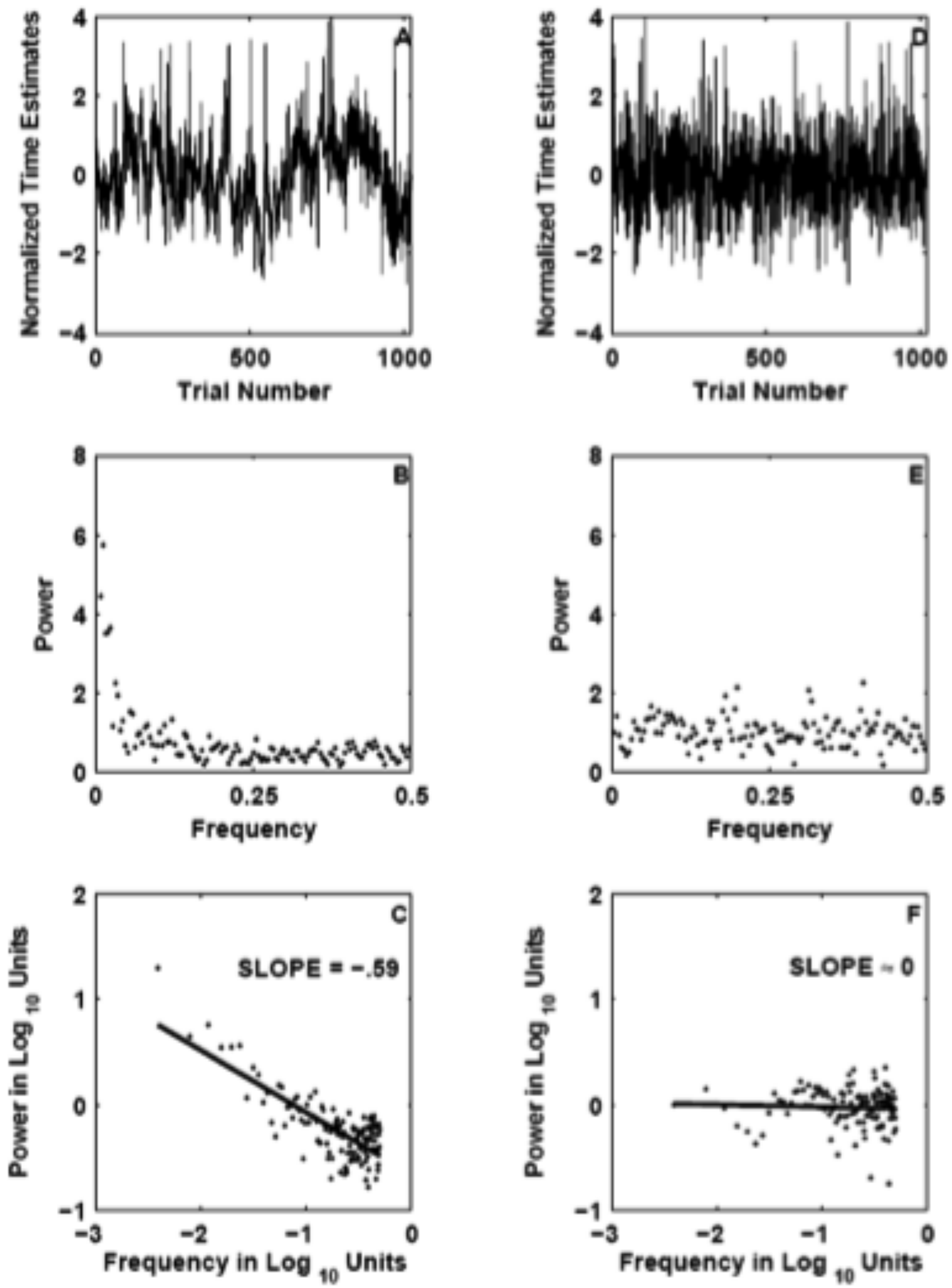
Important: The scaling relation exists only In the time series as it was recorded.

