

Scaling Phenomena in Complex Adaptive Systems with Internal State Dynamics

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<https://www.ru.nl/bsi/research/group-pages/complex-systems-group/>

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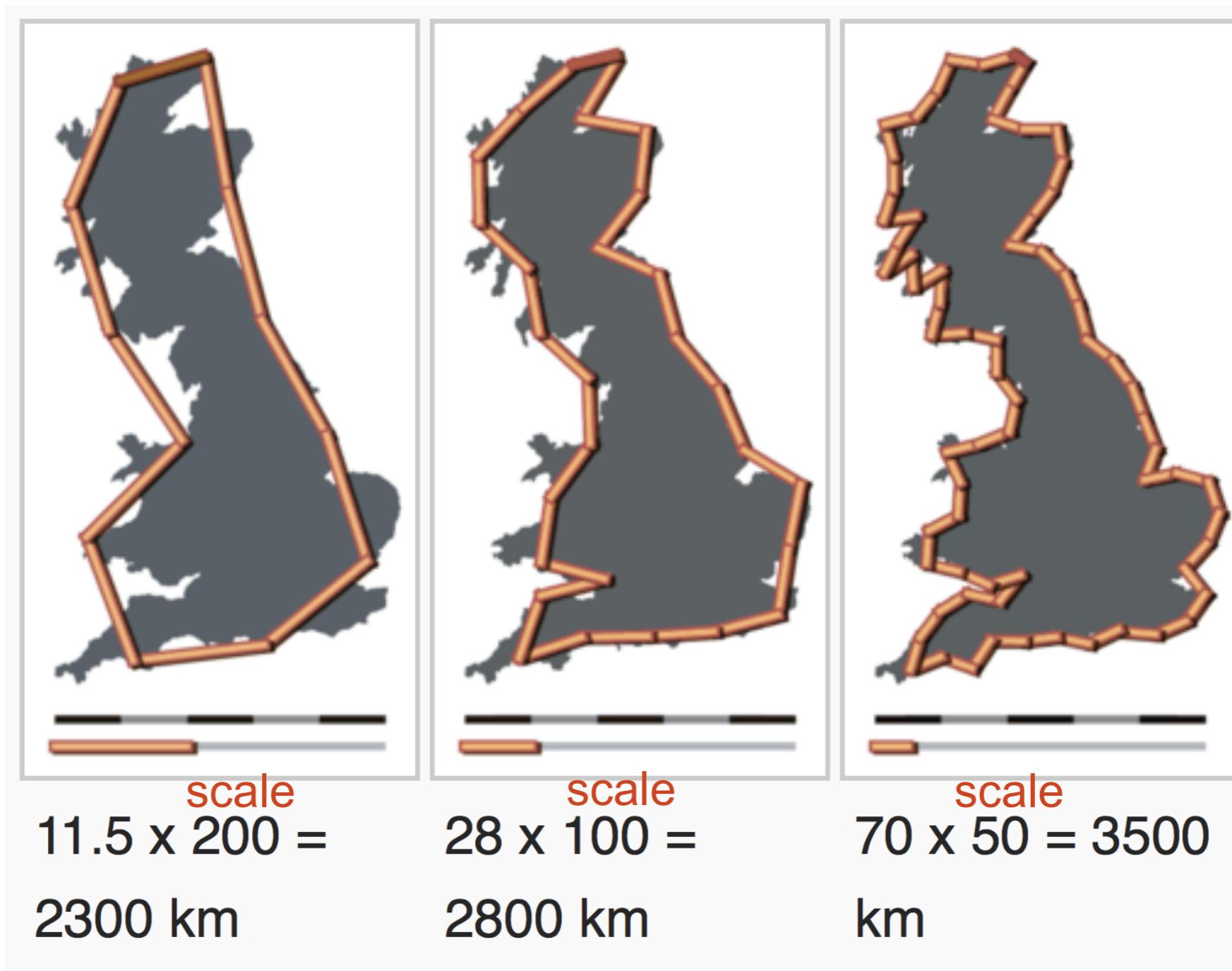


Scaling phenomena



How long is the coast of Great-Britain?

Scaling phenomena



Length systematically depends on the size of the measurement stick you use!

Scaling phenomena



“scaling of bulk with size”

(Theiler, 1990)

The formal answer to the question is:

“There is no characteristic scale at which the length of the coast of GB can be expressed”

Mandelbrot, B. B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science*, 156(3775), 636–8.

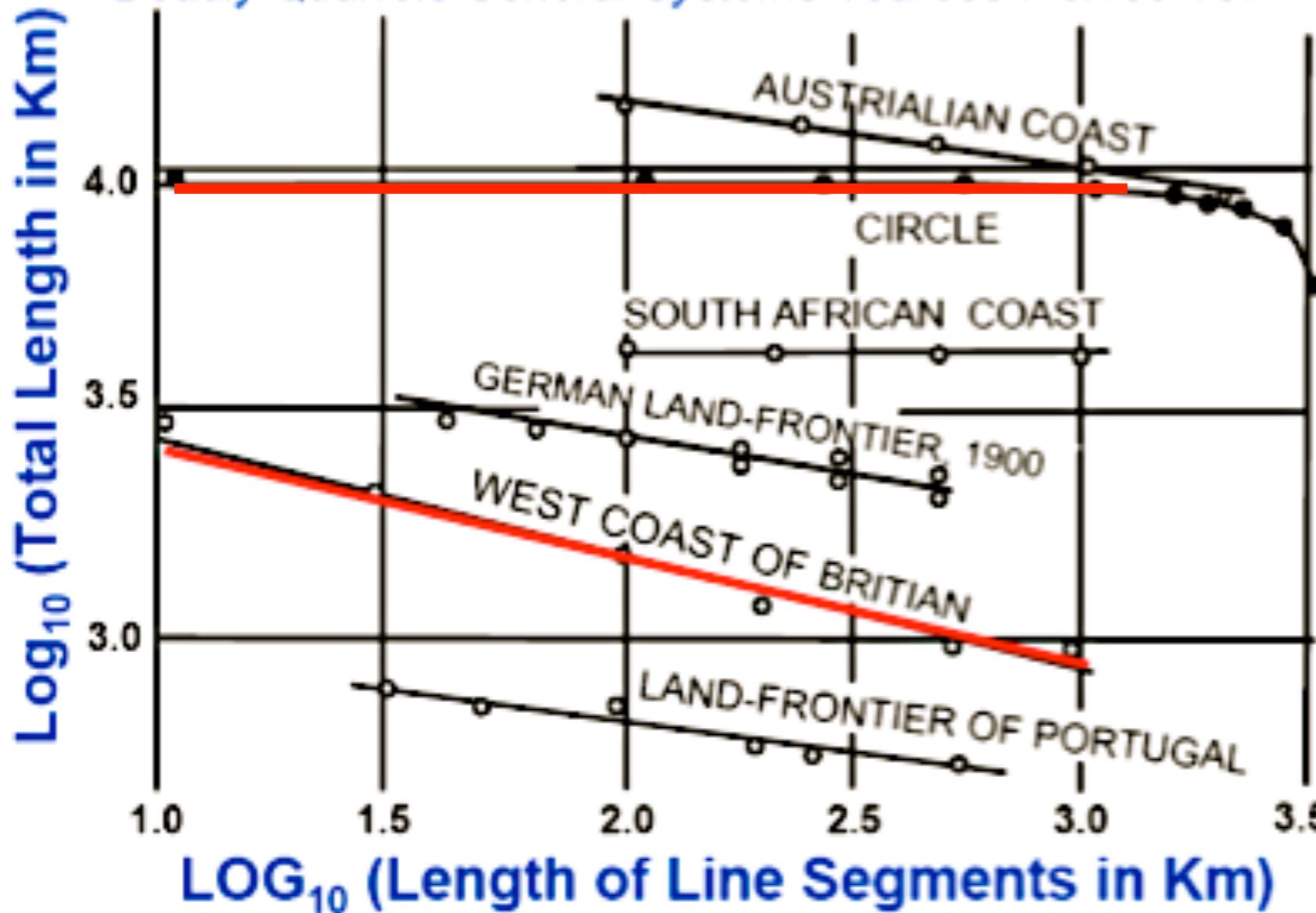
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How Long is the Coastline of Britain?

Richardson 1961 *The problem of contiguity: An Appendix to Statistics of Deadly Quarrels* General Systems Yearbook 6:139-187



Scale invariance...

A power law scaling relation (**LOG scale**):

There is no characteristic length, just an indication of **complexity**

Mandelbrot, B. B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science*, 156(3775), 636–8.

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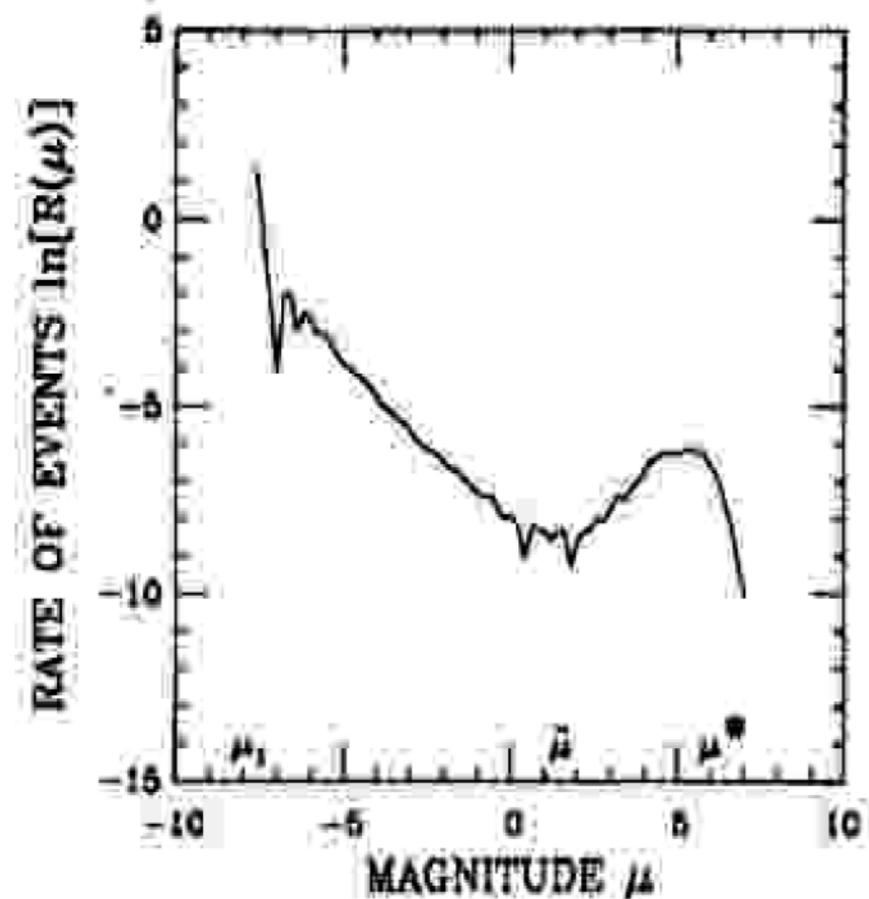
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Scaling phenomena

Scaling relations can emerge with all kinds of observables
They inform about properties of the process / system under scrutiny

Earthquakes (Richter-Law)
frequency of occurrence ~ magnitude



Distribution of mass in the Universe
resolution ~ density

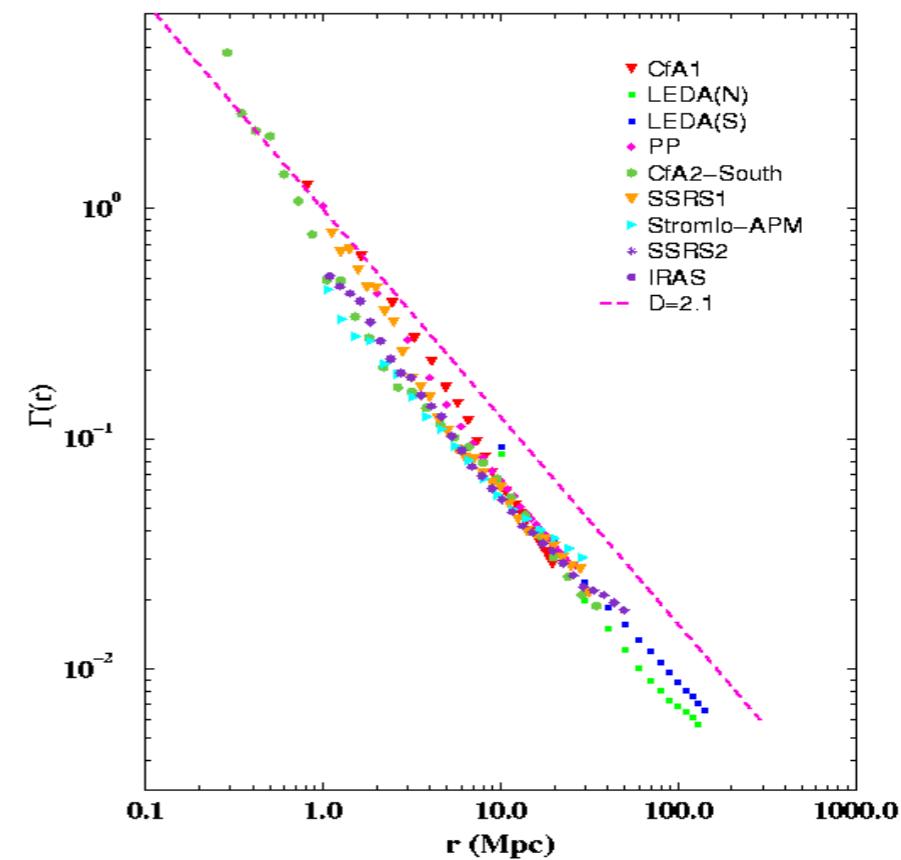
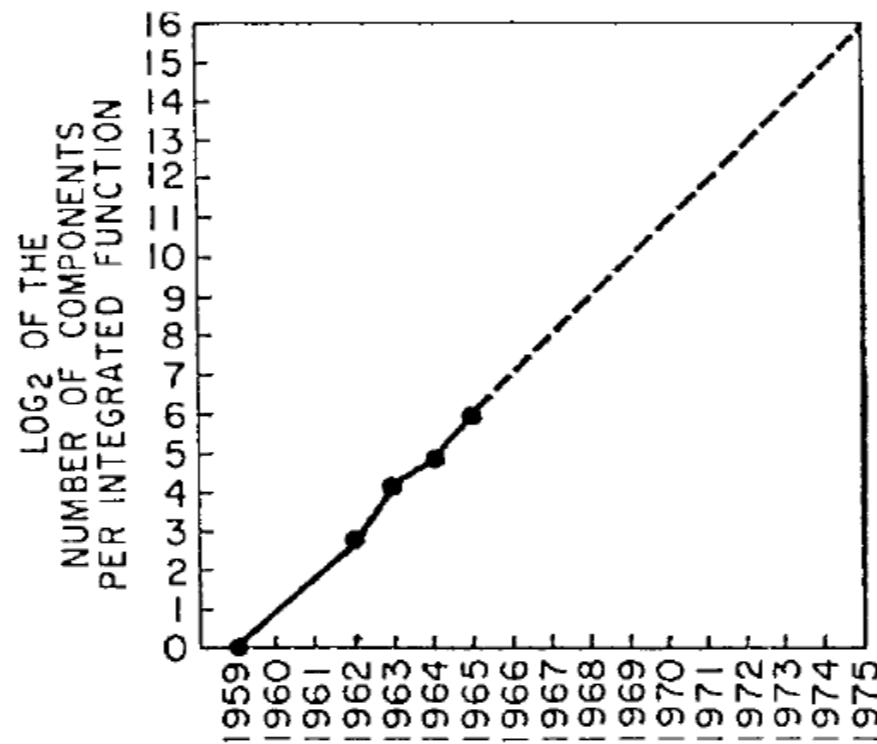


Figure 13: frequency distribution of the slip events (earthquakes) of magnitude μ taken from [53]. Notice the large bump that corresponds to an excess of events of high magnitude.

