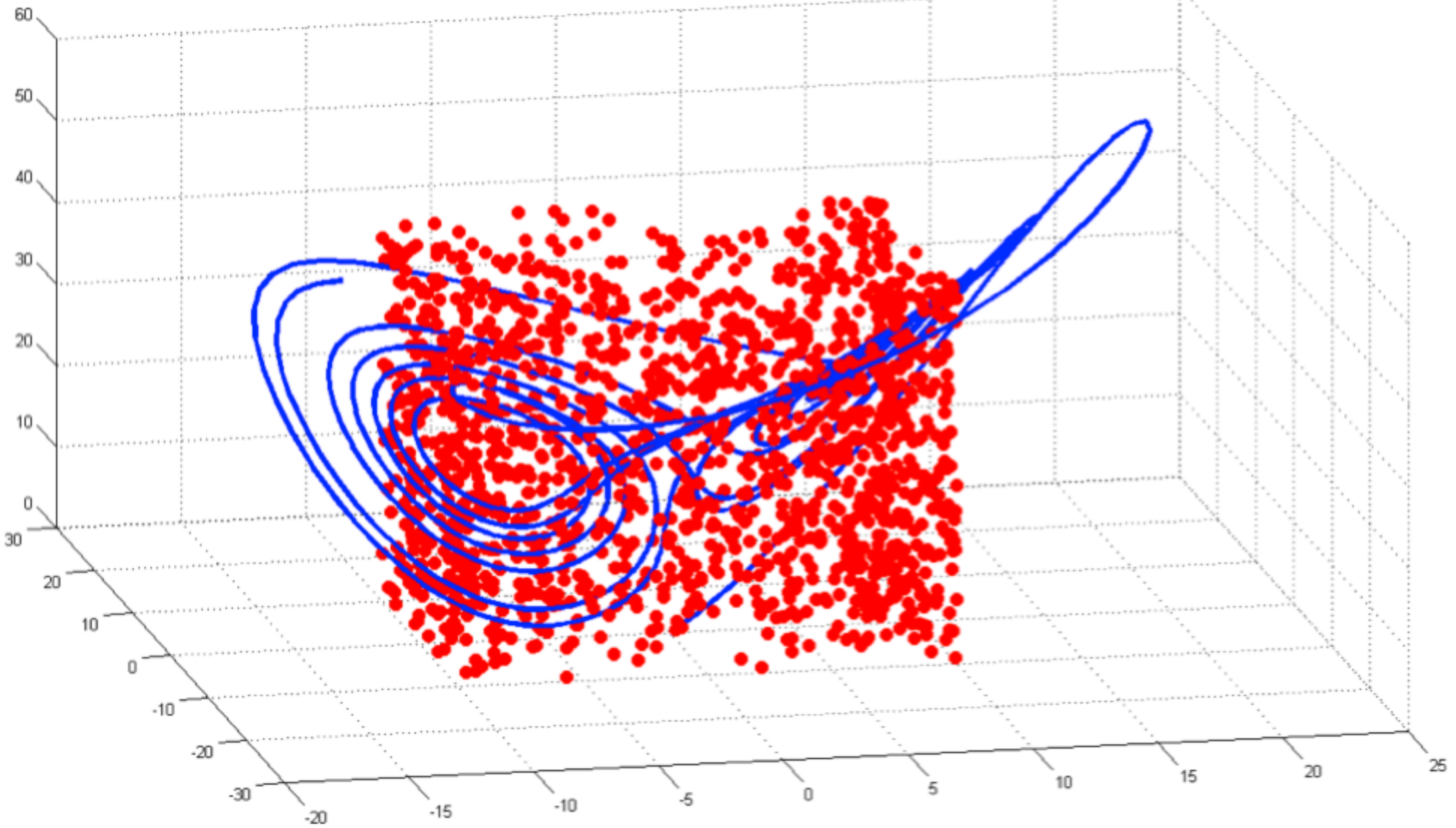




%REC = 4.4
%DET = 99.82
Lmax = 13
Entropy = 3.3
%LAM = 99.9
TT = 10.9

HOW REAL IS THIS COUPLING?

Do the shuffle…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20
10
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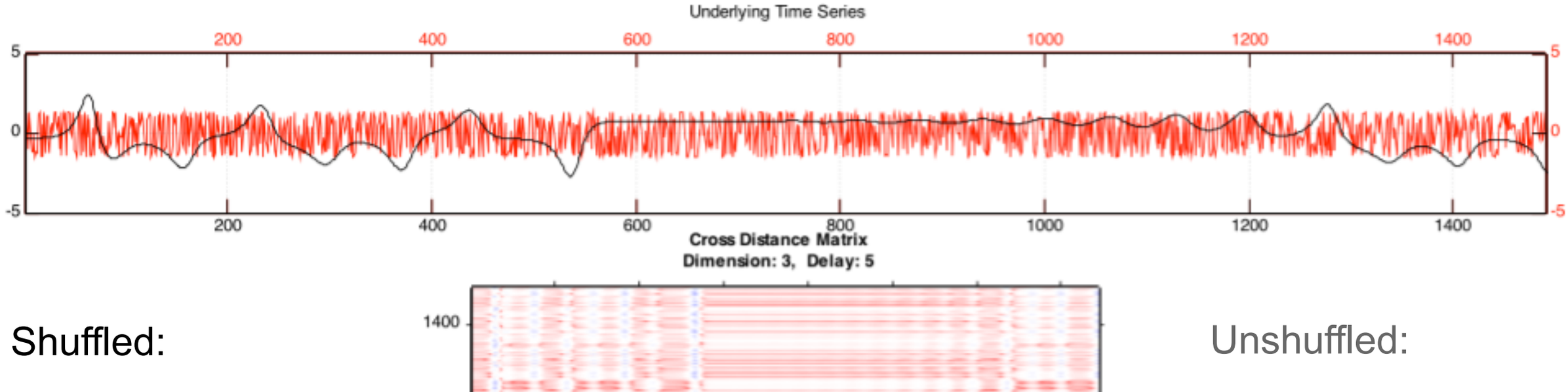
0
-10
-20
-30
-20
-15
-10
-5
0
5
10
15
20
25



Shuffled:

%REC = 4.4
%DET = 8.7
Lmax = 2
Entropy = 0.28
%LAM = 8.5
TT = 2.1

Threshold: 0.745

Unshuffled:

%REC = 4.4
%DET = 99.82
Lmax = 13
Entropy = 3.3
%LAM = 99.9
TT = 10.9

Threshold: 0.5

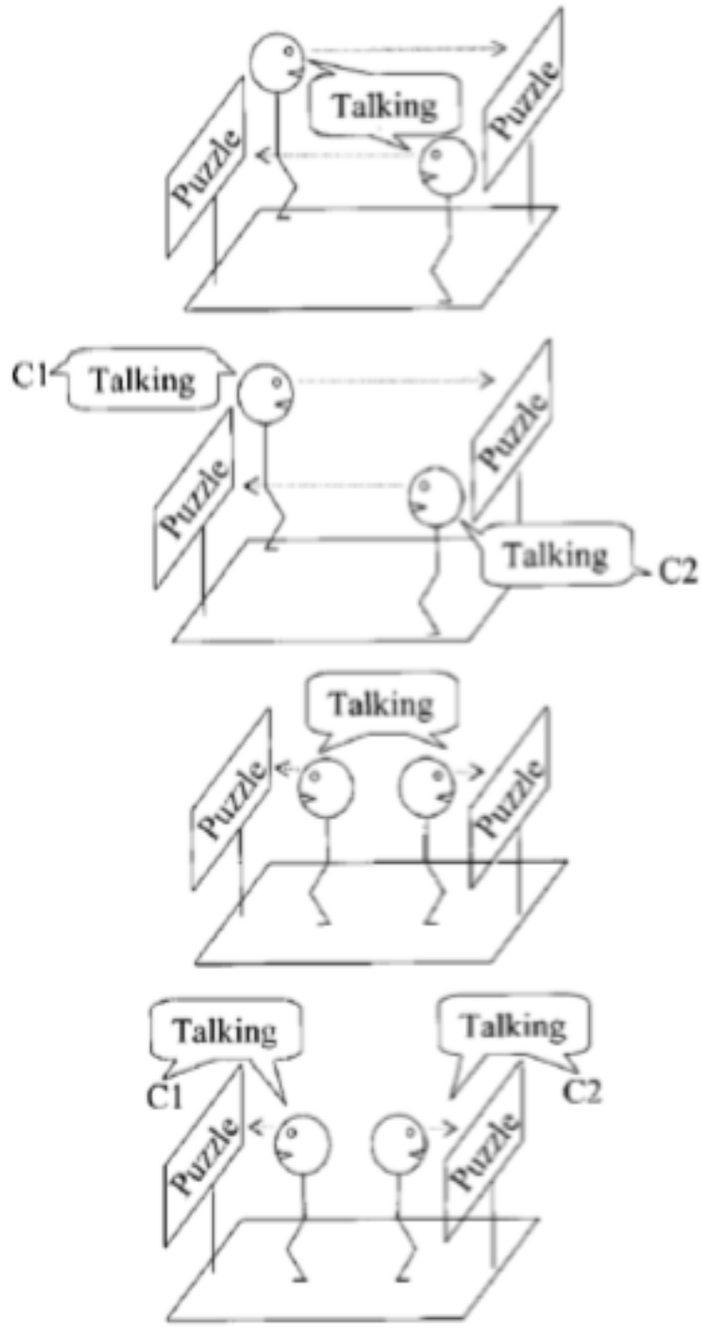
Some Applications

- Coupling of postural sway through communication
- Coupling of language development between infant and caretaker
- Coupling of eye movements to communication

Coupling of postural sway through communication

- Postural sway measured by force plate

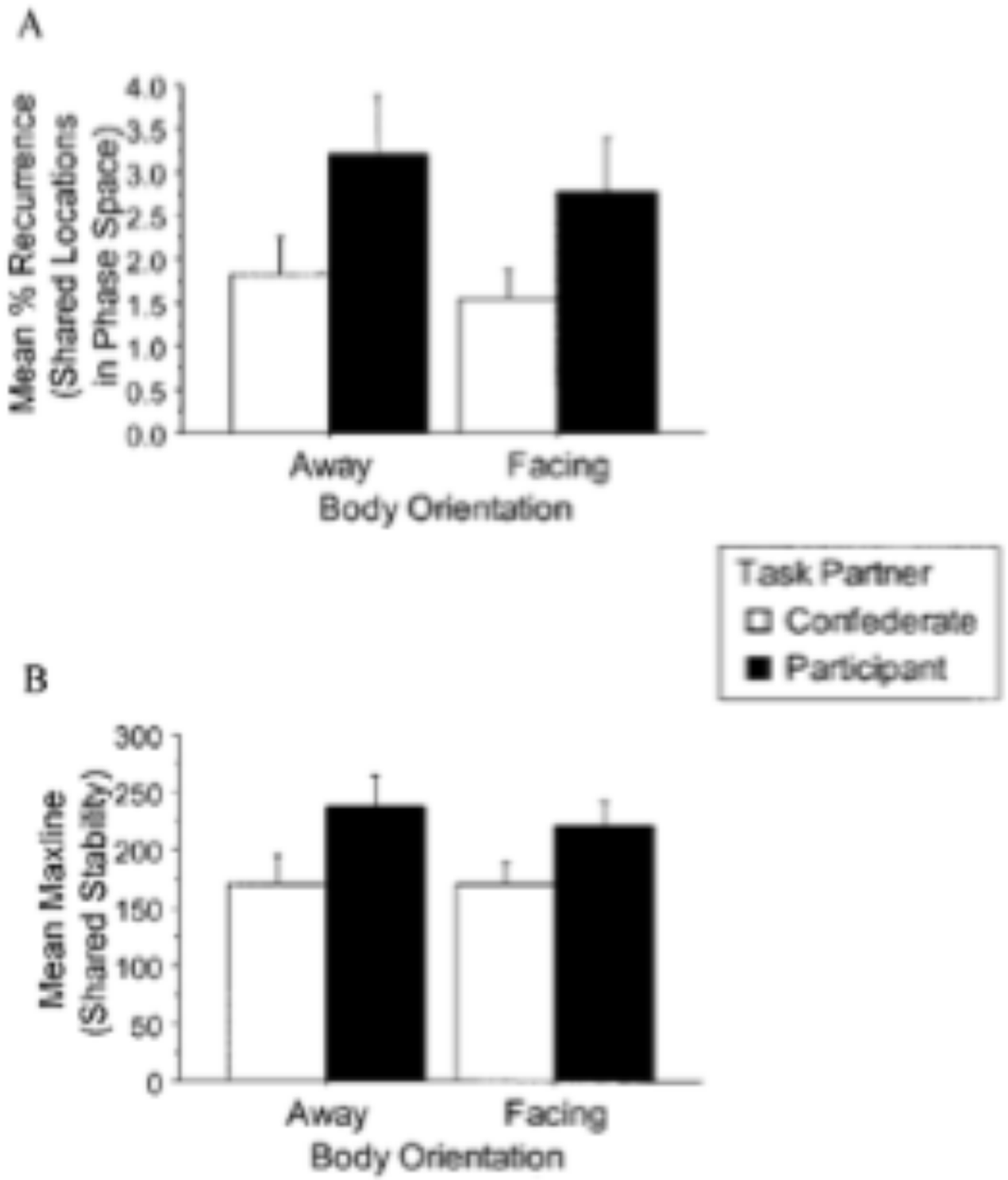
- Level of direct communication manipulated by talking directly or to confederate / visibility



Shockley, K., Santana, M-V., Fowler, C. (2003). Mutual Interpersonal Postural Constraints Are Involved in Cooperative Conversation. *Journal of Experimental Psychology: Human Perception and Performance, 29*, 326-323.

Coupling of postural sway through communication

Speech can be a “coupling tool” for coordination of previously autonomous bodies



Shockley, K., Santana, M-V., Fowler, C. (2003). Mutual Interpersonal Postural Constraints Are Involved in Cooperative Conversation. *Journal of Experimental Psychology: Human Perception and Performance, 29*, 326-323.

