The Complex Systems Approach to Behavioural Science

Fred Hasselman

2019

Inhoudsopgave

Course Guide

This is a companion book for a number of courses listed on this website: https://complexity-methods.github.io :

- Research Master Behavioural Science curriculum: **Dynamics of Complex Systems**
- Radboud University Summerschool: **Complexity Methods for Behavioural Science: A toolbox for studying change.**
- Shorter workshops, for example: 2.5 day course in Helsinki 2020

Image from Grip on Complexity

The Complex Systems Approach

Complexity research transcends the boundaries between the classical scientific disciplines and is a hot topic in physics, mathematics, biology, economy as well as psychology and the life sciences. This course will discuss techniques that allow for the study of human behaviour from the perspective of the Complexity Sciences, specifically, the study of complex physical systems that are alive and display complex adaptive behaviour such as learning, development and creativity. Contrary to what the term "complex" might suggest, complexity research is often about finding simple models/explanations that are able to describe a wide range of qualitatively different behavioural phenomena. "Complex" generally refers to the object of study: Complex systems are composed of many constituent parts that interact with one another across many different temporal and spatial scales to generate behaviour at the level of the system as a whole, in complex systems "everything is interacting with everything".

The idea behind many methods for studying the dynamics of complex systems is to exploit the fact that "everything is interacting" and quantify the degree of periodicity, nonlinearity, context sensitivity or resistance to perturbation (resilience) of system behaviour. Applications in the behavioural sciences are very diverse and concern analyses of continuous time or trial series data such as response times, heart rate variability or EEG to assess proficiency of skills, or health and well-being. Complexity methods can also be used for the analysis of categorical data, such as behaviour observation of dyadic interactions (client-therapist, child-caregiver), daily experience sampling, social and symptom networks. The complex systems approach to behavioural science often overlaps with the idiographical approach of "the science of the individual", that is, the goal is not to generalise properties or regularities to universal or statistical laws that hold at the level of infinitely large populations, but to apply general principles and universal laws that govern the adaptive behaviour of all complex systems to a specific case, in a specific context, at a specific moment in time.

The main focus of the course will be hands-on data-analysis. Practical sessions will follow after a lecture session in which a specific technique will be introduced.

We will cover the following topics:

- Theoretical background of phase transitions (self-organised criticality), synchronisation (coupling dynamics) and resilience (resistance to perturbation) in complex dynamical systems and networks.

- Simple models of linear and nonlinear dynamical behaviour (Linear & logistic growth, Predator-Prey dynamics, Deterministic chaos),

- Analysis of (multi-) scale dependence in time and trial series (Entropy, Relative roughness, Standardized Dispersion Analysis, (multi-fractal) Detrended Fluctuation Analysis).

- Quantification of temporal patterns in time and trial series including dyadic interactions (Phase Space Reconstruction, [Cross-] Recurrence Quantification Analysis).

- Dynamical network analyses for univariate (recurrence networks) and multivariate time series (multiplex recurrence networks).

- Using the method of surrogate data analysis (constrained realisations of time series data) to test hypotheses about the nature of the data generating process.

Learning outcomes

After completing a course you will able to:

- Simulate linear, nonlinear and coupled dynamics using simple models.
- Conduct (multi-fractal) Detrended Fluctuation Analysis and related techniques to quantify global and local scaling relations.
- Conduct Recurrence Quantification Analysis and related techniques to quantify temporal patterns, synchronisation and coupling direction.
- Conduct analyses on (multiplex) Recurrence Networks to quantify structure and dynamics of (multivariate) time series.

Naturally the (depth of) topics discussed will be limited by the duration of the course.

Level of participant

- Master
- PhD
- Post-doc
- Professional

For whom are these courses designed?

The courses are designed for all researchers who are interested in acquiring hands-on experience with applying research methods and analytic techniques to study human behaviour from the perspective of Complexity Science. Prior knowledge is not required, some experience using R is recommended.

Admission requirements

During the course we will mostly be using the R statistical software environment. Basic experience with R is highly recommended (e.g. installing packages, calling functions that run analyses, handling and plotting data). We also offer a module for the Jamovi software with which the most basic analyses can be conducted. Using Jamovi does not require any prior knowledge of R, but you will not be able to use more advanced features of certain analyses.

Please bring your own laptop to the course. We will help you to install the necessary open source software, all of which can run on Windows, MacOS and most likely also on common varieties of Unix/Linux. The specifications for your computer are simply this: You need to be able to connect to a wireless network (wifi) and you should be able to install and run R (https://www.r-project.org). In addition, you might want to be able to use RStudio (https://www.rstudio.com) and Jamovi (https://www.jamovi.org).

If you do not have the resources to bring a laptop that meets the required specifications, please let us know in advance so we can try to find an alternative solution.

Literature

Pre-course literature:

It will be helpful to read the following articles before the first day of the course:

- Molenaar, P. C., & Campbell, C. G. (2009). The new person-specific paradigm in psychology. Current directions in psychological science, 18(2), 112-117.
- Kello, C. T., Brown, G. D., Ferrer-i-Cancho, R., Holden, J. G., Linkenkaer-Hansen, K., Rhodes, T., & Van Orden, G. C. (2010). Scaling laws in cognitive sciences. Trends in cognitive sciences, 14(5), 223-232.
- Thelen, E., & Ulrich, B. D. (1991). Hidden skills: A dynamic systems analysis of treadmill stepping during the first year. *Monographs of the Society for Research in Child Devevelopment, 56(1)*, 1-98; discussion 99-104. Retrieved from https://www.ncbi.nlm.nih.gov/pubmed/1922136
- Lewis, M. D. (2000). The promise of dynamic systems approaches for an integrated account of human development. Child development, 71(1), 36-43.

Selected chapters from these books will be made available so you can make a personal copy:

- Friedenberg, J. (2009). Dynamical psychology: Complexity, self-organization and mind. ISCE Publishing.
- Kaplan, D., & Glass, L. (2012). Understanding nonlinear dynamics. Springer Science & Business Media.
- Rose, T. (2016). The end of average: How we succeed in a world that values sameness. Penguin UK.

Links to online materials on specific topics will be provided (*Study Materials*) that may provide additional explanation and information about key concepts. These materials are not obligatory, but highly recommended to study at least once.

Notes about this book and the assignments

The texts in the chapters of this book are somewhat of a work in progress, and are intended as a rough introductory guide to accompany the lectures. Sometimes, you will notice a paragraph or chapter rather resembles a set of lecture notes instead of a self-contained text. Do not hesitate to let us know if you think anything is unclear or too far out of context for you to understand.

An essential part of the course are the assignments that are available online and are linked to from the course pages, for example: https://complexity-methods.github.io/courses/helsinki-workshop-2020/day1_2/

The text inside these blocks provides important information about the course, the assignments, or the exam.

The text inside these blocks provides examples, or, information about a topic you should pay close attentiont to and try to understand.

The text inside these blocks provides a note, a comment, or observation.

The content in these blocks are often questions about a topic, or, suggestions about connections between different topics discussed in the book and the assignments. You should decide for yourself if you need to dig deeper to answer the questions or if you want to discuss the content. One way to find an answer or start a discussion is to open a thread in the discussion forum on Blackboard labelled *ThinkBox*.

The content in these blocks is provided as entertainment :)

Schedule

You can find detailed schedules on the course website: https://complexity-methods.github.io/courses/

Some Notes on Using R

You have probably heard many people say they should invest more time and effort to learn to use the R software environment for statistical computing… *and they were right*. However, what they probably meant to say is: "I tried it, but it's so damned complicated, I gave up"… *and they were right*. That is, they were right to note that this is not a point and click tool designed to accommodate any user. It was built for the niche market of scientists who use statistics, but in that segment it's actually the most useful tool I have encountered so far.

New to R?

Now that your struggles with getting a grip on R are fully acknowledged in advance, let's try to avoid the 'giving up' from happening. Try to follow these steps to get started:

1. **Get R and add some user comfort:** Install the latest R software *and* install a user interface like RStudio… *It's all free!* An R interface will make some things easier, e.g., searching and installing packages from repositories. R Studio will also add functionality, like git/svn version control, project management and more, like the tools to create html pages like this one (`knitr` and `Rmarkdown`). Another source of user comfort are the `packages`. R comes with some basic packages installed, but you'll soon need to fit generalised linear mixture models, or visualise social networks using graph theory and that means you'll be searching for packages that allow you to do such things. A good place to start *package hunting* are the CRAN task view pages.

2. **Learn by running example `code`:** Copy the commands in the `code` blocks you find on this page, or any other tutorial or help files (e.g., Rob Kabacoff's Quick R). Paste them into an `.R` script file in the script (or, source) editor. In R Studio You can run code by pressing `cmd` + `enter` when the cursor is on a single single line, or you can run multiple lines at once by selecting them first. If you get stuck remember that there are expert R users who probably have answered your question already when it was posted on a forum. Search for example through the Stack overflow site for questions tagged with R)

3. **Examine what happens… when you tell R to make something happen:** R stores variables (anything from numeric data to functions) in an `Environment`. There are in fact many different environments, but we'll focus on the main workspace for the current R session. If you run the command `x <- 1+1`, a variable `x` will appear in the `Environment` with the value `2` assigned to it. Examining what happens in the `Environment` is not the same as examining the output of a statistical analysis. Output in R will appear in the `Console` window. Note that in a basic set-up each new R session starts with an empty `Environment`. If you need data in another session, you can save the entire `Environment`, or just some selected variables, to a file (`.RData`).

4. **Learn about the properties of R objects:** Think of objects as containers designed for specific content. One way to characterize the different objects in R is by how picky they are about the content you can assign it. There are objects that hold `character` and `numeric` type data, a `matrix` for numeric data organised in rows and

columns, a `data.frame` is a matrix that allows different data types in columns, and least picky of all is the `list` object. It can carry any other object, you can have a `list` of which item 1 is an entire `data.frame` and item 2 is just a `character` vector of the letter `R`. The most difficult thing to master is how to efficiently work with these objects, how to assign values and query contents.

5. **Avoid repeating yourself:** The `R` language has some amazing properties that allow execution of many repetitive algorithmic operations using just a few lines of code at speeds up to warp 10. Naturally, you'll need to be at least half Vulcan to master these features properly and I catch myself copying code when I shouldn't on a daily basis. The first thing you will struggle with are the `apply` functions. These functions pass the contents of a `list` object to a function. Suppose we need to calculate the means of column variables in 40 different SPSS `.sav` files stored in the folder `DAT`. With the `foreign` package loaded we can execute the following commands:

```
data <- lapply(dir(/DAT/",pattern=".sav$"),read.spss)
out  <- sapply(data,colMeans)
```

   The first command applies read.spss to all files with a `.sav` extension found in the folder `/DAT`. It creates a data frame for each file which are all stored as elements of the list `data`. The second line applies the function `colMeans` to each element of `data` and puts the combined results in a matrix with dataset ID as columns (1-40), dataset variables as rows and the calculated column means as cells. This is just the beginning of the `R` magic, wait 'till you learn how to write functions that can create functions.

Getting started with R tutorials

- Tutorials on using **functions**:
  - Quick-R
  - Software Carpentry
  - Nicer Code
  - Advanced R
- Tutorials on using **conditionals** and **for loops**:
  - Quick-R
  - Software Carpentry
  - R-Bloggers
- Tutorials on the **-ply family** of functions: + R-bloggers + Nicer Code + R for Dummies
- **Plotting**, **plotting** and more **plotting**: + A Compendium of Clean Graphs in R + ggplot2 reference + ggplot2 extensions + patchwork, the ultimate ggplot2 extension + The R-graph gallery + Quick-R + Nicer Code
- Tutorial on **Effect Size Confidence Intervals** and more:
  - In this tutorial on estimating Effect Size Confidence Intervals (ESCI) there are a lot of examples on how to use R.
  - It was written as an addendum for a post on the **Open Science Collaboration Blog**, which contains many interesting entries on diverse subjects (like behavioural priming, theoretical amnesia and anonymous peer review)

Not new to R?

If you have been using R for a while, but do not consider yourself a master yet, I recommend learning to use the **tidyverse** packages and the accompanying web-book **R for data scientists**.

- Welcome to the **tidyverse**: + Install the tidyverse + Learn how to use the tidyverse + Learn how to use the tidyverse to do statistics + Learn how to use the tidyverse to create networks + How to make R purrr

Time series analyses in R

In this book you can find some tips on plotting time series (see section **Working with time series in R**) and we will be using package `casnet` as our main tool for analyses. However, if you really want a deep dive into everything related to time series in R be sure to check the CRAN task view on time series: https://cran.r-project.org/web/views/TimeSeries.html

casnet

To install `casnet` you need to have package `devtools` or `remotes` installed and call the following code from the commands line:

```
library(devtools)
devtools::install_github("FredHasselman/casnet", dependencies = TRUE)

# or equivalently
library(devtools)
remotes::install_github("FredHasselman/casnet", dependencies = TRUE)
```

If all goes well this should install the package and all the packages it depends on. If the vignette build fails, don't worry, you can access them through the casnet website under *Articles*.

We used R!

This text was transformed to HTML, PDF en ePUB using bookdown(**?**) in **RStudio**, the graphical user interface of the statistical language **R** (**?**). bookdown makes use of the R version of markdown called Rmarkdown (**?**), together with knitr (**?**) and pandoc.

We'll use some web applications made in Shiny (**?**)

Other R packages used are: DT (**?**), htmlTable (**?**), plyr (**?**), dplyr (**?**),tidyr (**?**), png (**?**), rio (**?**).

Deel I

Introduction

Chapter 1

A Quick Guide to Scientific Rigour

> "Meanwhile our eager-beaver researcher, undismayed by logic-of-science considerations and relying blissfully on the "exactitude" of modern statistical hypothesis-testing, has produced a long publication list and been promoted to a full professorship. In terms of his contribution to the enduring body of psychological knowledge, he has done hardly anything."
>
> —Paul Meehl (**?**, p. 114)

Before we can begin our introduction to the wonderful world of Complex Adaptive Systems and Complex Networks, we briefly discuss the philosophy of science and perspective on the goal of scientific inquiry that is used throughout this book. This will allow us to highlight some differences between the **Complex Systems Approach (CSA)** we propose for the scientific study of human nature and the perspective (implicitly) used in most disciplines of the social and life sciences, we will call the **Machine Metaphor Approach (MMA)**, **Cognitivism**, or, **Computationalism**.

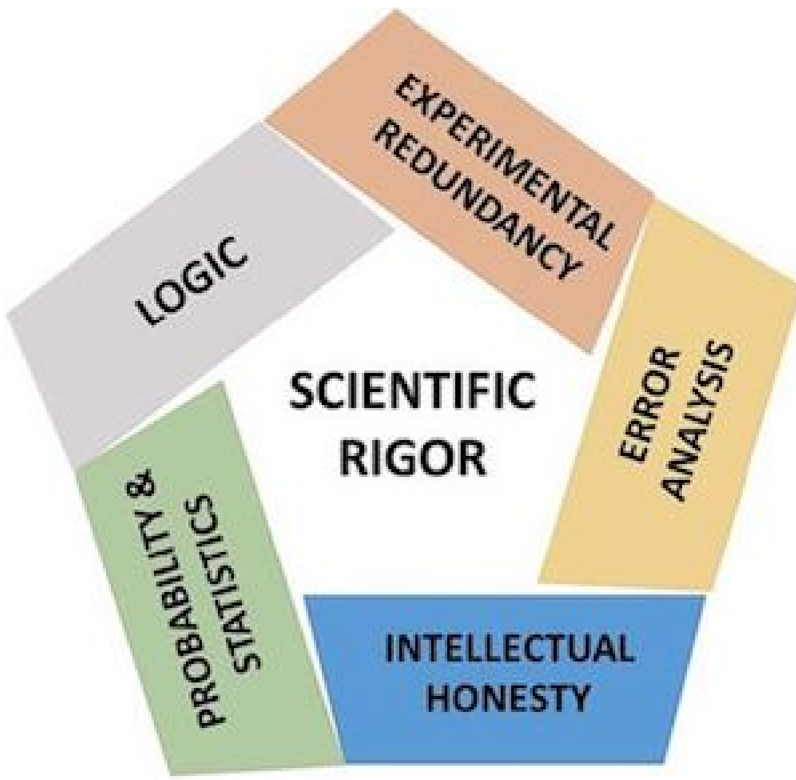
Use of the **scientific method** is what separates scientific, from non-scientific claims about the nature of reality. It consists of all philosophical, theoretical, and empirical tools that can be used to systematically evaluate the veracity of such explanatory claims. The repeated application of the scientific method, to study scientific questions, promises to generate **valid** (accurate) inferences and **reliable** (precise) facts about a certain explanatory domain. It does not guarantee that any kind of absolute 'truth' will be discovered.

The ‘**scientific method song**’ discusses the most important phases of the *empirical cycle*. Be aware that there is also a *theoretical cycle* and a *diagnotsic cycle*.

One factor affecting the perceived veracity of scientific inferences, is the quality of the body of scientific knowledge from which the inferences were deduced, induced or abducted. For example, when a *crisis of confidence* about the trustworthiness of facts in the scientific record generated by some sub disciplines of psychological science was suggested (**?**), the immediate consequence was that the veracity of all claims by psychological science was called into question.

1.1 Rigorous Open Science

Less tangible, but not less important for the perceived veracity of scientific knowledge are concepts such as *intellectual honesty* and *scientific integrity* of the scientists laying explanatory claims on some domain in reality. Merely checking whether the scientific method has been applied does not fully grasp all the prerequisites for generating a solid body of knowledge. We will use the term **rigorous open science** to denote the ideal set of conditions that should be in place to allow us to distinguish scientific claims that are likely to be false, from claims that are likely to be true, given the perceived *verisimilitude* (truth-likeness) of the knowledge accumulated in the scientific record.



Figuur 1.1 – Rigorous Science according to @casadevall2016a.

When a claim is based on **Scientific Rigour** (**?**), we mean it was posited based on the following set of principles:

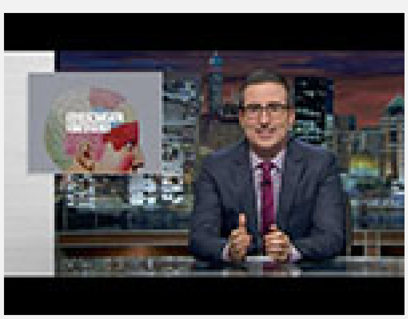
1. **Experimental Redundancy** - The claim has been examined by all methodological and analytical tools that are available and are appropriate given the context. Ri-

gorous Science does not rely on one type of experimental design or one type of statistical analysis.

2. **Recognition of Error** - Without failure there can be no progress, therefore we should carefully study failures and not just report success stories. Any sources of error should be carefully studied and reported to the scientific community.
3. **Sound Probability & Statistics** - Use of the most recent and appropriate statistical theories, models and analytical techniques. Statistical modelling techniques become more realistic over time and often the models that were taught in undergraduate statistics courses have long been replaced and should not be used any more.
4. **Efforts to Avoid Logical Traps** - When generating theories and defining constructs and laws, make sure logical inconsistencies are avoided. When making inferences, avoid the common logical traps such as *The Effect = Structure Fallacy* in null hypothesis significance testing (NHST).
5. **Intellectual Honesty** - Rigorous science is ethical, has integrity and thrives on critical reflection on scientific practice. The right mindset is *“Prove yourself wrong!”*, not *“Prove yourself right!”*

We add to the list that science must be open and transparent. This may seem like an obvious statement to a fresh student of human behaviour, but concepts that make up an essential part of the scientific debate in 2017, such as *open science*, *open data*, *reproducibility*, *Questionable Research Practices (QRPs)*, *Hypothesizing After the Results are Known (HARKing)* and *preregistration*, were practically unknown 5 years ago.

Comedian John Oliver discusses how and why media outlets so often report untrue or incomplete information as science:



Tabel **1.1**

Strong Inference according to @platt1964strong

	Strong inference consists of applying the following steps to every problem in science, formally and explicitly and regularly:
1.	Devising alternative hypotheses
2.	Devising a crucial experiment (or several of them), with alternative possible outcomes, each of which will, as nearly as possible, exclude one or more of the hypotheses
3.	Carrying out the experiment so as to get a clean result
1'	Recycling the procedure, making subhypotheses or sequential hypotheses to refine the possibilities that remain
...	and so on.

Strong Inference

> A difficulty of much psychological theorizing is vagueness in the terms employed. In this work, the above ideas have been studied in mathematical form throughout, the definitions and proofs being given corresponding precision.
>
> —W. R. Ashby in 'The Physical Origin of Adaptation by Trial adn Error' (**?**, p. 13)

The Effect = Structure Fallacy refers to the logical error that occurs a predicted effect is observed (i.e. a statistically significant test result leads to a rejection of the null hypothesis), it is not valid to infer the existence of the assumed cause was evidenced. NHST is based on *the falsification principle*, which means the perceived veracity of a scientific claim will increase only if it has resisted many rigorous attempts to prove it is wrong. If a scientific claim has a large track-record of resisting falsification attempts, we can call it plausible, or high in verisimilitude, but this could all change with one crucial experiment. Contrary to what some scholars suggest, falsifiability is not optional in a rigorous science (**?**).

An excellent recipe for a rigorous application of the scientific method was provided by **?**. Perhaps we should implement it and get us out of the curious situation in which so many different "theories" competing to explain the same pehnomena can be considered to be "true" at the same point in time.

1.2 Theoretical Tunnelvision

> "It is the theory that decides what we may observe"
>
> —Einstein (as quoted by Heisenberg)

Many of the initiatives proposed to improve the social and life sciences focus on improving methodology and statistics. This is understandable, it's where errors are easily made (and discovered) and it allows for relatively simple interventions, e.g. more stringent control on appropriate use of statistics by journals. However, the goal of generating empirical facts is ultimately because we want to find out which scientific claim about the structure of reality best explains why those empirical facts were observed.

The quote attributed to Einstein refers to an important, and grossly underestimated phenomenon one might call the *theoretical tunnelvision*. It is best explained by an example that is commonly encountered in the literature in psychological science and goes something like this:

1. A study tries to find independent causes (predictors) of a certain disease-entity, a pathological state or behavioural mode people can 'get stuck in'.
2. Typically, a statistical model fitted on a large, representative sample of individuals in which many different predictors were measured will yield associations between predictor and disease-entity that are significant but small (on average $r \approx 0.3$, or $\approx 9\%$ explained variance).
3. Often, if other known (non-clinical) covariates are included in a model, or, if the multivariate nature of the phenomenon is taken seriously by including repeated measurements and/or multiple dependent variables, these predictors will no longer explain any unique variance in the outcome measures.

Here's an example of a 'predictor' study (**?**) to find predictors of persistence of Major Depressive Disorder MDD 10 over the course of 10 years in a representative sample of 331 individuals who suffered MDD 10 years earlier:

> "Clinical variables in this analysis were not strongly associated with persistence of MDD over the course of 10 years. Comorbid generalized anxiety disorder, baseline depression severity, and taking a prescription for nerves, anxiety, or depression were significantly associated with persistent depression in the unadjusted logistic regression models, but the associations became non-significant when in the multivariate model. These findings are in contrast to the results from several other studies."

The study concludes by discussing three factors that play a statistically significant role in the persistence of MDD (text between brackets not in original):

- "*having two or more chronic medical conditions [in 1995-1996] contributes to experiencing depression ten years later.* [2.89 more likely] *However, only having one chronic medical condition did not increase the odds of being classified as having MDD in 2004–2006.*"
- "*days of activity limitation in 1995–1996 were significantly associated with a greater risk of depression ten years later,* [2.19 more likely] *independent of the number of chronic medical conditions a person had.*"
- "*Individuals who were in contact with family less than once a week* [in 1995-1996] *were more likely to have MDD in 2004–2006.* [2.07 more likely] *Likewise, people who were married were less likely to have persistent depression compared to those who have never married* [never married 2.42 more likely]"

So? What's wrong?

So what's wrong with these inferences? The study shows some previous assumptions about the relevance of clinical predictors should be reconsidered, and it adds to scientific record some facts about risk factors that might have eluded scientists, clinicians and health professionals. Let's look at the main conclusion of the study, in addition to a plea for more attention for people with two or more chronic medical conditions, **?** end the article with:

> Future research should continue to examine the complex nature of the relationship between chronic medical disorders and comorbid psychiatric conditions. Addressing these conditions and strengthening social support systems could be important strategies for reduce the burden of depression.

Here's what is odd from the perspective of *rigorous science*:

1. If clinical predictors play no role in explaining why some people remain depressed for such long periods of time, why isn't the main conclusion of the study that we must re-appraise the scientific theories laying explanatory claim on the aetiology of MDD? It is from these theories that the diagnostic tools, the medical, and psychological interventions to which these patients have been exposed, were derived.

2. Even though the authors acknowledge –and indeed show– that the propagation of a pathological state like MDD over many years is a very complex multivariate

phenomenon, their suggestion for future research is still based on an implicit assumption about causation that is extremely simple. The idea is that there is a chain of unique (efficient) causes, each contributing independently to the emergence, and persistence in time of the MDD state. The authors basically suggest some component causes have to be added to the aetiology. The metaphor is that of a *machine* of which the sum output of its constituent components is equal to the purpose or function of the machine as a whole. Should a component fail, then it can be repaired or replaced as long as it performs the same function as the defective part, thereby restoring the function of the machine as a whole. This is why the authors suggest that strengthening social support systems could be an intervention to reduce the burden of depression: The absence of a partner or visits by family members were predictors that explained some unique variance in the data on the persistence of MDD. Obviously, restoring this defective social support component should restore or at least facilitate the escape from the MDD state. Meanwhile, they seem to forget that they convincingly argued that MDD is a very complex phenomenon that cannot be dissected into neat, independent component causes.

3. Very much related to the previous point: The authors mention three important factors in the discussion and conclusion section, however, the results section contains another factor that was omitted, it is in fact the second most important predictor of the persistence of MDD:

> "Women had 2.48 the odds of remaining depressed compared to men"

Why did they ignore this predictor in the discussion? This is speculation, but could it be that this factor is not mentioned because it would have to be considered a 'deficient' component and suggesting any kind of 'treatment' intended to 'repair' it is of course beyond the realm of sane things to suggest. Nevertheless, it does seem rather important to figure out why women are 2.5 times more likely than men to still be depressed after 10 years. Perhaps *not* considering gender to be a unique causal component in a chain of *in*dependent predictors might help. Instead, gender could be considered a complex aggregate, or, contextual variable that is associated to the dependent variable through a vast network of *inter*dependent facts, events and states of affair. An obvious factor of importance is that effect-studies of medical interventions are mainly conducted on white, male, 20-30 year old, right-handed, subjects with above average SES. Also, it is likely that on average, the stability of mood over longer periods of time is more variable in women than in men due to fluctuations of hormone levels, but also due to antenatal and postnatal depression (**?**). It does not seem unreasonable to suggest this poses extra challenges for women who want to escape the MDD state.

No such thing as theory-free 'facts'

The analytical tools selected by the researchers (a generalized linear statistical model) restricts the kinds of associations we might observe in the data. In the the present case all associations will –after transformation– be linear compositions of independent components.[1] One never reads this valid equally valid conclusion: "*We conclude that the linear model is inadequate to describe the complexity of this phenomenon.*" The reason is that the implicit assumptions about causality underlying scientific claims never enter the empirical cycle and therefore escape falsification by the repeated application of the scientific method even though those causality assumptions are also based on a scientific theory about the structure of reality that is in principle falsifiable.

[1]Naturally, if one would use mixed models we can account for dependencies in the data, but they will still be limited to linear associations.

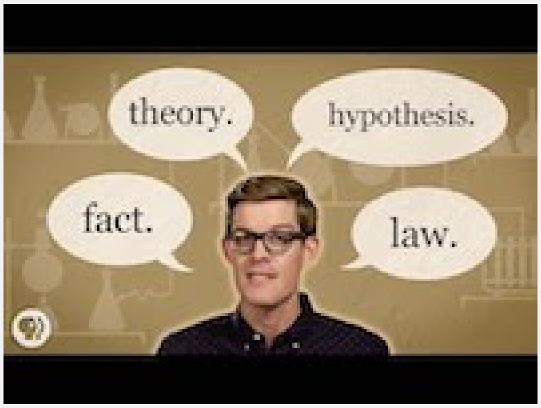
Study Materials

Phenomena, theories, facts and laws

> "All science is either physics or stamp collecting."
>
> —Ernest Rutherford (Physics Nobel Laureate, 1872-1937)

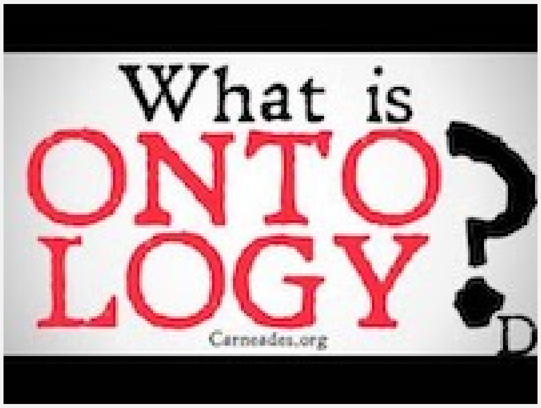
It's important to distinguish between phenomena, hypothesis, theory and law. For example, we will be discussing, nonlinear *phenomena*, catastrophe *theory* and power *law* scaling.

The video is provides a very clear explanation of the differences between these concepts.

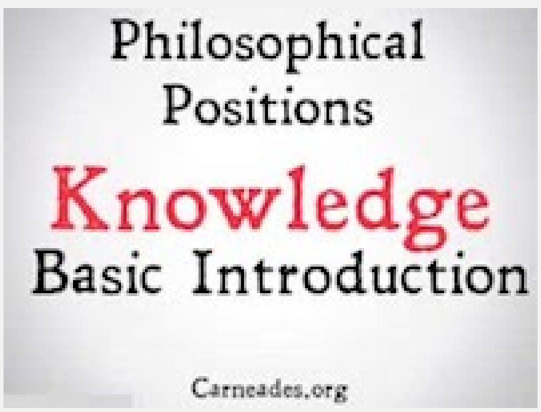


You might also want to refresh your knowledge about some important aspects of scientific theorising about reality: *Ontology* and *Epistomology*

Ontology.



Epistemology



Intellectual Honesty and *Epistemic Responisiblity*:

Epistemic Responsibility


14
conspiracy theories
& EPISTEMIC
RESPONSIBILITY

Chapter 2

Introduction to Complexity Methods

Psychological systems are biological systems which are physical systems that are alive. Therefore, any theory that lays explanatory claim to phenomena of the mind, ultimately must be a theory about how a physical system is able to accumulate non-random order into its internal structure that appears to codetermine its behaviour. Less formally stated, a science that studies the behaviour of physical systems that are alive, that appear to have a memory which makes their behaviour adaptive, future oriented and intelligent, should be grounded in physical and biological principles and laws.