Scaling & Growth

Moore's Law:

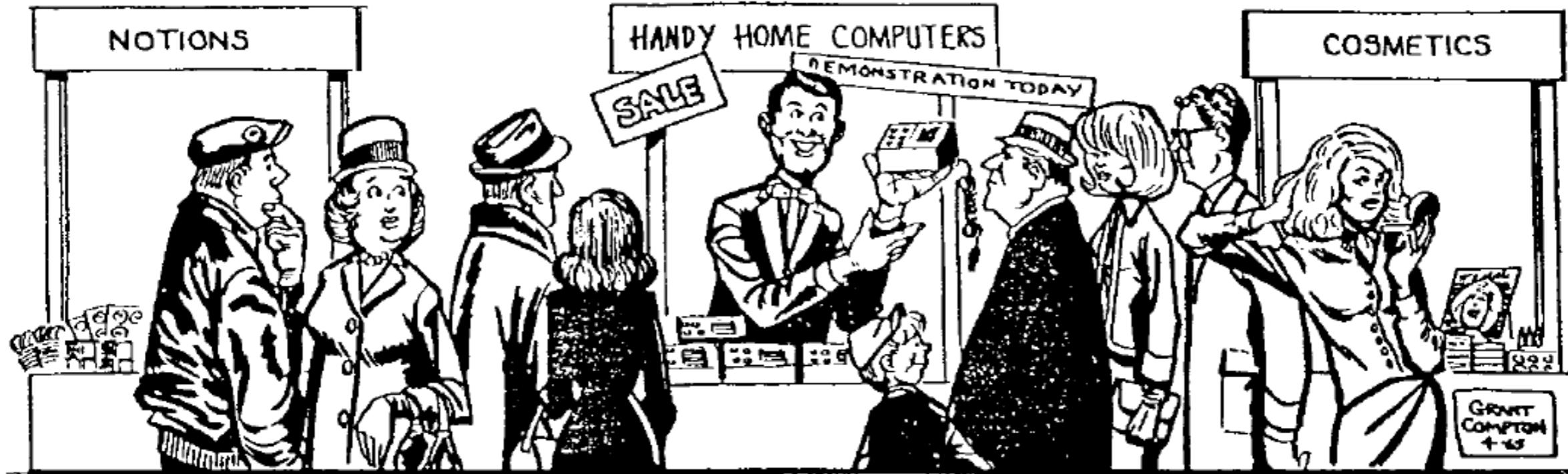
Predicted if speed of innovations in “cramming more components onto integrated circuits” kept up ...



Moore's Law:

... we would soon be buying computers at the local market ...

which apparently was a preposterous idea



Moore, Gordon E. (1965). "Cramming more components onto integrated circuits"

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VANAF WOENSDAG
26-02

**ONZE
AANBIEDINGEN**

Tablet PC cover



GSM AT-B26D
 - simlockvrije telefoon
 - GSM 900/1800 MHz
 - Dual-Sim
 - micro-SD-lezer
 - afmetingen: hoogte: 12.4 cm,
 breedte: 6.4 cm, dikte: 1.1 cm
 - gewicht: 140 g (incl. accu)
 - accu: 2100 mAh
 - kleurenscherm
 - 0.3MP-camera
 - zaklampfunctie
 - micro-USB

OP=OP
Per stuk
49.99* 2 Jaar GARANTIE



10" Tablet MD98516



16 GB GEHEUGEN

Per stuk

179.00* 3 Jaar GARANTIE

ALDI TALK

Wireless speaker adapter



Bevestigingsset

Ophangset voor lijsten, met o.a.
diverse muurhaken, schroeven
en spijkers.



Hortensia

Veel bloemknoppen en
kleurzijdende of open bloemen.
Blauw, roze/rood of wit.



Per set

2.99*

Per stuk

4.79*

Per stuk

14.99*

3 Jaar GARANTIE

ALDI

Basis voor soep
Tomaat, kip of rundvlees met groenten. 0.485 l
0.485 l 1.29* 2.861

Aperitiefbiscuits
Ham/kaassoesjes, kaaswafelbolletjes of Gouda kaasbiscuits. 70-125 g
70-125 g 0.99* 7.80-14.14/kg

DROPHAKERS HONING DROP
250 g 1.89* 3.36/kg

DROPHAKERS MUNTEN DROP
250 g 0.99* 3.36/kg

Munt- of honingdrop
250 g 0.99* 3.36/kg

Mini-stroopwafels
Bereid met echte roomboter. 300 g
300 g 1.89* 3.36/kg

Chips patatje kapsalon of hete kip
250 g 0.99* 3.36/kg

Smulgerechten
2-in-1 mix. Kebab met yoghurt-knoflooksaus gyros met paprikasaus of kipnuggets met BBQ-saus. Per pak

Kruidvat

Goed gekleed

DAMESJASJE
M t/m XL



true SPIRIT

Perfect haar

NIVEA HAARVERZORGING OF STYLING



voordeel
magazine

Stralende lippen

MAX FACTOR
LIPSTICK, -GLOSS
OF -LINER

ALLES VOOR
€ 10.00
PER STUK



Lekker schoon

ROBOTSTOFZUIGER



Dirt Devil

129.99 79.99

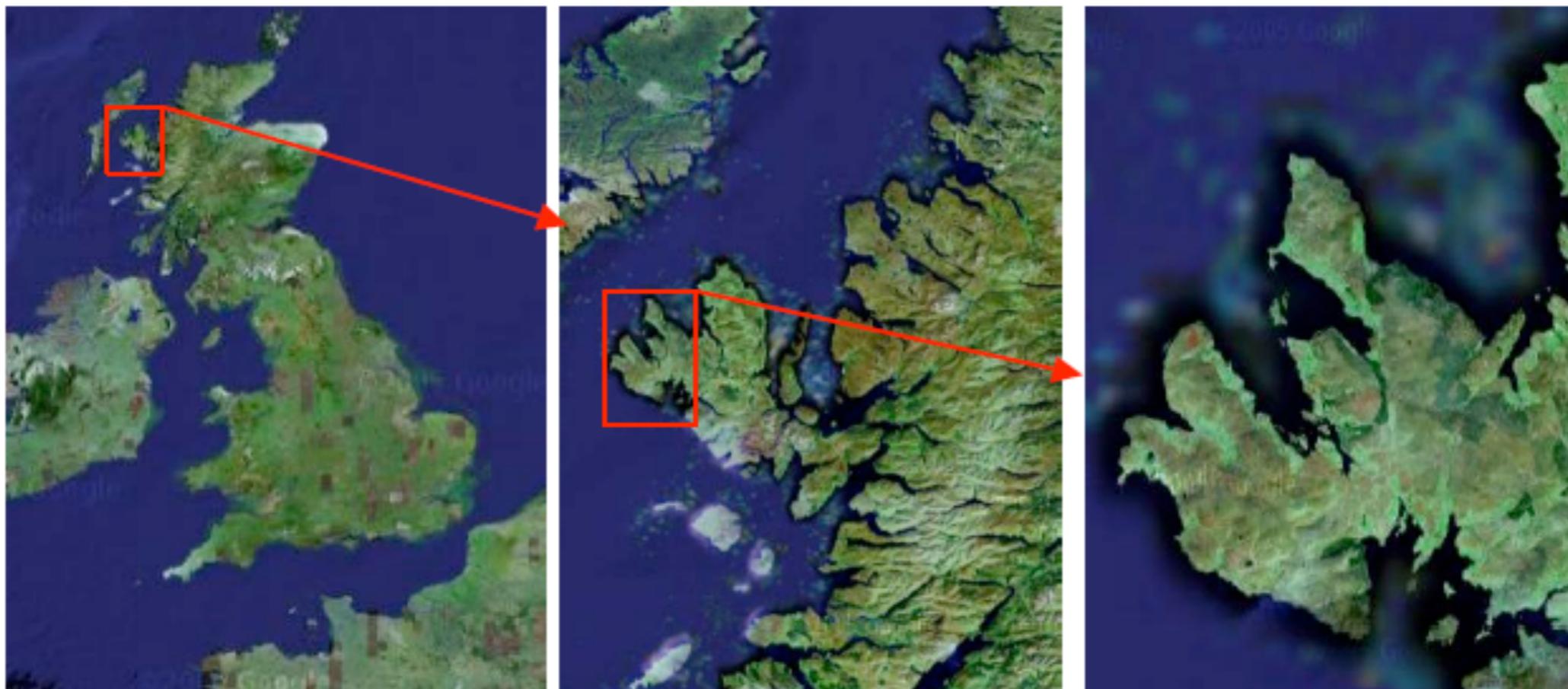
STEEDS VERRASSEND, ALTIJD VOORDELIG!

Geldig van dinsdag 25 februari t/m zondag 9 maart 2014

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What is scaling? Self-similarity & Self-affinity



Object looks roughly the same on all scales = (Statistical) **self-similarity** (“zoom similarity”)
(Statistical) self-similarity is observed after affine transformation = **self-affinity** (“warp similarity”)

Degree of invariance across scales = Dependencies/regularities/correlations across scales

aka: “Nested scales”

How to describe scaling relations: Calculate a “fractional” dimension, e.g. box-counting dimension