Coupling of language development between infant and caretaker

Dale, R., & Spivey, M.J. (2006). Unraveling the dyad: Using recurrence analysis to explore patterns of syntactic coordination between children and caregivers in conversation. *Language Learning, 56(3)*, 391–430

Rick Dale has introduced some interesting applications of Recurrence Analysis:

- CRQA on categorical/nominal data
- “LOS”-profile, as a measure of who’s leading and who’s trailing

Categorical (C)RQA:

- The RP’s of the poems are an example of recurrence plots on categorical data. The recurring values represent an arbitrary category.

- Dale examined transcriptions of conversations between children and caregivers (CHILDES). The unit of analysis was syntactic structure

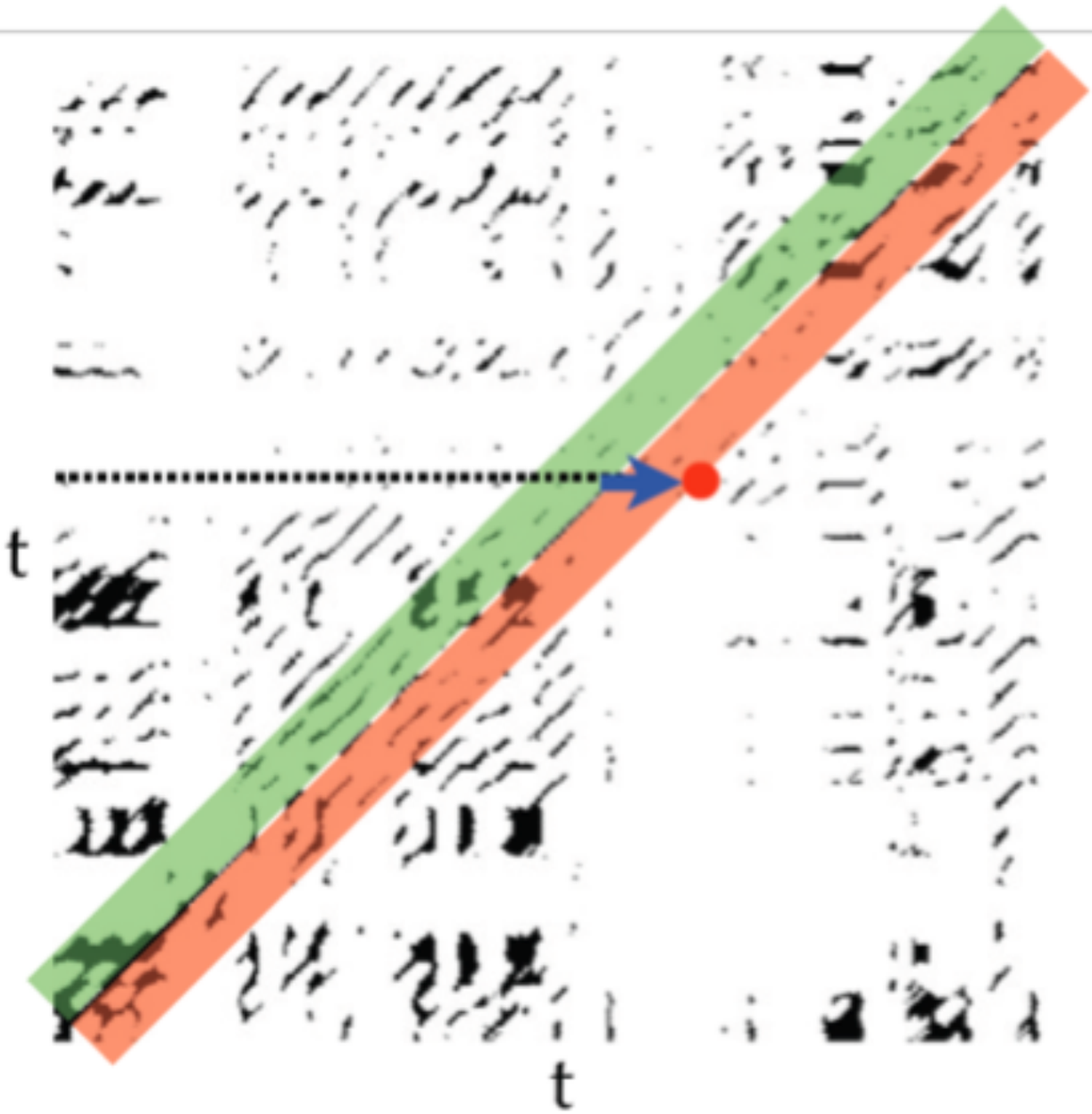
The RQA parameters become extremely simple, no need for estimation:

Lag = 1, Embedding = 1, Threshold / Radius = 0

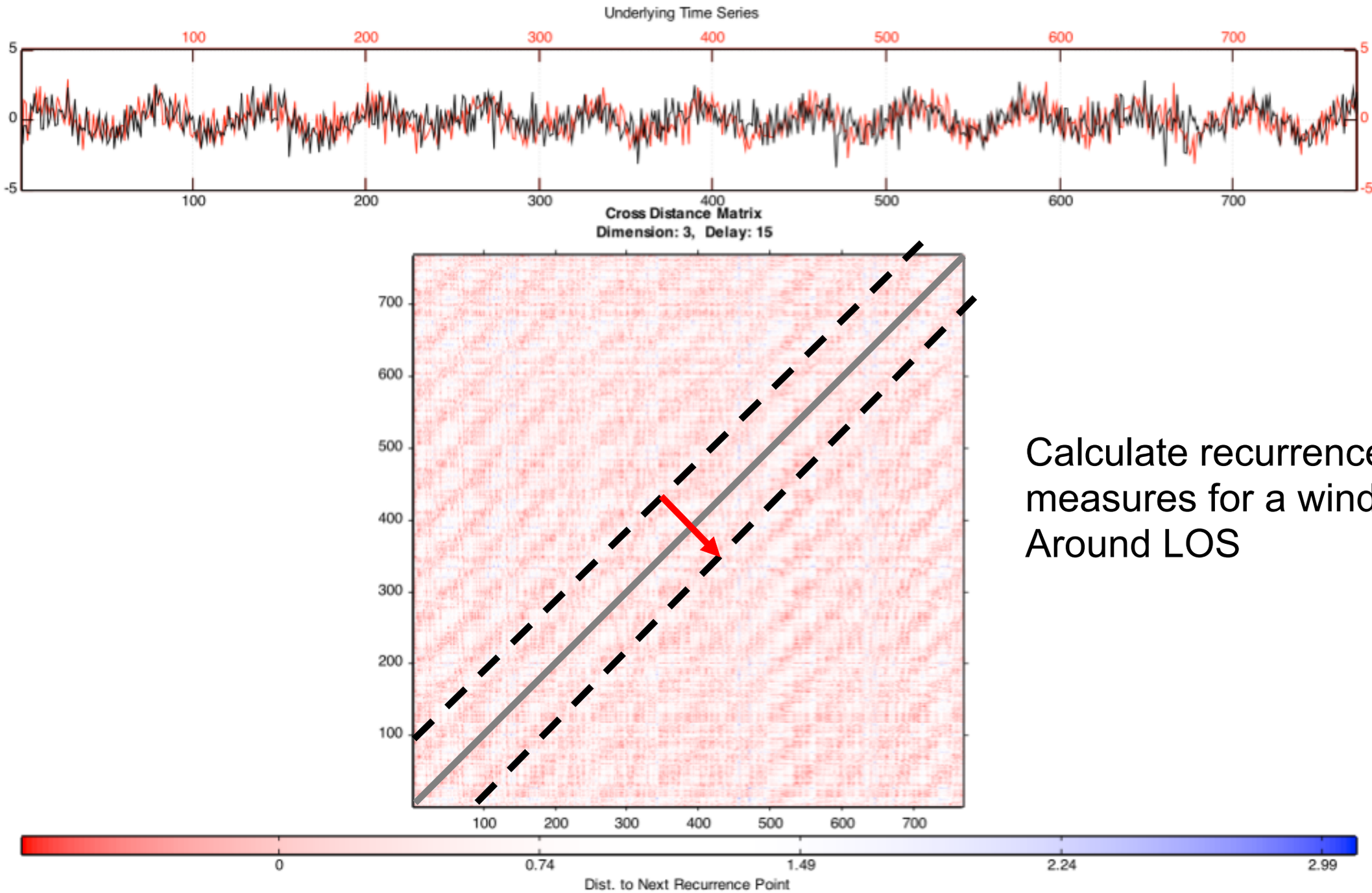
Who leads?

Time Series
On Y-axis
leads at red dot:

The category- / word- / syntactic- / pattern first occurred there,
in the X-axis series
it occurred later

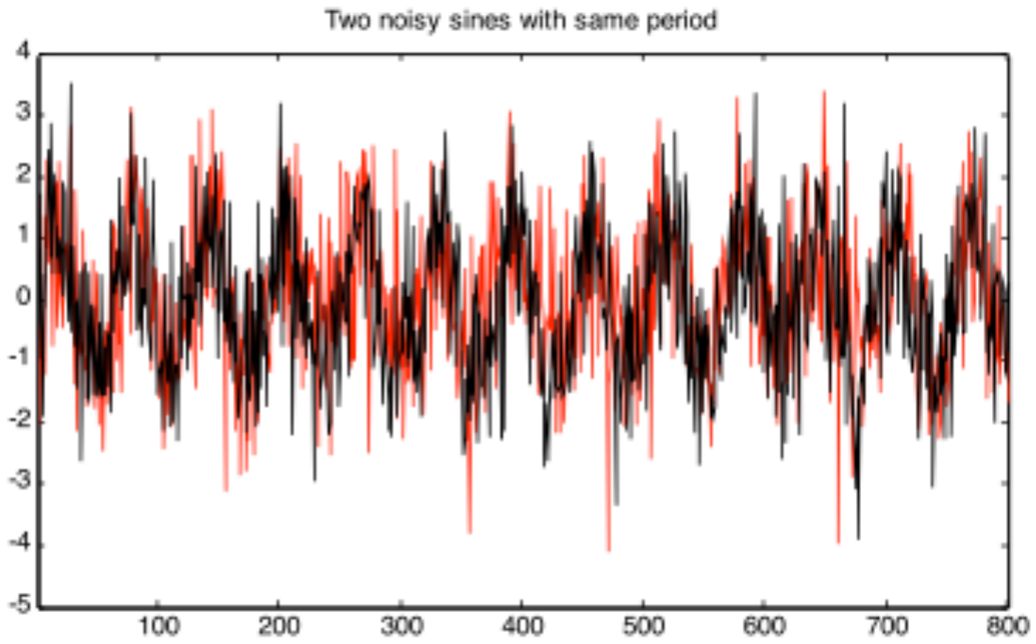


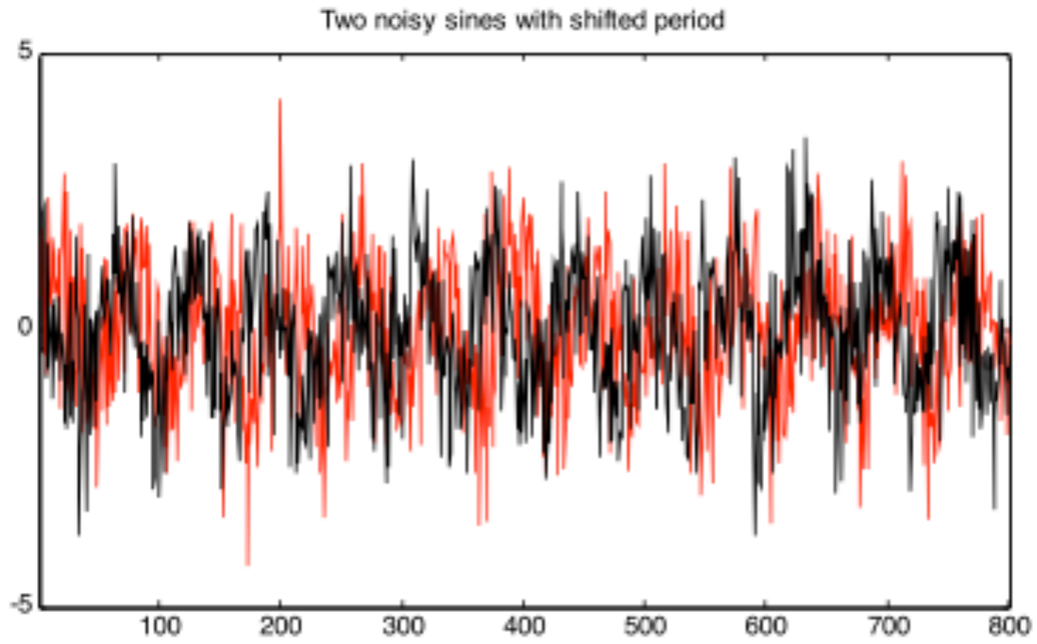
Diagonal Recurrence Profile

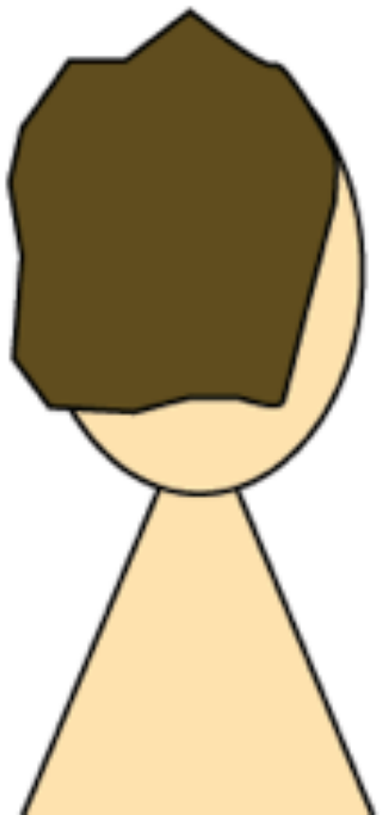


Calculate recurrence measures for a window Around LOS

Diagonal Recurrence Profile





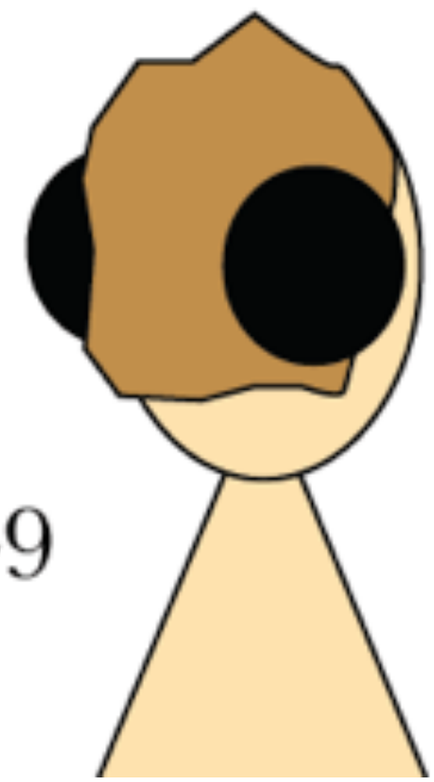


eye movement patterns...