For now, generating such a theory might be a bridge too far (however, see **?**), the least we may demand is that our current theories of human behaviour should *not* contradict highly corroborated theories of physics that describe (constituent components of) simple or complex dynamical systems. This is arguably not the case in current psychological theorising, theories assume internal, highly organised structures (such as mental representations) as causes for behaviour, without explaining where the order came from, or how it is maintained or increased. Well studied and formally defined constructs from other sicentitif disciplines are often imported at a metaphorical level, or are misinterpreted and essentially wrong. For example, *plasticity*, *holism*, *behavioural state/mode/change*, and especially, any concept related to the term *information* (computation, coding/decoding, information processing/storage/retrieval, entropy, etc.). Information is a formally defined quantity that resolves uncertainty about the states or properties of a theoretical object of measurement (e.g. a system, a signal) relative to its degrees of freedom, by assigning it (the uncertainty), a value. If a system represents 1 bit of information (e.g. a coin-toss system), this means it means it can be in 1 of 2 states, or have one of 2 distinct values.

This is clearly not the same as “meaning” with which it is often conflated in theorising about cognition and behaviour. Shannon lucidly explained this in his seminal paper, which was to be the start of a new scientific discipline, information theory:

> “The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have *meaning*; that is they refer to or are correlated according to some system with certain physical or *conceptual entities*. These *semantic aspects of communication* are irrelevant to the engineering problem. The significant aspect is that the actual message is one selected from a set of possible messages. The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design.”
>
> —Shannon (1948, p.379)

2.1 The Complex Systems Approach

The *Complex Systems Approach* to behavioural science departs from the assumption (which is probably not very constroversial) that human behaviour originates from a complex adaptive system.

A *system* is an entity that can be described as a composition of components, according to one or more organizing principles. The organizing prniciples can take many different forms, but essentially they decide three important features of systems that have to do with the relationship between parts and wholes and therefore whether we would call a system complex or not.

In order to find out what kind of system we are dealing with, we can ask three basic questions:

1. What are the relevant scales of observation of the system?
2. What are the relevant phenomena that may be observed at the different scales of observation, and are there any interactions across the relevant scales of observation that are needed to explain the relevant phenomena?
3. Can interactions with the internal and external environment of a system occur, and if so, do these interactions have any after-effects on the structure and/or behaviour of the system?

If the answer to the first question is "many" and to the second and third "yes" it is very likely we are dealing with a complex dynamical system.

So let's look at some properties of this system that generates human behaviour, it's a system:

- … which has many different constituent parts, and those parts are often also systems with many different constituent parts (the tRNA system, the prefrontal cortex, the respiratory system, the speech system, the endocrine system, the microbiome, etc.).
- … that is *open* and can exchange energy, matter and information with its internal and external environment, as a consequence, dissipating heat (disorder, entropy) back into the environment.
- … which has many different internal states that can have their own specific dynamics, sometimes appearing to be independent of, but oftentimes coupled to, the dynamics of other internal states (emotional states, motivational states, attentional states, physical fitness, general health, biological development, etc.).

- … in which there are many potential levels of organisation and, therefore, potentially many different levels of analysis (cognitive development, cognitive neuroscience, lifespan IQ, socio-cultural differences in IQ, etc.).
- … in which many processes operate on, and, interact across, many different spatial and temporal scales (Studying proficiency at playing chess: social/cultural/pedagogical/genetic contexts, development of social/emotional/motor/cognitive skills, availability/quality of education, motivation, personality, etc.).

So… that's probably a *yes* on complex dynamical system. To be more specific, we can state that most living organisms, including human beings, are **complex adaptive systems with internal state dynamics**.

2.2 Ergodicity and the Measurment Problem

It does not take an expert on *population statistics* to see there is probably a mismatch between the interesting behavioural pehnomena and the analytical toolbox most frequently used to study human behaviour in the social and life sciences. Anyone who took an introductory class in inferential statistics will remember the assumptions of statistical models require observations to be independent of one another, variances to be homogeneous (e.g. Levene's test), and measurement error to be essentially random in nature and normally distributed, not correlated to any other factors that might cause the phenmenon under scrutiny.

Given the nature of the phenomena of interest and the properties of the system under scrutiny, there are two main concerns about the scientific study of human behaviour:

1. The assumption that *the ergodic theorems apply* to the theoretical objects of measurement and data generating processes (**??**): Ensemble averages of variables observed in samples of sufficiently many individuals are expected to be arbitrarily similar to the time averages of variables evolving over a sufficiently long interval of time, from any single initial condition.
2. The assumption that the interpretation of outcomes of psychological measurement is, or should be, equivalent to *classical physical measurement* (**?**): It is considered unproblematic to interpret a measurement outcome as a property of the theoretical object of measurement confounded by some random additive measurement noise or sampling error.

The validity of the assumptions related to ergodicity (i.e. stationarity and homogeneity of central moments) are obviously important for making valid statistical inferences and generalizations. However, even if some of the core assumptions for an ergodic data generating process are formally valid, one cannot rely on parameter estimates to converge on a characteristic expected value within the time scale of observation, or, scale of fluctuation, as is the case when the process samples from a stable distribution with one or more undefined central moments like the Cauchy distribution. This has led some scholars to suggest that "*the very notion of probability may not make sense*" (**?**) when studying complex systems with internal state dynamics.

Recent observations of discrepancies between inferred properties at the ensemble level (inter-individual) and the individual level (intra-individual), have been suggested as a cause of the so-called reproducibility crisis in the social and life sciences (**???**). A study which observed a lack of 'group-to-individual generalizability' in the context of psychopathology described the phenomenon as a threat to human subjects research: "*In clini-*

cal research, diagnostic tests may be systematically biased and our classification systems may be at least partially invalid. In terms of theory development, we may have a misleading impression about the nature of psychological variables and their interactions." (**?**). A study of the neuroanatomical phenotypes of schizophrenia and bi-polar disorder (**?**) concluded: "*This study found that group-level differences disguised biological heterogeneity and interindividual differences among patients with the same diagnosis. This finding suggests that the idea of the average patient is a noninformative construct in psychiatry that falls apart when mapping abnormalities at the level of the individual patient.*"

The second concern is about the lack of a clear notion in psychology and the life sciences of how to incorporate the measurement context and the act of measurement into the description of a phenomenon (**?**). Psychological measurement is an interaction between a (prepared) theoretical object of measurement and the elements of the measurement procedure (experimental design, instruments, etc.). The very act of asking someone to project their current internal state of happiness onto an arbitrary ordinal scale will interfere with their "true" state of happiness (if such a thing even exists without the measurement context). There is no "happiness" equivalent for unobtrusive measurement of body temperature using an infrared camera.

Resolutions to these and other problems with psychological measurement have been proposed, for example the various types of conjoint measurement (**??**), or suggestions to adopt concepts from quantum measurement (**??**). However, when measurement and analysis of the temporal evolution of internal states is concerned, problems arise due to the fact that living systems are subject to *ageing* (loss of identity over time) and appear to be able to coordinate their current behavior relative to some record of previously experienced events. In more general terms, the behavior of a complex adaptive system will display after-effects of interactions with its internal or external environment that extend far beyond any timescale that might be understood as a simple stochastic process with autoregressive components. Time series of observables of living systems will often lack the memoryless-ness property (**??**), suggesting anomalous, rather than normal diffusion processes should be considered as a model for the data generating process (**?**).

2.3 Component- vs. Interaction-dominant Dynamics

> In summary, with conventional computing technology we often "torture" the physical substrate so that it implements desired computations (e.g., using continuous electronic processes to implement binary logic), whereas embodied computation "respects the medium," conforming to physical characteristics rather than working against them. The goal in embodied computation is to exploit the physics, not to circumvent it (which is costly)."
>
> —Bruce Maclennan (**?**, p.230)

intdom.png

A helpful framework for discussing the differences between a Complex Systems Approach and a Machine Metaphor Approach to the scientific study of human behaviour is to describe the causal ontology used to explain behaviour. Familiar "degrees of causation", or entailment are possible in component dominant dynamics, such as uniquely explained variance, beta weights or effect sizes. In general, a linear arrangement of partial causes always neatly sum up to produce the behaviour of interest. An alternative causal ontology is interaction dominant dynamics in which not the components themselves, but their interactions as a whole are the source of the observed behaviour (Ihlen & Vereijken, 2010; Kello, Beltz, Holden, & Van Orden, 2007; Van Orden, Holden & Turvey, 2003; Van Orden, Holden & Turvey, 2005; Wijnants, Cox, et al., 2012). Here the contribution of components is not additive, but multiplicative and nonlinear (Holden et al., 2009; van Rooij, Nash, Rajaraman, & Holden, 2013). Such interaction dominant dynamics render individual component behaviour (which are still posited to exist), such as poor performance on ability X, impaired representation of that feature Y, as a less interesting object of theoretical and empirical inquiry.

As a consequence, theoretical and empirical inquiry is aimed at identifying and understanding the contexts in which impaired behaviour emerged. Adopting such a perspective entails that all observable behaviour can only be understood relative to the context in which it was observed, that is, the measurement context (cf. Holden, Choi, Amazeen, & Van Orden, 2010; Van Orden, Kello, & Holden, 2010). Figure presents the fundamental differences between the two ontologies in their assumptions about the causes of behaviour and their assumed place of measurement. Figure may reveal why the nature of cognitive components and processes remain elusive in their causal role. They are inferred, not postulated, based on data from different places of measurement. Their causal structure does not incorporate the nested nature of both measurements as well as posited entities. Applying the concept of the complex conditional reveals hierarchical dependencies of one condition on another and such a complex, if it were composed of the correct conditionals, should be considered as a whole. As a consequence, impaired behaviour should be understood as emerging from the whole of constituent components, not from an individual component. The notion of a cause is somewhat more radical than the complex conditionals and is known as impredicative, circular causation (Chemero & Turvey, 2010; Freeman, 1999; Turvey, 2007), or nested causation.

Deel II

Mathematics of Change

Chapter 3

Introduction to the Mathematics of Change

The simplest non-trivial *iterative change process* can be described by the following *difference equation*:

$$Y_{t+1} = Y_{t=0} + a * Y_t$$

The equation describes the way in which the value of Y changes between two adjacent, **discrete** moments in time (hence the term difference equation, or recurrence relation). There are two parameters resembling an intercept and a slope:

1. The starting value Y_0 at $t = 0$, also called the *starting value*, or the *initial conditions*.
2. A rule for incrementing time, here the change in Y takes place over a discrete time step of 1: $t + 1$.

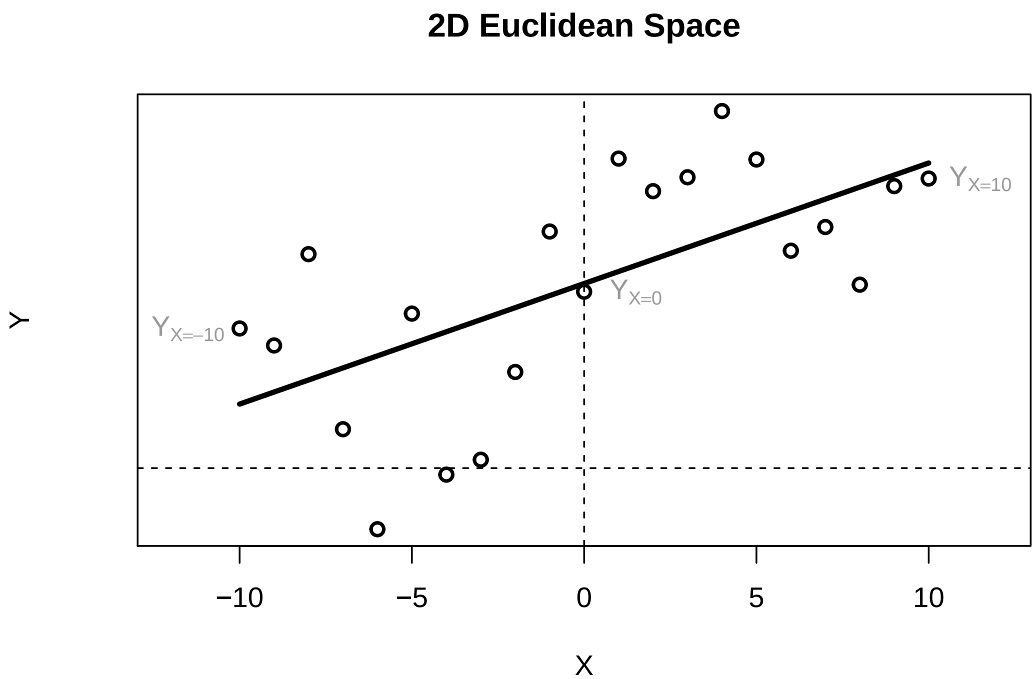
The values taken on by variable Y are considered to represent the states quantifiable observable alternative ways to describe the change of states :

- A dynamical rule describing the propagation of the states of a system observable measured by the values of variable Y through discrete time.
- A dynamic law describing the time-evolution of the states of a system observable measured by the variable Y.

These descriptions all refer to the change processes that govern system observables (properties of dynamical systems that can be observed through measurement).

3.1 It's a line! It's a plane!

The formula resembles the equation of a line. There is a constant value Y_0 which is added to a proportion of the value of Y at time t, given by parameter a. This is equivalent to the slope of a line. However, in a (X, Y) plane there are two 'spatial' (metric) dimensions representing the values two variables X and Y can take on (see figure).



The best fitting straight line would be called a statistical model of the linear relationship between the observed values of X and Y. It can be obtained by fitting a General Linear Model (GLM) to the data. If X were to represent repeated measurements the multivariate GLM for repeated measures would have to be fitted to the data. This can be very problematic, because statistical models rely on Ergodic theory:

> "… it is the study of the long term average behavior of systems evolving in time."

In other words: If you throw 1 die 100 times in a row, the average of the 100 numbers is the **time-average** of one of the observables of die-throwing systems. If this system is ergodic, then its **time-average** is expected to be

> similar to the average of the numbers that turn up if you throw 100 dice all at the same instance of time. The dice layed out on the table represent a spatial sample, a snapshot frozen in time, of the possible states the system can be in. Taking the average would be the **spatial average** this observable of die-throwing systems. This ergodic condiciotn is often implicitly assumed in Behavioural Science when studies claim to study change by taking different samples of individuals (snapshots of system states) and comparing if they are the same.

need to assume independence of measurements within and between subjects. These assumptions can be translated to certain conditions that must hold for the model to be valid, known as *Compound Symmetry* and *Sphericity*:

> The compound symmetry assumption requires that the variances (pooled within-group) and covariances (across subjects) of the different repeated measures are homogeneous (identical). This is a sufficient condition for the univariate F test for repeated measures to be valid (i.e., for the reported F values to actually follow the F distribution). However, it is not a necessary condition. The sphericity assumption is a necessary and sufficient condition for the F test to be valid; it states that the within-subject "model" consists of independent (orthogonal) components. The nature of these assumptions, and the effects of violations are usually not well-described in ANOVA textbooks; [1]

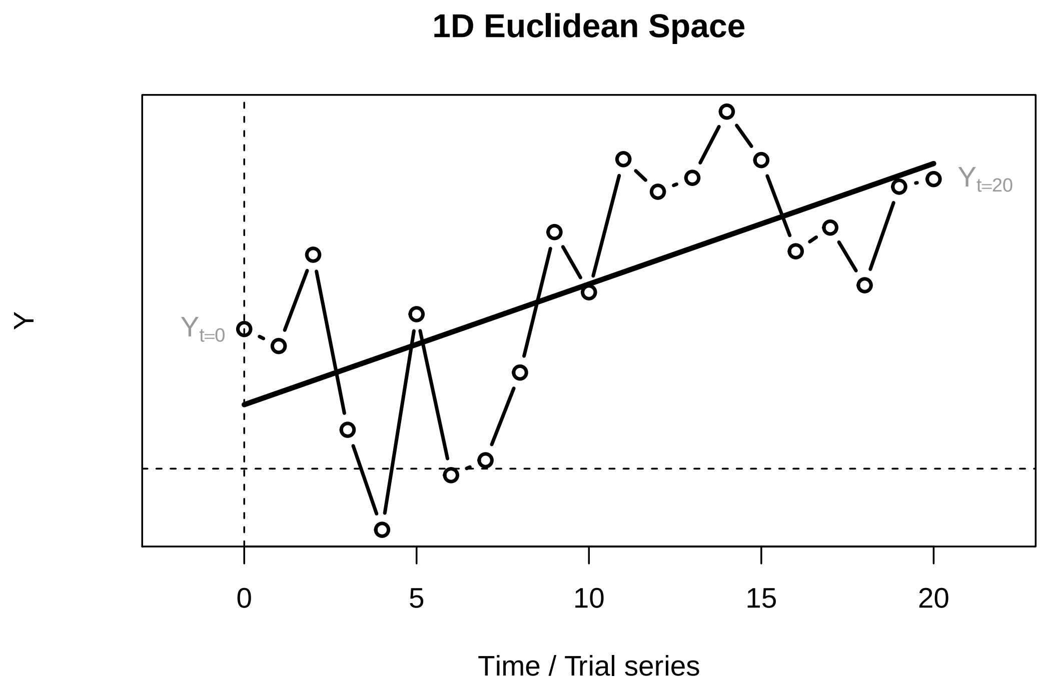
As you can read in the quoted text above, these conditions must hold in order to be able to identify unique independent components as the sources of variation of Y over time within a subject. This is the a clear example of:

If you choose to use GLM repeated measures to model change over time, you will only be able to infer independent components that are responsible for the time-evolution of Y. As is hinted in the last sentence of the quote, the validity of such inferences is not a common topic of discussion statistics textbooks.

[1] Retreived from www.statsoft.com

3.2 No! … It's a time series!

The important difference between a regular 2-dimensional Euclidean plane and the space in which we model change processes is that the X-axis represents the physical dimension **time**. In the case of the Linear Map we have a 1D space with one 'spatial' dimension Y and a time dimension t. This is called time series if Y is sampled as a continuous process, or a trial series if the time between subsequent observations is not relevant, just the fact that there was a temporal order (for example, a series of response latencies to trials in a psychological experiment in the order in which they were presented to the subject).



Time behaves different from a spatial dimension in that it is directional (time cannot be reversed), it cannot take on negative values, and, unless one is dealing with a truly random process, there will be a temporal correlation across one or more values of Y separated by an amount of time. In the linear difference equation this occurs because each value one step in the future is calculated based on the current value. If the values of Y represent an observable of a dynamical system, the system can be said to have a history, or a memory.

Ergodic systems do *not* have a history or a memory that extends across more than one time step. This is very convenient, because one can calculate the expected value of a system observable given infinite time, by making use of of the laws of probabilities of random events (or random fields). This means: The average of an observable of an Ergodic system measured across infinite time (its entire history, the **time-average**), will be the be the same value as the average of this observable measured at one instance in time, but in an infinite amount of systems of the same kind (the population, the **spatial average**) [^dice].

The simple linear difference equation will have a form of *perfect memory* across the smallest time scale (i.e., the increment of 1, $t + 1$). This 'memory' just concerns a correlation of 1 between values at adjacent time points (a short range temporal correlation, SRC), because the change from Y_t to Y_{t+1} is exactly equal to $a * Y_t$ at each iteration step. This is the meaning of deterministic, not that each value of Y is the same, but that the value of Y now can be perfectly explained form the value of Y one moment in the past.

Summarising, the most profound difference is not the fact that the equation of linear change is a deterministic model and the GLM is a probabilistic model with parameters fitted from data, this is something we can (and will) do for a as well. The profound difference between the models is the role given to the passage of time:

- The linear difference equation represents changes in Y as a function of the physical dimension *time* and Y itself.
- The GLM represents changes in Y as a function of a linear predictor composed of additive components that can be regarded as independent sources of variation that sum up to the observed values of Y.

3.3 Implementing iterative functions

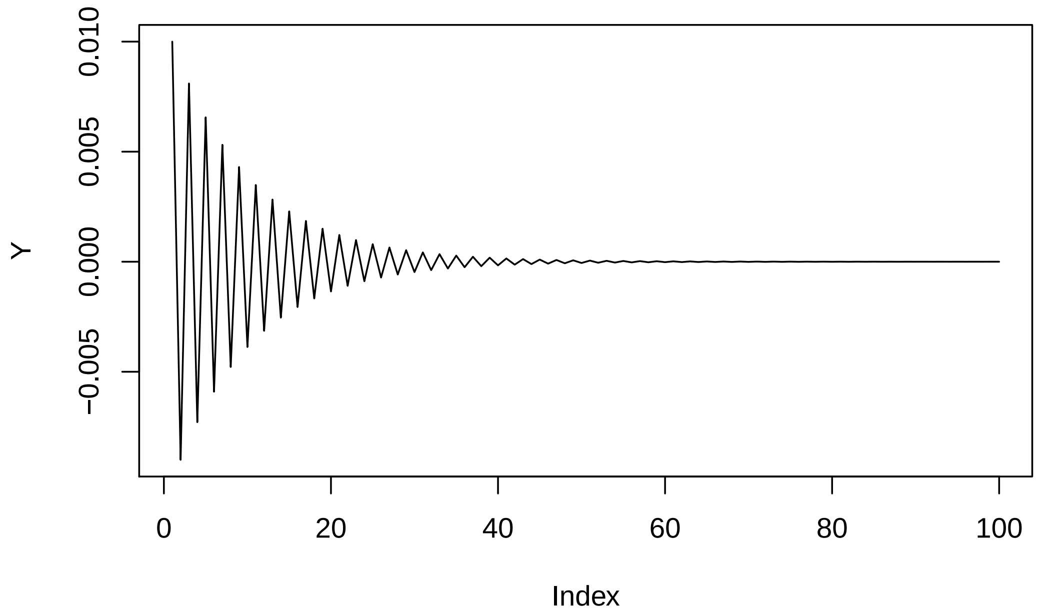
Coding change processes (difference equations) in `Matlab` and `R` is always easier than using a spreadsheet. One obvious way to do it is to use a counter variable representing the iterations of time in a `for ... next` loop (see tutorials). The iterations should run over a vector (which is the same concept as a row or a column in a spreadsheet: An indexed array of numbers or characters). The first entry should be the starting value, so the vector index 1 represents Y_0.

The loop can be implemented a number of ways, for example as a function which can be called from a script or the command or console window. In `R` working with **functions** is easy, and very much recommended (see tutorials), because it will speed up calculations considerably, and it will reduce the amount of code you need to write. You need to gain some experience with coding in `R` before you'll get it right. In order to get it lean and clean (and possibly even mean as well) you'll need a lot of experience with coding in `R`,therefore, we will (eventually) provide you the functions you'll need to complete the assignments in the **Answers** section of the assignments. If you get stuck, look at the answers. If you need to do something that reminds you of an assignment, figure out how to modify the answers to suit your specific needs.

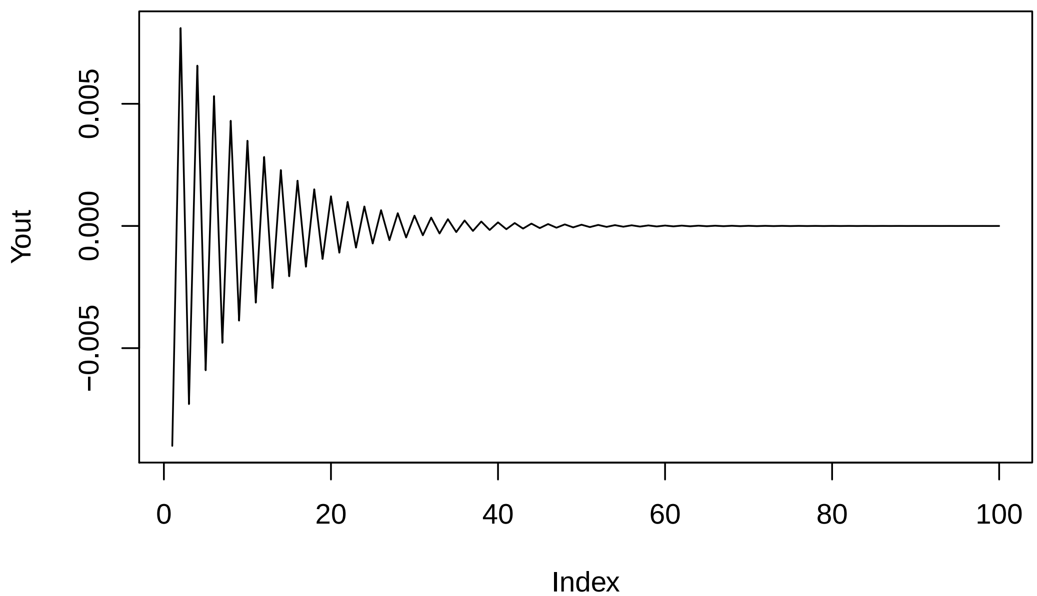
We'll use the linear map $Y_{i+1} = r * Y_i$ as an example and show three different ways to implement iterative processes:

1. The `for...` loop
2. The `-ply` family of functions
3. User defined `function()` with arguments

```
# for loop
N  <- 100
r  <- -.9
Y0 <- 0.01
Y  <- c(Y0,rep(NA,N-1))

for(i in 1:(N-1)){
  Y[i+1] <- r*Y[i]
}
plot(Y,type = "l")
```

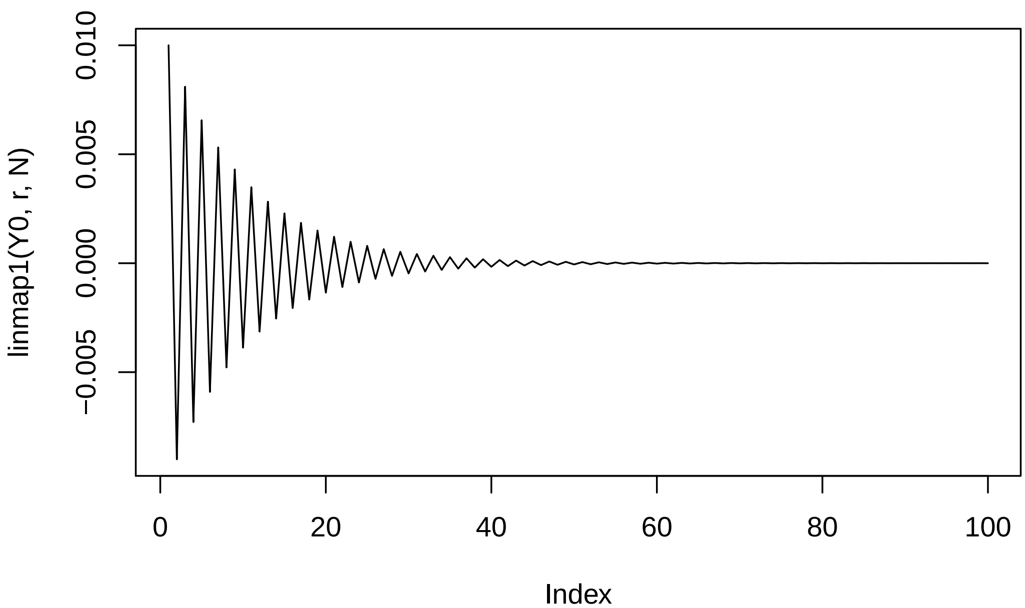



```
# -ply family: sapply
Yout <- sapply(seq_along(Y),function(t) r*Y[t])
plot(Yout,type = "l")
```




```
# function with for loop
linmap1 <- function(Y0,r,N){
```

```r
  Y  <- c(Y0,rep(NA,N-1))
  for(i in 1:(N-1)){
    Y[i+1] <- r*Y[i]
  }
  return(Y)
}
plot(linmap1(Y0,r,N),type = "l")
```



3.4 Numerical integration to simulate continuous time

In order to 'solve' a differential equation for continuous time using a method of numerical integration, one could code it like in the spreadsheet assignment below. For `R` and `Matlab` there are so-called *solvers* available, functions that will do the integration for you. For `R` look at the Examples in package `deSolve`.

Euler's method and more...

The result of applying a method of numerical integration is called a **numerical solution** of the differential equation. The **analytical solution** is the equation which will give you a value of Y for any point in time, given an initial value Y_0. Systems which have an analytical solution can be used to test the accuracy of **numerical solutions**.

Analytical solution

Remember that the analytical solution for the logistic equation is:

$$Y(t) = \frac{K * Y_0}{Y_0 + (K - Y_0) * e^{-r*t}}$$

This can be 'simplified' to

$$Y(t) = \frac{K}{1 + \left(\frac{K}{Y_0 - 1}\right) * e^{-r*t}}$$

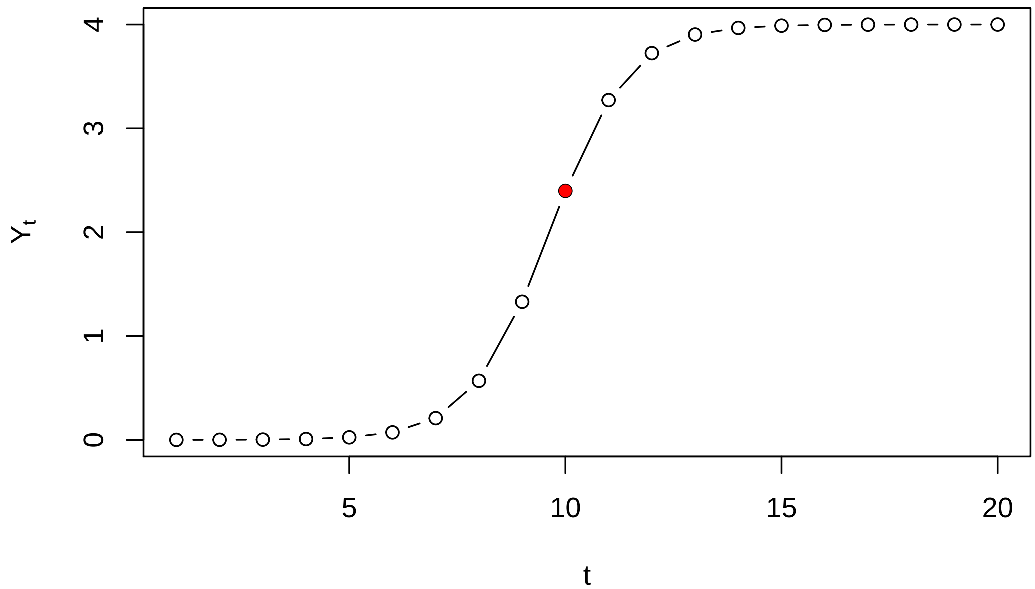
If we want to know the growth level Y_t at $t = 10$, with $Y_0 = .0001$, $r = 1.1$ and $K = 4$, we can just `fill it in`:

```
# Define a function for the solution
logSol <- function(Y0, r, K, t){K/(1+(K/Y0-1)*exp(-r*t))}

# Call the function
logSol(Y0=.0001, r=1.1, K=4, t=10)
```

```
## [1] 2.398008
```

We can pass a vector of time points to create the exact solution, the same we would get if we were to iterate the differential/difference equation.

```
# Plot from t=1 to t=100
plot(logSol(Y0=.0001, r=1.1, K=4, t=seq(1,20)), type = "b",
     ylab = expression(Y[t]), xlab = "t")
# Plot t=10 in red
points(10,logSol(Y0=.0001, r=1.1, K=4, t=10), col="red", pch=16)
```



Numerical solution (discrete)

If we would iterate the differential equation ...

