About the course….

Course site: https://complexity-methods.github.io

Dynamics of Complex Systems

Day 1: Intro to Complexity Science

Intro Mathematics of Change

Behavioural Science Institute
Radboud University Nijmegen

“Lack of Group to Individual Generalizability”

PNAS PNAS PNAS PNAS PNAS

Lack of group-to-individual generalizability is a threat to human subjects research

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Only for ergodic processes will inferences based on group-level data generalize to individual experience or behavior. Because human social and psychological processes typically have an individually variable and time-varying nature, they are unlikely to be ergodic. In this paper, six studies with a repeated-measure design were used for symmetric comparisons of interindividual and intraindividual variation. Our results delineate the potential scope and impact of nonergodic data in human subjects research. Analyses across six samples (with 87–94 participants and an equal number of assessments per participant) showed some degree of agreement in central tendency estimates (mean) between groups and individuals across constructs and data collection paradigms. However, the variance around the expected value was two to four times larger within individuals than within groups. This suggests that literatures in social and medical sciences may overestimate the accuracy of aggregated statistical estimates. This observation could have serious consequences for how we understand the consistency between group and individual correlations, and the generalizability of conclusions between domains. Researchers should explicitly test for equivalence of processes at the individual and group level across the social and medical sciences.

research methodology | replicability | idiographic science | generalizability | ecological fallacy

Inferences made in social and medical research typically result from statistical tests conducted on aggregated data. The implicit assumption is that group-derived estimates can be applied

consistency between individual and group variability before generalizing results across levels of analysis. We will refer to this latter condition as the “group-to-individual generalizability” of a given statistical estimate. However, whether couched in prosaic terms, or within formal mathematical theorems, researchers have not systematically examined such generalizability in extant literatures, despite a number of calls to do so throughout the years (cf. refs. 6–11). Hitherto, the highest-impact publications in medical and social sciences have been largely based on data aggregated across large samples, with best-practice guidelines almost exclusively based on statistical inferences from group designs. The worst-case scenario—a global, uniform absence of group-to-individual generalizability due to nonergodicity in the social and medical sciences—would undermine the validity of our scientific canon in these domains. However, even moderate incongruities between group and individual estimates could result in imprecise or potentially invalid conclusions. We argue that this possibility should be formally tested, wherever possible, to be ruled out.

Ergodicity, the Ecological Fallacy, and Simpson’s Paradox

The ergodic theorem is a general and formal mathematical expression that deals with the generalizability of statistical phenomena across levels and units of analysis. [While a more thorough explication of the ergodic theorem is outside of the scope of the present paper, readers are referred to Molenaar (1) for a comprehensive mathematical treatment of ergodicity in human subjects research.] Ergodic theory postulates that the

PSYCHOLOGICAL AND COGNITIVE SCIENCES

MEDICAL SCIENCES

There are two important takeaways from these findings:

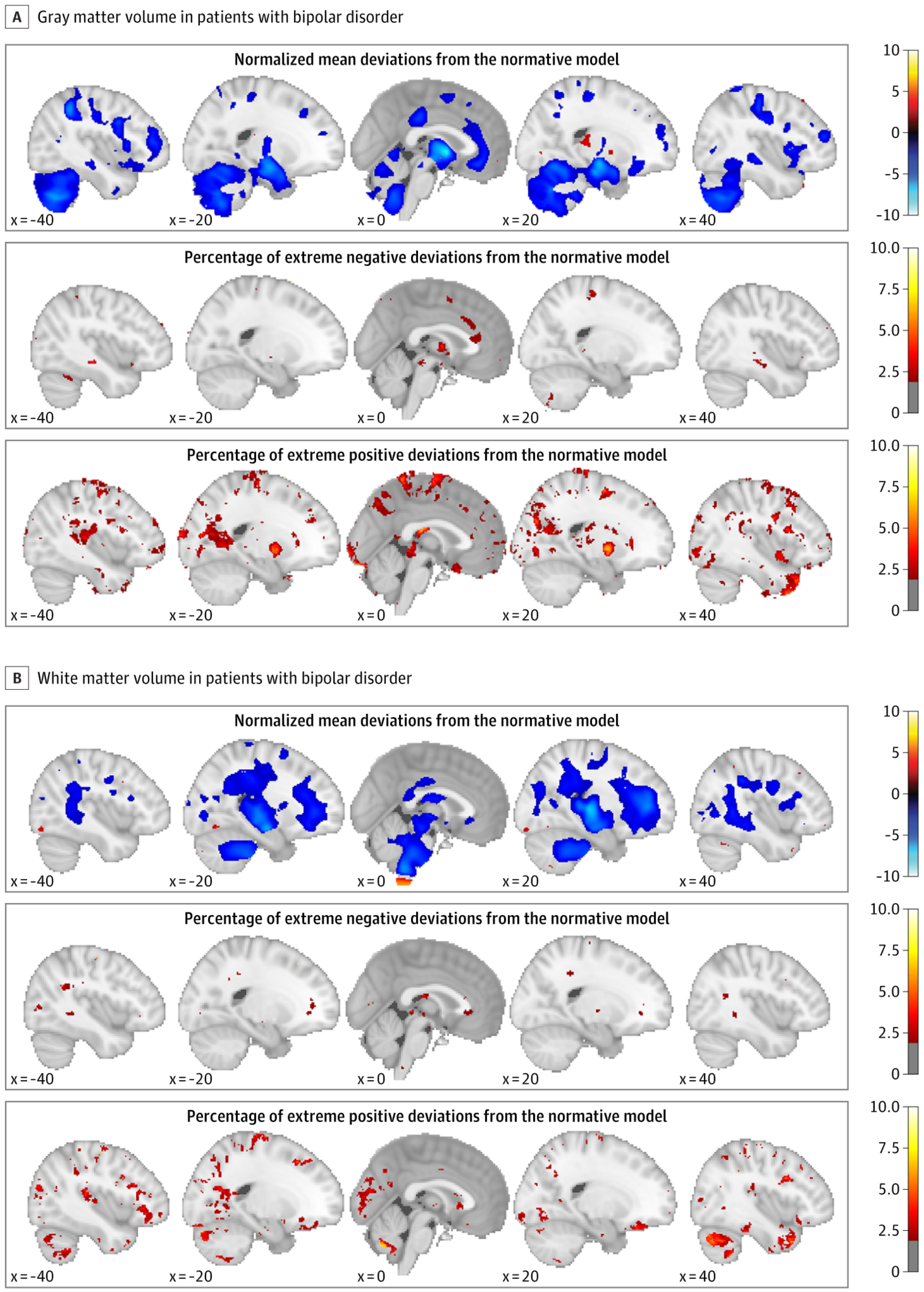
(i) **Aggregated estimates did not consistently agree with individual estimates […]** the present study provides an empirical demonstration of [*the implausibility of ergodicity in human subjects research*] across multiple settings and constructs.

(ii) even in the best-case scenario, **we should not think of a correlation in group data as an estimate that generalizes to any given individual in the population**. Only 68% of all individual correlational values fall within a range that would be predicted by group data to cover 99.7% of all possible correlations—a discrepancy of nearly 32%.

The worst-case scenario is clearly dire: It is plausible that **inattention to nonergodicity and a lack of group-to-individual generalizability threaten the veracity of countless studies, conclusions, and best-practice recommendations.**

Fisher, A. J., Medaglia, J. D., and Jeronimus, B. F. (2018). Lack of group-to-individual generalizability is a threat to human subjects research. *Proceedings of the National Academy of Sciences.*

The idea of the average patient is a noninformative construct



Question: Is the **focus on the average patient disguising interindividual differences** among patients with mental disorders?

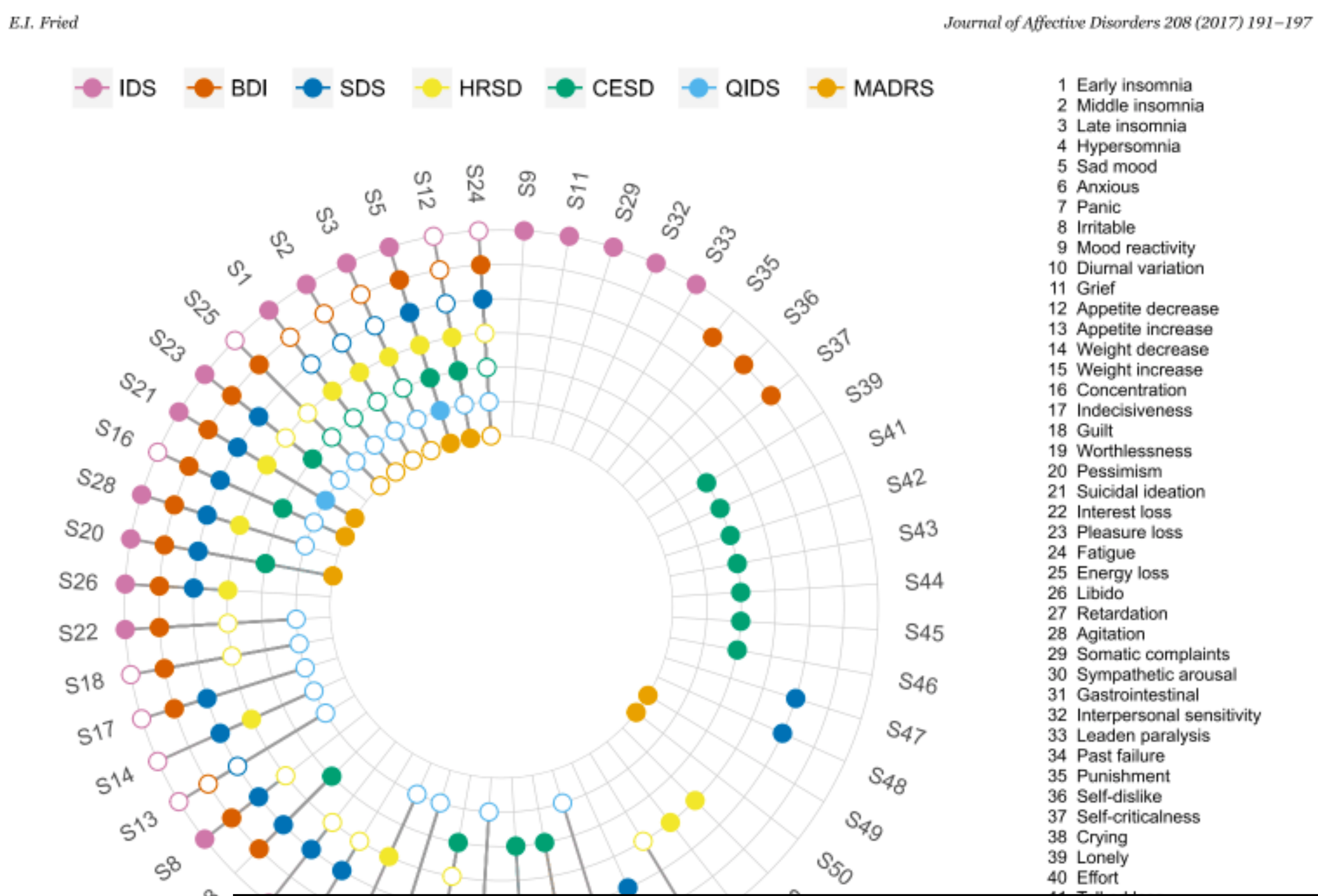
Findings: In this study of magnetic resonance imaging data from 218 patients with schizophrenia spectrum disorders and 256 healthy control individuals, mapping of interindividual differences in brain structure revealed that **only a few brain loci had the same abnormalities in more than 2% of patients with the same disorder despite robust group-level differences** in multiple brain regions between patients and control individuals.

Meaning: These findings suggest that **the idea of the average patient is a noninformative construct that falls apart when mapping interindividual differences** and provide a framework toward precision medicine in psychiatry.

https://www.ru.nl/donders/@1179539/very-few-similarities-between-brains-schizophrenia/

Wolfers, T., Doan, N. T., Kaufmann, T., Alnæs, D., Moberget, T., Agartz, I., ... & Andreassen, O. A. (2018). Mapping the heterogeneous phenotype of schizophrenia and bipolar disorder using normative models. *JAMA psychiatry*.

Problemen met assumpties van “sample-based statistical inference” & Gevolgen voor de praktijk



3700 patiënten with MDD (Fried et al., 2015):

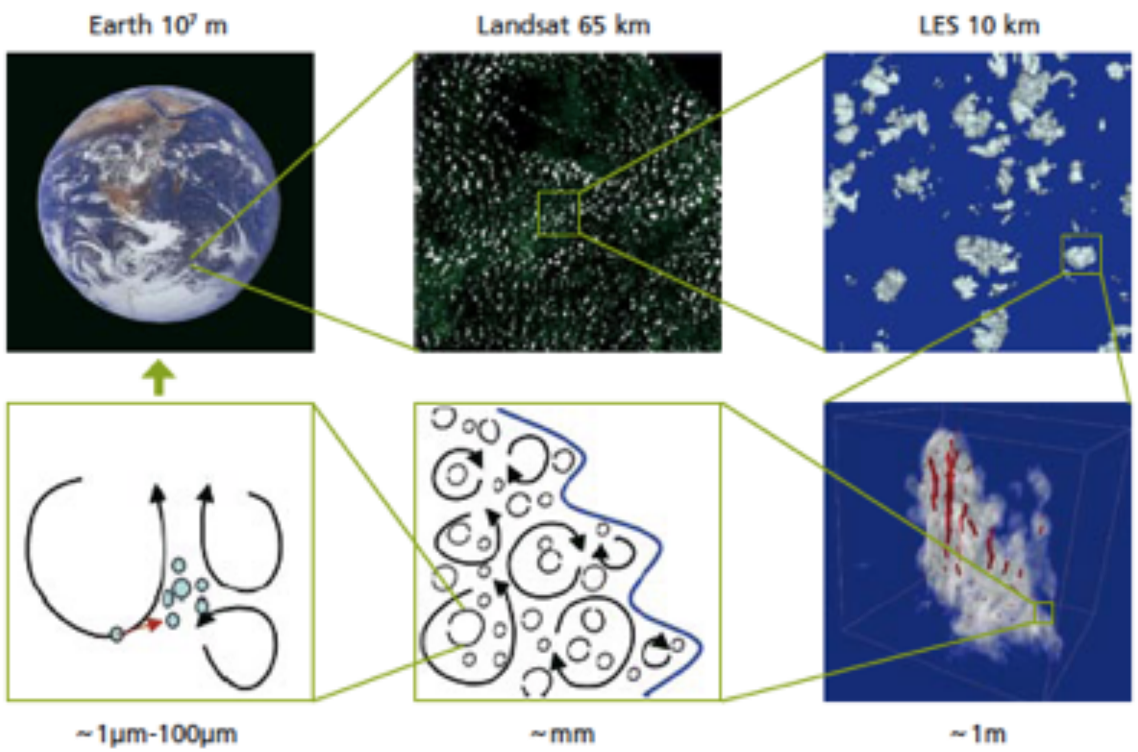
- “1030 unique symptom profiles”
- “864 profiles (83.9%) were endorsed by five or fewer subjects …”
- “501 profiles (48.6%) were endorsed by only one individual.”
- “The most common symptom profile exhibited a frequency of only 1.8%.”

“The substantial symptom variation among individuals who all qualify for one diagnosis **calls into question the status of MDD as a specific consistent syndrome** […]
We suggest that the **analysis of individual symptoms, their patterns, and their causal associations** will provide insights that could not be discovered in studies relying on only sum-scores.”

Fried, E. I., & Nesse, R. M. (2015). Depression is not a consistent syndrome: an investigation of unique symptom patterns in the STAR*D study. *Journal of affective disorders, 172*, 96-102.

Complexity Science

- Time! (Dynamics)
- Self-Organization
- Micro-Macro levels (Emergence)
- Scale invariance



Fair coin + Physics = Any outcome you like!

