Complexity Methods For Behavioural Science

Graph theory
Complex Networks
Symptom Networks

Networks of (Networks of) Complex Systems

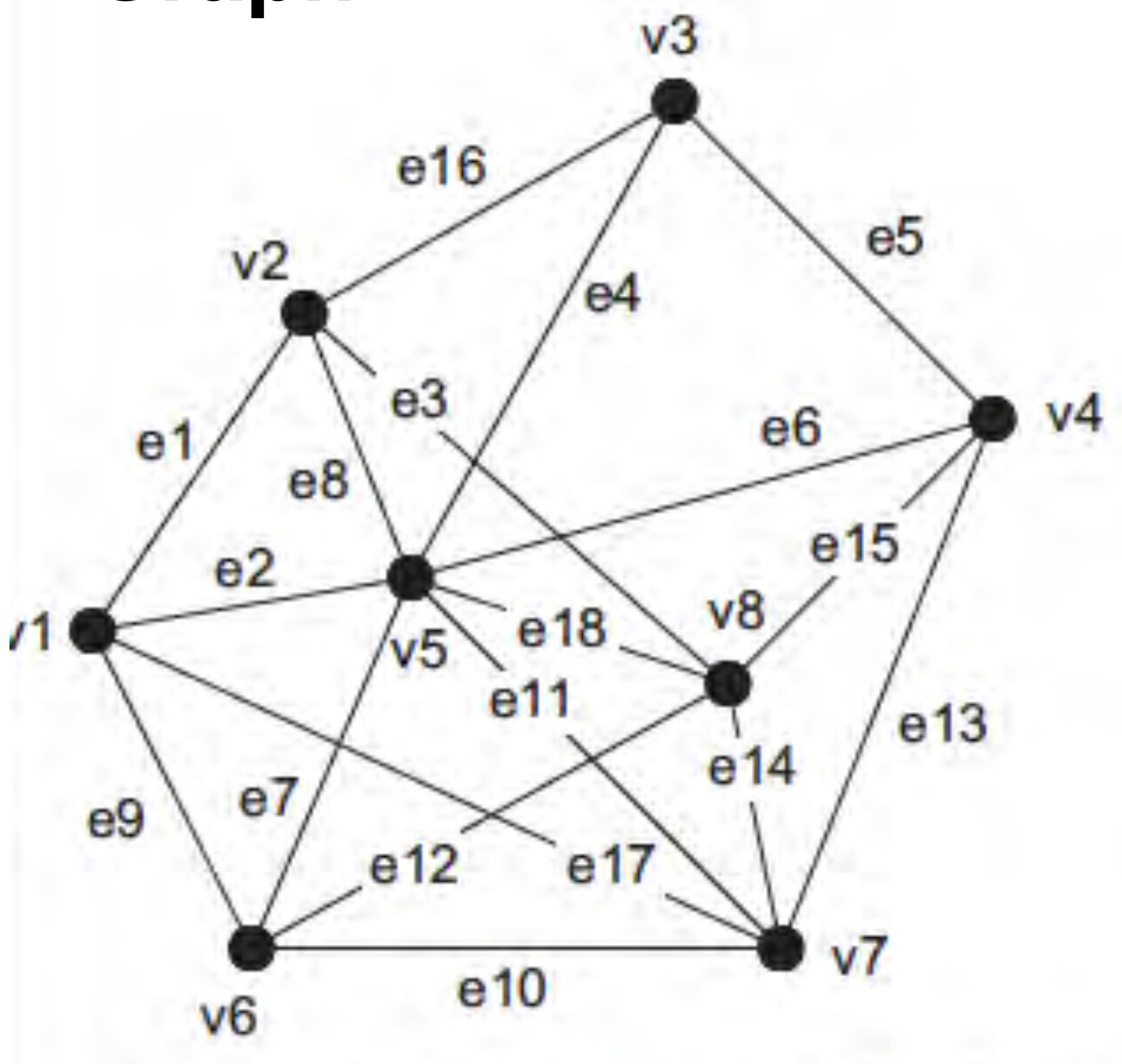
composition = topology

- Nodes (point, **vertex**)
- Connections between nodes (line, side, branch, arrow, **edge**)

Graph theory

- **Graph Theory:** Compositions of *edges* and *vertices*
- **Complexe network:** Many vertices and edges
- **Statistical netwerk models**

Graph



Formal

$$V(G) = \{v_1, \ldots, v_8\}$$
$$E(G) = \{e_1, \ldots, e_{18}\}$$

$$\begin{array}{ll}
e_1 = \langle v_1, v_2 \rangle & e_{10} = \langle v_6, v_7 \rangle \\
e_2 = \langle v_1, v_5 \rangle & e_{11} = \langle v_5, v_7 \rangle \\
e_3 = \langle v_2, v_8 \rangle & e_{12} = \langle v_6, v_8 \rangle \\
e_4 = \langle v_3, v_5 \rangle & e_{13} = \langle v_4, v_7 \rangle \\
e_5 = \langle v_3, v_4 \rangle & e_{14} = \langle v_7, v_8 \rangle \\
e_6 = \langle v_4, v_5 \rangle & e_{15} = \langle v_4, v_8 \rangle \\
e_7 = \langle v_5, v_6 \rangle & e_{16} = \langle v_2, v_3 \rangle \\
e_8 = \langle v_2, v_5 \rangle & e_{17} = \langle v_1, v_7 \rangle \\
e_9 = \langle v_1, v_6 \rangle & e_{18} = \langle v_5, v_8 \rangle
\end{array}$$

Figure 2.1: An example of a graph with eight vertices and 18 edges.

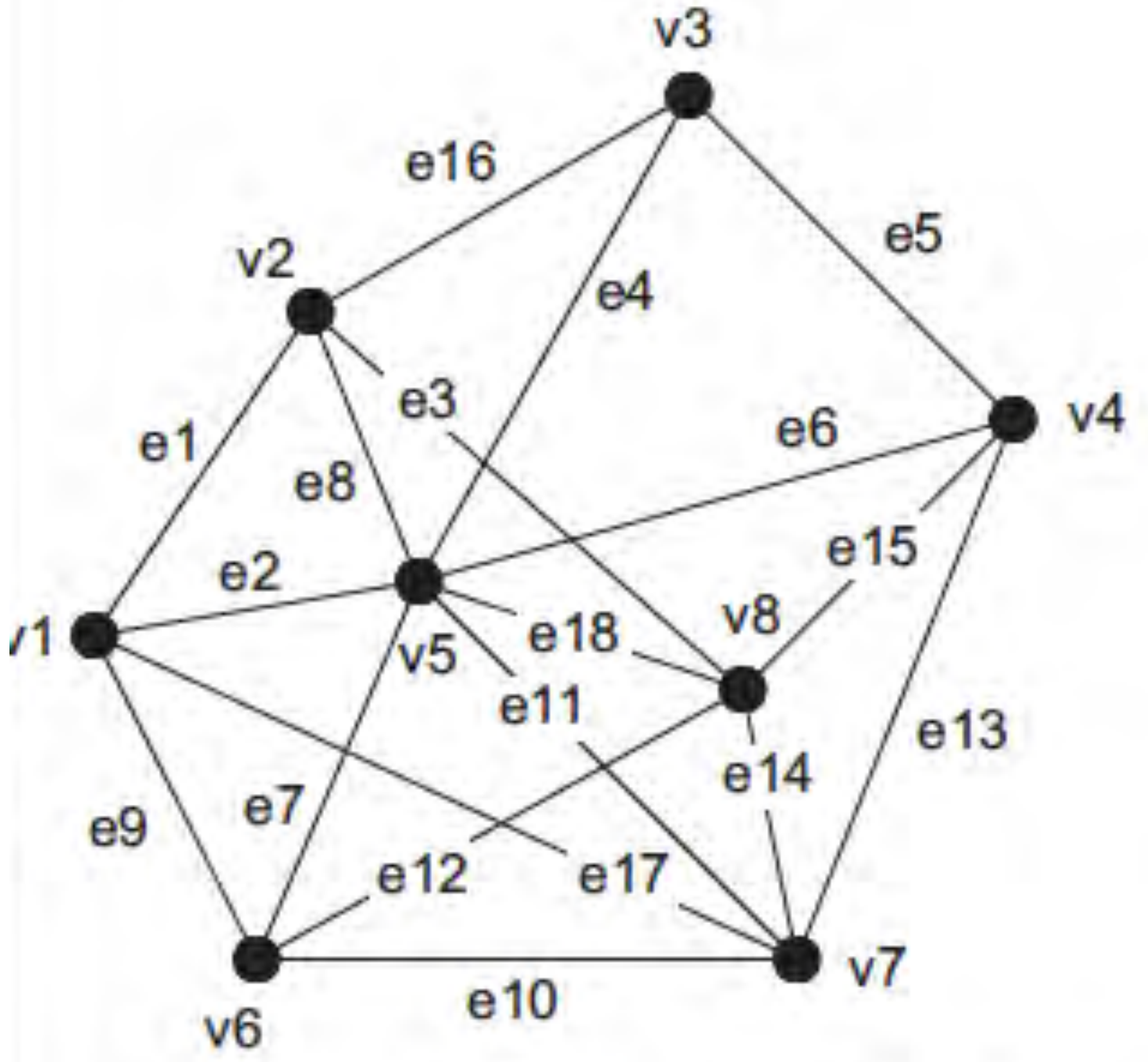
Adjacency matrix

	v1	v2	v3	v4	v5	v6	v7	v8
v1	0	1	0	0	1	1	1	0
v2	1	0	1	0	1	0	0	1
v3	0	1	0	1	1	0	0	0
v4	0	0	1	0	1	0	1	1
v5	1	1	1	1	0	1	1	1
v6	1	0	0	0	1	0	1	1
v7	1	0	0	1	1	1	0	1
v8	0	1	0	1	1	1	1	0

van Steen, M. (2010). Graph Theory and Complex Networks. An Introduction. Retrieved from: http://www.distributed-systems.net/

Graph theory

undirected graph

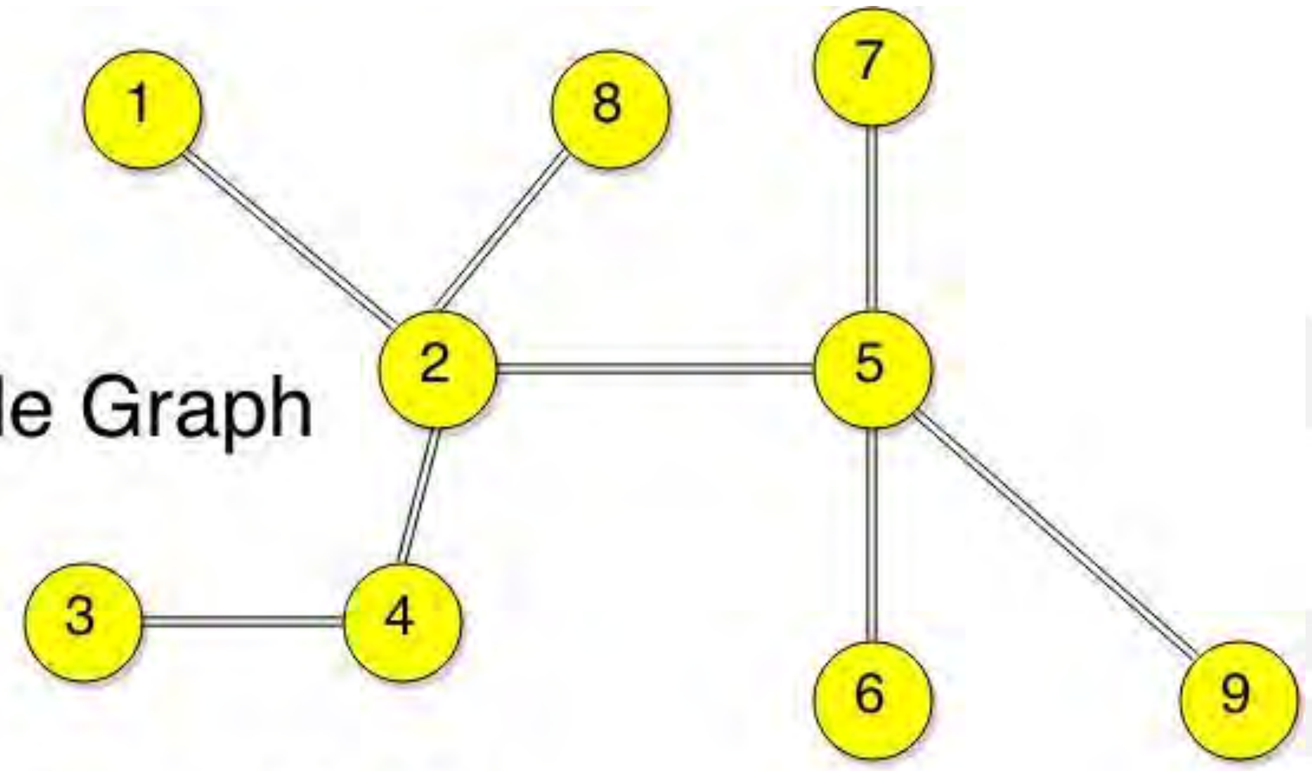


$$V(G) = \{v_1, \ldots, v_8\}$$
$$E(G) = \{e_1, \ldots, e_{18}\}$$

$$\begin{array}{ll}
e_1 = \langle v_1, v_2 \rangle & e_{10} = \langle v_6, v_7 \rangle \\
e_2 = \langle v_1, v_5 \rangle & e_{11} = \langle v_5, v_7 \rangle \\
e_3 = \langle v_2, v_8 \rangle & e_{12} = \langle v_6, v_8 \rangle \\
e_4 = \langle v_3, v_5 \rangle & e_{13} = \langle v_4, v_7 \rangle \\
e_5 = \langle v_3, v_4 \rangle & e_{14} = \langle v_7, v_8 \rangle \\
e_6 = \langle v_4, v_5 \rangle & e_{15} = \langle v_4, v_8 \rangle \\
e_7 = \langle v_5, v_6 \rangle & e_{16} = \langle v_2, v_3 \rangle \\
e_8 = \langle v_2, v_5 \rangle & e_{17} = \langle v_1, v_7 \rangle \\
e_9 = \langle v_1, v_6 \rangle & e_{18} = \langle v_5, v_8 \rangle
\end{array}$$

Figure 2.1: An example of a graph with eight vertices and 18 edges.

Simple Graph



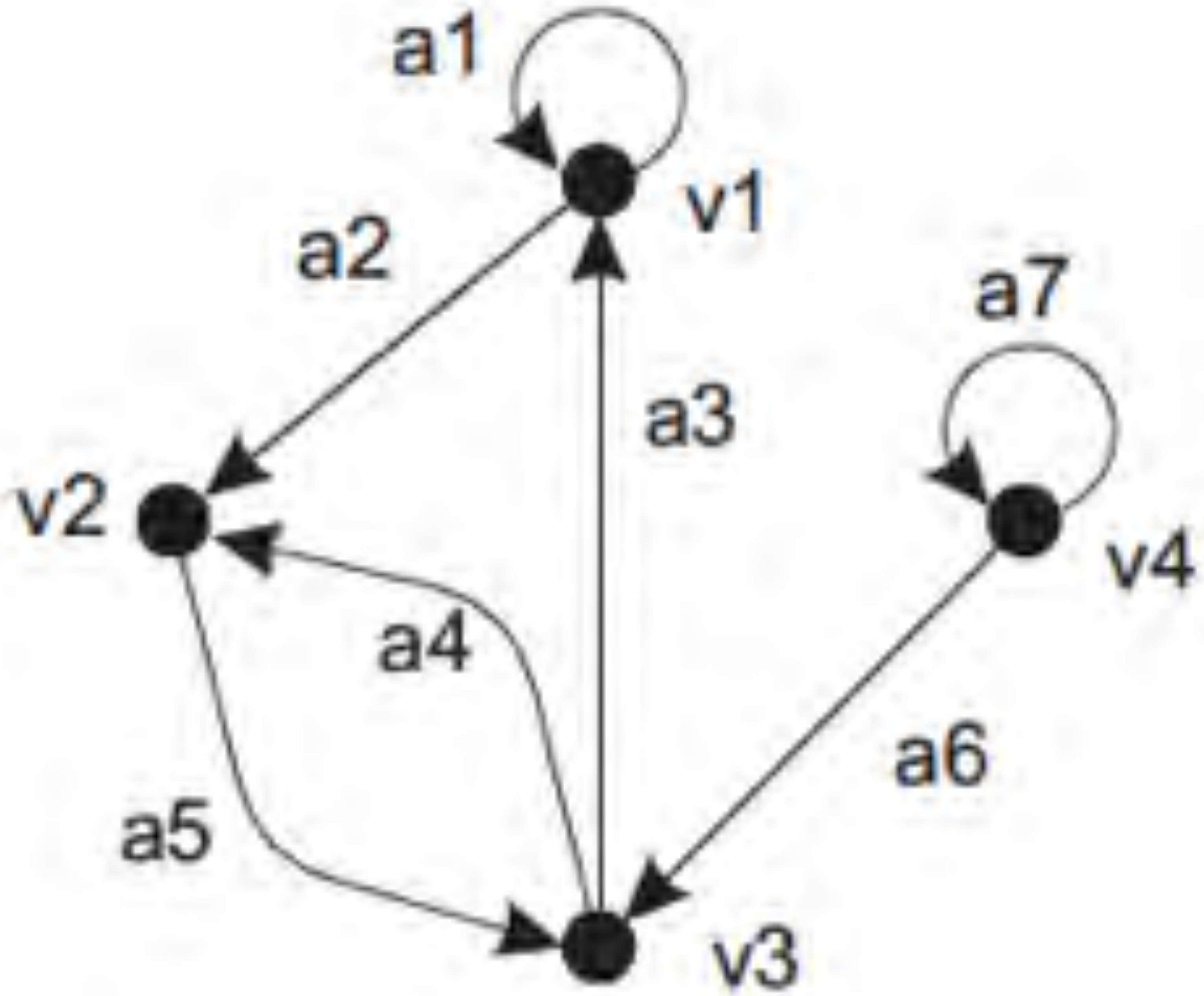
Adjacency Matrix

	Vertex 1	Vertex 2	Vertex 3	Vertex 4	Vertex 5	Vertex 6	Vertex 7	Vertex 8	Vertex 9
Vertex 1	0	1	0	0	0	0	0	0	0
Vertex 2	1	0	0	1	1	0	0	1	0
Vertex 3	0	0	0	1	0	0	0	0	0
Vertex 4	0	1	1	0	0	0	0	0	0
Vertex 5	0	1	0	0	0	1	1	0	1
Vertex 6	0	0	0	0	1	0	0	0	0
Vertex 7	0	0	0	0	1	0	0	0	0
Vertex 8	0	1	0	0	0	0	0	0	0
Vertex 9	0	0	0	0	1	0	0	0	0