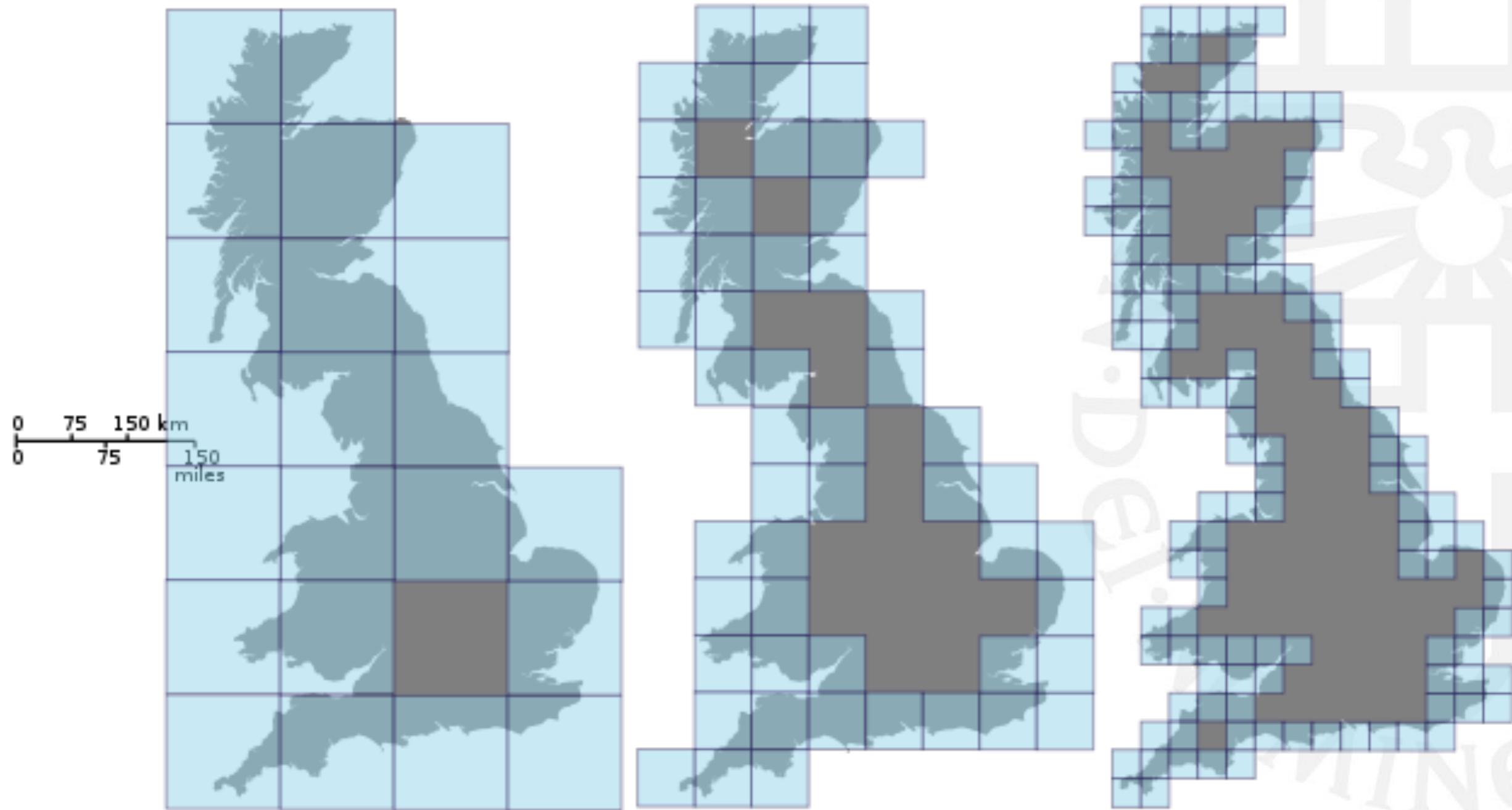


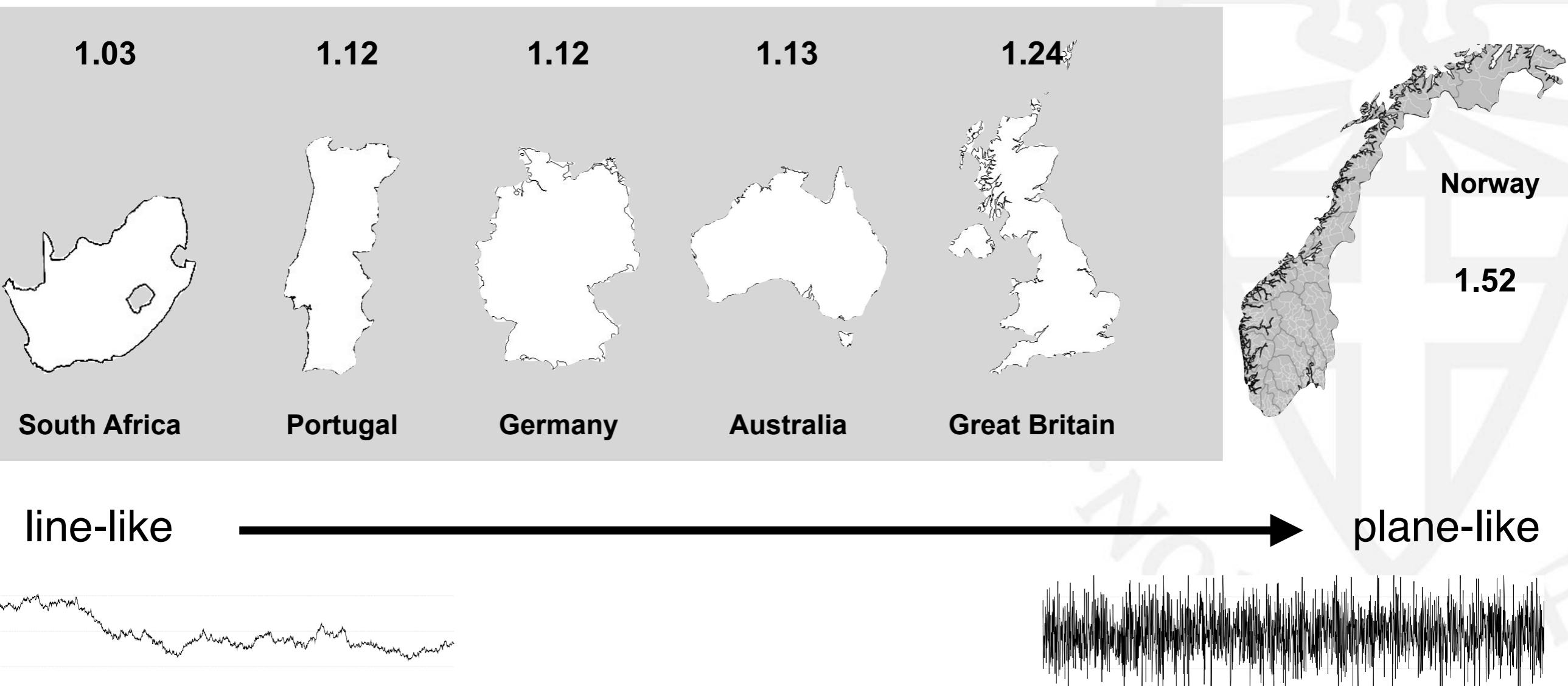
Image by Prokofiev - Own work, CC BY-SA 3.0,
<https://commons.wikimedia.org/w/index.php?curid=12042116>

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Scaling phenomena

Fractional dimension = “spill over” into next dimension
Associated to Processes & Properties



Scaling phenomena

“Optimised” packing/filling

Packing Cubes or Spheres and Wrapping Blankets:

2D ~ 3D spatial scaling relations in nature:

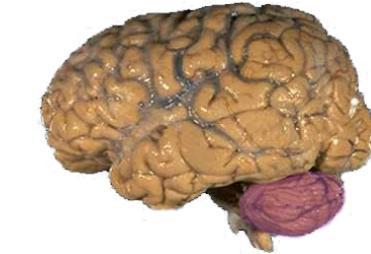
Cauliflower fractal dimension = 2.33



Surface of human lungs: 2.97

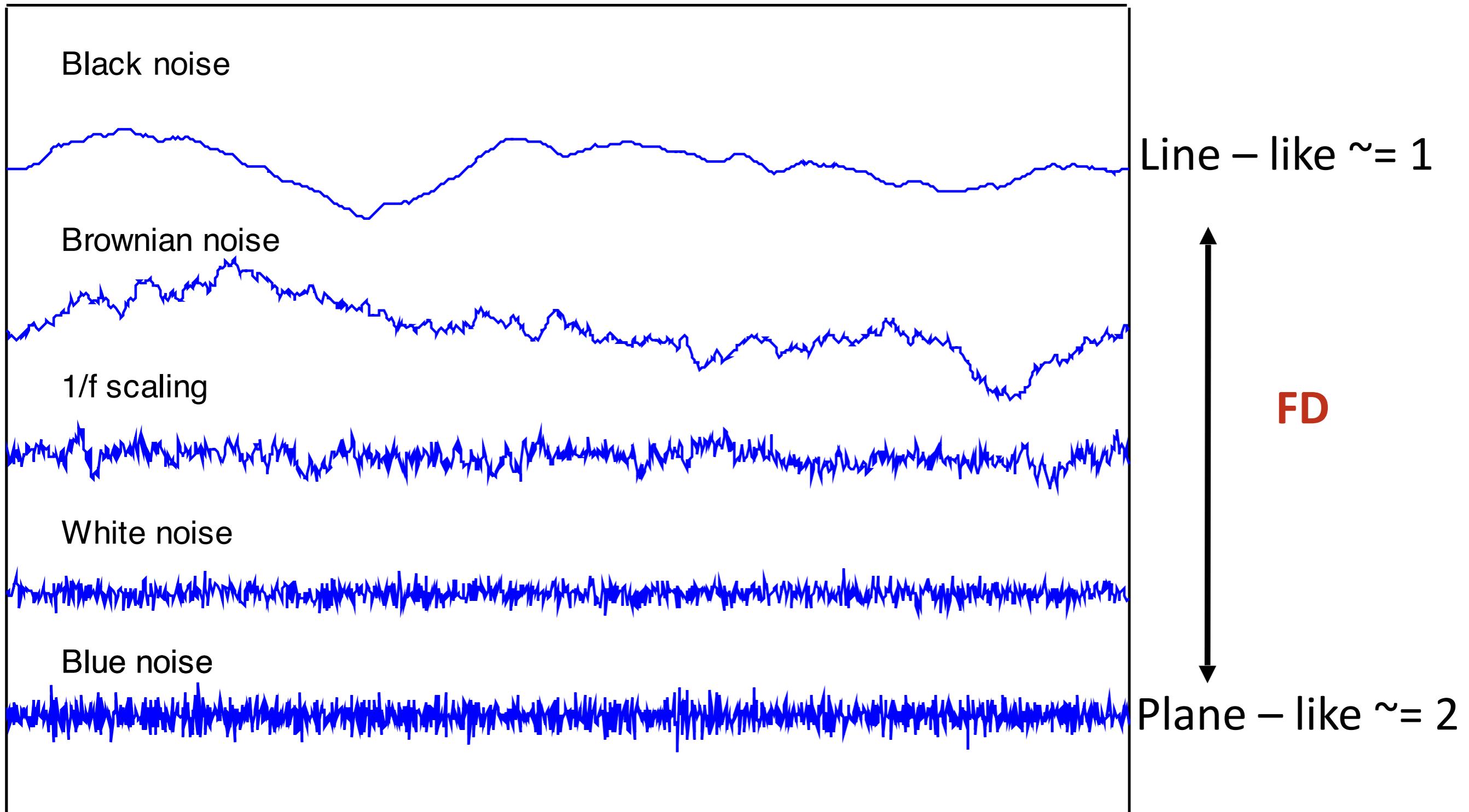


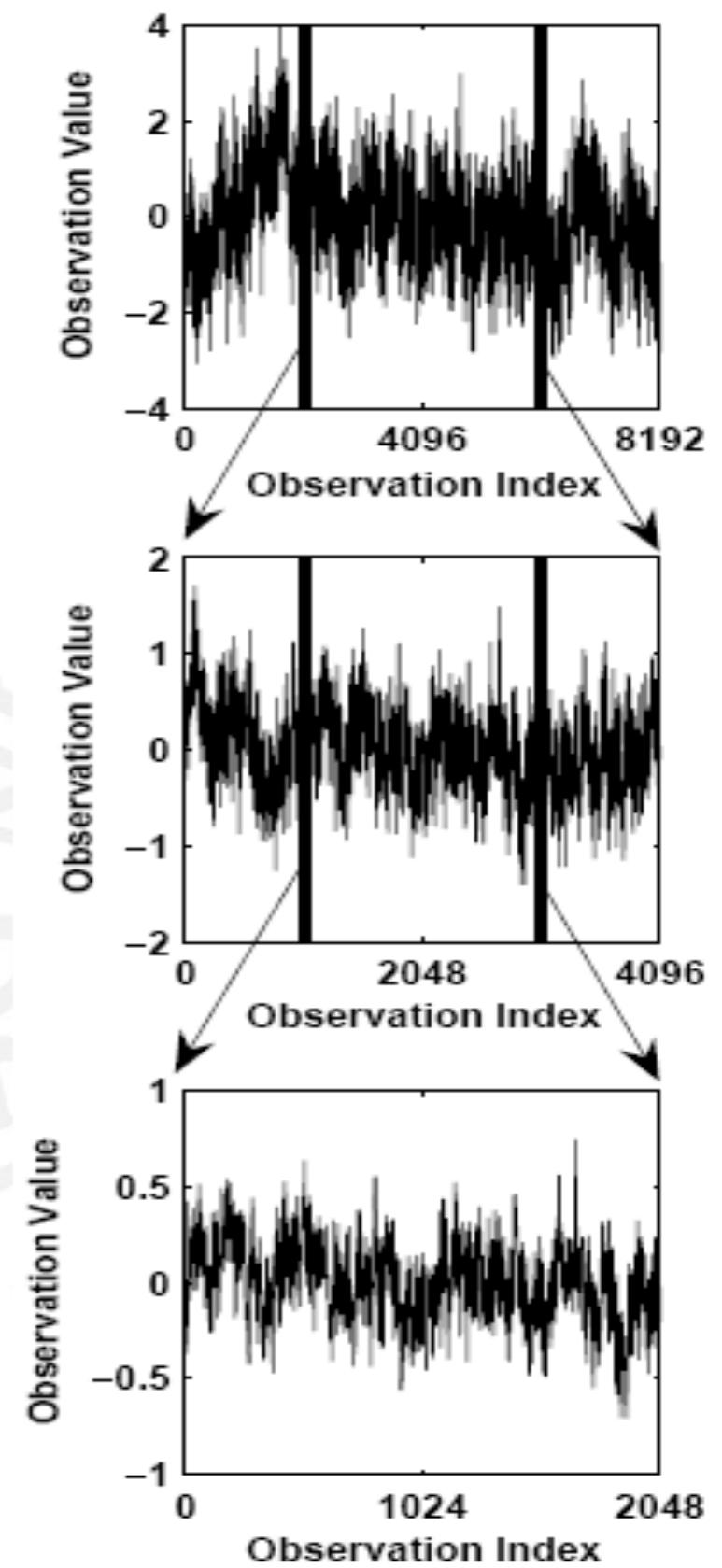
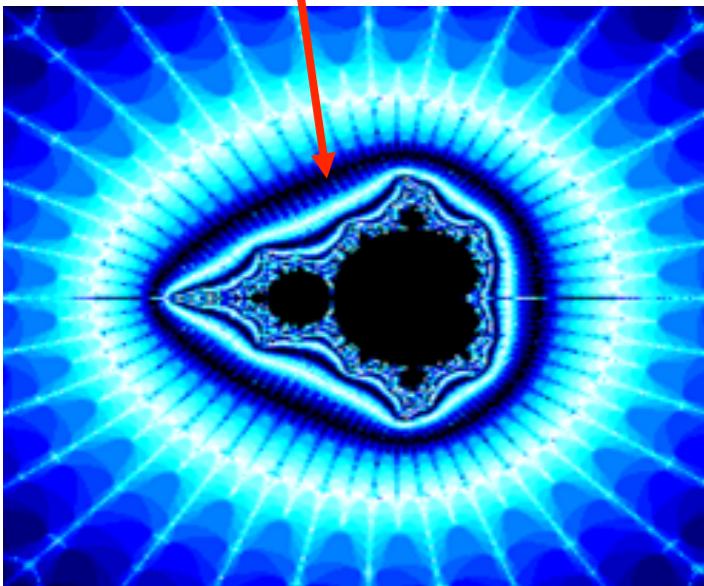
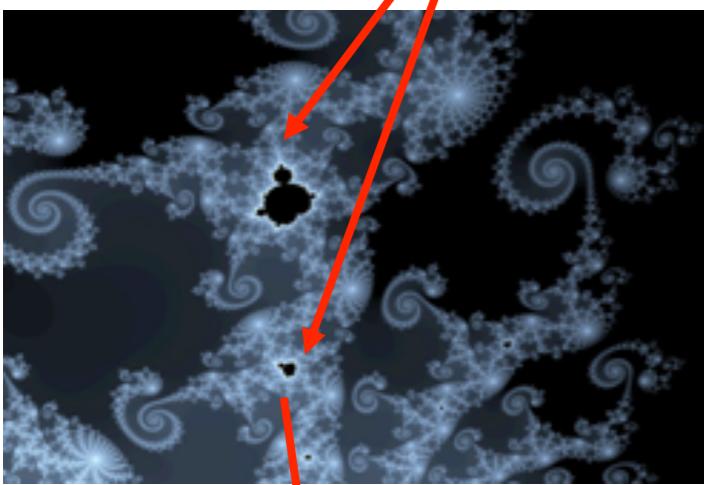
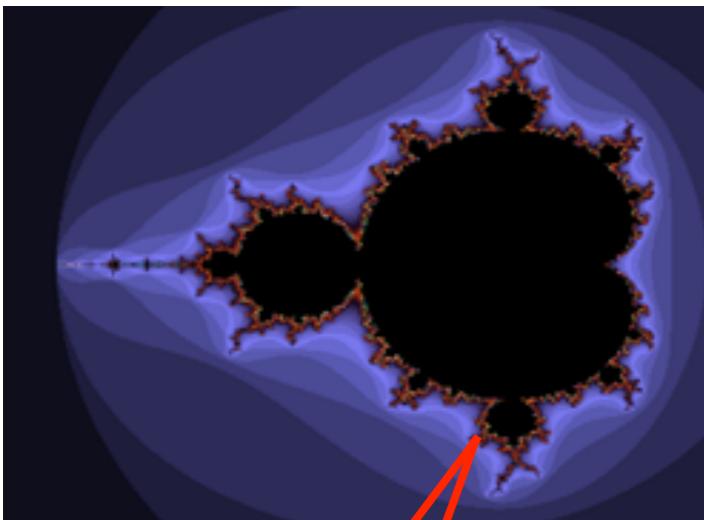
Surface of human brain: 2.79