How random is a coin toss? - Numberphile

https://youtu.be/AYnJv68T3MM

SIAM REVIEW
Vol. 49, No. 2, pp. 211–235

Dynamical Bias in the Coin Toss*

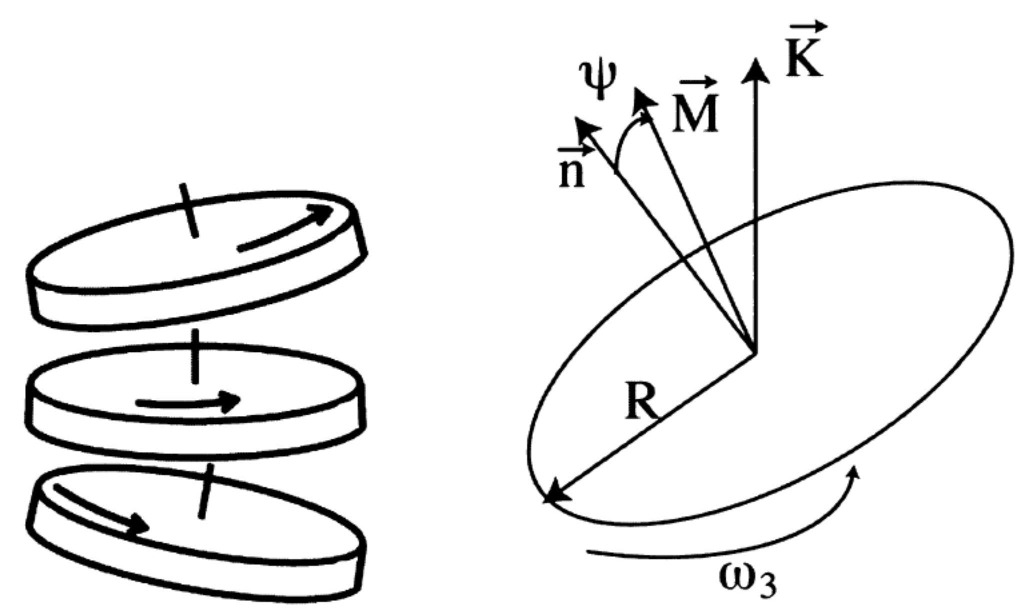
Persi Diaconis†
Susan Holmes‡
Richard Montgomery§

Abstract. We analyze the natural process of flipping a coin which is caught in the hand. We show that vigorously flipped coins tend to come up the same way they started. The limiting chance of coming up this way depends on a single parameter, the angle between the normal to the coin and the angular momentum vector. Measurements of this parameter based on high-speed photography are reported. For natural flips, the chance of coming up as started is about .51.

Key words. Berry phase, randomness, precession, image analysis

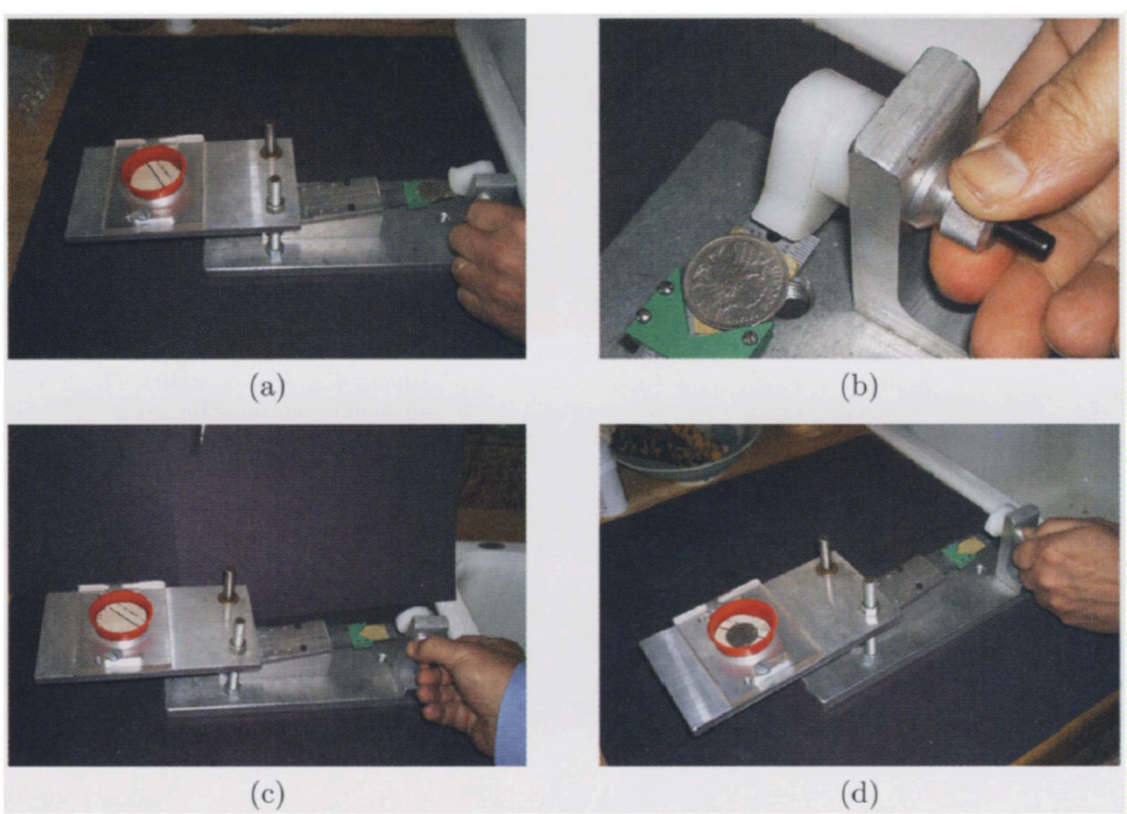
AMS subject classifications. 62A01, 70B10, 60A99

Fig. 1



(a) (b)

Fig. 2 *(a) Diagram of a precessing coin. (b) Coordinates of precessing coin: $\vec{K}$ is the upward direction, $\vec{n}$ is the normal to the coin, $\vec{M}$ is the angular momentum vector, and ω_3 is the rate of rotation around the normal $\vec{n}$.*



(a) (b) (c) (d)

Fig. 1

Complexity Methods for Behavioural Science

Day 1: Intro to Complexity Science
Intro Mathematics of Change

Time in the Social Sciences

All psychological processes are about *change*:

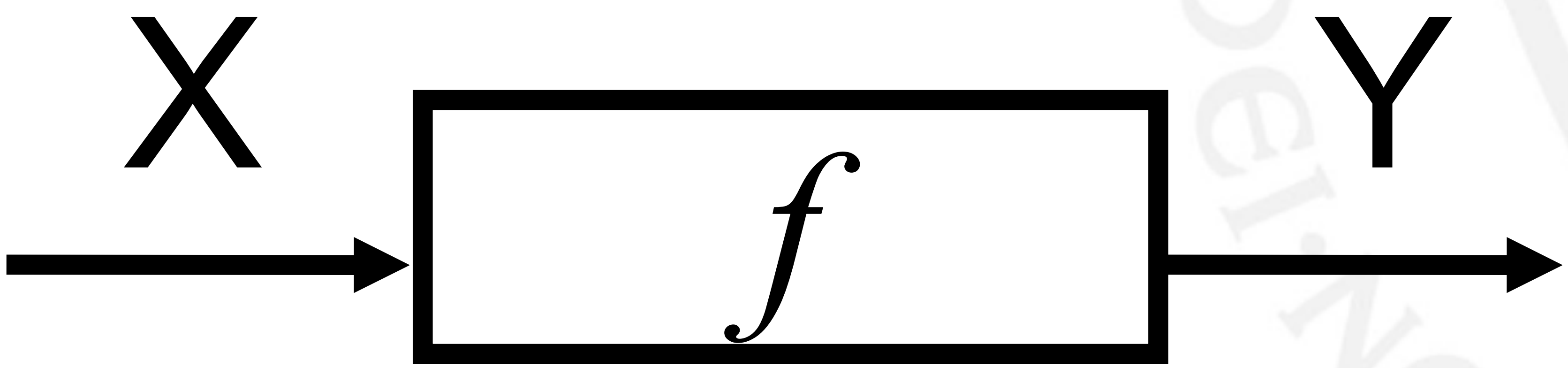
These processes involve the (in)stability of some behaviour, feature or variable over time:

- Sometimes gradual
- Sometimes abrupt/sudden
- Sometimes requiring a lot of effort or energy (resistance to change!)
- Sometimes seemingly without any resistance at all
- Think of common topics in psychological science:
  - *Developmental change*
  - *Change of behaviour between conditions in an experiment*
  - *Change after therapy or intervention*
  - *Stability of personality across the life span*

The mathematics of change

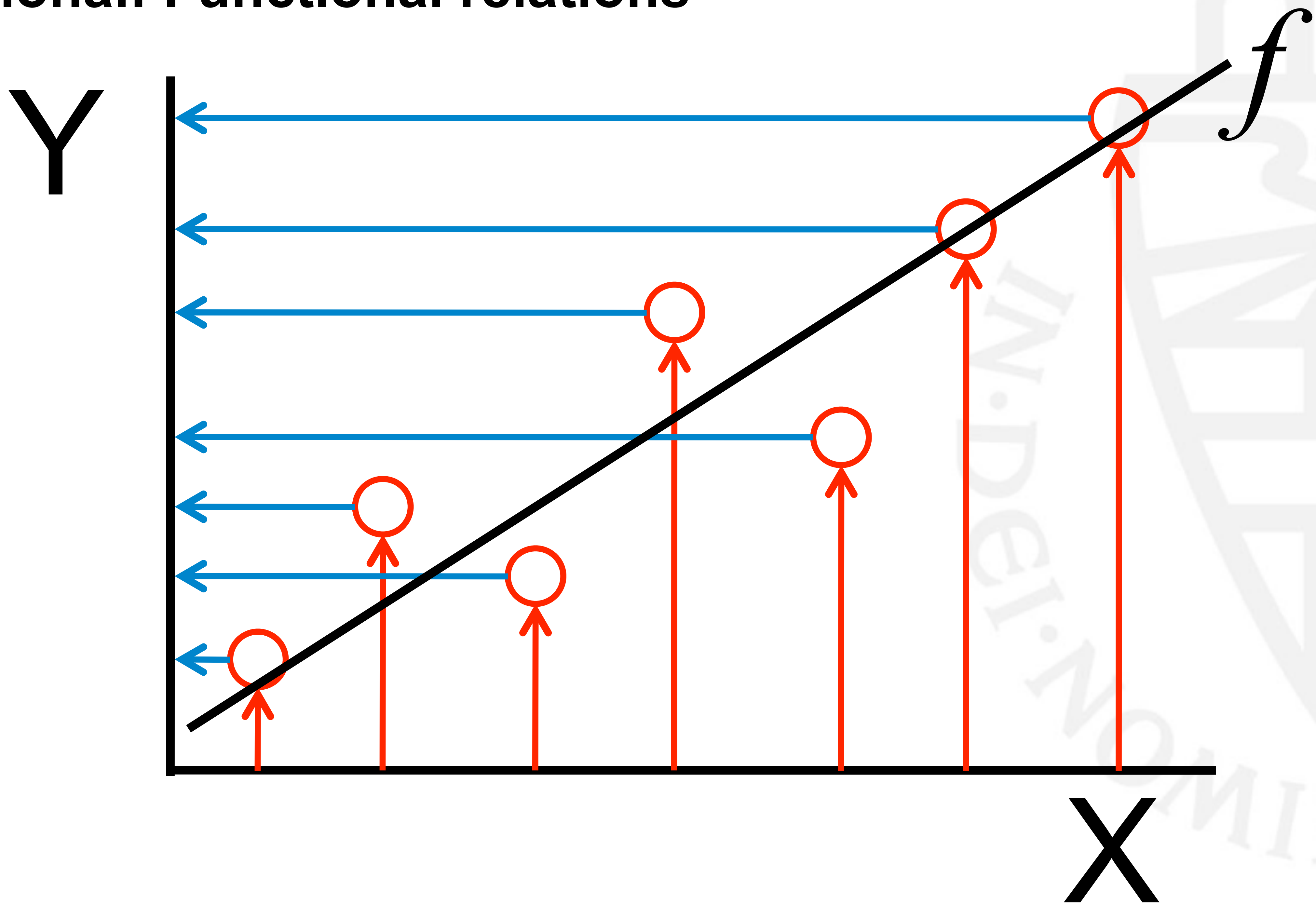
Traditional: Functional relations

$$Y = f(X)$$



The mathematics of change

Traditional: Functional relations



The mathematics of change

Complex systems however:

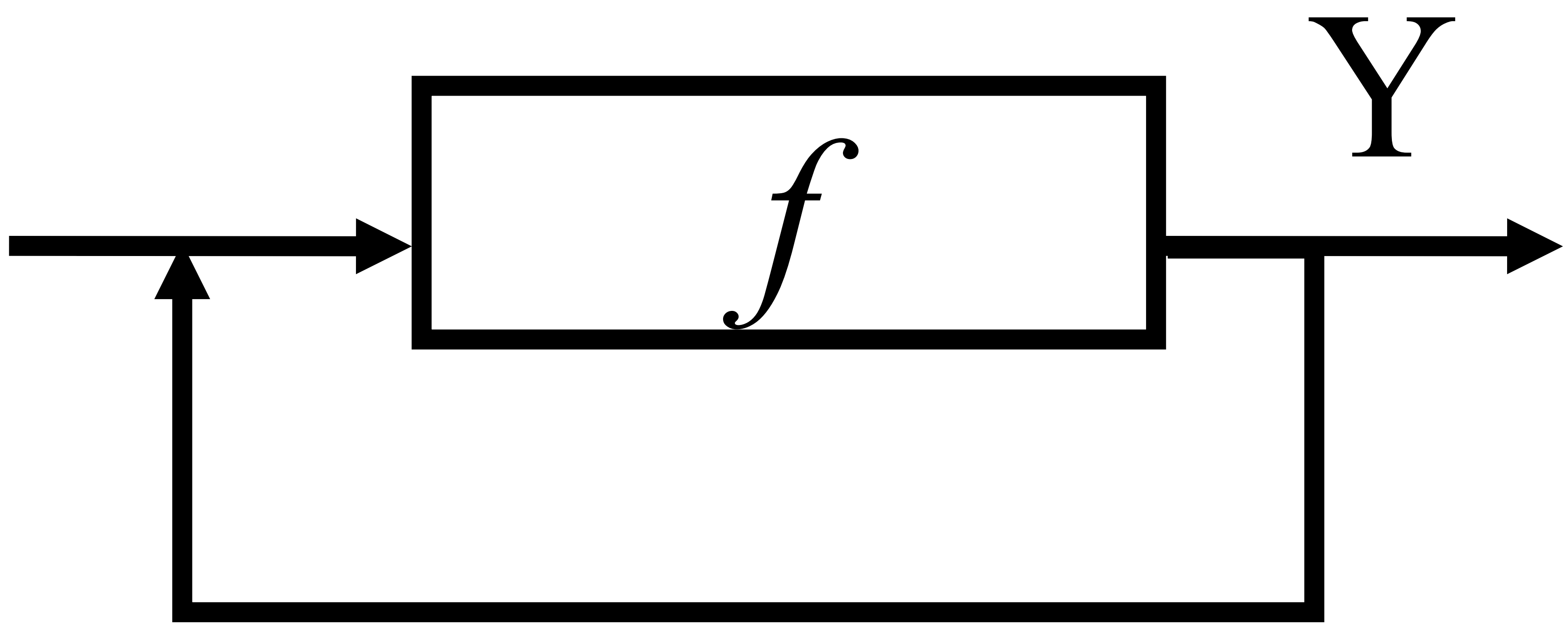
- Consist of feedback loops
- Are recurrent / recursive
- Have history
- Are characterised by multiplicative interactions between components