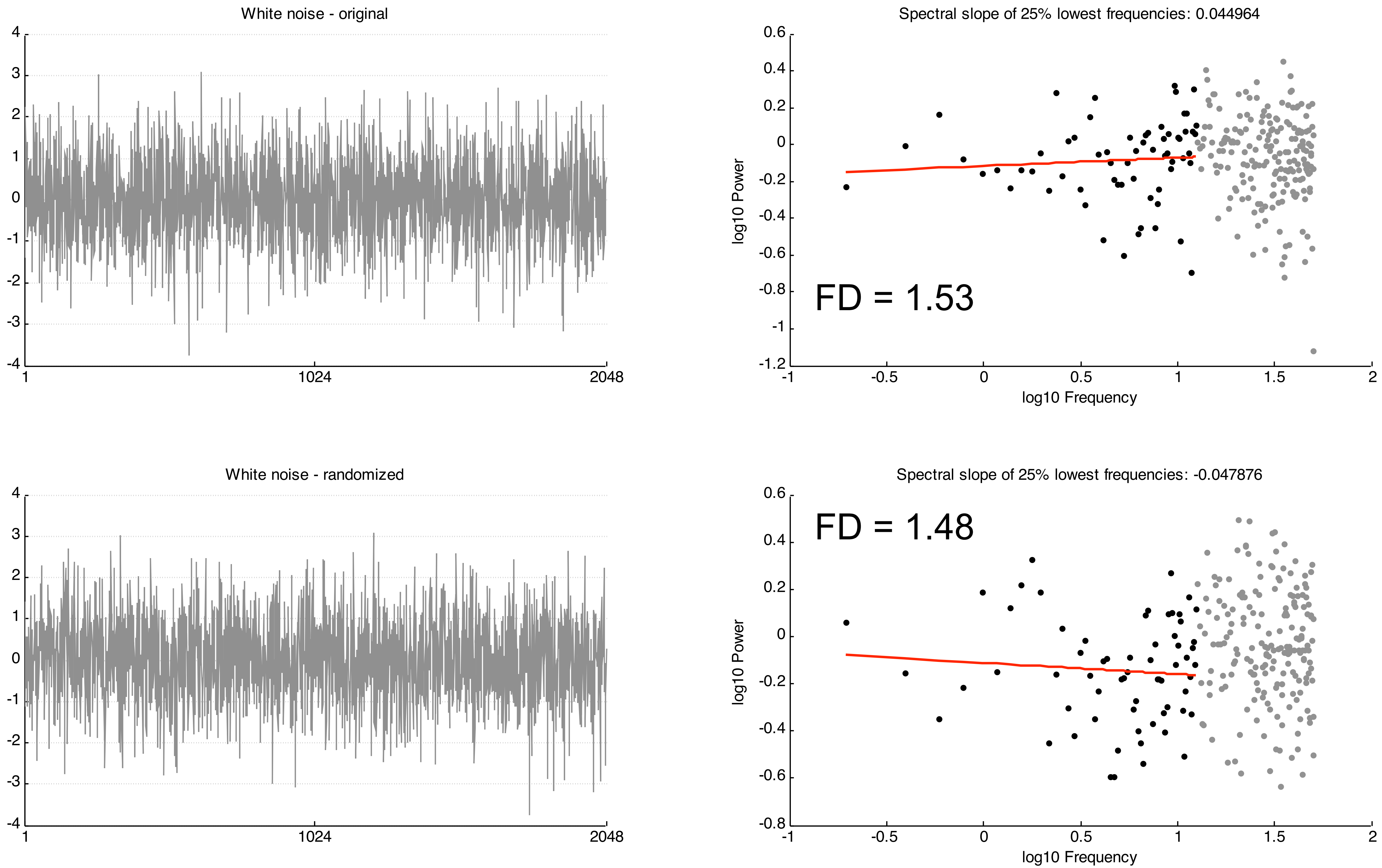
If you randomise the timeseries after it was recorded and loose the temporal structure (dynamics) white noise appears!

So even if you randomised your stimuli, the noise is in the temporal, dynamical structure of the time series.

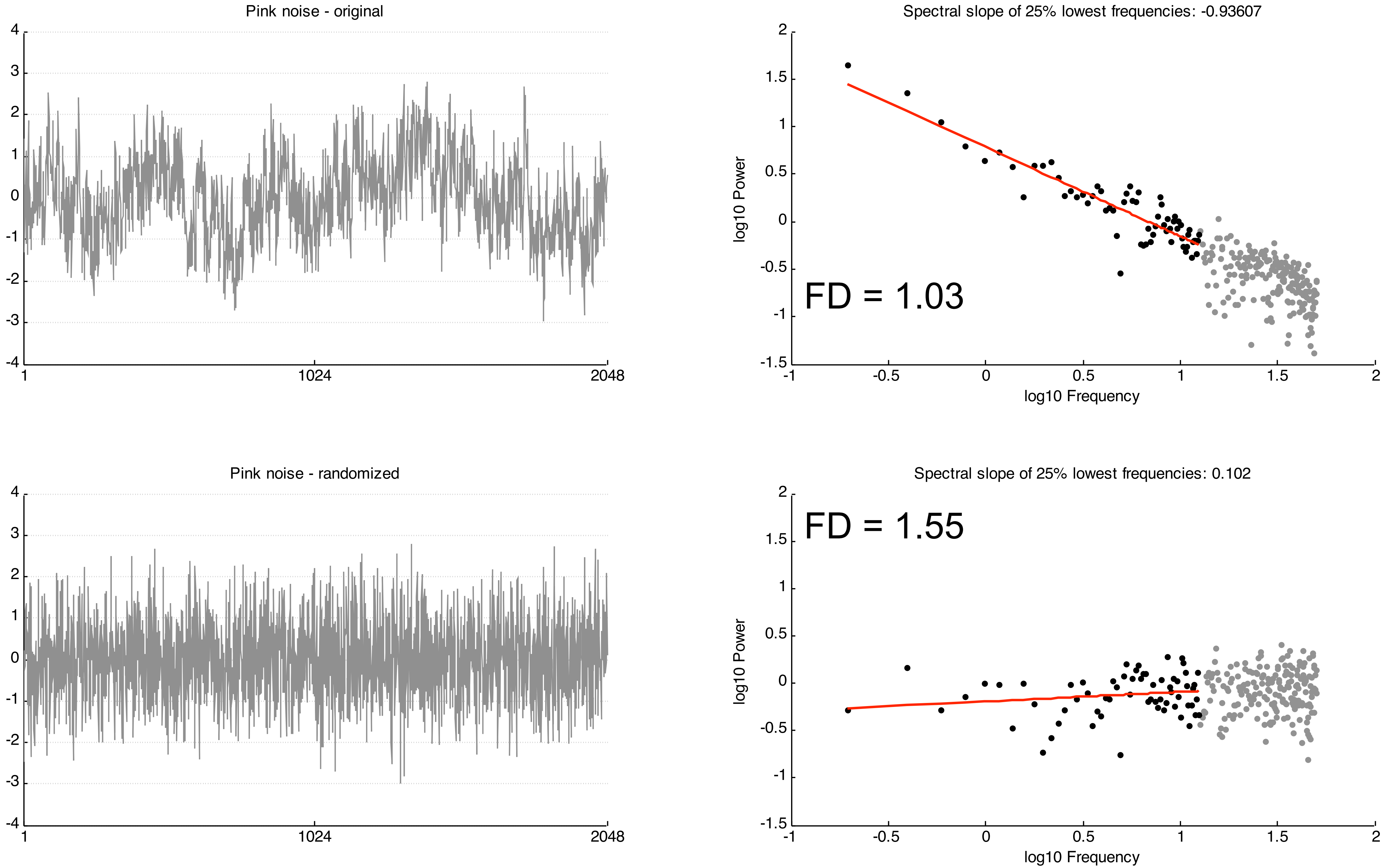
Test wether your fractal slope deviates from white noise by calculating a lot of random slopes.