http://theoryofprogramming.com/tag/adjacency-matrix/

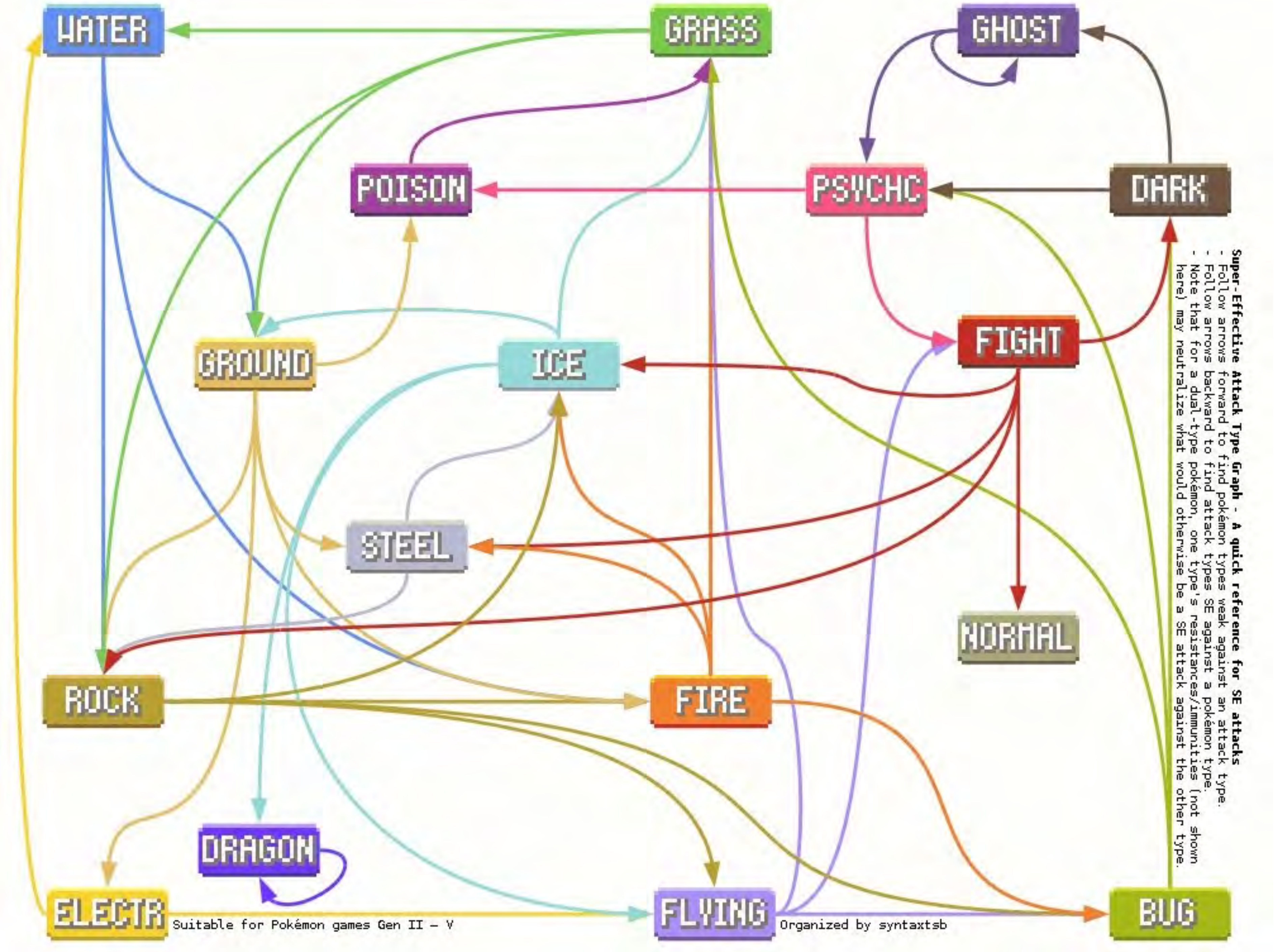
van Steen, M. (2010). Graph Theory and Complex Networks. An Introduction. Retrieved from: http://www.distributed-systems.net/

directed graph (digraph)



With *loops* en *arcs*

	v_1	v_2	v_3	v_4	OUT $\sum$
v_1	1	1	0	0	2
v_2	0	0	1	0	1
v_3	1	1	0	0	2
v_4	0	0	1	1	2
IN $\sum$	2	2	2	1	7



directed network

Effectivity of
Pokemon attacks
By species

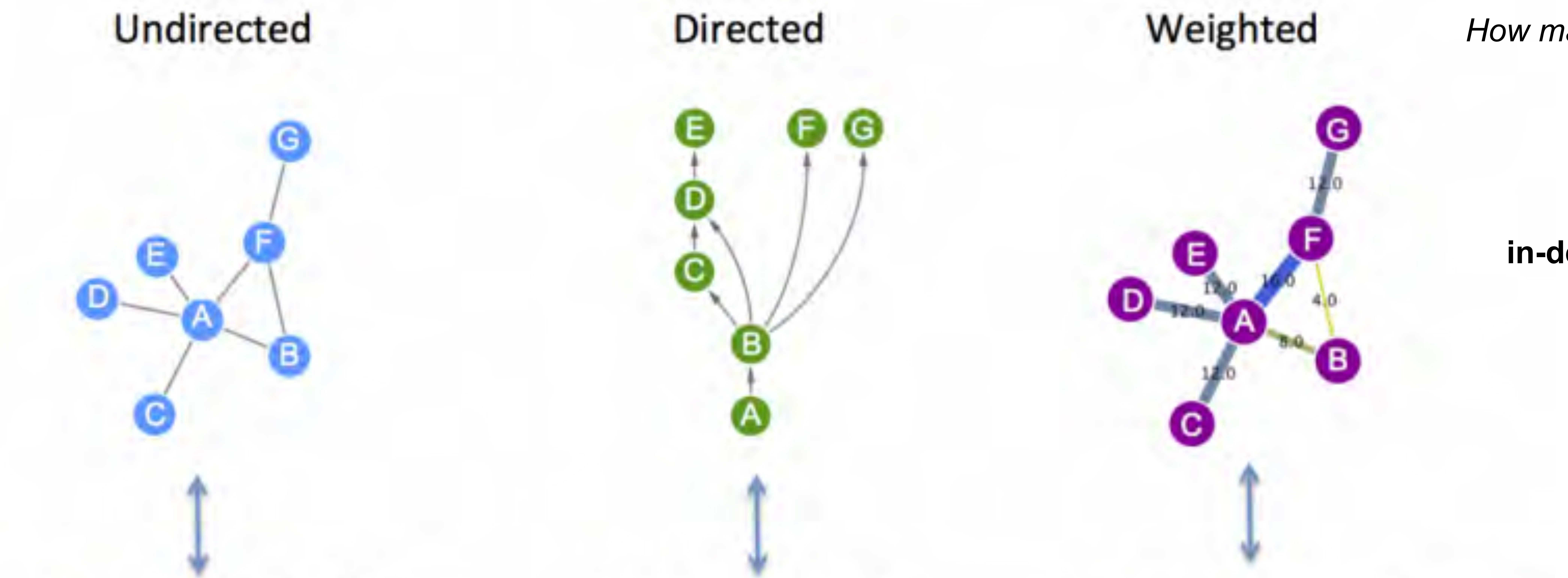
Graph theory

Degree:

How many edges connect to a vertex/node

directed graphs:

in-degree, out-degree



	A	B	C	D	E	F	G	Degree
A	0	1	1	1	1	1	0	5
B	1	0	0	0	0	1	0	2
C	1	0	0	0	0	0	0	1
D	1	0	0	0	0	0	0	1
E	1	0	0	0	0	0	0	1
F	1	1	0	0	0	0	1	3
G	0	0	0	0	0	1	0	1

	A	B	C	D	E	F	G	Out-degree
A	0	1	0	0	0	0	0	1
B	0	0	1	1	0	1	1	4
C	0	0	0	1	0	0	0	1
D	0	0	0	0	1	0	0	1
E	0	0	0	0	0	0	0	0
F	0	0	0	0	0	0	0	0
G	0	0	0	0	0	0	0	0

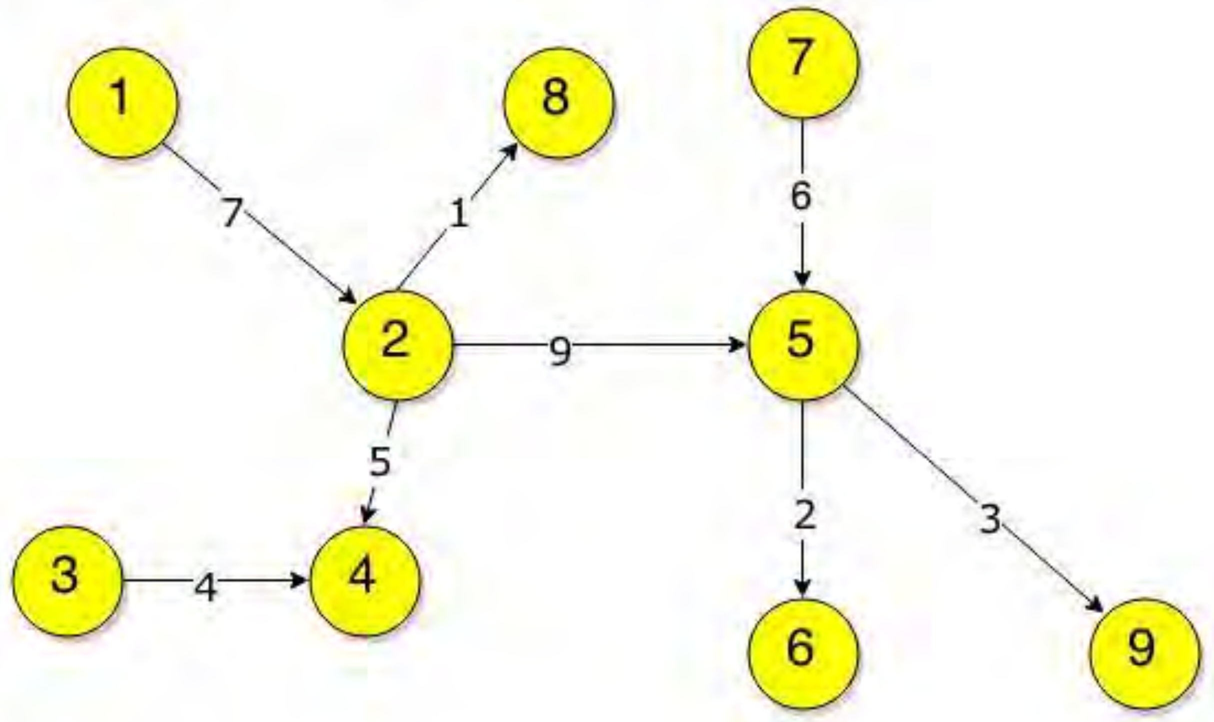
	A	B	C	D	E	F	G	Degree
A	0	8	12	12	12	16	12	72
B	8	0	0	0	0	4	0	12
C	12	0	0	0	0	0	0	12
D	12	0	0	0	0	0	0	12
E	12	0	0	0	0	0	0	12
F	16	4	0	0	0	0	12	32
G	12	0	0	0	0	12	0	24

Adjacency matrices

Graph theory

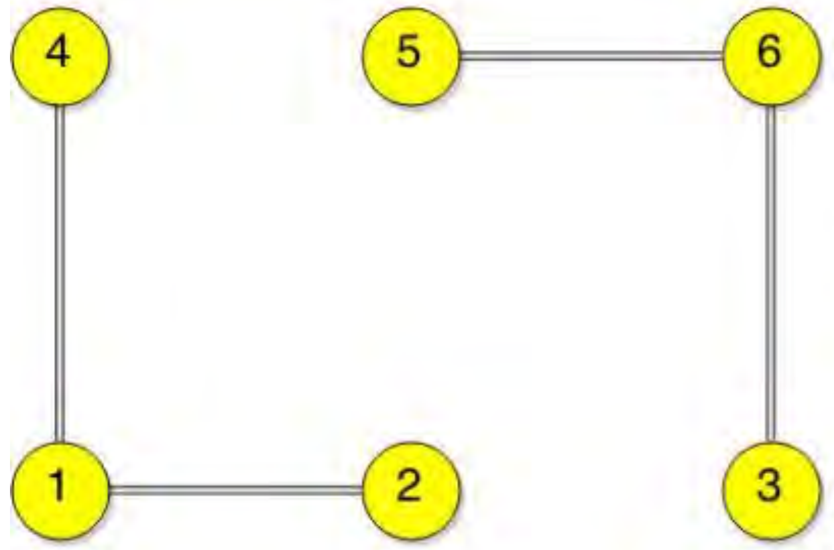
weighted directed graph

Weighted Directed Graph

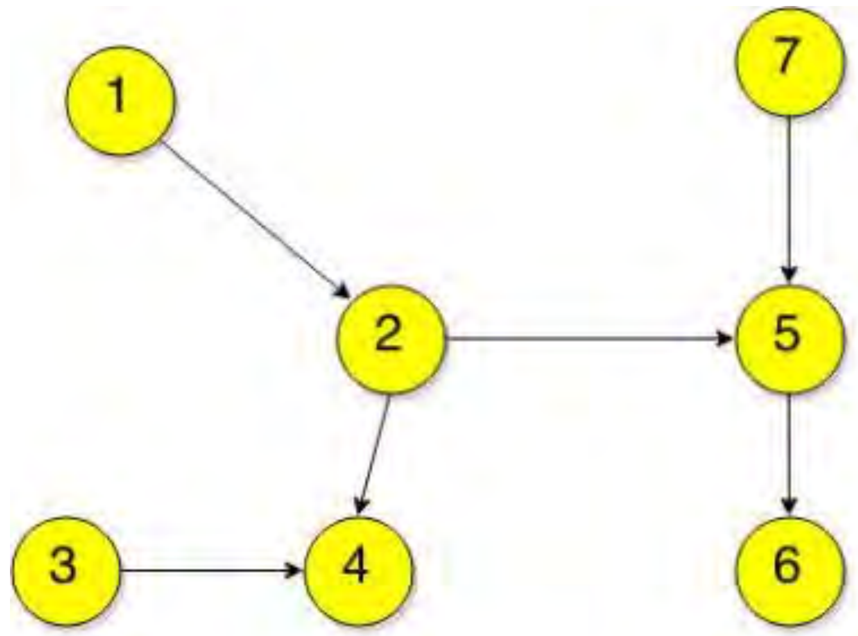


Adjacency Matrix

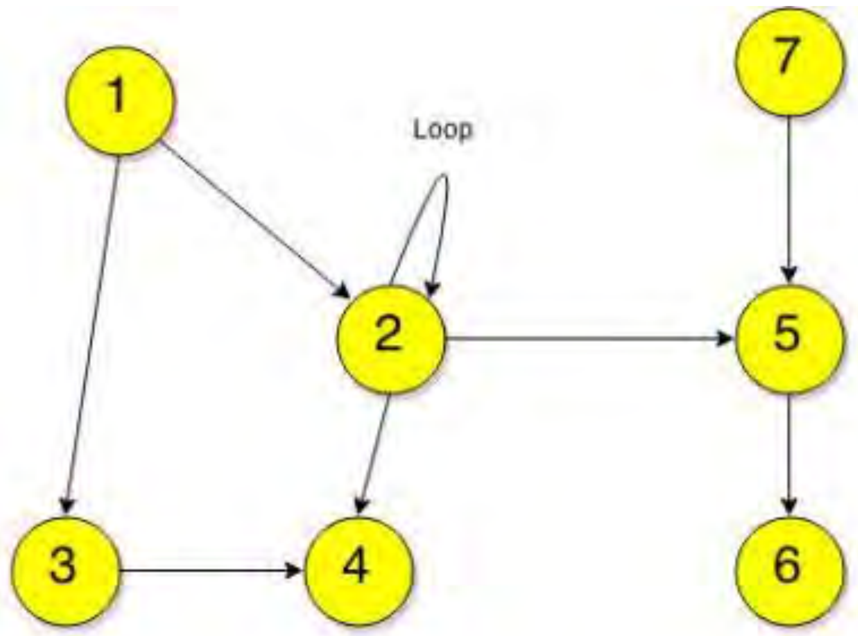
to	Vertex 1	Vertex 2	Vertex 3	Vertex 4	Vertex 5	Vertex 6	Vertex 7	Vertex 8	Vertex 9
Vertex 1	0	7	0	0	0	0	0	0	0
Vertex 2	0	0	0	5	9	0	0	1	0
Vertex 3	0	0	0	4	0	0	0	0	0
Vertex 4	0	0	0	0	0	0	0	0	0
Vertex 5	0	0	0	0	0	2	0	0	3
Vertex 6	0	0	0	0	0	0	0	0	0
Vertex 7	0	0	0	0	6	0	0	0	0
Vertex 8	0	0	0	0	0	0	0	0	0
Vertex 9	0	0	0	0	0	0	0	0	0