[1]refs

The mathematics of change

Complex systems: Recurrent processes / Feedback

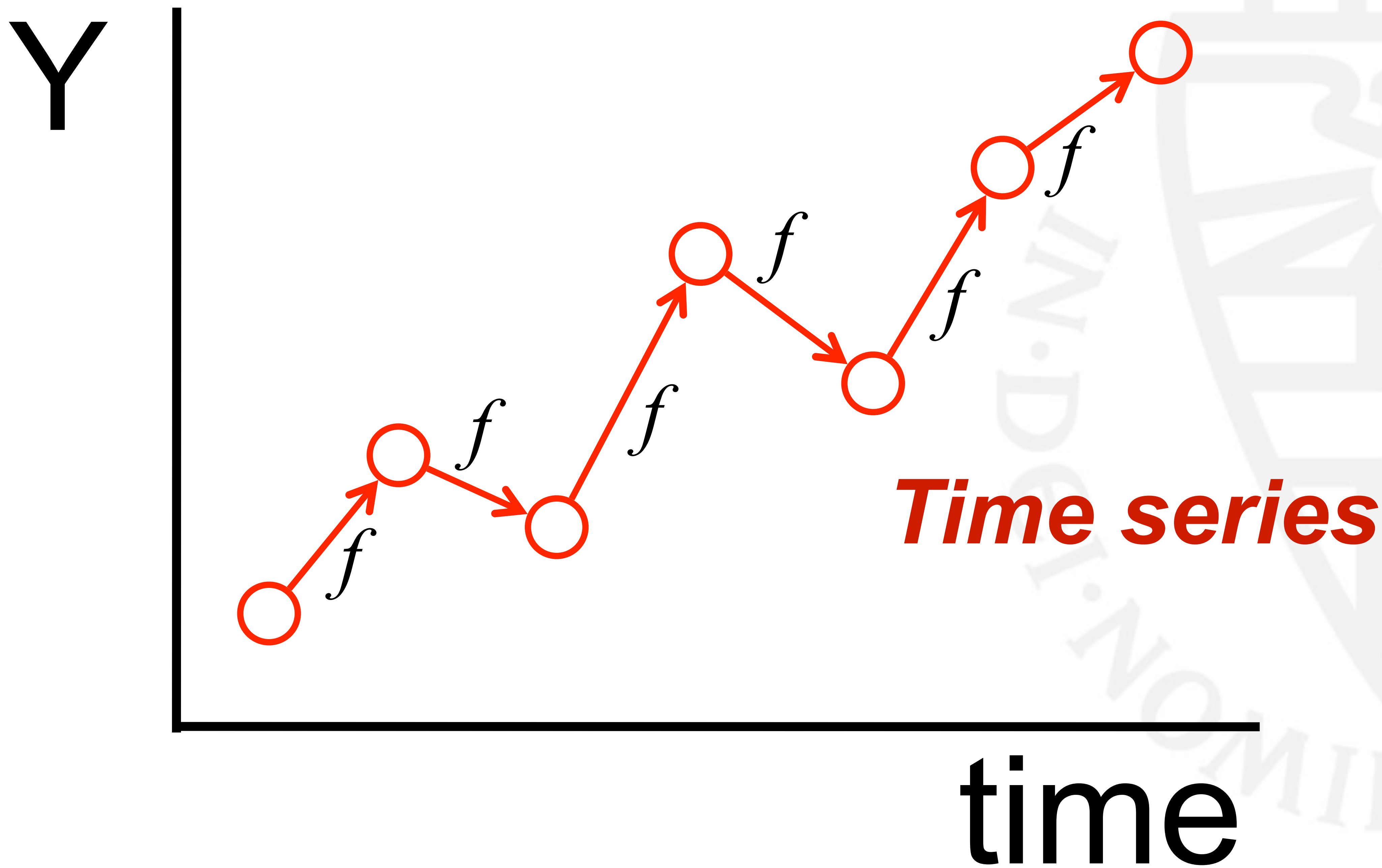
$$\hat{Y} = f(Y)$$



1refs

The mathematics of change

Complex systems: Recurrent processes / Feedback



[1]refs

Two Flavors: Flows & Maps

Dynamical models of psychological processes can be formulated in:

'Clock' time	**'Metronome' time**
Continuous System ~ Flow ~ (Differen*tial* equation)	Discrete System … Map ... (Differen*ce* equation)

1refs

PARAMETERS & BIFURCATIONS

EXAMPLE 1:

The Linear Map

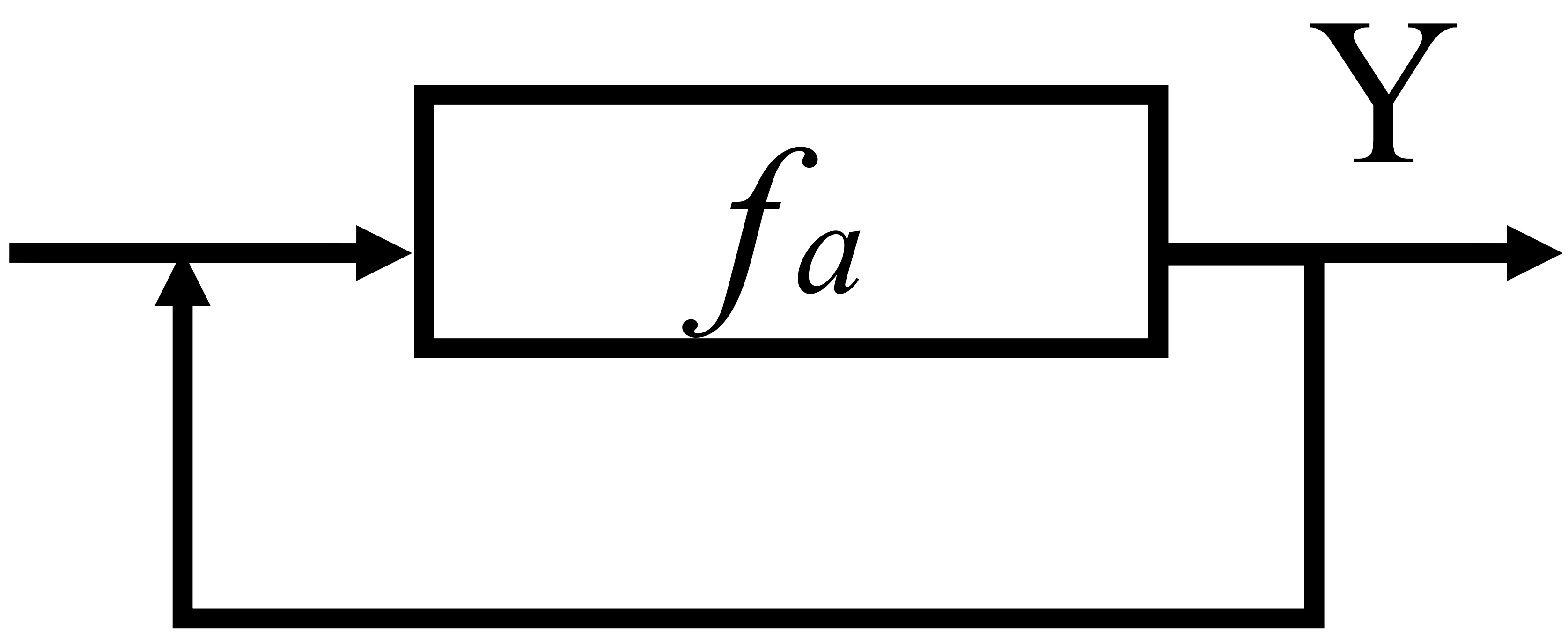
(Linear Growth)

1refs

The linear map

Dynamic Models: Parameter

$$\hat{Y} = f_a(Y)$$



1refs

The Linear Map …

The (rate of) change of the state of a system is proportional to its current state:

$$\boxed{Y_i{+}1 = a \cdot Y_i}$$

...Iteration...

[1]refs

The Linear Map

Initial value: Y_0

$Y_1 = f(Y_0)$

$Y_2 = f(Y_1)$

$Y_3 = f(Y_2)$

Iteration in general just means applying the function over and over again starting with an initial value

and subsequently to the result of the previous step

[1]refs

The Linear Map

$$\boxed{Y_{i+1} = f(Y_i)}$$

$i = 0$: $\quad Y_0 \rightarrow Y_1 = f(Y_0)$

$i = 1$: $\quad Y_1 \rightarrow Y_2 = f(Y_1) = f(f(Y_0)) = f^2(Y_0)$

$i = 2$: $\quad Y_2 \rightarrow Y_3 = f(Y_2) = \ldots = f^3(Y_0)$

$\vdots \qquad \vdots$

[1]refs

$i = n$: $\quad Y_n \rightarrow Y_{n+1} = f(Y_n) = \ldots = f^n(Y_0)$

Linear Map: Iteration with a parameter

$$\boxed{Y_{i+1} = a \cdot Y_i}$$

i = 0: $Y_0 \rightarrow Y_1 = a \cdot Y_0$

i = 1: $Y_1 \rightarrow Y_2 = a \cdot Y_1 = a \cdot a \cdot Y_0 = a^2 \cdot Y_0$

i = 2: $Y_2 \rightarrow Y_3 = a \cdot Y_2 = \ldots = a^3 \cdot Y_0$

$\vdots \qquad \vdots$

i = n: $Y_n \rightarrow Y_{n+1} = a \cdot Y_n = \ldots = a^{n+1} \cdot Y_0$

[1]refs

Linear Map: Iteration with a Parameter

$$Y_{i+1} = a \cdot Y_i$$

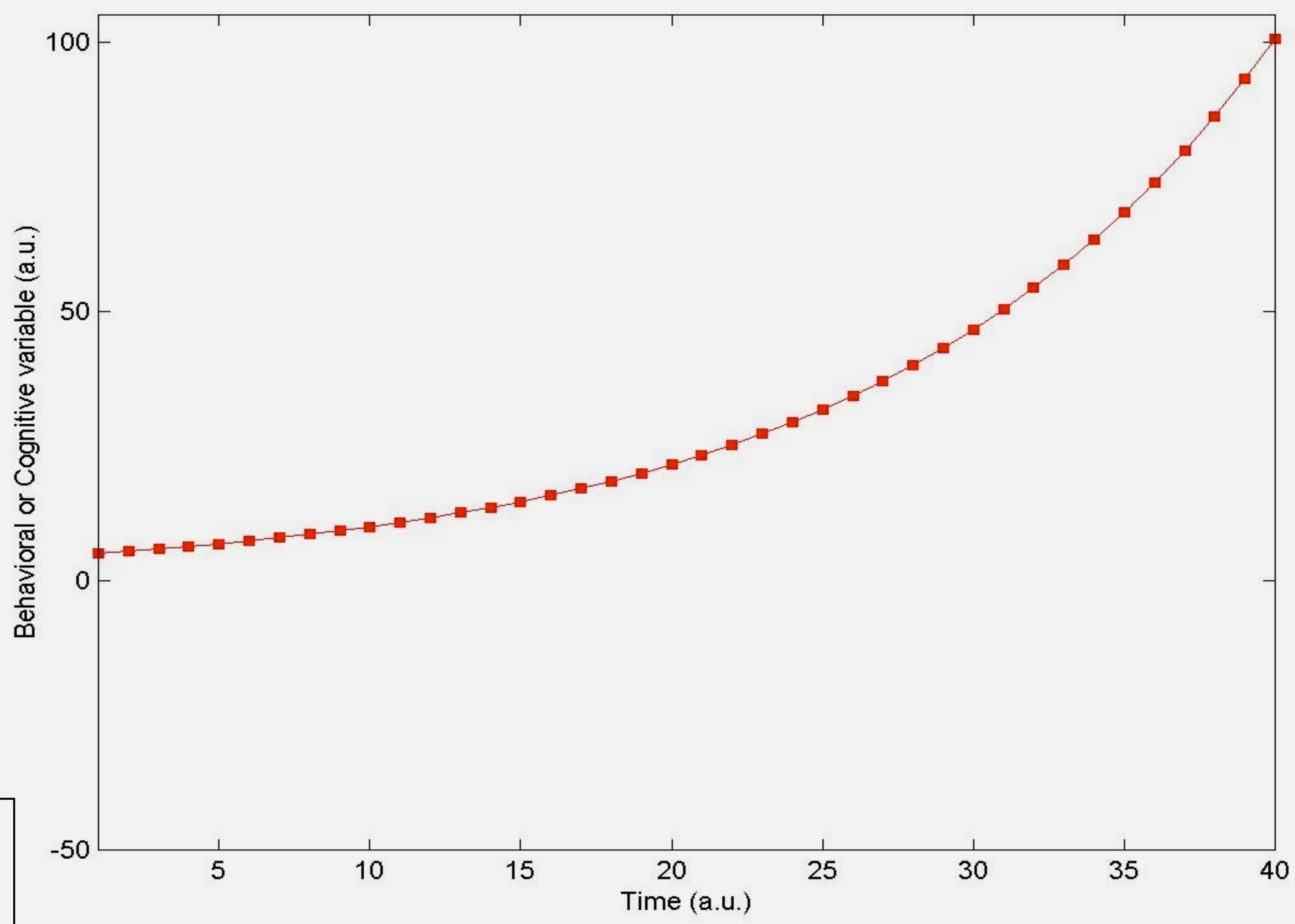
$0 < a < 1$	$-1 < a < 0$
$a > 1$	$a < -1$
$a = 1$	$a = -1$