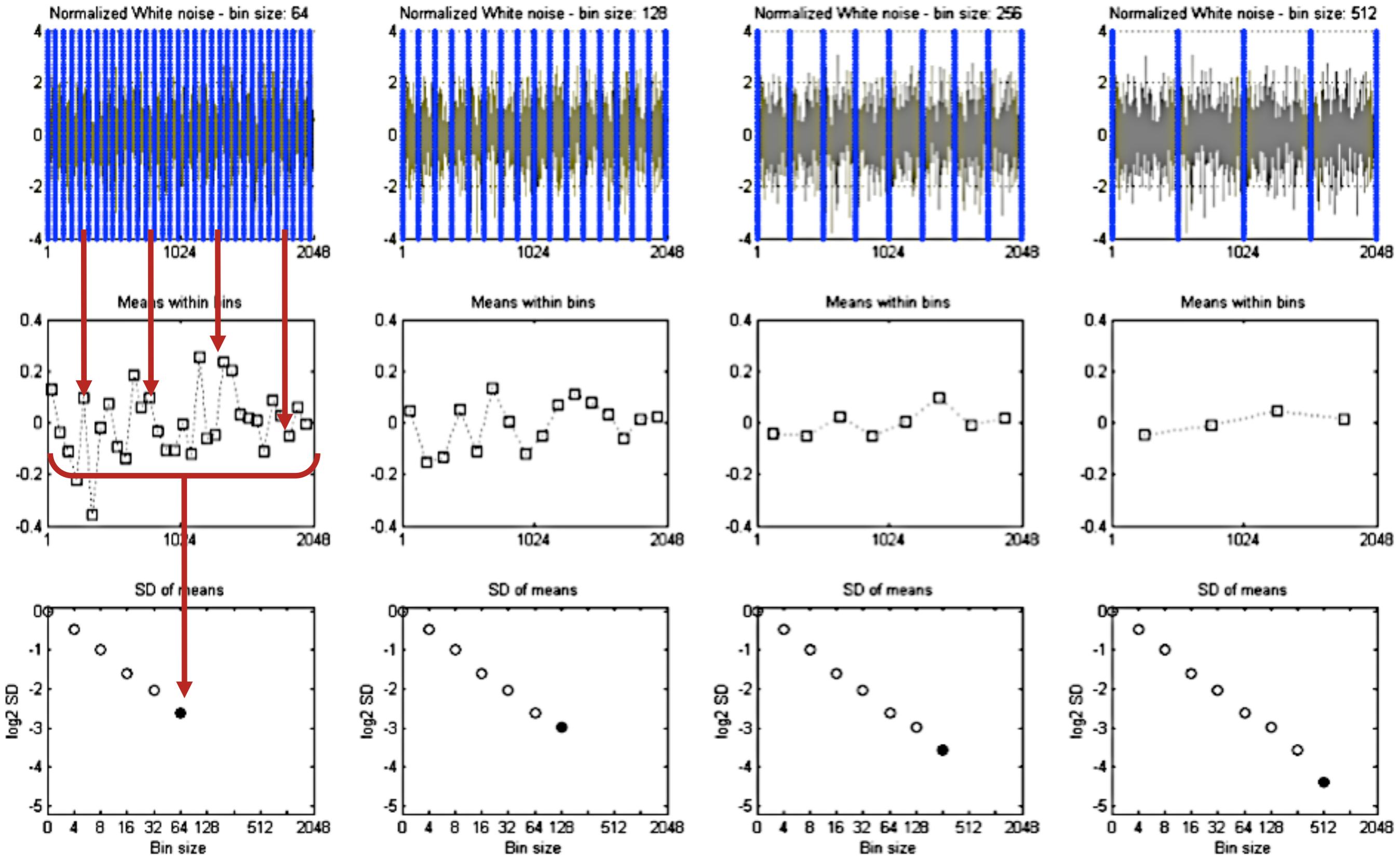
WHY OPTIMISED SURFACE AREA
and not VOLUME?

Scaling exponents reveal properties of data generating processes

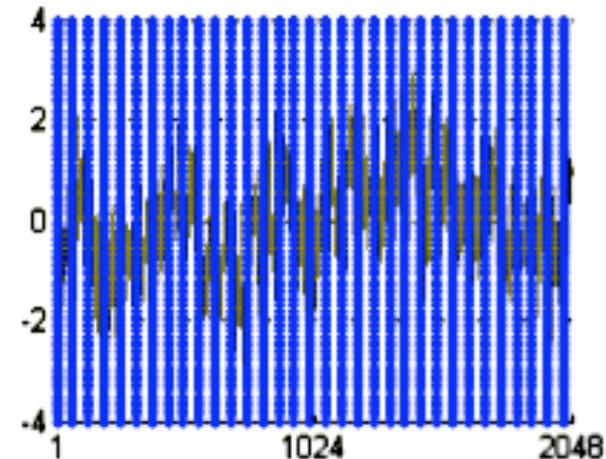




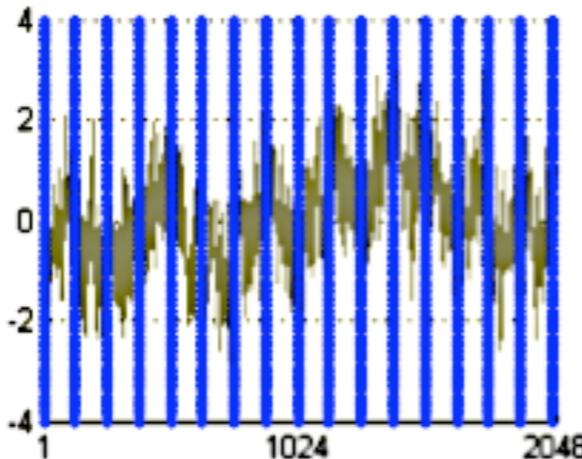
aka: “Fractal scaling”