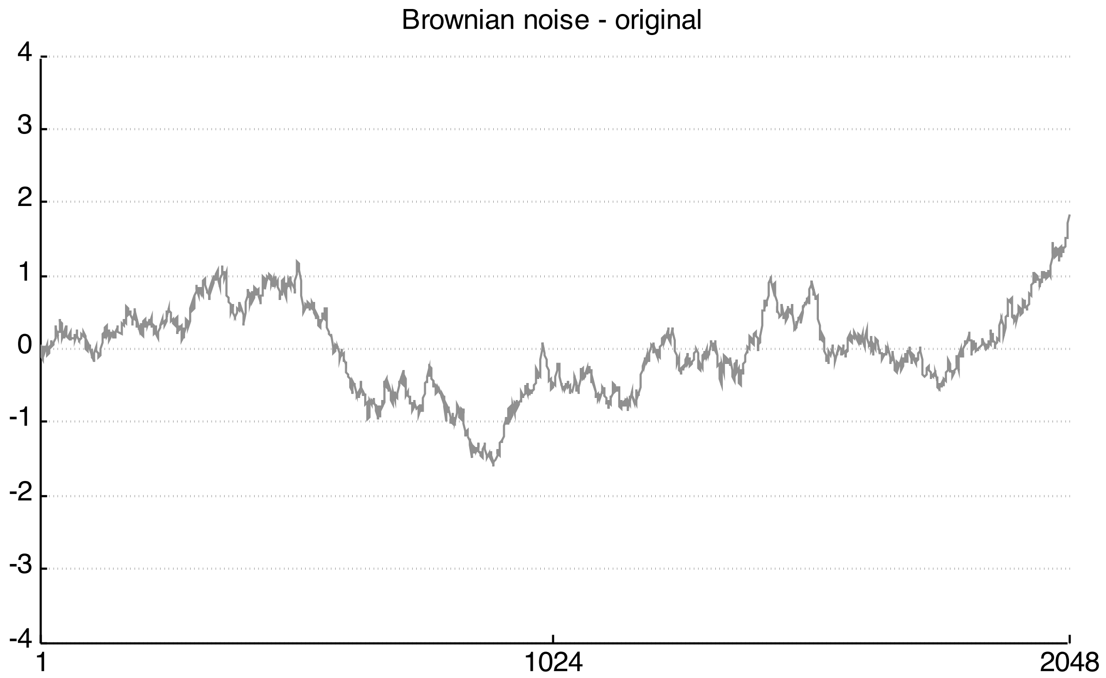
Randomization removes temporal structure