Y_0 nonspecific

[1]refs

Linear Map: Iteration with a Parameter

a = 1.08
Y0 = 5

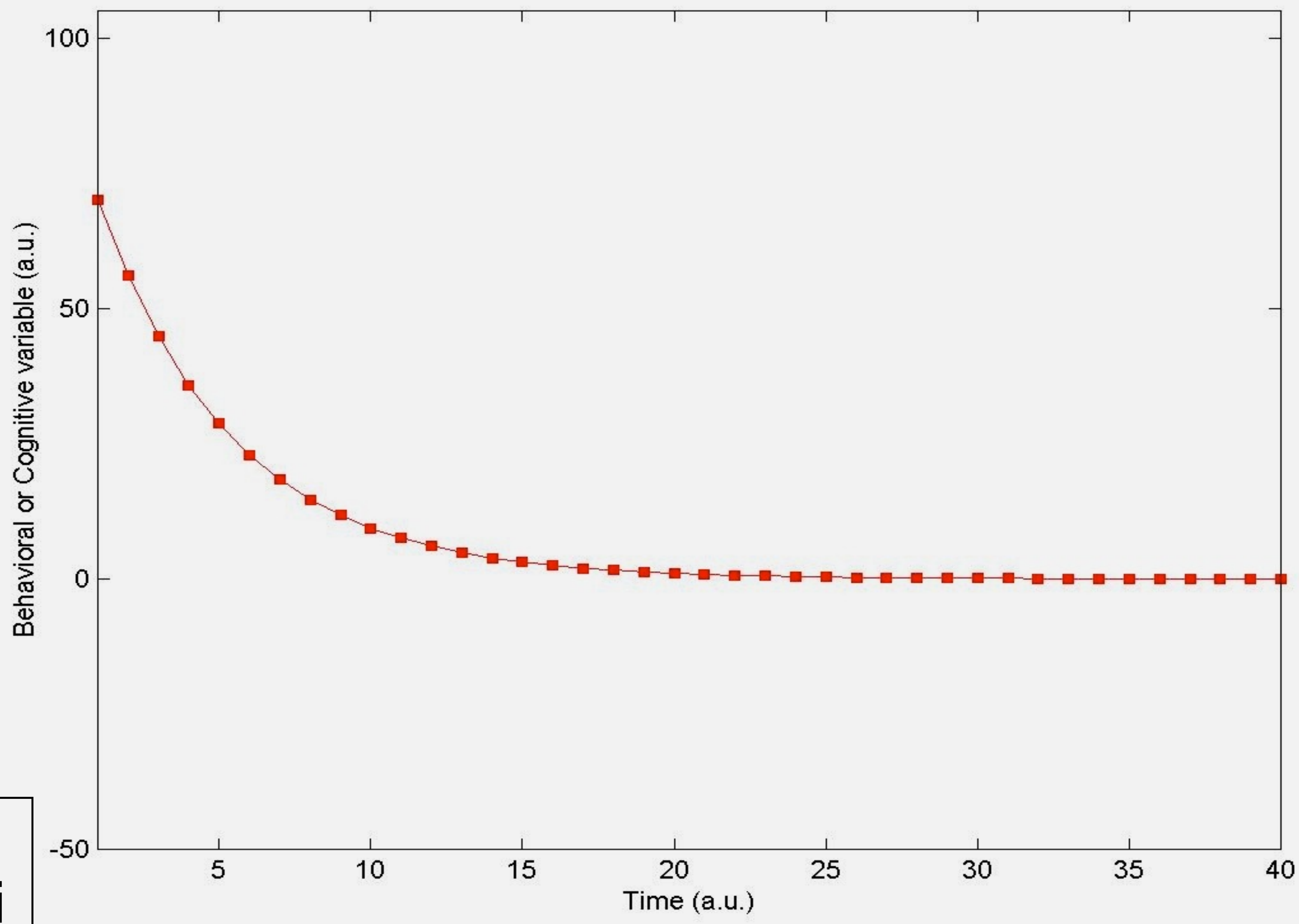


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

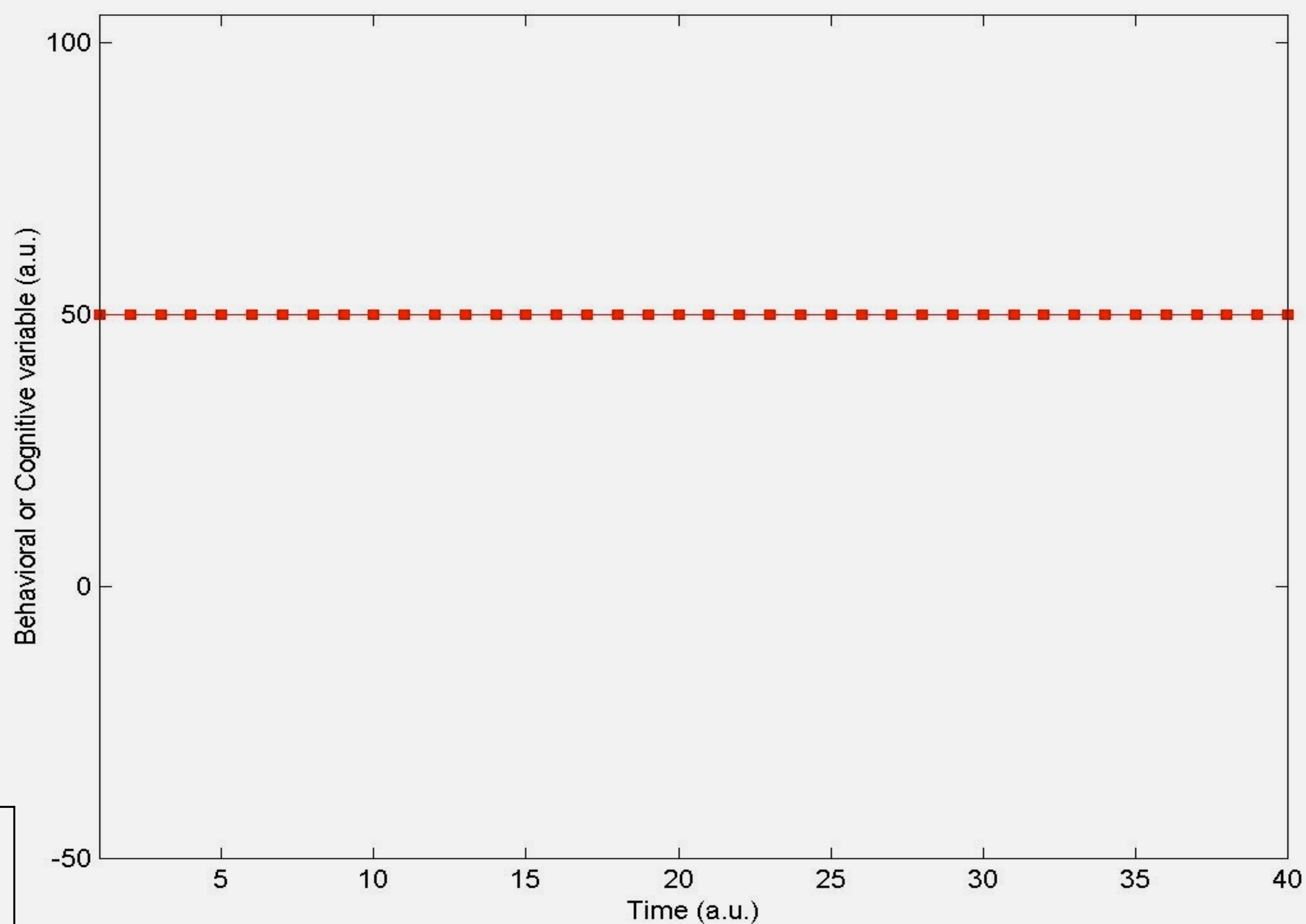
a = 0.8
Y0 = 70



$$Y_{i+1} = a \cdot Y_i$$

Linear Map: Iteration with a Parameter

a = 1.00
Y0 = 50

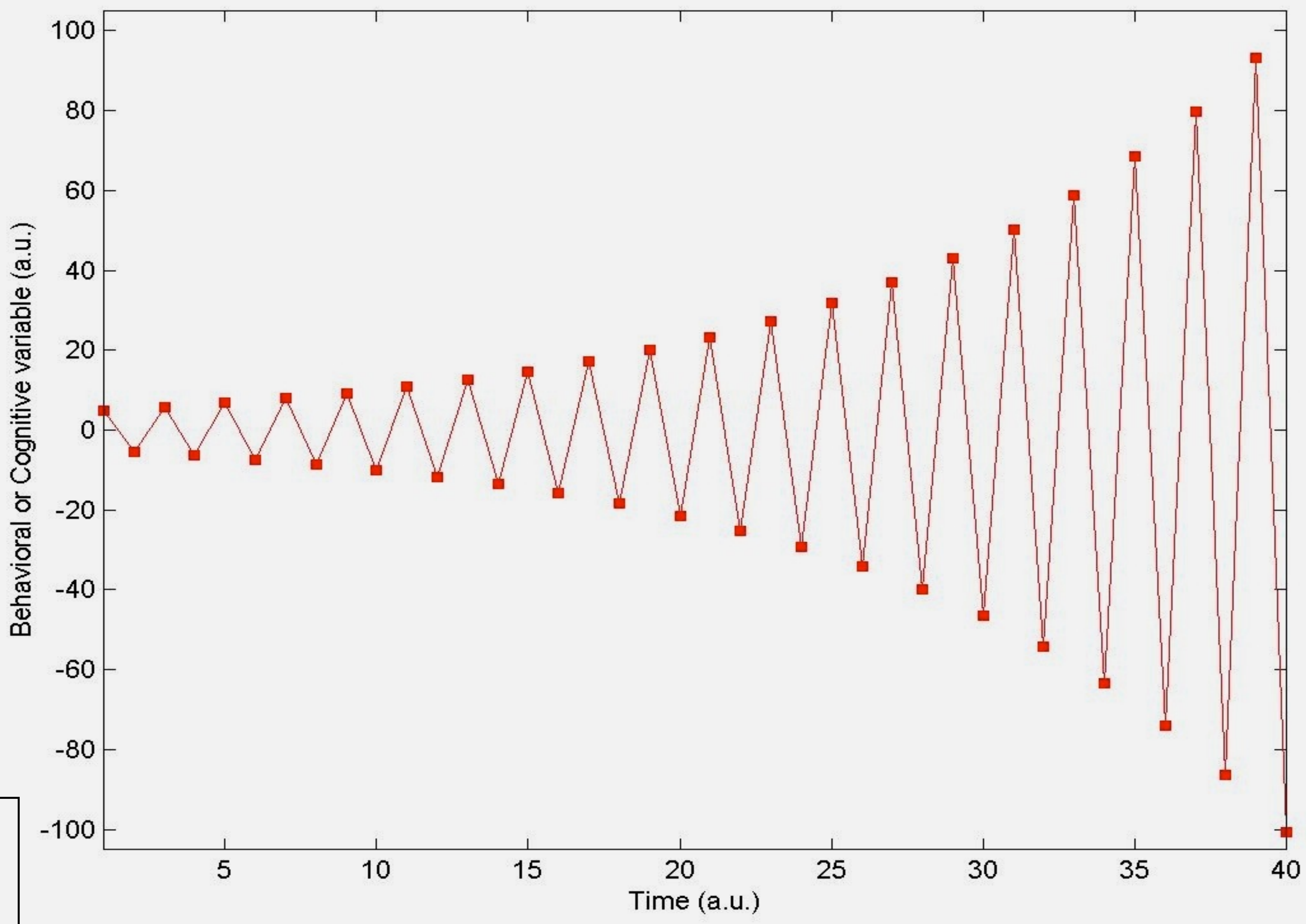


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -1.08
Y0 = 5

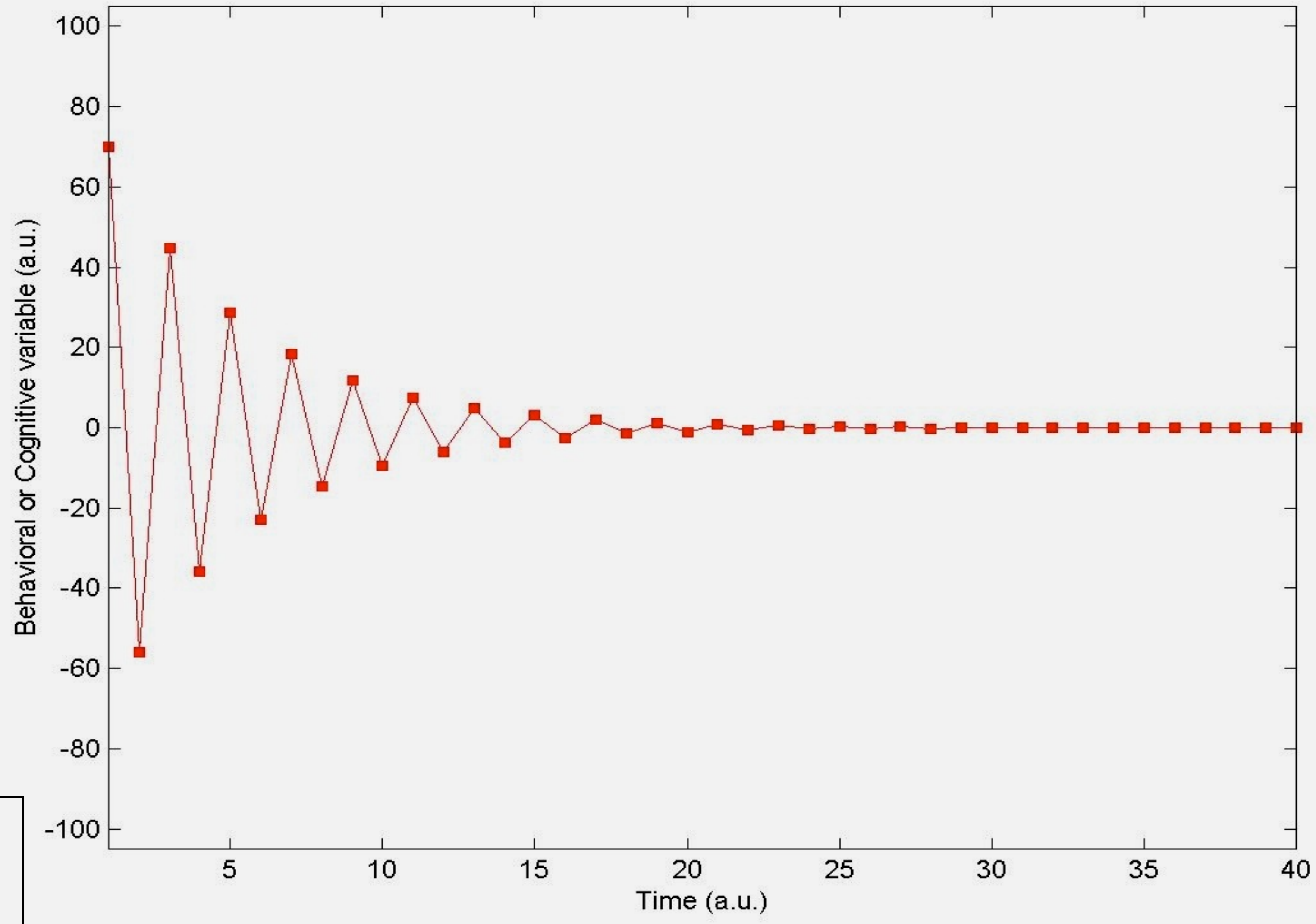


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -0.8
Y0 = 70

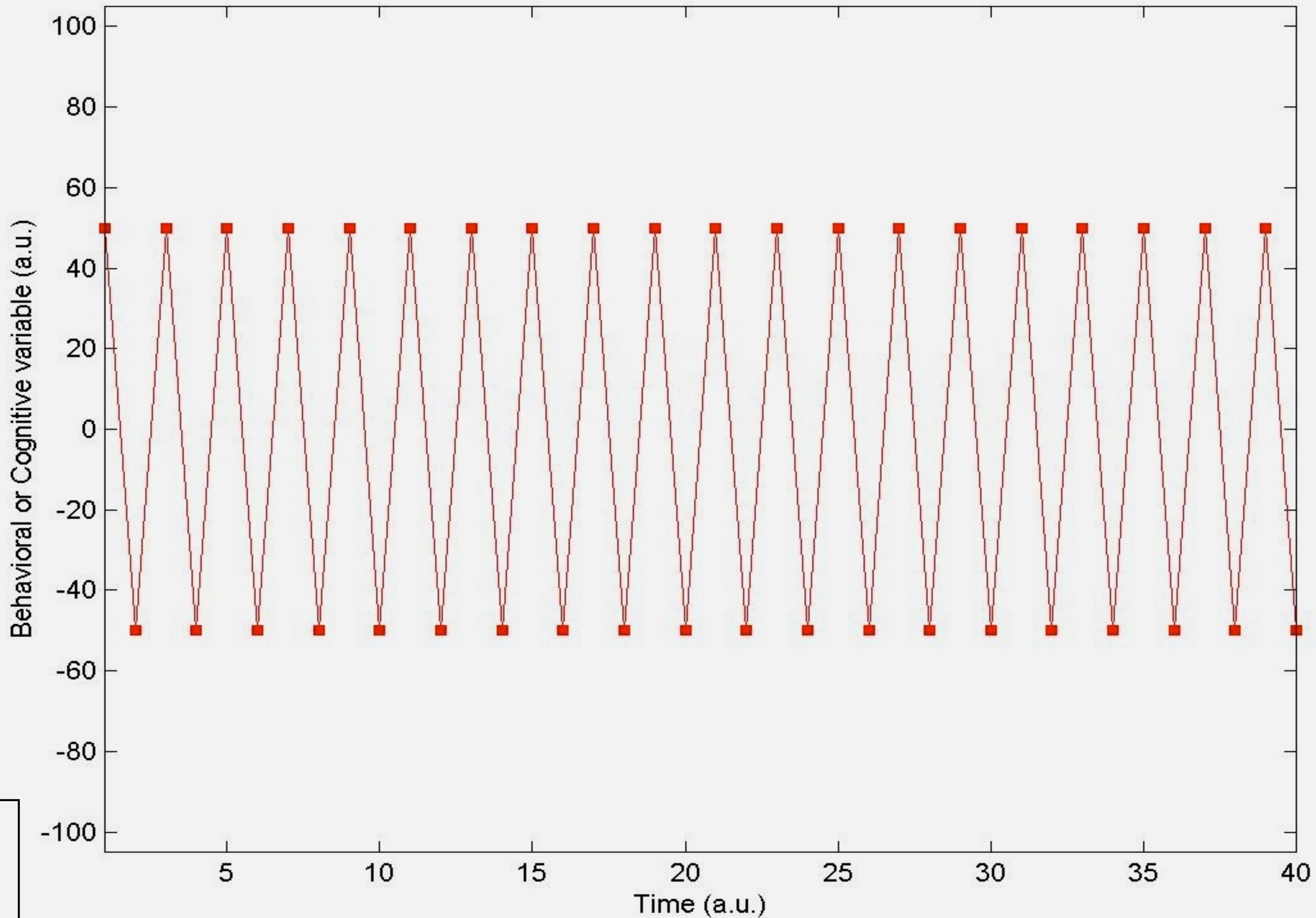


$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

a = -1.00
Y0 = 50



$$Y_{i+1} = a \cdot Y_i$$

[1]refs

Linear Map: Iteration with a Parameter

Some interesting differences compared to a linear model:

- Change of behaviour over iterations
  - *Simple model vs. "time" or "occasion" as a predictor*
- Qualitatively different behaviour
  - *One model produces at least four different types of behaviour*
  - *Not by adding predictors (components), by changing one parameter*

[1]refs

Parameters & Bifurcations

Example 2:

The Logistic Map

(restricted growth)

Logistic Map …

$$\boxed{L_{i+1} = r\,L_i\,(1 - L_i)}$$

- Simplest nontrivial model often used as an introduction to DST and Chaos theory.
- Well-known model in ecology, physics, economics and social sciences.
- ‘Styled’ version of Van Geert’s model for language growth. (*Next meeting*)