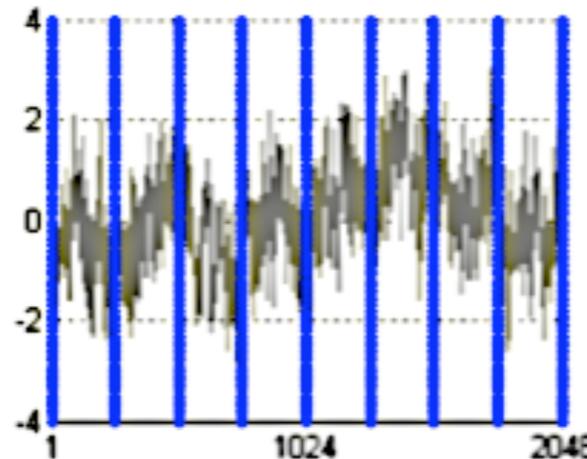
Normalized Pink noise - bin size: 64



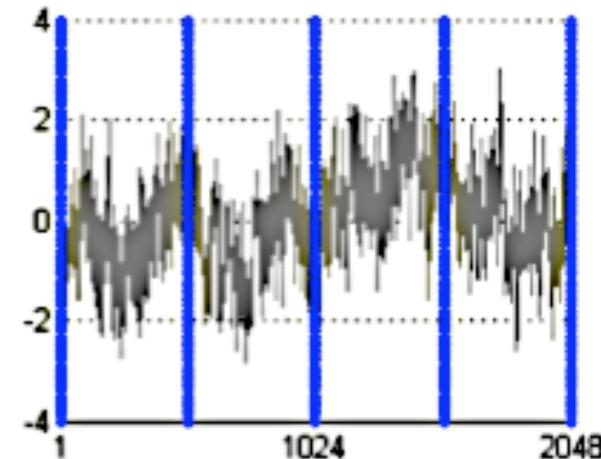
Normalized Pink noise - bin size: 128



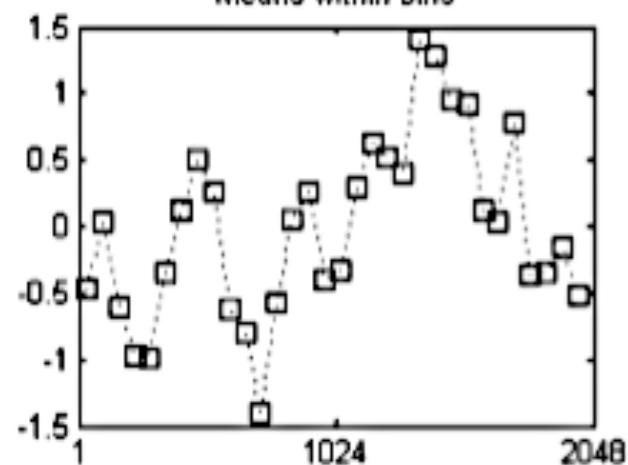
Normalized Pink noise - bin size: 256



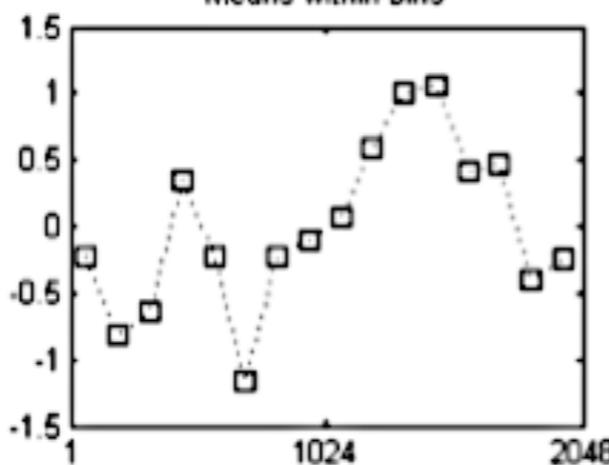
Normalized Pink noise - bin size: 512



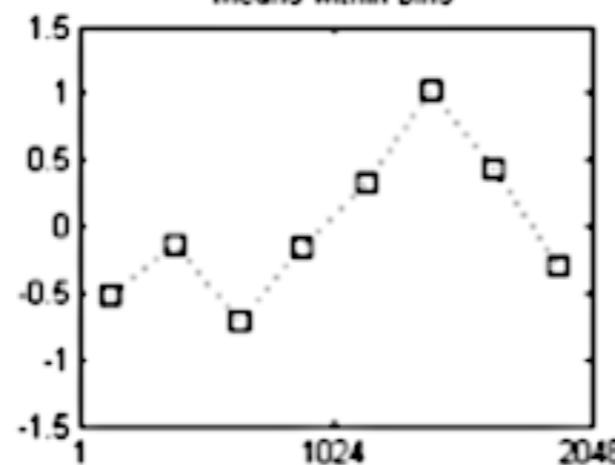
Means within bins



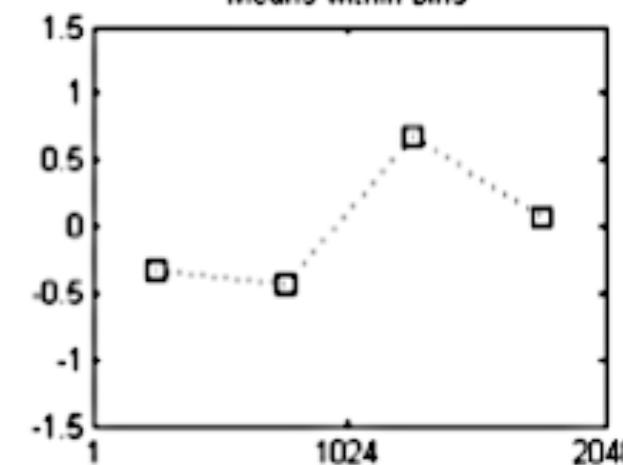
Means within bins



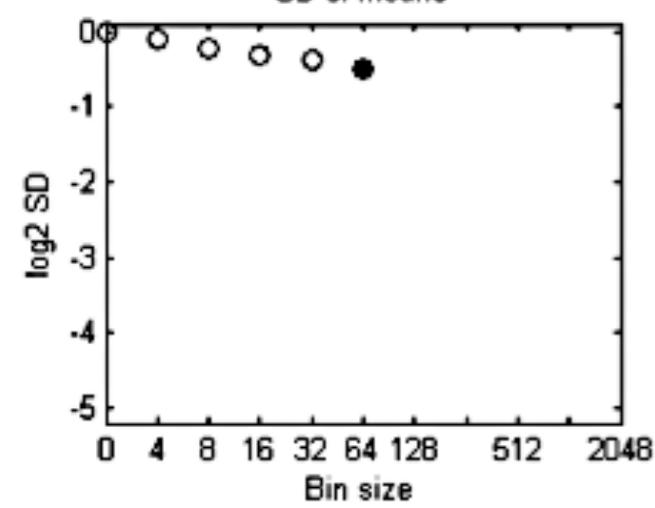
Means within bins



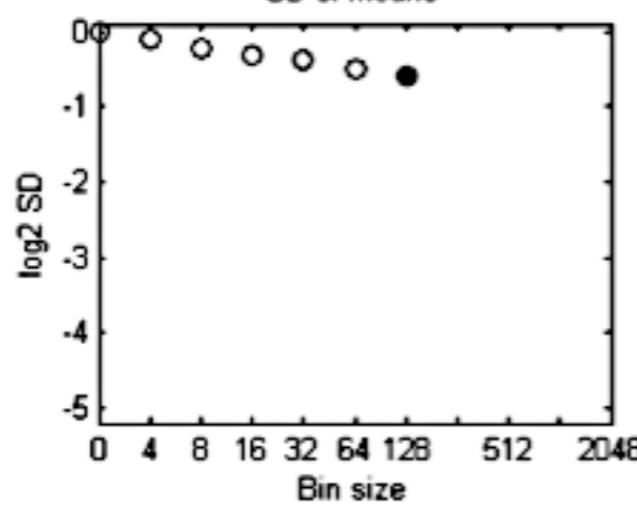
Means within bins



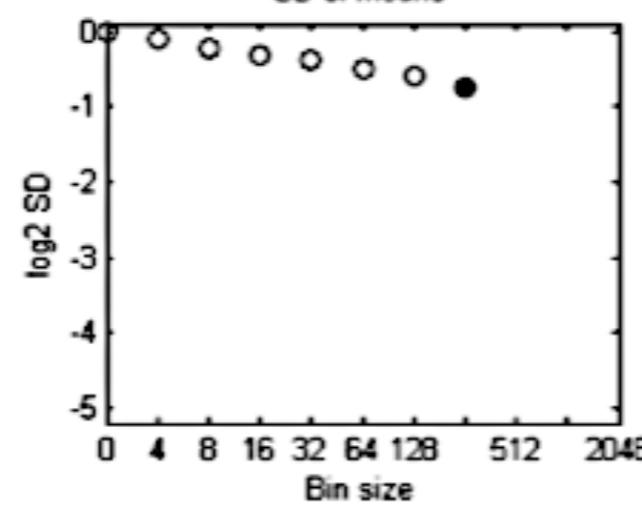
SD of means



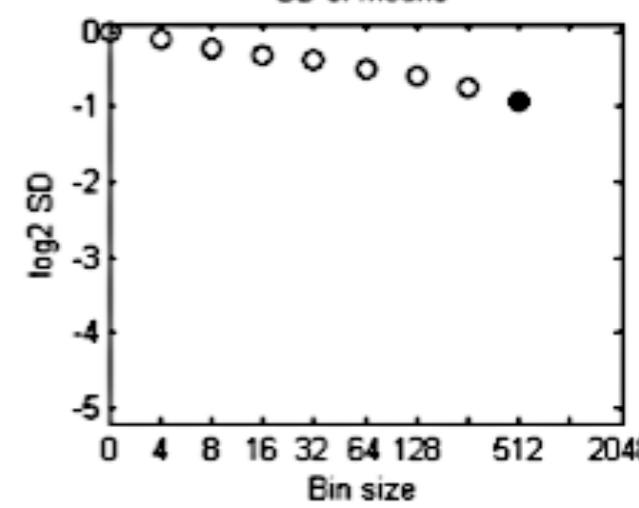
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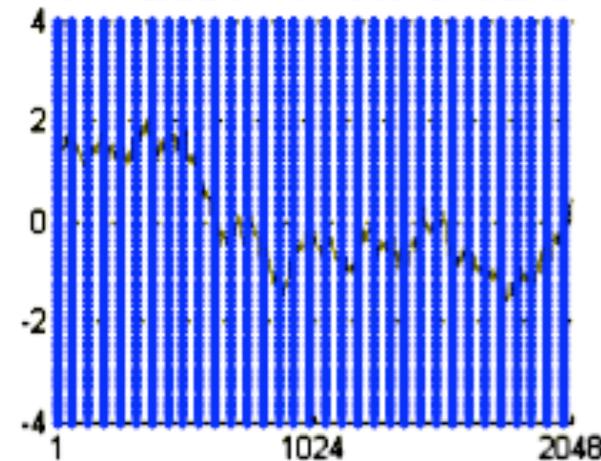
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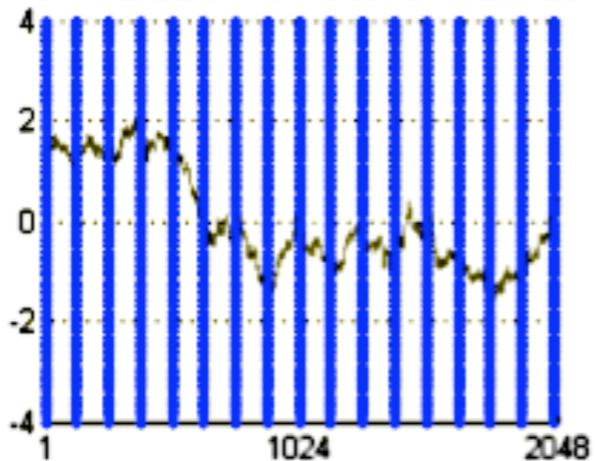
SD of means



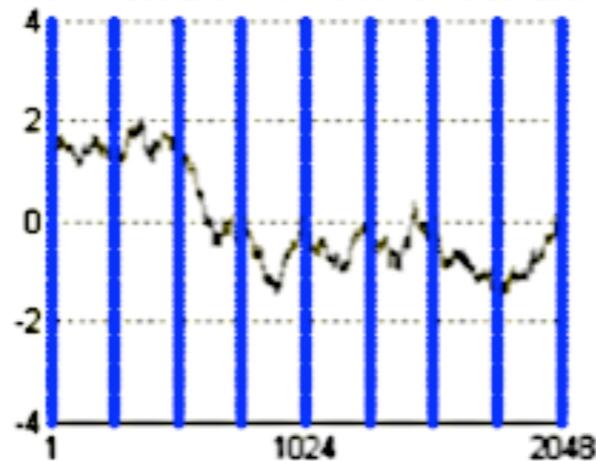
Normalized Brownian noise - bin size: 64



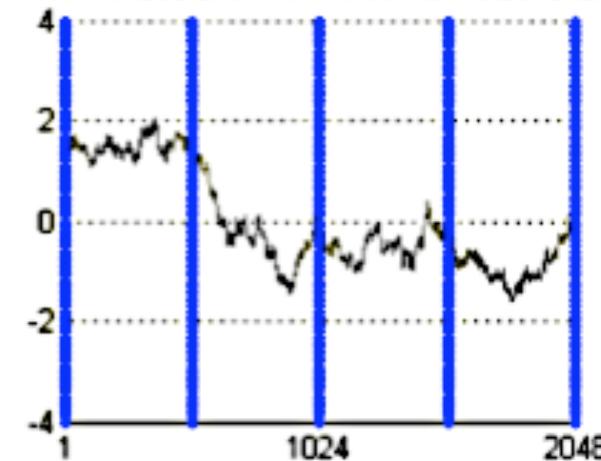
Normalized Brownian noise - bin size: 128



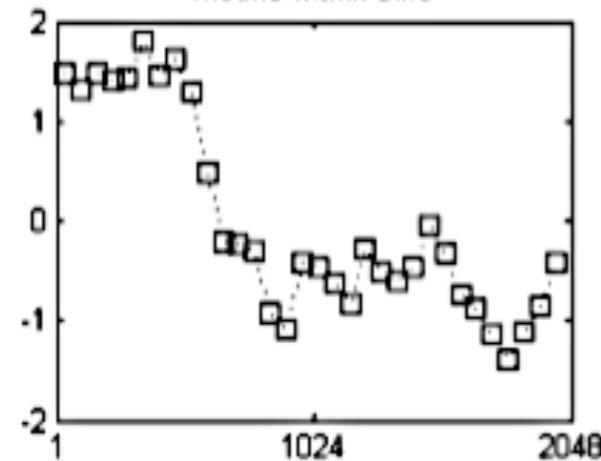
Normalized Brownian noise - bin size: 256



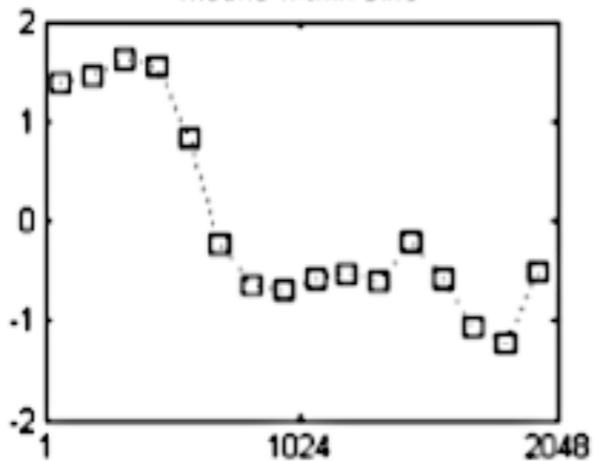
Normalized Brownian noise - bin size: 512



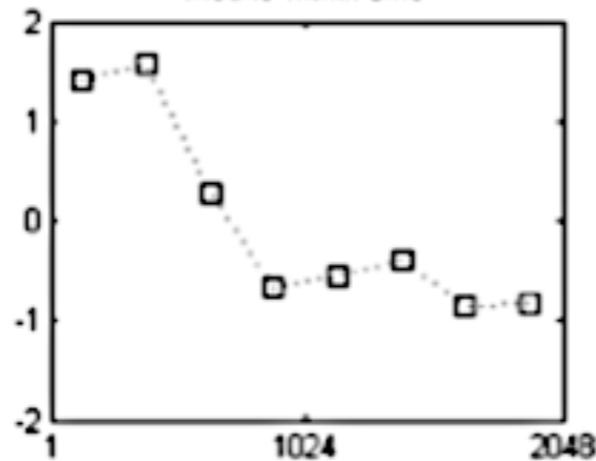
Means within bins



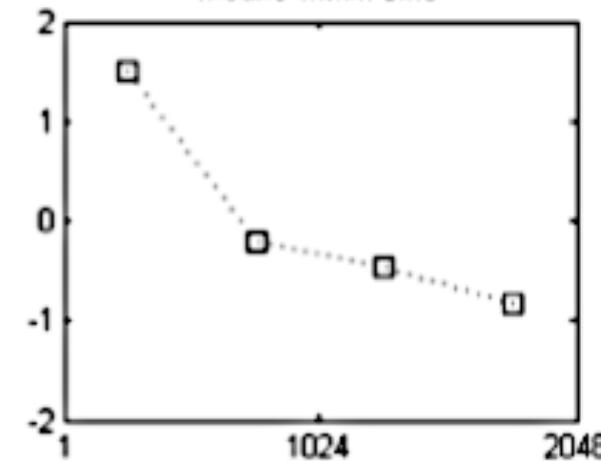
Means within bins



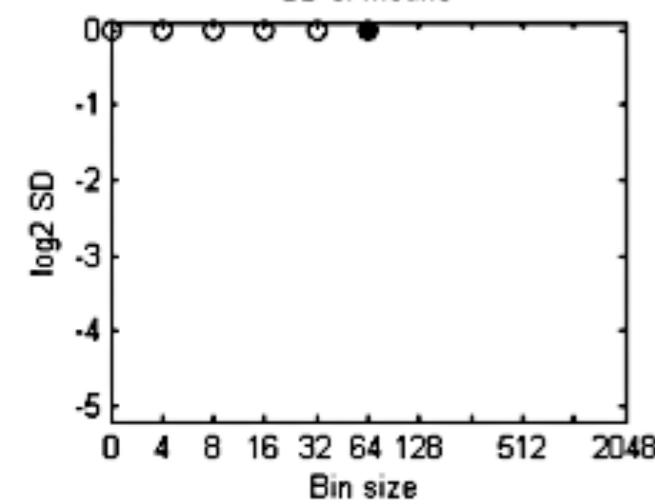
Means within bins



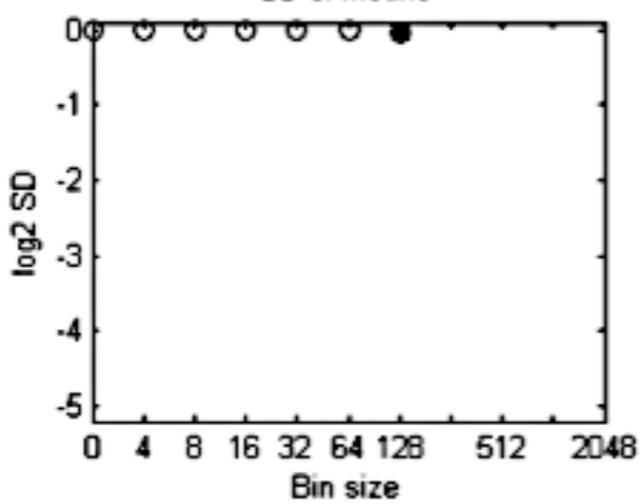
Means within bins



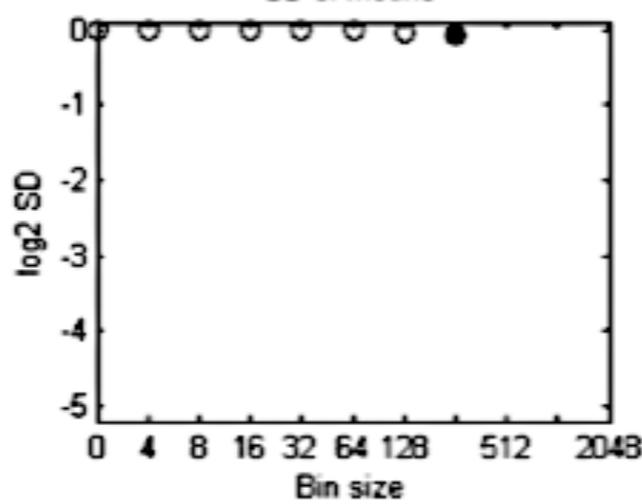
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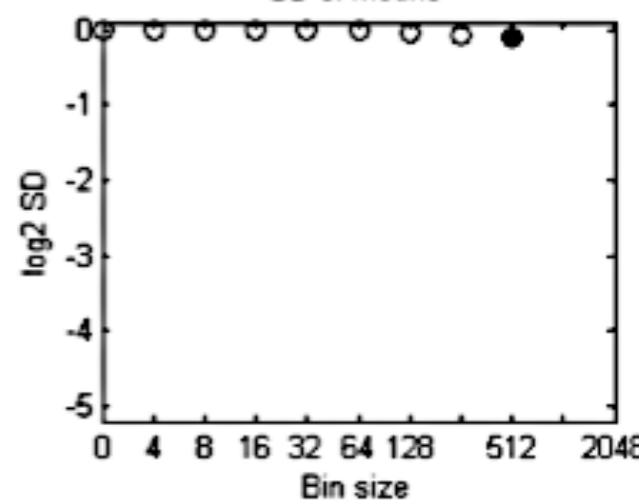
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SD of means