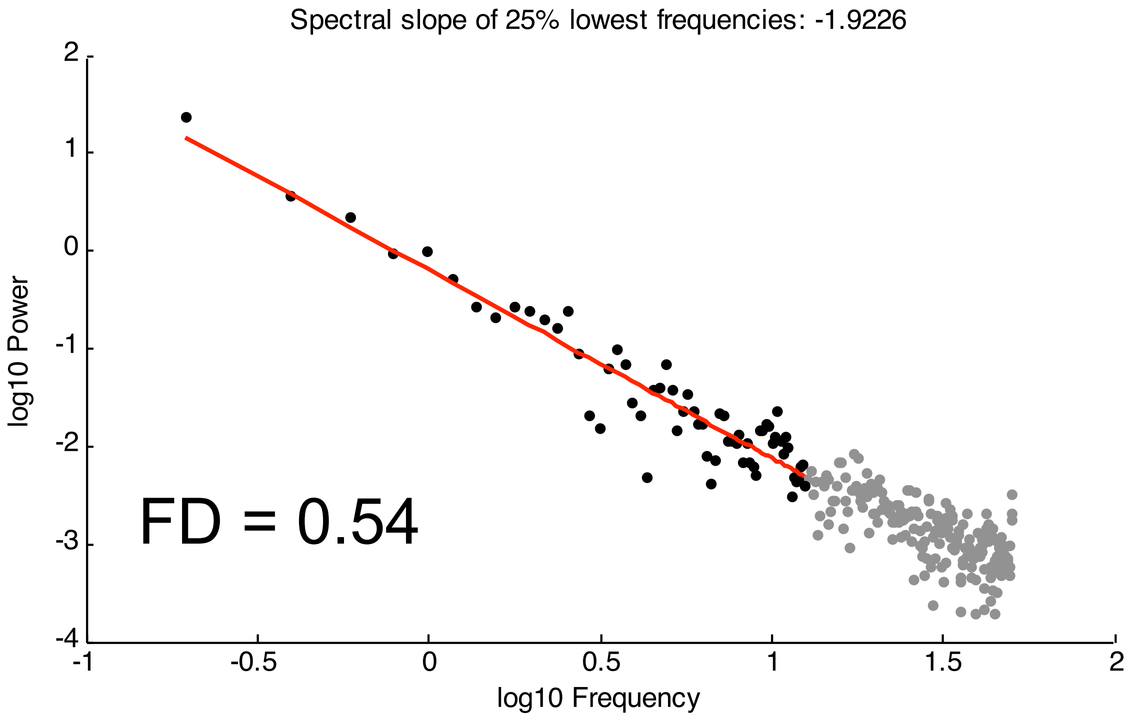


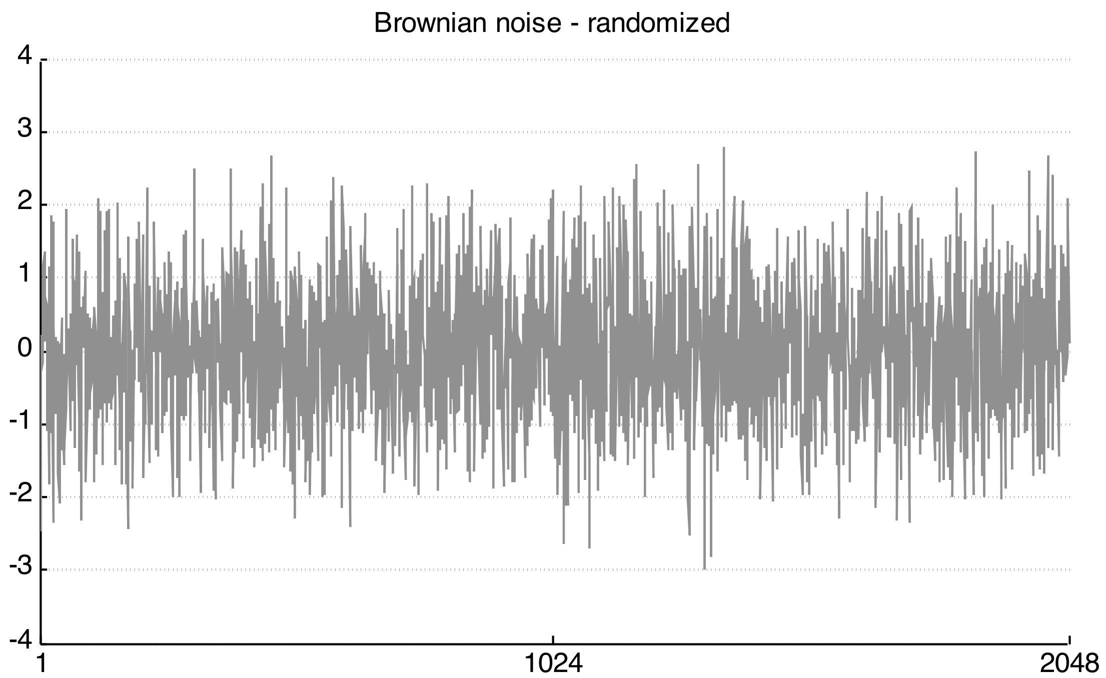
Randomization removes temporal structure