1, 2, 2, 2, 2, 4, 4, 5, ...

“blah blah blah...”

N = 4



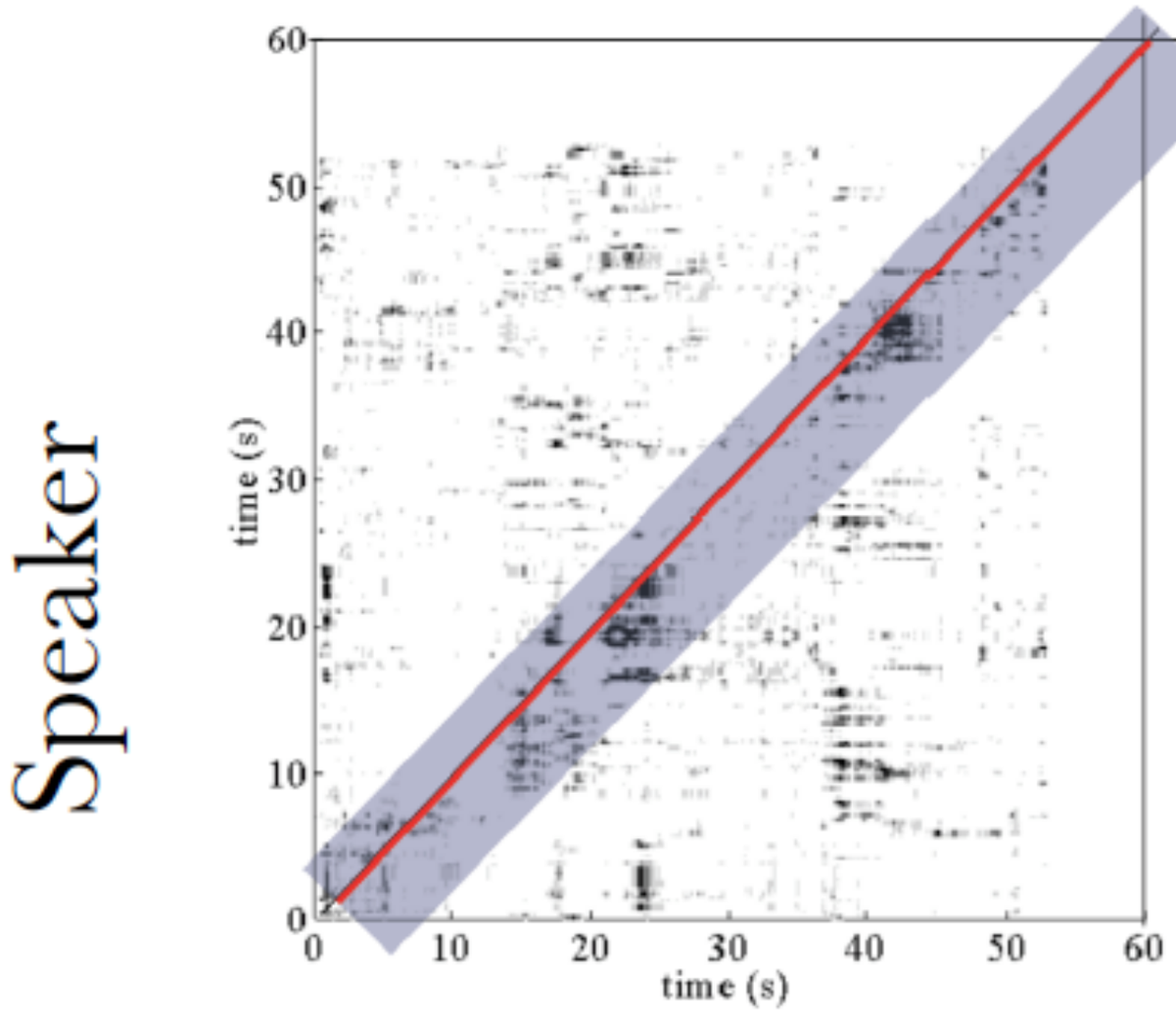
eye movement patterns...
1, 1, 1, 2, 2, 2, 2, 4, ...

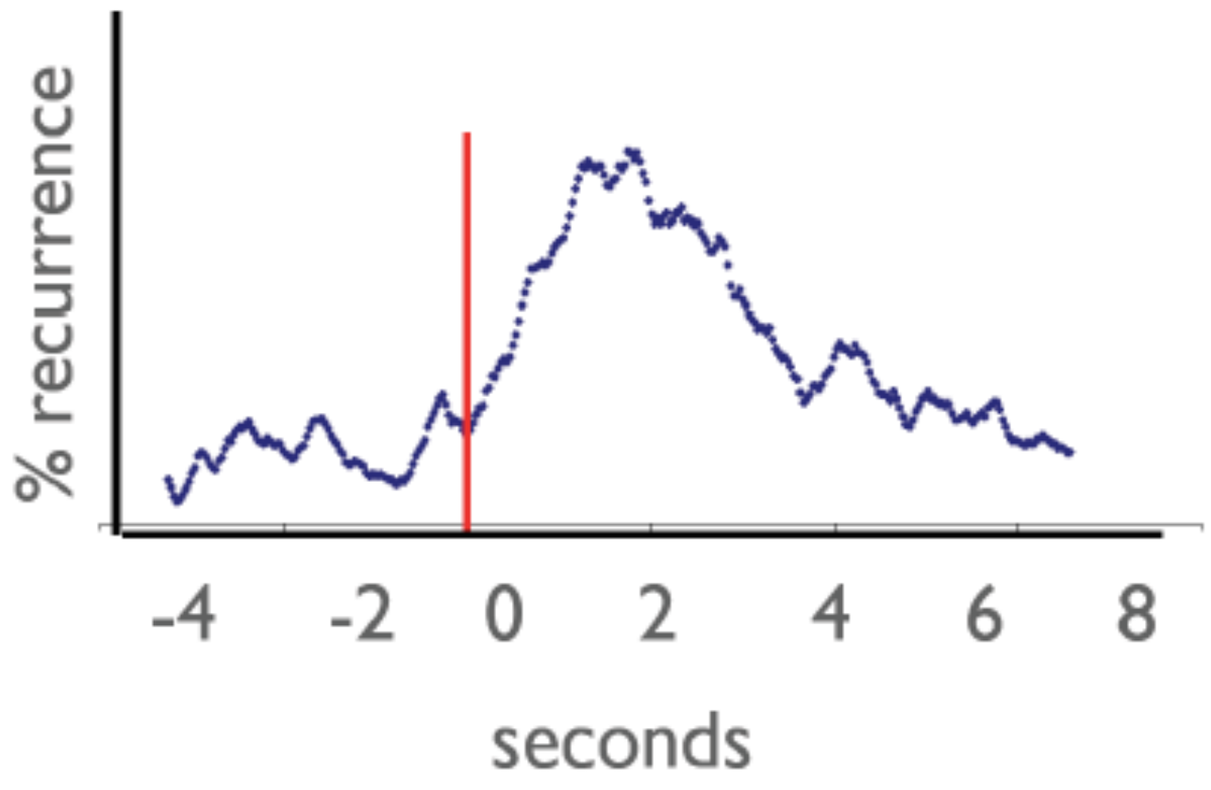
“blah blah blah...”

N = 49

Coupling of eye movements to communication

Richardson, D.C., Dale, R., Kirkham, N.Z. (2007). The art of conversation is coordination. *Psychological Science, 18*, 407-413.



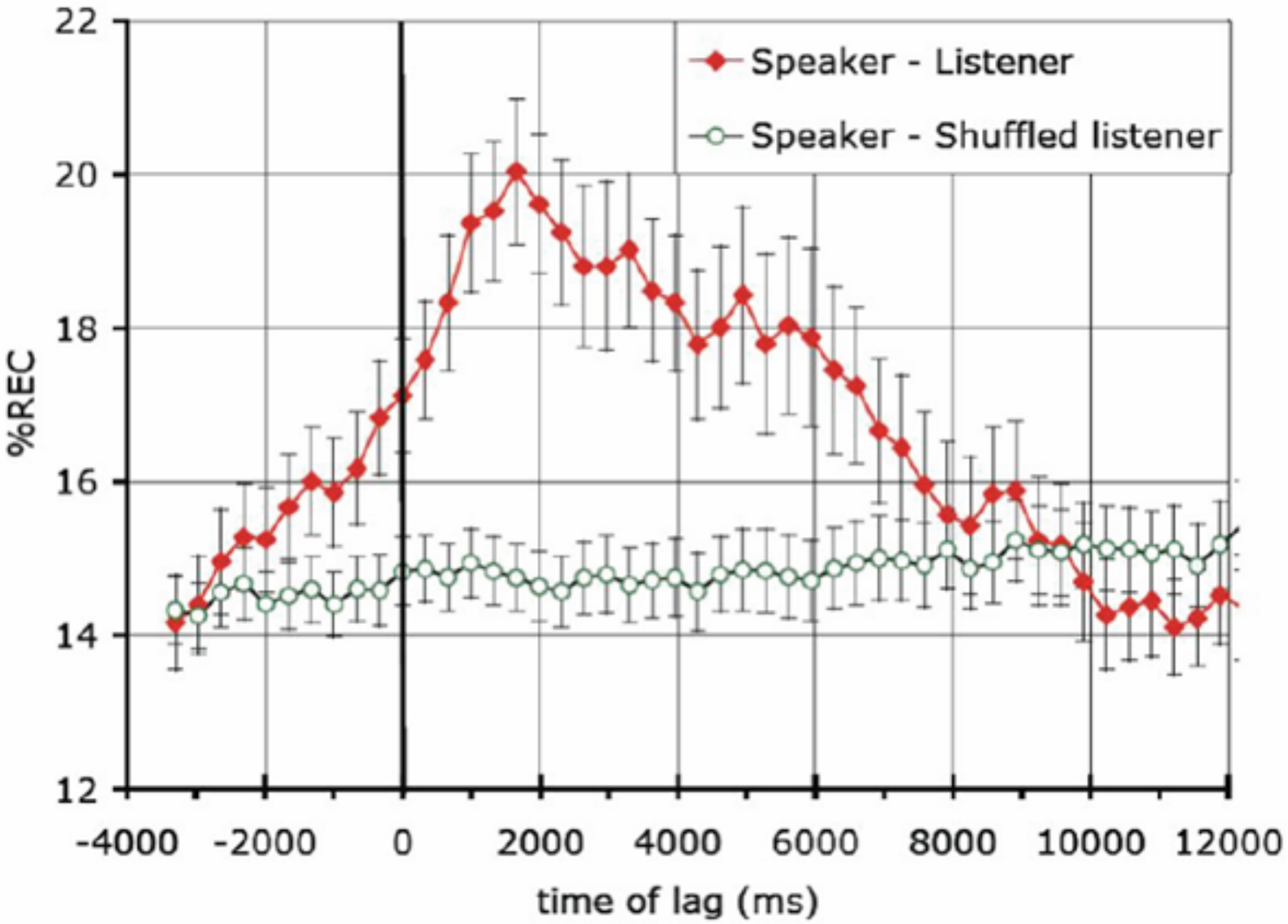


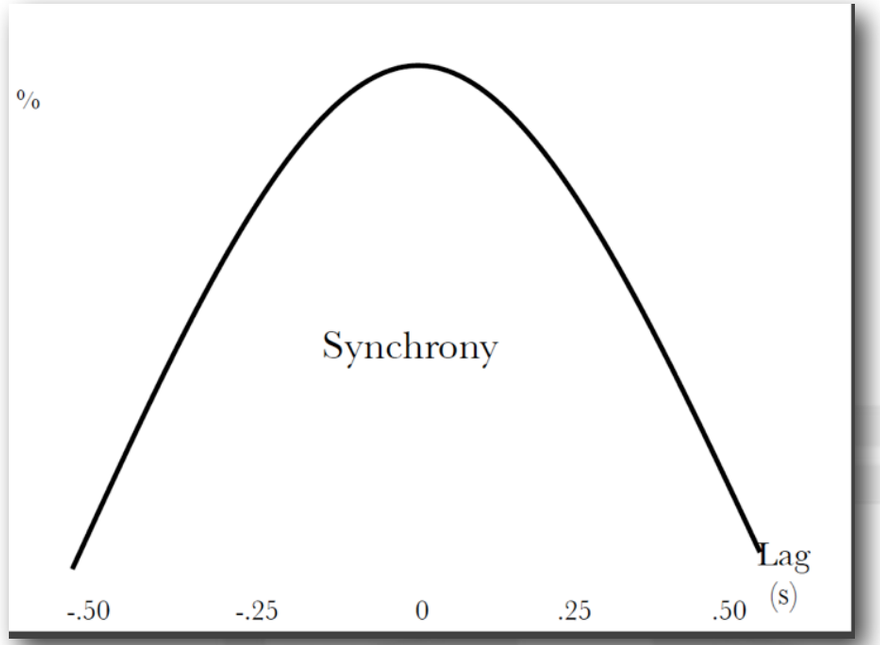
Richardson & Dale, 2005

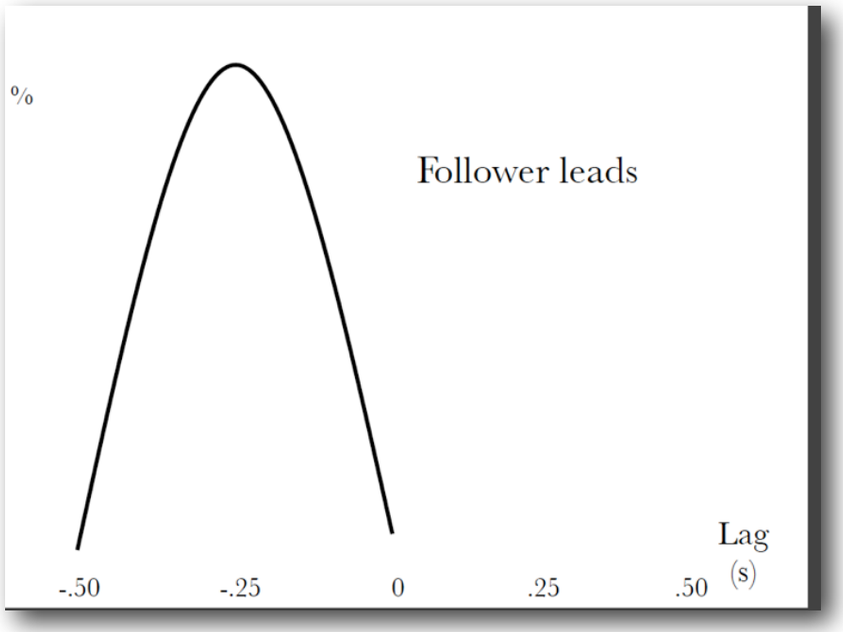
Coupling of eye movements to communication