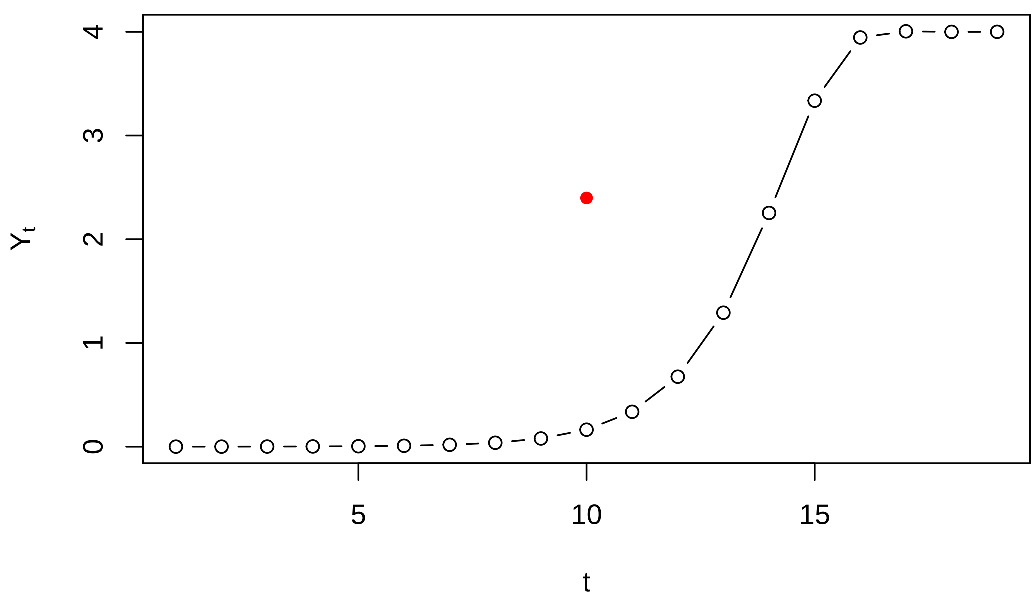
$$\frac{dY}{dt} = Y_t * (1 + r - r * \frac{Y_t}{K})$$

... as if it were a difference equation, we are *not* simulating continuous time, but a discrete time version of the model:

$$Y_{i+1} = Y_i * (1 + r - r * \frac{Y_i}{K})$$

```
logIter <-  function(Y0,r,K,t){
  N <- length(t)
  Y <- as.numeric(c(Y0, rep(NA,N-2)))
```

```
  sapply(seq_along(Y), function(t){ Y[[t+1]] <<- Y[t] * (1 + r - r * Y[t] / K)})
  }

# Plot from t=1 to t=100
plot(logIter(Y0=.0001, r=1.1, K=4, t=seq(1,20)), type = "b",
     ylab = expression(Y[t]), xlab = "t")
# Plot t=10 in red
points(10,logSol(Y0=.0001, r=1.1, K=4, t=10), col="red", pch=16)
```



3.4.1 Euler vs. Runge-Kutta

The method developped by Runge and Kutta takes a harmonic mean over a number of points, R-K4 takes 4 points, R-K6 takes 6, but there are many more variants.

Here's an exampkle with **Predator-Prey dynamics** comparing Euler's method to R-K4.

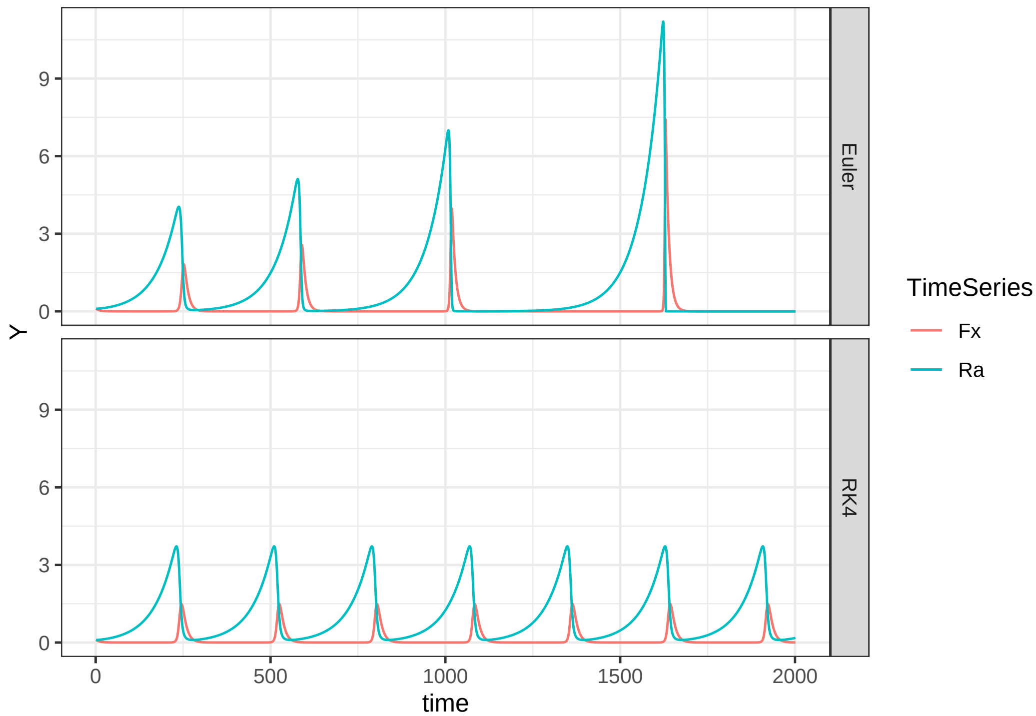
```
library(plyr)
library(tidyverse)
library(lattice)

# Lotka-Volterra Euler
lvEuler <- function(R0,F0,N,a,b,c,d,h){
```

```
  # Init vector
  Ra <- as.numeric(c(R0, rep(NA,N-1)))
  Fx <- as.numeric(c(F0, rep(NA,N-1)))

  for(t in 1:N){
  # Euler numerical solution of the predator-prey model
  Ra[t+1] <- Ra[t] + (a - b * Fx[t]) * Ra[t] * h
  Fx[t+1] <- Fx[t] + (c * Ra[t] - d) * Fx[t] * h
  }

  return(data.frame(time=1:NROW(Ra),Ra=Ra,Fx=Fx,method="Euler"))
}

# Lotka-Volterra Runge Kutta 4
lvRK4 <- function(R0,F0,N,a,b,c,d,h){

  # Init vector
  Ra <- as.numeric(c(R0, rep(NA,N-1)))
  Fx <- as.numeric(c(F0, rep(NA,N-1)))

  for(t in 1:N){
  # RK4 numerical solution of the predator-prey model
  k1_R=(a - b * Fx[t]) * Ra[t]
  k1_F=(c * Ra[t] - d) * Fx[t]

  k2_R=(a - b * (Fx[t]+h*k1_F/2)) * (Ra[t]+h*k1_R/2)
  k2_F=(c * (Ra[t]+h*k1_R/2) - d) * (Fx[t]+h*k1_F/2)

  k3_R=(a - b * (Fx[t]+h*k2_F/2)) * (Ra[t]+h*k2_R/2)
  k3_F=(c * (Ra[t]+h*k2_R/2) - d) * (Fx[t]+h*k2_F/2)

  k4_R=(a - b * (Fx[t]+h*k3_F)) * (Ra[t]+h*k3_R)
  k4_F=(c * (Ra[t]+h*k3_R) - d) * (Fx[t]+h*k3_F)

  # Iterative process
  Ra[t+1] <- Ra[t] + (1/6)*h*(k1_R+2*k2_R+2*k3_R+k4_R)
  Fx[t+1] <- Fx[t] + (1/6)*h*(k1_F+2*k2_F+2*k3_F+k4_F)
```

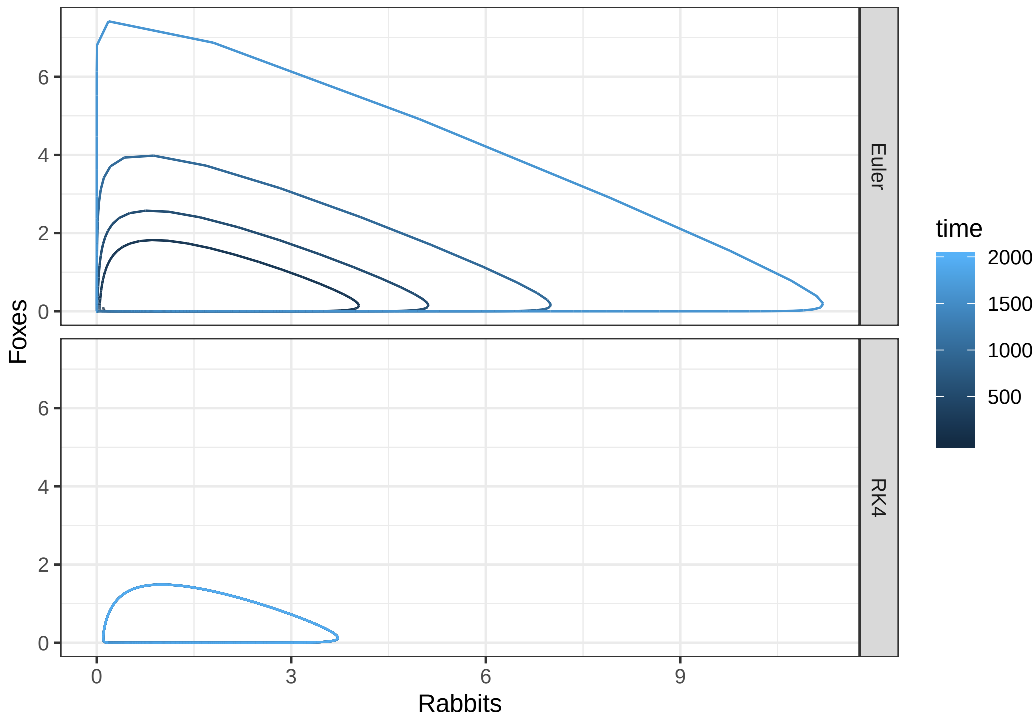
```
  }

  return(data.frame(time=1:NROW(Ra),Ra=Ra,Fx=Fx,method="RK4"))
}
```

Now that we have the fuctions, we'll plot the numerical solutions for the same set of parameters. The continuous mathematics (= if you do some calculations to find the fixed points of the system) ensure us that the system should be in an equilibrium state in which the populations keep going around in the same cycle of growth and collapse. Let's see what happens…

```
# Parameters
N  <- 2000

# Equilibrium
a  <- 1/6
b  <- 4/3
c  <- d  <- 1
R0 <- F0 <- 0.1

# Time constant
h <- 0.1

# Get the results
pp1 <- lvEuler(R0,F0,N,a,b,c,d,h)
pp2 <- lvRK4(R0,F0,N,a,b,c,d,h)

# Make a long dataframe
pp <- rbind(pp1,pp2)

pp.long <- pp %>%
  gather(key = TimeSeries, value = Y, -c("time","method"))

# Time series plots
ggplot(pp.long, aes(x=time,y=Y,colour=TimeSeries)) +
  geom_line() +
  facet_grid(method~.) +
  theme_bw()
```




```r
# Phase plane plots
ggplot(pp, aes(x=Ra,y=Fx,colour=time)) +
  geom_path() +
  facet_grid(method~.) +
  xlab("Rabbits") + ylab("Foxes") +
  theme_bw()
```



Using the Euler method predator and prey populations do not 'die out', but in phase space they seem to occupy different behavioural regimes. This looks like an unstable periodic orbit, or an unstable limit cycle, but it is in fact caused by the inaccuarcy of Euler's method. Here *RK4* clearly outperforms *Euler*.

3.5 Modeling interactions between processes and agents

3.5.1 The Competetive Lottka-Volterra Equations

The coupled predator-prey dynamics in the previous assignment are not a very realistic model of an actual ecological system. Both equations are exponential growth functions, but **R**abbits for example, also have to eat! One way to increase realism is to consider coupled logistic growth by introducing a carrying capacity.

- Follow the link to the Wiki page and try to model the system!

> This is what *interaction dynamics* refers to, modeling mutual dependiencies using the `if ... then` conditional rules isn't really about interaction, or coupling between processes.

3.5.2 Predator-Prey (and other) dynamics as Agent Based Models

Agent-Based models are an expansion of the idea of "connected growers" that includes a spatial location of the things that is subject to change over time.

Have a look at some of the NETlogo demo's:

- Rabbits Weeds Grass
- Wolf Sheep Grass

3.5.3 The dynamic field model

Probably the most impressive modelling example in developmental psychology is the Dynamic Field Model for infant perservative reaching, also known as the *A-not-B error*:

Thelen, E., Schöner, G., Scheier, C., & Smith, L. (2001). The dynamics of embodiment: A field theory of infant perseverative reaching. Behavioral and Brain Sciences, 24(1), 1-34. doi:10.1017/S0140525X01003910

The model makes some very interesting predictions that have been confirmed and it has been generalized to other phenomena and scientific disciplines as well

Smith, L. B., & Thelen, E. (2003). Development as a dynamic system. Trends in cognitive sciences, 7(8), 343-348.

Schöner, G., & Thelen, E. (2006). Using dynamic field theory to rethink infant habituation. Psychological review, 113(2), 273.

TWOMEY, K. E., & HORST, J. S. (2014). TESTING A DYNAMIC NEURAL FIELD MODEL OF CHILDREN'S CATEGORY LABELLING. In Computational Models of Cognitive Processes: Proceedings of the 13th Neural Computation and Psychology Workshop (pp. 83-94).

You can learn about it on the Dynamic Field Theory website centered around the book:

Schöner, G., & Spencer, J. (2015). Dynamic thinking: A primer on dynamic field theory. Oxford University Press.

Deel III

Basic (Nonlinear) Time Series Analysis

Chapter 4

Basic Time Series Analysis

4.1 Always plot your data!

4.2 Correlation Functions

4.3 Autoregressive models

Chapter 5

Basic Nonlinear Time Series Analysis

5.1 An intuitive notion fractal dimension

5.2 Relative Roughness

5.3 Entropy

5.4 Other measures in *casnet*

lkjlkj

Deel IV

Scaling Phenomena - Fluctuation Analyses

Chapter 6

Fluctuation Analyses: Global Scaling

> "If you have not found the 1/f spectrum, it is because you have not waited long enough. You have not looked at low enough frequencies."
> - Machlup (1981)

As **?** noted

6.1 The Box-Counting Dimension

6.2 Fractal Geometry in Time Series

6.3 Spectral Analysis (PSD)

6.4 Standardised Dispersion Analysis (SDA)

6.5 Detrended Fluctuation Analysis (DFA)

6.6 Other varieties of fluctuation analysis

Chapter 7

Fluctuation Analyses: Local Scaling

7.1 Multi-fractal geometry in time series

7.2 Multi-fractal DFA

7.3 The Wavelet Transform Modulus Maxima (WTMM)

7.4 Multi-fractal Spectrum Measures

Deel V

Recurrence Quantification Analysis

lklk

Chapter 8

Categorical Auto-Recurrence Analysis

lklk

8.1 An R interface to Marwan's commandline recurrence plots

The `rp_cl()` function is a wrapper for the commandline Recurrence Plots executables provided by Norbert Marwan.

The `rp` executable is installed when the function is called for the first time:

- It is renamed to `rp` from a platform specific filename located in the directory: [path to casnet]/commandline_rp/
- The file is copied to the directory: [path to casnet]/exec/
- The latter location is stored as an option and can be read by calling `getOption("casnet.path_to_rp")`

Note that the platform specific `rp` command line executables were created by Norbert Marwan and obtained under a Creative Commons License from the website of the Potsdam Institute for Climate Impact Research at: http://tocsy.pik-potsdam.de/

The full copyright statement on the website is as follows:

> © 2004-2017 SOME RIGHTS RESERVED
> University of Potsdam, Interdisciplinary Center for Dynamics of Complex Systems, Germany
> Potsdam Institute for Climate Impact Research, Transdisciplinary Concepts and Methods, Germany
> This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivs 2.0 Germany License.

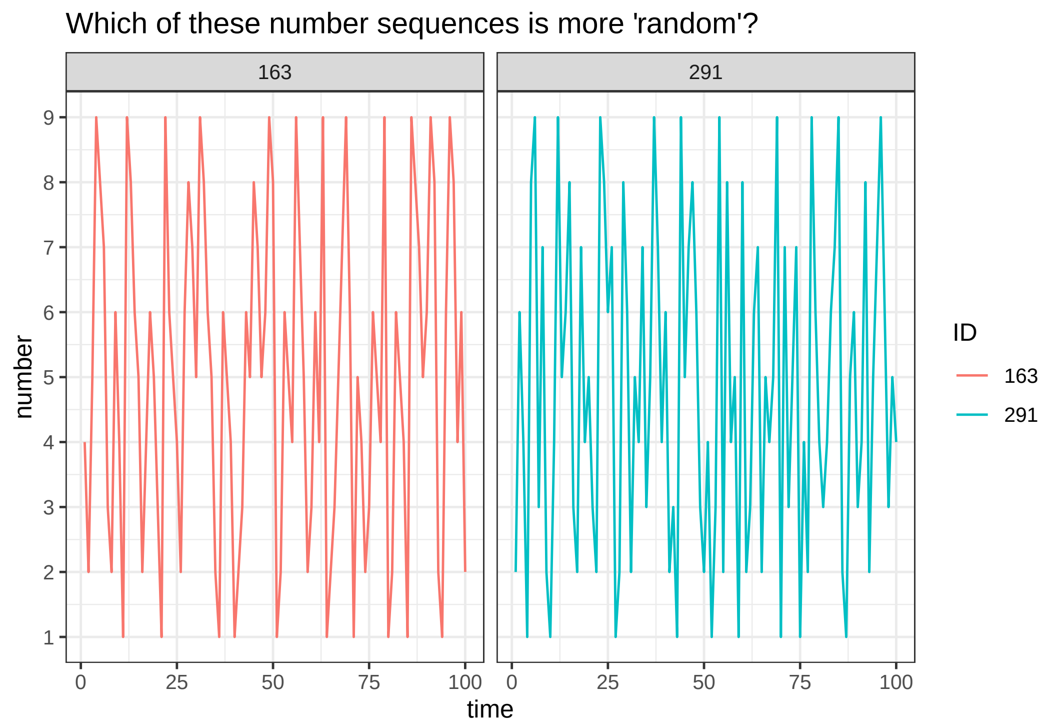
More information about recurrence quantification analysis can be found on the Recurrence Plot website.

> If you are using macOS Catalina, the `rp_cl()` will not work because it is identified as a 32-bit application!!!.
>
> Please use function `rp_measures()` instead.

8.2 Auto RQA

We'll use data from **?** in which 242 students were asked to generate random sequences of 100 numbers between 1 and 9 (for details see the article).

```
library(tidyverse)
library(invctr)
library(casnet)

# Load the random number sequence data from Oomens et al. (2015)
data(RNG)

# Select a subject
IDs <- RNG$ID%in%c(163,291)

# Look at the sequence
ggplot(RNG[IDs,],aes(x=time,y=number,group=ID)) +
  geom_line(aes(colour=ID))+
  facet_grid(~ID) +
  scale_y_continuous(breaks = 1:9) +
  ggtitle("Which of these number sequences is more 'random'?") +
  theme_bw()
```



In order to answer the question in the Figure title, we'll run a Recurrence Quantification Analysis.

The data are unordered categorical, that is, the differences between the integers are meaningless in the context of generating random number sequences. This means the RQA parameters can be set to quantify recurrences of the same value:

- Embedding lag = 1
- Embedding dimension = 1
- Radius = 0 (any number ≤ 1 will do)

In the code block below the functions `rp()`, `rp_measures()`, and `rp_plot()` are used to perform RQA on 2 participants in the dataset.

```
# Run the RQA analysis
y_1  <- RNG$number[RNG$ID==163]
y_2 <-  RNG$number[RNG$ID==291]

# This no longer works on macOS Catalina
# crqa_1 <- rp_cl(y1 = y_1, emDim  = 1, emLag = 1, emRad= 1)
# crqa_2 <- rp_cl(y1 = y_2, emDim  = 1, emLag = 1, emRad= 1)
```

```r
# Get the recurrence matrix and measures
rp_1 <- rp(y1=y_1, emDim = 1, emLag = 1, emRad = 1)
rp_2 <- rp(y1=y_2, emDim = 1, emLag = 1, emRad = 1)

# The matrix returned by rp() contains all the parameter settings as attributes.
crqa_1 <- rp_measures(rp_1)
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix           9900         3104       0.3135354            1324
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.174157          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate      Mean Max
## 1   Diagonal     668              1780  Determinism 0.5734536 2.664671   9
## 2   Vertical     959              2090 V Laminarity 0.6733247 2.179353   3
## 3 Horizontal     959              2090 H Laminarity 0.6733247 2.179353   3
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          1.1141896        0.2424724      0.3923046
## 2          0.4704117        0.1023720      0.1761294
## 3          0.4704117        0.1023720      0.1761294
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```r
crqa_2 <- rp_measures(rp_2)
```

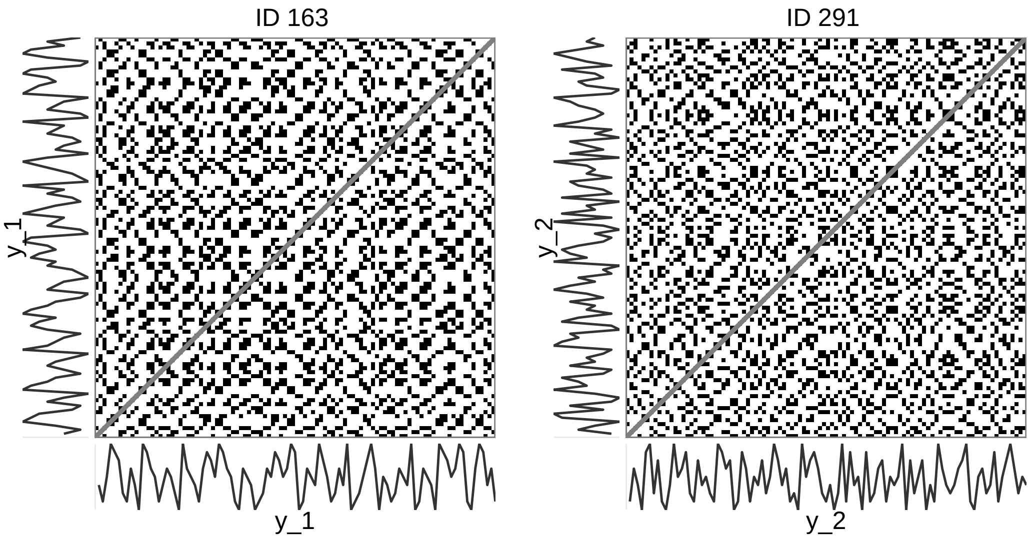
```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix           9900         3094       0.3125253            1456
##   Divergence Repetitiveness Anisotropy
```

```
## 1  0.1428571      0.9010989          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate      Mean Max
## 1   Diagonal     668              1638  Determinism 0.5294118 2.452096   7
## 2   Vertical     682              1476 V Laminarity 0.4770524 2.164223   3
## 3 Horizontal     682              1476 H Laminarity 0.4770524 2.164223   3
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8901225       0.19371040      0.3195282
## 2          0.4466065       0.09719148      0.1713084
## 3          0.4466065       0.09719148      0.1713084
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
## Plot the recurrence matrix

# Get the plots
g_1 <- rp_plot(rp_1, plotDimensions = TRUE, returnOnlyObject = TRUE, title = "ID 163")
g_2 <- rp_plot(rp_2, plotDimensions = TRUE, returnOnlyObject = TRUE, title = "ID 291")
```

Below the data and plots are rearranged for ease of comparison.

```
library(cowplot)
# The recurrence plots
cowplot::plot_grid(g_1, g_2)
```




```
# The RQA measures side by side
data.frame(subj163=t(crqa_1), subj291=t(crqa_2))
```

```
##                       X1         X1.1
## emRad          1.0000000 1.000000e+00
## RP_N        3104.0000000 3.094000e+03
## RR             0.3135354 3.125253e-01
## SING_N      1324.0000000 1.456000e+03
## SING_rate      0.4265464 4.705882e-01
## DIV_dl         0.1111111 1.428571e-01
## REP_av         1.1741573 9.010989e-01
## ANI            1.0000000 1.000000e+00
## N_dl         668.0000000 6.680000e+02
## N_dlp       1780.0000000 1.638000e+03
## DET            0.5734536 5.294118e-01
## MEAN_dl        2.6646707 2.452096e+00
## MAX_dl         9.0000000 7.000000e+00
## ENT_dl         1.1141896 8.901225e-01
## ENTrel_dl      0.2424724 1.937104e-01
## CoV_dl         0.3923046 3.195282e-01
## N_vl         959.0000000 6.820000e+02
## N_vlp       2090.0000000 1.476000e+03
## LAM_vl         0.6733247 4.770524e-01
## TT_vl          2.1793535 2.164223e+00
```

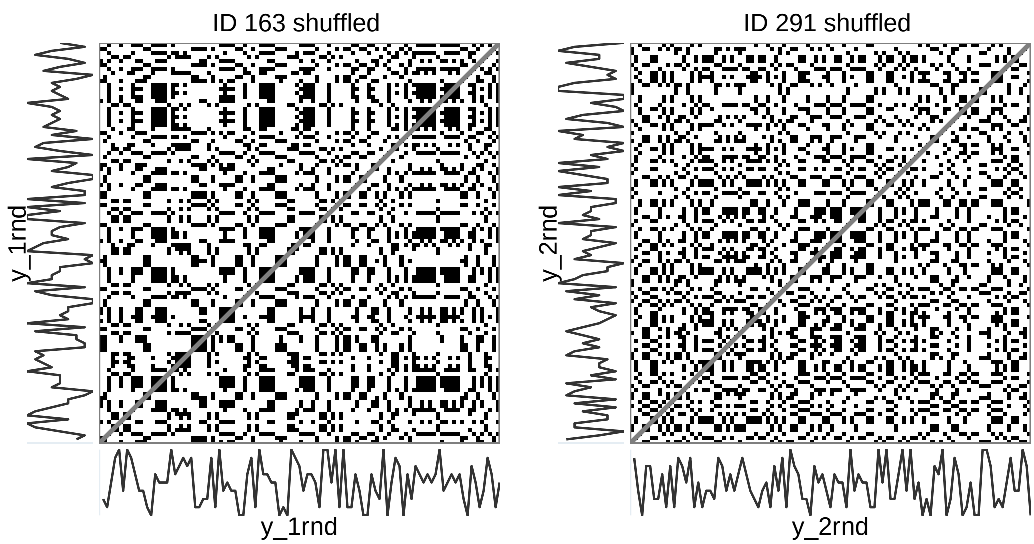
```
## MAX_vl       3.0000000 3.000000e+00
## ENT_vl       0.4704117 4.466065e-01
## ENTrel_vl    0.1023720 9.719148e-02
## CoV_vl       0.1761294 1.713084e-01
## REP_vl       1.1741573 9.010989e-01
## N_hlp     2090.0000000 1.476000e+03
## N_hl       959.0000000 6.820000e+02
## LAM_hl       0.6733247 4.770524e-01
## TT_hl        2.1793535 2.164223e+00
## MAX_hl       3.0000000 3.000000e+00
## ENT_hl       0.4704117 4.466065e-01
## ENTrel_hl    0.1023720 9.719148e-02
## CoV_hl       0.1761294 1.713084e-01
## REP_hl       1.1741573 9.010989e-01
```

The sequence generated by participant 163 has a higher **DET**erminism (`DET = .40`) than the sequence by particpant 291 (`DET = .19`). The ratio of points on a diagonal line to the total number of recurrent point also quantifies this difference (`DET_RR`). Also interesting to note, both participants have a **LAM**inarity score of `0`. This implies they avoided to produce patterns in which the exact same numbers were repeated in succession. This is a tell-tale sign of the *non-random* origins of these sequences.

8.3 Hypothesis testing using constrained data realisations

A simple strategy to get some more certainty about the differences between the two sequences is to randomise the observed series, thus removing any temporal correlations that might give rise to recurring patterns in the sequences and re-run the RQA. If the repeated patterns generated by participant 163 are non-random one would expect the **DET**erminism to drop. If they do not drop this could indicate some random autoregressive process is causing apparent deterministic temporal patterns.

```r
# Reproduce the same randomisation
set.seed(123456789)

# Randomise the number sequences
y_1rnd <- y_1[sample(1:NROW(y_1),size = NROW(y_1))]
y_2rnd <- y_2[sample(1:NROW(y_2),size = NROW(y_2))]

# Calculate RQA measures using rp_cl()
# crqa_1rnd <- rp_cl(y1 = y_1rnd, emDim  = 1, emLag = 1, emRad= 1)
# crqa_2rnd <- rp_cl(y1 = y_2rnd, emDim  = 1, emLag = 1, emRad= 1)

# Create the recurrence matrix
rp_1 <- rp(y1=y_1rnd, emDim = 1, emLag = 1,emRad = 1)
rp_2 <- rp(y1=y_2rnd, emDim = 1, emLag = 1,emRad = 1)


# If `returnMeasures = TRUE`, the RQA measures are an attribute of the rp object,
# They can also be calculated using function rp_measures()
crqa_1rnd <- attributes(rp_1)$measures
crqa_2rnd <- attributes(rp_2)$measures

# Look at the RPs
g_1rnd <- rp_plot(rp_1, plotDimensions = TRUE, returnOnlyObject = TRUE, title = "ID 163 shuffled")
g_2rnd <- rp_plot(rp_2, plotDimensions = TRUE, returnOnlyObject = TRUE, title = "ID 291 shuffled")

# Display recurrence plots
cowplot::plot_grid(g_1rnd, g_2rnd, align = "h")
```




```r
# Display the RQA measures for ID 163
cbind.data.frame(subj163=t(crqa_1), subj163rnd=t(crqa_1rnd))
```

```
##                        1 subj163rnd
## emRad          1.0000000         NA
## RP_N        3104.0000000         NA
## RR             0.3135354         NA
## SING_N      1324.0000000         NA
## SING_rate      0.4265464         NA
## DIV_dl         0.1111111         NA
## REP_av         1.1741573         NA
## ANI            1.0000000         NA
## N_dl         668.0000000         NA
## N_dlp       1780.0000000         NA
## DET            0.5734536         NA
## MEAN_dl        2.6646707         NA
## MAX_dl         9.0000000         NA
## ENT_dl         1.1141896         NA
## ENTrel_dl      0.2424724         NA
## CoV_dl         0.3923046         NA
## N_vl         959.0000000         NA
## N_vlp       2090.0000000         NA
## LAM_vl         0.6733247         NA
```

```
## TT_vl        2.1793535          NA
## MAX_vl       3.0000000          NA
## ENT_vl       0.4704117          NA
## ENTrel_vl    0.1023720          NA
## CoV_vl       0.1761294          NA
## REP_vl       1.1741573          NA
## N_hlp     2090.0000000          NA
## N_hl       959.0000000          NA
## LAM_hl       0.6733247          NA
## TT_hl        2.1793535          NA
## MAX_hl       3.0000000          NA
## ENT_hl       0.4704117          NA
## ENTrel_hl    0.1023720          NA
## CoV_hl       0.1761294          NA
## REP_hl       1.1741573          NA
```

```
# Display the RQA measures for ID 291
cbind.data.frame(subj291=t(crqa_2),  subj291rnd=t(crqa_2rnd))
```

```
##                      1 subj291rnd
## emRad     1.000000e+00          NA
## RP_N      3.094000e+03          NA
## RR        3.125253e-01          NA
## SING_N    1.456000e+03          NA
## SING_rate 4.705882e-01          NA
## DIV_dl    1.428571e-01          NA
## REP_av    9.010989e-01          NA
## ANI       1.000000e+00          NA
## N_dl      6.680000e+02          NA
## N_dlp     1.638000e+03          NA
## DET       5.294118e-01          NA
## MEAN_dl   2.452096e+00          NA
## MAX_dl    7.000000e+00          NA
## ENT_dl    8.901225e-01          NA
## ENTrel_dl 1.937104e-01          NA
## CoV_dl    3.195282e-01          NA
## N_vl      6.820000e+02          NA
## N_vlp     1.476000e+03          NA
```

```
## LAM_vl    4.770524e-01         NA
## TT_vl     2.164223e+00         NA
## MAX_vl    3.000000e+00         NA
## ENT_vl    4.466065e-01         NA
## ENTrel_vl 9.719148e-02         NA
## CoV_vl    1.713084e-01         NA
## REP_vl    9.010989e-01         NA
## N_hlp     1.476000e+03         NA
## N_hl      6.820000e+02         NA
## LAM_hl    4.770524e-01         NA
## TT_hl     2.164223e+00         NA
## MAX_hl    3.000000e+00         NA
## ENT_hl    4.466065e-01         NA
## ENTrel_hl 9.719148e-02         NA
## CoV_hl    1.713084e-01         NA
## REP_hl    9.010989e-01         NA
```

Note that the number of recurrent points (`RR`) does not change whe we shuffle the data. What changes is the number of recurrent points that form line structures in the recurrence plot. Randomising the number sequences causes vertical line structures to appear in the recurrence plot (`LAM`, `V_max`, `V_entr`, `TT`), this is what we would expect if the data generating process were indeed a random process. Having no such structures means there were hardly any sequences consisting of repetitions of the same number. Participants may have adopted a strategy to avoid such sequences because they erroneously believed this to be a feature of non-random sequences.