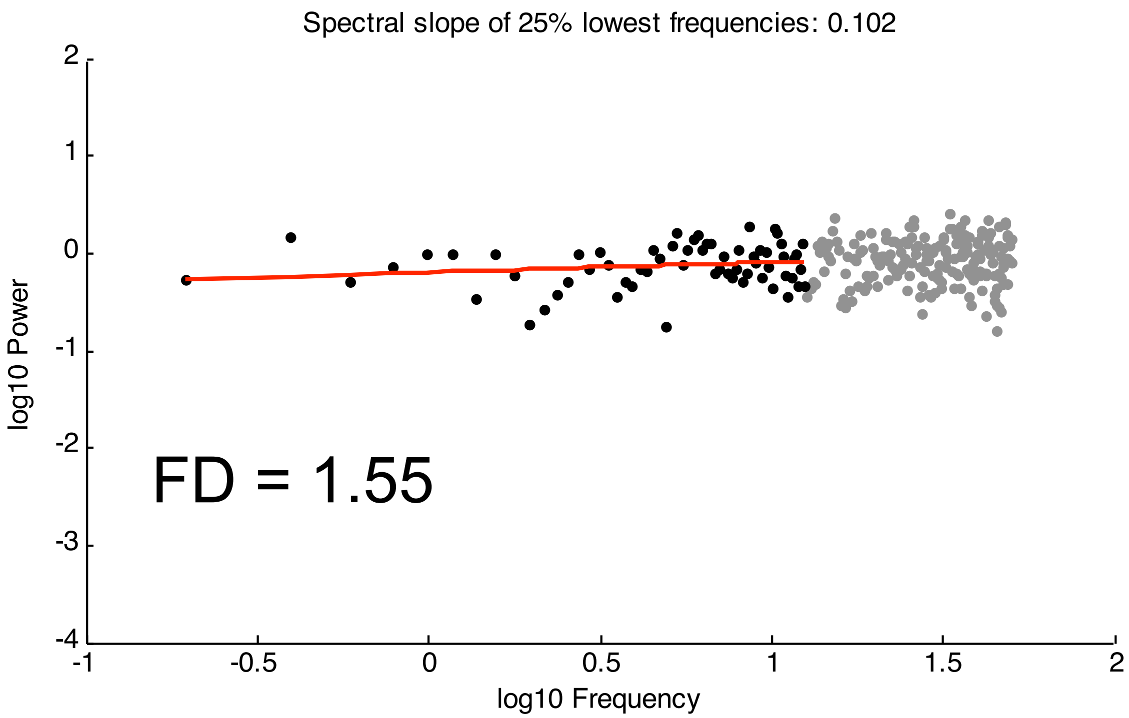


Randomization removes temporal structure









About doing the shuffle…

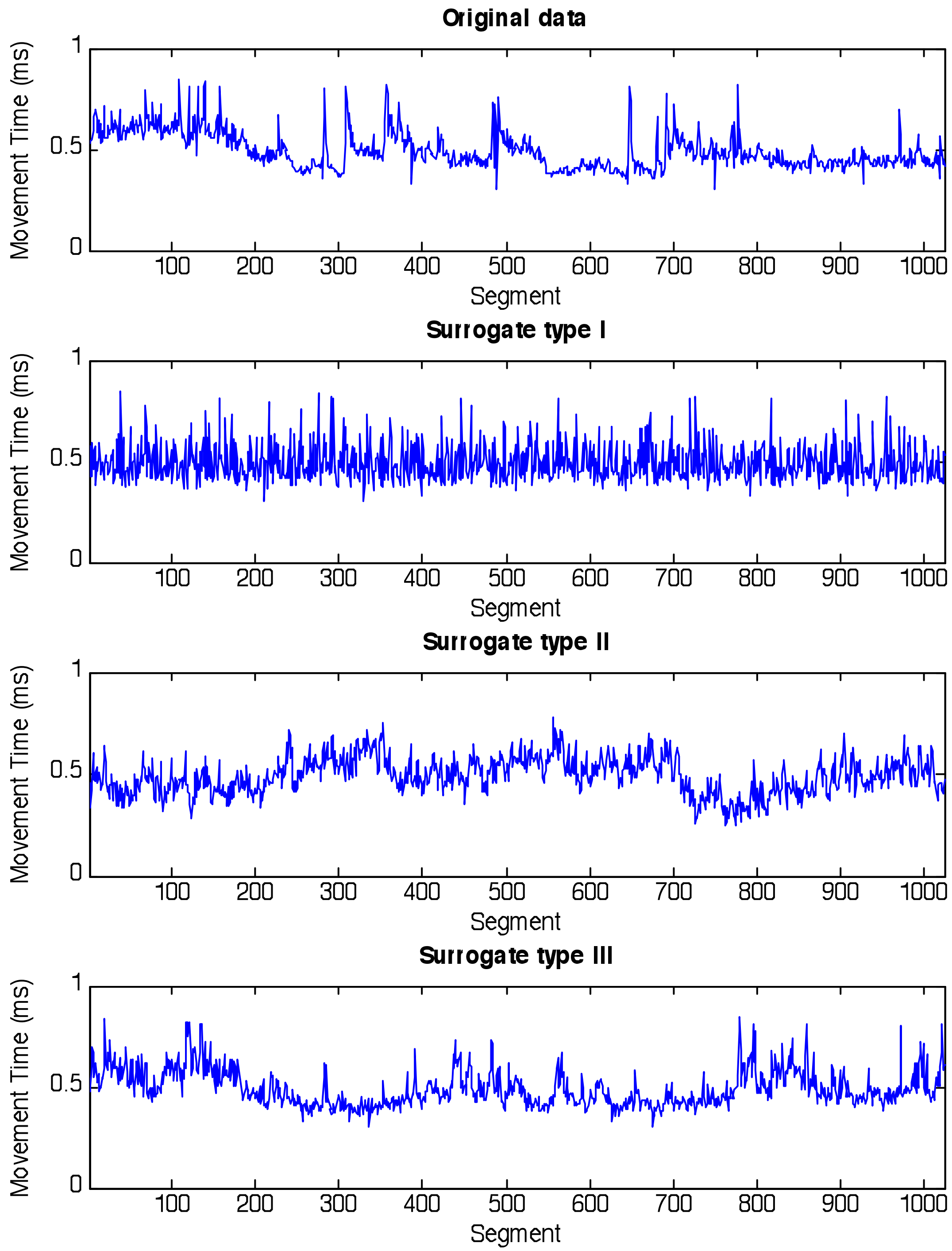
- “Post-hoc” randomisation suggests that the power laws and FD are not some coincidental temporal structure, but are actually in the signal as it was measured.
- Power laws do not disappear if you randomise your stimuli. As we saw last week, pink noise does tend to flatten towards a white noise slope if the stimulus presentation becomes unpredictable… but it doesn’t disappear in all the measures! (key-contact duration)
- Shuffling your data is not a “real” test of (the type) of long-range correlations, or the process underlying the correlations. It is a test of whether there are any correlations in the data at all!
- A solution is surrogate data analysis

Test a hypothesis by creating surrogate data and bootstrapping your favourite nonlinear measure

It is possible to create more sophisticated hypotheses by generating surrogate data based on the spectrum / distribution of the observed data.