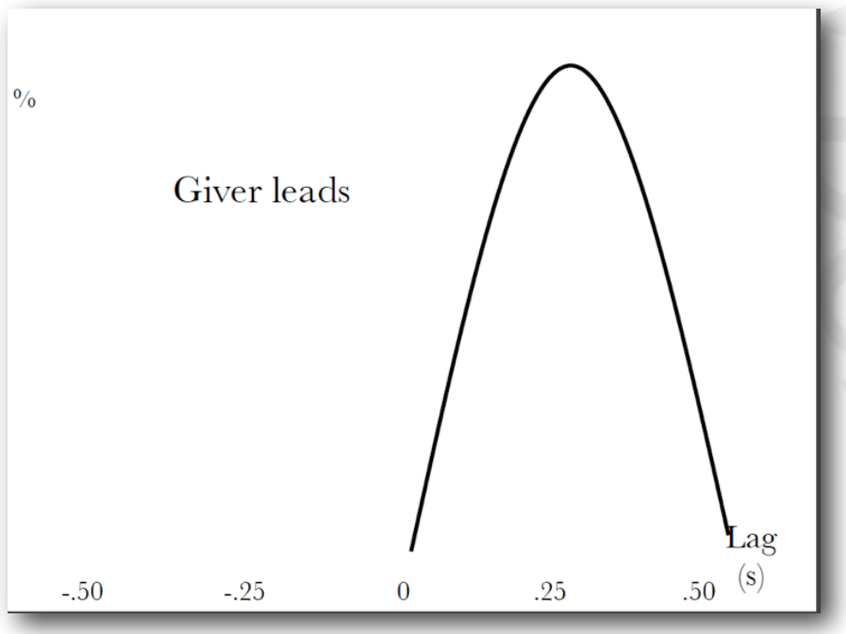
Richardson, D.C., Dale, R., Kirkham, N.Z. (2007). The art of conversation is coordination. *Psychological Science, 18*, 407-413.

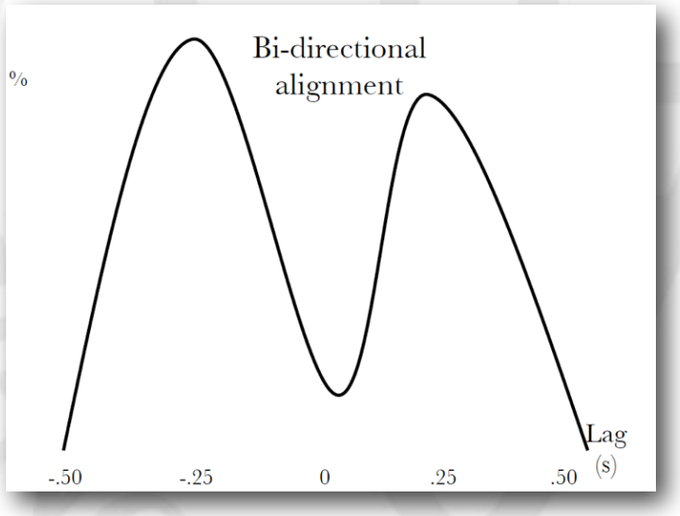
Listeners eye movements are coupled and lagging depending on level of interaction in conversation



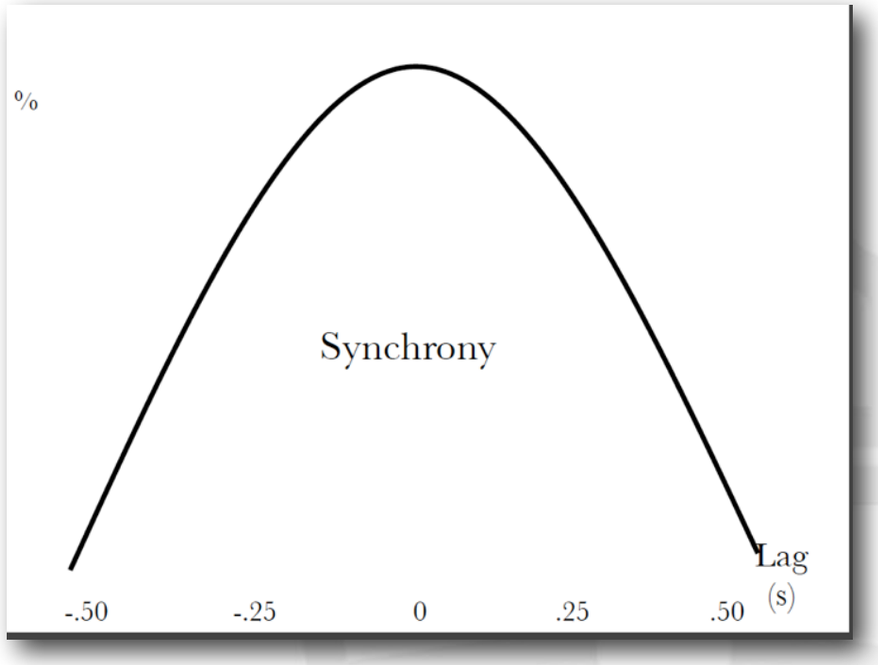


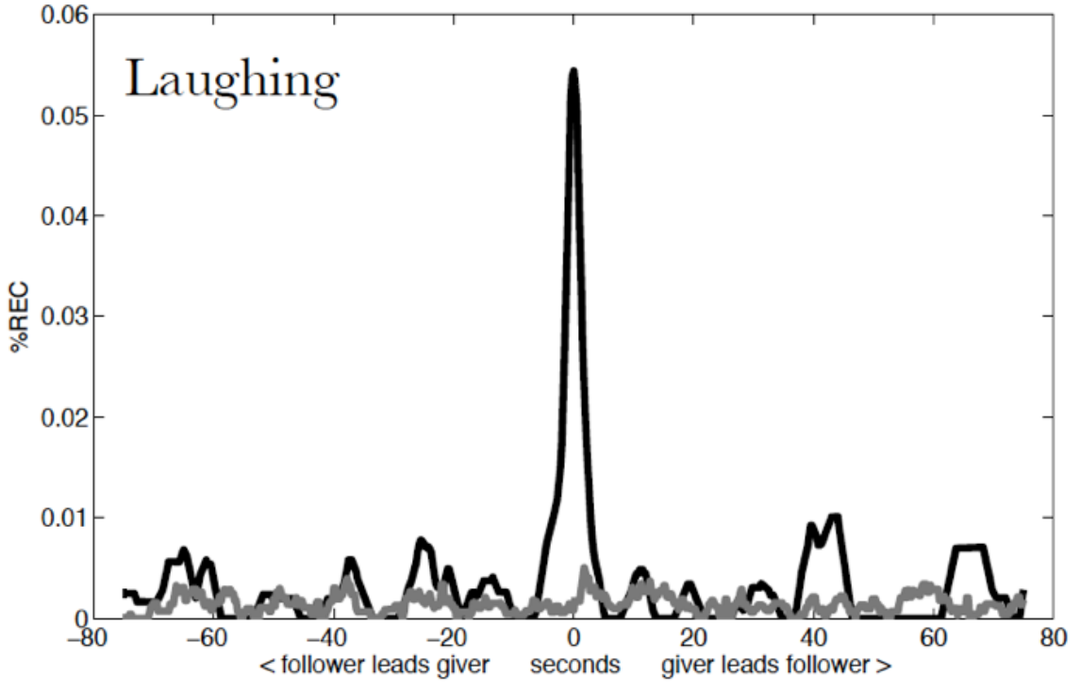






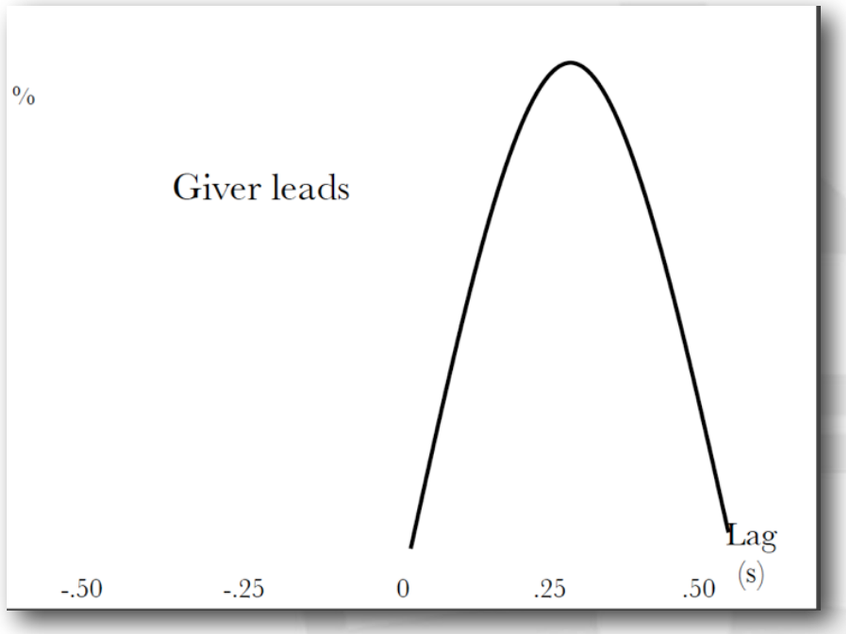
Louwerse, M. M., Dale, R., Bard, E. G., & Jeuniaux, P. (2012). Behavior matching in multimodal communication is synchronized. *Cognitive science*, *36*(8), 1404–26. doi:10.1111/j.1551-6709.2012.01269.x