8.3.1 A permutation test with surrogate time series

In order to get an idea about the meanignfulness of these differences, we can construct a surrogate data test for each participant. If we want a one-sided test with $\alpha = .05$, the formula for the number of constrained realisations M we minimally need is:

$$M = \frac{1}{\alpha} - 1 = 19$$

. Add the observed value and we have a sample size of $N = 20$. For a two sided test we would use

$$M = \frac{2}{\alpha} - 1 = 39$$

.

Of course, if there are no computational constraints on generating surrogate time series, we can go much higher, If we want $N = 100$, the test will be an evaluation of H_0 at $\alpha = .01$.

1. Create `99` realisations that reflect a test of the hypothesis $H_0 : X_i \sim \mathcal{U}(1, 9)$ at $\alpha = .01$.
2. Calculate the measure of interest, e.g. `DET`
3. If the observed `DET` value is at the extremes of the distribution of values representing H_0, the observed value was probably not generated by drawing from a discrete uniform distribution with finite elements `1` through `9`.

```
library(plyr)
library(dplyr)

set.seed(123456789)

y_1rnd_sur <- ldply(1:99, function(s) y_1[sample(1:NROW(y_1),size = NROW(y_1))])
y_2rnd_sur <- ldply(1:99, function(s) y_2[sample(1:NROW(y_2),size = NROW(y_2))])

# crqa_1rnd_sur <- ldply(seq_along(y_1rnd_sur$V1), function(r) rp_cl(y1 = as.numeric(y_1rnd_sur[r,]), em
# crqa_1rnd_sur[NROW(crqa_1rnd_sur)+1,] <- crqa_1
# crqa_2rnd_sur <- ldply(seq_along(y_2rnd_sur$V1), function(r) rp_cl(y1 = as.numeric(y_2rnd_sur[r,]), em
# crqa_2rnd_sur[NROW(crqa_2rnd_sur)+1,] <- crqa_2

crqa_1rnd_sur <- ldply(seq_along(y_1rnd_sur$V1), function(r) rp_measures(rp(y1 = as.numeric(y_1rnd_sur[r
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1474
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.142945          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1630  Determinism 0.5251289 2.500000   6
```

```
## 2   Vertical     673              1863 V Laminarity 0.6001933 2.768202   6
## 3 Horizontal     673              1863 H Laminarity 0.6001933 2.768202   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9496474        0.2066643      0.3299676
## 2          1.1353416        0.2470755      0.3826106
## 3          1.1353416        0.2470755      0.3826106
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1532
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.080153          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     662              1572  Determinism 0.5064433 2.374622   6
## 2   Vertical     711              1698 V Laminarity 0.5470361 2.388186   5
## 3 Horizontal     711              1698 H Laminarity 0.5470361 2.388186   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8016346        0.1744535      0.2998388
## 2          0.8097987        0.1762302      0.2806101
## 3          0.8097987        0.1762302      0.2806101
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1570
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9556714          1
##
```

```
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     592              1534  Determinism 0.4942010 2.591216   7
## 2   Vertical     651              1466 V Laminarity 0.4722938 2.251920   6
## 3 Horizontal     651              1466 H Laminarity 0.4722938 2.251920   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          1.0377648        0.2258406      0.3780268
## 2          0.6052616        0.1317183      0.2768727
## 3          0.6052616        0.1317183      0.2768727
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1608
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.088904          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     610              1496  Determinism 0.4819588 2.452459   6
## 2   Vertical     661              1629 V Laminarity 0.5248067 2.464448   6
## 3 Horizontal     661              1629 H Laminarity 0.5248067 2.464448   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8927877        0.1942904      0.3042756
## 2          0.9102961        0.1981006      0.3104811
## 3          0.9102961        0.1981006      0.3104811
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
```

```
## 1 Recurrence Matrix          9900        3104      0.3135354           1500
##   Divergence Repetitiveness Anisotropy
## 1        0.2       0.915212          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     670              1604  Determinism 0.5167526 2.394030   5
## 2   Vertical     616              1468 V Laminarity 0.4729381 2.383117   6
## 3 Horizontal     616              1468 H Laminarity 0.4729381 2.383117   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8206086        0.1785826      0.2977704
## 2          0.7975358        0.1735615      0.2967558
## 3          0.7975358        0.1735615      0.2967558
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1516
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9049118          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     658              1588  Determinism 0.5115979 2.413374   6
## 2   Vertical     556              1437 V Laminarity 0.4629510 2.584532   6
## 3 Horizontal     556              1437 H Laminarity 0.4629510 2.584532   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8518589        0.1853834      0.3082351
## 2          1.0176877        0.2214714      0.3698172
## 3          1.0176877        0.2214714      0.3698172
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1504
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.004375          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1600  Determinism 0.5154639 2.424242   6
## 2   Vertical     648              1607 V Laminarity 0.5177191 2.479938   5
## 3 Horizontal     648              1607 H Laminarity 0.5177191 2.479938   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.862682        0.1877387      0.3019945
## 2           0.888703        0.1934015      0.3508241
## 3           0.888703        0.1934015      0.3508241
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1452
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.071429          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     668              1652  Determinism 0.5322165 2.473054   8
## 2   Vertical     644              1770 V Laminarity 0.5702320 2.748447   6
## 3 Horizontal     644              1770 H Laminarity 0.5702320 2.748447   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9170476        0.1995699      0.3424927
## 2          1.1525852        0.2508281      0.3906511
```

```
## 3          1.1525852        0.2508281      0.3906511
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1454
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571        1.02303          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     672              1650  Determinism 0.5315722 2.455357   7
## 2   Vertical     660              1688 V Laminarity 0.5438144 2.557576   7
## 3 Horizontal     660              1688 H Laminarity 0.5438144 2.557576   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8930781        0.1943536      0.3107592
## 2          0.9487190        0.2064623      0.3979966
## 3          0.9487190        0.2064623      0.3979966
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1520
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.054293          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     634              1584  Determinism 0.5103093 2.498423   8
## 2   Vertical     636              1670 V Laminarity 0.5380155 2.625786   9
```

```
## 3 Horizontal     636              1670 H Laminarity 0.5380155 2.625786   9
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9459457        0.2058588      0.3566150
## 2          0.9904936        0.2155534      0.4964229
## 3          0.9904936        0.2155534      0.4964229
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1302
##   Divergence Repetitiveness Anisotropy
## 1        0.1       1.022198          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     732              1802  Determinism 0.5805412 2.461749  10
## 2   Vertical     691              1842 V Laminarity 0.5934278 2.665702   7
## 3 Horizontal     691              1842 H Laminarity 0.5934278 2.665702   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8920705        0.1941343      0.3661549
## 2          1.0934871        0.2379671      0.3645973
## 3          1.0934871        0.2379671      0.3645973
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1502
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.053683          1
##
##
```

```
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     640              1602  Determinism 0.5161082 2.503125   8
## 2   Vertical     644              1688 V Laminarity 0.5438144 2.621118   6
## 3 Horizontal     644              1688 H Laminarity 0.5438144 2.621118   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9448253        0.2056149      0.3686073
## 2          1.0650500        0.2317785      0.3507751
## 3          1.0650500        0.2317785      0.3507751
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1412
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.010638          1
##
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     686              1692  Determinism 0.5451031 2.466472   7
## 2   Vertical     659              1710 V Laminarity 0.5509021 2.594841   7
## 3 Horizontal     659              1710 H Laminarity 0.5509021 2.594841   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.914963        0.1991162      0.3375214
## 2           1.002124        0.2180844      0.4154863
## 3           1.002124        0.2180844      0.4154863
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1460
```

```
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.077859          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674              1644  Determinism 0.5296392 2.439169   6
## 2   Vertical     756              1772 V Laminarity 0.5708763 2.343915   5
## 3 Horizontal     756              1772 H Laminarity 0.5708763 2.343915   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8822886        0.1920056      0.3152792
## 2          0.7484042        0.1628694      0.3021096
## 3          0.7484042        0.1628694      0.3021096
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1442
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.043923          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     698              1662  Determinism 0.5354381 2.381089   6
## 2   Vertical     641              1735 V Laminarity 0.5589562 2.706708   8
## 3 Horizontal     641              1735 H Laminarity 0.5589562 2.706708   8
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8089361        0.1760424      0.2883778
## 2          1.1051250        0.2404997      0.4234779
## 3          1.1051250        0.2404997      0.4234779
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1490
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.155514          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1614  Determinism 0.5199742 2.445455   9
## 2   Vertical     758              1865 V Laminarity 0.6008376 2.460422   5
## 3 Horizontal     758              1865 H Laminarity 0.6008376 2.460422   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8798297        0.1914705      0.3471851
## 2          0.8665501        0.1885805      0.2668452
## 3          0.8665501        0.1885805      0.2668452
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1474
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.130675          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     630              1630  Determinism 0.5251289 2.587302   8
## 2   Vertical     651              1843 V Laminarity 0.5937500 2.831029   7
## 3 Horizontal     651              1843 H Laminarity 0.5937500 2.831029   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           1.037190        0.2257156      0.3596712
## 2           1.226479        0.2669090      0.4397469
## 3           1.226479        0.2669090      0.4397469
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1496
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.218284          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     672              1608  Determinism 0.5180412 2.392857   7
## 2   Vertical     784              1959 V Laminarity 0.6311211 2.498724   7
## 3 Horizontal     784              1959 H Laminarity 0.6311211 2.498724   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8140217        0.1771492      0.3195385
## 2          0.8900119        0.1936863      0.3735017
## 3          0.8900119        0.1936863      0.3735017
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1462
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.160171          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     678              1642  Determinism 0.5289948 2.421829   7
## 2   Vertical     727              1905 V Laminarity 0.6137242 2.620358   5
## 3 Horizontal     727              1905 H Laminarity 0.6137242 2.620358   5
```

```
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8566649        0.1864293      0.3452682
## 2          1.0449435        0.2274029      0.3189839
## 3          1.0449435        0.2274029      0.3189839
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1552
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.099227          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     628              1552  Determinism 0.5000000 2.471338   9
## 2   Vertical     686              1706 V Laminarity 0.5496134 2.486880   6
## 3 Horizontal     686              1706 H Laminarity 0.5496134 2.486880   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9108676        0.1982250      0.3756865
## 2          0.9084244        0.1976933      0.3342763
## 3          0.9084244        0.1976933      0.3342763
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1506
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571        1.14393          1
##
##
##  Line-based Measures
```

```
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     666              1598  Determinism 0.5148196 2.399399   7
## 2   Vertical     703              1828 V Laminarity 0.5889175 2.600284   6
## 3 Horizontal     703              1828 H Laminarity 0.5889175 2.600284   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8325816        0.1811882      0.3099337
## 2          1.0222793        0.2224707      0.3521682
## 3          1.0222793        0.2224707      0.3521682
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1434
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9694611          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     686              1670  Determinism 0.5380155 2.434402   7
## 2   Vertical     612              1619 V Laminarity 0.5215851 2.645425   6
## 3 Horizontal     612              1619 H Laminarity 0.5215851 2.645425   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8737947        0.1901571      0.3294269
## 2          1.0835339        0.2358010      0.3644853
## 3          1.0835339        0.2358010      0.3644853
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1502
##   Divergence Repetitiveness Anisotropy
```

```
## 1      0.125       1.202871          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     644              1602  Determinism 0.5161082 2.487578   8
## 2   Vertical     796              1927 V Laminarity 0.6208119 2.420854   6
## 3 Horizontal     796              1927 H Laminarity 0.6208119 2.420854   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9305274        0.2025034      0.3373800
## 2          0.8310139        0.1808471      0.3009616
## 3          0.8310139        0.1808471      0.3009616
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1484
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9919753          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     676              1620  Determinism 0.5219072 2.396450   6
## 2   Vertical     628              1607 V Laminarity 0.5177191 2.558917   5
## 3 Horizontal     628              1607 H Laminarity 0.5177191 2.558917   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8299387        0.1806131      0.2951742
## 2          0.9718675        0.2114999      0.3298955
## 3          0.9718675        0.2114999      0.3298955
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1530
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.083863          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     638              1574  Determinism 0.5070876 2.467085   6
## 2   Vertical     653              1706 V Laminarity 0.5496134 2.612557   5
## 3 Horizontal     653              1706 H Laminarity 0.5496134 2.612557   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9115156        0.1983660      0.3109218
## 2          0.9604610        0.2090176      0.2660418
## 3          0.9604610        0.2090176      0.2660418
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1554
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9503226          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     622              1550  Determinism 0.4993557 2.491961   6
## 2   Vertical     528              1473 V Laminarity 0.4745490 2.789773   7
## 3 Horizontal     528              1473 H Laminarity 0.4745490 2.789773   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9396176        0.2044816      0.3391095
## 2          1.1799781        0.2567894      0.4094626
## 3          1.1799781        0.2567894      0.4094626
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1548
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.037918          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     636              1556  Determinism 0.5012887 2.446541   6
## 2   Vertical     588              1615 V Laminarity 0.5202964 2.746599   6
## 3 Horizontal     588              1615 H Laminarity 0.5202964 2.746599   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8897234        0.1936235      0.3248864
## 2          1.1502626        0.2503227      0.3424295
## 3          1.1502626        0.2503227      0.3424295
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1532
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.010814          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     638              1572  Determinism 0.5064433 2.463950   6
## 2   Vertical     602              1589 V Laminarity 0.5119201 2.639535   7
## 3 Horizontal     602              1589 H Laminarity 0.5119201 2.639535   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
```

```
## 1          0.9028595        0.1964823      0.3145655
## 2          1.0843971        0.2359889      0.3954420
## 3          1.0843971        0.2359889      0.3954420
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1552
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.180412          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     640              1552  Determinism 0.5000000 2.425000   7
## 2   Vertical     762              1832 V Laminarity 0.5902062 2.404199   5
## 3 Horizontal     762              1832 H Laminarity 0.5902062 2.404199   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8652749         0.188303      0.3114060
## 2          0.8226920         0.179036      0.3051178
## 3          0.8226920         0.179036      0.3051178
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1480
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.012315          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
```

```
## 1   Diagonal     664              1624  Determinism 0.5231959 2.445783   7
## 2   Vertical     632              1644 V Laminarity 0.5296392 2.601266   7
## 3 Horizontal     632              1644 H Laminarity 0.5296392 2.601266   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8900316        0.1936906      0.3160089
## 2          0.9642939        0.2098517      0.4261837
## 3          0.9642939        0.2098517      0.4261837
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1500
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667        1.11596          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     666              1604  Determinism 0.5167526 2.408408   6
## 2   Vertical     766              1790 V Laminarity 0.5766753 2.336815   5
## 3 Horizontal     766              1790 H Laminarity 0.5766753 2.336815   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8414948        0.1831279      0.3142475
## 2          0.7479880        0.1627788      0.2849116
## 3          0.7479880        0.1627788      0.2849116
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1516
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.010076          1
```

```
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     658              1588  Determinism 0.5115979 2.413374   8
## 2   Vertical     650              1604 V Laminarity 0.5167526 2.467692   5
## 3 Horizontal     650              1604 H Laminarity 0.5167526 2.467692   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8425515        0.1833579      0.3247491
## 2          0.9014084        0.1961665      0.3008984
## 3          0.9014084        0.1961665      0.3008984
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1466
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.8968254          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     664              1638  Determinism 0.5277062 2.466867   8
## 2   Vertical     552              1469 V Laminarity 0.4732603 2.661232   9
## 3 Horizontal     552              1469 H Laminarity 0.4732603 2.661232   9
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8946311        0.1946916      0.3885999
## 2          1.0167126        0.2212592      0.5298799
## 3          1.0167126        0.2212592      0.5298799
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
```

```
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354           1488
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.019802          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1616  Determinism 0.5206186 2.448485   6
## 2   Vertical     660              1648 V Laminarity 0.5309278 2.496970   5
## 3 Horizontal     660              1648 H Laminarity 0.5309278 2.496970   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8864585        0.1929130      0.3424261
## 2          0.9241537        0.2011163      0.2829684
## 3          0.9241537        0.2011163      0.2829684
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354           1540
##   Divergence Repetitiveness Anisotropy
## 1        0.2       1.054987          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1564  Determinism 0.5038660 2.398773   5
## 2   Vertical     748              1650 V Laminarity 0.5315722 2.205882   5
## 3 Horizontal     748              1650 H Laminarity 0.5315722 2.205882   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8261516        0.1797889      0.2996374
## 2          0.5060857        0.1101355      0.2726940
## 3          0.5060857        0.1101355      0.2726940
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1460
##   Divergence Repetitiveness Anisotropy
## 1        0.2       1.161192          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     688              1644  Determinism 0.5296392 2.389535   5
## 2   Vertical     755              1909 V Laminarity 0.6150129 2.528477   7
## 3 Horizontal     755              1909 H Laminarity 0.6150129 2.528477   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8126801        0.1768572      0.2976810
## 2          0.9394468        0.2044445      0.3551864
## 3          0.9394468        0.2044445      0.3551864
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1510
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.050816          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     662              1594  Determinism 0.5135309 2.407855   8
## 2   Vertical     665              1675 V Laminarity 0.5396263 2.518797   6
## 3 Horizontal     665              1675 H Laminarity 0.5396263 2.518797   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8346869        0.1816464      0.3538628
```

```
## 2          0.9065868        0.1972934      0.4005908
## 3          0.9065868        0.1972934      0.4005908
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1498
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111      0.8711083          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     646              1606  Determinism 0.5173969 2.486068   9
## 2   Vertical     567              1399 V Laminarity 0.4507088 2.467372   6
## 3 Horizontal     567              1399 H Laminarity 0.4507088 2.467372   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9283747        0.2020349      0.3574586
## 2          0.8911409        0.1939320      0.3210412
## 3          0.8911409        0.1939320      0.3210412
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1524
##   Divergence Repetitiveness Anisotropy
## 1      0.125       0.978481          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     650              1580  Determinism 0.5090206 2.430769   8
```

```
## 2   Vertical     565              1546 V Laminarity 0.4980670 2.736283   6
## 3 Horizontal     565              1546 H Laminarity 0.4980670 2.736283   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8685096        0.1890070      0.3415659
## 2          1.1330581        0.2465786      0.4105346
## 3          1.1330581        0.2465786      0.4105346
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1542
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9679898          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     636              1562  Determinism 0.5032216 2.455975   6
## 2   Vertical     631              1512 V Laminarity 0.4871134 2.396197   5
## 3 Horizontal     631              1512 H Laminarity 0.4871134 2.396197   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8897350        0.1936261      0.3426756
## 2          0.8111777        0.1765303      0.2796809
## 3          0.8111777        0.1765303      0.2796809
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1524
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.8822785          1
##
```

```
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     664              1580  Determinism 0.5090206 2.379518   7
## 2   Vertical     589              1394 V Laminarity 0.4490979 2.366723   5
## 3 Horizontal     589              1394 H Laminarity 0.4490979 2.366723   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8096781        0.1762039      0.3046276
## 2          0.7805987        0.1698756      0.2813892
## 3          0.7805987        0.1698756      0.2813892
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354           1490
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.027261          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     680              1614  Determinism 0.5199742 2.373529   7
## 2   Vertical     662              1658 V Laminarity 0.5341495 2.504532   5
## 3 Horizontal     662              1658 H Laminarity 0.5341495 2.504532   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8000412        0.1741067      0.3148189
## 2          0.9429019        0.2051964      0.3180766
## 3          0.9429019        0.2051964      0.3180766
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
```

```
## 1 Recurrence Matrix          9900        3104      0.3135354           1366
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.067894          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     696              1738  Determinism 0.5599227 2.497126   6
## 2   Vertical     687              1856 V Laminarity 0.5979381 2.701601   7
## 3 Horizontal     687              1856 H Laminarity 0.5979381 2.701601   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9427453        0.2051623      0.3348749
## 2          1.1245945        0.2447367      0.4387494
## 3          1.1245945        0.2447367      0.4387494
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1530
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.007624          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1574  Determinism 0.5070876 2.414110   6
## 2   Vertical     658              1586 V Laminarity 0.5109536 2.410334   6
## 3 Horizontal     658              1586 H Laminarity 0.5109536 2.410334   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8511663        0.1852327      0.3003213
## 2          0.8440570        0.1836855      0.3242095
## 3          0.8440570        0.1836855      0.3242095
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104      0.3135354            1532
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9777354          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     650              1572  Determinism 0.5064433 2.418462   7
## 2   Vertical     614              1537 V Laminarity 0.4951675 2.503257   6
## 3 Horizontal     614              1537 H Laminarity 0.4951675 2.503257   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8471002        0.1843478      0.3287191
## 2          0.8891883        0.1935071      0.3910802
## 3          0.8891883        0.1935071      0.3910802
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104      0.3135354            1544
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.002564          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     638              1560  Determinism 0.5025773 2.445141   8
## 2   Vertical     623              1564 V Laminarity 0.5038660 2.510433   6
## 3 Horizontal     623              1564 H Laminarity 0.5038660 2.510433   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8874723        0.1931337      0.3327104
## 2          0.9435345        0.2053340      0.3534380
```

```
## 3          0.9435345        0.2053340      0.3534380
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1488
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.8391089          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1616  Determinism 0.5206186 2.448485   7
## 2   Vertical     539              1356 V Laminarity 0.4368557 2.515770   4
## 3 Horizontal     539              1356 H Laminarity 0.4368557 2.515770   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8925514        0.1942390      0.3146924
## 2          0.8862420        0.1928659      0.2510550
## 3          0.8862420        0.1928659      0.2510550
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1564
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9753247          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     630              1540  Determinism 0.4961340 2.444444   7
## 2   Vertical     627              1502 V Laminarity 0.4838918 2.395534   5
```

```
## 3 Horizontal     627              1502 H Laminarity 0.4838918 2.395534   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8860456        0.1928232      0.3075730
## 2          0.7491294        0.1630272      0.3546788
## 3          0.7491294        0.1630272      0.3546788
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1448
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.9021739          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     680              1656  Determinism 0.5335052 2.435294   8
## 2   Vertical     581              1494 V Laminarity 0.4813144 2.571429   5
## 3 Horizontal     581              1494 H Laminarity 0.4813144 2.571429   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8721395        0.1897969      0.3361959
## 2          0.9834178        0.2140135      0.3286711
## 3          0.9834178        0.2140135      0.3286711
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1500
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.023691          1
##
##
```

```
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1604  Determinism 0.5167526 2.460123   7
## 2   Vertical     612              1642 V Laminarity 0.5289948 2.683007   6
## 3 Horizontal     612              1642 H Laminarity 0.5289948 2.683007   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.903191        0.1965544      0.3080826
## 2           1.098652        0.2390911      0.3816088
## 3           1.098652        0.2390911      0.3816088
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1560
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.056995          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     622              1544  Determinism 0.4974227 2.482315   8
## 2   Vertical     604              1632 V Laminarity 0.5257732 2.701987   8
## 3 Horizontal     604              1632 H Laminarity 0.5257732 2.701987   8
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9273709        0.2018165      0.3524388
## 2          1.0832395        0.2357369      0.4557390
## 3          1.0832395        0.2357369      0.4557390
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1396
```

```
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       0.912178          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     702              1708  Determinism 0.5502577 2.433048   7
## 2   Vertical     644              1558 V Laminarity 0.5019330 2.419255   5
## 3 Horizontal     644              1558 H Laminarity 0.5019330 2.419255   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8740153        0.1902051      0.3182713
## 2          0.7981361        0.1736921      0.3271586
## 3          0.7981361        0.1736921      0.3271586
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1570
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.148631          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     630              1534  Determinism 0.4942010 2.434921   6
## 2   Vertical     722              1762 V Laminarity 0.5676546 2.440443   5
## 3 Horizontal     722              1762 H Laminarity 0.5676546 2.440443   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8763468        0.1907125      0.3049645
## 2          0.8552021        0.1861110      0.2911240
## 3          0.8552021        0.1861110      0.2911240
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1466
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.128205          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     666              1638  Determinism 0.5277062 2.459459   8
## 2   Vertical     708              1848 V Laminarity 0.5953608 2.610169   6
## 3 Horizontal     708              1848 H Laminarity 0.5953608 2.610169   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8958142        0.1949490      0.3516453
## 2          1.0409292        0.2265293      0.3361889
## 3          1.0409292        0.2265293      0.3361889
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1574
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9908497          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     634              1530  Determinism 0.4929124 2.413249   6
## 2   Vertical     602              1516 V Laminarity 0.4884021 2.518272   5
## 3 Horizontal     602              1516 H Laminarity 0.4884021 2.518272   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8449839        0.1838872      0.2990710
## 2          0.9476654        0.2062330      0.3031653
## 3          0.9476654        0.2062330      0.3031653
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1508
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       0.947995          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1596  Determinism 0.5141753 2.418182   7
## 2   Vertical     633              1513 V Laminarity 0.4874356 2.390205   5
## 3 Horizontal     633              1513 H Laminarity 0.4874356 2.390205   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8537455        0.1857940      0.3191784
## 2          0.7744145        0.1685298      0.3372298
## 3          0.7744145        0.1685298      0.3372298
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1554
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.068387          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     644              1550  Determinism 0.4993557 2.406832   9
## 2   Vertical     628              1656 V Laminarity 0.5335052 2.636943   6
## 3 Horizontal     628              1656 H Laminarity 0.5335052 2.636943   6
```

```
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8360015        0.1819325      0.3339544
## 2          1.0516505        0.2288625      0.3663753
## 3          1.0516505        0.2288625      0.3663753
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1546
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571        1.09371          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     646              1558  Determinism 0.5019330 2.411765   7
## 2   Vertical     742              1704 V Laminarity 0.5489691 2.296496   4
## 3 Horizontal     742              1704 H Laminarity 0.5489691 2.296496   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8477702        0.1844936      0.3267161
## 2          0.6574334        0.1430721      0.2150748
## 3          0.6574334        0.1430721      0.2150748
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1524
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.114557          1
##
##
##  Line-based Measures
```

```
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     646              1580  Determinism 0.5090206 2.445820   6
## 2   Vertical     748              1761 V Laminarity 0.5673325 2.354278   4
## 3 Horizontal     748              1761 H Laminarity 0.5673325 2.354278   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8871512        0.1930638      0.3250234
## 2          0.7491623        0.1630343      0.2423198
## 3          0.7491623        0.1630343      0.2423198
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1548
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.218509          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     624              1556  Determinism 0.5012887 2.493590   7
## 2   Vertical     759              1896 V Laminarity 0.6108247 2.498024   5
## 3 Horizontal     759              1896 H Laminarity 0.6108247 2.498024   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9385827        0.2042564      0.3549192
## 2          0.9087331        0.1977605      0.3519037
## 3          0.9087331        0.1977605      0.3519037
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1474
##   Divergence Repetitiveness Anisotropy
```

```
## 1  0.1428571       1.046626          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     646              1630  Determinism 0.5251289 2.523220   7
## 2   Vertical     684              1706 V Laminarity 0.5496134 2.494152   6
## 3 Horizontal     684              1706 H Laminarity 0.5496134 2.494152   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9771000        0.2126386      0.3380039
## 2          0.8892125        0.1935124      0.3723195
## 3          0.8892125        0.1935124      0.3723195
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1502
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.008739          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     656              1602  Determinism 0.5161082 2.442073   7
## 2   Vertical     698              1616 V Laminarity 0.5206186 2.315186   5
## 3 Horizontal     698              1616 H Laminarity 0.5206186 2.315186   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8838186        0.1923385      0.3301588
## 2          0.7147236        0.1555397      0.2808391
## 3          0.7147236        0.1555397      0.2808391
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1464
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9073171          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     680              1640  Determinism 0.5283505 2.411765   7
## 2   Vertical     623              1488 V Laminarity 0.4793814 2.388443   5
## 3 Horizontal     623              1488 H Laminarity 0.4793814 2.388443   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8446774        0.1838205      0.3400545
## 2          0.7885166        0.1715987      0.3237871
## 3          0.7885166        0.1715987      0.3237871
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1548
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.090617          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     642              1556  Determinism 0.5012887 2.423676   6
## 2   Vertical     695              1697 V Laminarity 0.5467139 2.441727   7
## 3 Horizontal     695              1697 H Laminarity 0.5467139 2.441727   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8580292        0.1867262      0.3129453
## 2          0.8144396        0.1772401      0.4134782
## 3          0.8144396        0.1772401      0.4134782
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1466
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9957265          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     684              1638  Determinism 0.5277062 2.394737   6
## 2   Vertical     696              1631 V Laminarity 0.5254510 2.343391   6
## 3 Horizontal     696              1631 H Laminarity 0.5254510 2.343391   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8251830        0.1795781      0.3062766
## 2          0.7108378        0.1546941      0.3406671
## 3          0.7108378        0.1546941      0.3406671
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1506
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.046934          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     658              1598  Determinism 0.5148196 2.428571   7
## 2   Vertical     731              1673 V Laminarity 0.5389820 2.288646   4
## 3 Horizontal     731              1673 H Laminarity 0.5389820 2.288646   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
```

```
## 1          0.8686189        0.1890307      0.3296713
## 2          0.6703989        0.1458937      0.2332876
## 3          0.6703989        0.1458937      0.2332876
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1396
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.149297          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     694              1708  Determinism 0.5502577 2.461095   8
## 2   Vertical     746              1963 V Laminarity 0.6324098 2.631367   5
## 3 Horizontal     746              1963 H Laminarity 0.6324098 2.631367   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9079516        0.1975904      0.3397264
## 2          1.0433653        0.2270594      0.3460216
## 3          1.0433653        0.2270594      0.3460216
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1464
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571        1.14939          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
```

```
## 1   Diagonal     670              1640  Determinism 0.5283505 2.447761   7
## 2   Vertical     716              1885 V Laminarity 0.6072809 2.632682   6
## 3 Horizontal     716              1885 H Laminarity 0.6072809 2.632682   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8914837        0.1940066      0.3349389
## 2          1.0721419        0.2333218      0.3748664
## 3          1.0721419        0.2333218      0.3748664
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1490
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.049566          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     664              1614  Determinism 0.5199742 2.430723   7
## 2   Vertical     659              1694 V Laminarity 0.5457474 2.570561   6
## 3 Horizontal     659              1694 H Laminarity 0.5457474 2.570561   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8630264        0.1878137      0.3407410
## 2          0.9660112        0.2102255      0.4107238
## 3          0.9660112        0.2102255      0.4107238
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1476
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.127764          1
```

```
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     684              1628  Determinism 0.5244845 2.380117   6
## 2   Vertical     728              1836 V Laminarity 0.5914948 2.521978   7
## 3 Horizontal     728              1836 H Laminarity 0.5914948 2.521978   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8076871        0.1757706      0.3038286
## 2          0.9352370        0.2035283      0.3441457
## 3          0.9352370        0.2035283      0.3441457
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1456
##   Divergence Repetitiveness Anisotropy
## 1 0.09090909       1.040049          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     668              1648  Determinism 0.5309278 2.467066  11
## 2   Vertical     628              1714 V Laminarity 0.5521907 2.729299   7
## 3 Horizontal     628              1714 H Laminarity 0.5521907 2.729299   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9006037        0.1959913      0.3517445
## 2          1.0770149        0.2343823      0.4128254
## 3          1.0770149        0.2343823      0.4128254
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
```

```
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1536
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9387755          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1568  Determinism 0.5051546 2.375758   6
## 2   Vertical     618              1472 V Laminarity 0.4742268 2.381877   6
## 3 Horizontal     618              1472 H Laminarity 0.4742268 2.381877   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.7997650        0.1740466      0.3088548
## 2          0.7869108        0.1712492      0.3471611
## 3          0.7869108        0.1712492      0.3471611
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1476
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.038698          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     678              1628  Determinism 0.5244845 2.401180   6
## 2   Vertical     723              1691 V Laminarity 0.5447809 2.338866   4
## 3 Horizontal     723              1691 H Laminarity 0.5447809 2.338866   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8355550        0.1818353      0.2929038
## 2          0.7298745        0.1588369      0.2381380
## 3          0.7298745        0.1588369      0.2381380
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1412
##   Divergence Repetitiveness Anisotropy
## 1        0.1       1.172577          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     690              1692  Determinism 0.5451031 2.452174  10
## 2   Vertical     752              1984 V Laminarity 0.6391753 2.638298   7
## 3 Horizontal     752              1984 H Laminarity 0.6391753 2.638298   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8838044        0.1923354      0.3357076
## 2          1.0634814        0.2314371      0.4092732
## 3          1.0634814        0.2314371      0.4092732
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1462
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.083435          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     658              1642  Determinism 0.5289948 2.495441   6
## 2   Vertical     682              1779 V Laminarity 0.5731314 2.608504   6
## 3 Horizontal     682              1779 H Laminarity 0.5731314 2.608504   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9387649        0.2042961      0.3354891
```

```
## 2          1.0380931        0.2259121      0.3854585
## 3          1.0380931        0.2259121      0.3854585
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1406
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9322733          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     698              1698  Determinism 0.5470361 2.432665   7
## 2   Vertical     645              1583 V Laminarity 0.5099871 2.454264   5
## 3 Horizontal     645              1583 H Laminarity 0.5099871 2.454264   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8698608        0.1893010      0.3322515
## 2          0.8619430        0.1875779      0.3458187
## 3          0.8619430        0.1875779      0.3458187
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1496
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9402985          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     666              1608  Determinism 0.5180412 2.414414   7
```

```
## 2   Vertical     615              1512 V Laminarity 0.4871134 2.458537   4
## 3 Horizontal     615              1512 H Laminarity 0.4871134 2.458537   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8496057        0.1848930      0.3003146
## 2          0.8647550        0.1881899      0.2720587
## 3          0.8647550        0.1881899      0.2720587
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1478
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9686347          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674              1626  Determinism 0.5238402 2.412463   6
## 2   Vertical     669              1575 V Laminarity 0.5074098 2.354260   6
## 3 Horizontal     669              1575 H Laminarity 0.5074098 2.354260   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8484970        0.1846518      0.3079037
## 2          0.6818829        0.1483928      0.3574427
## 3          0.6818829        0.1483928      0.3574427
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1504
##   Divergence Repetitiveness Anisotropy
## 1      0.125       0.905625          1
##
```

```
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652               1600  Determinism 0.5154639 2.453988   8
## 2   Vertical     573               1449 V Laminarity 0.4668170 2.528796   6
## 3 Horizontal     573               1449 H Laminarity 0.4668170 2.528796   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8947401        0.1947153      0.3184718
## 2          0.9640019        0.2097882      0.3751430
## 3          0.9640019        0.2097882      0.3751430
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1494
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.004969          1
##
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     662               1610  Determinism 0.5186856 2.432024   8
## 2   Vertical     643               1618 V Laminarity 0.5212629 2.516330   6
## 3 Horizontal     643               1618 H Laminarity 0.5212629 2.516330   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8661154        0.1884859      0.3394301
## 2          0.9315726        0.2027309      0.3336660
## 3          0.9315726        0.2027309      0.3336660
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
```

```
## 1 Recurrence Matrix          9900        3104      0.3135354           1470
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.252754          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     668              1634  Determinism 0.5264175 2.446108   6
## 2   Vertical     844              2047 V Laminarity 0.6594716 2.425355   7
## 3 Horizontal     844              2047 H Laminarity 0.6594716 2.425355   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8851463        0.1926275      0.3185819
## 2          0.8256144        0.1796720      0.3414966
## 3          0.8256144        0.1796720      0.3414966
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1536
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.014668          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     638              1568  Determinism 0.5051546 2.457680   8
## 2   Vertical     666              1591 V Laminarity 0.5125644 2.388889   6
## 3 Horizontal     666              1591 H Laminarity 0.5125644 2.388889   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8980235        0.1954298      0.3481483
## 2          0.7471831        0.1626036      0.3616227
## 3          0.7471831        0.1626036      0.3616227
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1490
##   Divergence Repetitiveness Anisotropy
## 1        0.2      0.9572491          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     678              1614  Determinism 0.5199742 2.380531   5
## 2   Vertical     584              1545 V Laminarity 0.4977448 2.645548   5
## 3 Horizontal     584              1545 H Laminarity 0.4977448 2.645548   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8055216        0.1752994      0.2814117
## 2          0.9842539        0.2141955      0.2599035
## 3          0.9842539        0.2141955      0.2599035
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354            1492
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9962779          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     650              1612  Determinism 0.5193299 2.480000   6
## 2   Vertical     613              1606 V Laminarity 0.5173969 2.619902   6
## 3 Horizontal     613              1606 H Laminarity 0.5173969 2.619902   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9125333        0.1985875      0.3504535
## 2          1.0222160        0.2224569      0.4181350
```

```
## 3          1.0222160        0.2224569      0.4181350
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1464
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.167683          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate    Mean Max
## 1   Diagonal     656              1640  Determinism 0.5283505 2.50000   6
## 2   Vertical     797              1915 V Laminarity 0.6169459 2.40276   5
## 3 Horizontal     797              1915 H Laminarity 0.6169459 2.40276   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9459908        0.2058686      0.3409909
## 2          0.7866839        0.1711999      0.3029298
## 3          0.7866839        0.1711999      0.3029298
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1340
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       0.803288          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     704              1764  Determinism 0.5682990 2.505682   7
## 2   Vertical     584              1417 V Laminarity 0.4565077 2.426370   5
```

```
## 3 Horizontal     584              1417 H Laminarity 0.4565077 2.426370   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9587647        0.2086485      0.3432327
## 2          0.8473553        0.1844033      0.2989926
## 3          0.8473553        0.1844033      0.2989926
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1530
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.022872          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     654              1574  Determinism 0.5070876 2.406728   6
## 2   Vertical     678              1610 V Laminarity 0.5186856 2.374631   4
## 3 Horizontal     678              1610 H Laminarity 0.5186856 2.374631   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8396586        0.1827283      0.3023677
## 2          0.7578759        0.1649306      0.2350118
## 3          0.7578759        0.1649306      0.2350118
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1538
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9514687          1
##
##
```

```
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     650              1566  Determinism 0.5045103 2.409231   6
## 2   Vertical     625              1490 V Laminarity 0.4800258 2.384000   5
## 3 Horizontal     625              1490 H Laminarity 0.4800258 2.384000   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8450850        0.1839092      0.2973827
## 2          0.7858265        0.1710133      0.3255389
## 3          0.7858265        0.1710133      0.3255389
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1366
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.196203          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     706              1738  Determinism 0.5599227 2.461756   6
## 2   Vertical     774              2079 V Laminarity 0.6697809 2.686047   6
## 3 Horizontal     774              2079 H Laminarity 0.6697809 2.686047   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9049484        0.1969368      0.3148258
## 2          1.1071656        0.2409438      0.3352199
## 3          1.1071656        0.2409438      0.3352199
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1438
```

```
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9801921          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     692              1666  Determinism 0.5367268 2.407514   7
## 2   Vertical     668              1633 V Laminarity 0.5260954 2.444611   4
## 3 Horizontal     668              1633 H Laminarity 0.5260954 2.444611   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8447203        0.1838299      0.3188104
## 2          0.8432899        0.1835186      0.2863970
## 3          0.8432899        0.1835186      0.2863970
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354           1454
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.224242          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     672              1650  Determinism 0.5315722 2.455357   8
## 2   Vertical     833              2020 V Laminarity 0.6507732 2.424970   4
## 3 Horizontal     833              2020 H Laminarity 0.6507732 2.424970   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8928458        0.1943030      0.3525020
## 2          0.8134226        0.1770188      0.2449519
## 3          0.8134226        0.1770188      0.2449519
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1488
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.053218          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     668              1616  Determinism 0.5206186 2.419162   7
## 2   Vertical     641              1702 V Laminarity 0.5483247 2.655226   7
## 3 Horizontal     641              1702 H Laminarity 0.5483247 2.655226   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8578221        0.1866811      0.3211647
## 2          1.0807309        0.2351910      0.4347438
## 3          1.0807309        0.2351910      0.4347438
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3104       0.3135354            1506
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.9561952          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     622              1598  Determinism 0.5148196 2.569132   8
## 2   Vertical     634              1528 V Laminarity 0.4922680 2.410095   6
## 3 Horizontal     634              1528 H Laminarity 0.4922680 2.410095   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          1.0089238        0.2195642      0.3684776
## 2          0.7827693        0.1703480      0.3588119
## 3          0.7827693        0.1703480      0.3588119
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1482
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.054254          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     654              1622  Determinism 0.5225515 2.480122   8
## 2   Vertical     697              1710 V Laminarity 0.5509021 2.453372   6
## 3 Horizontal     697              1710 H Laminarity 0.5509021 2.453372   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9240005        0.2010830      0.3410681
## 2          0.8846981        0.1925299      0.3316328
## 3          0.8846981        0.1925299      0.3316328
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1478
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9618696          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     666              1626  Determinism 0.5238402 2.441441   7
## 2   Vertical     645              1564 V Laminarity 0.5038660 2.424806   5
## 3 Horizontal     645              1564 H Laminarity 0.5038660 2.424806   5
```

```
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8854213        0.1926873      0.3161615
## 2          0.8572504        0.1865567      0.3038559
## 3          0.8572504        0.1865567      0.3038559
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1498
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.123288          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674              1606  Determinism 0.5173969 2.382789   6
## 2   Vertical     695              1804 V Laminarity 0.5811856 2.595683   6
## 3 Horizontal     695              1804 H Laminarity 0.5811856 2.595683   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8022059        0.1745778      0.3150388
## 2          1.0261884        0.2233214      0.3440195
## 3          1.0261884        0.2233214      0.3440195
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104      0.3135354           1542
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111      0.9487836          1
##
##
##  Line-based Measures
```

```
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     634              1562  Determinism 0.5032216 2.463722   9
## 2   Vertical     601              1482 V Laminarity 0.4774485 2.465890   6
## 3 Horizontal     601              1482 H Laminarity 0.4774485 2.465890   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9031779        0.1965515      0.3627756
## 2          0.8467435        0.1842702      0.3902227
## 3          0.8467435        0.1842702      0.3902227
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1530
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9911055          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1574  Determinism 0.5070876 2.384848   7