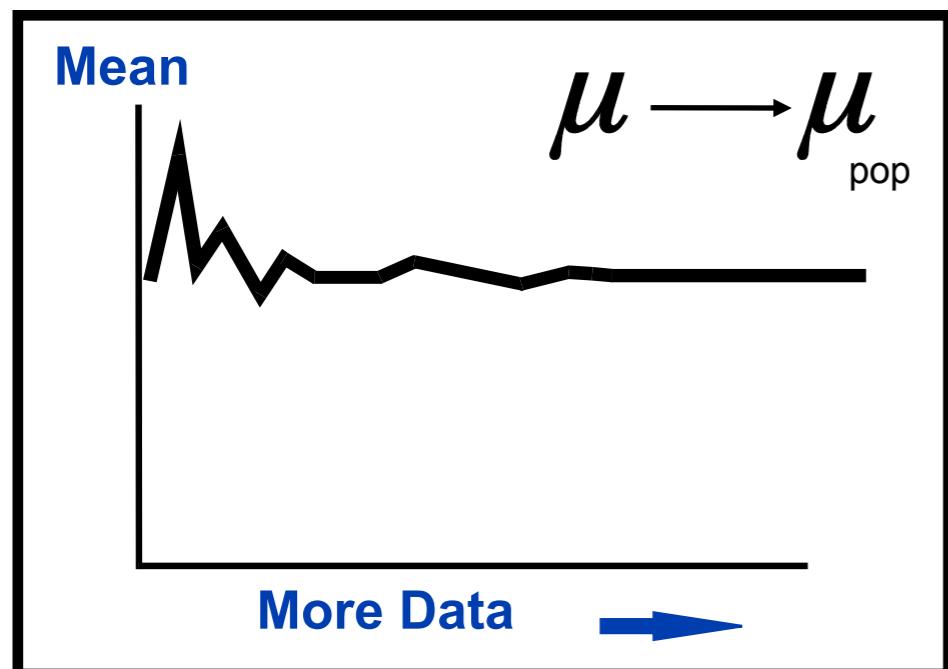
SD of means



Scaling phenomena: Time scales

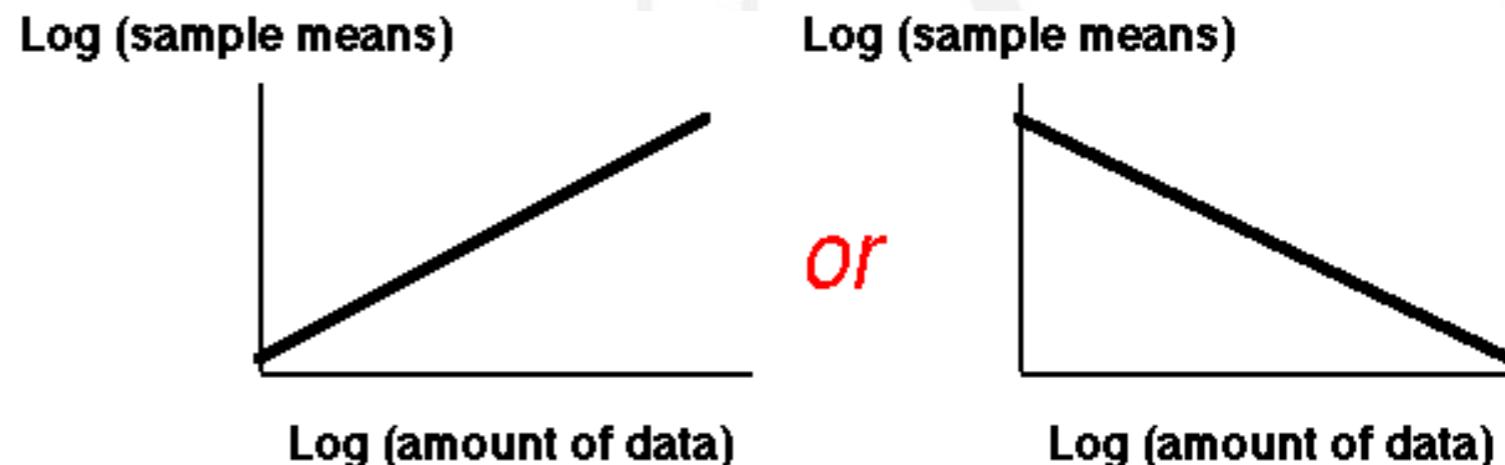
Independent observations of random variables

$\mu \pm \sigma$ are sufficient to characterise absence of dependencies in the data:
e.g. Expected value of μ for $N = 100$, given σ
 $N = \text{ensemble size}$

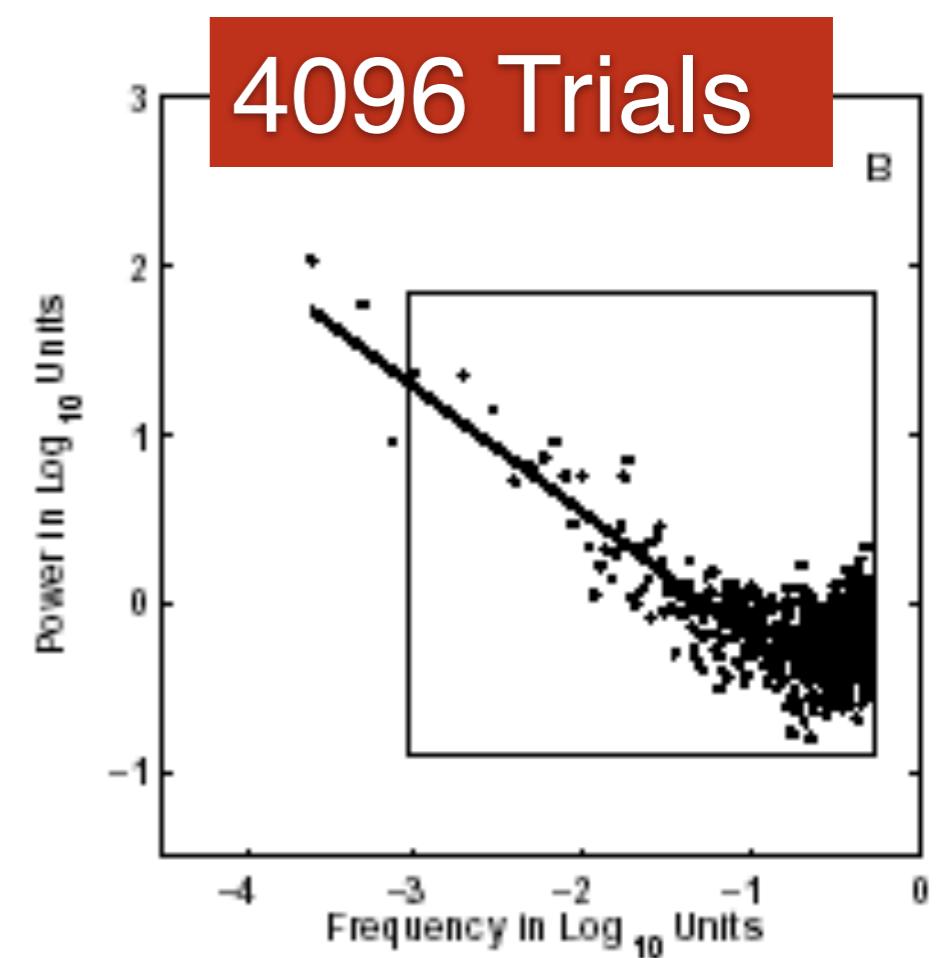
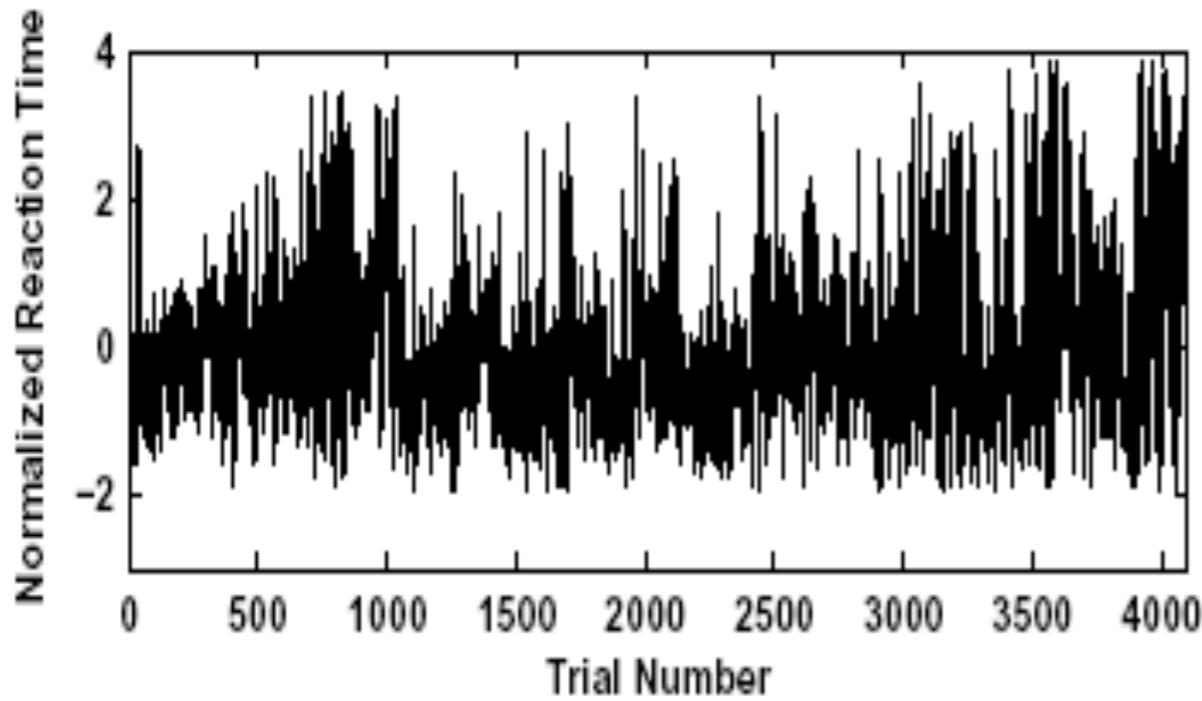
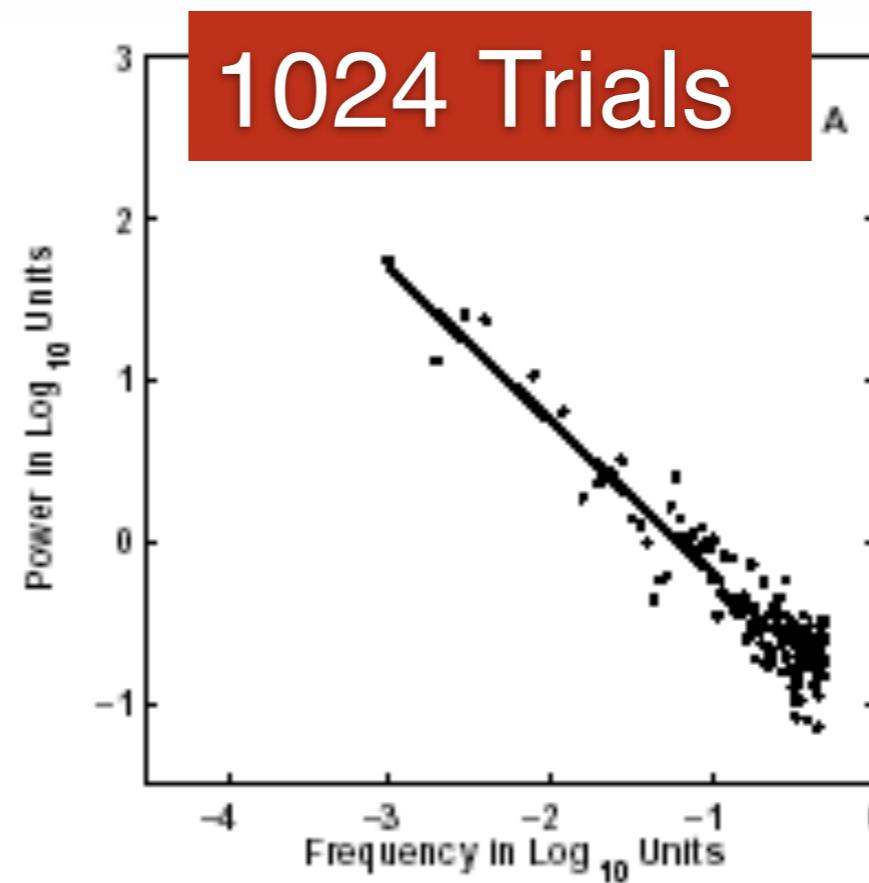
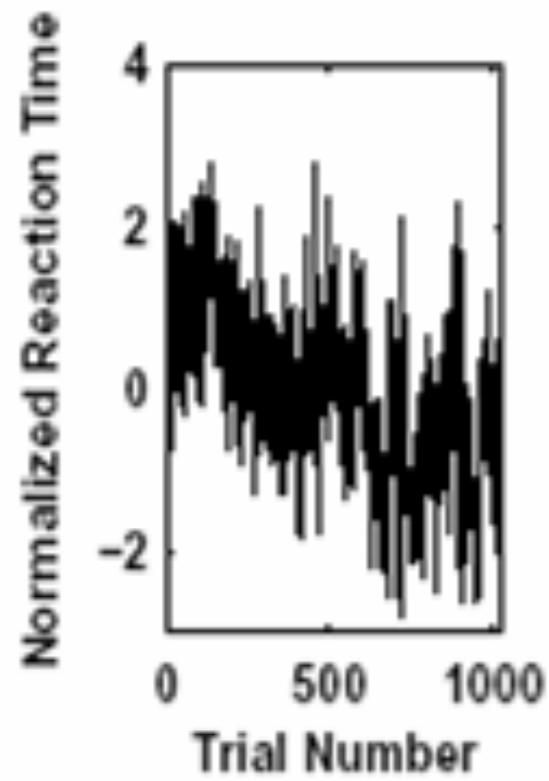