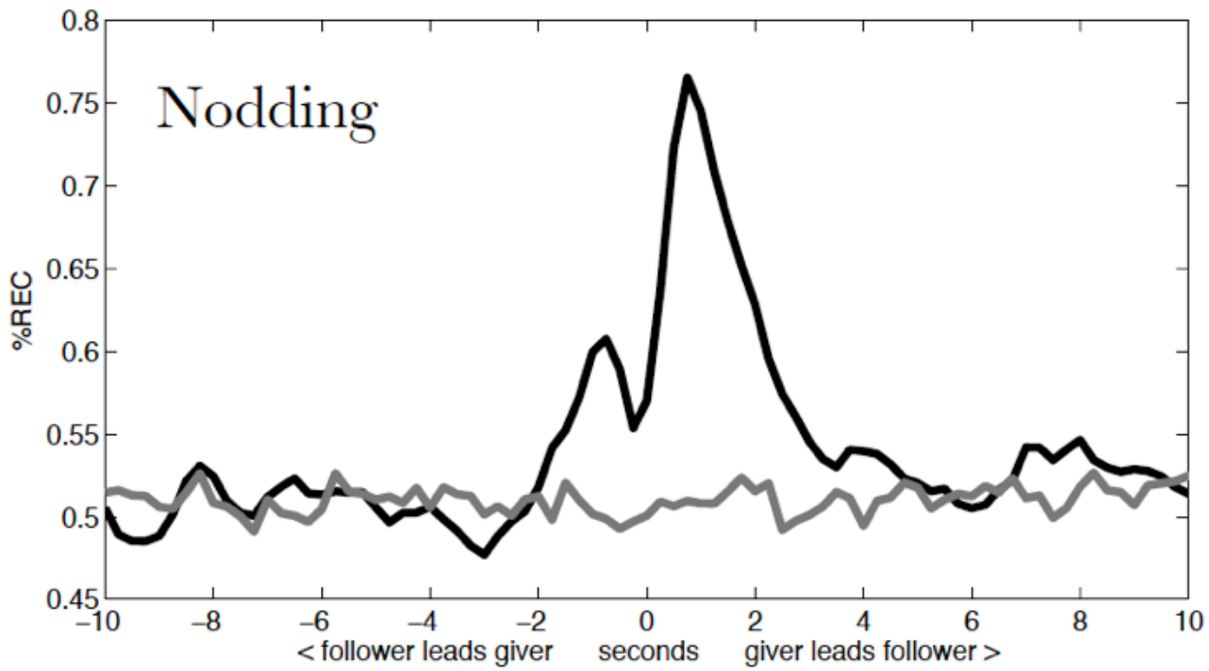




Louwerse, Dale, et al., in prep

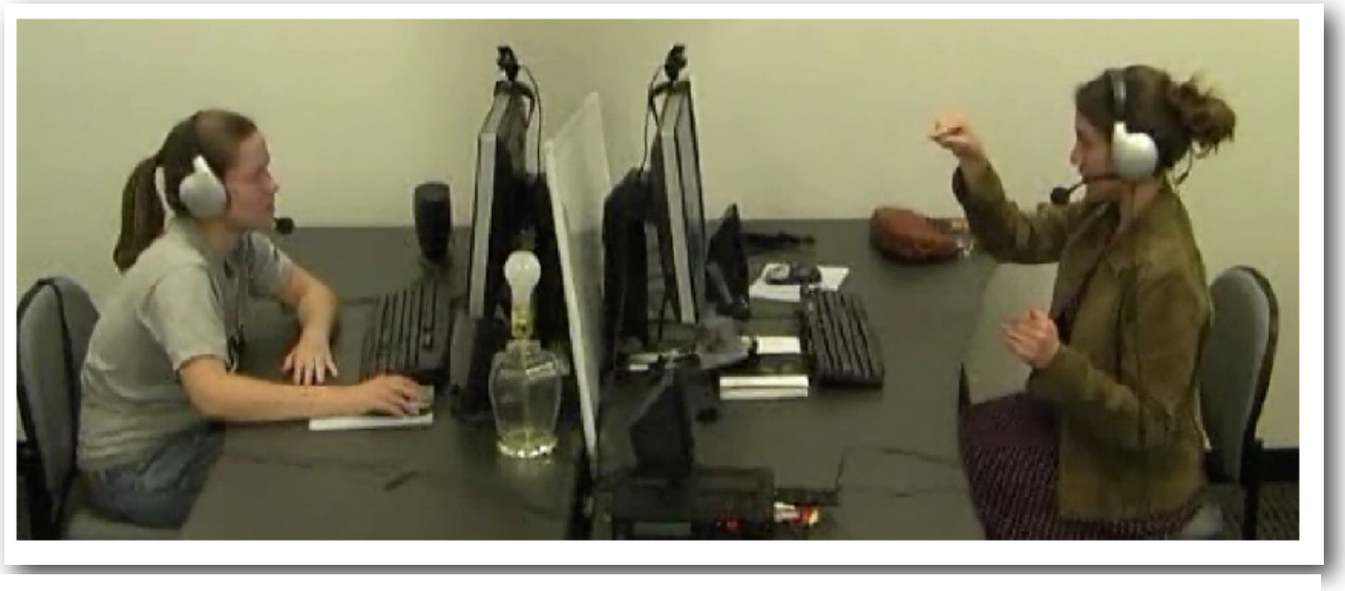
Louwerse, M. M., Dale, R., Bard, E. G., & Jeuniaux, P. (2012). Behavior matching in multimodal communication is synchronized. *Cognitive science*, *36*(8), 1404–26. doi:10.1111/j.1551-6709.2012.01269.x

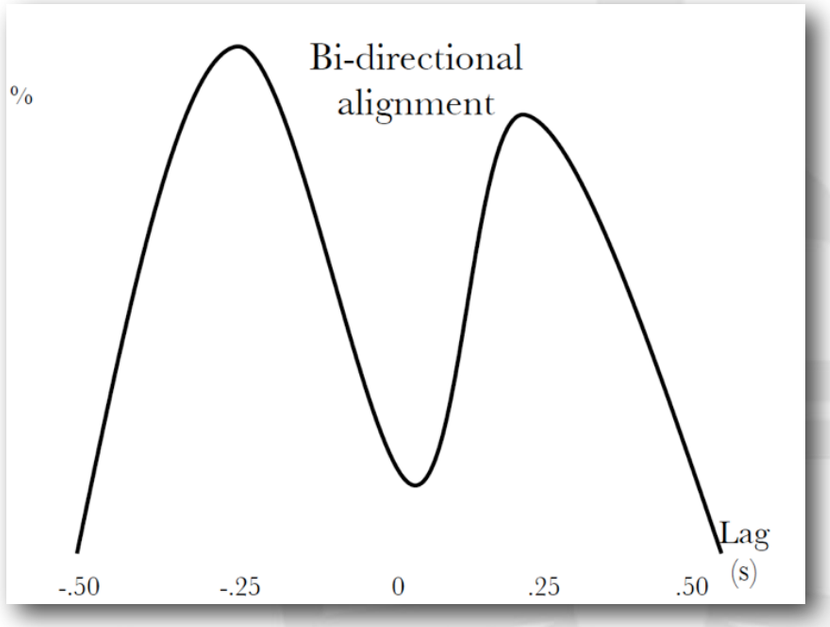


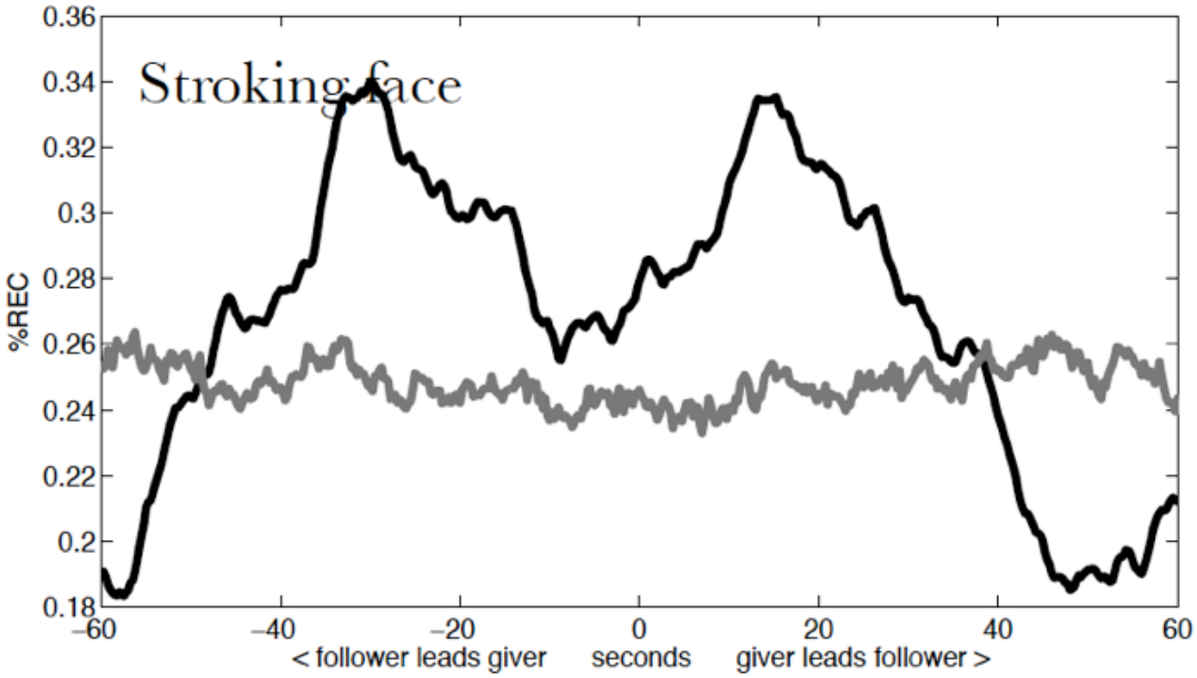


Louwerse, M. M., Dale, R., Bard, E. G., & Jeuniaux, P. (2012). Behavior matching in multimodal communication is synchronized. *Cognitive science*, *36*(8), 1404–26. doi:10.1111/j.1551-6709.2012.01269.x

Louwerse, Dale, et al., in prep







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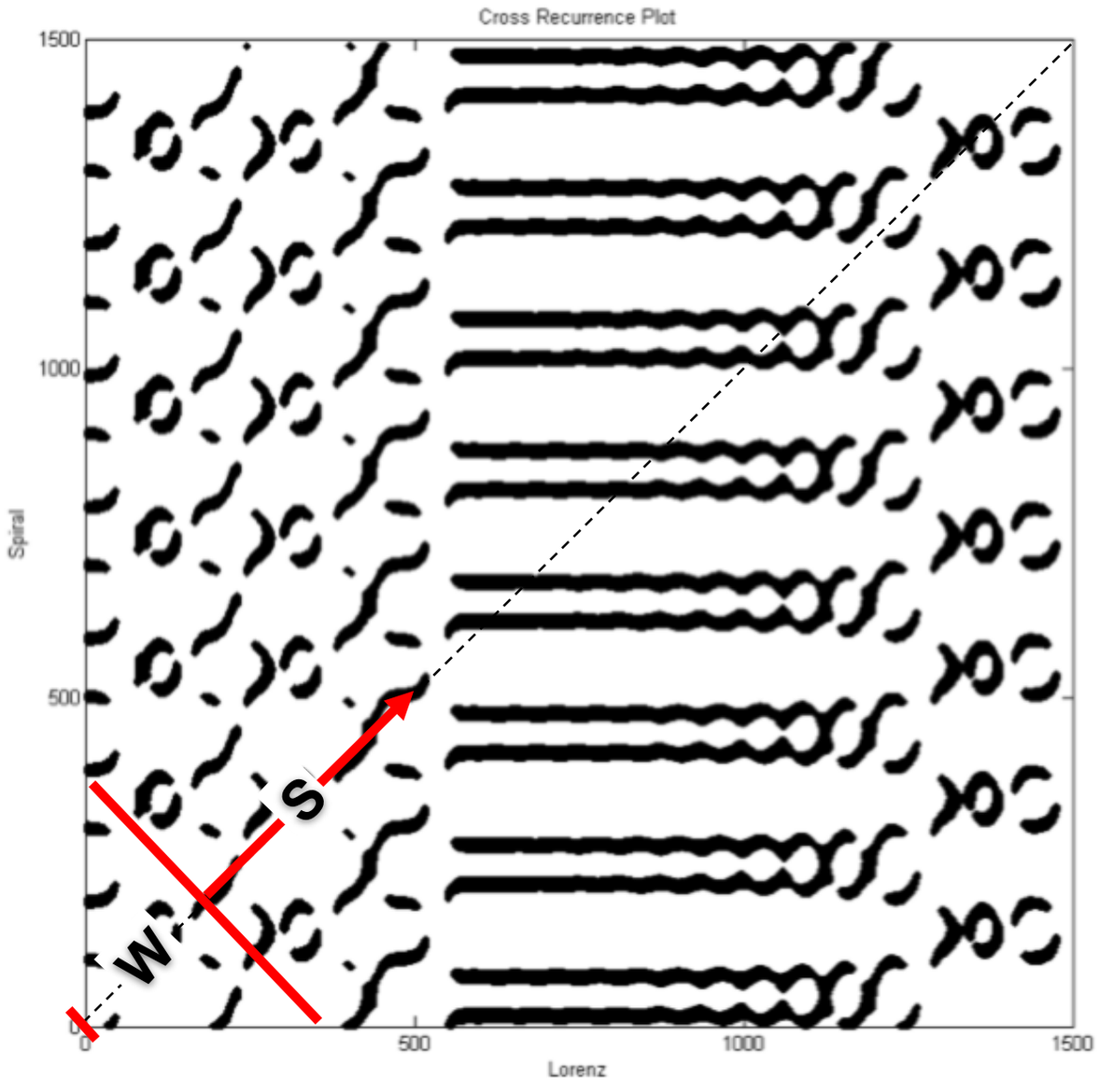
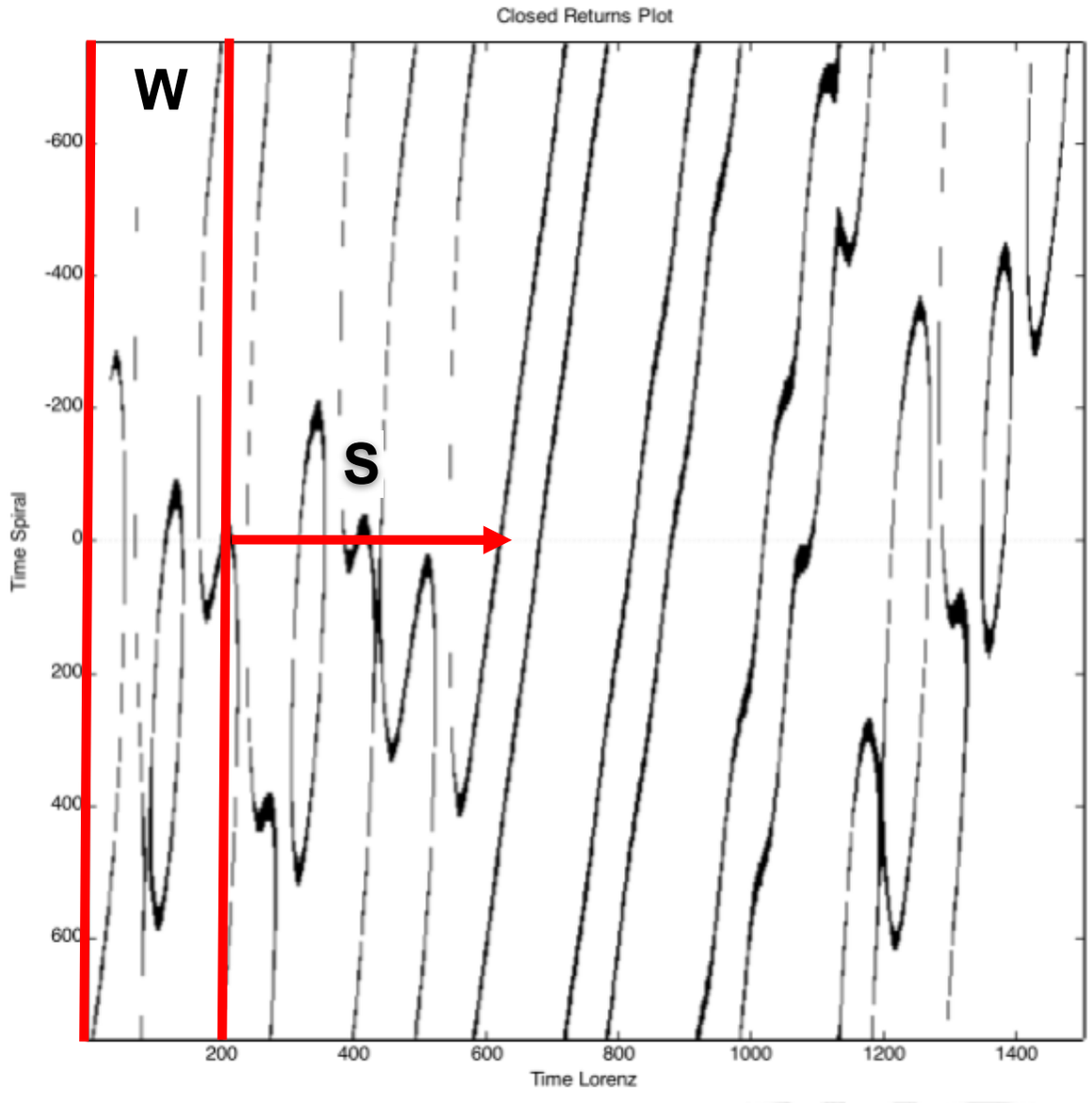
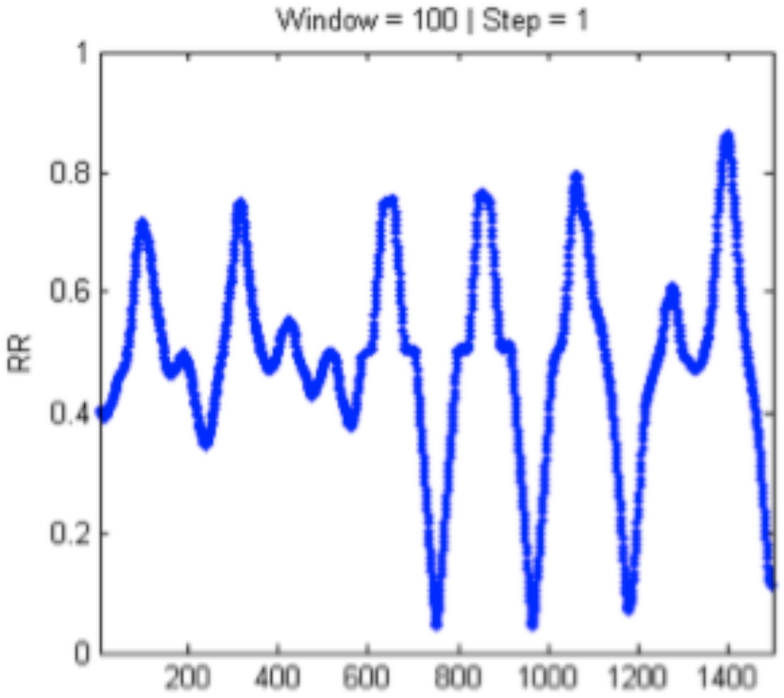
Louwerse, M. M., Dale, R., Bard, E. G., & Jeuniaux, P. (2012). Behavior matching in multimodal communication is synchronized. *Cognitive science*, *36*(8), 1404–26. doi:10.1111/j.1551-6709.2012.01269.x