Disconnected Graph

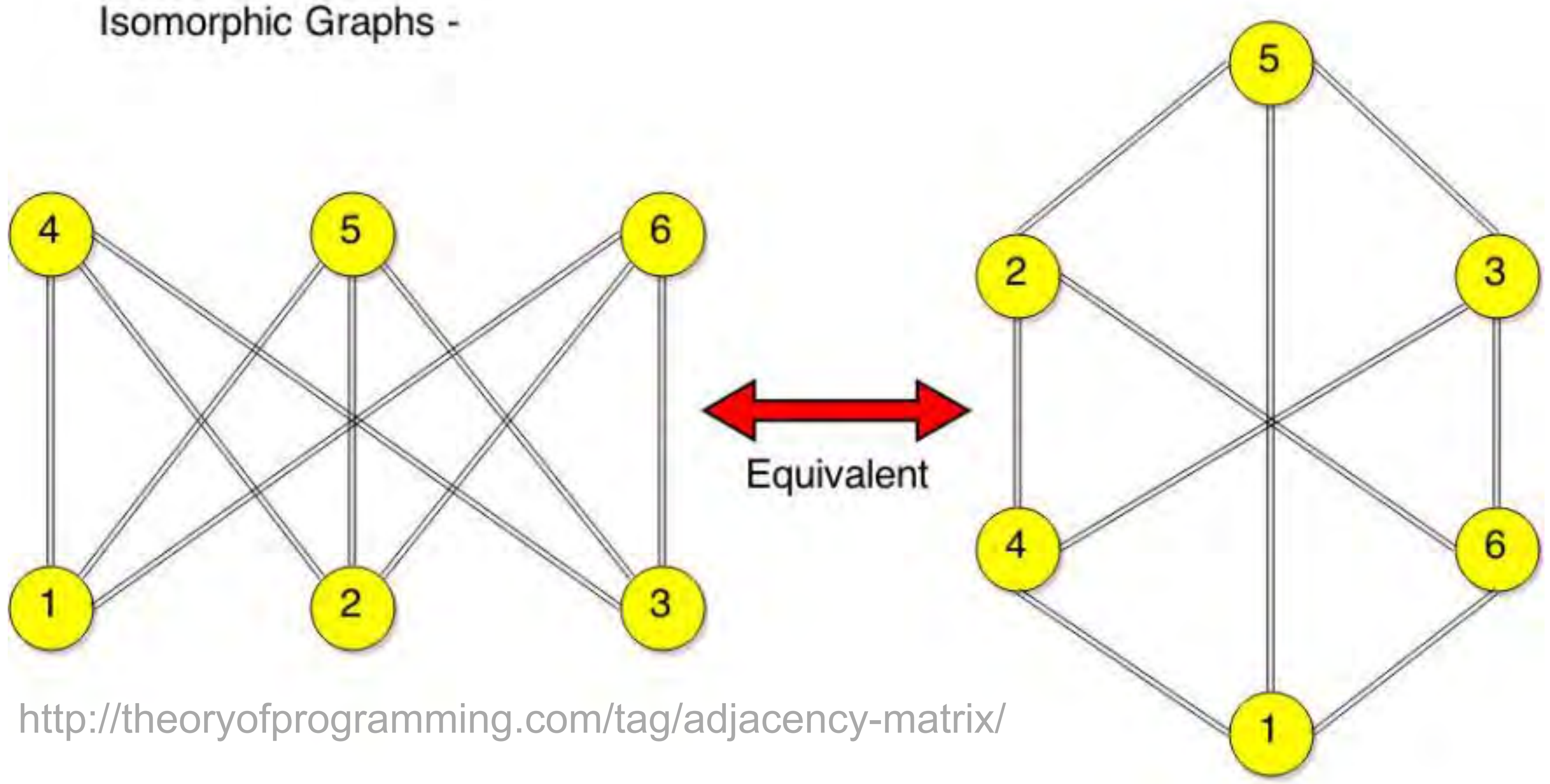


Simple Graph



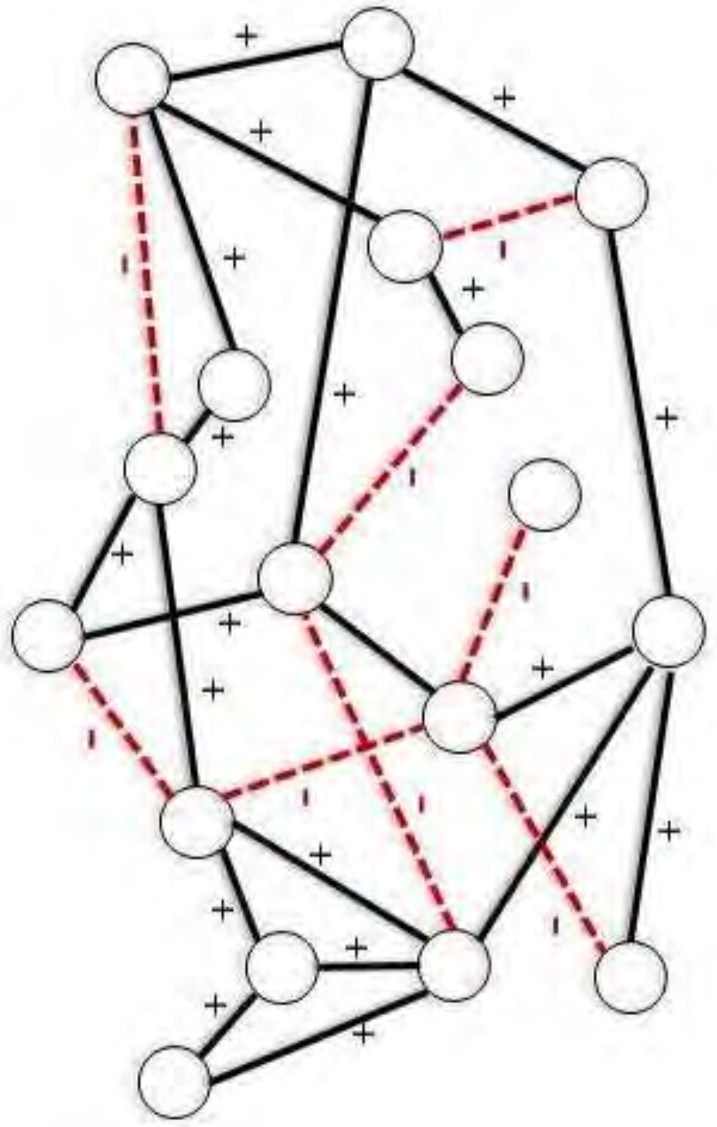
Non - Simple Graph

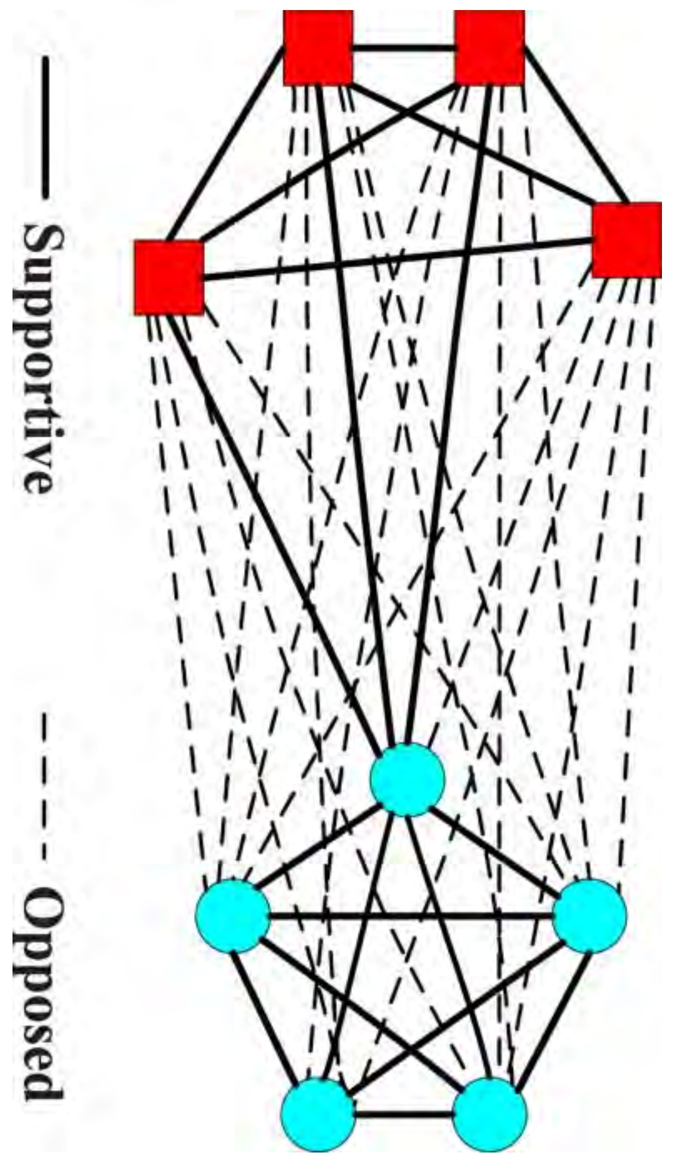
Isomorphic Graphs -



http://theoryofprogramming.com/tag/adjacency-matrix/

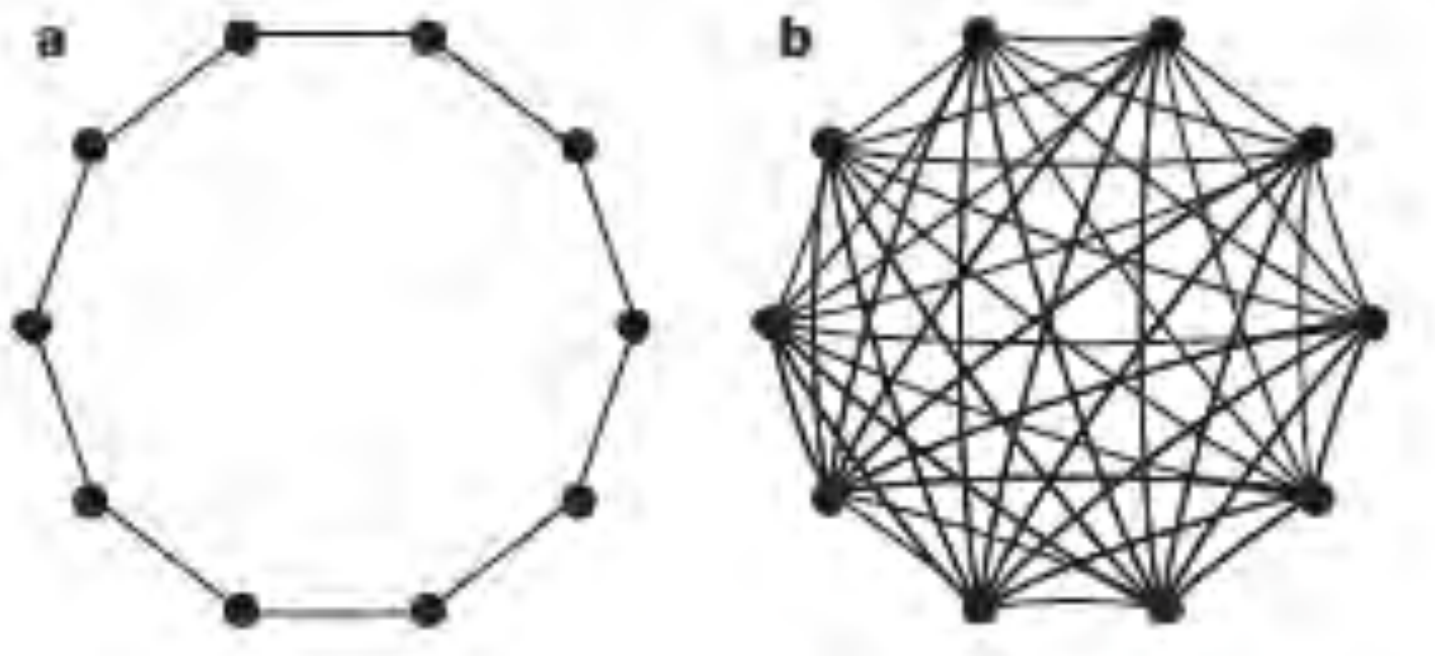
signed graphs

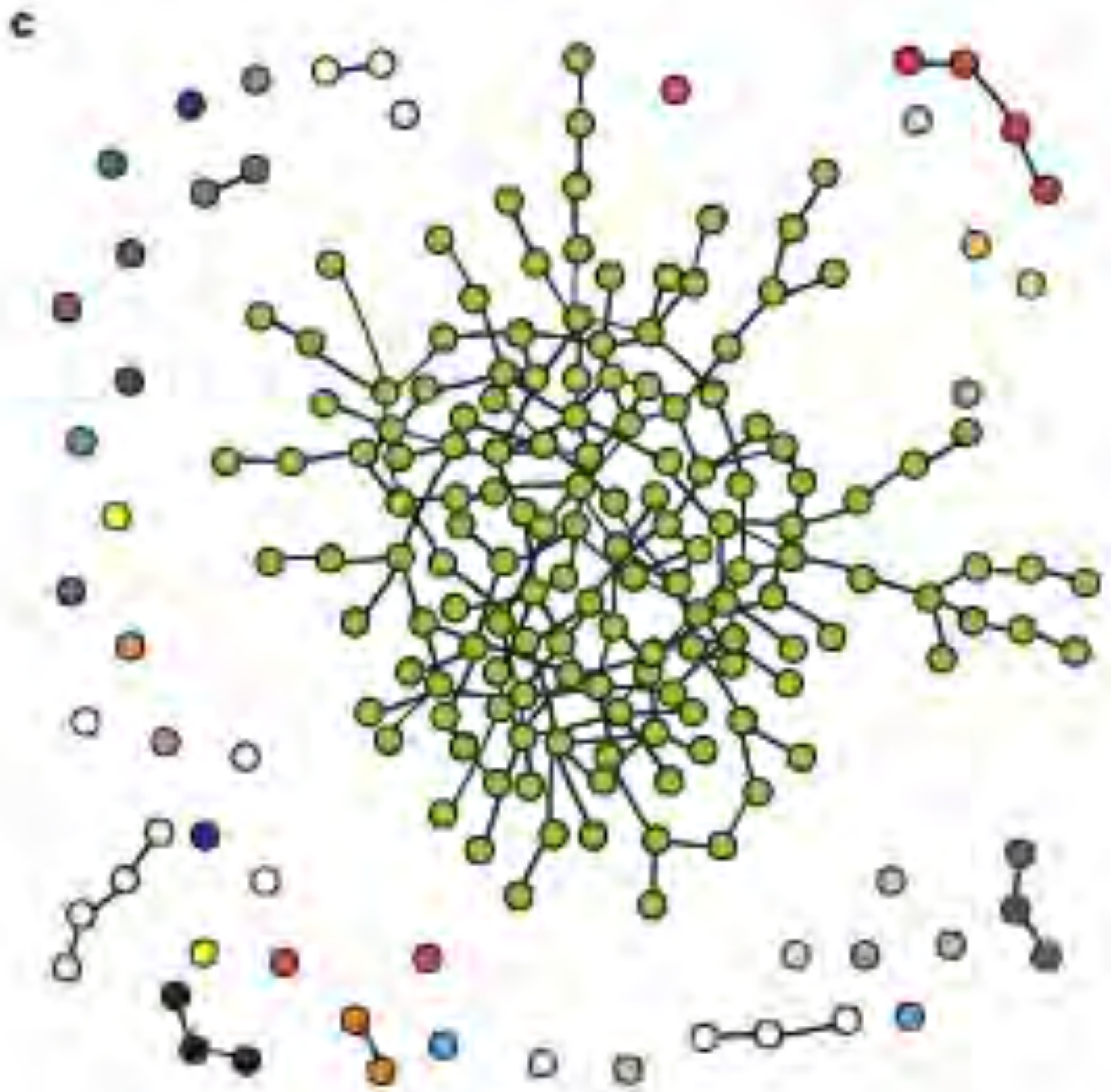




nearest neighbour ring

complete network





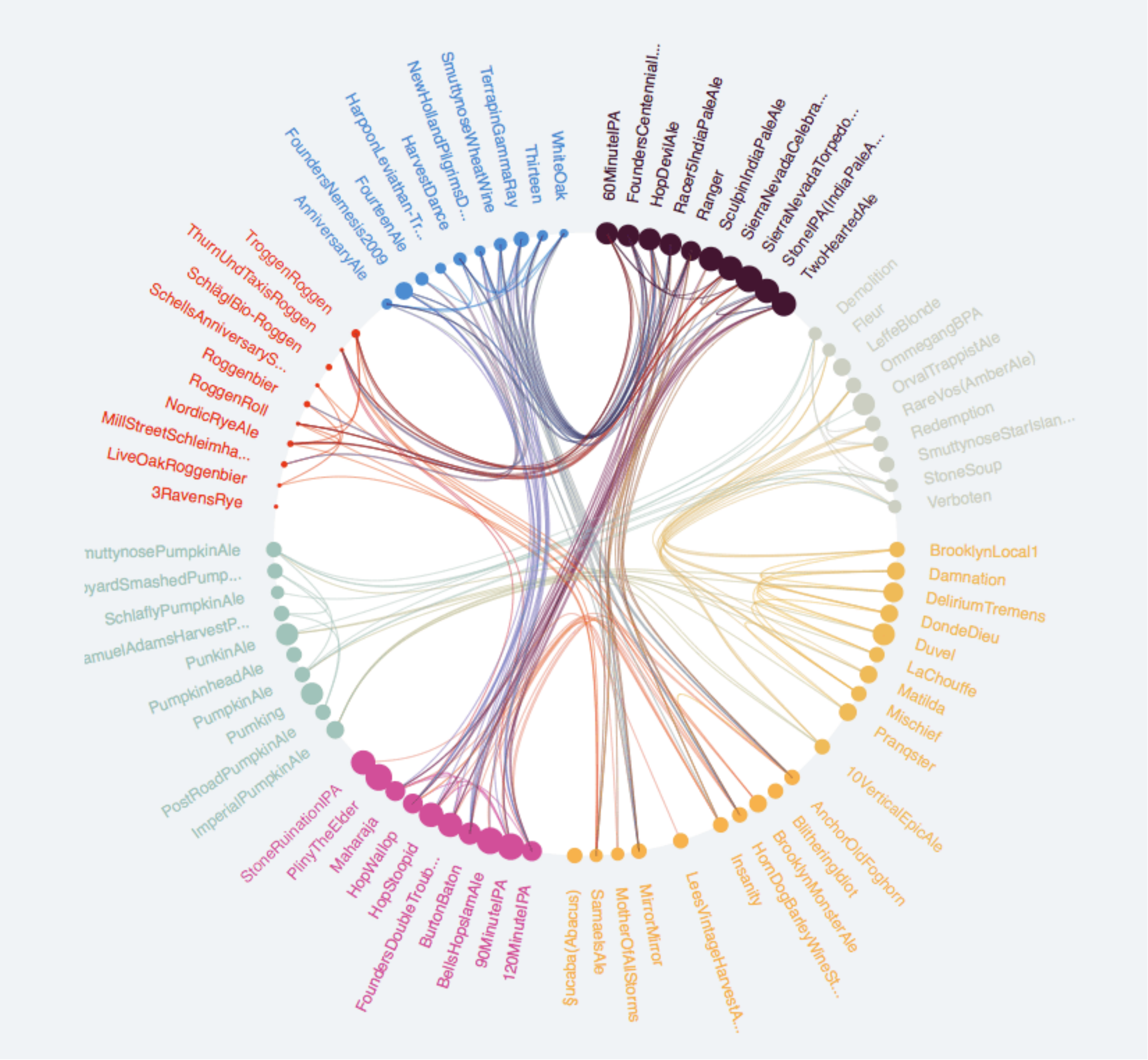
random network

(random edges)

Strogatz, S. H. (2001). Exploring complex networks. Nature, 410(6825), 268-76. doi: 10.1038/35065725.