[1]refs

Logistic Map: Iteration

$$\boxed{L_{i+1} = r L_i (1 - L_i)}$$

i = 0: $L_0 \rightarrow L_1 = r L_0 (1 - L_0)$

i = 1: $L_1 \rightarrow L_2 = r L_1 (1 - L_1)$

$$= r\, rL_0(1-L_0)\,(1- rL_0(1-L_0)\,)$$

$$= -r^3 L_0^4 + 2r^3 L_0^3 - r^2(1+r)L_0^2 + r^2 L_0$$

[1]refs

Logistic Map: Parameter

$$L_{i+1} = r L_i (1 - L_i)$$

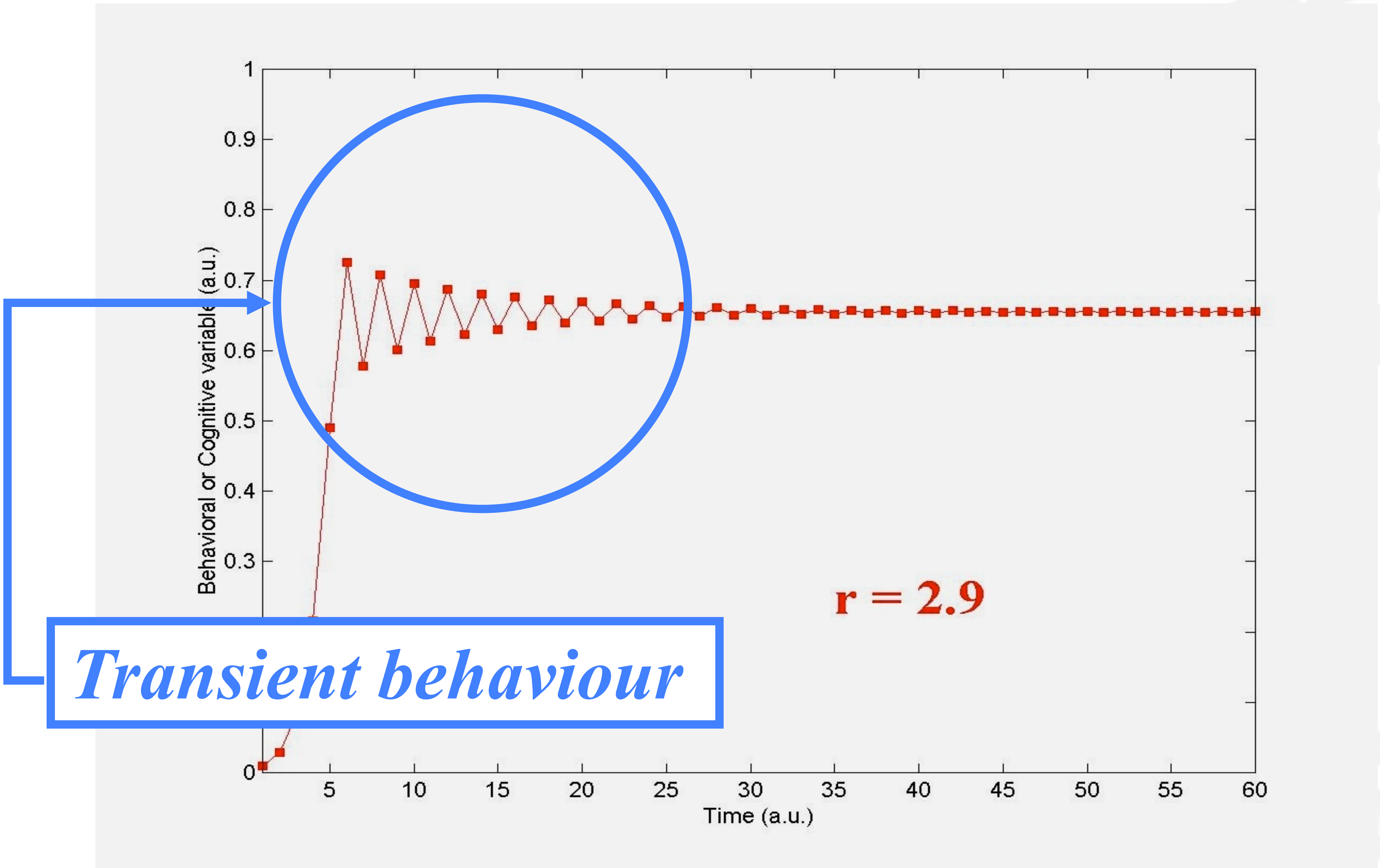
r = 0.90	r = 3.30
r = 1.90	r = 3.52
r = 2.90	r = 3.90

L_0 small

[1]refs

Logistic Map: Graphs

$$L_{i+1} = r L_i (1 - L_i)$$



[1]refs

An ecology of growth models?
Same principle!

Basic Growth Models: Exponential + Restricted Growth

$$Population = rN \times (\frac{K - N}{K})$$

Additional Parameter: Carrying Capacity

$$CognitiveGrowth = L_i(1 + r \times \frac{K - L_i}{K})$$

$$StylizedLogistic = rY_i \times (\frac{1 - Y_i}{1})$$

Bifurcation Diagram

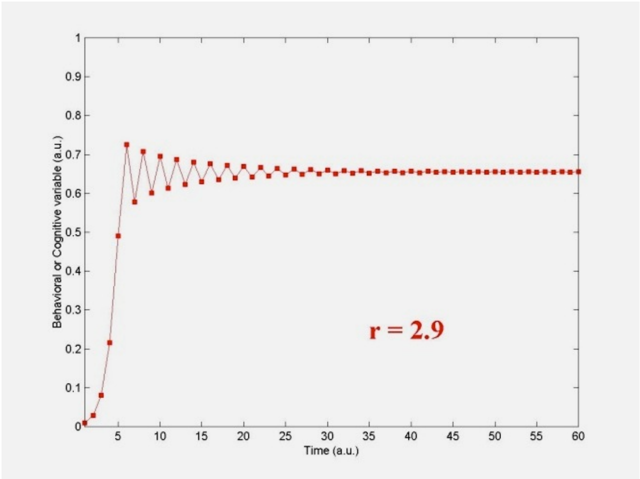
Bifurcation Diagram - Phase Diagram

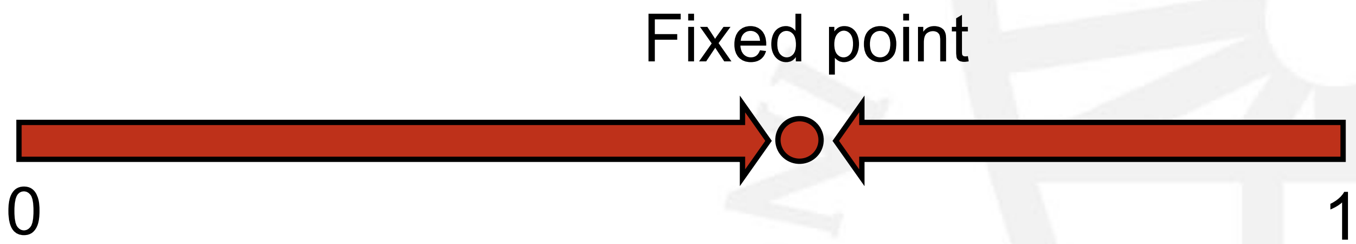
A graphical representation of the possible states a dynamical system can end up in for different values of one or more parameters.

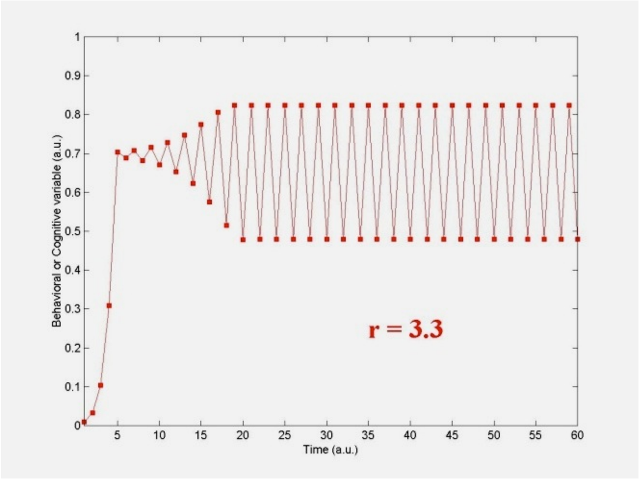
- The parameter is called the ***control parameter***.
- The end states are called ***attractors***.
- The change from one attractor (or set) to another is called a ***bifurcation***.

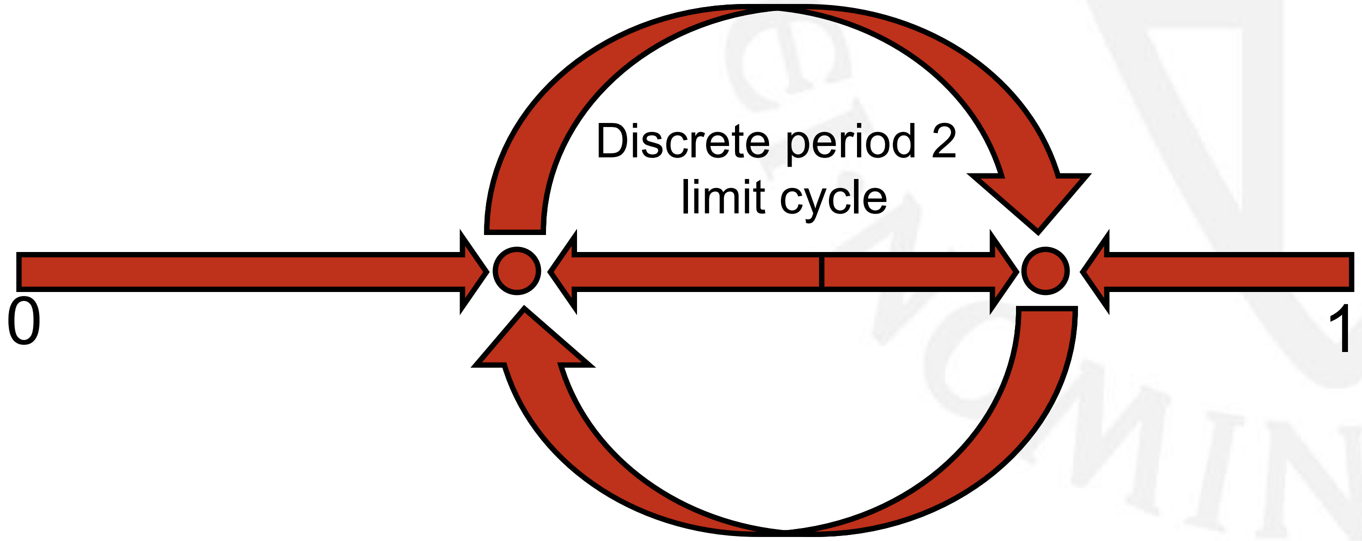
End states are attractors in state space: Attractor types

State Space is an abstract space used to represent the behaviour of a system. Its dimensions are the variables of the system. Thus a point in the phase space defines a potential state of the system. The points actually achieved by a system depend on its iterative function and initial condition (starting point).









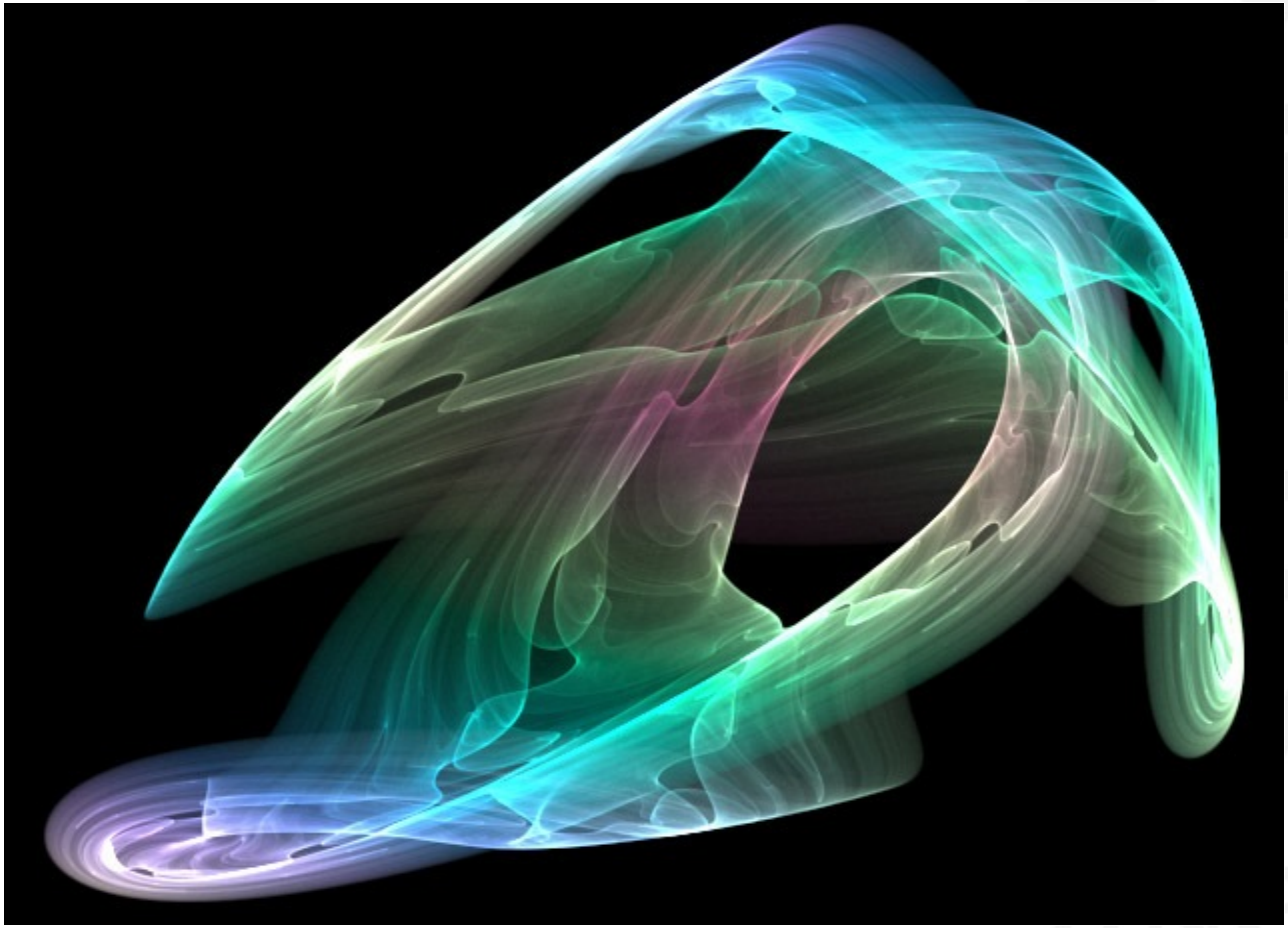
State space, Attractor types

“Saturn”
attractor

Strange attractors
are quasi periodic
and bounded

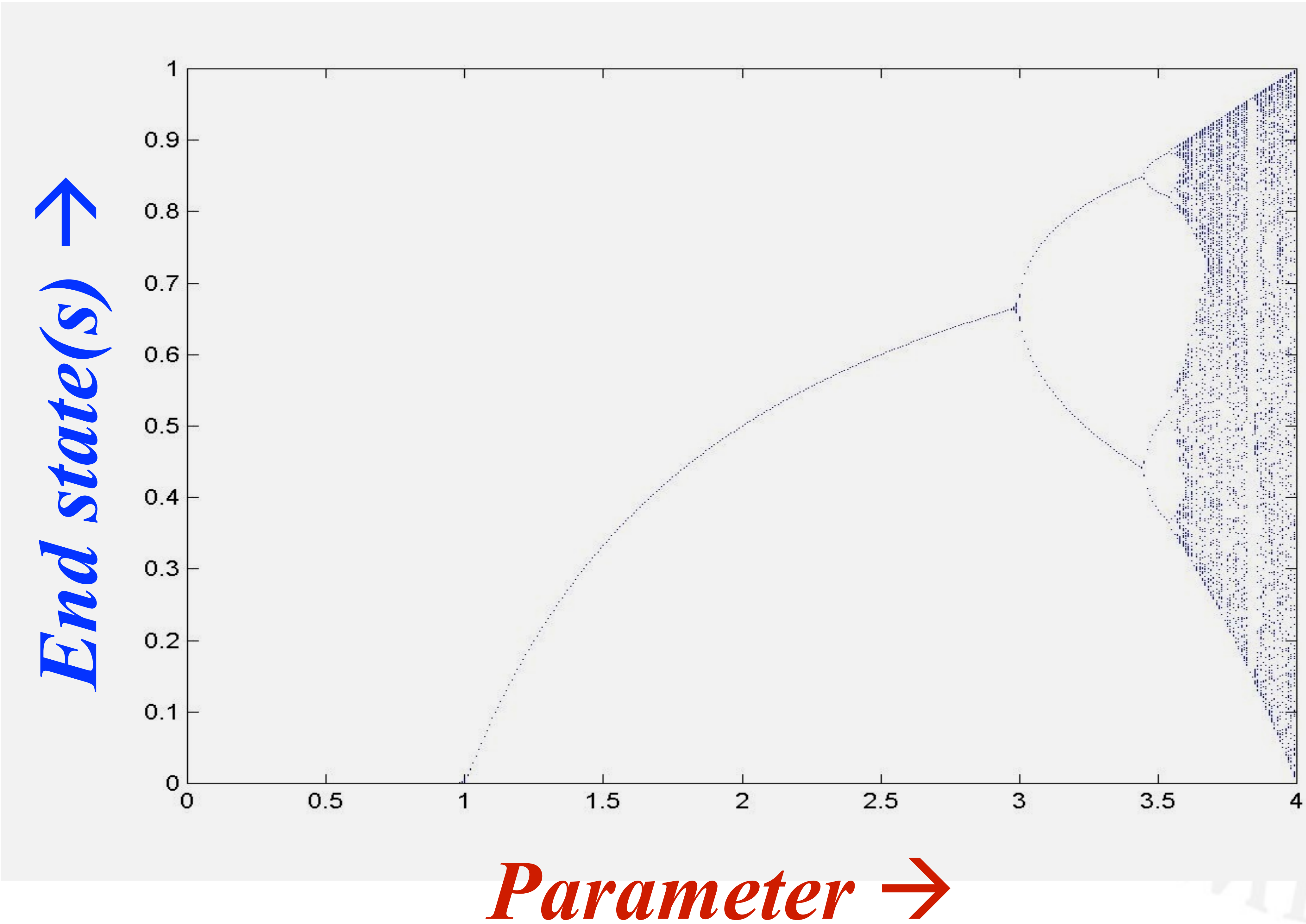
Bottom line:

An attractor means
a limited region
of state space
is visited.
Not all DF actually
available
to the system
are used.