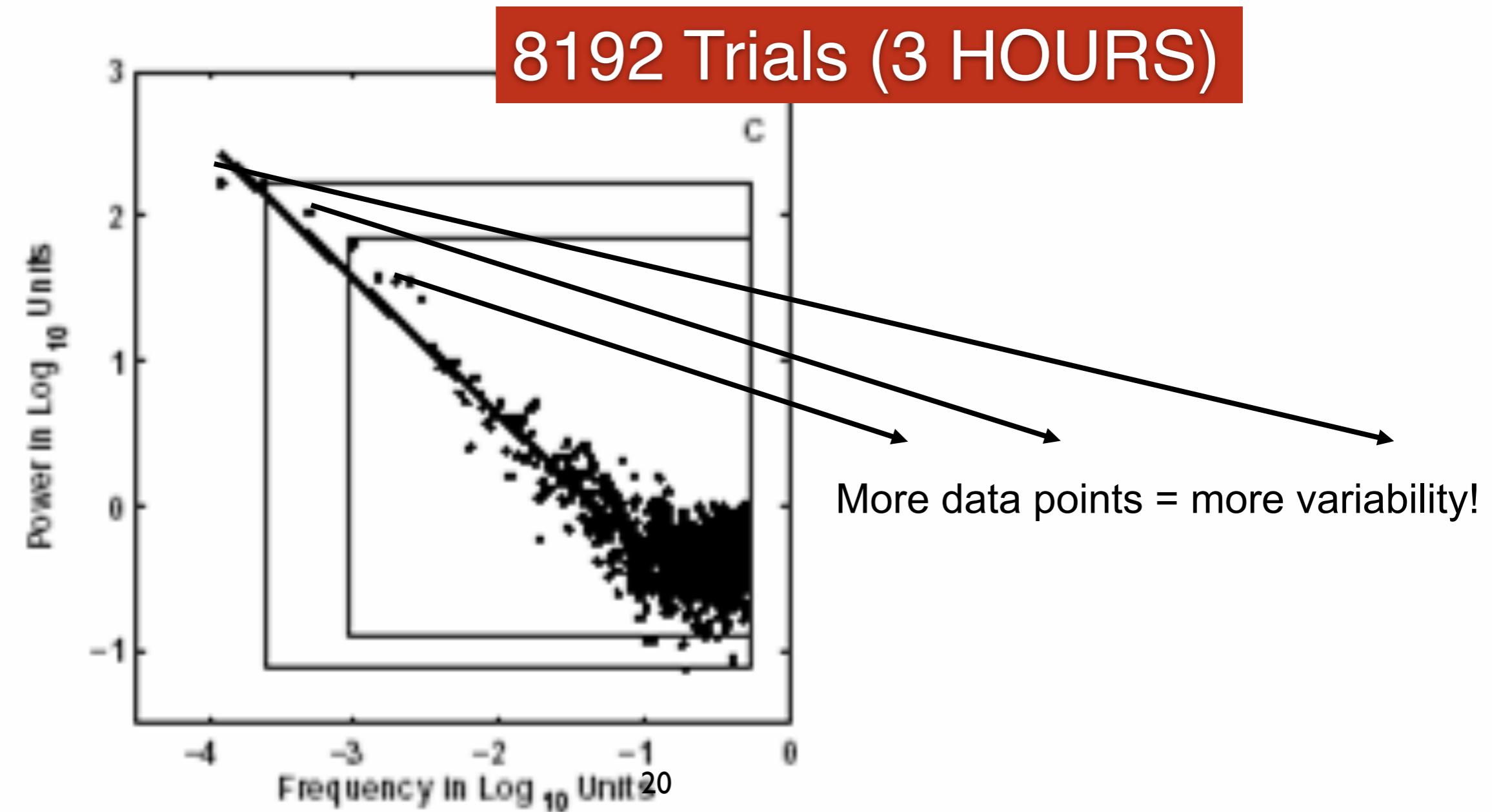
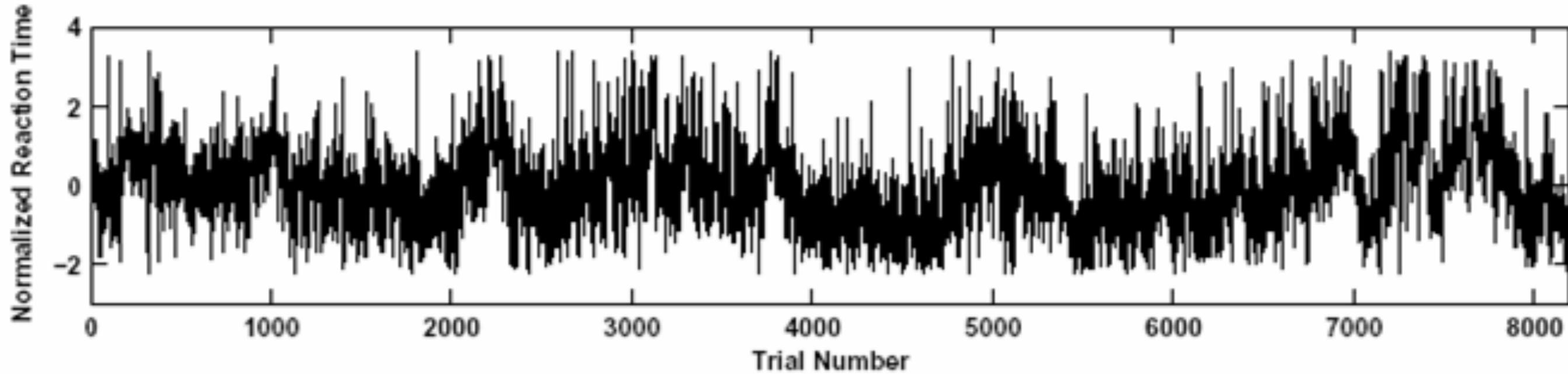
Interdependent observations across different scales

$\mu \pm \sigma$ are insufficient to characterise dependencies in the data:
e.g. Sample estimates of μ change with N
 $N = \text{observation time}$



“Statistics”: More data = more variance





Scaling exponents reveal properties of data generating processes

Sixth International Conference
on Noise in Physical Systems

FOREWORD

Proceedings of a conference held
at the National Bureau of Standards,
Gaithersburg, MD, April 6-10, 1981

The study of fluctuations (or noise) in a physical system provides insights, not available by any other technique, into the microscopic dynamic behavior of that system. Besides being a source of information, noise can also be a source of irritation, in that it limits the performance of numerous devices. The study of noise is of prime importance for the testing of physical theories as well as for the development of improved physical measurements and improved performance of devices. Therefore, the Conference has as one of its goals an improved understanding of noise in devices and its influence on the error budget of a measurement. Indeed, progress in relieving or minimizing noise in some devices was reported (e.g., the relationship of "burst noise" to the metallurgical condition of the sample).

Strong emphasis was given in this Conference to new topics for which the noise spectra proved to be particularly helpful in characterizing the underlying system dynamics. Papers discussed, for example, the transition from periodic to chaotic behavior in chemical systems and turbulent fluid flow, entropy generation in the computer process, the existence and implications of quantum mechanical noise, and noise spectra occurring in electrochemical processes.

Judging from the number of contributions and the intensity of the discussions following their presentations, the topic of $1/f$ noise remains as a very interesting one. It has resisted most, if not all theoretical attempts to explain it. An invited paper by T. Musha gave even more evidence to its ubiquity in nature. One of the most interesting developments here has been the connection between $1/f$ noise and human comfort. Extending beyond the observation that noise exhibiting a $1/f$ spectrum is pleasing to the listener, clinical evidence now suggests that electronic alleviation of pain in humans is improved when the electrical shocks are given a $1/f$ component.

Scaling phenomena: Time scales



1/f Noise in Human Cognition

D. L. Gilden,* T. Thornton, M. W. Mallon

When a person attempts to produce from memory a given spatial or temporal interval, there is inevitably some error associated with the estimate. The time course of this error was measured in a series of experiments where subjects repeatedly attempted to replicate given target intervals. Sequences of the errors in both spatial and temporal replications were found to fluctuate as 1/f noises. 1/f noise is encountered in a wide variety of physical systems and is theorized to be a characteristic signature of complexity.

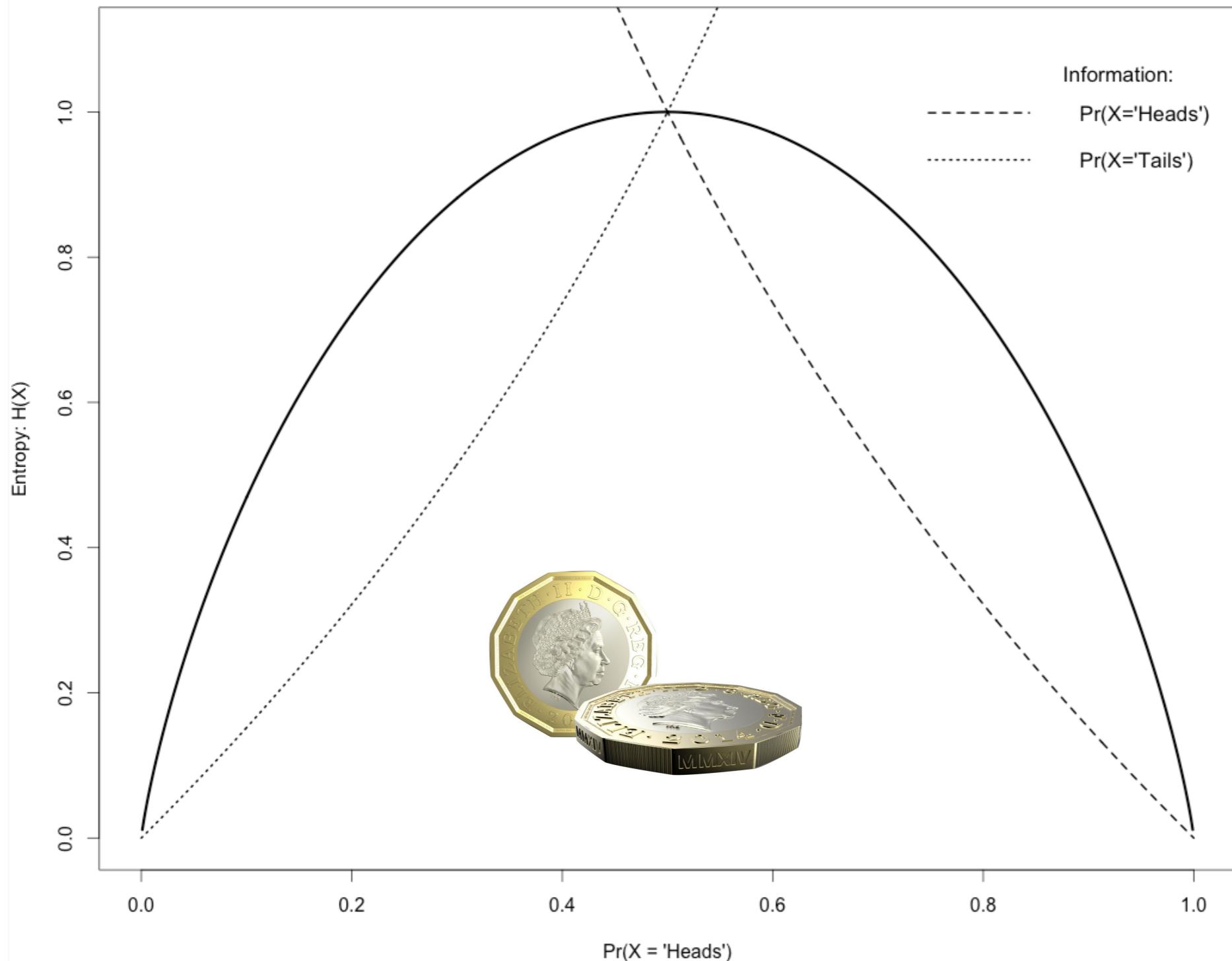
SCIENCE • VOL. 267 • 24 MARCH 1995

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Entropy





Entropy

Information

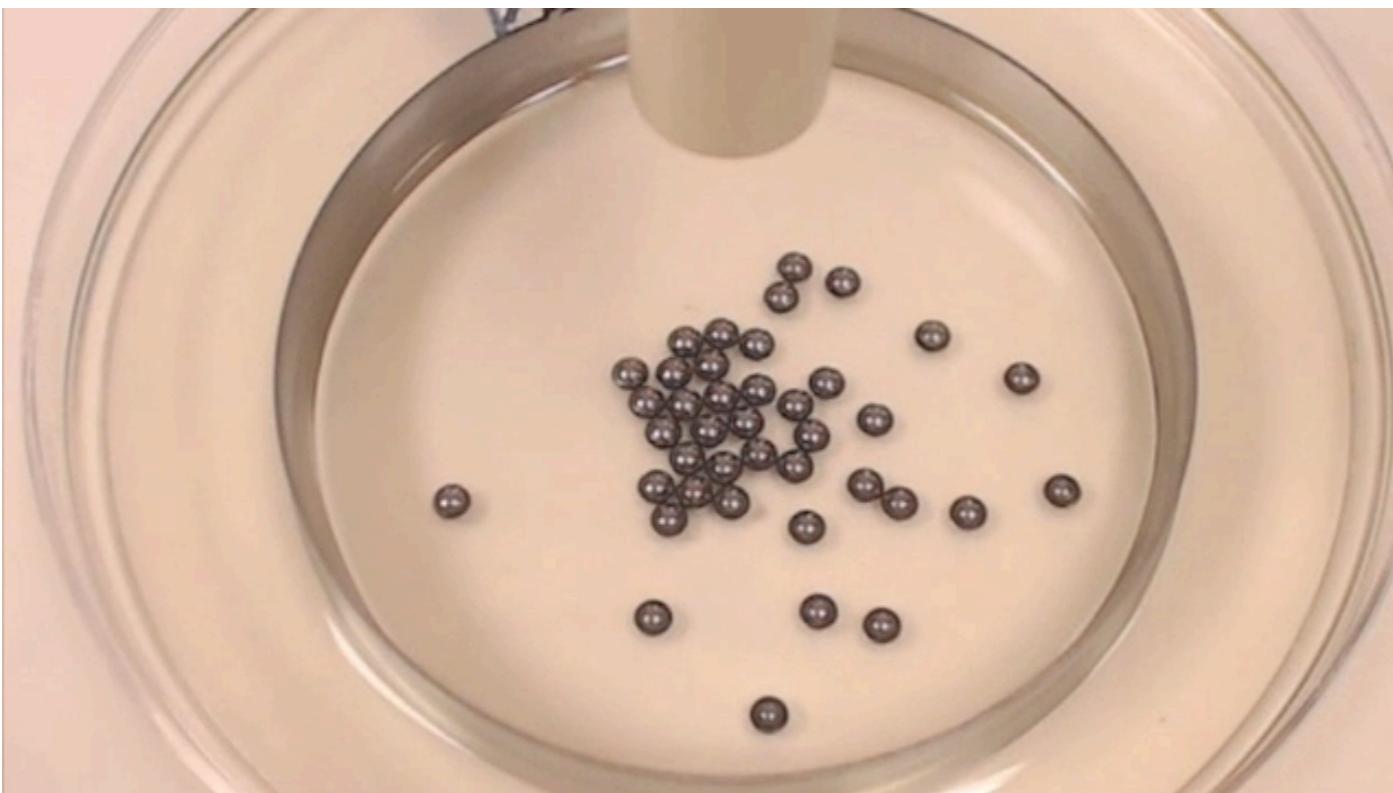
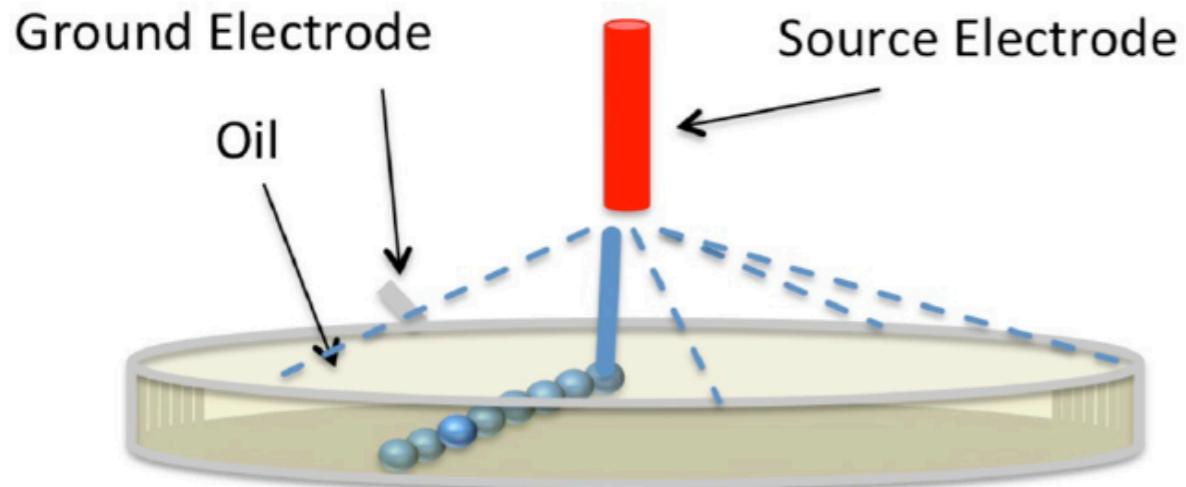
Uncertainty