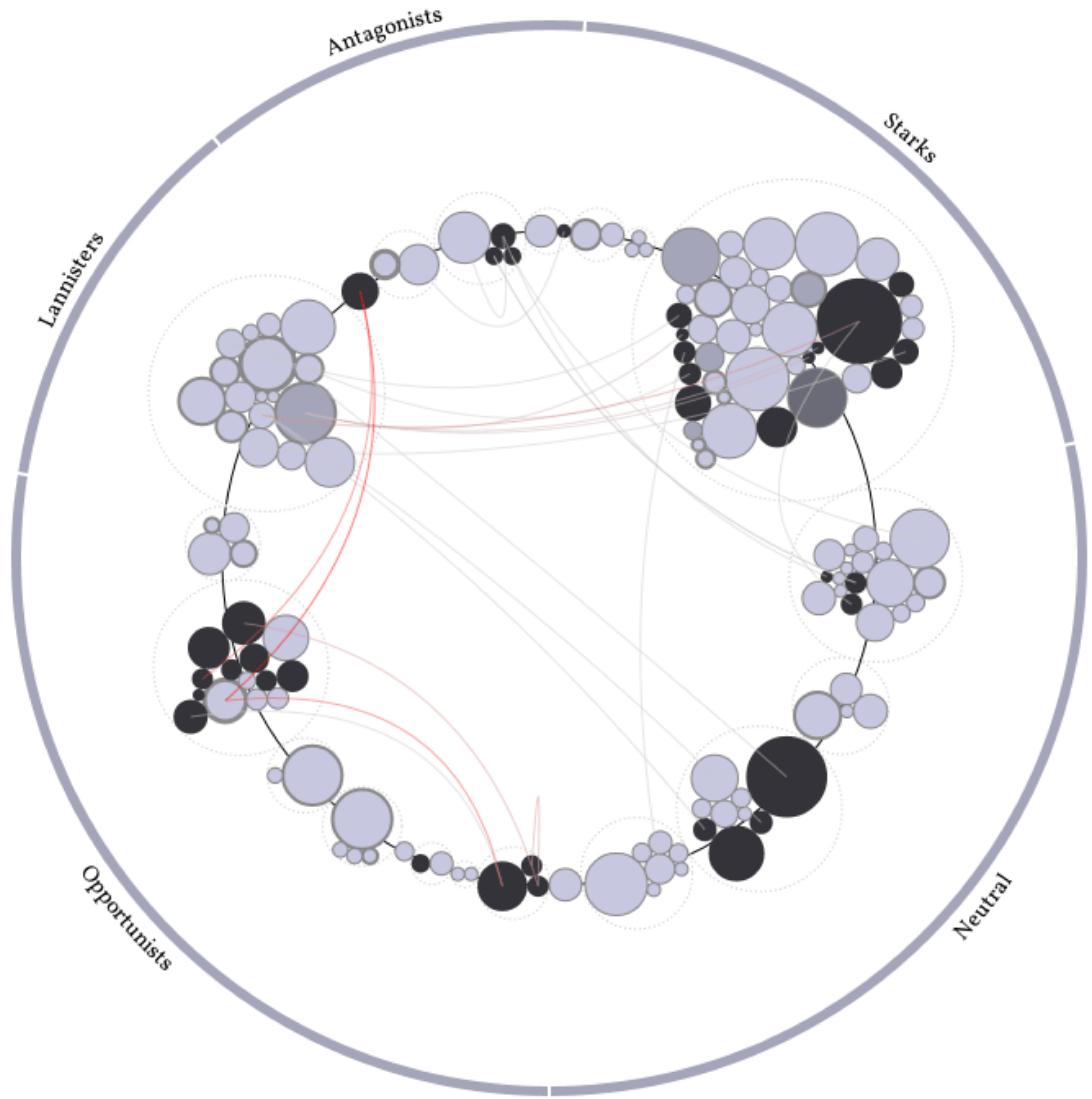
Graph theory

Netwerk that helps you find new beers based on your taste preference



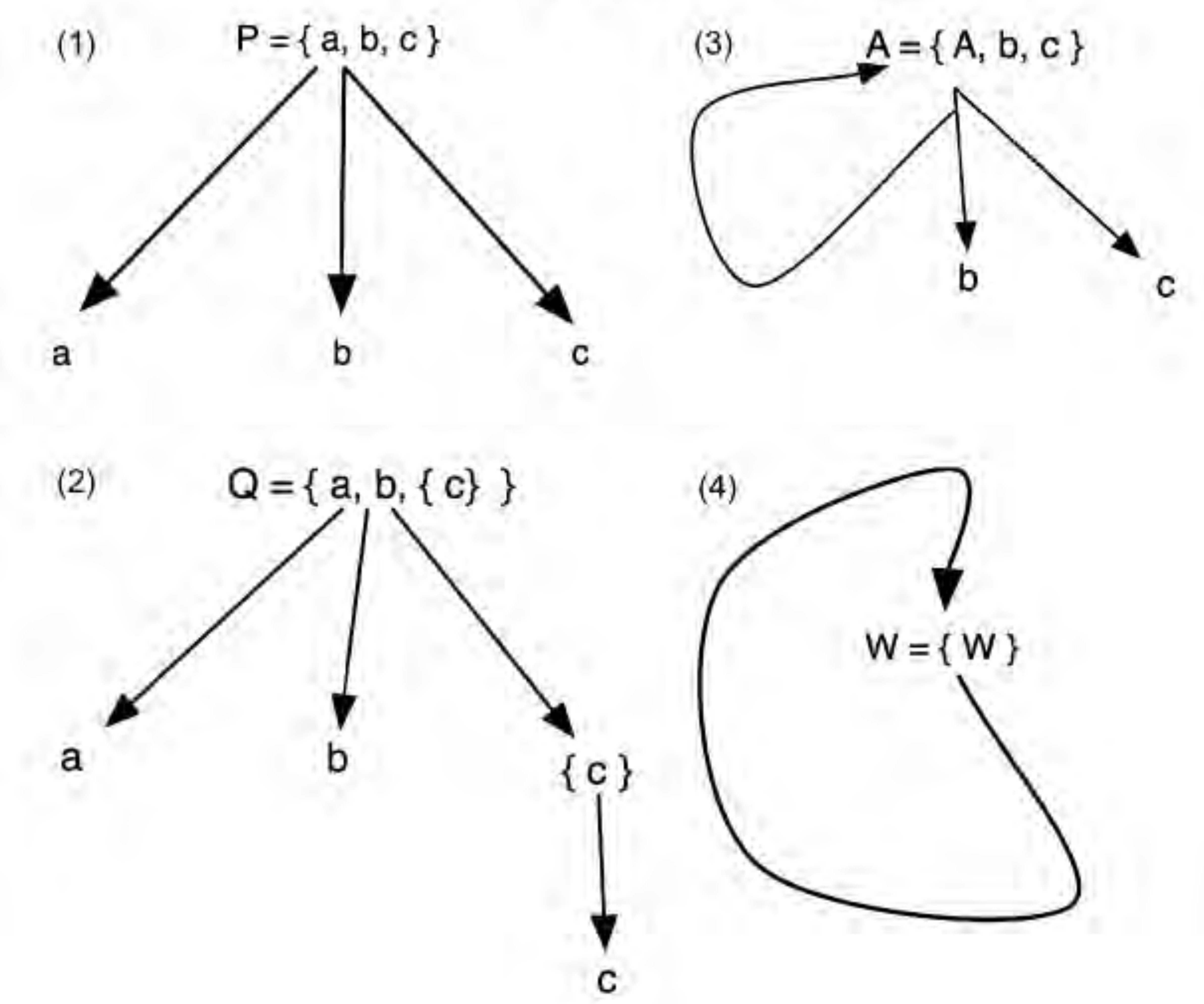
"Relation" between Game of Thrones characters

http://www.jeromecukier.net/projects/agot/events.html



Hyperset theory + Graph theory = Hyperset Graphs (Impredicative Logic)

Aczel's Anti-Foundation Axiom (1988)
(hyperset theory, circular causality, complexity analysis)



Non well-founded sets:

Definition of a set can contain itself

Aczel, P. (1988). *Non-well-founded sets. Foreword by Jon Barwise*. CSLI Lecture Notes, 14.

Hyperset theory + Graph theory = Hyperset Graphs (Impredicative Logic)

Impredicative loop

Hyperset loop

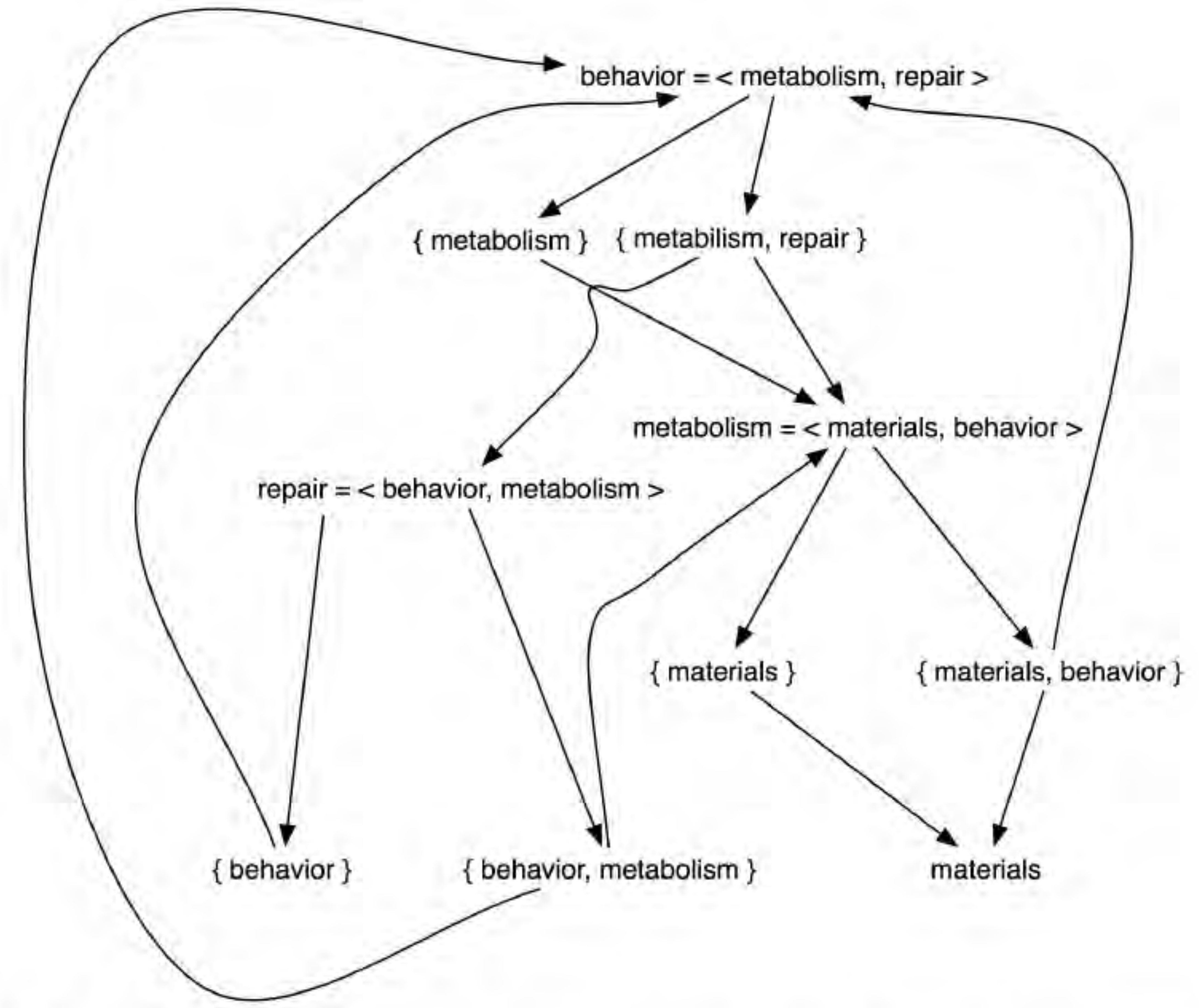


Fig. 6. Hyperset diagram of Rosen's metabolism-repair system. Functions are represented as ordered pairs containing their domain and range. So f (a) = b is represented as f = < a, b >.

Rosen's definition of a living system
(metabolism-repair-system)

Chemero, A., & Turvey, M. T. (2010). Is Life Computable? *Advances in modeling adaptive and cognitive systems*, 29-37.

Social networks

Moreno, 1930

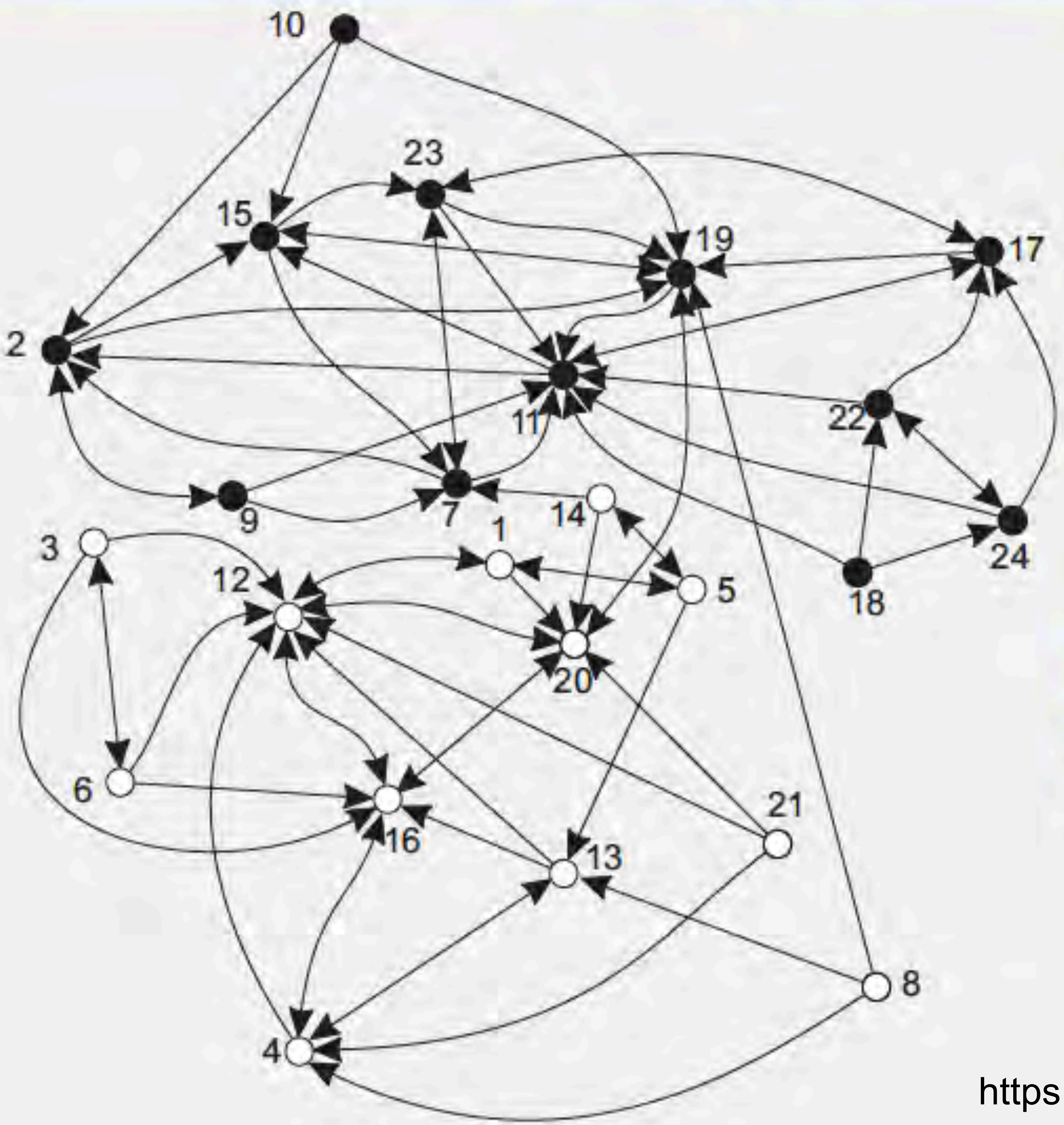
sociogram

3 “most liked”
3 “most disliked”

Sex	ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
F	1	■				+				−	−		+								+	−			
M	2	−	■							+						+				+			−	−	
F	3			■			+	−			−		+				+					−			
F	4				■						−		+	+			+			−	−				
F	5	+				■					−			+	+					−	−				
F	6	−		+			■						+			−	+						−		
M	7		+					■			−	+								−		−		+	
F	8				+		−		■					+				−		+					−
M	9		+					+		■		+		−		−									−
M	10		+					−			■	−				+				+		−			
M	11		+								−	■				+		+		−	−				
F	12	+						−			−		■			−	+				+				
F	13				+								+	■			+			−	−	−			
F	14					+	−	+		−					■						+		−		
M	15							+			−				−	■				+				+	−
F	16				+						−		+				■				+		−	−	
M	17				−							+						■		+	−	−		+	
M	18							−				+							■	−		−	+		+
M	19		−									+	−			+				■	+	−			
F	20						−			−			+		−		+			+	■				
F	21	−	−		+								+							−	+	■			
M	22						−			−		+					−	+					■		+
M	23	−						+				+						−		+		−		■	
M	24											+						+			−	−	+	−	■
	+	2	4	1	4	2	1	4	0	1	0	8	8	3	1	4	6	3	0	7	6	0	2	3	2
	−	4	2	0	1	0	4	4	0	4	9	1	1	1	2	3	1	2	0	7	6	10	4	3	3

https://www.distributed-systems.net/index.php/books/gtcn/

Classroom example - positive nominations



- Clear distinction between boys ("•") and girls ("o")
- Relation between 19 and 20 is important
- There are a few "isolated" children (8 & 10)

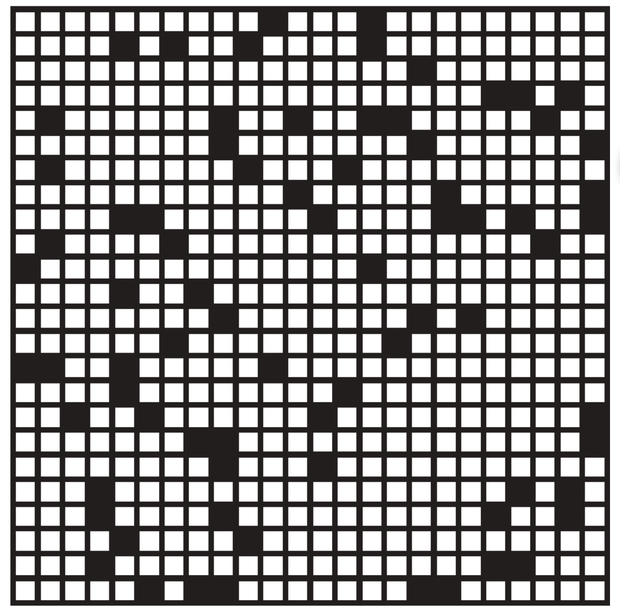
Issue

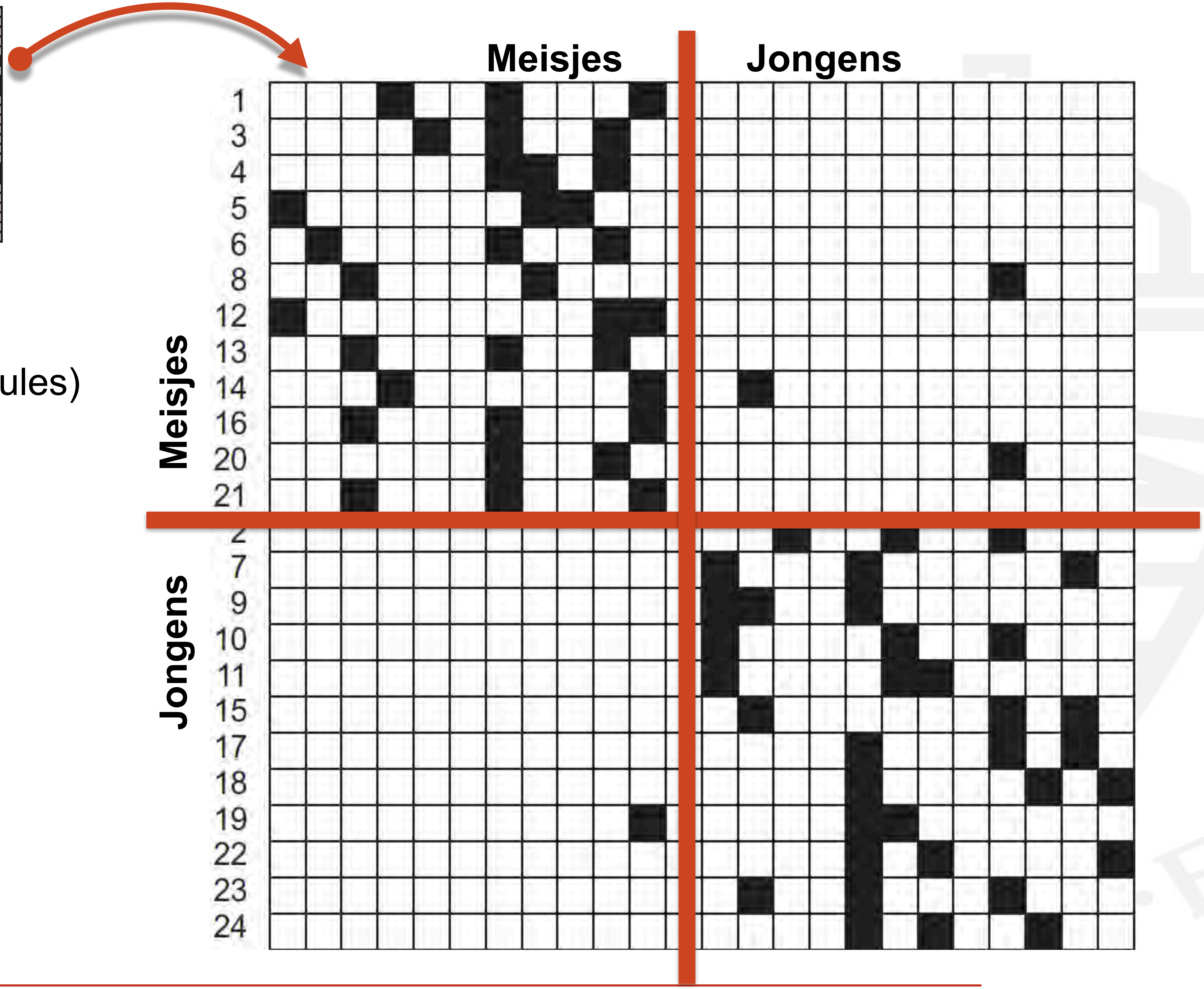
Can we discover these properties **mathematically**?