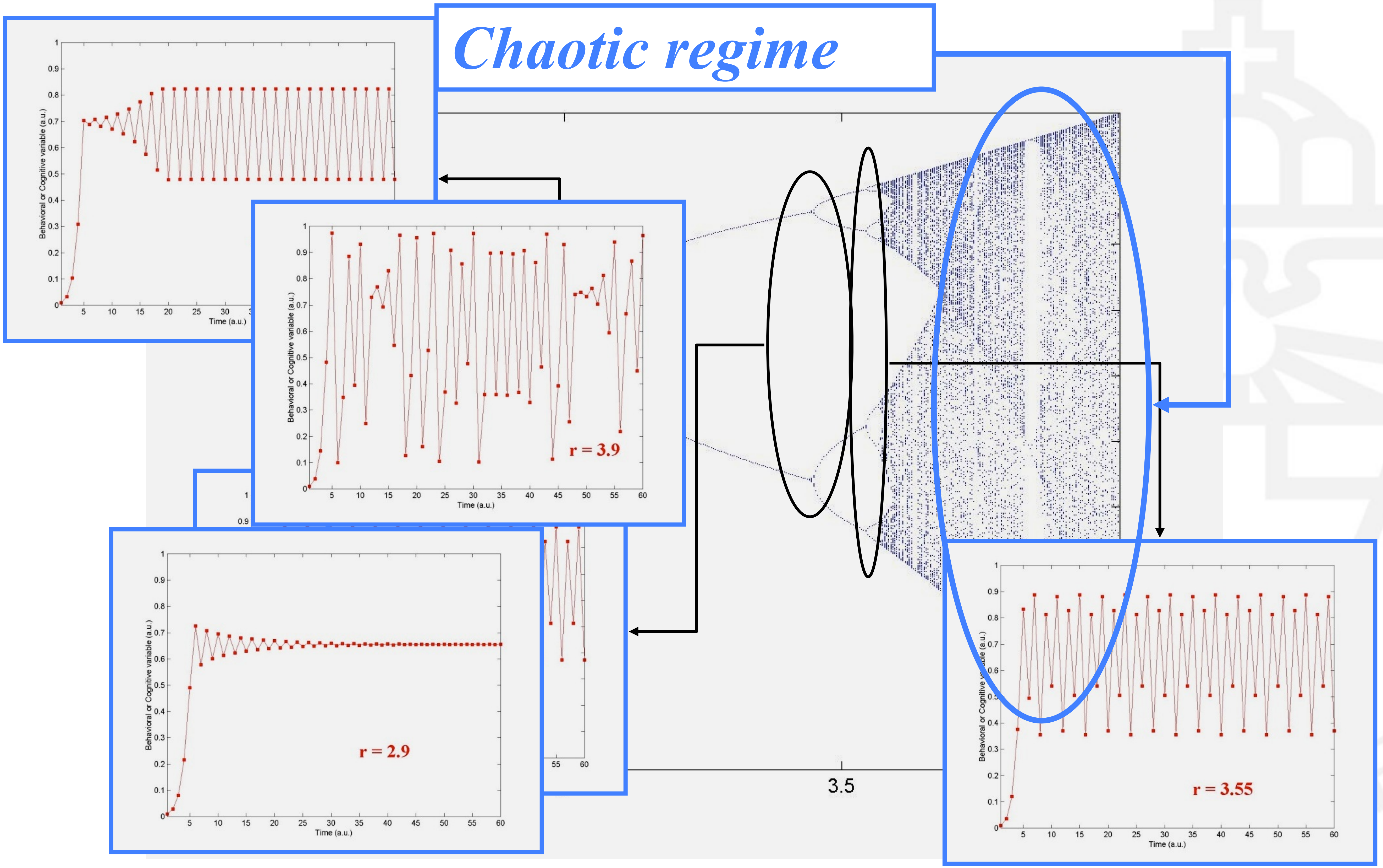
http://www.da4ga.nl/wp-content/uploads/2012/03/PastedGraphic-2-1.jpg

Logistic Map: Bifurcation Diagram

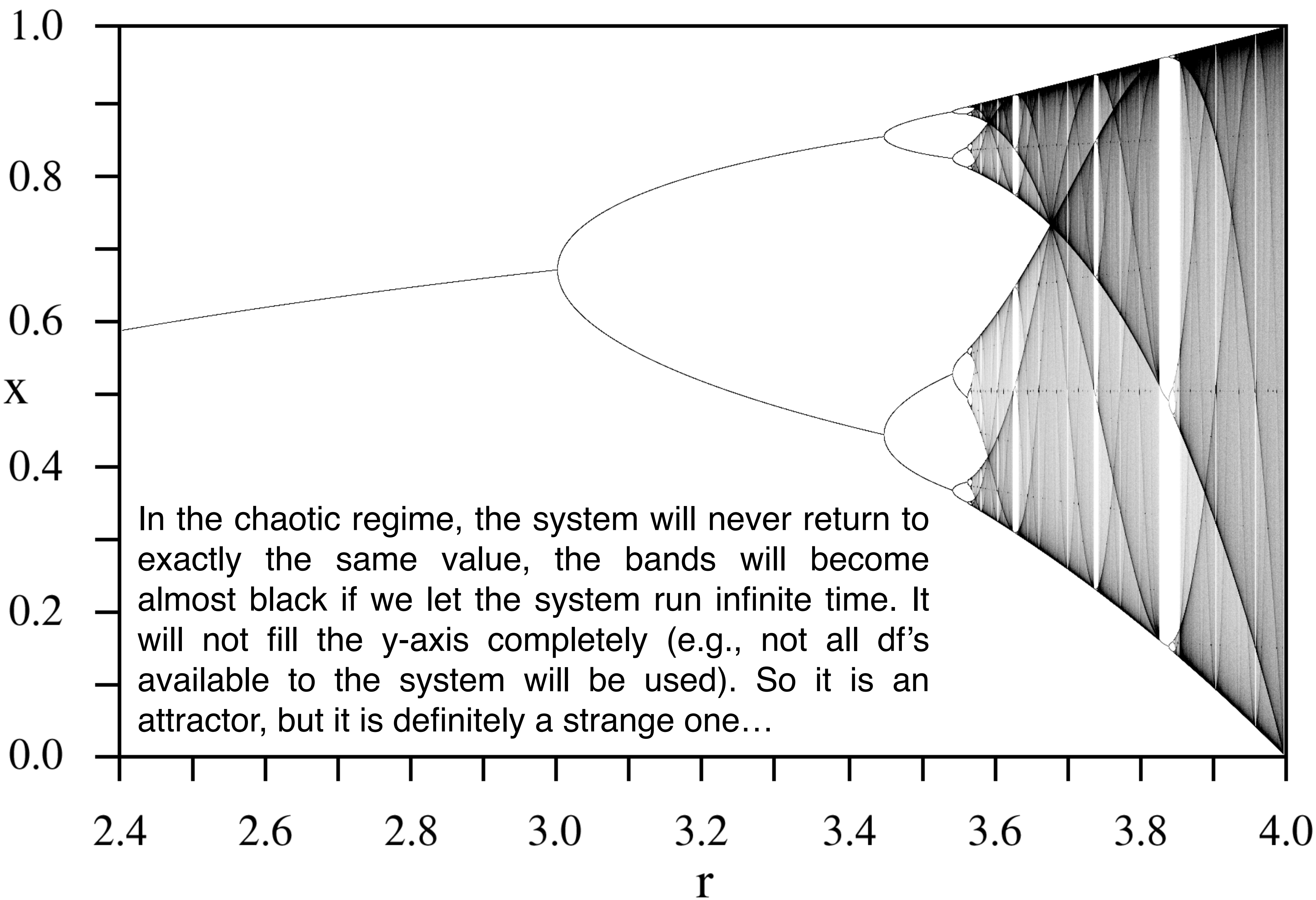


[1]refs

Chaotic regime



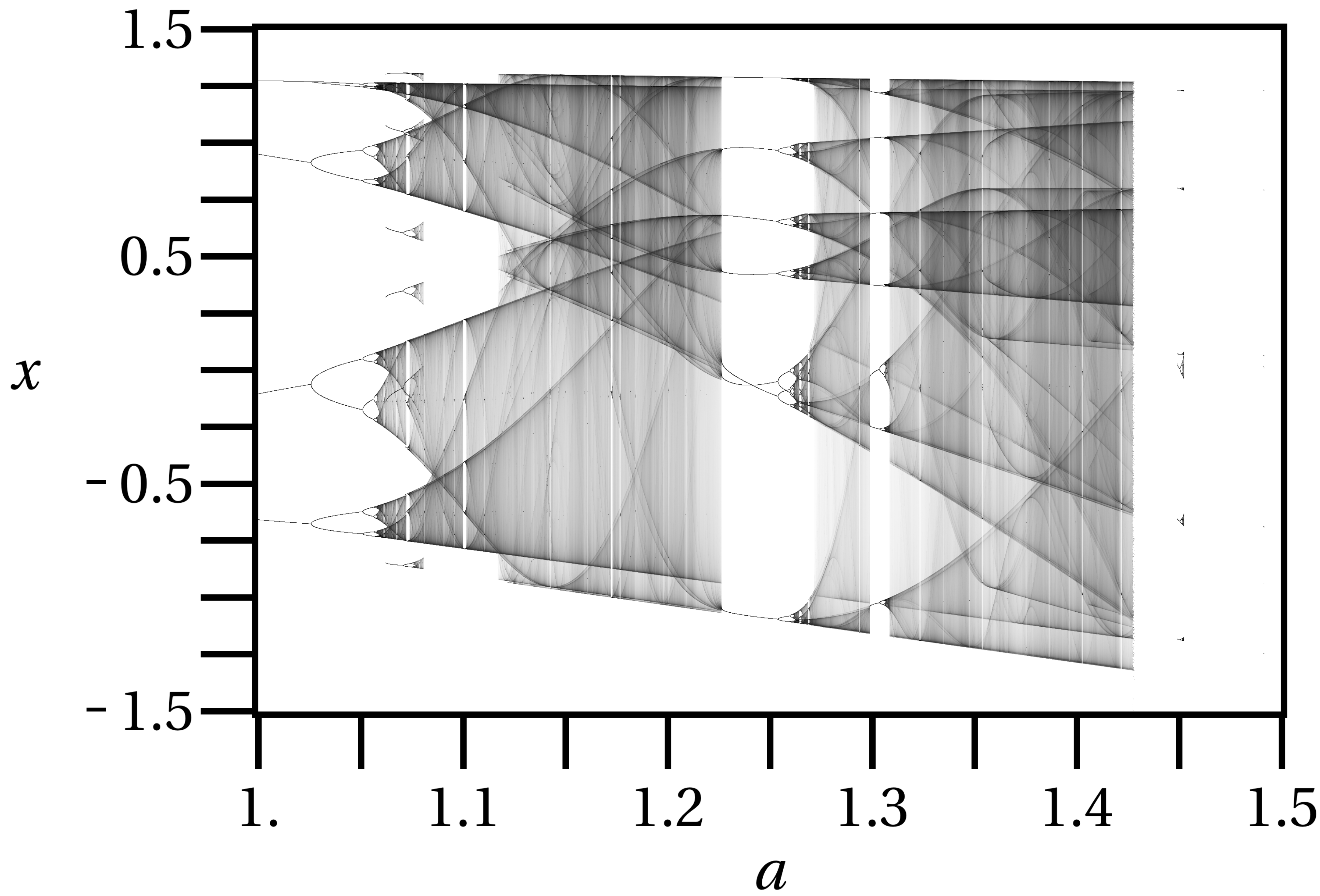
Logistic Map: Bifurcation Diagram



In the chaotic regime, the system will never return to exactly the same value, the bands will become almost black if we let the system run infinite time. It will not fill the y-axis completely (e.g., not all df's available to the system will be used). So it is an attractor, but it is definitely a strange one…

http://upload.wikimedia.org/wikipedia/commons/7/7d/LogisticMap_BifurcationDiagram.png

Henon Map: Bifurcation Diagram



http://upload.wikimedia.org/wikipedia/commons/c/cd/Henon_bifurcation_map_b%3D0.3.png

Deterministic Chaos

1refs

CHAOS, TURBULENCE
and other unsolved mysteries

"Turbulence is the most important unsolved problem of classical physics"

\- Richard Feynman (1918 - 1988)

"I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment:

One is quantum electrodynamics,
and the other is the turbulent motion of fluids.

And about the former I am rather optimistic."

\- Horace Lamb (1849 - 1934)

Turbulent

Laminar

Deterministic Chaos

Table 12-1. Summary of the Hierarchy of Dynamic Systems.

Type	Constraints	Description
Zero	Absolute	Constant state
I	Analytic integrals	Solvable dynamic system
II	Approximate analytic integrals	Amenable to perturbation theory
III	Quasi-deterministic; smooth but erratic trajectory	Chaotic dynamic system
IV	Rigorously defined only by averages over time or state space	Turbulent/stochastic

Table 12-2. A few examples of the types of dynamic systems.

Type	Examples
Zero	Images, gravity models, structures
I	Gear trains, 2-body problem, physical pendulum
II	Satellite orbits, lunar and planetary theories
III	Climatology, Lorenz equations, discrete logistic equation
IV	Quantum mechanics, turbulent flow, statistical mechanics

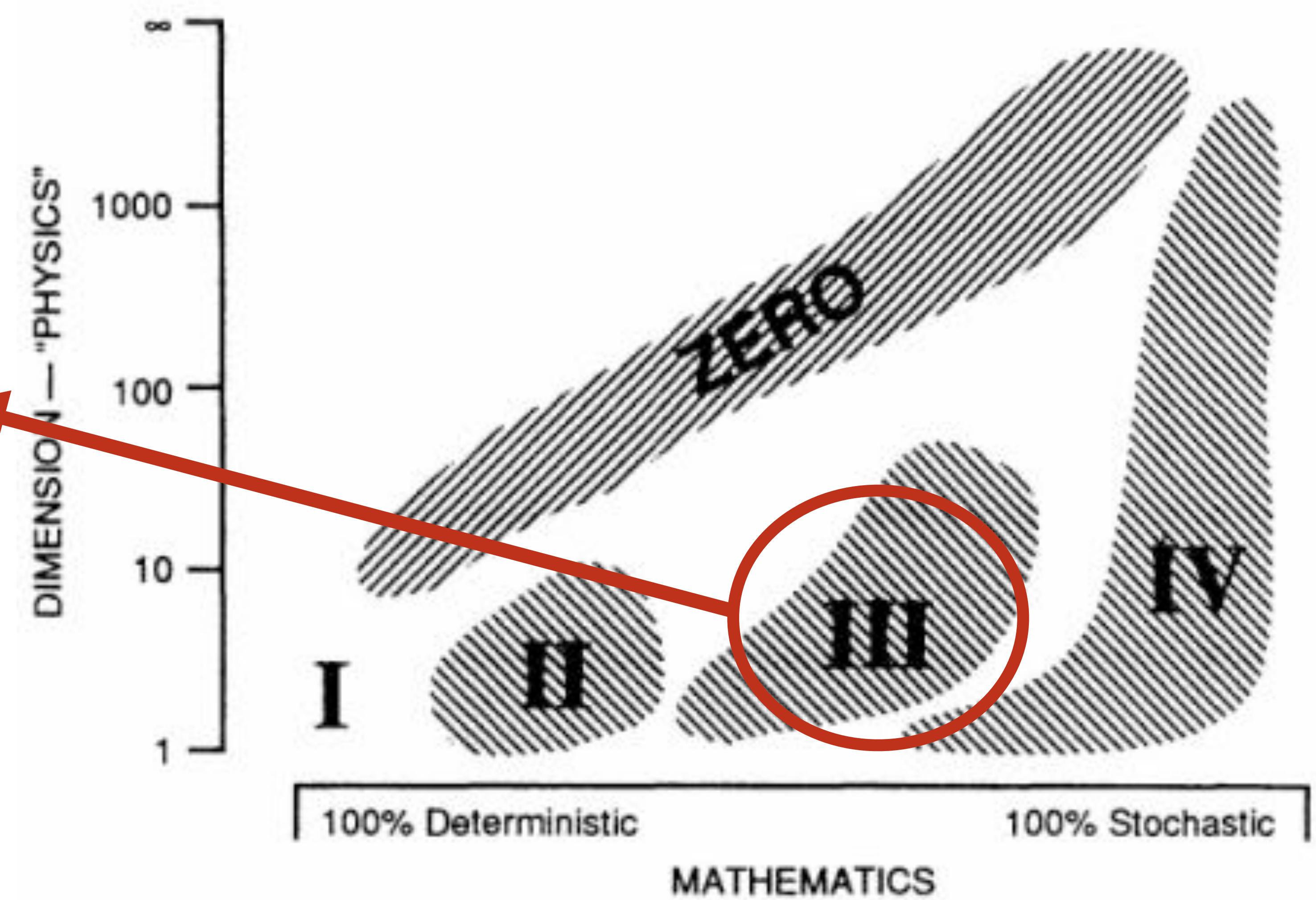


Figure 12-1. Schematic representation of the Hierarchy of Dynamic Systems.