Lagged RQA
move a window of size W in S steps across LOS



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RR
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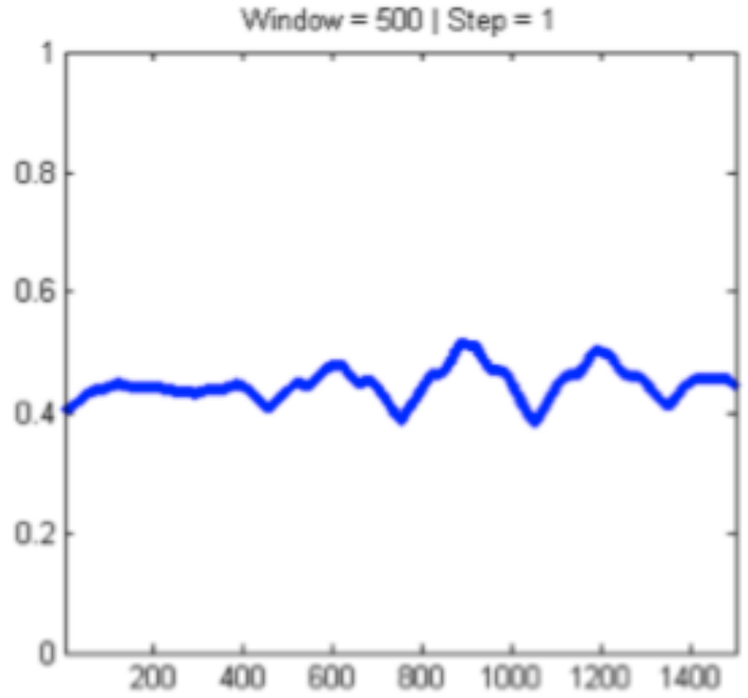
0.8
0.6
0.4
0.2
0
200
400
600
800
1000
1200
1400


Window = 500 | Step = 1
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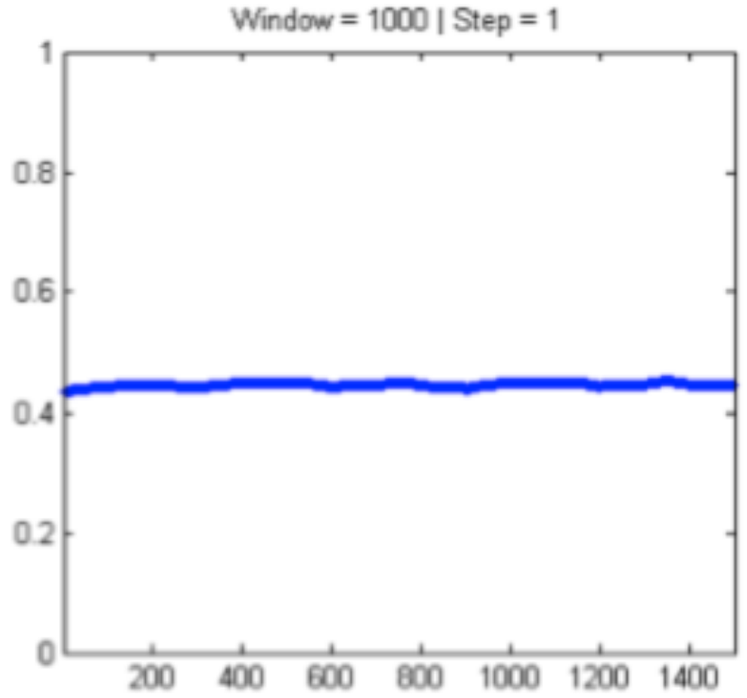
0.8
0.6
0.4
0.2
0
200
400
600
800
1000
1200
1400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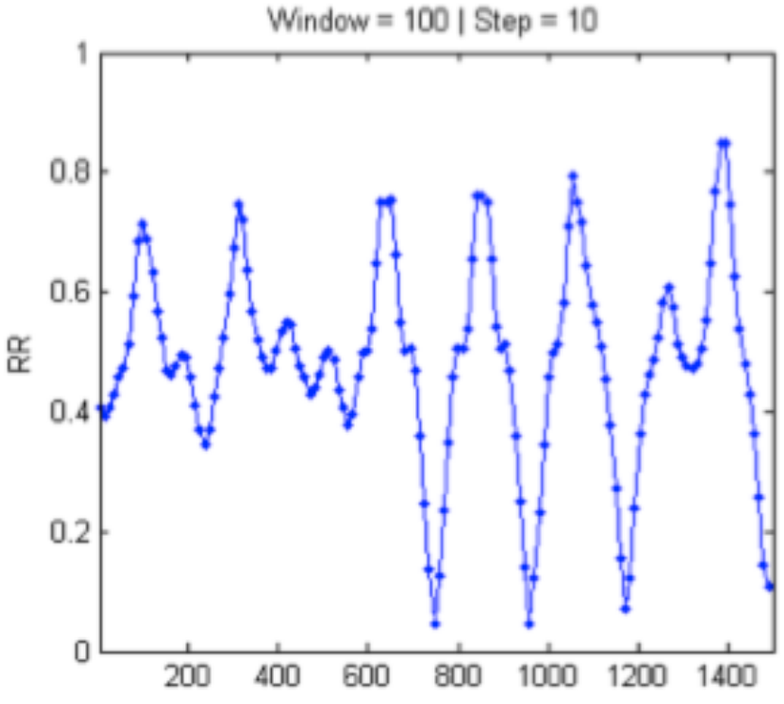
1
0.8
0.6
0.4
0.2
0
200
400
600
800
1000
1200
1400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RR
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0.8
0.6
0.4
0.2
0
200
400
600
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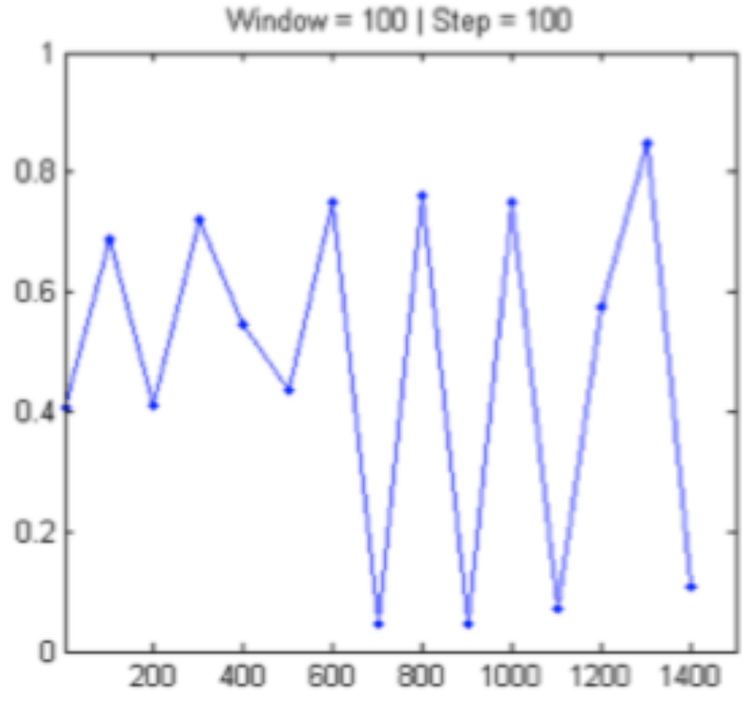
1000
1200
1400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1
0.8
0.6
0.4
0.2
0
200
400
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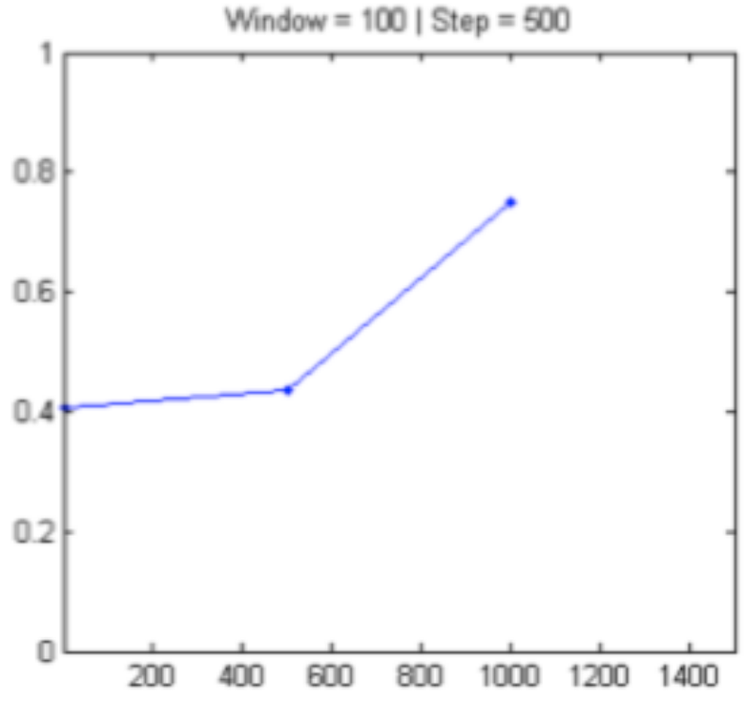
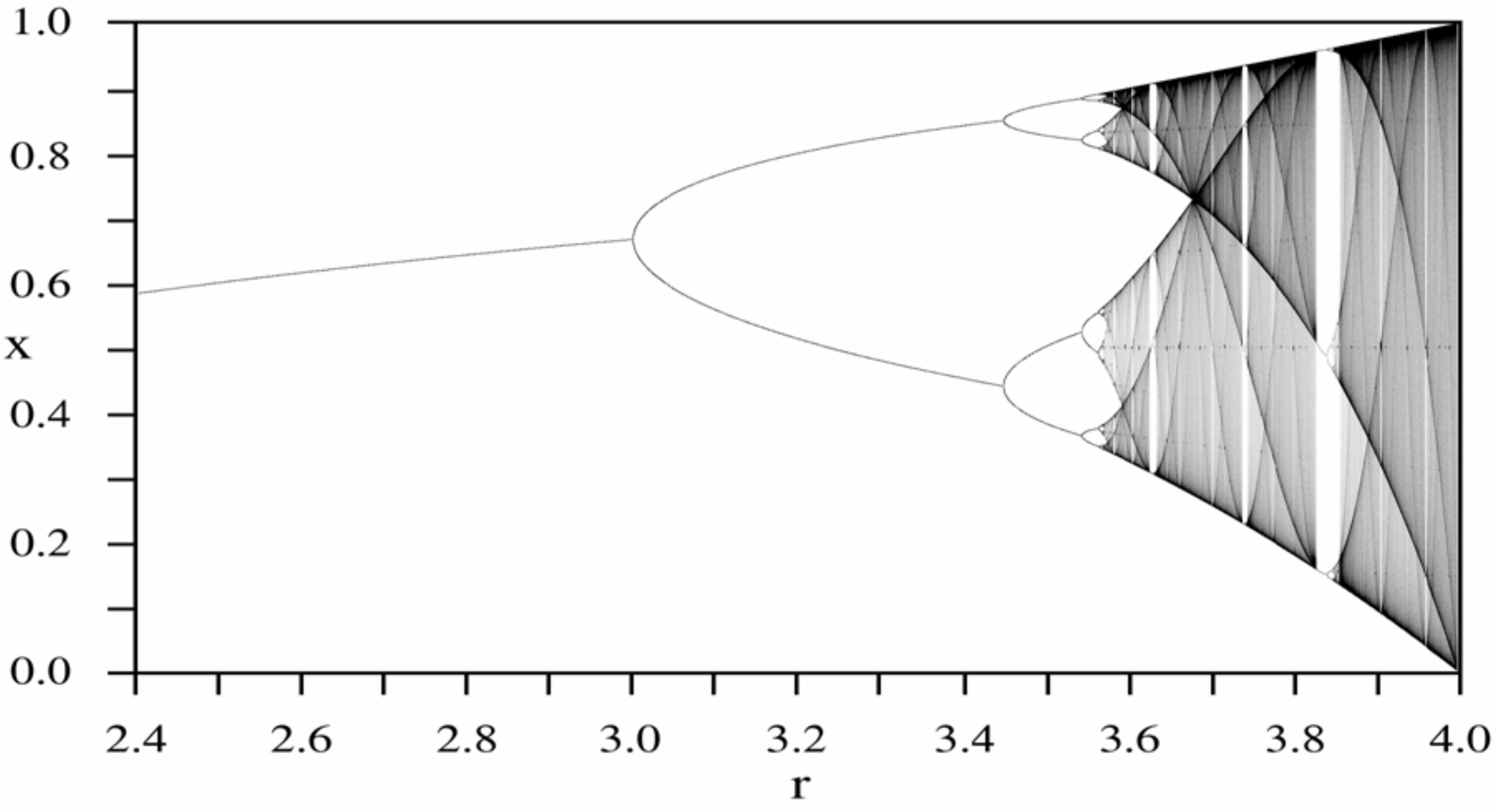
1400