Degree, Centrality, Closeness, Eccentricity, Betweenness,

https://www.distributed-systems.net/index.php/books/gtcn/

Sociale networks





Clusters (communities, subgraphs, modules)

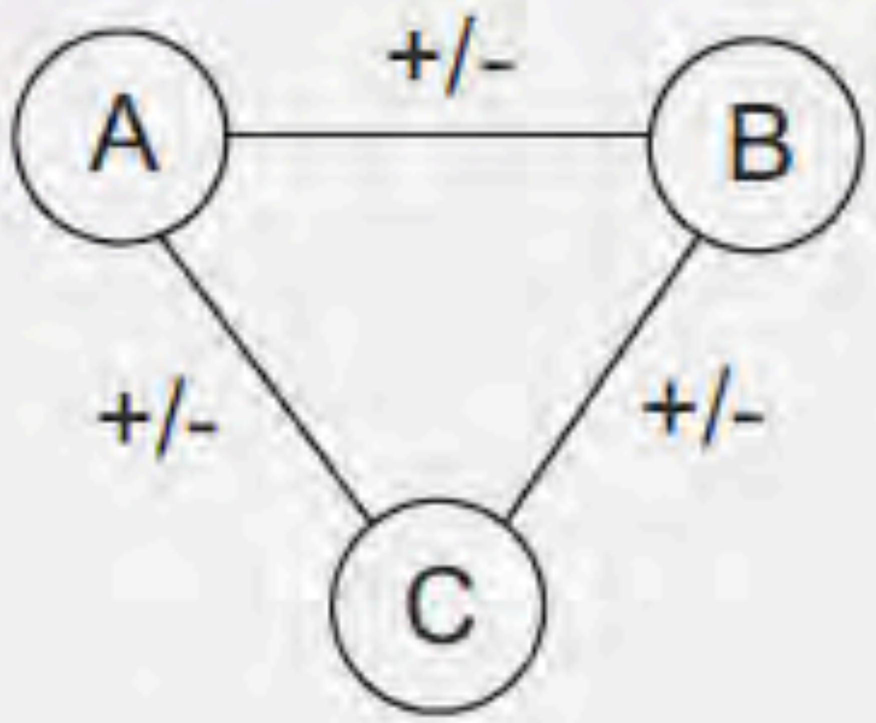
"hubs"

https://www.distributed-systems.net/index.php/books/gtcn/

Structural balance

https://www.distributed-systems.net/index.php/books/gtcn/

Motifs (signed)
Motieven

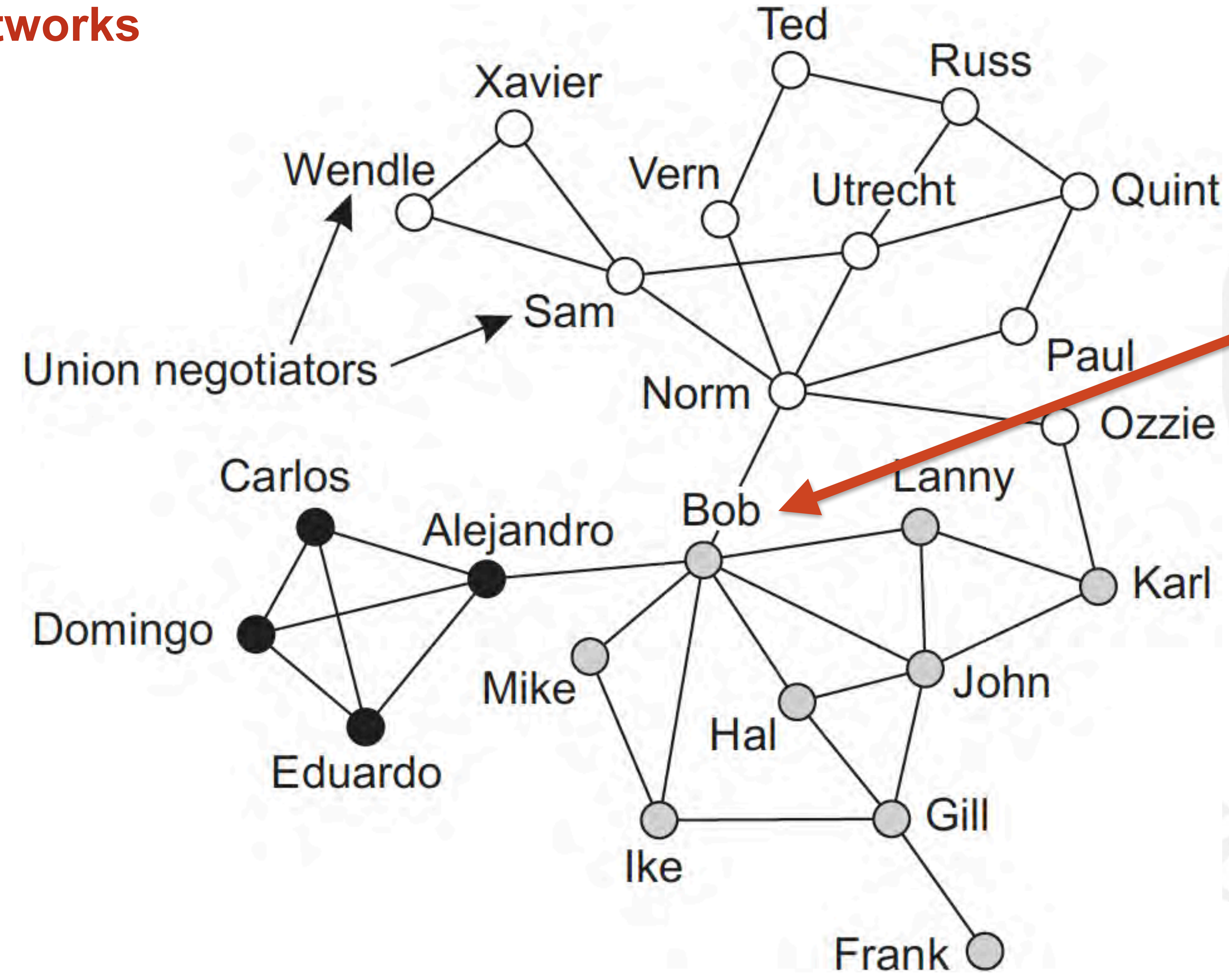


Basic idea

Consider **triads**: potential relationships between triples of social entities, and label every relationship as positive or negative. We then consider **balanced** triads.

A–B	B–C	A–C	B/I	Description
+	+	+	B	Everyone likes each other
+	+	−	I	Dislike A–C stresses relation B has with either of them
+	−	+	I	Dislike B–C stresses relation A has with either of them
+	−	−	B	A and B like each other, and both dislike C
−	+	+	I	Dislike A–B stresses relation C has with either of them
−	+	−	B	B and C like each other, and both dislike A
−	−	+	B	A and C like each other, and both dislike B
−	−	−	I	Nobody likes each other

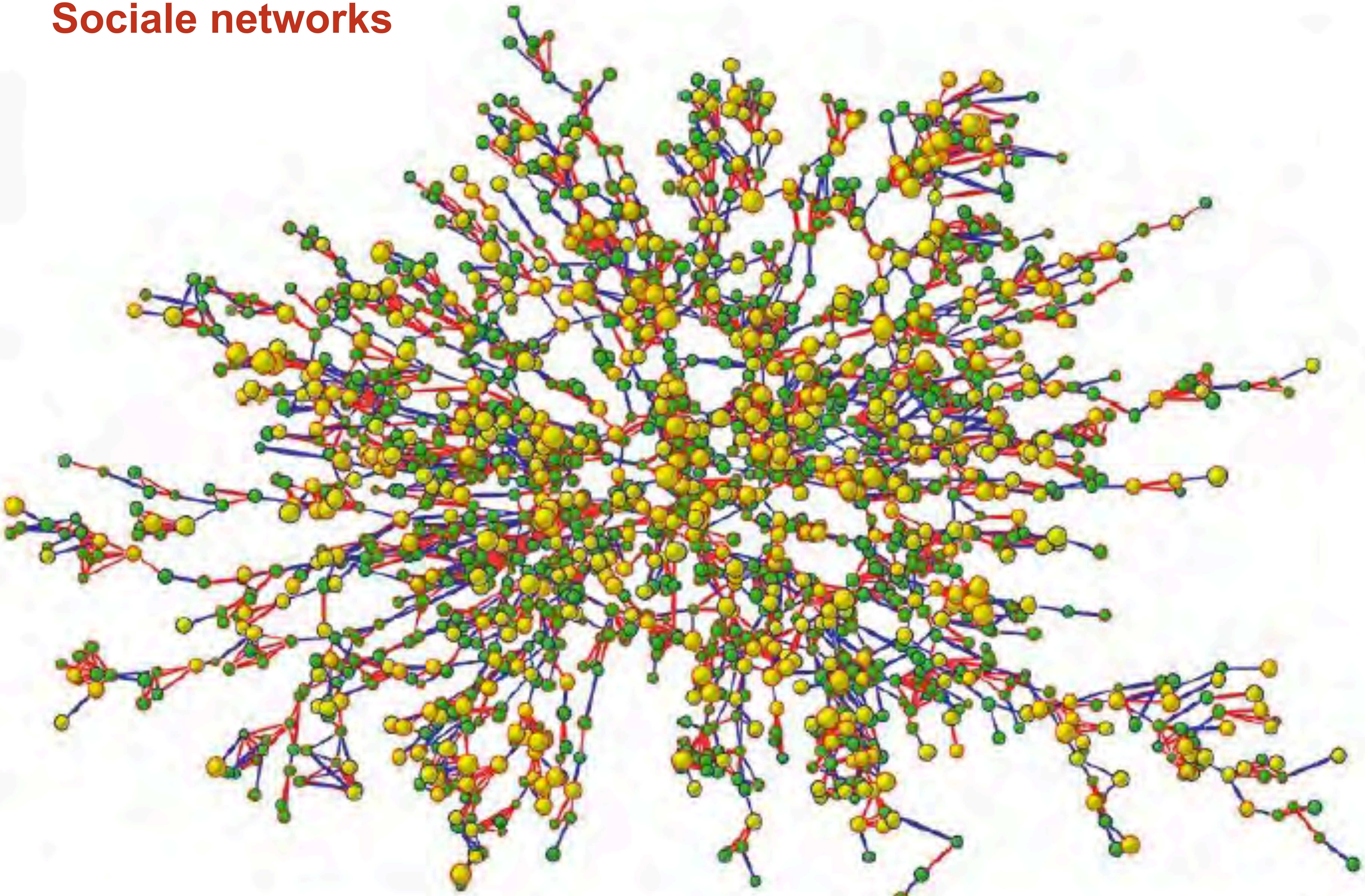
Sociale networks



Bob is a 'connector hub'
For 3 clusters

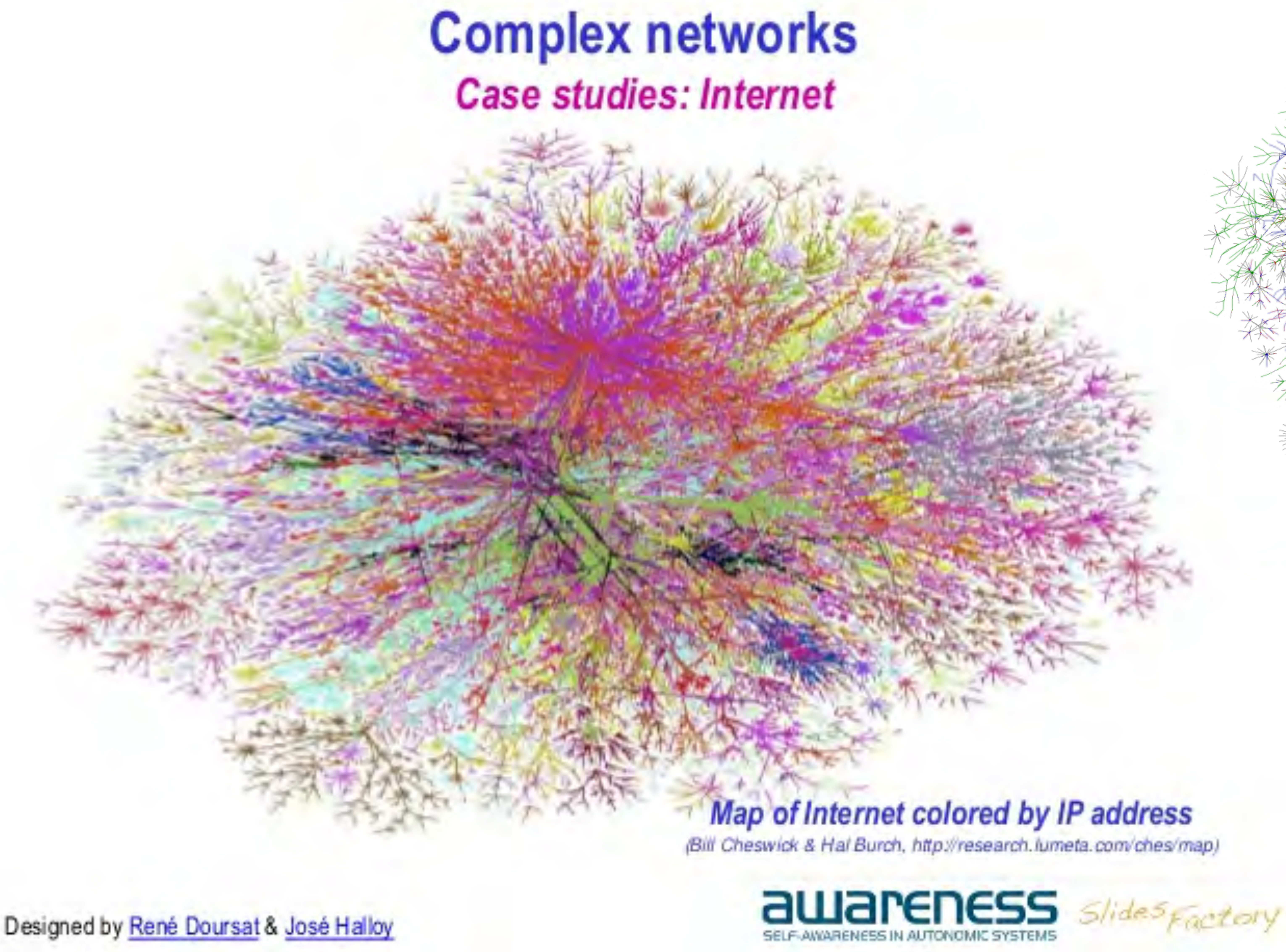
http://www.distributed-systems.net/gtcn/