H_0: Signal is generated by	**Surrogate type**	**Method**
Independent random numbers	I. Random temporal structure with same distribution as observed data	Randomise samples without replacement
Gaussian linear stochastic process	II. Random spectral phases	Multiply Fourier transform of the sample by random phases and convert back to time domain
Rescaled Gaussian linear stochastic process	III. Rescaled to a Gaussian distribution, phase randomised and rescaled to the empirical distribution	- Amplitude Adjusted Fourier Transform (AAFT) (biased!) - Itratively Refined Surrogates
Correlated linear stochastic process with time dependent mean and variance	IV. Preserve correlation structure and nonstationarity of mean and variance	General constrained randomisation by a cost function minimised by simulated annealing
Same dynamical system at different initial conditions	V. Exact copy of phase space, different initial conditions	Twin surrogates: Generate from recurrence matrix

Data displaying 1/f spectral slope of -1



Observed

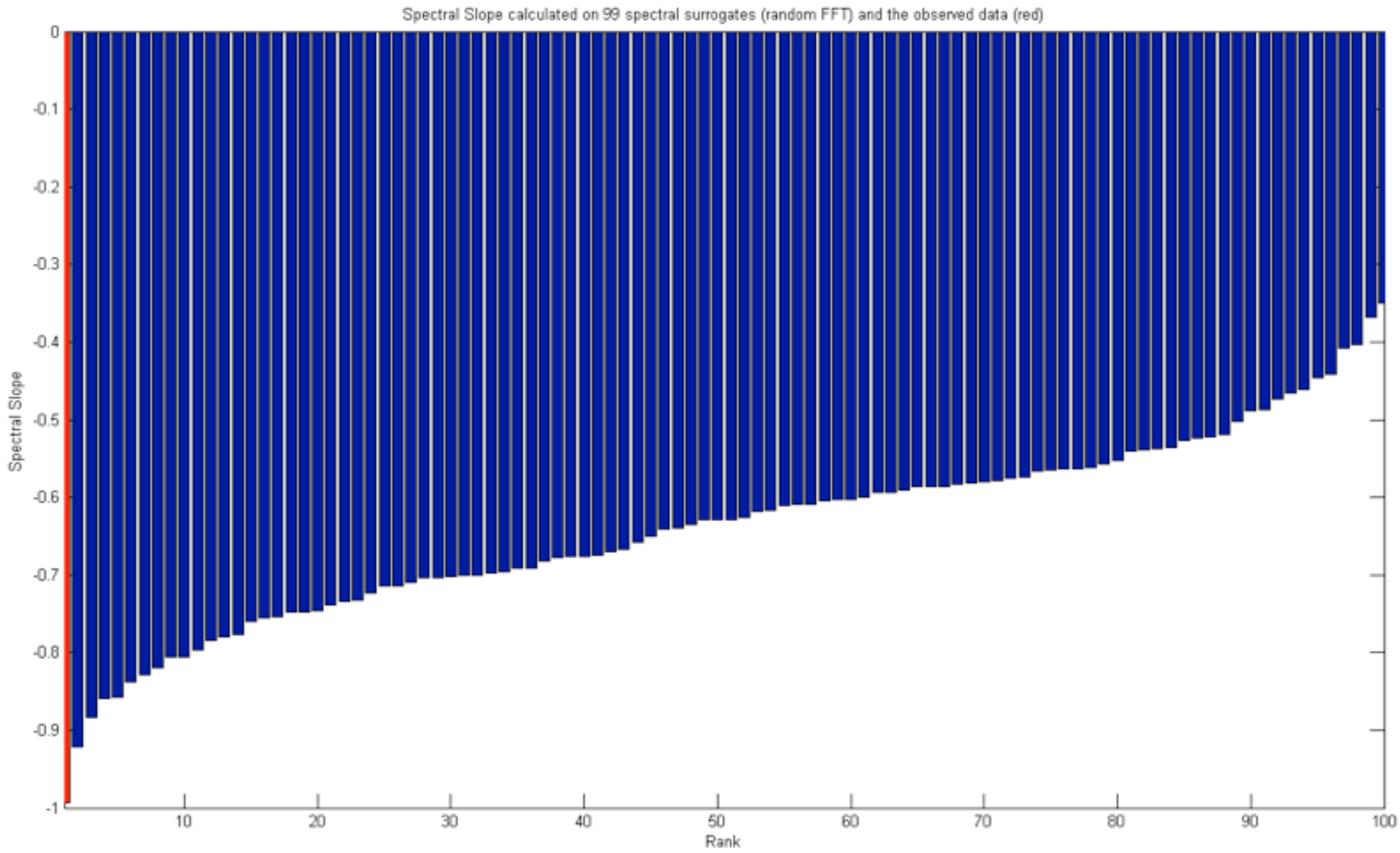
Random shuffle

Phase randomization of FFT

Amplitude Adjusted FFT

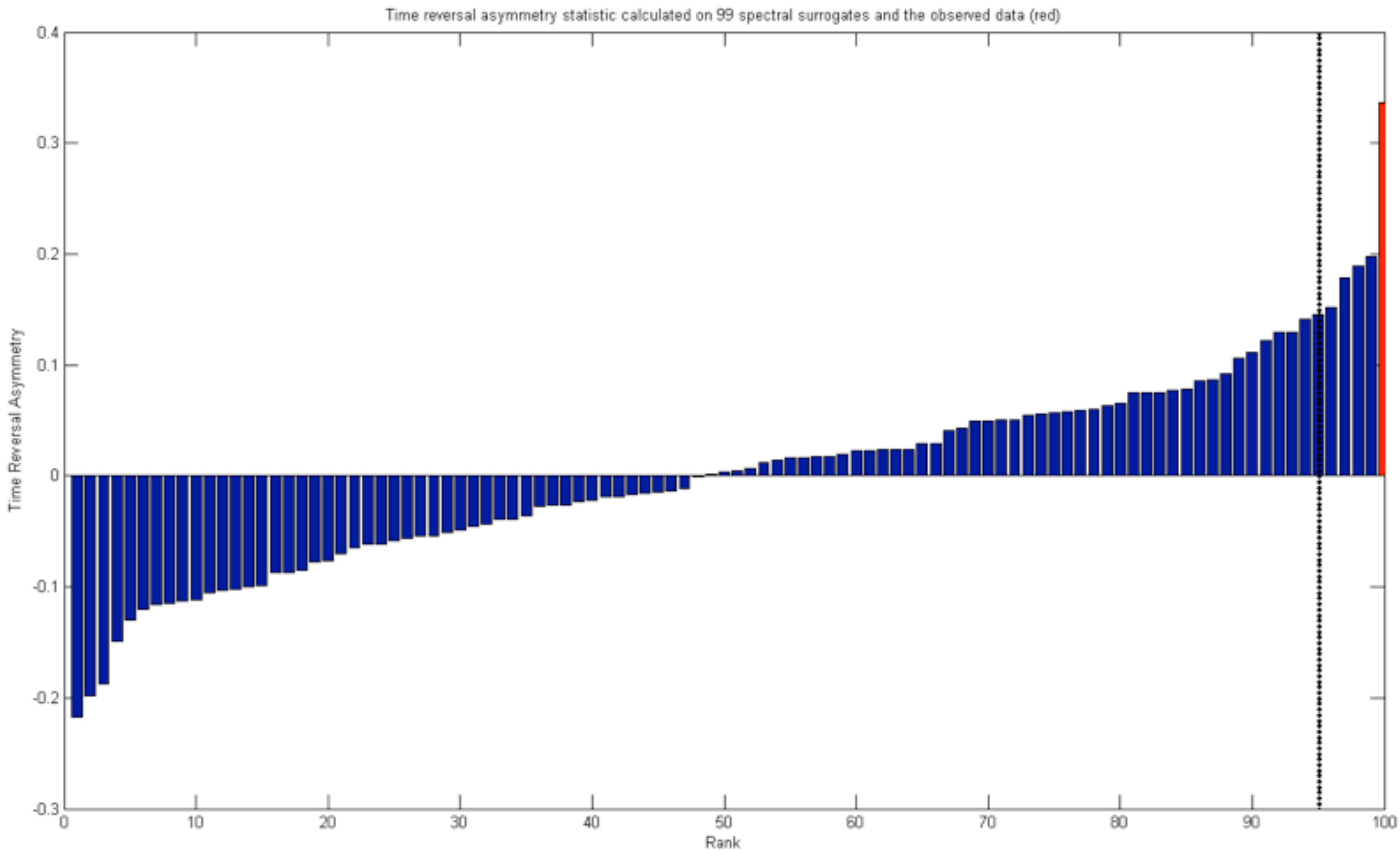
H0: Data originates from a linear stochastic Gaussian process

Reject with with p < .01 for Spectral Slope



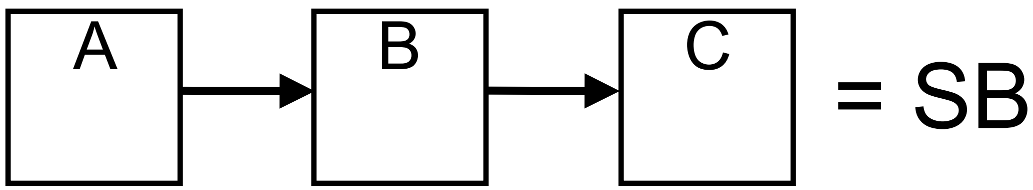
H0: Data originates from a linear stochastic Gaussian process

Reject with with p < .01 for Time reversal asymmetry