Deterministic Chaos

There is no real definition of chaos, but there are at least four ingredients:

*The dynamics is **a-periodic** and **bounded**, and the system is **deterministic** and **sensitively depends on initial conditions**.*

[1]refs

Deterministic Chaos… Paradox?

Something that is ***deterministic***, is:

- *Mathematically exact;*
- *Predictable*.

Something that is ***'chaotic'***, shows:

- *Disorderly behaviour;*
- *Extreme sensitivity*.

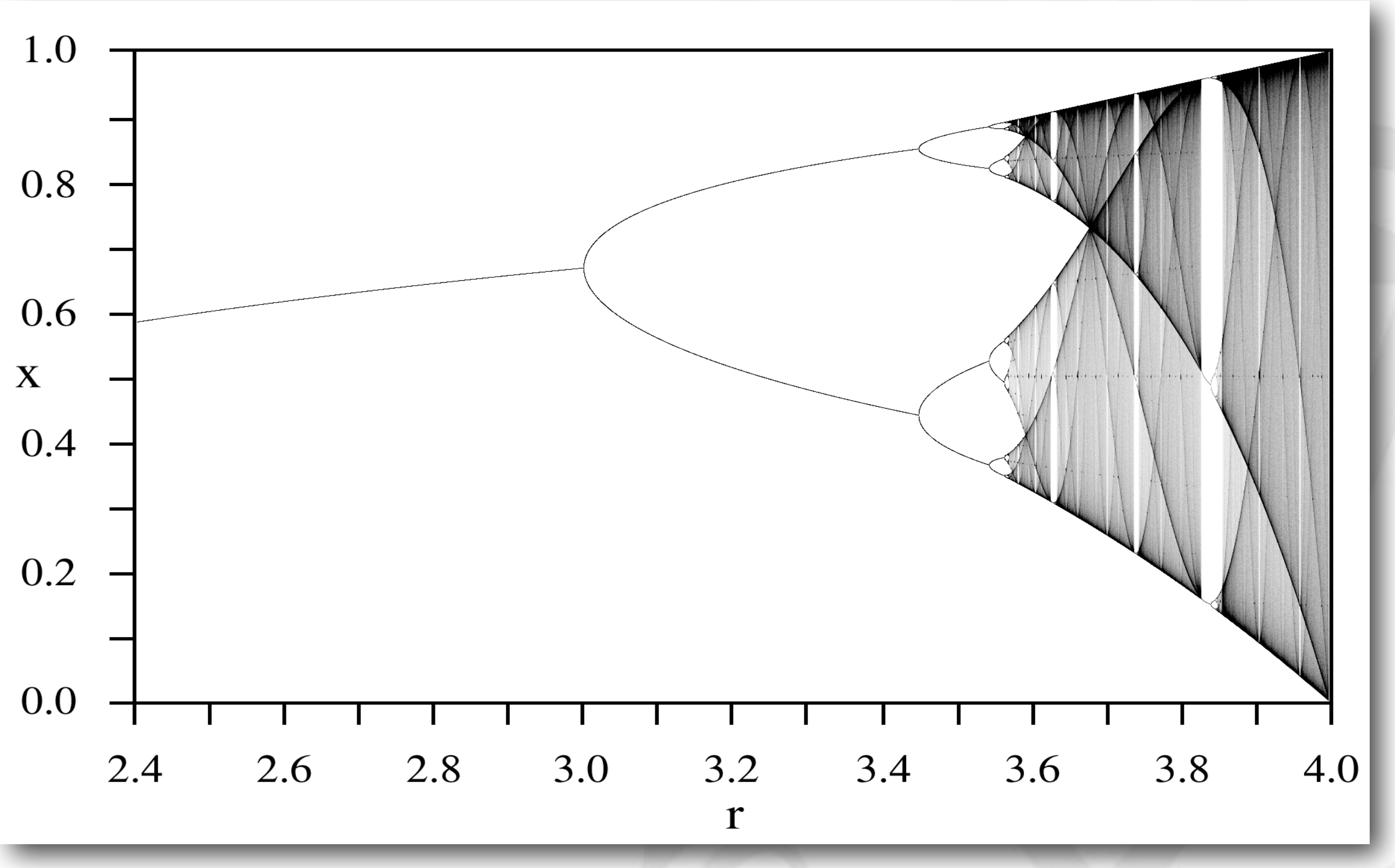
[1]refs

CHAOS, TURBULENCE
and other unsolved mysteries

Chaotic regime of the logistic map represented by the bifurcation diagram

Transitions between regimes:

- Order to Order
- Order to Chaos
- Chaos to Order
- Chaos to Chaos



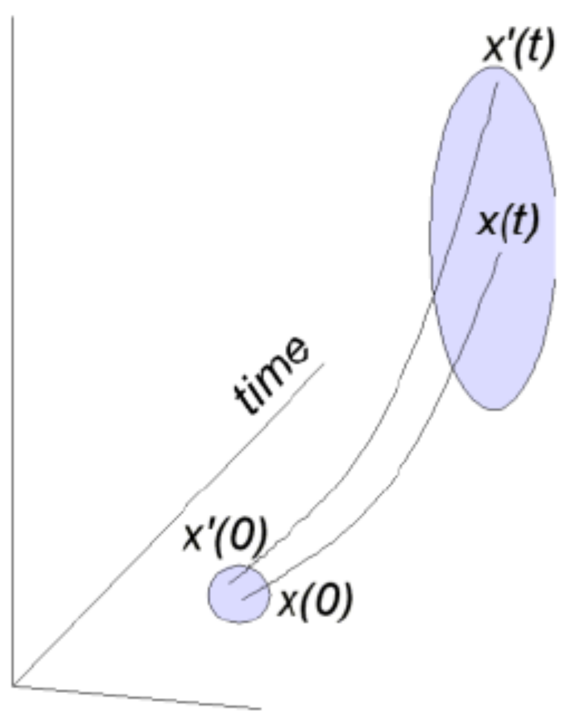
Why this happens at these parameter settings is…. unknown

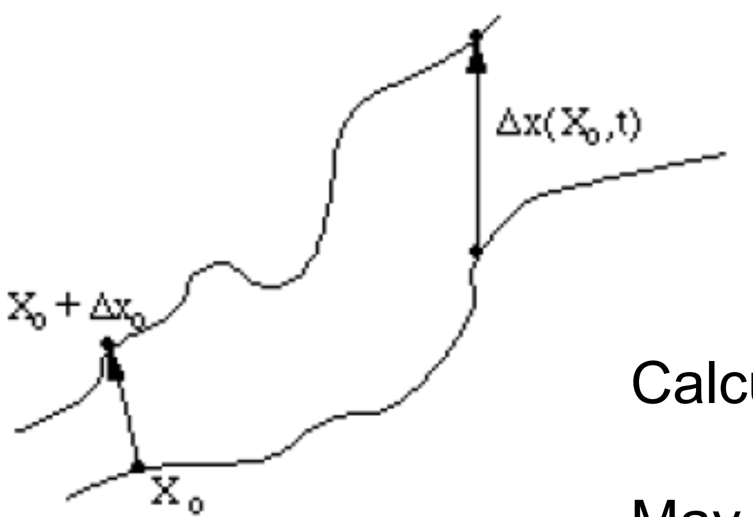
CHAOS, TURBULENCE
and other unsolved mysteries

What *can* we say about chaos?

4. Sensitive dependence on initial conditions

The *Lyapounov Exponent* characterises (quantifies) the rate of separation of two infinitesimally close trajectories in state space.





Calculate if you have a model

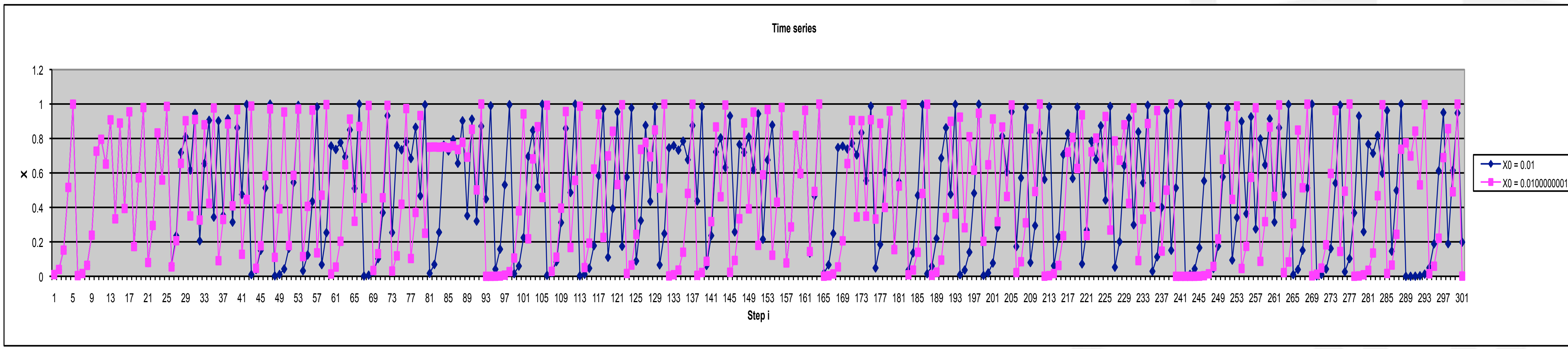
May be experimentally accessible

Analytic techniques (in R) are available

Sensitive Dependence on Initial Conditions

What *can* we say about deterministic chaos and complexity?

$X_0 = 0.01$ **$X_0 = 0.01000000001$**



Tiny differences in initial conditions can yield diverging time-evolutions of system states

Lorenz observed this in his models of the upper atmosphere:
The divergence was so extreme it resembled a butterfly flapping its wings -or not- could be the difference between weather developing as a hurricane or a summer breeze

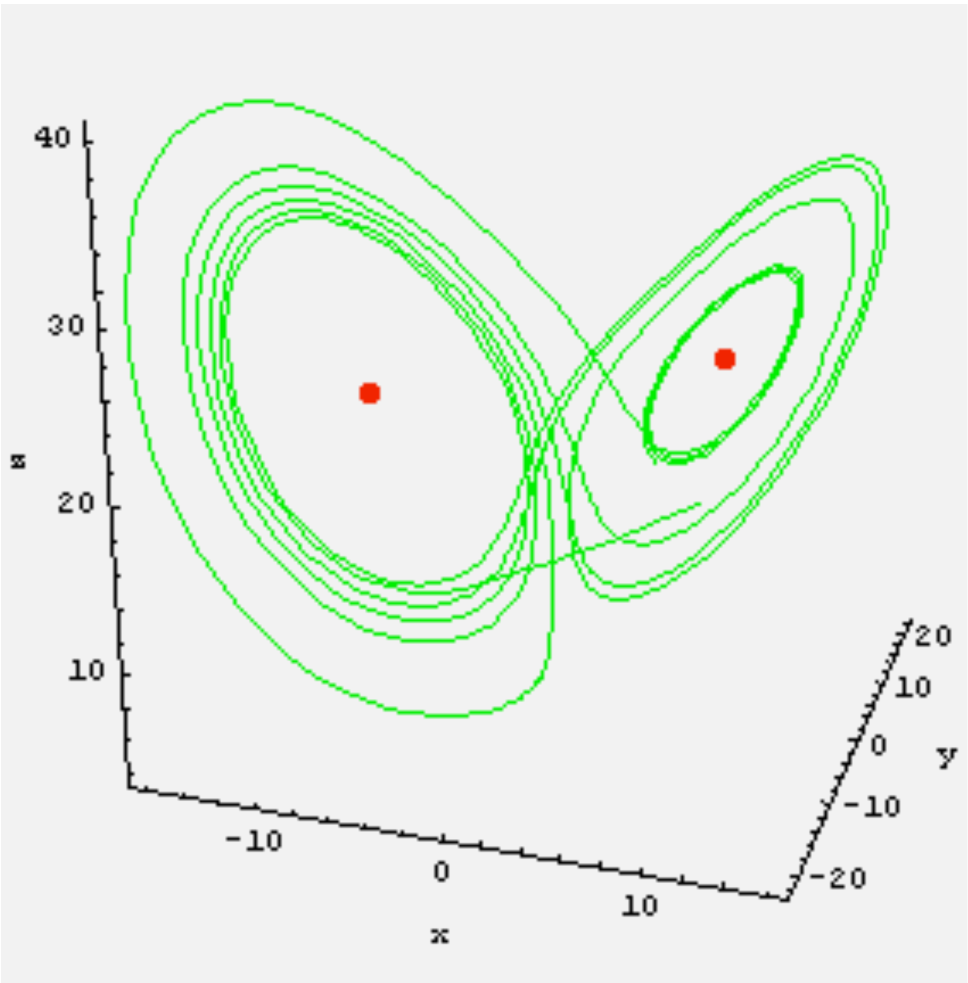
Lorenz Attractor

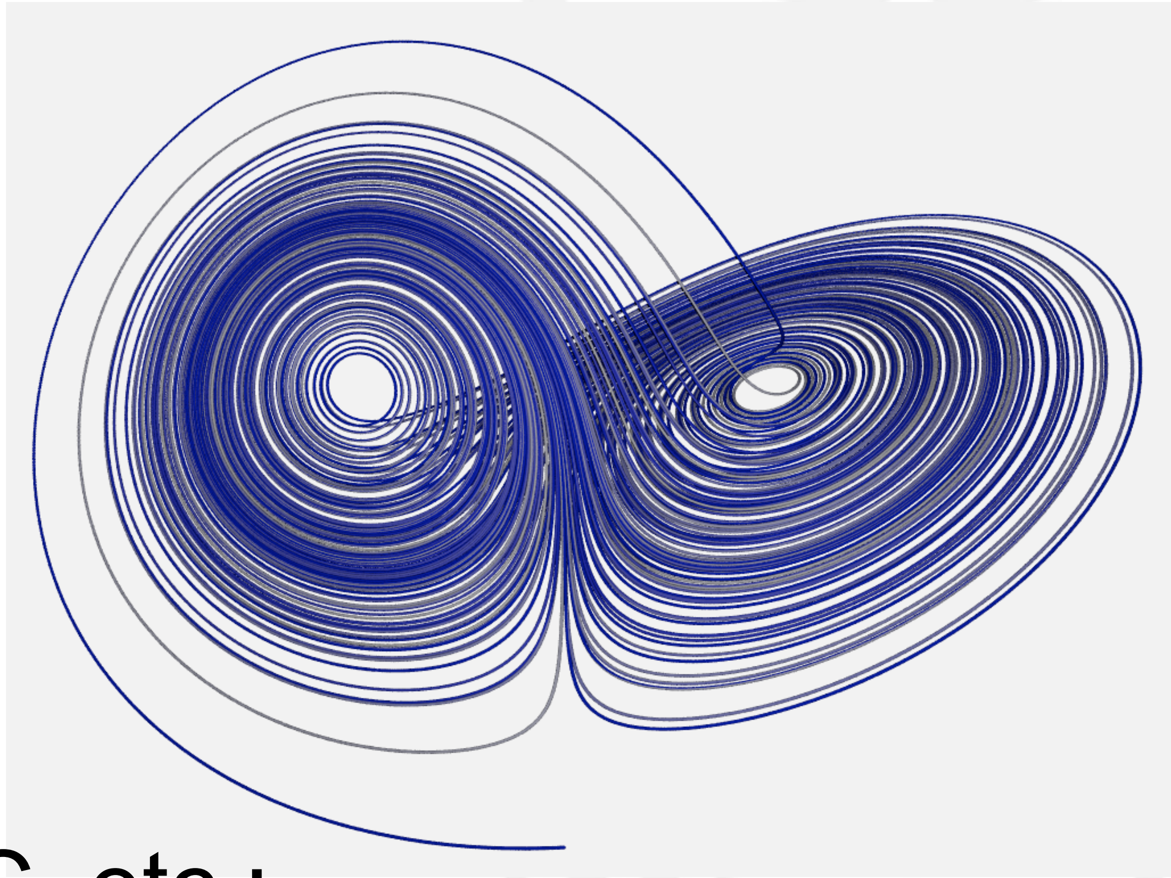
$$\frac{dx}{dt} = a(y - x),$$
$$\frac{dy}{dt} = x(b - z) - y,$$
$$\frac{dz}{dt} = xy - cz.$$