Sociale networks

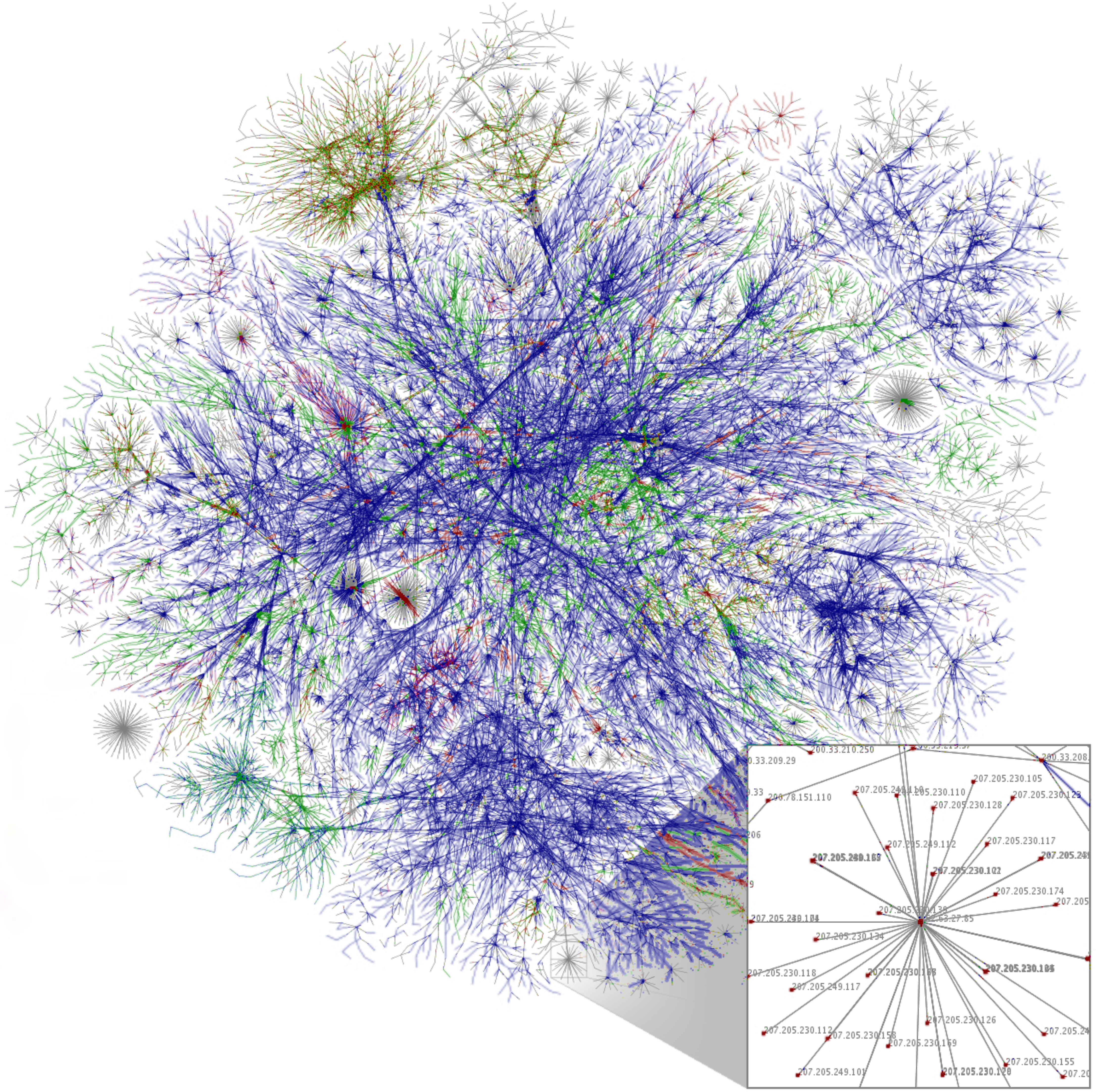


Yellow: obese | Green: nonobese | Purple: friend/marriage | Red: family

Complexe ‘sociale’ networks

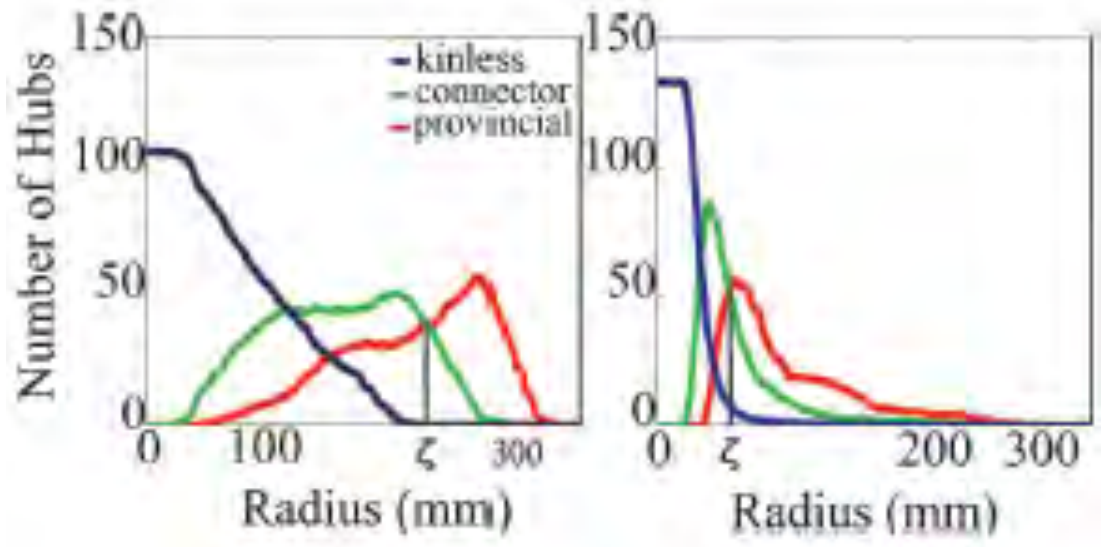


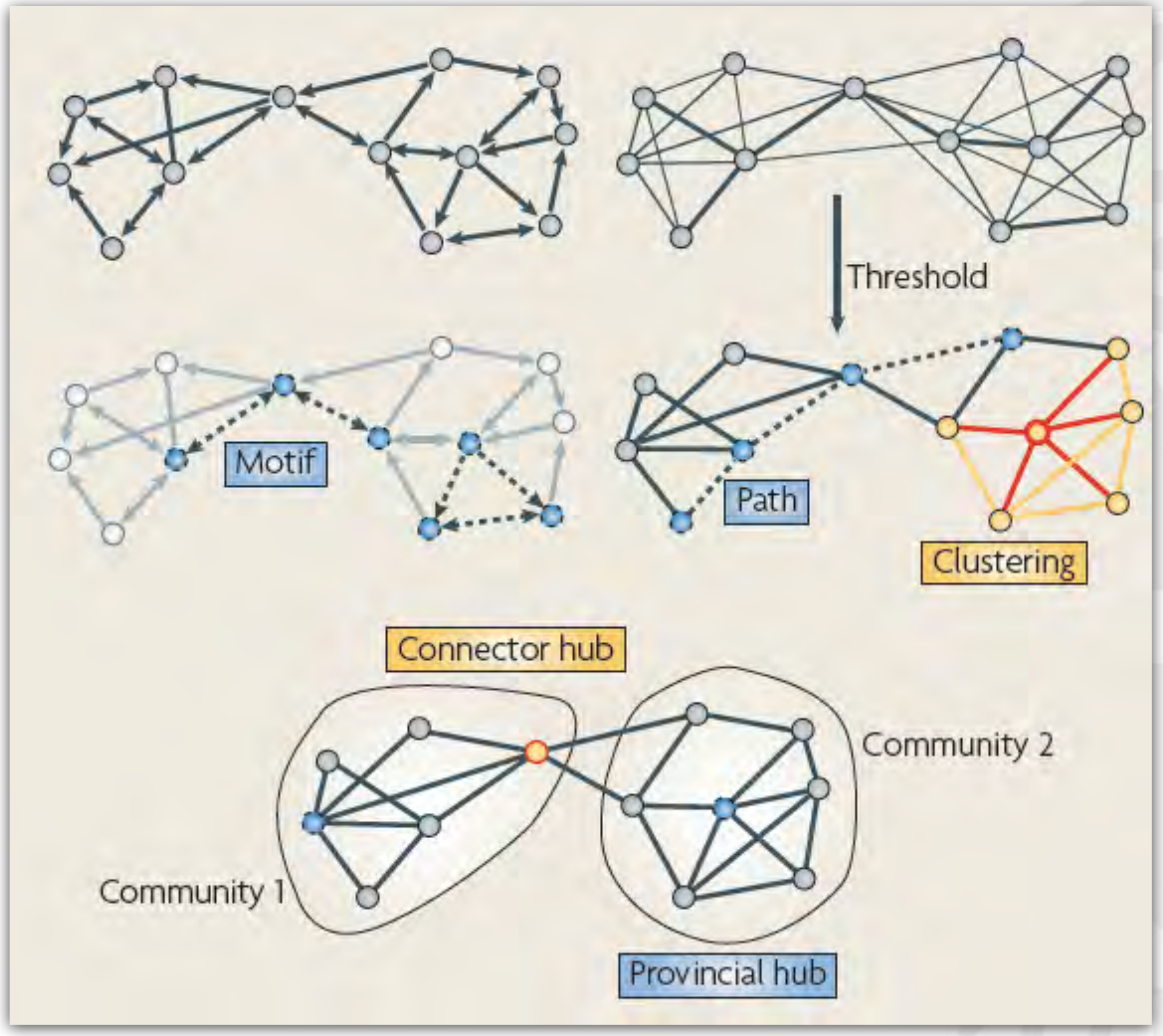
An Opte Project visualization of routing paths through a portion of the Internet



A brand new zoo of complexity measures!

- Node degree
- Degree distribution
- Assortativity
- Clustering coefficient
- Motifs
- Path length
- Path efficiency
- Connection density or cost
- Hubs
- Centrality
- Robustness
- Modularity





Bullmore, E., & Sporns, O. (2009). Complex brain networks: graph theoretical analysis of structural and functional systems. Nature reviews. Neuroscience, 10(3), 186-98. doi: 10.1038/nrn2575.

Table A1 (*continued*)

Measure	Binary and undirected definitions	Weighted and directed definitions
Modularity	Modularity of the network (Newman, 2004b), $$Q = \sum_{u \in M}\left[e_{uu} - \left(\sum_{v \in M} e_{uv}\right)^2\right],$$ where the network is fully subdivided into a set of nonoverlapping modules M, and e_{uv} is the proportion of all links that connect nodes in module u with nodes in module v. An equivalent alternative formulation of the modularity (Newman, 2006) is given by $Q = \frac{1}{l}\sum_{i,j \in N}\left(a_{ij} - \frac{k_i k_j}{l}\right)\delta_{m_i,m_j}$, where m_i is the module containing node i, and $\delta_{m_i,m_j} = 1$ if $m_i = m_j$, and 0 otherwise.	Weighted modularity (Newman, 2004), $Q^w = \frac{1}{l^w}\sum_{i,j \in N}\left[w_{ij} - \frac{k_i^w k_j^w}{l^w}\right]\delta_{m_i,m_j}$. Directed modularity (Leicht and Newman, 2008), $Q^{\rightarrow} = \frac{1}{l}\sum_{i,j \in N}\left[a_{ij} - \frac{k_i^{out} k_j^{in}}{l}\right]\delta_{m_i,m_j}$.
Measures of centrality		
Closeness centrality	Closeness centrality of node i (e.g. Freeman, 1978), $$L_i^{-1} = \frac{n-1}{\sum_{j \in N, j \neq i} d_{ij}}.$$	Weighted closeness centrality, $\left(L_i^w\right)^{-1} = \frac{n-1}{\sum_{j \in N, j \neq i} d_{ij}^w}$. Directed closeness centrality, $\left(L_i^{\rightarrow}\right)^{-1} = \frac{n-1}{\sum_{j \in N, j \neq i} d_{ij}^{\rightarrow}}$.
Betweenness centrality	Betweenness centrality of node i (e.g., Freeman, 1978), $$b_i = \frac{1}{(n-1)(n-2)} \sum_{\substack{h,j \in N \\ h \neq j, h \neq i, j \neq i}} \frac{\rho_{hj}(i)}{\rho_{hj}},$$ where ρ_{hj} is the number of shortest paths between h and j, and $\rho_{hj}(i)$ is the number of shortest paths between h and j that pass through i.	Betweenness centrality is computed equivalently on weighted and directed networks, provided that path lengths are computed on respective weighted or directed paths.
Within-module degree z-score	Within-module degree z-score of node i (Guimera and Amaral, 2005), $$z_i = \frac{k_i(m_i) - \bar{k}(m_i)}{\sigma^{k(m_i)}},$$ where m_i is the module containing node i, $k_i(m_i)$ is the within-module degree of i (the number of links between i and all other nodes in m_i), and $\bar{k}(m_i)$ and $\sigma^{k(m_i)}$ are the respective mean and standard deviation of the within-module m_i degree distribution.	Weighted within-module degree z-score, $z_i^w = \frac{k_i^w(m_i) - \bar{k}^w(m_i)}{\sigma^{k^w(m_i)}}$. Within-module out-degree z-score, $z_i^{out} = \frac{k_i^{out}(m_i) - \bar{k}^{out}(m_i)}{\sigma^{k^{out}(m_i)}}$. Within-module in-degree z-score, $z_i^{in} = \frac{k_i^{in}(m_i) - \bar{k}^{in}(m_i)}{\sigma^{k^{in}(m_i)}}$.

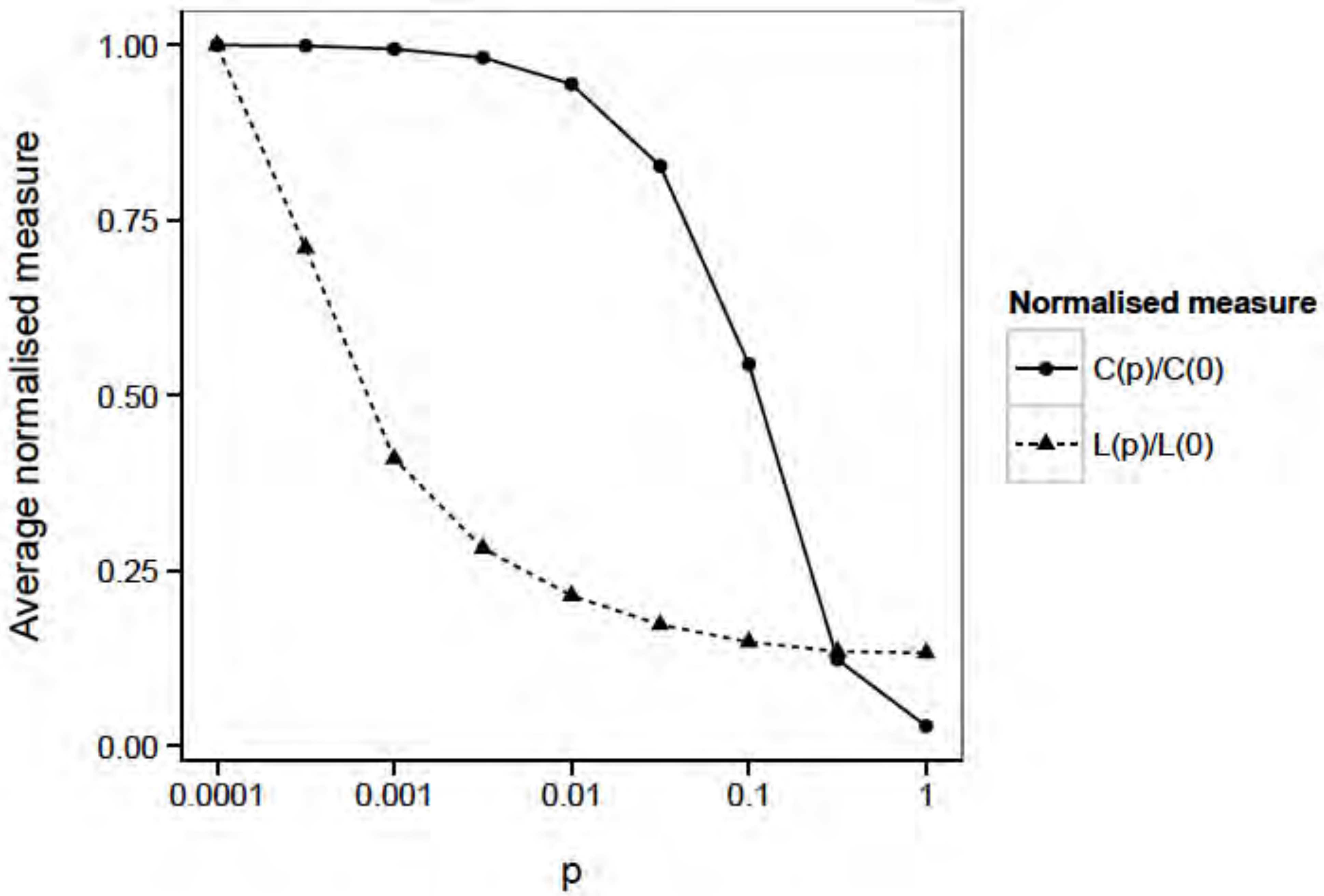
Network / Graph topology: It's a Small World After All

"small-world" test:

Average path length (L)

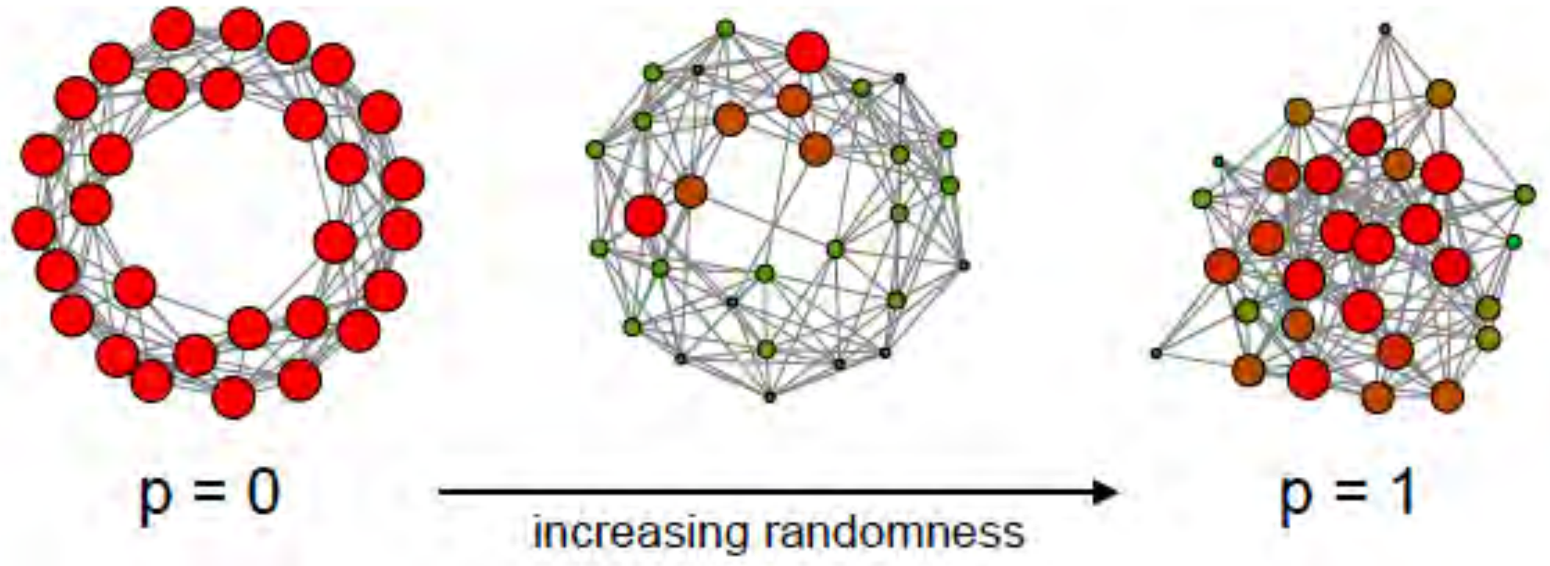
Clustering coefficient (C)

Compare to randomly rewired version



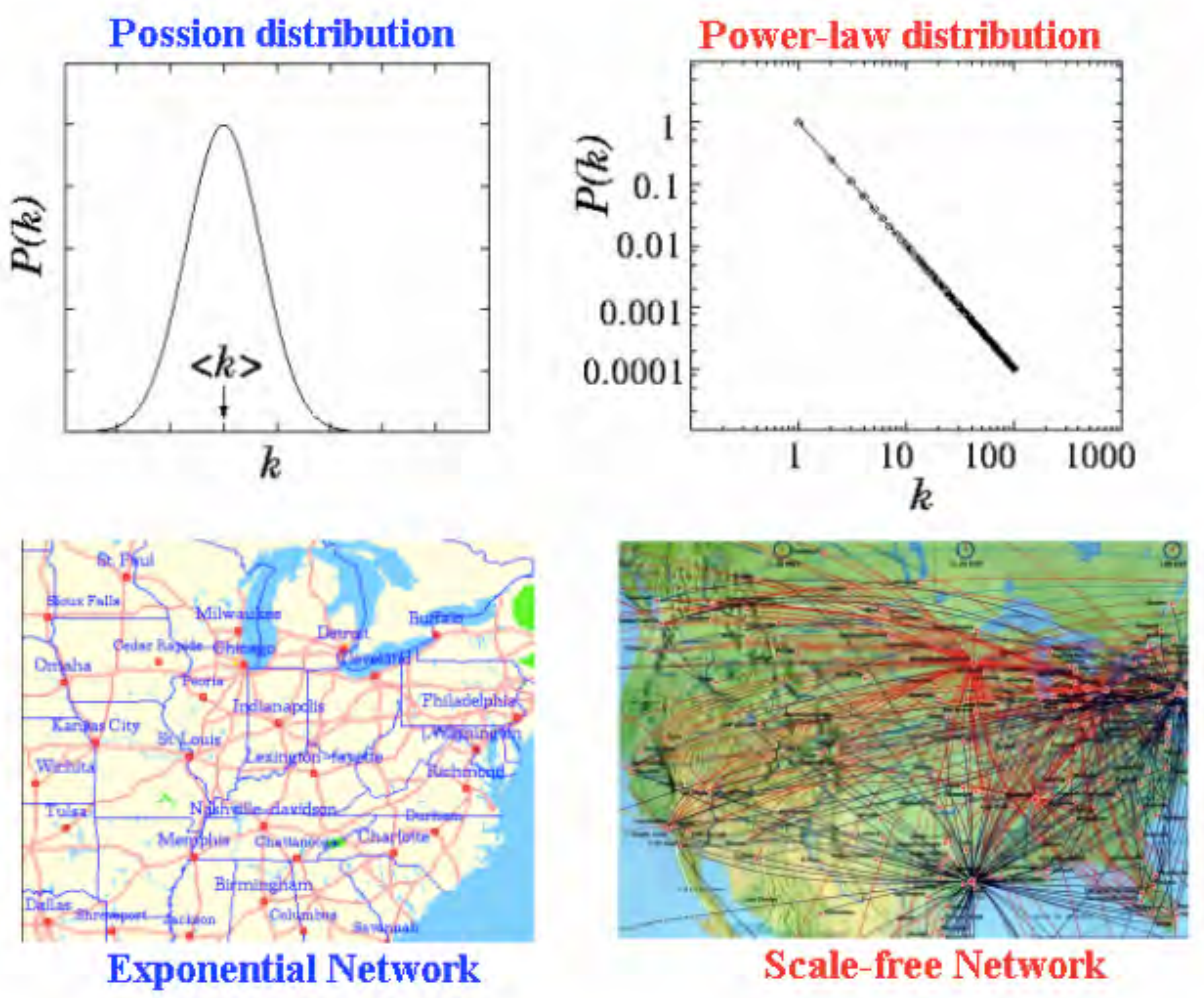
Sound familiar?