## 2   Vertical     637              1560 V Laminarity 0.5025773 2.448980   6
## 3 Horizontal     637              1560 H Laminarity 0.5025773 2.448980   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8099223        0.1762571      0.2904763
## 2          0.8724371        0.1898617      0.3640712
## 3          0.8724371        0.1898617      0.3640712
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3104       0.3135354            1568
##   Divergence Repetitiveness Anisotropy
```

```
## 1  0.1428571       1.050781          1
##
##
##   Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate      Mean Max
## 1   Diagonal     622              1536  Determinism 0.4948454 2.469453   7
## 2   Vertical     615              1614 V Laminarity 0.5199742 2.624390   9
## 3 Horizontal     615              1614 H Laminarity 0.5199742 2.624390   9
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.9136976        0.1988409      0.3526361
## 2           0.9557886        0.2080008      0.5484928
## 3           0.9557886        0.2080008      0.5484928
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
crqa_1rnd_sur[NROW(crqa_1rnd_sur)+1,] <- crqa_1
crqa_2rnd_sur <- ldply(seq_along(y_2rnd_sur$V1), function(r) rp_measures(rp(y1 = as.numeric(y_2
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##   Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253           1518
##    Divergence Repetitiveness Anisotropy
## 1        0.2       1.031091          1
##
##
##   Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate      Mean Max
## 1   Diagonal     654              1576  Determinism 0.5093730 2.409786   5
## 2   Vertical     656              1625 V Laminarity 0.5252101 2.477134   6
## 3 Horizontal     656              1625 H Laminarity 0.5252101 2.477134   6
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8411673        0.1830567      0.2986326
## 2           0.9172874        0.1996221      0.3511725
## 3           0.9172874        0.1996221      0.3511725
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1448
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111      0.9495747          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     686              1646  Determinism 0.5319974 2.399417   9
## 2   Vertical     637              1563 V Laminarity 0.5051713 2.453689   4
## 3 Horizontal     637              1563 H Laminarity 0.5051713 2.453689   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8192488        0.1782867      0.3171052
## 2          0.8559189        0.1862669      0.2835602
## 3          0.8559189        0.1862669      0.2835602
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1502
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.026382          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     644              1592  Determinism 0.5145443 2.472050   6
## 2   Vertical     613              1634 V Laminarity 0.5281189 2.665579   5
## 3 Horizontal     613              1634 H Laminarity 0.5281189 2.665579   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
```

```
## 1          0.9081967        0.1976437      0.3511420
## 2          1.0581520        0.2302773      0.2950077
## 3          1.0581520        0.2302773      0.2950077
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1538
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.031491          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     640              1556  Determinism 0.5029089 2.431250   9
## 2   Vertical     672              1605 V Laminarity 0.5187460 2.388393   6
## 3 Horizontal     672              1605 H Laminarity 0.5187460 2.388393   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8611892        0.1874139      0.3290549
## 2          0.7934629        0.1726751      0.3174082
## 3          0.7934629        0.1726751      0.3174082
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1428
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9093637          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
```

```
## 1   Diagonal     688              1666  Determinism 0.5384615 2.421512   7
## 2   Vertical     619              1515 V Laminarity 0.4896574 2.447496   7
## 3 Horizontal     619              1515 H Laminarity 0.4896574 2.447496   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8587873        0.1868912      0.3021470
## 2          0.8551763        0.1861053      0.3794593
## 3          0.8551763        0.1861053      0.3794593
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1402
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.8339243          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     696              1692  Determinism 0.5468649 2.431034   7
## 2   Vertical     622              1411 V Laminarity 0.4560440 2.268489   4
## 3 Horizontal     622              1411 H Laminarity 0.4560440 2.268489   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8666267        0.1885972      0.3162084
## 2          0.6386008        0.1389737      0.2464905
## 3          0.6386008        0.1389737      0.2464905
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1522
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.143766          1
```

```
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     648              1572  Determinism 0.5080802 2.425926   8
## 2   Vertical     719              1798 V Laminarity 0.5811248 2.500695   5
## 3 Horizontal     719              1798 H Laminarity 0.5811248 2.500695   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8619880        0.1875877      0.3152618
## 2          0.9322622        0.2028809      0.2946252
## 3          0.9322622        0.2028809      0.2946252
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094      0.3125253           1480
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       0.889715          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     676              1614  Determinism 0.5216548 2.387574   6
## 2   Vertical     576              1436 V Laminarity 0.4641241 2.493056   8
## 3 Horizontal     576              1436 H Laminarity 0.4641241 2.493056   8
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8179512        0.1780043      0.3094453
## 2          0.9004224        0.1959519      0.3901418
## 3          0.9004224        0.1959519      0.3901418
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
```

```
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253            1448
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       0.945322          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     694              1646  Determinism 0.5319974 2.371758   7
## 2   Vertical     639              1556 V Laminarity 0.5029089 2.435055   5
## 3 Horizontal     639              1556 H Laminarity 0.5029089 2.435055   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.7915812        0.1722656      0.3097769
## 2          0.8432492        0.1835097      0.2696438
## 3          0.8432492        0.1835097      0.2696438
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253            1542
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9323454          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     640              1552  Determinism 0.5016160 2.425000   6
## 2   Vertical     567              1447 V Laminarity 0.4676794 2.552028   6
## 3 Horizontal     567              1447 H Laminarity 0.4676794 2.552028   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8640431        0.1880349      0.3062357
## 2          0.8974380        0.1953024      0.4021683
## 3          0.8974380        0.1953024      0.4021683
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1482
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9621588          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     658              1612  Determinism 0.5210084 2.449848   7
## 2   Vertical     637              1551 V Laminarity 0.5012928 2.434851   6
## 3 Horizontal     637              1551 H Laminarity 0.5012928 2.434851   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8799547        0.1914977      0.3455550
## 2          0.8260718        0.1797715      0.3855714
## 3          0.8260718        0.1797715      0.3855714
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1516
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.229404          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     634              1578  Determinism 0.5100194 2.488959   6
## 2   Vertical     805              1940 V Laminarity 0.6270200 2.409938   5
## 3 Horizontal     805              1940 H Laminarity 0.6270200 2.409938   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9310283        0.2026124      0.3040102
```

```
## 2          0.8235318        0.1792188      0.2802423
## 3          0.8235318        0.1792188      0.2802423
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1468
##   Divergence Repetitiveness Anisotropy
## 1      0.125        1.01599          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     668              1626  Determinism 0.5255333 2.434132   8
## 2   Vertical     640              1652 V Laminarity 0.5339367 2.581250   6
## 3 Horizontal     640              1652 H Laminarity 0.5339367 2.581250   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8754392        0.1905150      0.3397222
## 2          1.0242744        0.2229048      0.3342876
## 3          1.0242744        0.2229048      0.3342876
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1462
##   Divergence Repetitiveness Anisotropy
## 1        0.2       1.109681          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     676              1632  Determinism 0.5274725 2.414201   5
```

```
## 2   Vertical     749              1811 V Laminarity 0.5853264 2.417891   6
## 3 Horizontal     749              1811 H Laminarity 0.5853264 2.417891   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8370413        0.1821588      0.2761547
## 2          0.8454335        0.1839851      0.3402309
## 3          0.8454335        0.1839851      0.3402309
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1566
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.010471          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     606              1528  Determinism 0.4938591 2.521452   8
## 2   Vertical     633              1544 V Laminarity 0.4990304 2.439179   5
## 3 Horizontal     633              1544 H Laminarity 0.4990304 2.439179   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9635181        0.2096829      0.3716287
## 2          0.8715880        0.1896769      0.2861598
## 3          0.8715880        0.1896769      0.2861598
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1390
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.7987089          1
##
```

```
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     704              1704  Determinism 0.5507434 2.420455   6
## 2   Vertical     569              1361 V Laminarity 0.4398836 2.391916   5
## 3 Horizontal     569              1361 H Laminarity 0.4398836 2.391916   5
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8591872        0.1869782      0.3051644
## 2           0.8093551        0.1761336      0.2771435
## 3           0.8093551        0.1761336      0.2771435
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1484
##    Divergence Repetitiveness Anisotropy
## 1        0.2       1.023602          1
##
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     672              1610  Determinism 0.5203620 2.395833   5
## 2   Vertical     671              1648 V Laminarity 0.5326438 2.456036   5
## 3 Horizontal     671              1648 H Laminarity 0.5326438 2.456036   5
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8249938         0.179537      0.2921376
## 2           0.8926112         0.194252      0.3068163
## 3           0.8926112         0.194252      0.3068163
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
```

```
## 1 Recurrence Matrix          9900        3094      0.3125253           1486
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.8439055          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     676              1608  Determinism 0.5197156 2.378698   6
## 2   Vertical     570              1357 V Laminarity 0.4385908 2.380702   4
## 3 Horizontal     570              1357 H Laminarity 0.4385908 2.380702   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8050501        0.1751968      0.2979495
## 2          0.7801118        0.1697696      0.2741537
## 3          0.7801118        0.1697696      0.2741537
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1576
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.131094          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     626              1518  Determinism 0.4906270 2.424920   8
## 2   Vertical     759              1717 V Laminarity 0.5549451 2.262187   5
## 3 Horizontal     759              1717 H Laminarity 0.5549451 2.262187   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8585420        0.1868378      0.3116586
## 2          0.5895951        0.1283090      0.2775525
## 3          0.5895951        0.1283090      0.2775525
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1482
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9931762          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1612  Determinism 0.5210084 2.442424   6
## 2   Vertical     674              1601 V Laminarity 0.5174531 2.375371   5
## 3 Horizontal     674              1601 H Laminarity 0.5174531 2.375371   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8829933        0.1921589      0.3004245
## 2          0.7737697        0.1683895      0.2857352
## 3          0.7737697        0.1683895      0.2857352
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1550
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.158031          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     624              1544  Determinism 0.4990304 2.474359   8
## 2   Vertical     722              1788 V Laminarity 0.5778927 2.476454   8
## 3 Horizontal     722              1788 H Laminarity 0.5778927 2.476454   8
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9170940        0.1995800      0.3486263
## 2          0.8765978        0.1907671      0.4290826
```

```
## 3          0.8765978        0.1907671      0.4290826
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094      0.3125253           1516
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9873257          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1578  Determinism 0.5100194 2.420245   7
## 2   Vertical     654              1558 V Laminarity 0.5035553 2.382263   5
## 3 Horizontal     654              1558 H Laminarity 0.5035553 2.382263   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8478155        0.1845035      0.2945557
## 2          0.8070712        0.1756366      0.2903575
## 3          0.8070712        0.1756366      0.2903575
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094      0.3125253           1490
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.9763092          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     648              1604  Determinism 0.5184228 2.475309   8
## 2   Vertical     641              1566 V Laminarity 0.5061409 2.443058   6
```

```
## 3 Horizontal     641              1566 H Laminarity 0.5061409 2.443058   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9147739        0.1990751      0.3456421
## 2          0.8475845        0.1844532      0.3316574
## 3          0.8475845        0.1844532      0.3316574
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1534
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.045513          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     642              1560  Determinism 0.5042017 2.429907   8
## 2   Vertical     633              1631 V Laminarity 0.5271493 2.576619   5
## 3 Horizontal     633              1631 H Laminarity 0.5271493 2.576619   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8667572        0.1886256      0.3272610
## 2          1.0125411        0.2203514      0.3195831
## 3          1.0125411        0.2203514      0.3195831
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1482
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.075682          1
##
##
```

```
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     676              1612  Determinism 0.5210084 2.384615   7
## 2   Vertical     651              1734 V Laminarity 0.5604396 2.663594   6
## 3 Horizontal     651              1734 H Laminarity 0.5604396 2.663594   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8106556        0.1764166      0.2993705
## 2          1.0921524        0.2376766      0.3960432
## 3          1.0921524        0.2376766      0.3960432
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1510
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667        1.05303          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     636              1584  Determinism 0.5119586 2.490566   6
## 2   Vertical     742              1668 V Laminarity 0.5391080 2.247978   4
## 3 Horizontal     742              1668 H Laminarity 0.5391080 2.247978   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9366110        0.2038273      0.3323008
## 2          0.6115748        0.1330922      0.2347600
## 3          0.6115748        0.1330922      0.2347600
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1484
```

```
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.9298137          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674              1610  Determinism 0.5203620 2.388724   8
## 2   Vertical     624              1497 V Laminarity 0.4838397 2.399038   5
## 3 Horizontal     624              1497 H Laminarity 0.4838397 2.399038   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8018991         0.174511      0.3370129
## 2          0.8272225         0.180022      0.2842154
## 3          0.8272225         0.180022      0.2842154
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253           1516
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9816223          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     656              1578  Determinism 0.5100194 2.405488   6
## 2   Vertical     656              1549 V Laminarity 0.5006464 2.361280   6
## 3 Horizontal     656              1549 H Laminarity 0.5006464 2.361280   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8411751        0.1830584      0.2951382
## 2          0.7651826        0.1665207      0.3128202
## 3          0.7651826        0.1665207      0.3128202
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253           1506
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.110831          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     648              1588  Determinism 0.5132515 2.450617   8
## 2   Vertical     687              1764 V Laminarity 0.5701357 2.567686   7
## 3 Horizontal     687              1764 H Laminarity 0.5701357 2.567686   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8910201        0.1939057      0.3631494
## 2          0.9857381        0.2145185      0.4127731
## 3          0.9857381        0.2145185      0.4127731
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253           1436
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9457177          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     692              1658  Determinism 0.5358759 2.395954   6
## 2   Vertical     646              1568 V Laminarity 0.5067873 2.427245   7
## 3 Horizontal     646              1568 H Laminarity 0.5067873 2.427245   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8292532        0.1804639      0.2968325
## 2          0.8478112        0.1845025      0.3509675
## 3          0.8478112        0.1845025      0.3509675
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1562
##   Divergence Repetitiveness Anisotropy
## 1      0.125        0.95953          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     624              1532  Determinism 0.4951519 2.455128   8
## 2   Vertical     614              1470 V Laminarity 0.4751131 2.394137   5
## 3 Horizontal     614              1470 H Laminarity 0.4751131 2.394137   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8962773        0.1950498      0.3291108
## 2          0.8218531        0.1788535      0.2875959
## 3          0.8218531        0.1788535      0.2875959
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1522
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.048346          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1572  Determinism 0.5080802 2.381818   7
## 2   Vertical     635              1648 V Laminarity 0.5326438 2.595276   5
## 3 Horizontal     635              1648 H Laminarity 0.5326438 2.595276   5
```

```
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8093107        0.1761240      0.3084896
## 2          1.0229365        0.2226137      0.3432020
## 3          1.0229365        0.2226137      0.3432020
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1516
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9296578          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     642              1578  Determinism 0.5100194 2.457944   6
## 2   Vertical     656              1467 V Laminarity 0.4741435 2.236280   4
## 3 Horizontal     656              1467 H Laminarity 0.4741435 2.236280   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9011533        0.1961109      0.3063164
## 2          0.5900576        0.1284096      0.2359595
## 3          0.5900576        0.1284096      0.2359595
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1518
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.9936548          1
##
##
##  Line-based Measures
```

```
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     630              1576  Determinism 0.5093730 2.501587   8
## 2   Vertical     614              1566 V Laminarity 0.5061409 2.550489   6
## 3 Horizontal     614              1566 H Laminarity 0.5061409 2.550489   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9401953        0.2046074      0.3555125
## 2          0.9738724        0.2119362      0.3787212
## 3          0.9738724        0.2119362      0.3787212
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1536
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.089217          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     622              1558  Determinism 0.5035553 2.504823   9
## 2   Vertical     715              1697 V Laminarity 0.5484809 2.373427   4
## 3 Horizontal     715              1697 H Laminarity 0.5484809 2.373427   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9501325        0.2067699      0.3762220
## 2          0.7749208        0.1686400      0.2509507
## 3          0.7749208        0.1686400      0.2509507
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1532
##   Divergence Repetitiveness Anisotropy
```

```
## 1      0.125       1.033291          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     636              1562  Determinism 0.5048481 2.455975   8
## 2   Vertical     685              1614 V Laminarity 0.5216548 2.356204   4
## 3 Horizontal     685              1614 H Laminarity 0.5216548 2.356204   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8897299        0.1936250      0.3662456
## 2          0.7463169        0.1624151      0.2379936
## 3          0.7463169        0.1624151      0.2379936
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253          1480
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9653036          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     658              1614  Determinism 0.5216548 2.452888   6
## 2   Vertical     640              1558 V Laminarity 0.5035553 2.434375   6
## 3 Horizontal     640              1558 H Laminarity 0.5035553 2.434375   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8925643        0.1942418      0.2980010
## 2          0.8659443        0.1884487      0.3421239
## 3          0.8659443        0.1884487      0.3421239
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1556
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.061769          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     626              1538  Determinism 0.4970911 2.456869   6
## 2   Vertical     678              1633 V Laminarity 0.5277957 2.408555   5
## 3 Horizontal     678              1633 H Laminarity 0.5277957 2.408555   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9014535        0.1961763      0.3170952
## 2          0.7889622        0.1716957      0.2522155
## 3          0.7889622        0.1716957      0.2522155
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1374
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9895349          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     712              1720  Determinism 0.5559147 2.415730   6
## 2   Vertical     676              1702 V Laminarity 0.5500970 2.517751   4
## 3 Horizontal     676              1702 H Laminarity 0.5500970 2.517751   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8526885        0.1855639      0.3077386
## 2          0.8771576        0.1908889      0.2459271
## 3          0.8771576        0.1908889      0.2459271
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1520
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571        1.01906          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1574  Determinism 0.5087266 2.414110   7
## 2   Vertical     682              1604 V Laminarity 0.5184228 2.351906   4
## 3 Horizontal     682              1604 H Laminarity 0.5184228 2.351906   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8519687        0.1854073      0.3304093
## 2          0.7387495        0.1607683      0.2753226
## 3          0.7387495        0.1607683      0.2753226
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1592
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.147137          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     598              1502  Determinism 0.4854557 2.511706   8
## 2   Vertical     763              1723 V Laminarity 0.5568843 2.258191   4
## 3 Horizontal     763              1723 H Laminarity 0.5568843 2.258191   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
```

```
## 1          0.9596434        0.2088397      0.3663275
## 2          0.6244069        0.1358848      0.2200382
## 3          0.6244069        0.1358848      0.2200382
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1502
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571        1.00691          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     638              1592  Determinism 0.5145443 2.495298   7
## 2   Vertical     596              1603 V Laminarity 0.5180995 2.689597   7
## 3 Horizontal     596              1603 H Laminarity 0.5180995 2.689597   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9469984        0.2060879      0.3358904
## 2          1.1096548        0.2414855      0.4054531
## 3          1.1096548        0.2414855      0.4054531
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1490
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.043641          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
```

```
## 1   Diagonal     648              1604  Determinism 0.5184228 2.475309   7
## 2   Vertical     681              1674 V Laminarity 0.5410472 2.458150   9
## 3 Horizontal     681              1674 H Laminarity 0.5410472 2.458150   9
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9140695        0.1989218      0.3441794
## 2          0.8483221        0.1846137      0.4292414
## 3          0.8483221        0.1846137      0.4292414
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1540
##   Divergence Repetitiveness Anisotropy
## 1      0.125       0.992278          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     644              1554  Determinism 0.5022624 2.413043   8
## 2   Vertical     566              1542 V Laminarity 0.4983840 2.724382   7
## 3 Horizontal     566              1542 H Laminarity 0.4983840 2.724382   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8462244        0.1841572      0.3135669
## 2          1.1516725        0.2506295      0.3946077
## 3          1.1516725        0.2506295      0.3946077
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1488
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9339975          1
```

```
##
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     662              1606  Determinism 0.5190692 2.425982   7
## 2   Vertical     604              1500 V Laminarity 0.4848093 2.483444   5
## 3 Horizontal     604              1500 H Laminarity 0.4848093 2.483444   5
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8586303        0.1868570      0.3293083
## 2           0.9227248        0.2008054      0.3071412
## 3           0.9227248        0.2008054      0.3071412
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1496
##    Divergence Repetitiveness Anisotropy
## 1         0.1       1.118899          1
##
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1598  Determinism 0.5164835 2.450920  10
## 2   Vertical     709              1788 V Laminarity 0.5778927 2.521862   6
## 3 Horizontal     709              1788 H Laminarity 0.5778927 2.521862   6
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8811796        0.1917642      0.3665536
## 2           0.9702336        0.2111443      0.3328693
## 3           0.9702336        0.2111443      0.3328693
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
```

```
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253           1498
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.8784461          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     622              1596  Determinism 0.5158371 2.565916   7
## 2   Vertical     519              1402 V Laminarity 0.4531351 2.701349   7
## 3 Horizontal     519              1402 H Laminarity 0.4531351 2.701349   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           1.004952        0.2186998      0.3884034
## 2           1.135037        0.2470092      0.3899227
## 3           1.135037        0.2470092      0.3899227
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253           1496
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.8904881          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     646              1598  Determinism 0.5164835 2.473684   7
## 2   Vertical     645              1423 V Laminarity 0.4599224 2.206202   7
## 3 Horizontal     645              1423 H Laminarity 0.4599224 2.206202   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9137372       0.19884948      0.3155740
## 2          0.4547888       0.09897212      0.3281687
## 3          0.4547888       0.09897212      0.3281687
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1434
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.8554217          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     670              1660  Determinism 0.5365223 2.477612   8
## 2   Vertical     564              1420 V Laminarity 0.4589528 2.517730   5
## 3 Horizontal     564              1420 H Laminarity 0.4589528 2.517730   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9184792        0.1998814      0.3650165
## 2          0.9229201        0.2008479      0.3240780
## 3          0.9229201        0.2008479      0.3240780
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1510
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.184975          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     648              1584  Determinism 0.5119586 2.444444   7
## 2   Vertical     792              1877 V Laminarity 0.6066580 2.369949   5
## 3 Horizontal     792              1877 H Laminarity 0.6066580 2.369949   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8875566        0.1931520      0.3101981
```

```
## 2          0.7877812        0.1714387      0.2942253
## 3          0.7877812        0.1714387      0.2942253
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1510
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.039773          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     644              1584  Determinism 0.5119586 2.459627   7
## 2   Vertical     648              1647 V Laminarity 0.5323206 2.541667   7
## 3 Horizontal     648              1647 H Laminarity 0.5323206 2.541667   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9075849        0.1975106      0.3238388
## 2          0.9748667        0.2121526      0.3727955
## 3          0.9748667        0.2121526      0.3727955
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1502
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.114322          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1592  Determinism 0.5145443 2.441718   6
```

```
## 2   Vertical     714              1774 V Laminarity 0.5733678 2.484594   6
## 3 Horizontal     714              1774 H Laminarity 0.5733678 2.484594   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8821831        0.1919826      0.3148642
## 2          0.9183452        0.1998523      0.3529564
## 3          0.9183452        0.1998523      0.3529564
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1490
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.043641          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674              1604  Determinism 0.5184228 2.379822   6
## 2   Vertical     714              1674 V Laminarity 0.5410472 2.344538   4
## 3 Horizontal     714              1674 H Laminarity 0.5410472 2.344538   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8097298        0.1762152      0.3033587
## 2          0.7439866        0.1619080      0.2561948
## 3          0.7439866        0.1619080      0.2561948
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1432
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.8128761          1
##
```

```
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674              1662  Determinism 0.5371687 2.465875   8
## 2   Vertical     572              1351 V Laminarity 0.4366516 2.361888   4
## 3 Horizontal     572              1351 H Laminarity 0.4366516 2.361888   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9078305        0.1975641      0.3671798
## 2          0.7604676        0.1654946      0.2468444
## 3          0.7604676        0.1654946      0.2468444
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253           1518
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.119924          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure     Rate     Mean Max
## 1   Diagonal     650              1576  Determinism 0.509373 2.424615   6
## 2   Vertical     685              1765 V Laminarity 0.570459 2.576642   9
## 3 Horizontal     685              1765 H Laminarity 0.570459 2.576642   9
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8630371        0.1878160      0.2944568
## 2          1.0022354        0.2181087      0.4007273
## 3          1.0022354        0.2181087      0.4007273
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
```

```
## 1 Recurrence Matrix          9900        3094      0.3125253           1492
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.078652          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     650              1602  Determinism 0.5177763 2.464615   7
## 2   Vertical     783              1728 V Laminarity 0.5585003 2.206897   4
## 3 Horizontal     783              1728 H Laminarity 0.5585003 2.206897   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9129159        0.1986707      0.3347229
## 2          0.5473284        0.1191108      0.2128078
## 3          0.5473284        0.1191108      0.2128078
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1436
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9716526          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     680              1658  Determinism 0.5358759 2.438235   6
## 2   Vertical     632              1611 V Laminarity 0.5206852 2.549051   6
## 3 Horizontal     632              1611 H Laminarity 0.5206852 2.549051   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8796675        0.1914352      0.3049580
## 2          0.9876548        0.2149356      0.3380276
## 3          0.9876548        0.2149356      0.3380276
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253            1514
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.9892405          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1580  Determinism 0.5106658 2.393939   6
## 2   Vertical     620              1563 V Laminarity 0.5051713 2.520968   7
## 3 Horizontal     620              1563 H Laminarity 0.5051713 2.520968   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8242539        0.1793759      0.2936095
## 2          0.9395881        0.2044752      0.3808919
## 3          0.9395881        0.2044752      0.3808919
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253            1492
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.041823          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     664              1602  Determinism 0.5177763 2.412651   6
## 2   Vertical     704              1669 V Laminarity 0.5394312 2.370739   5
## 3 Horizontal     704              1669 H Laminarity 0.5394312 2.370739   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8499776        0.1849740      0.3006909
## 2          0.7865864        0.1711787      0.2899765
```

```
## 3          0.7865864        0.1711787      0.2899765
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1386
##   Divergence Repetitiveness Anisotropy
## 1        0.1      0.7540984          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     712              1708  Determinism 0.5520362 2.398876  10
## 2   Vertical     546              1288 V Laminarity 0.4162896 2.358974   4
## 3 Horizontal     546              1288 H Laminarity 0.4162896 2.358974   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8223212        0.1789553      0.3319023
## 2          0.7587166        0.1651136      0.2487382
## 3          0.7587166        0.1651136      0.2487382
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1494
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.055625          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     638              1600  Determinism 0.5171299 2.507837   8
## 2   Vertical     716              1689 V Laminarity 0.5458953 2.358939   5
```

```
## 3 Horizontal     716             1689 H Laminarity 0.5458953 2.358939   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9568620        0.2082344      0.3573034
## 2          0.7490979        0.1630203      0.3181630
## 3          0.7490979        0.1630203      0.3181630
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1416
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.173421          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure     Rate     Mean Max
## 1   Diagonal     696             1678  Determinism 0.542340 2.410920   7
## 2   Vertical     792             1969 V Laminarity 0.636393 2.486111   7
## 3 Horizontal     792             1969 H Laminarity 0.636393 2.486111   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8486646        0.1846882      0.3256782
## 2          0.8925688        0.1942428      0.3968199
## 3          0.8925688        0.1942428      0.3968199
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1402
##   Divergence Repetitiveness Anisotropy
## 1        0.2      0.9964539          1
##
##
```

```
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     702              1692  Determinism 0.5468649 2.410256   5
## 2   Vertical     688              1686 V Laminarity 0.5449257 2.450581   7
## 3 Horizontal     688              1686 H Laminarity 0.5449257 2.450581   7
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8398155        0.1827625      0.2897298
## 2           0.8703340        0.1894040      0.3383534
## 3           0.8703340        0.1894040      0.3383534
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1440
##    Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.026602          1
##
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     684              1654  Determinism 0.5345831 2.418129   6
## 2   Vertical     742              1698 V Laminarity 0.5488041 2.288410   4
## 3 Horizontal     742              1698 H Laminarity 0.5488041 2.288410   4
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8542202        0.1858973      0.2945159
## 2           0.6710869        0.1460434      0.2446602
## 3           0.6710869        0.1460434      0.2446602
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1552
```

```
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.214656          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     622              1542  Determinism 0.4983840 2.479100   8
## 2   Vertical     758              1873 V Laminarity 0.6053652 2.470976   5
## 3 Horizontal     758              1873 H Laminarity 0.6053652 2.470976   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9255727        0.2014252      0.3438416
## 2          0.8754908        0.1905262      0.3081394
## 3          0.8754908        0.1905262      0.3081394
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1472
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9556104          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     666              1622  Determinism 0.5242405 2.435435   7
## 2   Vertical     616              1550 V Laminarity 0.5009696 2.516234   5
## 3 Horizontal     616              1550 H Laminarity 0.5009696 2.516234   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8761150        0.1906621      0.3277585
## 2          0.9370075        0.2039136      0.2829930
## 3          0.9370075        0.2039136      0.2829930
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1572
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.028252          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     620              1522  Determinism 0.4919198 2.454839   6
## 2   Vertical     634              1565 V Laminarity 0.5058177 2.468454   4
## 3 Horizontal     634              1565 H Laminarity 0.5058177 2.468454   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8967900        0.1951614      0.3314237
## 2          0.8719909        0.1897646      0.2674389
## 3          0.8719909        0.1897646      0.2674389
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1460
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.050796          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     662              1634  Determinism 0.5281189 2.468278   8
## 2   Vertical     695              1717 V Laminarity 0.5549451 2.470504   6
## 3 Horizontal     695              1717 H Laminarity 0.5549451 2.470504   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9023830        0.1963786      0.3691694
## 2          0.8866984        0.1929652      0.3183002
## 3          0.8866984        0.1929652      0.3183002
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253            1426
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.7398082          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674              1668  Determinism 0.5391080 2.474777   7
## 2   Vertical     500              1234 V Laminarity 0.3988365 2.468000   5
## 3 Horizontal     500              1234 H Laminarity 0.3988365 2.468000   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9164306        0.1994356      0.3426975
## 2          0.9060138        0.1971687      0.3059402
## 3          0.9060138        0.1971687      0.3059402
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253            1540
##   Divergence Repetitiveness Anisotropy
## 1      0.125      0.9646075          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     618              1554  Determinism 0.5022624 2.514563   8
## 2   Vertical     611              1499 V Laminarity 0.4844861 2.453355   5
## 3 Horizontal     611              1499 H Laminarity 0.4844861 2.453355   5
```

```
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9591338        0.2087288      0.3863616
## 2          0.8756506        0.1905610      0.2912382
## 3          0.8756506        0.1905610      0.2912382
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1426
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.051559          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     680              1668  Determinism 0.5391080 2.452941   9
## 2   Vertical     685              1754 V Laminarity 0.5669037 2.560584   6
## 3 Horizontal     685              1754 H Laminarity 0.5669037 2.560584   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8912646        0.1939589      0.3283583
## 2          1.0049806        0.2187061      0.3361427
## 3          1.0049806        0.2187061      0.3361427
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1420
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.8799283          1
##
##
##  Line-based Measures
```

```
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     716              1674  Determinism 0.5410472 2.337989   7
## 2   Vertical     619              1473 V Laminarity 0.4760827 2.379645   4
## 3 Horizontal     619              1473 H Laminarity 0.4760827 2.379645   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.7445163        0.1620233      0.3069378
## 2          0.7756124        0.1687905      0.2448383
## 3          0.7756124        0.1687905      0.2448383
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1500
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9598494          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     660              1594  Determinism 0.5151907 2.415152   7
## 2   Vertical     590              1530 V Laminarity 0.4945055 2.593220   5
## 3 Horizontal     590              1530 H Laminarity 0.4945055 2.593220   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8494509        0.1848594      0.3306481
## 2          0.9986831        0.2173356      0.3341929
## 3          0.9986831        0.2173356      0.3341929
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1484
##   Divergence Repetitiveness Anisotropy
```

```
## 1  0.1111111       1.057764          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     656              1610  Determinism 0.5203620 2.454268   9
## 2   Vertical     693              1703 V Laminarity 0.5504202 2.457431   5
## 3 Horizontal     693              1703 H Laminarity 0.5504202 2.457431   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8791975        0.1913329      0.3694892
## 2          0.8925927        0.1942480      0.3023388
## 3          0.8925927        0.1942480      0.3023388
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1486
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.099502          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     640              1608  Determinism 0.5197156 2.512500   7
## 2   Vertical     727              1768 V Laminarity 0.5714286 2.431912   4
## 3 Horizontal     727              1768 H Laminarity 0.5714286 2.431912   4
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9649179        0.2099875      0.3548400
## 2          0.8238694        0.1792923      0.2897103
## 3          0.8238694        0.1792923      0.2897103
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253          1538
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.098329          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     638              1556  Determinism 0.5029089 2.438871   7
## 2   Vertical     726              1709 V Laminarity 0.5523594 2.353994   5
## 3 Horizontal     726              1709 H Laminarity 0.5523594 2.353994   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8818079        0.1919010      0.3188138
## 2          0.7642083        0.1663087      0.2857446
## 3          0.7642083        0.1663087      0.2857446
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253          1560
##   Divergence Repetitiveness Anisotropy
## 1        0.1       1.078879          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     620              1534  Determinism 0.4957983 2.474194  10
## 2   Vertical     683              1655 V Laminarity 0.5349063 2.423133   6
## 3 Horizontal     683              1655 H Laminarity 0.5349063 2.423133   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9102935        0.1981001      0.3713645
## 2          0.8579925        0.1867182      0.3272010
## 3          0.8579925        0.1867182      0.3272010
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1406
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.096564          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     662              1688  Determinism 0.5455721 2.549849   8
## 2   Vertical     649              1851 V Laminarity 0.5982547 2.852080   6
## 3 Horizontal     649              1851 H Laminarity 0.5982547 2.852080   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9975354        0.2170858      0.3673199
## 2          1.2394881        0.2697401      0.3705612
## 3          1.2394881        0.2697401      0.3705612
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1426
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.096523          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     696              1668  Determinism 0.5391080 2.396552   7
## 2   Vertical     714              1829 V Laminarity 0.5911441 2.561625   6
## 3 Horizontal     714              1829 H Laminarity 0.5911441 2.561625   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
```

```
## 1          0.8293498        0.1804849      0.3222628
## 2          1.0053358        0.2187834      0.3384047
## 3          1.0053358        0.2187834      0.3384047
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1556
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9629389          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     630              1538  Determinism 0.4970911 2.441270   7
## 2   Vertical     609              1481 V Laminarity 0.4786684 2.431856   5
## 3 Horizontal     609              1481 H Laminarity 0.4786684 2.431856   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8843521        0.1924546      0.3197743
## 2          0.8628567        0.1877767      0.3008388
## 3          0.8628567        0.1877767      0.3008388
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1436
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.030157          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
```

```
## 1   Diagonal     684              1658  Determinism 0.5358759 2.423977   9
## 2   Vertical     694              1708 V Laminarity 0.5520362 2.461095   6
## 3 Horizontal     694              1708 H Laminarity 0.5520362 2.461095   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8513728        0.1852776      0.3526402
## 2          0.8854363        0.1926906      0.3581603
## 3          0.8854363        0.1926906      0.3581603
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1484
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.180124          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     656              1610  Determinism 0.5203620 2.454268   6
## 2   Vertical     787              1900 V Laminarity 0.6140918 2.414231   5
## 3 Horizontal     787              1900 H Laminarity 0.6140918 2.414231   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8929078        0.1943165      0.3272928
## 2          0.8179250        0.1779986      0.3102485
## 3          0.8179250        0.1779986      0.3102485
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253            1454
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.8847561          1
```

```
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     666              1640  Determinism 0.5300582 2.462462   6
## 2   Vertical     617              1451 V Laminarity 0.4689722 2.351702   7
## 3 Horizontal     617              1451 H Laminarity 0.4689722 2.351702   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8972001        0.1952506      0.3397881
## 2          0.7344431        0.1598311      0.3455049
## 3          0.7344431        0.1598311      0.3455049
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094      0.3125253            1526
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.072066          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     630              1568  Determinism 0.5067873 2.488889   6
## 2   Vertical     684              1681 V Laminarity 0.5433096 2.457602   6
## 3 Horizontal     684              1681 H Laminarity 0.5433096 2.457602   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9348060        0.2034345      0.3241566
## 2          0.8781521        0.1911054      0.3603856
## 3          0.8781521        0.1911054      0.3603856
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
```

```
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1486
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571       1.090174          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     652              1608  Determinism 0.5197156 2.466258   7
## 2   Vertical     656              1753 V Laminarity 0.5665805 2.672256   7
## 3 Horizontal     656              1753 H Laminarity 0.5665805 2.672256   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9071799        0.1974225      0.3504324
## 2          1.0824960        0.2355751      0.4493765
## 3          1.0824960        0.2355751      0.4493765
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1512
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.165613          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     644              1582  Determinism 0.5113122 2.456522   6
## 2   Vertical     754              1844 V Laminarity 0.5959922 2.445623   5
## 3 Horizontal     754              1844 H Laminarity 0.5959922 2.445623   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8970541        0.1952189      0.3381887
## 2          0.8799897        0.1915053      0.3054762
## 3          0.8799897        0.1915053      0.3054762
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1534
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667        1.05641          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     636              1560  Determinism 0.5042017 2.452830   6
## 2   Vertical     686              1648 V Laminarity 0.5326438 2.402332   7
## 3 Horizontal     686              1648 H Laminarity 0.5326438 2.402332   7
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8958511        0.1949571      0.3176916
## 2          0.7801436        0.1697766      0.3705988
## 3          0.7801436        0.1697766      0.3705988
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1506
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.9880353          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     640              1588  Determinism 0.5132515 2.481250   7
## 2   Vertical     628              1569 V Laminarity 0.5071105 2.498408   5
## 3 Horizontal     628              1569 H Laminarity 0.5071105 2.498408   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9288463        0.2021376      0.3373719
```

```
## 2          0.9211450        0.2004616      0.2791591
## 3          0.9211450        0.2004616      0.2791591
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1492
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.091136          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     632              1602  Determinism 0.5177763 2.534810   8
## 2   Vertical     752              1748 V Laminarity 0.5649644 2.324468   5
## 3 Horizontal     752              1748 H Laminarity 0.5649644 2.324468   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9783248        0.2129052      0.3838064
## 2          0.7325875        0.1594273      0.2686359
## 3          0.7325875        0.1594273      0.2686359
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1476
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.038319          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     644              1618  Determinism 0.5229476 2.512422   6
```

```
## 2   Vertical     704             1680 V Laminarity 0.5429864 2.386364   5
## 3 Horizontal     704             1680 H Laminarity 0.5429864 2.386364   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9606254        0.2090534      0.3369811
## 2          0.8104772        0.1763778      0.2866313
## 3          0.8104772        0.1763778      0.2866313
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1462
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667      0.8670343          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674             1632  Determinism 0.5274725 2.421365   6
## 2   Vertical     575             1415 V Laminarity 0.4573368 2.460870   5
## 3 Horizontal     575             1415 H Laminarity 0.4573368 2.460870   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8614562        0.1874720      0.3104659
## 2          0.8921251        0.1941462      0.3140653
## 3          0.8921251        0.1941462      0.3140653
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1496
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.8904881          1
##
```

```
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     668              1598  Determinism 0.5164835 2.392216   7
## 2   Vertical     595              1423 V Laminarity 0.4599224 2.391597   5
## 3 Horizontal     595              1423 H Laminarity 0.4599224 2.391597   5
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8216121        0.1788010      0.3050685
## 2           0.8151545        0.1773957      0.2807308
## 3           0.8151545        0.1773957      0.2807308
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##            Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1478
##    Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.065594          1
##
##
##  Line-based Measures
##    Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     674              1616  Determinism 0.5223012 2.397626   6
## 2   Vertical     697              1722 V Laminarity 0.5565611 2.470588   6
## 3 Horizontal     697              1722 H Laminarity 0.5565611 2.470588   6
##    Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1           0.8324592        0.1811616      0.3039557
## 2           0.9133860        0.1987731      0.3310728
## 3           0.9133860        0.1987731      0.3310728
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##            Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
```

```
## 1 Recurrence Matrix          9900        3094      0.3125253           1406
##   Divergence Repetitiveness Anisotropy
## 1  0.1428571      0.8981043          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     664              1688  Determinism 0.5455721 2.542169   7
## 2   Vertical     658              1516 V Laminarity 0.4899806 2.303951   5
## 3 Horizontal     658              1516 H Laminarity 0.4899806 2.303951   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9957143        0.2166895      0.3446043
## 2          0.6965217        0.1515786      0.2849693
## 3          0.6965217        0.1515786      0.2849693
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094      0.3125253           1502
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111      0.8360553          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     654              1592  Determinism 0.5145443 2.434251   9
## 2   Vertical     557              1331 V Laminarity 0.4301875 2.389587   5
## 3 Horizontal     557              1331 H Laminarity 0.4301875 2.389587   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8614060         0.187461      0.3328442
## 2          0.7764283         0.168968      0.3167138
## 3          0.7764283         0.168968      0.3167138
##
## ~~~o~~o~~casnet~~o~~o~~~
##
```

```
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253            1506
##   Divergence Repetitiveness Anisotropy
## 1      0.125       1.244962          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     632              1588  Determinism 0.5132515 2.512658   8
## 2   Vertical     744              1977 V Laminarity 0.6389787 2.657258   9
## 3 Horizontal     744              1977 H Laminarity 0.6389787 2.657258   9
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9657774        0.2101746      0.3549441
## 2          1.0724322        0.2333850      0.4299009
## 3          1.0724322        0.2333850      0.4299009
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##          Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900        3094       0.3125253            1300
##   Divergence Repetitiveness Anisotropy
## 1  0.1111111       1.029543          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     716              1794  Determinism 0.5798319 2.505587   9
## 2   Vertical     655              1847 V Laminarity 0.5969619 2.819847   6
## 3 Horizontal     655              1847 H Laminarity 0.5969619 2.819847   6
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.9442834        0.2054970      0.3796489
## 2          1.1902500        0.2590248      0.3230593
```