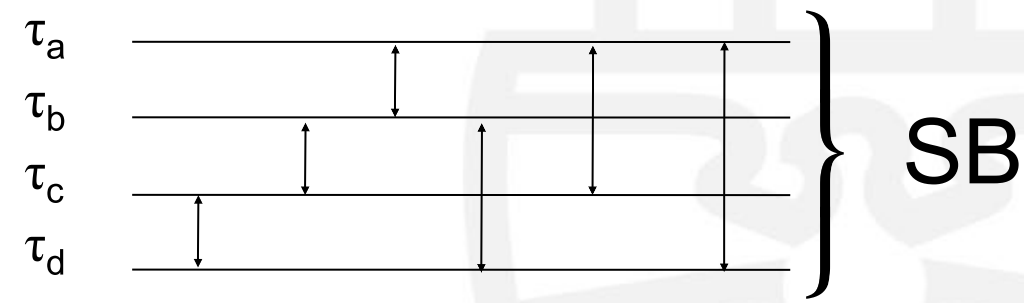


Self-organization with and without criticality

Component dominant dynamics



Interaction dominant dynamics



Additive interactions:
Independent components

Random processes
Brownian motion

One characteristic scale
one scale and location parameter

Likely: Stationary of the central moments

Multiplicative interactions:
Interdependence among components

Self-organizing processes
Multi-scale dynamics

Scale invariance
the scale parameter is not a constant
nor is the location parameter

Likely: Fluctuations of the central moments

Additive versus Multiplicative Interactions

Review

Trends in Cognitive Sciences Vol.xxx No.x

Box 1. Additive versus multiplicative effects

When measurements are independent and measured values are essentially sums of independent effects, the central limit theorem leads one to expect a normal distribution of values (i.e. a Gaussian probability function; Figure I blue). Illustrative examples are distributions of organism size in a population, such as height or weight, and distributions of scores on various tests of cognitive ability, such as the IQ test. Each observation of size or IQ is independent of other observations, and although factors affecting these measures are myriad and poorly understood, they are assumed to make largely independent and additive contributions to each individual's size or IQ.