In between
fully ordered
&
completely random
=
optimal



Watts, D. J., & Strogatz, S. H. (1998). Collective dynamics of 'small-world'networks. N*ature*, *393*(6684), 440-442.

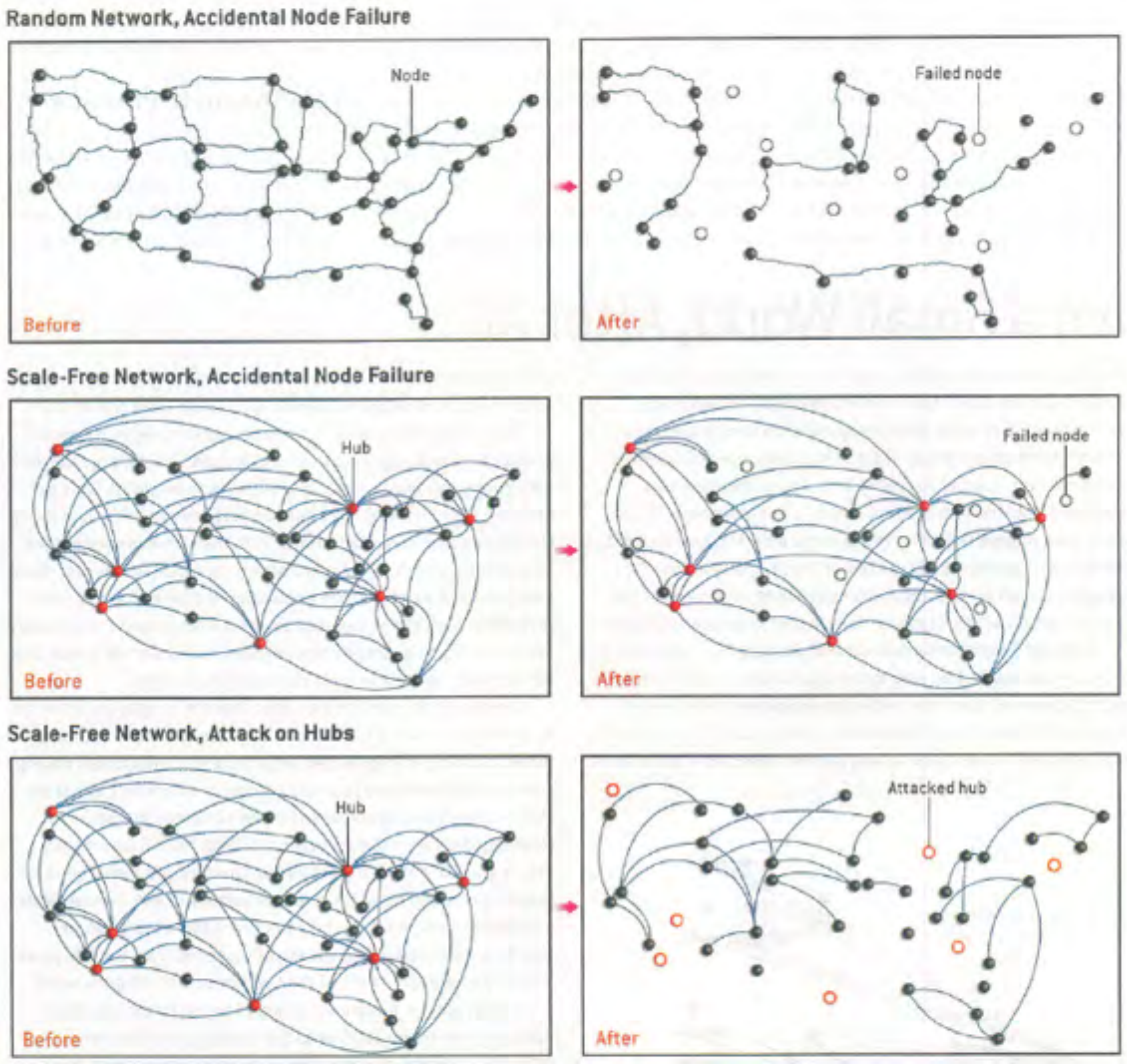
Network / Graph topology: It's a Scale Free World After All



Number of connections a node in the network has: degree (δ)

Scale-free network: degree distribution is a power law!

Wang, X. F., & Chen, G. (2003). Complex Networks: small-world, scale-free and beyond. Circuits and Systems Magazine, IEEE, 3(1), 6–20. IEEE. Retrieved May 18, 2011, from http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1228503.



Scale free networks are resilient to random attacks on nodes or node failures
(cf. internet on 9/11)

when more hub nodes fail though....

targeted attack!

Bonabeau, E., with Albert-László. (2003). Scale-free networks. *Scientific American, 288(5)*, 50–9.

Effectiveness / Connectivity: 6 degrees of separation

Kevin Bacon number (Erdős number)

- Node degree
- Degree distribution
- Assortativity
- Clustering coefficient
- Motifs
- Path length
- Path efficiency
- Connection density or cost
- Hubs
- Centrality
- Robustness
- Modularity

Degree of separation
(from Kevin Bacon)

'6 degrees of separation'

http://en.wikipedia.org/wiki/Bacon_number#Bacon_numbers

Effectiveness / Connectivity: 6 degrees of separation

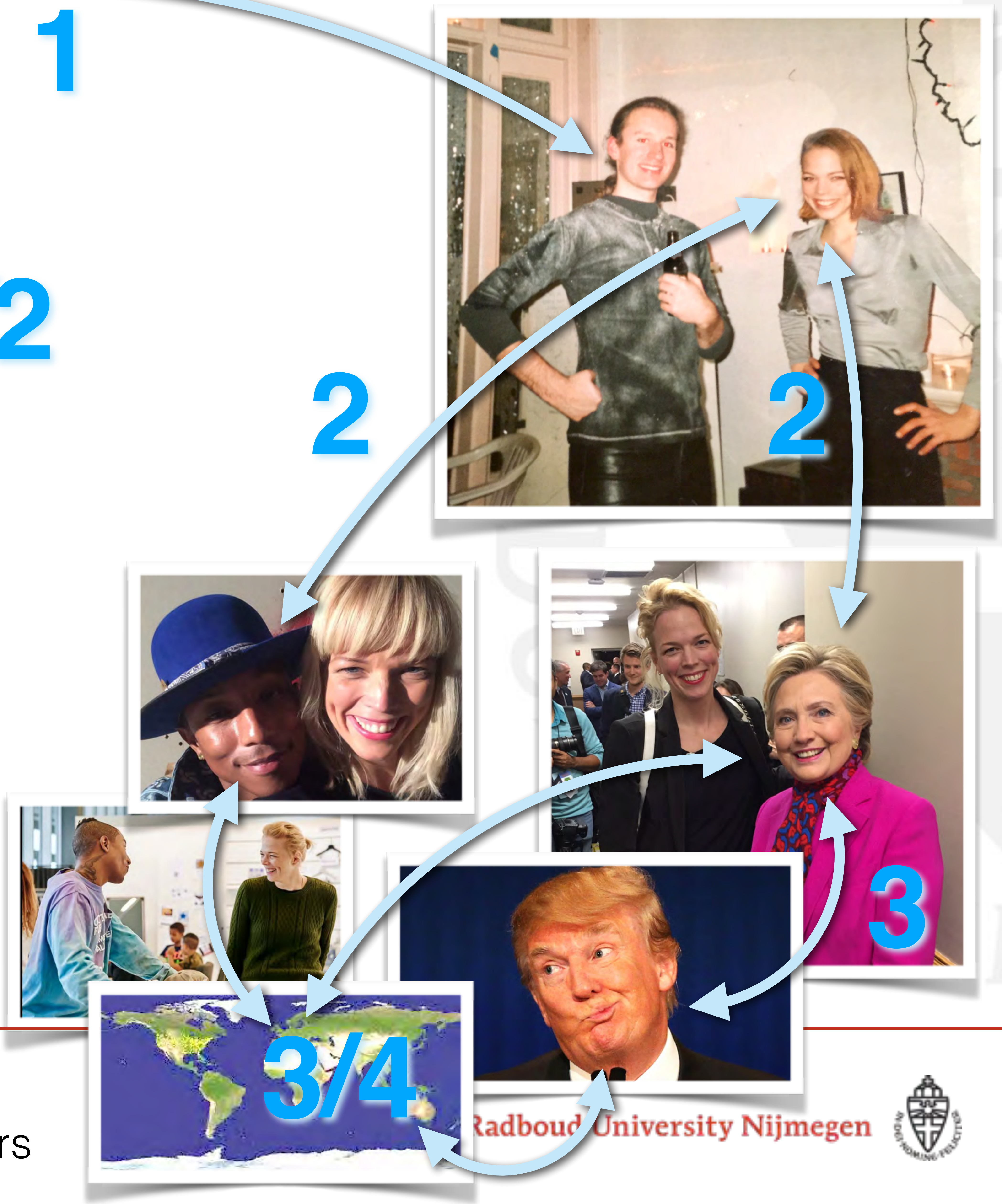
1

Your degree of separation from:

Nobel Laureate **1/2**

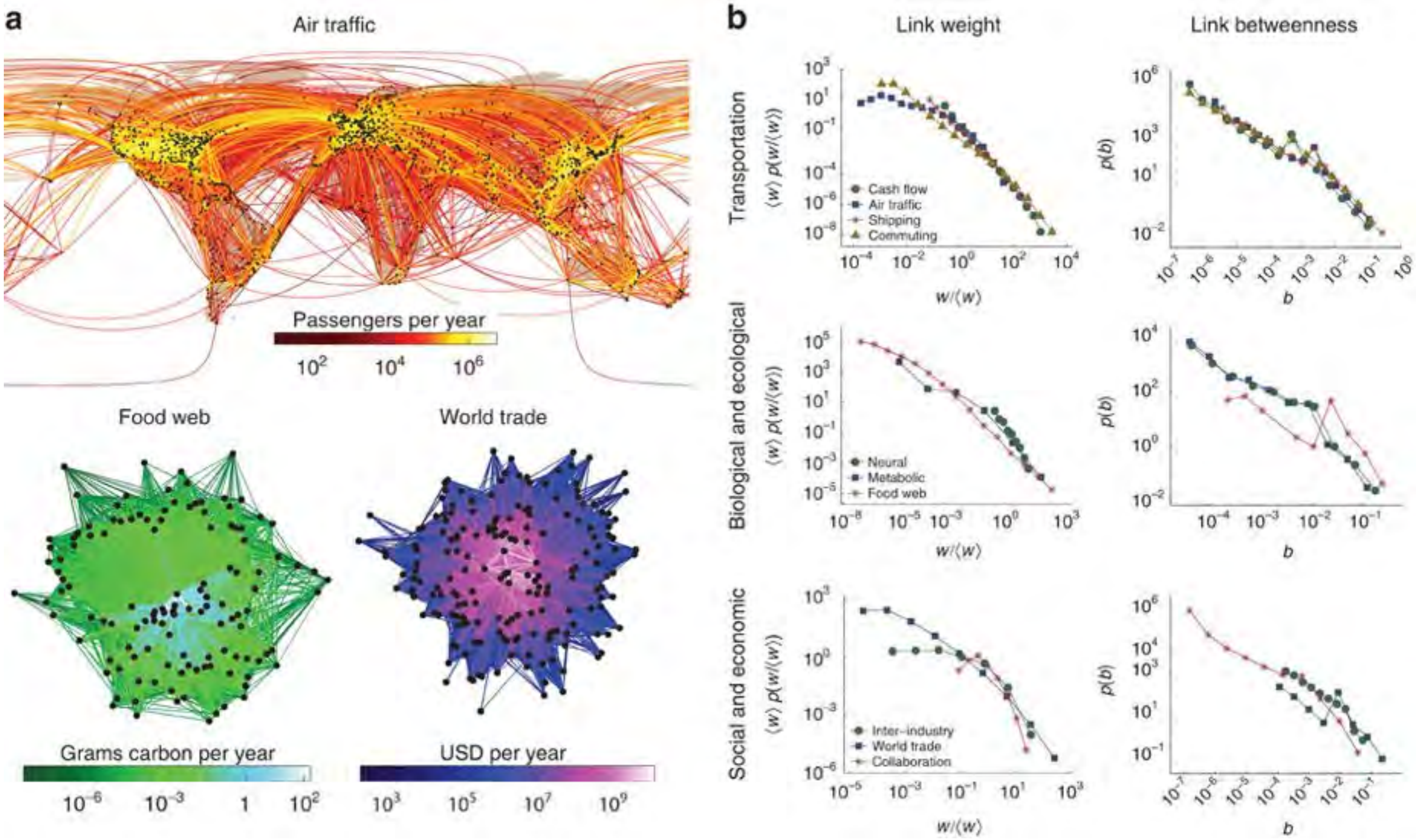
Pharrell Williams

Hillary Clinton
Donald Trump



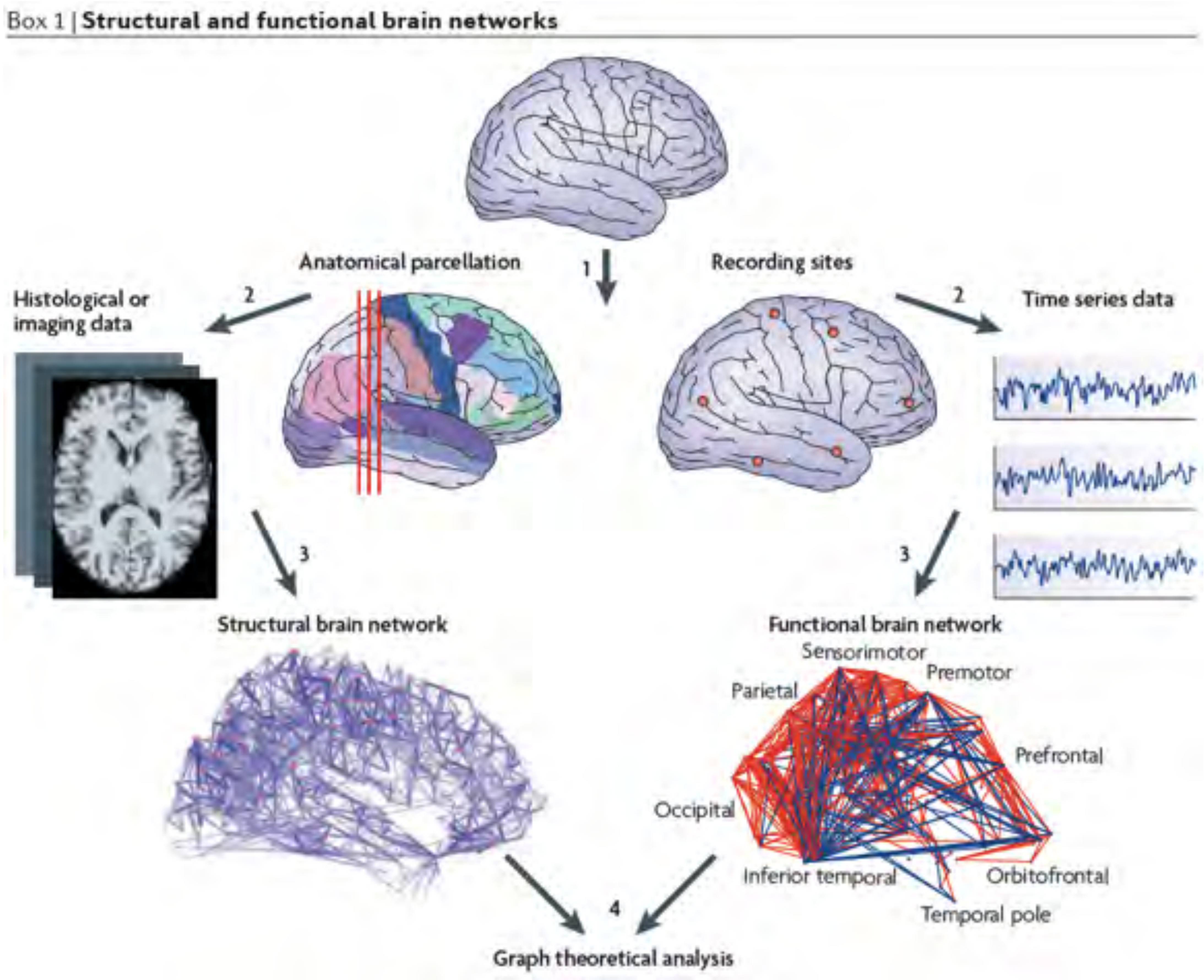
http://en.wikipedia.org/wiki/Bacon_number#Bacon_numbers