Redundancy

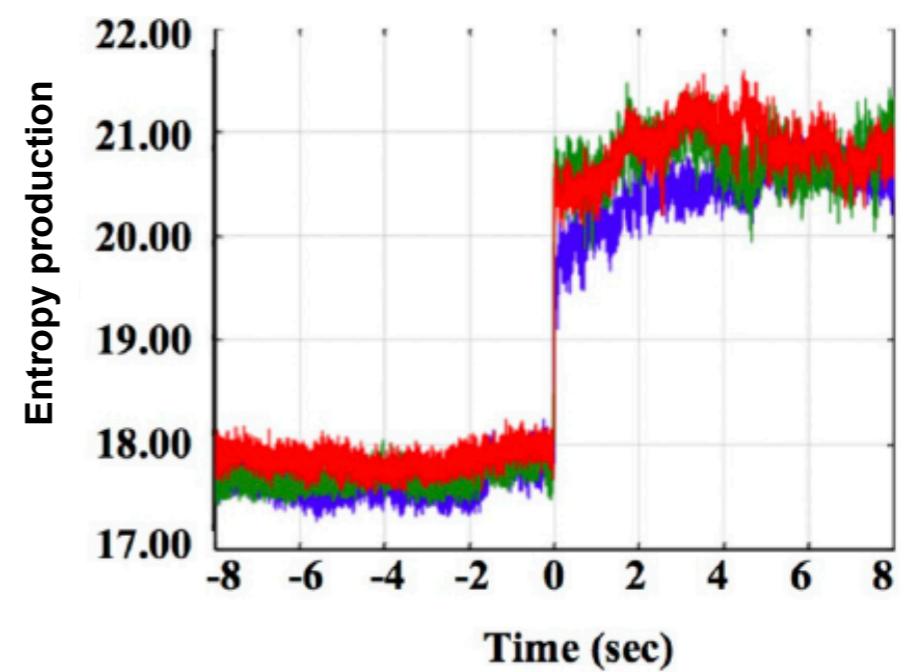
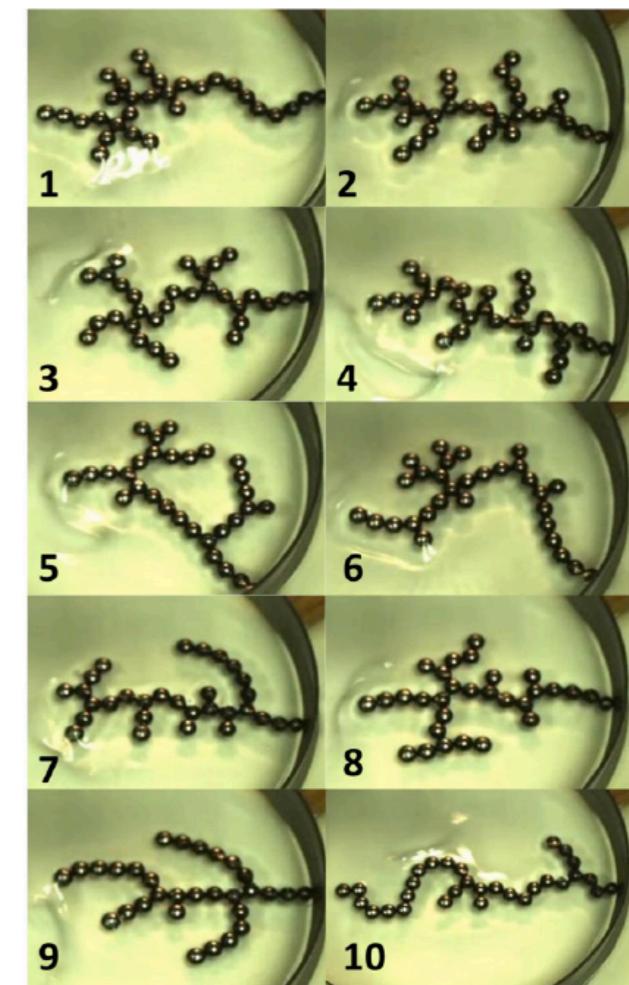
Probability

Symmetry
breaking

Complex behaviour from (physical) principles & laws



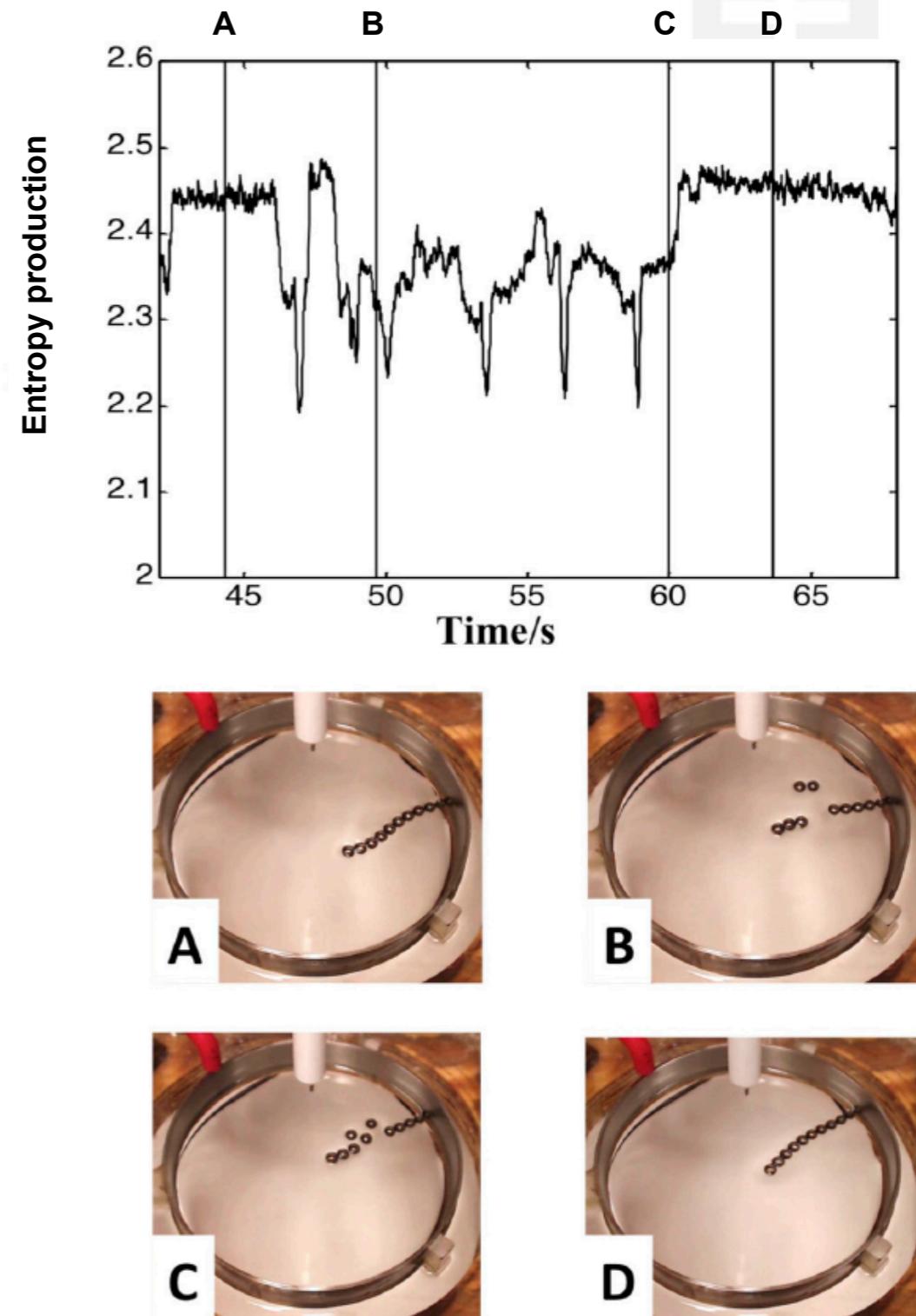
**self-organisation:
Tree formation**



Complex behaviour from (physical) principles & laws (bottom-up)



**self-repair:
Resilience to perturbation**



Complex behaviour from (physical) principles & laws (bottom-up)

END DIRECTED EVOLUTION TO STATES OF HIGHER ENTROPY PRODUCTION

More properties:

Memory

Classical conditioning (aversion / preference)

Memristors

[memristor.org]

“memory resistors”, are a type of passive circuit elements that maintain a relationship between the time integrals of current and voltage across a two terminal element. Thus, a memristors’ resistance varies according to a devices memristance function, allowing, via tiny read charges, access to a “history” of applied voltage

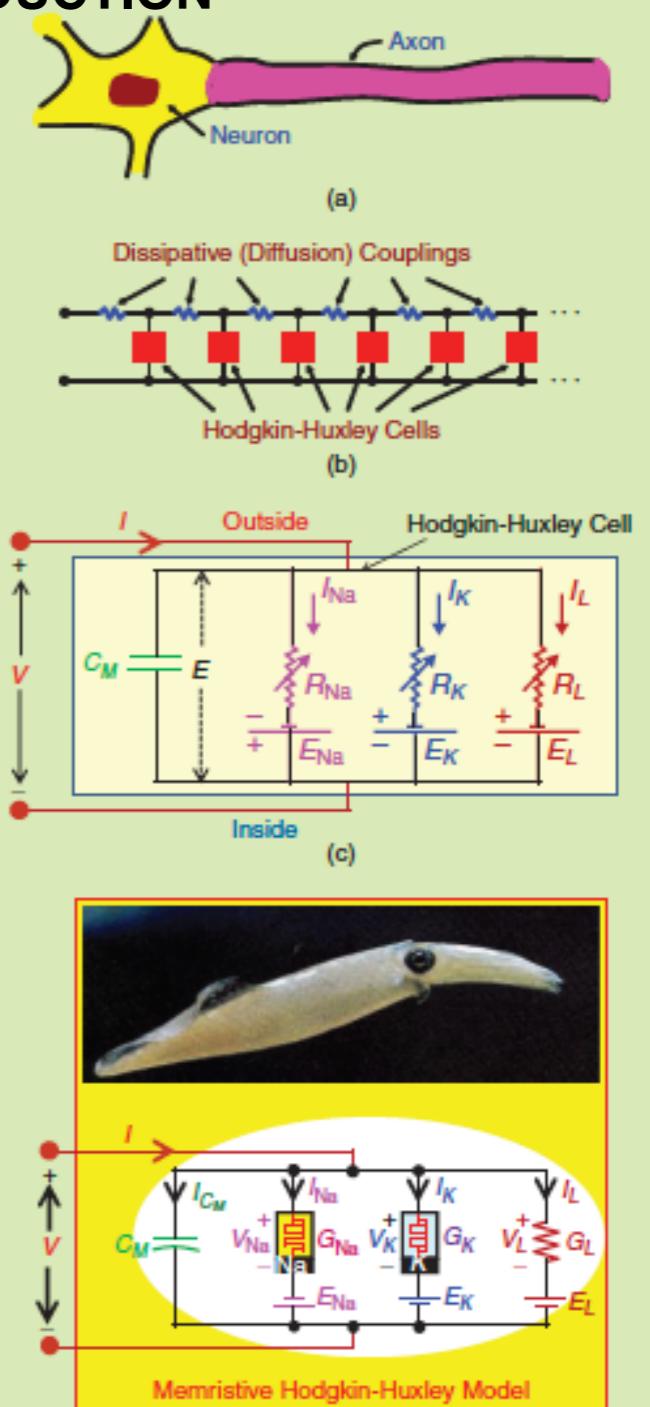


Figure 1. (a) Schematic of a neuron and its axon. (b) One-dimensional axon model made of resistively coupled Hodgkin-Huxley cells. (c) Hodgkin-Huxley circuit model made of a capacitor C_M , a resistor R_L , three batteries E_{Na^+} , E_K^- , and E_L^- , a time-varying potassium resistor R_K , and a time-varying sodium resistor R_{Na^+} . (d) Memristive Hodgkin-Huxley axon circuit model.