Normal distributions are not expected when measured values reflect multiplicative combinations of effects. An illustrative example is distributions of city population sizes. Cities appear to grow multiplicatively (i.e. bigger cities are more likely to have larger growth rates than smaller cities [82]). The consequence is that city populations appear to be power-law distributed over a wide range of sizes [10,80]. Multiplicative effects can also lead to lognormal distributions (Figure I red), and simple multiplicative models have been shown to generate either lognormal or power-law distributions depending on small parametric changes [79]. Lognormal and power-law distributions are both heavy-tailed, and hence heavy-tailed distributions are often interpreted as evidence for multiplicative processes. An important difference between heavy-tailed and normal distributions is that moments of the former (e.g. mean and variance) poorly characterize the distribution (in fact, they are undefined for certain power-law distributions).

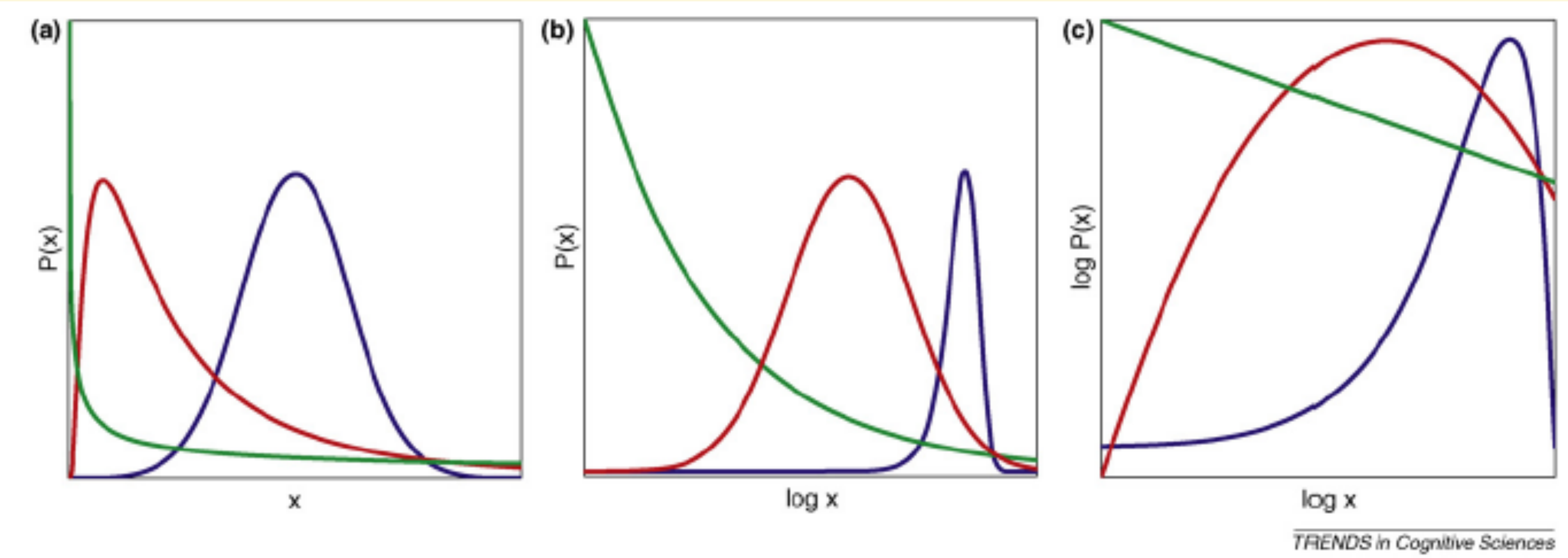


Figure I. Idealized normal (blue), lognormal (red), and power law (green) probability functions are plotted in raw (left), semi-log (middle) and log–log (right) coordinates.

Kello, C. T., Brown, G. D. A., Ferrer-i-Cancho, R., Holden, J. G., Linkenkaer-Hansen, K., Rhodes, T. (2010). Scaling laws in cognitive sciences. *Trends in Cognitive Sciences*, *14*(5), 223-232.

Additive versus Multiplicative Interactions

The difference between symmetric and skewed distributions is that the variables in play are either additive (i.e., single causes) or multiplicative (i.e., multiple causation), leading to *normal* or *log-normal* distributions, respectively.

The basic principles of additive and multiplicative effects can be demonstrated with two dice. Adding the two numbers on each throw leads to a symmetrical distribution on a longer run. Multiplying the two numbers, however, leads to a highly skewed distribution.

In the latter case, the symmetry has moved to the multiplicative level, which equals addition in the logarithmic domain (*log-normal*). Although these examples are imperfect, they do indicate that additive and multiplicative effects give rise to different distributions.

$$x1 * x2 = \log(x1) + \log(x2)$$