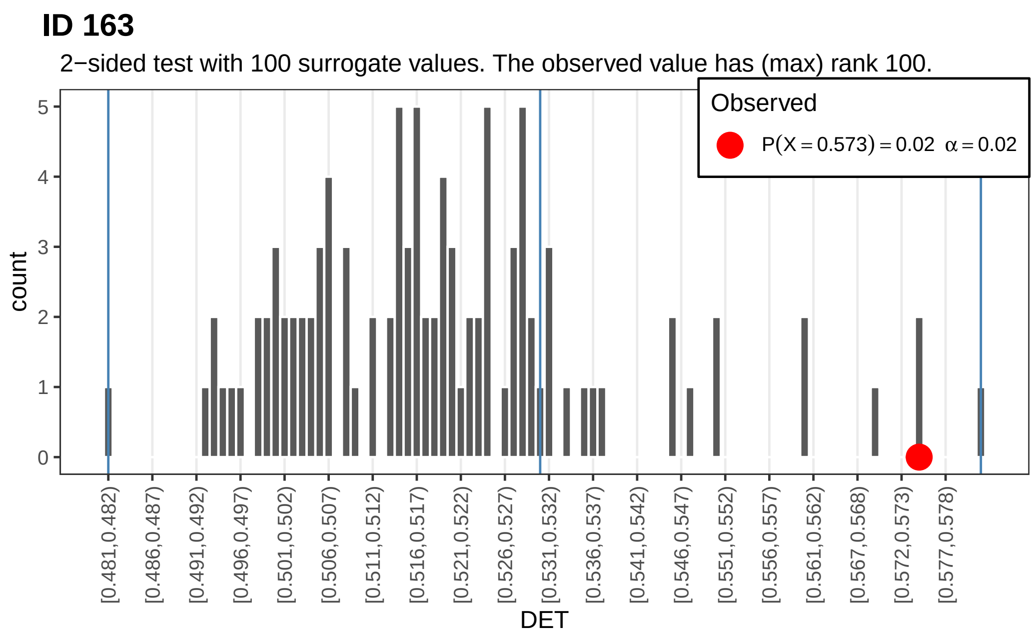
```
## 3          1.1902500        0.2590248      0.3230593
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1500
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.006274          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure      Rate     Mean Max
## 1   Diagonal     662              1594  Determinism 0.5151907 2.407855   6
## 2   Vertical     634              1604 V Laminarity 0.5184228 2.529968   5
## 3 Horizontal     634              1604 H Laminarity 0.5184228 2.529968   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8412765        0.1830804      0.3047335
## 2          0.9486399        0.2064451      0.3126026
## 3          0.9486399        0.2064451      0.3126026
##
## ~~~o~~o~~casnet~~o~~o~~~
##
## ~~~o~~o~~casnet~~o~~o~~~
##
##  Global Measures
##           Global Max.rec.points N.rec.points Recurrence.Rate Singular.points
## 1 Recurrence Matrix          9900         3094       0.3125253            1508
##   Divergence Repetitiveness Anisotropy
## 1  0.1666667       1.039723          1
##
##
##  Line-based Measures
##   Line.based N.lines N.points.on.lines      Measure     Rate     Mean Max
## 1   Diagonal     654              1586  Determinism 0.512605 2.425076   6
## 2   Vertical     635              1649 V Laminarity 0.532967 2.596850   5
```

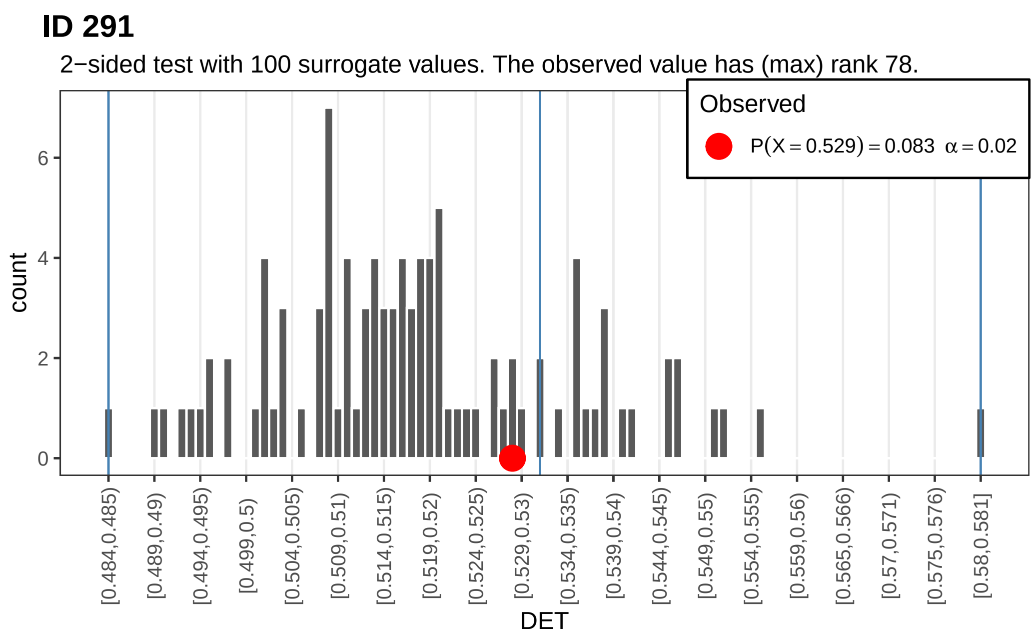
```
## 3 Horizontal     635              1649 H Laminarity 0.532967 2.596850   5
##   Entropy.of.lengths Relative.entropy CoV.of.lengths
## 1          0.8651645         0.188279      0.3061018
## 2          0.9861403         0.214606      0.3290047
## 3          0.9861403         0.214606      0.3290047
##
## ~~~o~~o~~casnet~~o~~o~~~
```

```
crqa_2rnd_sur[NROW(crqa_2rnd_sur)+1,] <- crqa_2
```

Use function `plorSUR_hist()` to get a p-value and plot the distributions. The red dots indicate the observed values.

```
# Get point estimates for p-values based on rank of observation (discrete distribution)
#   99 = (1 / alpha) - 1
# 99+1 = (1 / alpha)
alpha = 1/100

p_1 <- plotSUR_hist(surrogateValues = crqa_1rnd_sur$DET, observedValue = crqa_1$DET, measureName = "DET"
p_2 <- plotSUR_hist(surrogateValues = crqa_2rnd_sur$DET, observedValue = crqa_2$DET, measureName = "DET"

cowplot::plot_grid(p_1$surrogates_plot, p_2$surrogates_plot, labels = c("ID 163","ID 291"), ncol = 1)
```

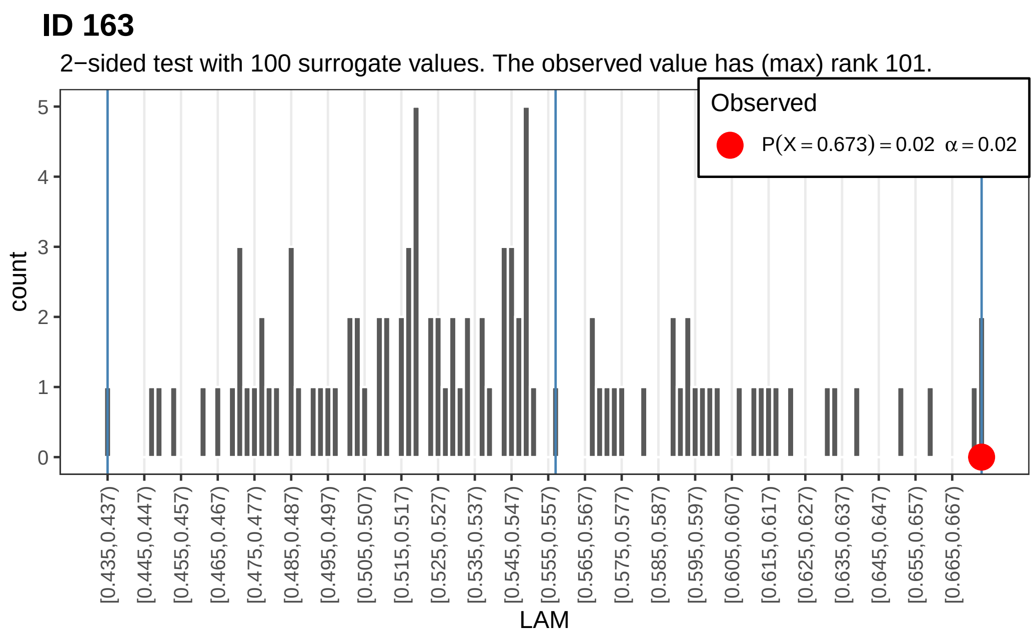


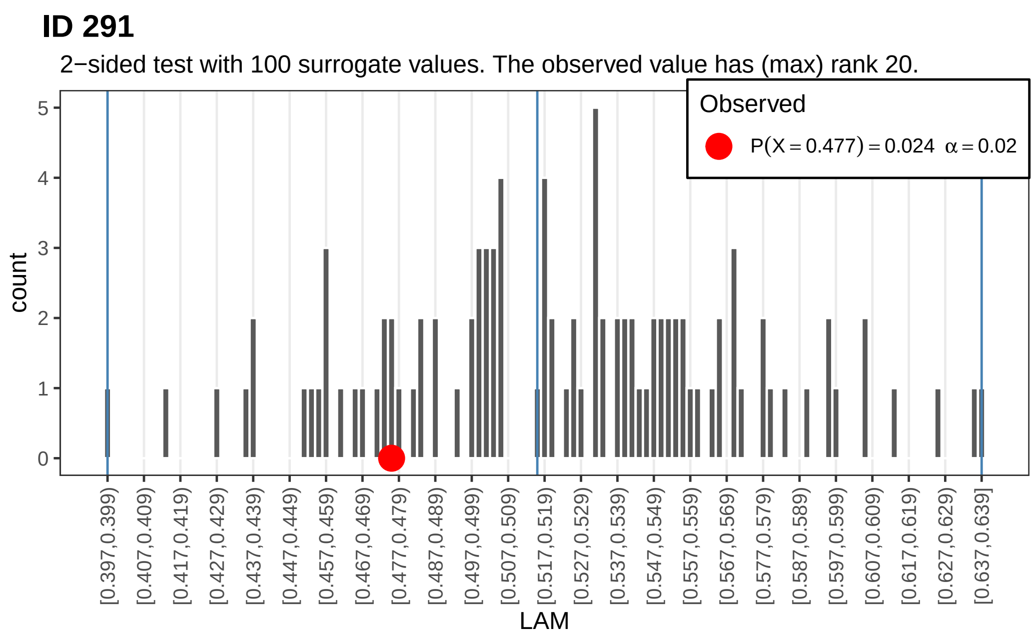


To get the full picture, let's look at those missing repetitions of the same numbers.

```
# Get point estimates for p-values based on rank of observation (discrete distribution)
#   99 = (1 / alpha) - 1
# 99+1 = (1 / alpha)
```

```
alpha = 1/100

p_1 <- plotSUR_hist(surrogateValues = crqa_1rnd_sur$LAM_vl, observedValue = crqa_1$LAM_vl, measureName =
p_2 <- plotSUR_hist(surrogateValues = crqa_2rnd_sur$LAM_vl, observedValue = crqa_2$LAM_vl, measureName =

cowplot::plot_grid(p_1$surrogates_plot, p_2$surrogates_plot, labels = c("ID 163","ID 291"), ncol = 1)
```





If we were naive to the orgin of these number sequences, the results for **LAM**inarity should make us doubt that they represent indendent draws from a discrete uniform distribution of the type $X \sim \mathcal{U}(1,9)$. If we had to decide which sequence was more, or, less random, then based on the **DET**erminism result, we would conclude that participant

163 produced a sequence that is less random than participant 291, the observed value of the former is at the right extreme of a distribution of DET values calculated from 99 realisations of the data constrained by H_0.