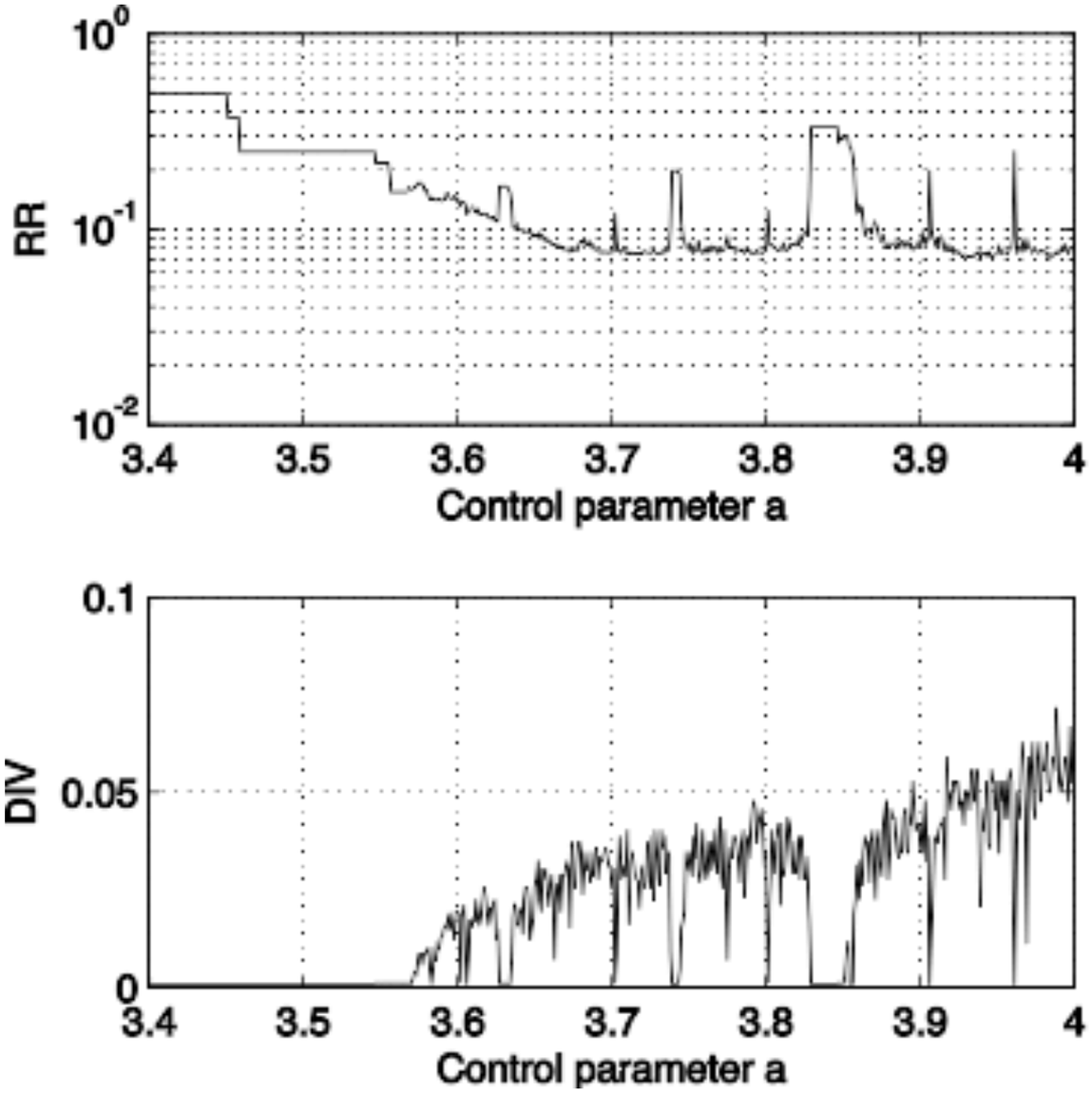

Window = 100 | Step = 500
1
0.8
0.6
0.4
0.2
0
200
400
600
800
1000
1200
1400

Logistic map – Transitions revealed by lagged RQA



Logistic map – Transitions revealed by lagged RQA

note: a = r in bifurcation diagram on previous slide



Marwan, N., Romano, M. C., Thiel, M., & Kurths, J. (2007). Recurrence plots for the analysis of complex systems. *Physics reports*, *438*(5-6), 237-329.

Further reading

- The paper by Marwan et al in Physics Reports tells you everything you wanted to know… and more.

- Paper by Rick Dale in psych. science gives a good explanation of CRQA on categorical data

Table 2
Comparison of RQA measures based on diagonal (*DET*, *L* and L_{max}) and vertical structures (*LAM*, *TT* and V_{max}) regarding periodic-chaos/chaos–periodic transitions (PC/CP), chaos–chaos transitions (band merging—BM and inner crisis—IC) and laminar states

Measure	PC/CP transitions	BM and IC	Laminar states
DET	Increases	—	—
L	Increases	—	—
L_{max}	Increases	—	—
LAM	Drops to zero	—	Increases
TT	Drops to zero	Increases	Increases
V_{max}	Drops to zero	Increases	Increases

Marwan, N., Romano, M. C., Thiel, M., & Kurths, J. (2007). Recurrence plots for the analysis of complex systems. *Physics reports*, *438*(5-6), 237-329.

Order Patterns Recurrence Plot

- Sort of “filter”: not recurrences of values, but order patterns

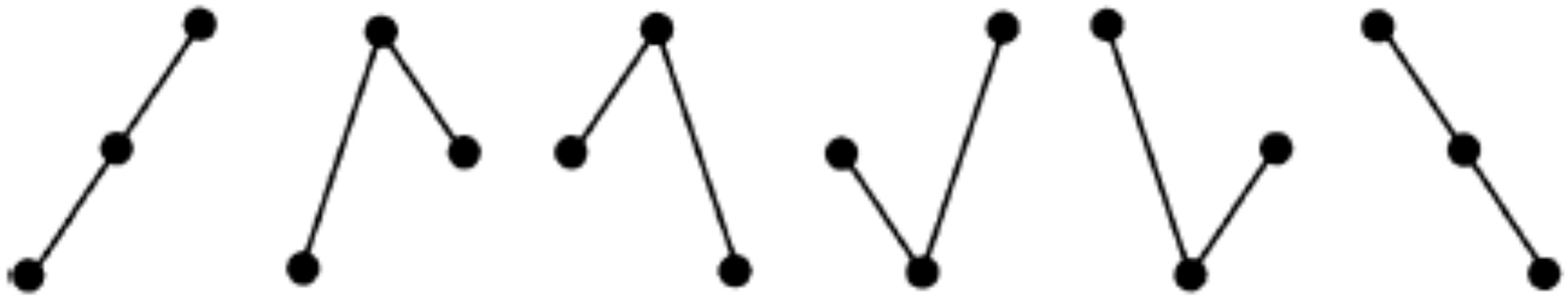
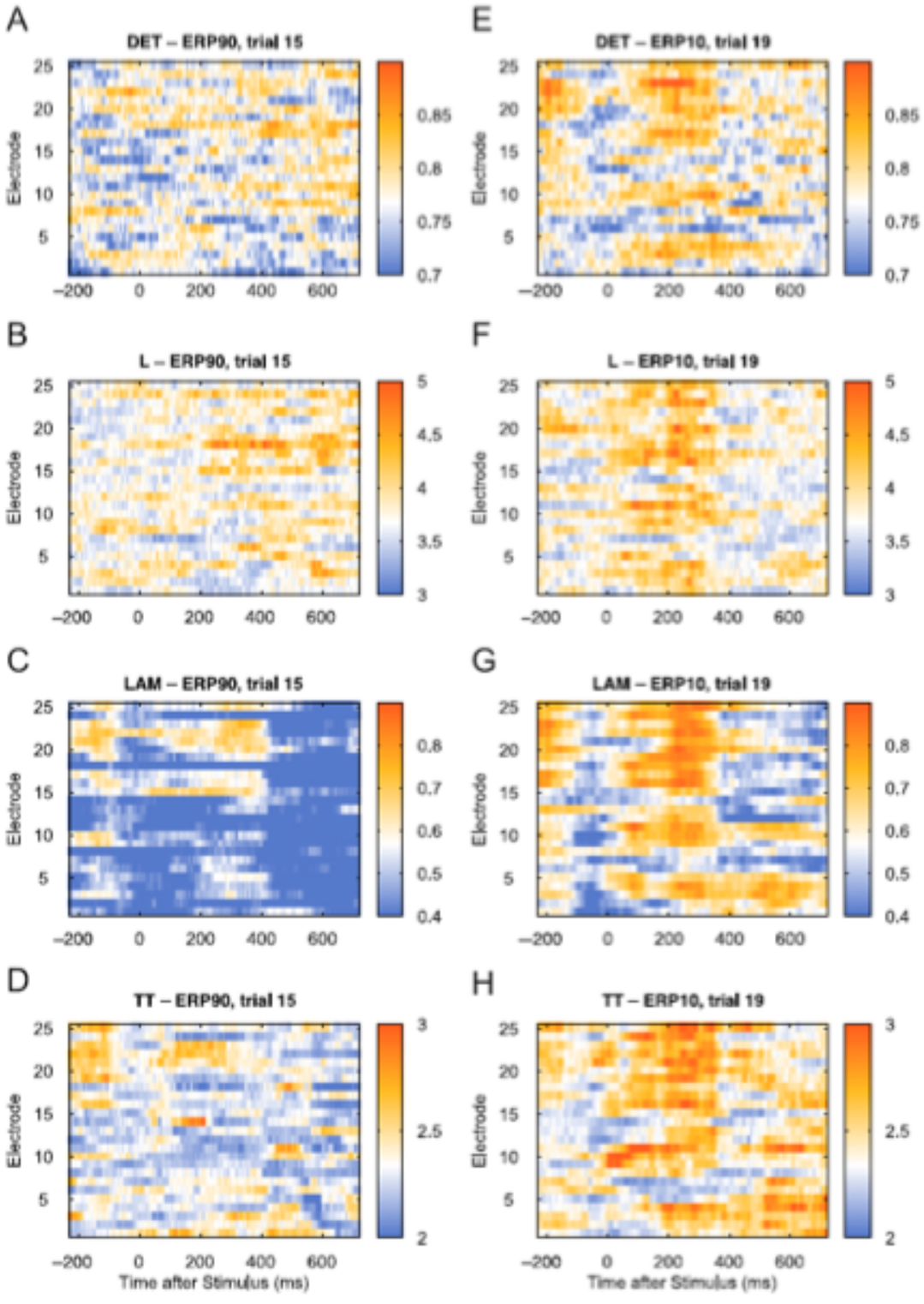


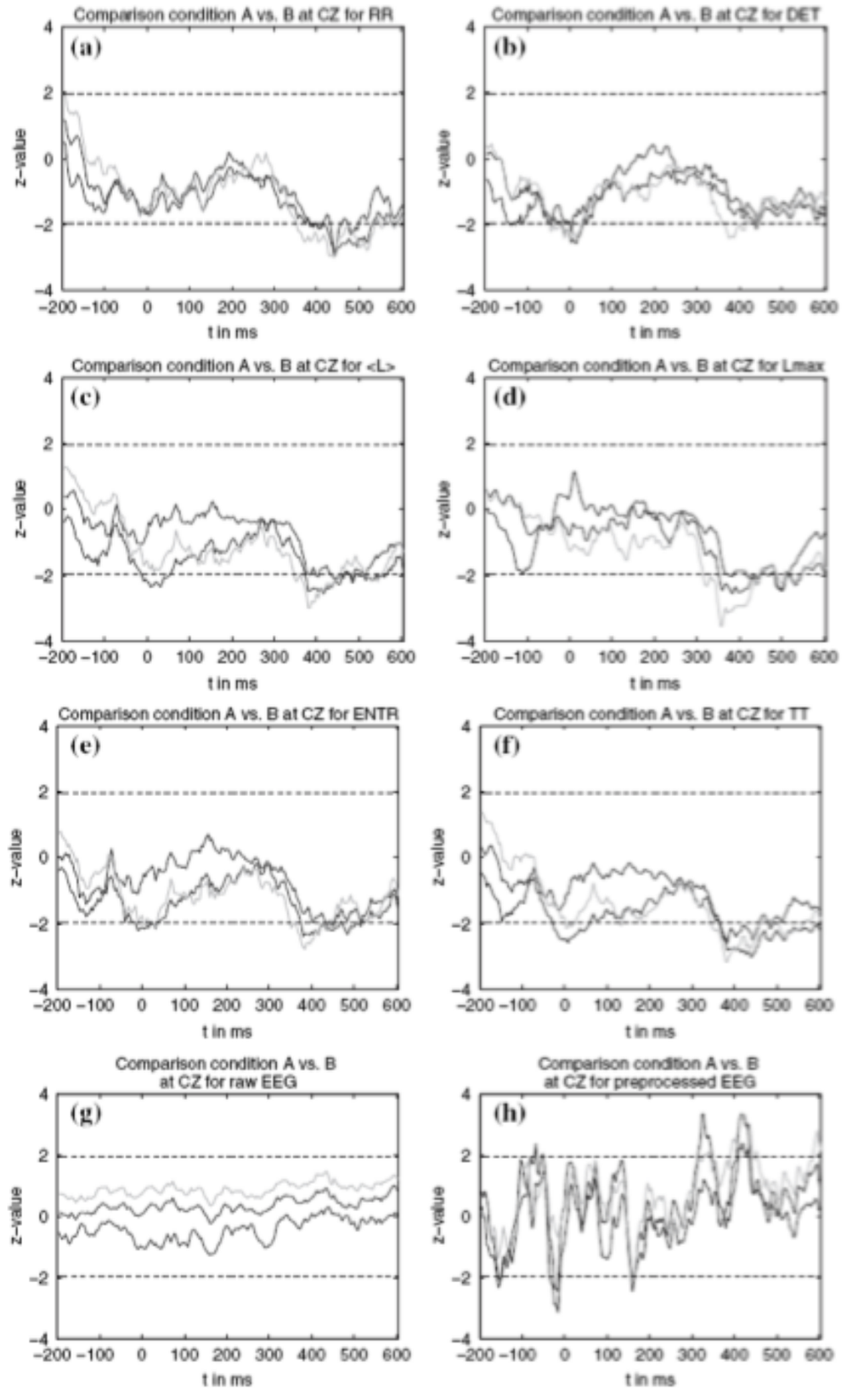
Fig. 2 Order patterns for dimension $d = 3$ (tied ranks $u_i = u_{i+\tau}$ are assumed to be rare)