Deterministic Chaos

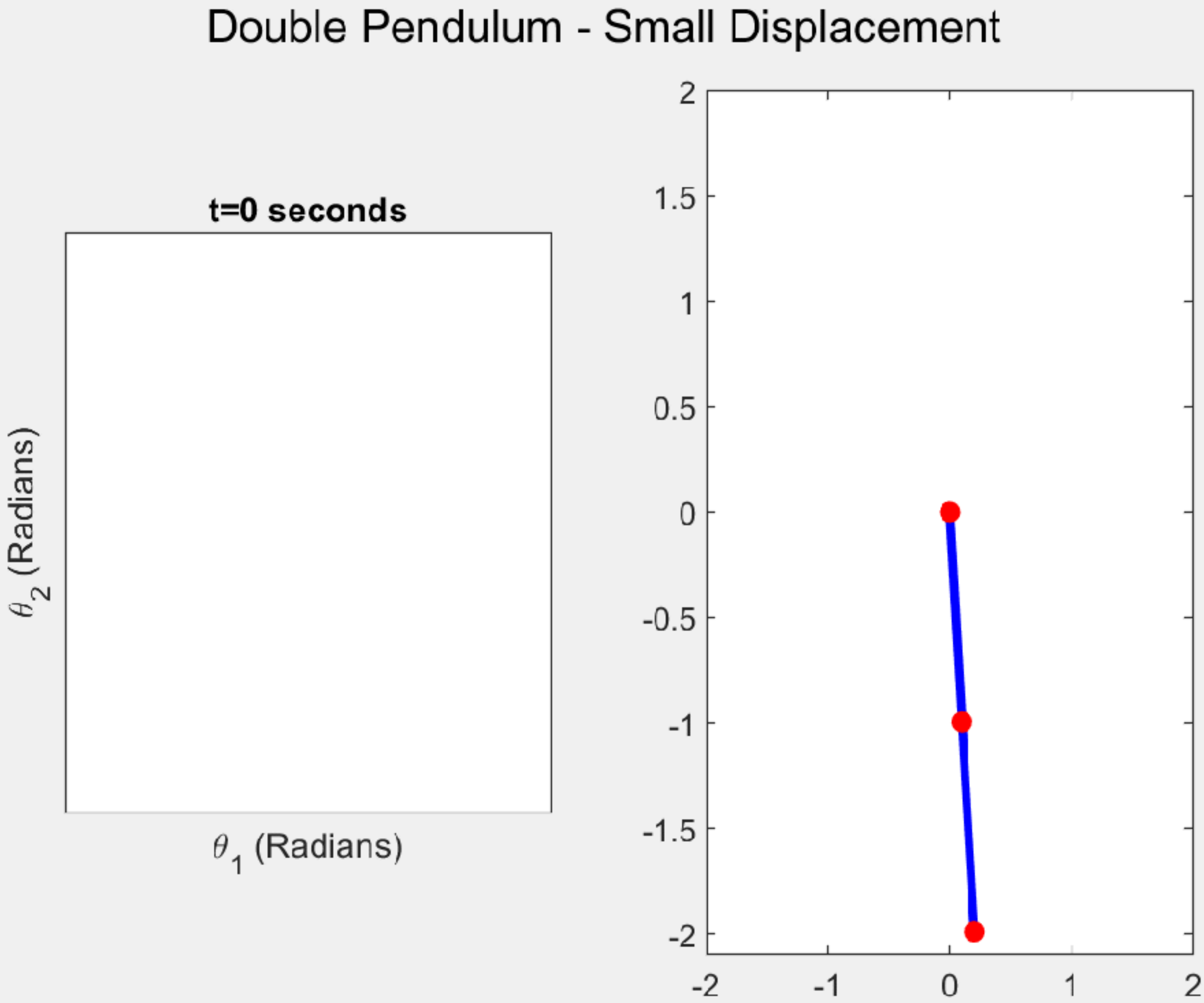
Maps: linear map, 1D state space

Flows: Need 3 coupled ODEs (ordinary differential equations) Minimum is 3D state space



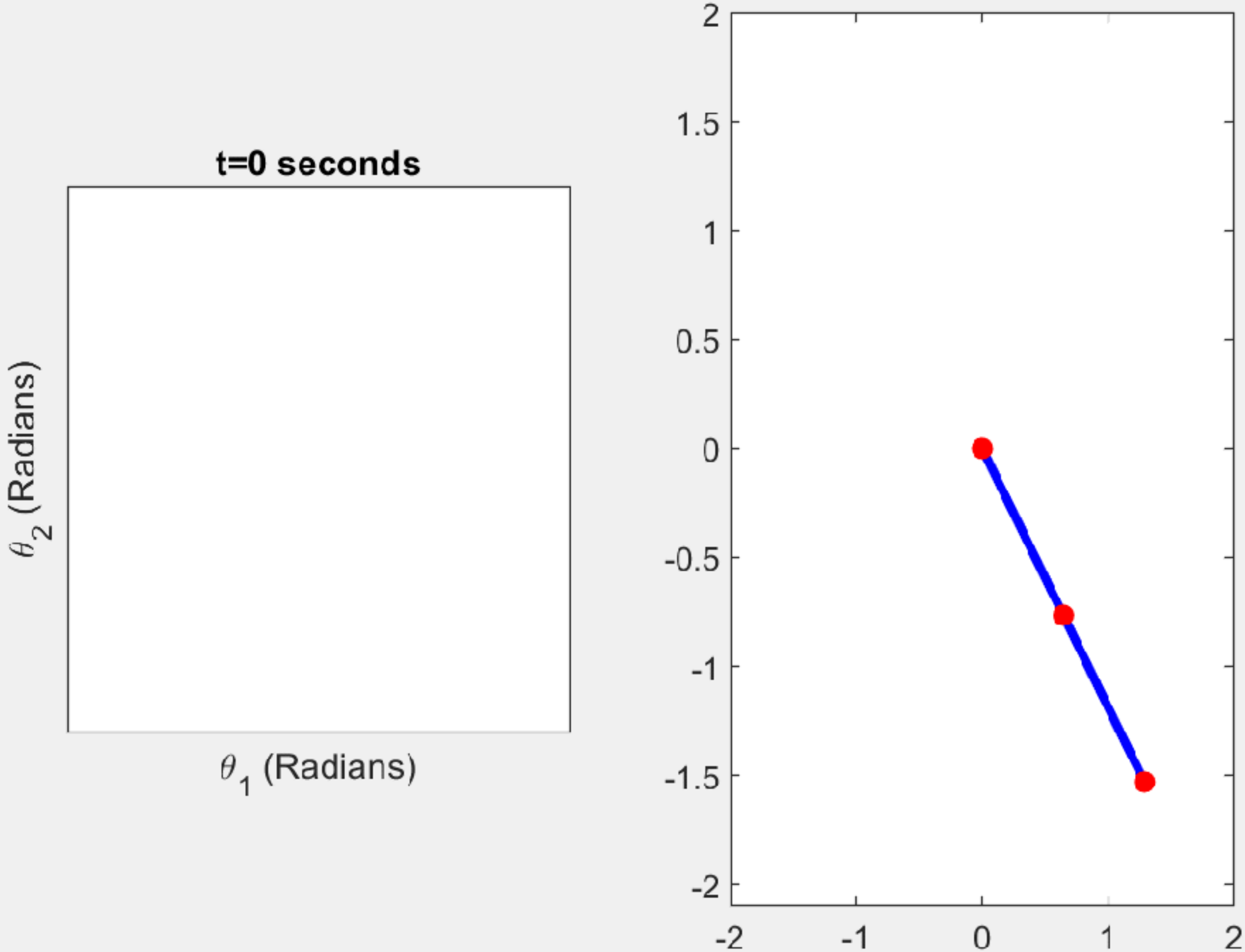


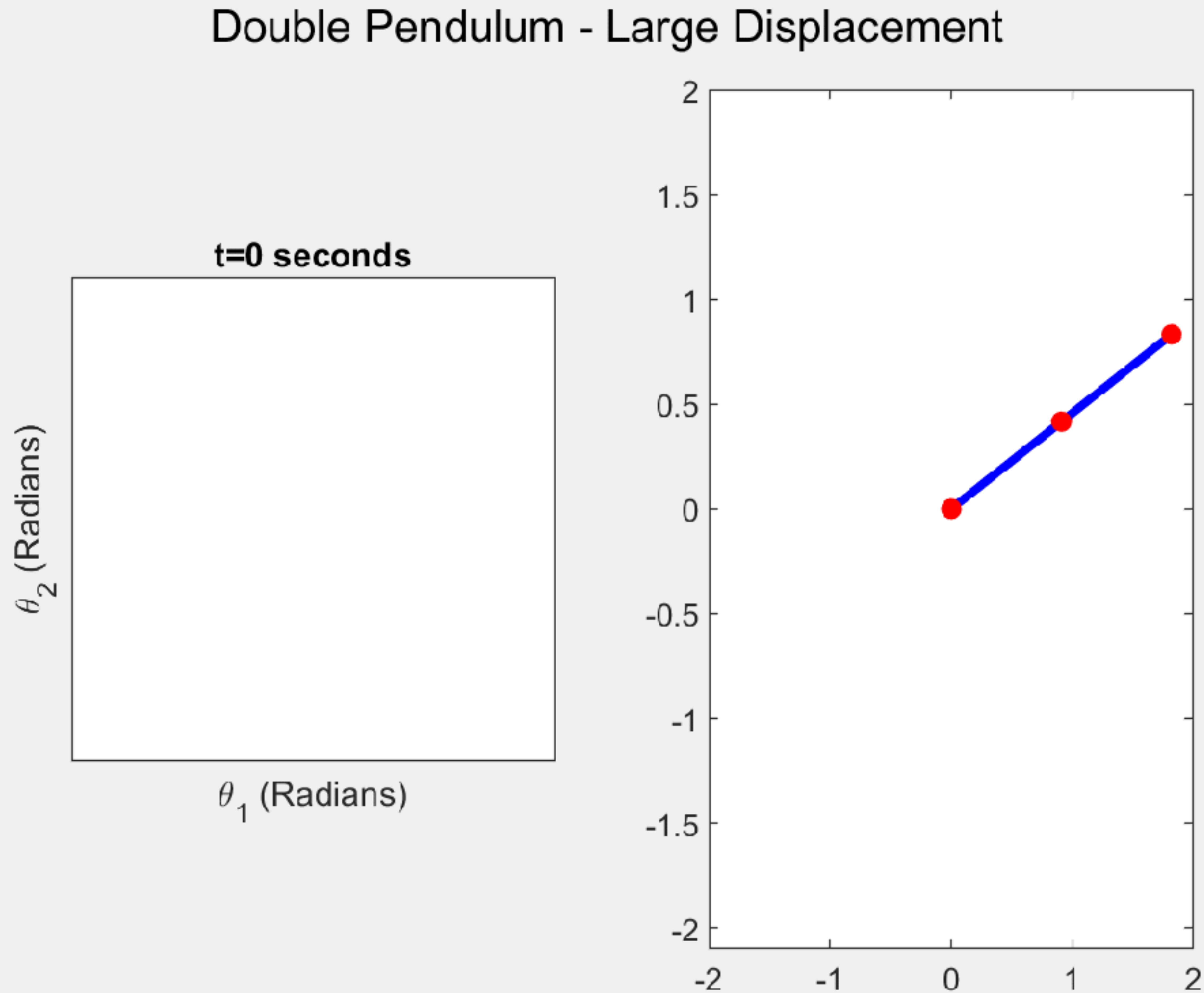
Lorenz about chaos, fractals, SOC, etc.:
"*Study of things that look random -but are not*"



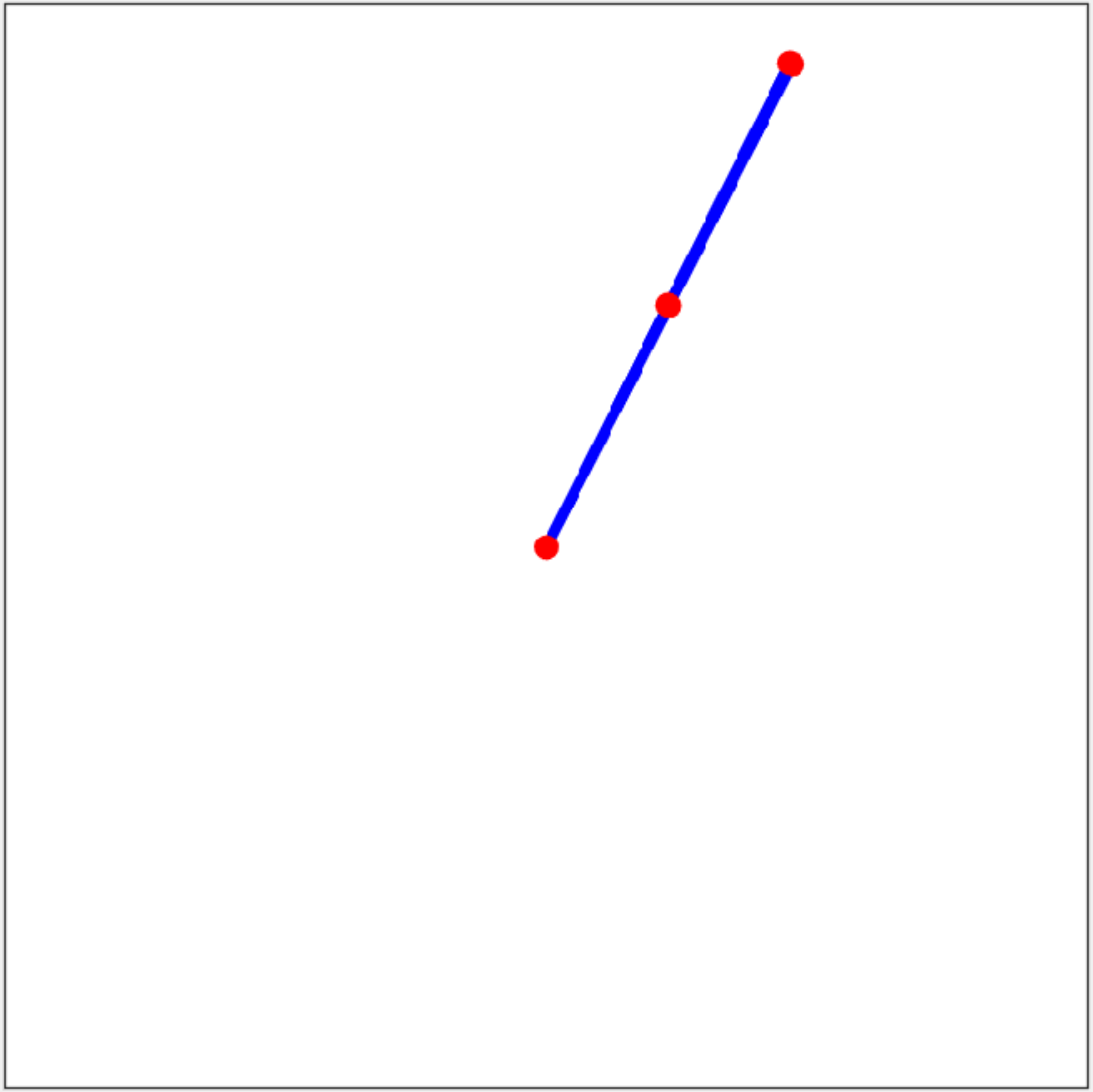
Double Pendulum - Small Displacement
t=0 seconds
θ_2 (Radians)
θ_1 (Radians)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2

Double Pendulum - Medium Displacement




Double Pendulum - Large Displacement
t=0 seconds
θ_2 (Radians)
θ_1 (Radians)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1
0
1
2

Double Pendulum at t=0 seconds



https://youtu.be/PrPYeu3GRLg?t=68

[1]refs