Chapter 9

Continuous Auto-Recurrence Analysis

lklkl

9.1 Estimating Parameters for Optimal Phase Space Reconstruction

lklkl

Chapter 10

Categorical Cross-Recurrence Analysis

kjkjkjkjk

Chapter 11

Continuous Cross-Recurrence Analysis

lklklklk

11.0.1 Diagonal Recurrence Profiles

Chapter 12

Other flavours of RQA

12.1 Lagged RQA: Sliding window analyses

12.2 Chromatic RQA

12.3 Anisotropic RQA

12.4 Multidimensional RQA

12.5 References

Deel VI

Complex Networks and Multivariate Timeseries

Chapter 13

Vector Auto Regression (VAR)

Chapter 14

Dynamic Complexity

kjkjkjk

Chapter 15

Graph Theory and Complex Network Analysis

kjkjkjkj

15.1 Recurrence Networks

kjkjkjk

15.2 Multiplex Recurrence Networks

kjkjkj

Bijlage A

Working with time series in R

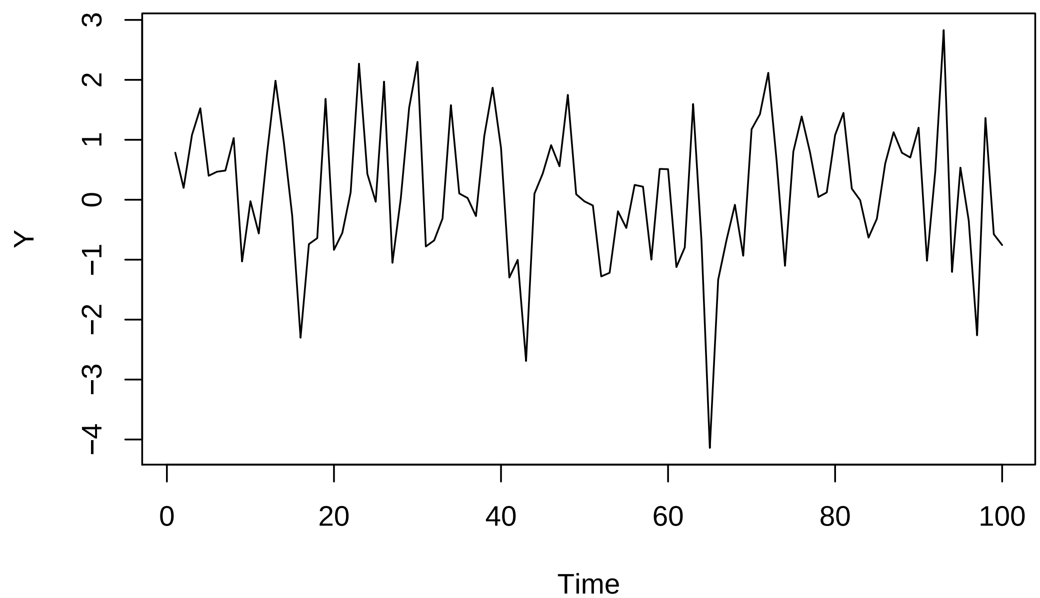
There are many ways to handle time series in `R`, this appendix provides some examples and suggest some best practices, based on the function `ts()`, which creates a time series object.

A time series object is expected to have a time-dimension on the x-axis. This is very convenient, because `R` will generate the time axis for you by looking at the *t*ime *s*eries *p*roperties attribute of the object. Even though we are not working with measurement outcomes, consider a value at a time-index in a time series object a **sample**:

- `Start` - The value of time at the first sample in the series (e.g., 0, or 1905)
- `End` - The value of time at the last sample in the series (e.g., 100, or 2005)
- `Frequency` - The amount of time that passed between two samples, or, the sample rate (e.g., 0.5, or 10)

Examples of using the time series object.

```r
set.seed(2718282)
# Get a timeseries of 100 random numbers
Y <- ts(rnorm(100))
# plot.ts
plot(Y)
```




```
# Get sample rate info
tsp(Y)
```

```
## [1]   1 100   1
```

```
# Extract the time vector
time(Y)
```

```
## Time Series:
## Start = 1
## End = 100
## Frequency = 1
##   [1]   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18
##  [19]  19  20  21  22  23  24  25  26  27  28  29  30  31  32  33  34  35  36
##  [37]  37  38  39  40  41  42  43  44  45  46  47  48  49  50  51  52  53  54
##  [55]  55  56  57  58  59  60  61  62  63  64  65  66  67  68  69  70  71  72
##  [73]  73  74  75  76  77  78  79  80  81  82  83  84  85  86  87  88  89  90
##  [91]  91  92  93  94  95  96  97  98  99 100
```

For now, these values are in principle all arbitrary units (`a.u.`). These settings only make sense if they represent the parameters of an actual measurement procedure.

It is easy to adjust the time vector, by assigning new values using `tsp()` (values have to be possible given the time series length). For example, suppose the sampling frequency was 0.1 instead of 1 and the Start time was 10 and End time was 1000.

```
# Assign new values
(tsp(Y) <- c(10, 1000, .1))
```

```
## [1] 1e+01 1e+03 1e-01
```

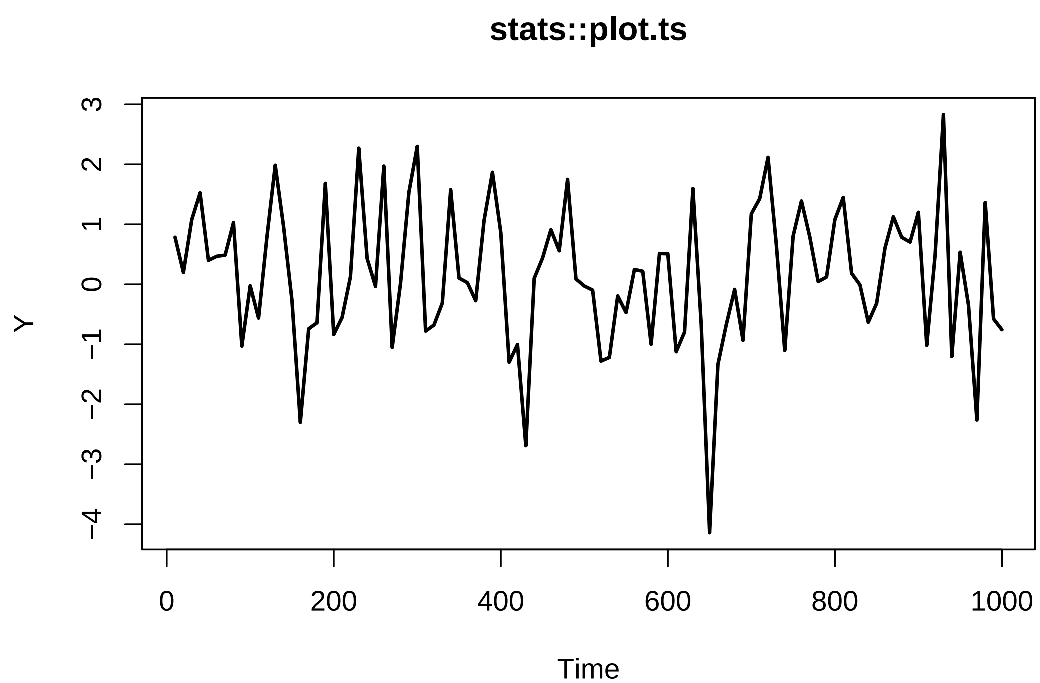
```
# Time axis is automatically adjusted
time(Y)
```

```
## Time Series:
## Start = 10
## End = 1000
## Frequency = 0.1
##   [1]   10   20   30   40   50   60   70   80   90  100  110  120  130  140  150
##  [16]  160  170  180  190  200  210  220  230  240  250  260  270  280  290  300
##  [31]  310  320  330  340  350  360  370  380  390  400  410  420  430  440  450
##  [46]  460  470  480  490  500  510  520  530  540  550  560  570  580  590  600
##  [61]  610  620  630  640  650  660  670  680  690  700  710  720  730  740  750
##  [76]  760  770  780  790  800  810  820  830  840  850  860  870  880  890  900
##  [91]  910  920  930  940  950  960  970  980  990 1000
```

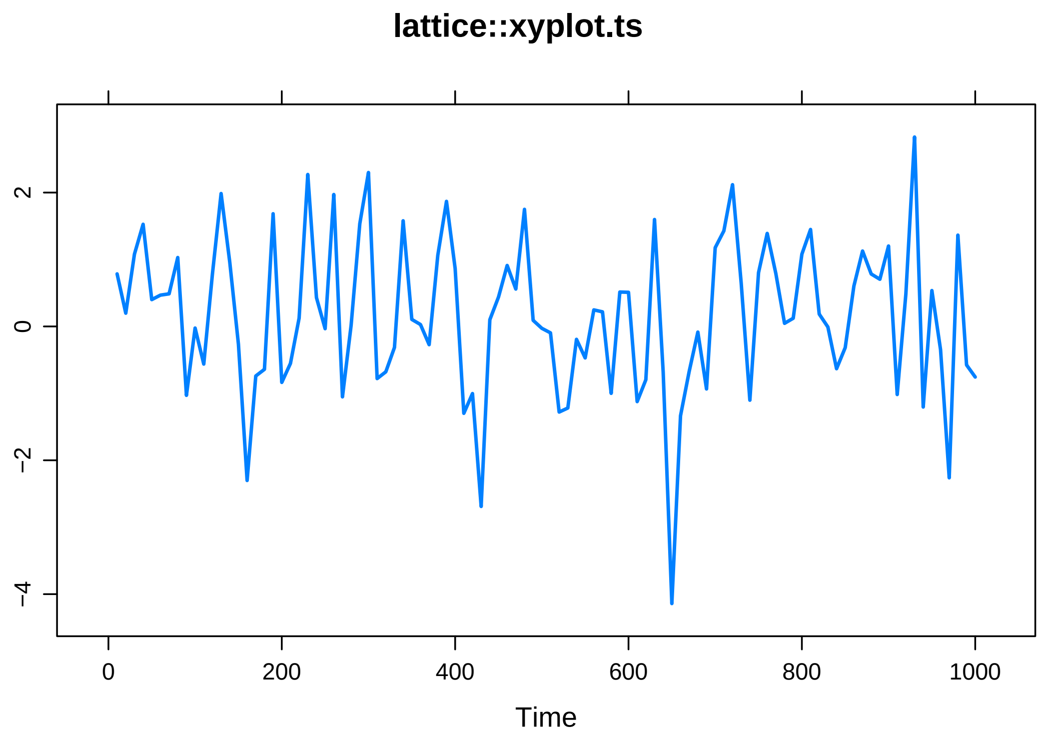
A.0.1 Plotting a `ts` object as a time series

Depending on which packages you use, there will be different settings applied to time series objects created by `ts()`. Below are some examples of differences between plotting routines.

```
require(lattice)       # Needed for plotting
require(latticeExtra)  # Needed for plotting
require(casnet)        # Need for ts_center()

# stats::plot.ts
plot(Y, lwd = 2, main = "stats::plot.ts")
```



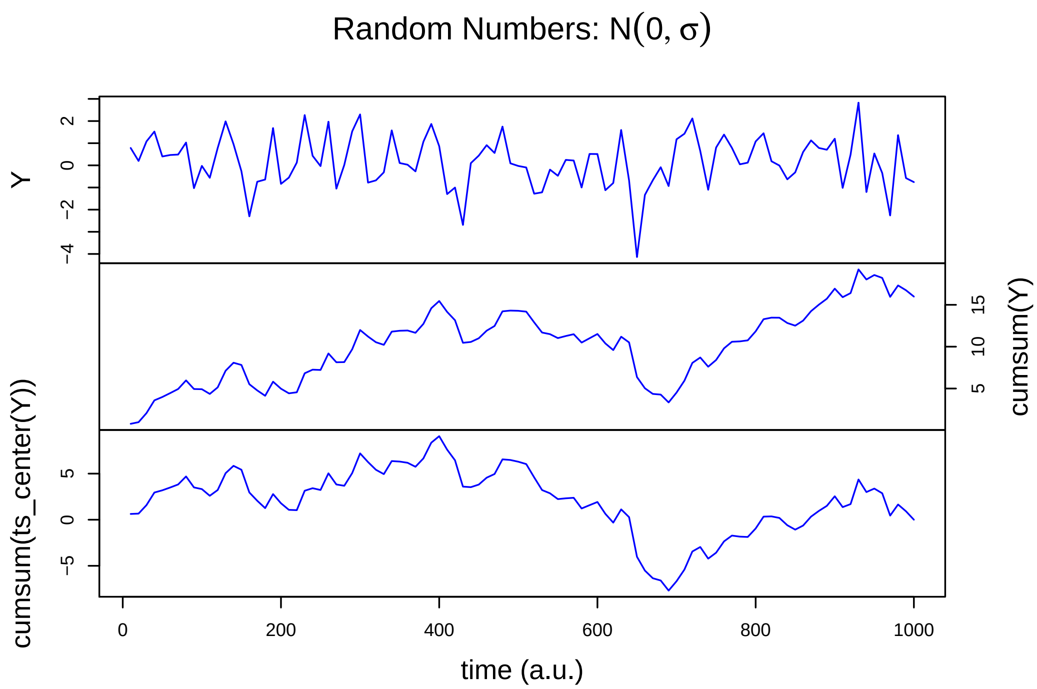

```r
# lattice::xyplot.ts
xyplot(Y, lwd = 2, main = "lattice::xyplot.ts")
```



A.0.2 Plotting multiple time series in one figure

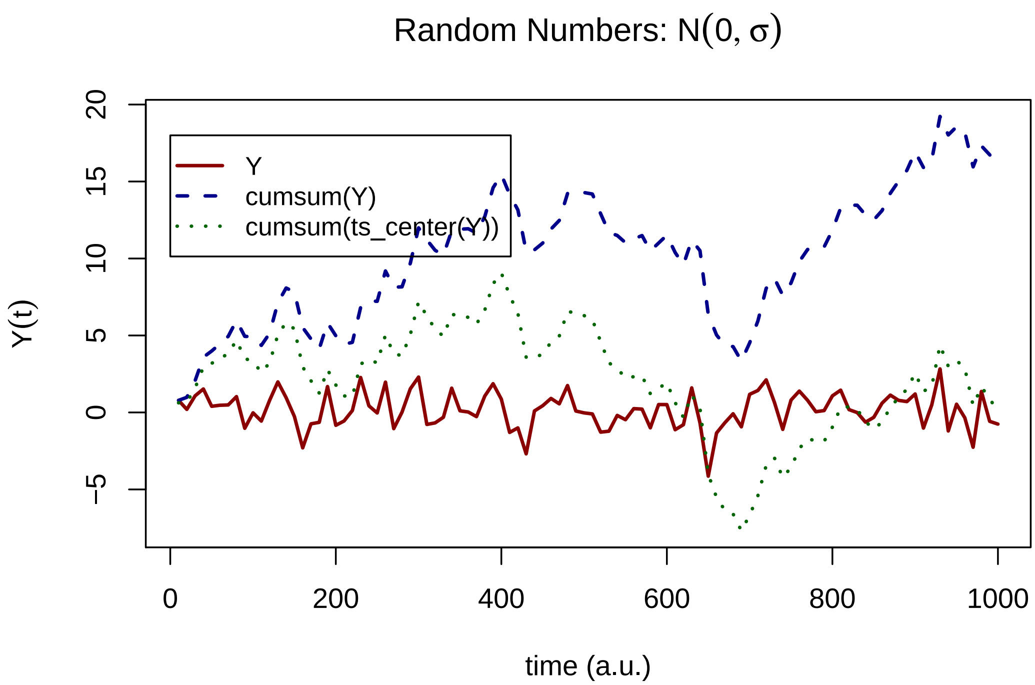
Plot multiple time series in frames with `plot.ts()` in `package::stats`. This function takes a matrix as input, here we use `cbind( ... )`.

```
# stats::plot.ts
plot(cbind(Y,
           cumsum(Y),
           cumsum(ts_center(Y))
           ),
     yax.flip = TRUE, col = "blue", frame.plot = TRUE,
     main = expression(paste("Random Numbers: ",N(0,sigma))), xlab = "time (a.u.)")
```



Plot multiple time series in one graph with `ts.plot()` in `package::graphics`. This function can handle multiple `ts` objects as arguments.

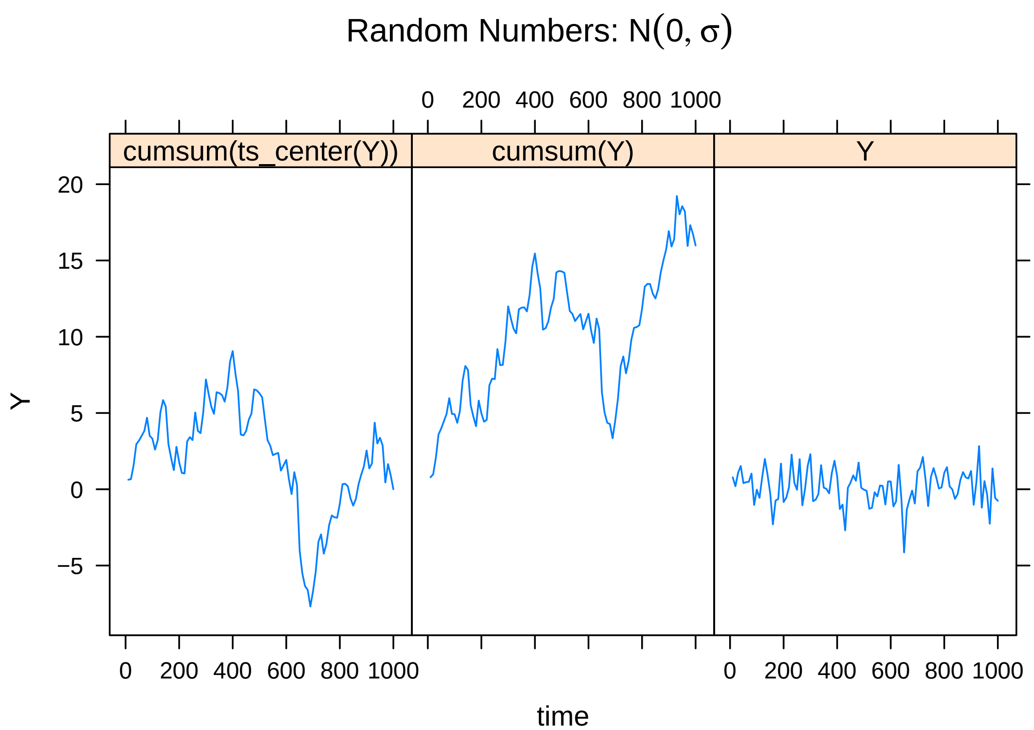
```
# graphics::ts.plot
ts.plot(Y,
        cumsum(Y),
        cumsum(ts_center(Y)),
        gpars = list(xlab = "time (a.u.)",
                     ylab = expression(Y(t)),
                     main = expression(paste("Random Numbers: ",N(0,sigma))),
                     lwd = rep(2,3),
                     lty = c(1:3),
                     col = c("darkred","darkblue","darkgreen")
                     )
        )
legend(0, 18, c("Y","cumsum(Y)", "cumsum(ts_center(Y))"), lwd = rep(2,3), lty = c(1:3), col = c
```



Use `xyplot()` in `package::lattice` to create a plot with panels. The easiest way to do this is to create a dataset in so-called "long" format. This means the variable to plot is in 1 column and other variables indicate different levels, or conditions under which the variable was observed or simulated.

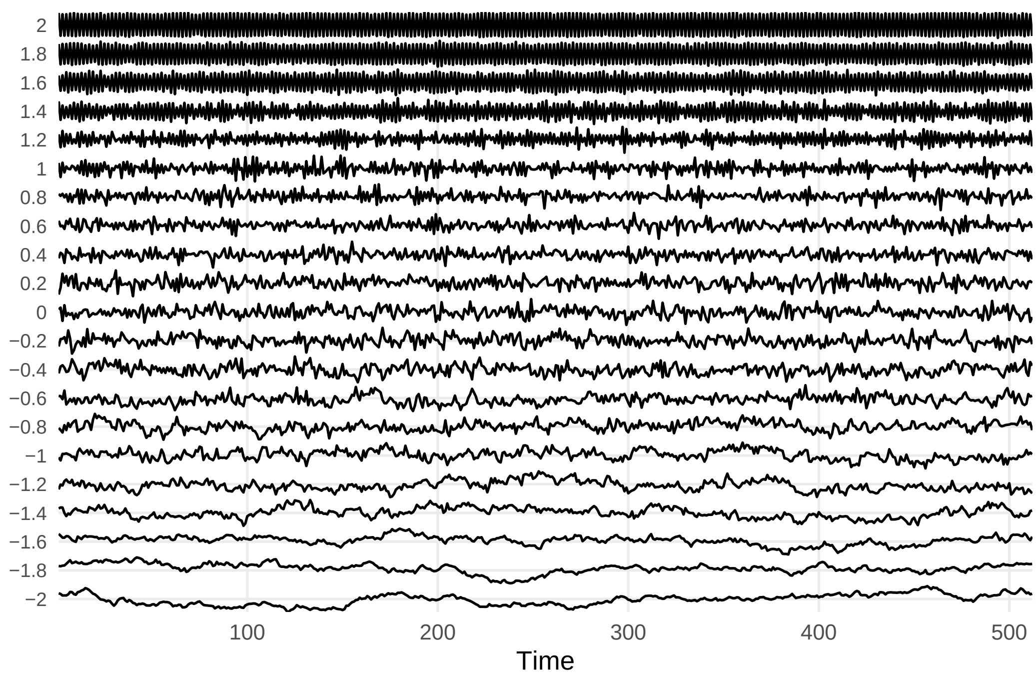
Function `ldply()` is used to generate Y for three different settings of r. The values of r are passed as a **l**ist and after a function is applied the result is returned as a **d**ataframe.

```
require(plyr)          # Needed for function ldply()

# Create a long format dataframe for various values for `r`
data <- cbind.data.frame(Y     = c(as.numeric(Y), cumsum(Y), cumsum(ts_center(Y))),
                         time  = c(time(Y), time(Y), time(Y)),
                         label = factor(c(rep("Y",length(Y)),  rep("cumsum(Y)",length(Y)), rep("cumsum(t
                         )
# Plot using the formula interface
xyplot(Y ~ time | label, data = data, type = "l", main = expression(paste("Random Numbers: ",N(0,sigma))
```



Or, if you have very may time series, you can use the function `PLOT()` in `casnet`.

```
# Create a data frame with time series
# Generate some coloured noise
N <- 512
noises <- seq(-2,2,by=.2)
y <- data.frame(matrix(rep(NA,length(noises)*N), ncol=length(noises)))

for(c in seq_along(noises)){y[,c] <- noise_powerlaw(N=N, alpha = noises[c])}
colnames(y) <- paste0(noises)

plotTS_multi(y)
```



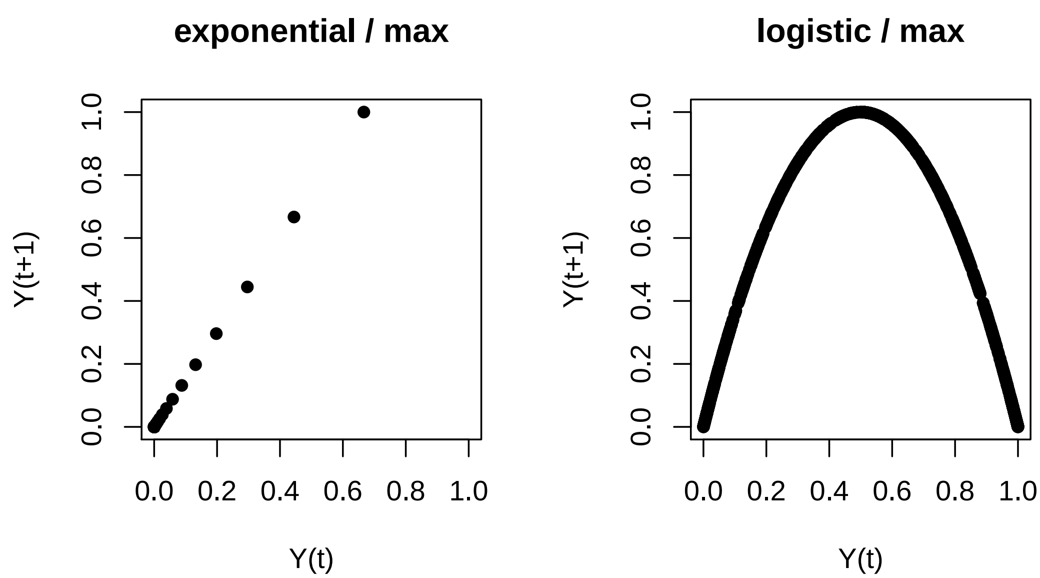
Note that the y-axis is rescaled for each series and does not reflect magnitude differences between the series.