Order patterns recurrence plots in the analysis of ERP data

Stefan Schinkel · Norbert Marwan · Jürgen Kurths

Schinkel, S., Marwan, N., & Kurths, J. (2007). Order patterns recurrence plots in the analysis of ERP data. *Cognitive neurodynamics*, *1*(4), 317-25. doi:10.1007/s11571-007-9023-z





Schinkel, S., Marwan, N., & Kurths, J. (2009). Brain signal analysis based on recurrences. *Journal of physiology, Paris*, *103*(6), 315-23. Elsevier Ltd. doi:10.1016/j.jphysparis.2009.05.007

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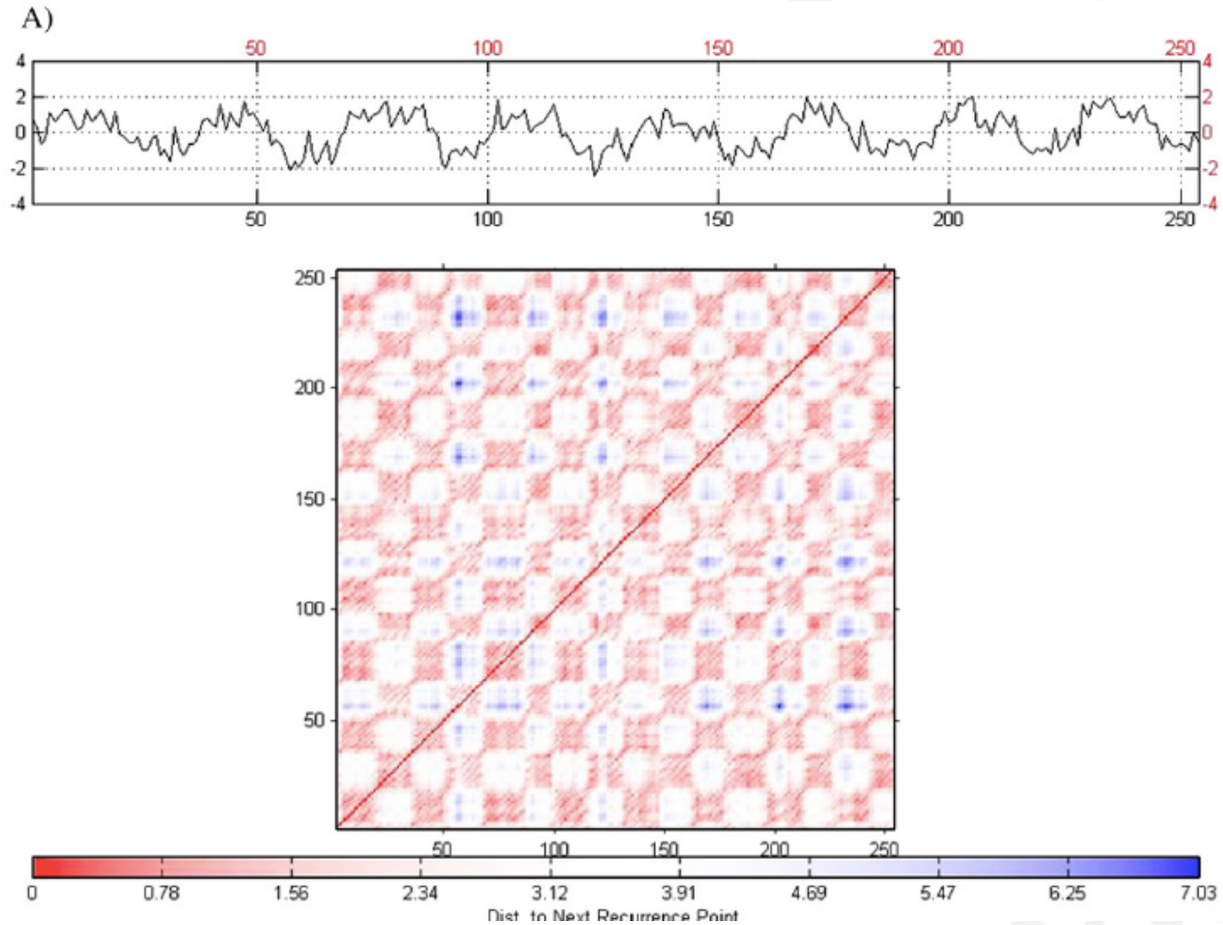
Model-free analysis of brain fMRI data by recurrence quantification

Marta Bianciardi,[a] Paolo Sirabella,[b] Gisela E. Hagberg,[a] Alessandro Giuliani,[c]
Joseph P. Zbilut,[d] and Alfredo Colosimo[b,*]



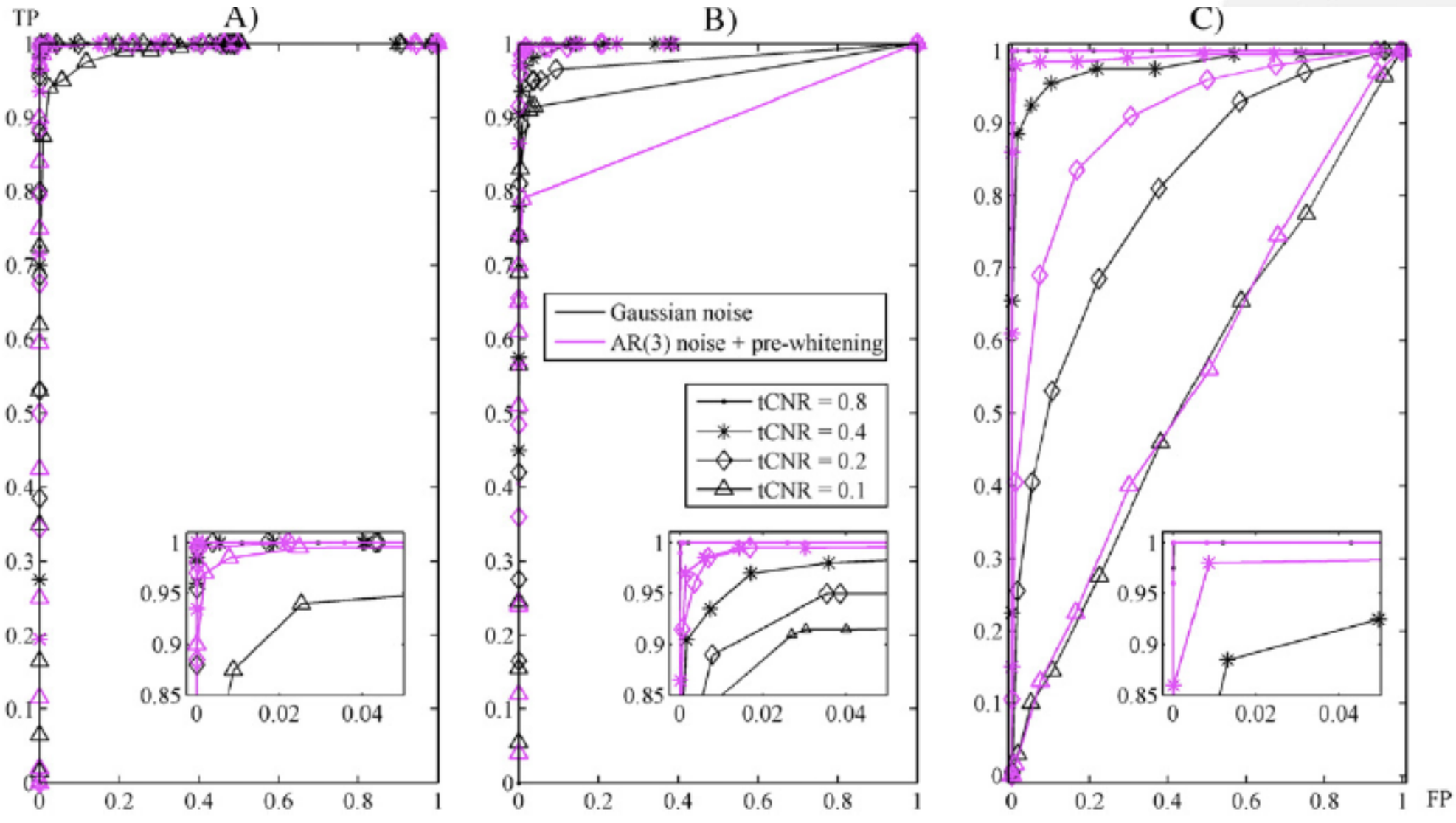
Comparison of RQA with GLM and P-ICA

Ability of three analyses to distinguish between noises in fMRI signal (ROC analysis)

GLM

P-ICA

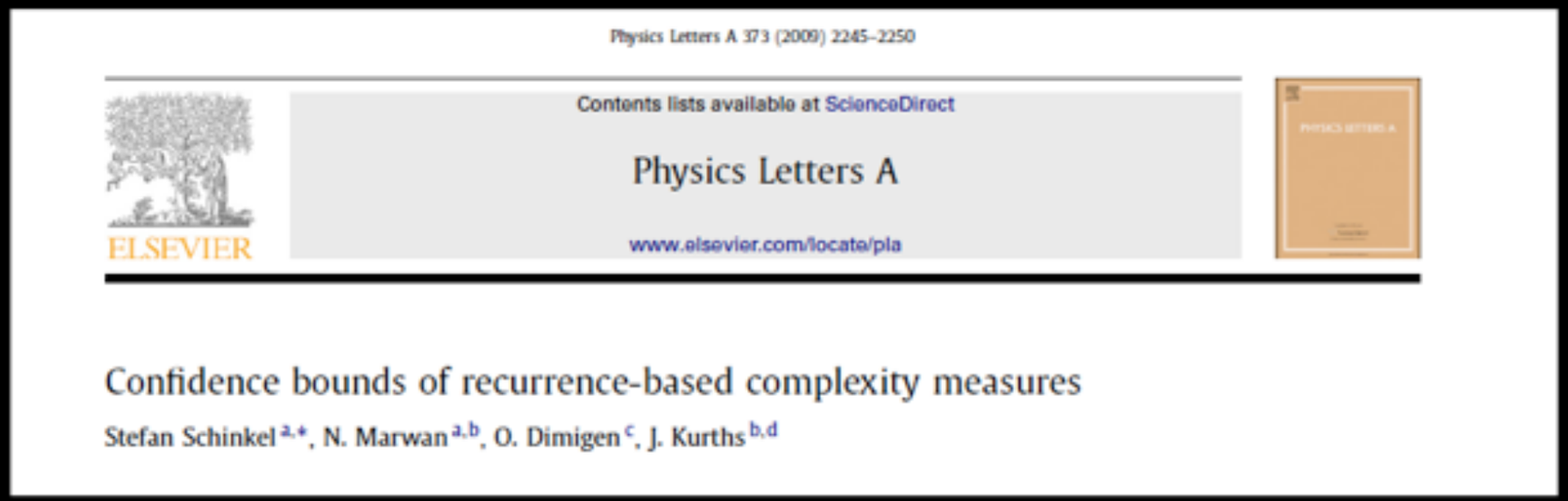
RQA



Bianciardi, M., Sirabella, P., Hagberg, G. E., Giuliani, A., Zbilut, J. P., & Colosimo, A. (2007). Model-free analysis of brain fMRI data by recurrence quantification. *NeuroImage*, *37*(2), 489-503. doi:10.1016/j.neuroimage.2007.05.025

Summary

RQA(™) - now comes with an errorbar



Physics Letters A 373 (2009) 2245–2250

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Confidence bounds of recurrence-based complexity measures

Stefan Schinkel [a,*], N. Marwan [a,b], O. Dimigen [c], J. Kurths [b,d]

http://www.agnld.uni-potsdam.de/~schinkel

Schinkel, S., Marwan, N., Dimigen, O., & Kurths, J. (2009). Confidence bounds of recurrence-based complexity measures. *Physics Letters A*, *373*(26), 2245-2250. Elsevier B.V. doi:10.1016/j.physleta.2009.04.045

Next session

- Complex Networks

- I am my connectome TED video

- Borsboom D, Cramer AOJ, Schmittmann VD, Epskamp S, Waldorp LJ (2011) The Small World of Psychopathology. *PLoS ONE 6(11)*: e27407. doi:10.1371/journal.pone.0027407 http://www.plosone.org/article/info:doi/10.1371/journal.pone.0027407