A.0.3 The return plot

To create a return plot the values of Y have to be shifted by a certain lag. The functions `lead()` and `lag()` in `package::dplyr` are excellent for this purpose (note that `dplyr::lag()` behaves different from `stats::lag()`).

```
# Function lag() and lead()
library(dplyr)
library(casnet)

# Get exponential growth
YY <- growth_ac(N=1000,r=1.5,type = "driving")
Y1 <- as.numeric(YY/max(YY))
# Get logistic growth in the chaotic regime
Y2 <- as.numeric(growth_ac(N=1000,r=4,type = "logistic"))
# Use the `lag` function from package `dplyr`
op <- par(mfrow = c(1,2), pty = "s")
```

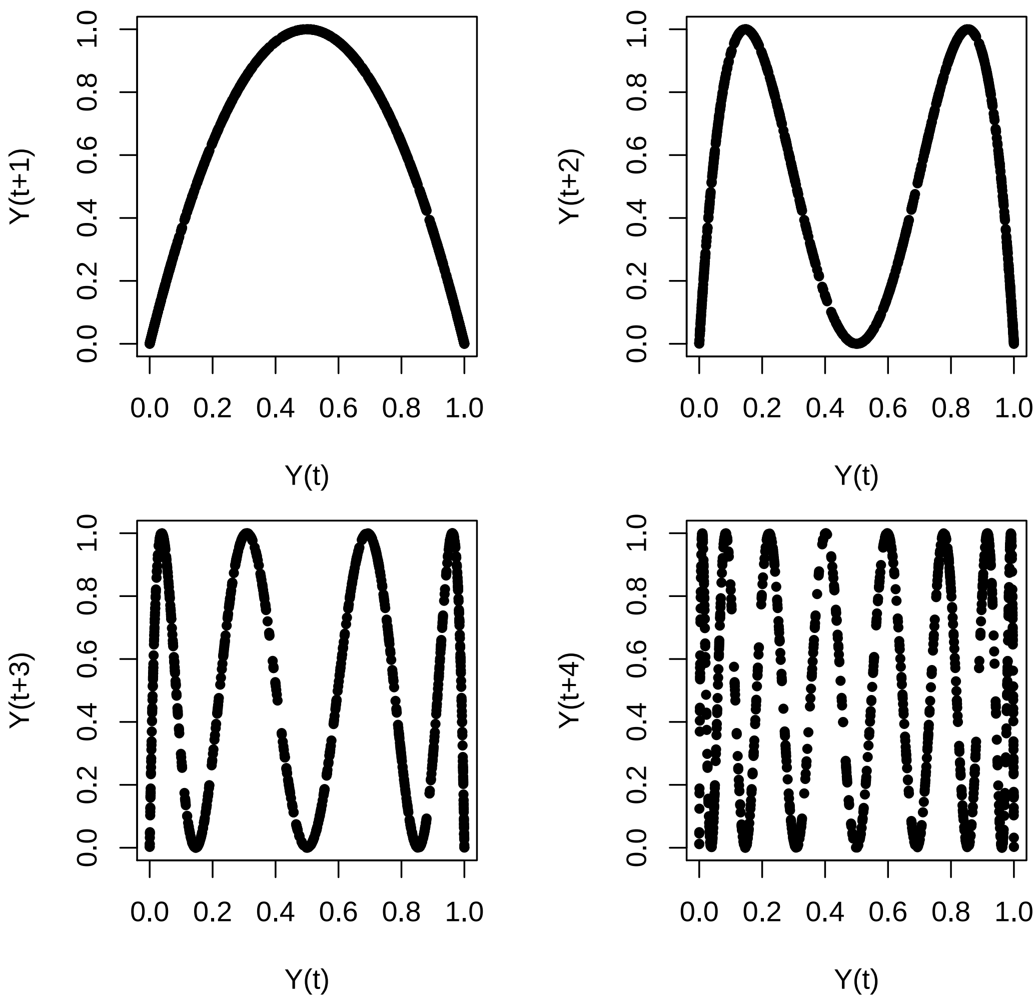
```r
plot(dplyr::lag(Y1), Y1, xy.labels = FALSE, pch = 16, xlim = c(0,1), ylim = c(0,1), xlab = "Y(t
     main = "exponential / max")
plot(dplyr::lag(Y2), Y2, xy.labels = FALSE, pch = 16, xlim = c(0,1), ylim = c(0,1), xlab = "Y(t
     main = "logistic / max")
```




```r
par(op)
```

Use `l_ply()` from `package::plyr` to create return plots with different lags. The **l_** before **ply** means the function will take a **l**ist as input to a function, but it will not expect any data to be returned, for example in the case of a function that is used to plot something.

```r
# Explore different lags
op <- par(mfrow = c(1,2), pty = "s")
plyr::l_ply(1:4, function(l) plot(dplyr::lag(Y2, n = l), Y2, xy.labels = FALSE, pch = 16, xlim =
```




```r
par(op)
```

A.0.4 Using `ggplot2`

Becoming proficient at `ggplot2` can take some time, but it does pay off. One of the problems with plotting time series data is that `ggplot2` wants tidy data in *long* format. Tidy data is:

> Tidy data is a standard way of mapping the meaning of a dataset to its structure. A dataset is messy or tidy depending on how rows, columns and tables are matched up with observations, variables and types. In tidy data: 1. Each variable forms a column. 2. Each observation forms a row. 3. Each

type of observational unit forms a table.

—**?**

So if we have a set of time series as in the previous examples, we need to change it to long format.

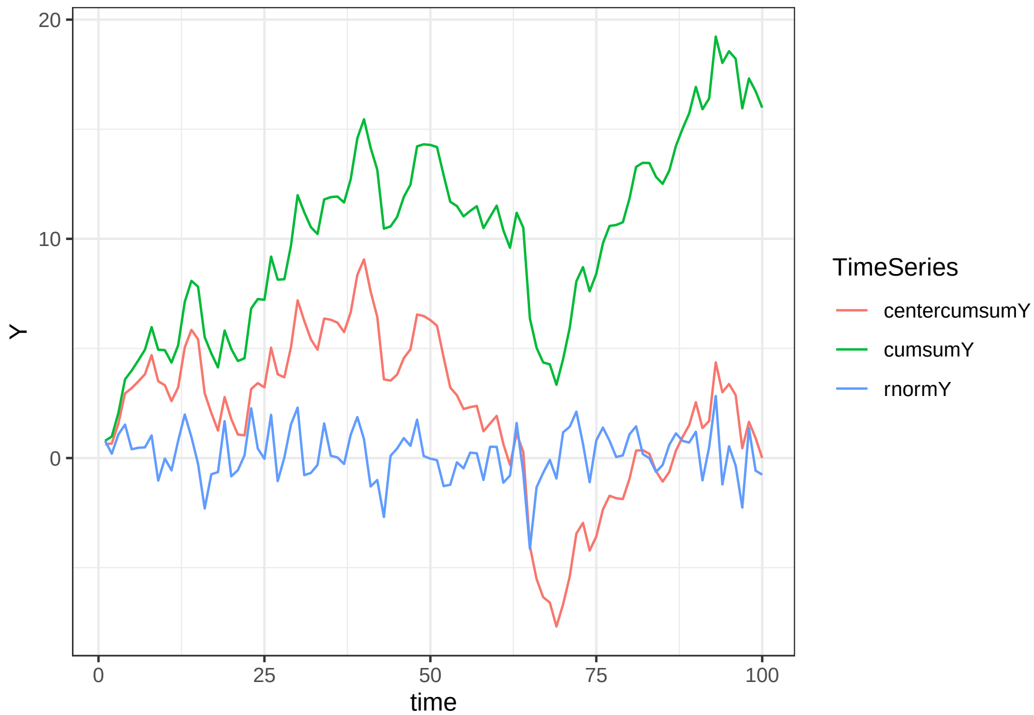
```
library(tidyverse)

# A wide data frame
df.wide <- data.frame(rnormY         = Y,
                      cumsumY        = cumsum(Y),
                      centercumsumY = cumsum(ts_center(Y)),
                      time           = seq_along(Y)
                      )

glimpse(df.wide)
```

```
## Observations: 100
## Variables: 4
## $ rnormY        <dbl> 0.78482166, 0.19776074, 1.07957851, 1.52605836, 0.400...
## $ cumsumY       <dbl> 0.7848217, 0.9825824, 2.0621609, 3.5882193, 3.9884794...
## $ centercumsumY <dbl> 0.6249966, 0.6629322, 1.5826857, 2.9489189, 3.1893540...
## $ time          <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16...
```

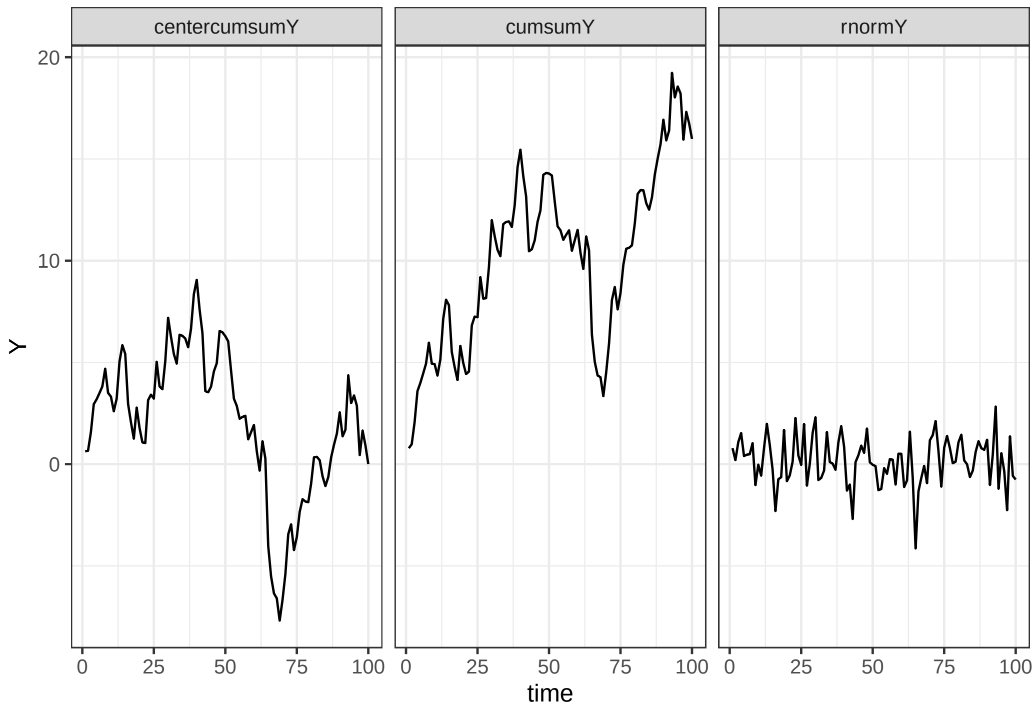
```
# Create a long dataframe using gather()
df.long <- df.wide %>%
  gather(key=TimeSeries,value=Y,-"time")

glimpse(df.long)
```

```
## Observations: 300
## Variables: 3
## $ time       <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 1...
## $ TimeSeries <chr> "rnormY", "rnormY", "rnormY", "rnormY", "rnormY", "rnorm...
## $ Y          <dbl> 0.78482166, 0.19776074, 1.07957851, 1.52605836, 0.400260...
```

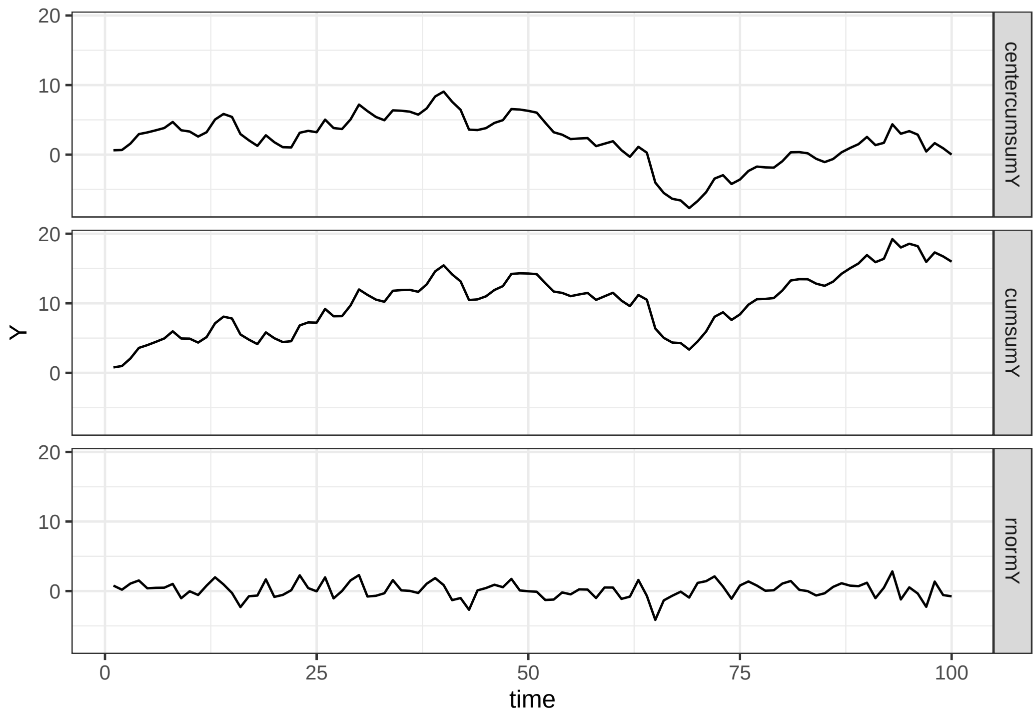
```
# 1 plot
ggplot(df.long, aes(x=time, y=Y, colour=TimeSeries)) +
  geom_line() +
  theme_bw()
```




```
# using facets
ggplot(df.long, aes(x=time,y=Y)) +
  geom_line() +
  facet_wrap(~TimeSeries) +
  theme_bw()
```

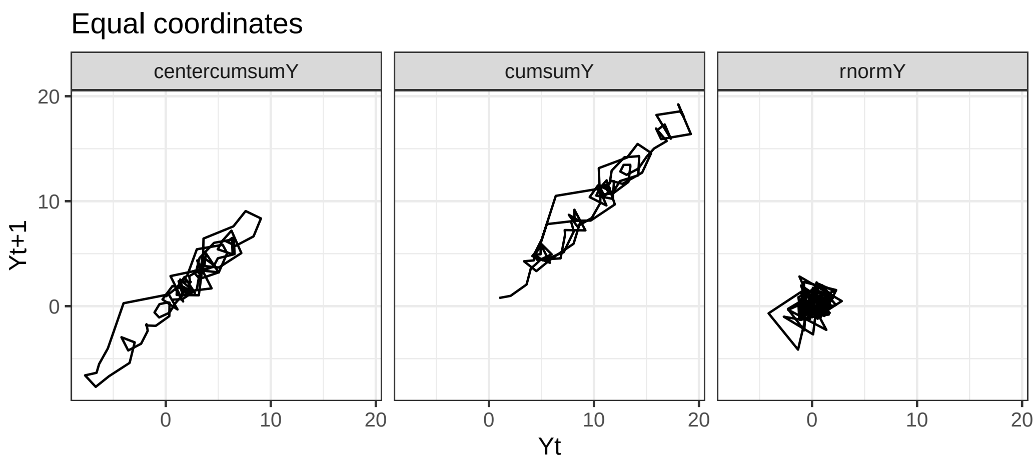



```
# using facets
ggplot(df.long, aes(x=time,y=Y)) +
  geom_line() +
  facet_grid(TimeSeries~.) +
  theme_bw()
```

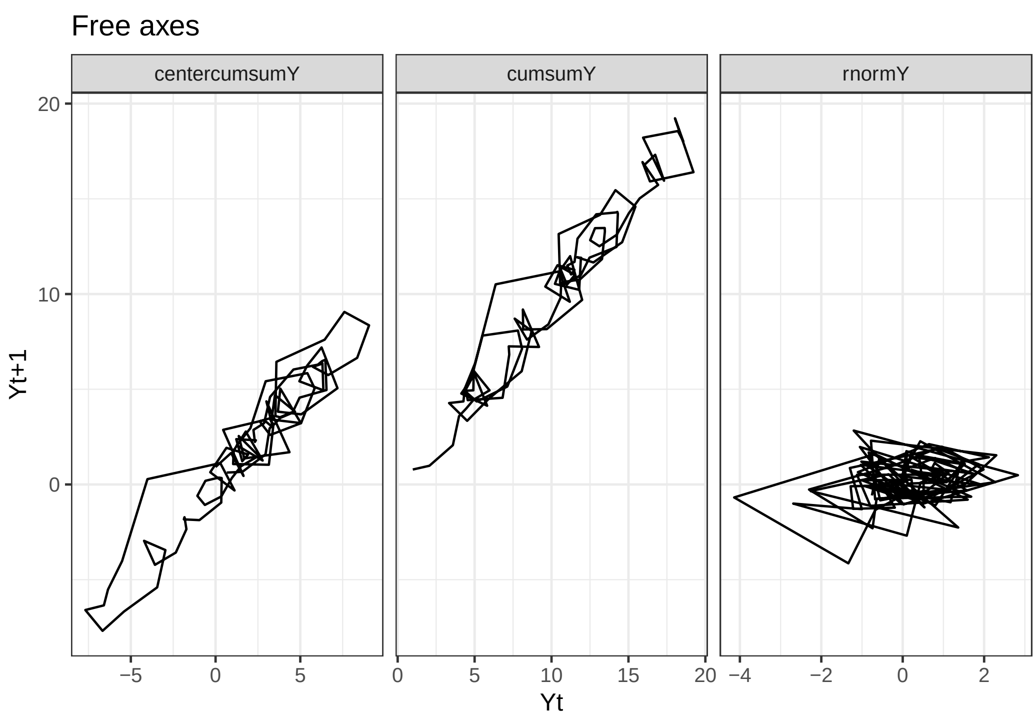


To create a return plot you can use `geom_path()` instead of `geom_line()` and make the area square using `coord_equal()`.

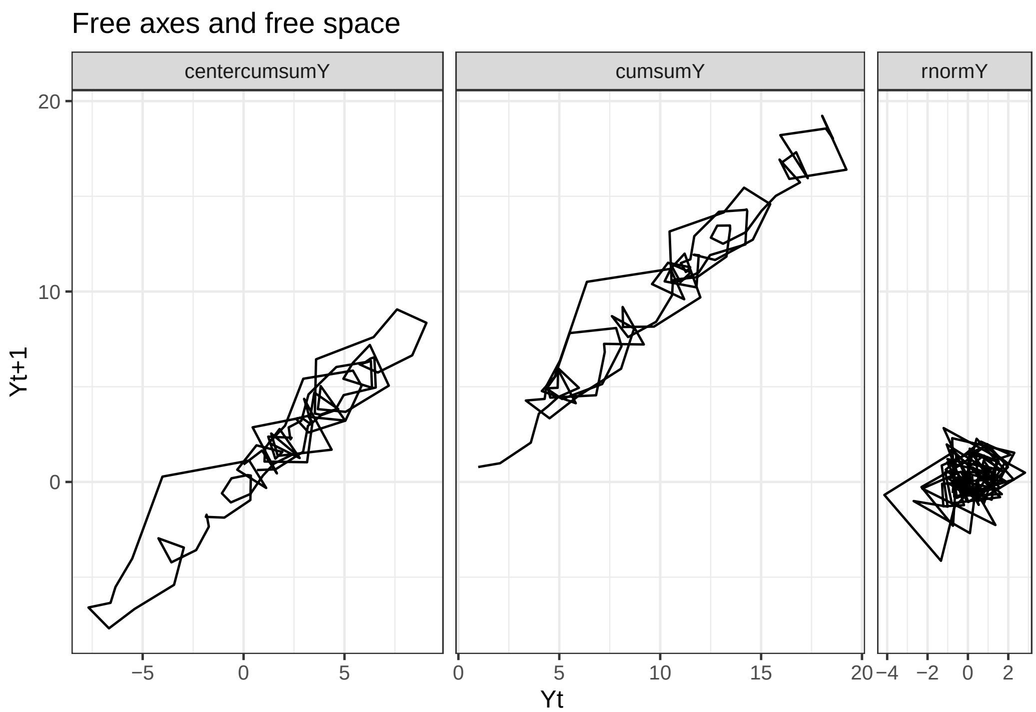
```
# Add a lagged variable
df.long <- df.long %>%
  group_by(TimeSeries) %>%
  mutate(Ylag = dplyr::lag(Y))

# Use geom-path()
ggplot(df.long, aes(x=Y,y=Ylag,group=TimeSeries)) +
  geom_path() +
  facet_grid(.~TimeSeries) +
  theme_bw() +
  labs(title = "Equal coordinates", x="Yt",y="Yt+1") +
  coord_equal()
```




```r
# You could also have free axes
ggplot(df.long, aes(x=Y,y=Ylag,group=TimeSeries)) +
  geom_path() +
  facet_grid(.~TimeSeries, scales = 'free') +
  labs(title="Free axes", x="Yt",y="Yt+1") +
  theme_bw()
```



```
# Or free axes and a free space
ggplot(df.long, aes(x=Y,y=Ylag,group=TimeSeries)) +
  geom_path() +
  facet_grid(.~TimeSeries, scales = 'free', space = 'free') +
  labs(title="Free axes and free space", x="Yt",y="Yt+1") +
  theme_bw()
```



Bijlage B

List of terms

Adaptive Behaviour

- System behaviour that appears to be (partially) coordinated by previously 'experienced events'

Analytic solution

- The solution to a difference or differential equation allows one to find any state of the system without the need to iterate the model starting from some initial condition. There are very few (systems of) equations for which analytical solutions exist

Attractor

- The status that a dynamic system eventually "settles down to". An attractor is a set of values in the phase space to which a system migrates over time, or iterations. Attractors can have as many dimensions as the number of variables that influence its system

Basin of attraction

- A region in phase space associated with a given attractor. The basin of attraction of an attractor is the set of all (initial) points that eventually end up in that attractor

Behaviour (of a dynamic system)

- The temporal evolution of states of a system according to one or more rules (also known as state propagation rules, or, iterative processes). Models of the behaviour of dynamic systems use difference or differential equations to describe the iterative processes hypothesized to underlie the temporal evolution

Bifurcation

- A clearly observable qualitative change in the behavioural mode (attractor state) of a dynamic system associated with continuous change in one or more control parameters (also known as Phase-, State-, or Order- Transition). The value of a control parameter at which a bifurcation occurs is often non-specific, or trivial

Bifurcation diagram

- Visual summary of the succession of period-doubling bifurcations produced by gradual changes in the control parameter(s)

Catastrophe flags

- Markers indicative for a physical system that is described by a catastrophe. There are 5 'classical' flags and 3 'diagnostic' flags. Classical: bimodality, sudden jumps, inaccessibility, sensitivity & hysteresis. Diagnostic: divergence from linear response, critical slowing down and critical fluctuations. Diagnostic flags can be used as early-warning signals.

Catastrophe theory

- Mathematical research program describing how gradual change in some parameters can lead to disproportionately large changes in another parameter, called catastrophes (similar to bifurcations, Phase-, State-, or Order-transitions). 'This kind of behaviour has been summarized succinctly in the phrase "the straw that broke the camel's back".' (Gilmore, 1992).

Complex Network

- A network with of many nodes and likely many substructures depending on that nature and distribution of connections between nodes

Complex system

- Spatially and/or temporally extended nonlinear systems characterized by emergent properties and self-organised behavioural modes at a global, or, macro-level (the system as a whole), that is often different from the characteristic behaviour at a local, or, micro-level (behaviour of the individual parts that constitute the whole)

Complexity science

- Complexity science studies how systems that consist of many components can generate relatively simple and stable (non-random) behaviour. Important behavioural phenomena studied in Complexity Science are synchronisation, adaptation and coordination of behaviour across many different temporal and spatial scales, emergent properties and collective behaviour, holism and self-organisation

Component dominant dynamics

- A causal ontology in which observed behaviour is explained by assuming it is the result of a chain of independent efficient causes (components)

Control parameter

- A variable that controls the global behaviour of a dynamic system. For certain values of the parameter, transitions between qualitatively different behavioural modes (orders) can occur.

Critical fluctuations

- An early warning signal for a phase transition that is characterised by an increase in fluctuations (variability) of the behaviour of the system. The increase occurs because the self-organised transition from one state to another relaxes the constraints on the degrees of freedom a system has available to generate its behaviour, allowing states and behavioural modes to appear that were previously inaccessible.

Critical slowing down

- An early warning signal for a phase transition that is characterised by an increase in the duration of relaxation times. If it takes longer for the system to return to the state it was perturbed from, this implies the emergence of a new stable state is imminent

Deterministic Chaos

- Behaviour of a dynamic system that "looks random, but is not" (Lorenz, 1973). The dynamics can be characterised as follows: 1) A-periodic, no point or trajectory in state space will exactly recur; 2) Sensitive dependence in initial conditions; 3) Bounded, not all theoretically possible degrees of freedom are available to the system; 4) The origin of this behaviour is deterministic, not stochastic

Difference equation

- A function specifying the underlying change process in a variable from one discrete point in time to another

Differential equation

- A function specifying the underlying change process of a variable in continuous time

Dimension

- See embedding dimension, box-counting dimension, correlation dimension, information dimension, dimension of a system

Dimensions of a system

- The set of variables that define a system. Iterative processes operate on the dimensions of a system

Dynamic system

- A set of equations specifying how certain variables change over time. The equations specify how to determine (compute) the new values as a function of their current values and control parameters. The functions, when explicit, are either difference equations or differential equations. Dynamic systems may be stochastic or deterministic. In a stochastic system, new values come from a probability distribution. In a deterministic system, a single new value is associated with any current value

Early warning signals

- Critical slowing down and critical fluctuations. Early-warning signals indicate instability in the existing state which may result in a qualitative shift towards a new state (phase transition / catastrophe). Early-warning signals are similar to diagnostic catastrophe flags.

Effective Complexity

- "The effective complexity of an entity is the length of a highly compressed description of its regularities." (Gell-man & Lloyd, 2004)

Embedding Dimension

- Successive N-tuples of points in a time series are treated as points in N dimensional space. The points are said to reside in embedding dimensions of size N, for N = 1, 2, 3, 4, … etc.

Emergence

- A complex system can generate emergent behaviour or display emergent properties that are novel and unexpected, that is, they are not predictable from the behaviour and properties of the components of the system

Entropy

- Relative absence of order/redundancy in a system. The degrees of freedom a system has available for generating its behaviour: Possibility

Epigenetic landscape (potential landscape)

- A hypothetical landscape describing the relative stability of behavioural modes of a system over time

Experienced event

- An interaction of a system with its environment that changed the internal structure/organization of the system such that it can be said to display adaptive behaviour. “Interaction with after-effects”. Random behaviour is “Interaction without after-effects”.

flow ~

- A differential equation

Fractal

- An irregular shape with self-similarity. It has infinite detail, and cannot be differentiated. “Wherever chaos, turbulence, and disorder are found, fractal geometry is at play” (Briggs and Peat, 1989).

Fractal Dimension

- A measure of a geometric object that can take on fractional values. At first used as a synonym to Hausdorff dimension, fractal dimension is currently used as a more general term for a measure of how fast length, area, or volume increases with decrease in scale. (Peitgen, Jurgens, & Saupe, 1992a).

Graph theory

- Models in which associations between mathematical objects are defined as edges (connections) between vertices (nodes)

Hausdorff Dimension

- A measure of a geometric object that can take on fractional values. (see fractal dimension).

Holism (epistemic)

- "some property of a whole would be holistic if, according to the theory in question, there is no way we can find out about it using only local means, i.e., by using only all possible non-holistic resources available to an agent." (Seevinck, 2002)

Idiographic approach

- Scientific explanation in which the goal is to generate knowledge about specific facts, events or entities. The goal is not to generalize to universal laws and first principles.

Information (quantity)

- A measurable quantity that resolves uncertainty about the state of a system by assigning a value to the uncertainty.

Initial condition

- The starting point of a dynamic system, the initial state of a system from which it evolved to the current state.

Interaction dominant dynamics

- A causal ontology in which observed behaviour is explained by assuming it is the result of interactions between processes across many temporal and spatial scales

Iteration

- The repeated application of a function, using its output from one application as its input for the next.

Iterative function

- A function used to calculate the new state of a dynamic system.

Iterative system

- A system in which one or more functions are iterated to define the system.

Largest Lyapunov exponent

- The value of the largest exponent in a spectrum of exponents (the Lyapunov spectrum), coefficients of time, that reflect the rate of departure (divergence) of dynamic orbits of a system. The largest exponent indicates the extent to which the behaviour of a system is sensitive to initial conditions.

Limit cycle

- An attractor that is periodic in time, that is, that cycles periodically through an ordered sequence of states.

Limit points

- Points in phase space. There are three kinds: attractors, repellors, and saddle points. A system moves away from repellors and towards attractors. A saddle point is both an attractor and a repellor, it attracts a system in certain regions, and repels the system to other regions.

Linear function of predictors

- A linear equation is of predictors is of the form y=a*x(i)+b, in which variable y varies 'linearly' with other variables x(i). In this equation, 'a' determines the slope of the line and 'b' reflects the y-intercept, the value y obtains when all x(i) equal zero.

Linear function of time

- A linear function of time is of the form $\hat{y}(t) = a*y(t) + b$, in which variable y varies 'linearly' with time 't', that is, with itself at an earlier moment in time. In this equation 'a' determines the rate with which 'y' will change as time passes, 'b' reflects the initial condition, the value y obtains when t equals zero.

map …

- A difference equation

Nonlinear dynamics

- The study of dynamic systems whose functions specify that change is not a linear function of time.

Orbit (trajectory)

- A sequence of coordinates (a path) through the phase space of a system.

Order

- "order is essentially the arrival of redundancy in a system, a reduction of possibilities"(Von Förster, 2003). Any form of non-random association or dependency that exists between parts of a system, its behaviour over time and/or its environment is a form of order. In scientific explanation of behaviour, the presence of order in non-artificial systems must be explained and should not be (implicitly) assumed.

Order Parameter

- A nominal variable that indexes qualitatively different behavioural modes of a system, for example the phases of matter (gas, liquid, solid, plasma)

Period-doubling

- The change in dynamics in which a N-point attractor is replaced by a 2N-point attractor.

Phase portrait

- The collection of all trajectories from all possible starting points in the phase space of a dynamic system.

Phase space

- An abstract space used to represent the behaviour of a system. Its dimensions are the variables of the system. Thus a point in the phase space defines a potential state of the system. The points actually achieved by a system depend on its iterative function and initial condition (starting point).

Phase transition

- A transition between qualitatively different behavioural modes

Potential function

- A function that describes the order parameter of a system, that is, it describes the relative stability of the potential end-states (attractor states) a system can settle into. The parameters of the potential function include the control parameter.

Power-law scaling

- A relationship between two variables that is linear on doubly logarithmic coordinates, meaning the law is expressed in increments that represent 'power'

Recursive process

- For our purposes, "recursive" and "iterative" are synonyms. Thus recursive processes are iterative processes, and recursive functions are iterative functions.

Relaxation time

- The time it takes for a system to return to a stable state after it was perturbed enough to leave that state. A characteristic warning signal of an imminent phase transition is an increase relaxation times, also known as critical slowing down.

Repellors

- One type of limit point. A point in phase space that a system moves away from.

Return map

- Plot of time series values vs. a delayed copy of itself. A return plot can be used to get an idea about the functional form of the iterative process, it is a simple variant of delay embedding.

Saddle point

- A point, usually in three dimensional state space, that both attracts and repels, attracting in one dimension and repelling to another.

Scale free network

- A network in which the distribution of the number of connections of a node and their frequency of occurrence follows a power-law in which there are just a few nodes with many connections and many nodes with just a few connections

Self-affinity

- An infinite nesting of characteristic structure on all scales. Strict self-affinity refers to a form of which all substructures are affine transformation, which means the different dimensions of the system can be scaled by their own exponent. Statistical self-affinity refers to an approximate equivalence of form at all scales.

Self-similarity

- An infinite nesting of characteristic structure on all scales. Strict self-similarity refers to a form of which all substructures can be considered scaling transformations, larger or smaller copies scaled by a single exponent for all dimensions of the structure. Statistical self-similarity refers to an approximate equivalence of scaled structure.

Sensitive dependence on initial conditions

- A property of chaotic systems. A dynamic system has sensitivity to initial conditions when very small differences in starting values result in very different behaviour. If the orbits of nearby starting points diverge, the system has sensitivity to initial conditions.

Small world network

- Many real-world networks have a small average shortest path length, but also a clustering coefficient that is significantly higher than expected by chance. These networks are extremely efficient, each node in a very large network can still be reach in just a few steps (the ’six degrees of separation‘ phenomenon).

State

- A coordinate in state space designating the current status of a dynamic system. The elements of the coordinates are values on the dimensions of the system that span the state space.

State space

- A hypothetical space spanned by the dimensions of the system. Each combination of values of variables that represent the dimension is a state of the system, it is a coordinate in state space.

State space (phase space)

- An abstract space used to represent the behaviour of a system. Its dimensions are the variables of the system. Thus a point in the phase space defines a potential state of the system.

Strange attractor

- An attractor state representing chaotic dynamics: a-periodic, bounded, and sensitive dependence on initial conditions

System

- An entity that can be described as a composition of components according to some organising principle. Organising principles describe how parts of the system relate to the whole and.

Time series

- A record of observations (data points) of behaviour over time.

Trajectory (orbit)

- A sequence of positions (path) of a system in its phase space. The path from its starting point (initial condition) to and within its attractor.

Transient time (transient behaviour)

- The time it takes for a system to transition from one stable state (behavioural mode, attractor state) into another, during which the system displays transient behaviour