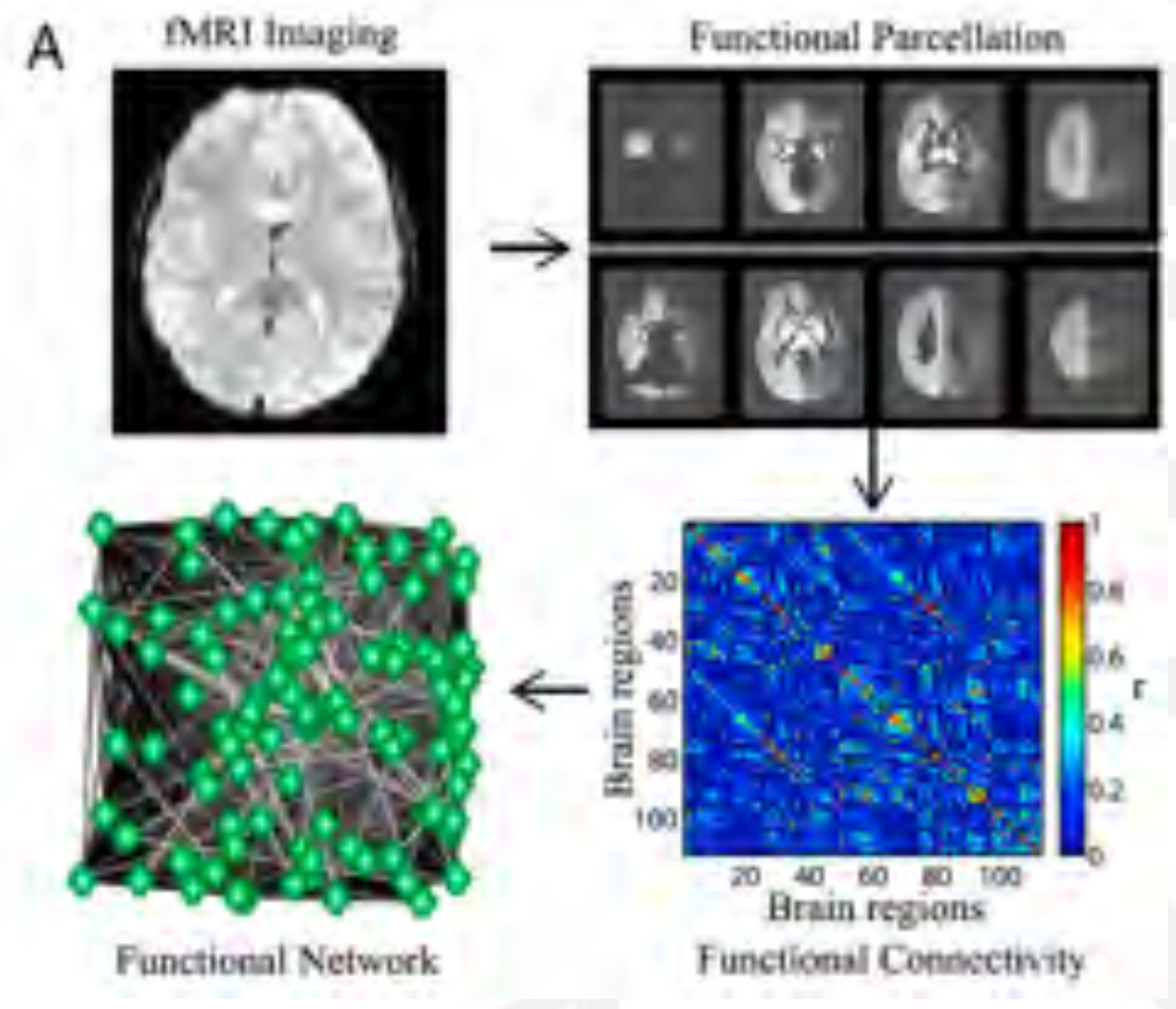
Scale-free structure



Network / Graph topology

Functional vs. Structural networks

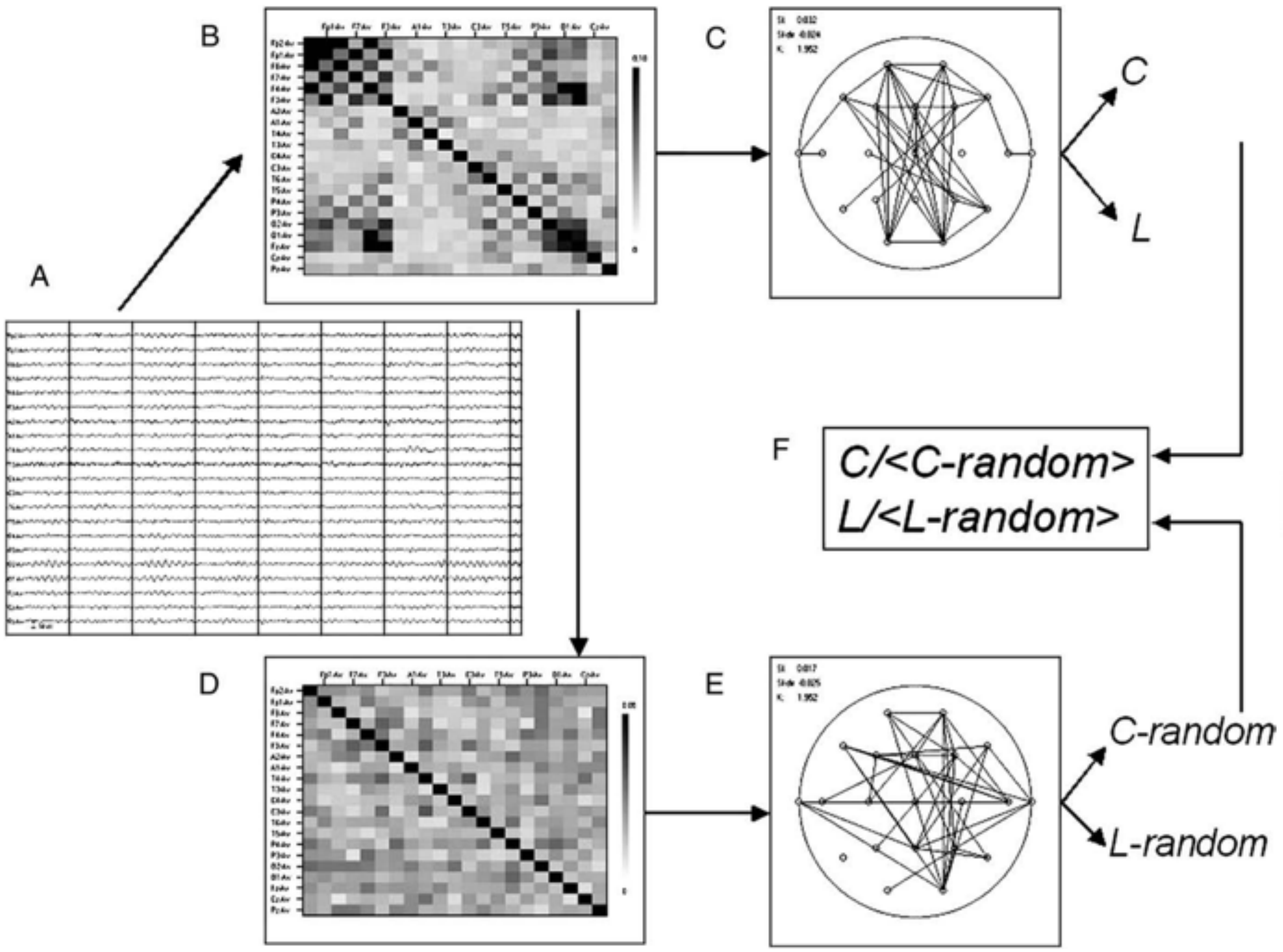




Bullmore, E., & Sporns, O. (2009). Complex brain networks: graph theoretical analysis of structural and functional systems. Nature reviews. Neuroscience, 10(3), 186-98. doi: 10.1038/nrn2575.

Network / Graph topology

How to get the matrices



Adjacency matrix and weighted graph can be extracted from resting state recordings:

5 min. eyes closed

find 4096 samples without artefacts

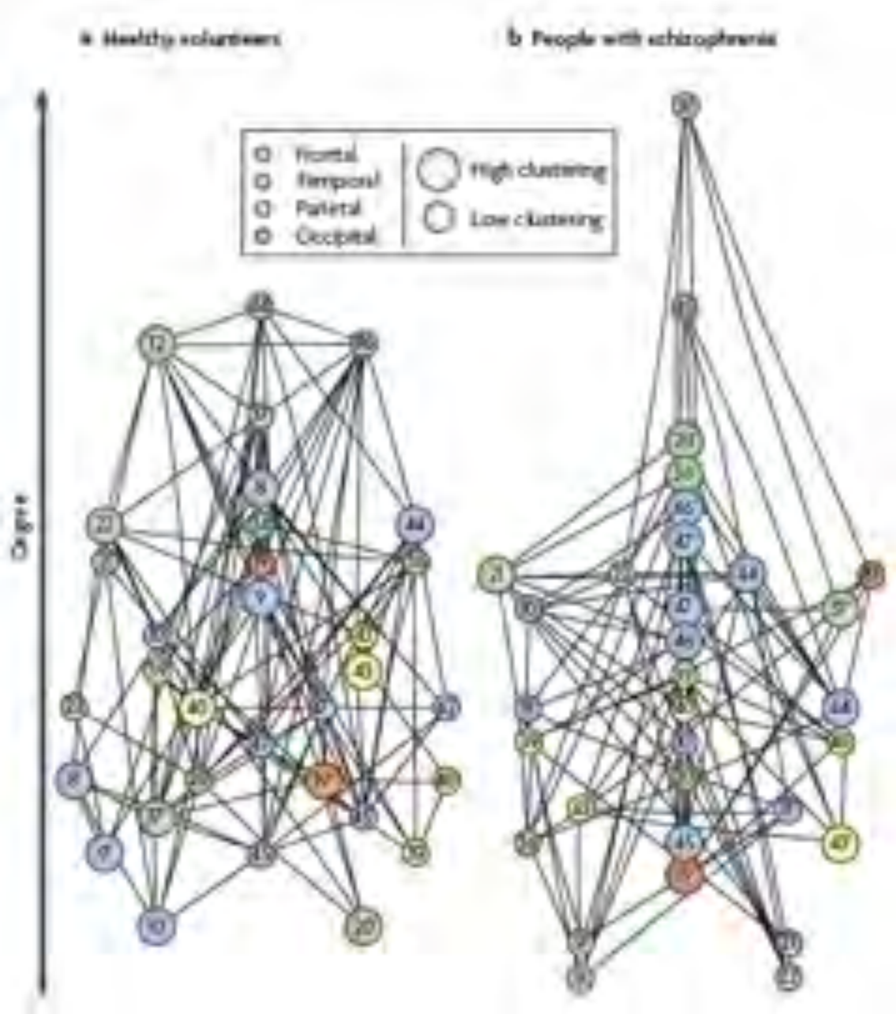
That's about 6-7 seconds!

Stam, C. J. (2010). Characterization of anatomical and functional connectivity in the brain: A complex networks perspective. International journal of psychophysiology : official journal of the International Organization of Psychophysiology, 77(3), 186-194. Elsevier B.V. doi: 10.1016/j.ijpsycho.2010.06.024.

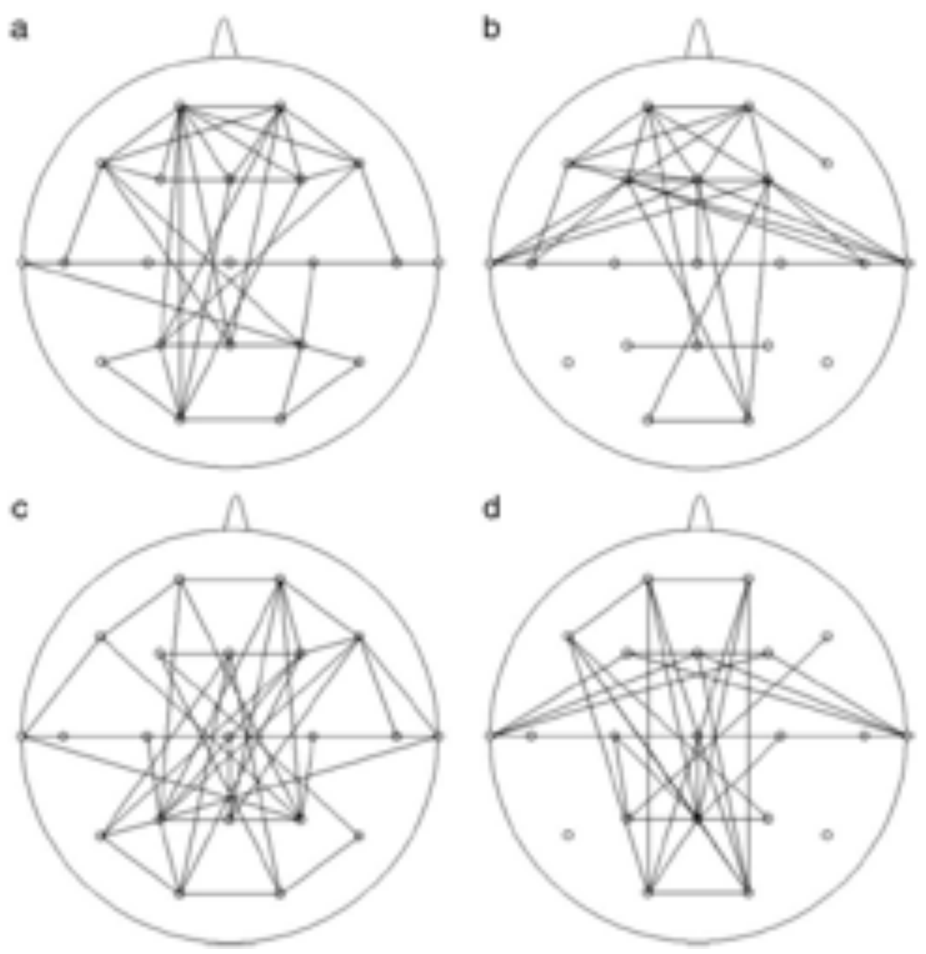
Network / Graph topology

Pathology studies

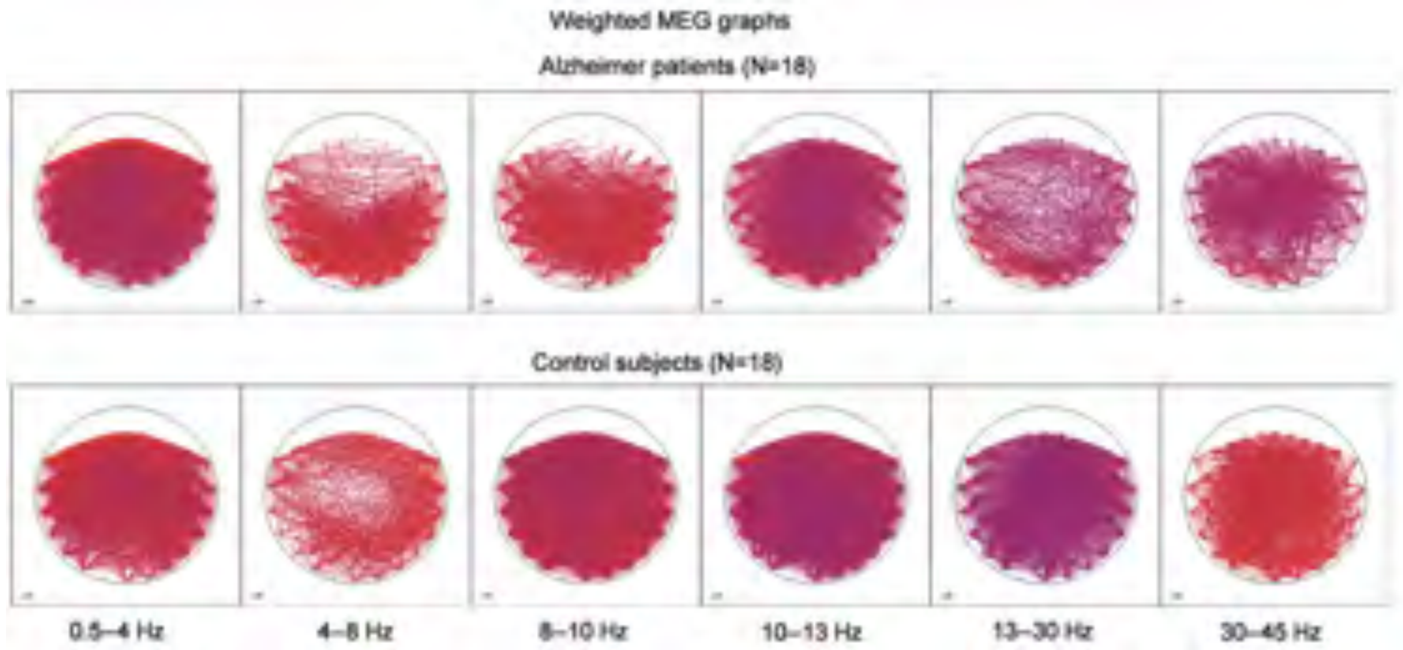
Based on a few samples we can distinguish healthy subjects from patients:



Schizophrenia



Absence seizure

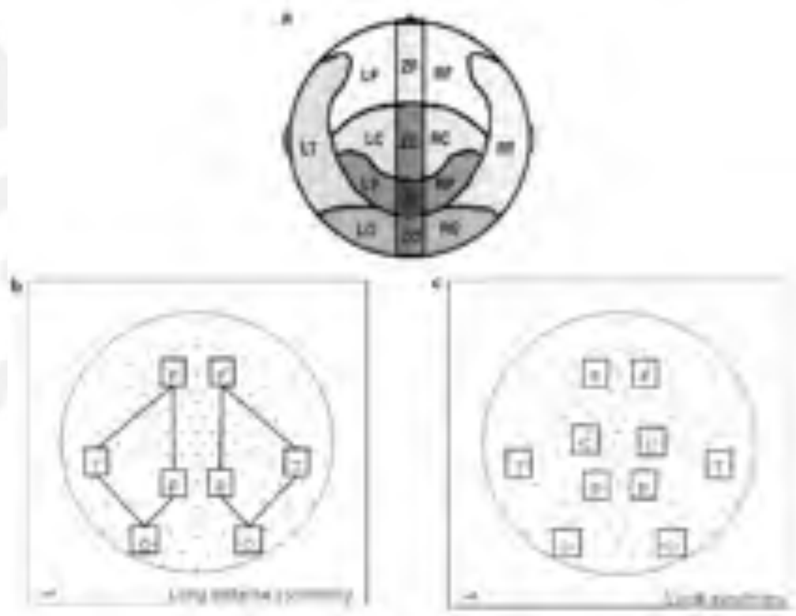


Alzheimer's disease: Targeted attack on hubs!

Parkinson's

Major depressive disorder

Epilepsy



brain tumor patients

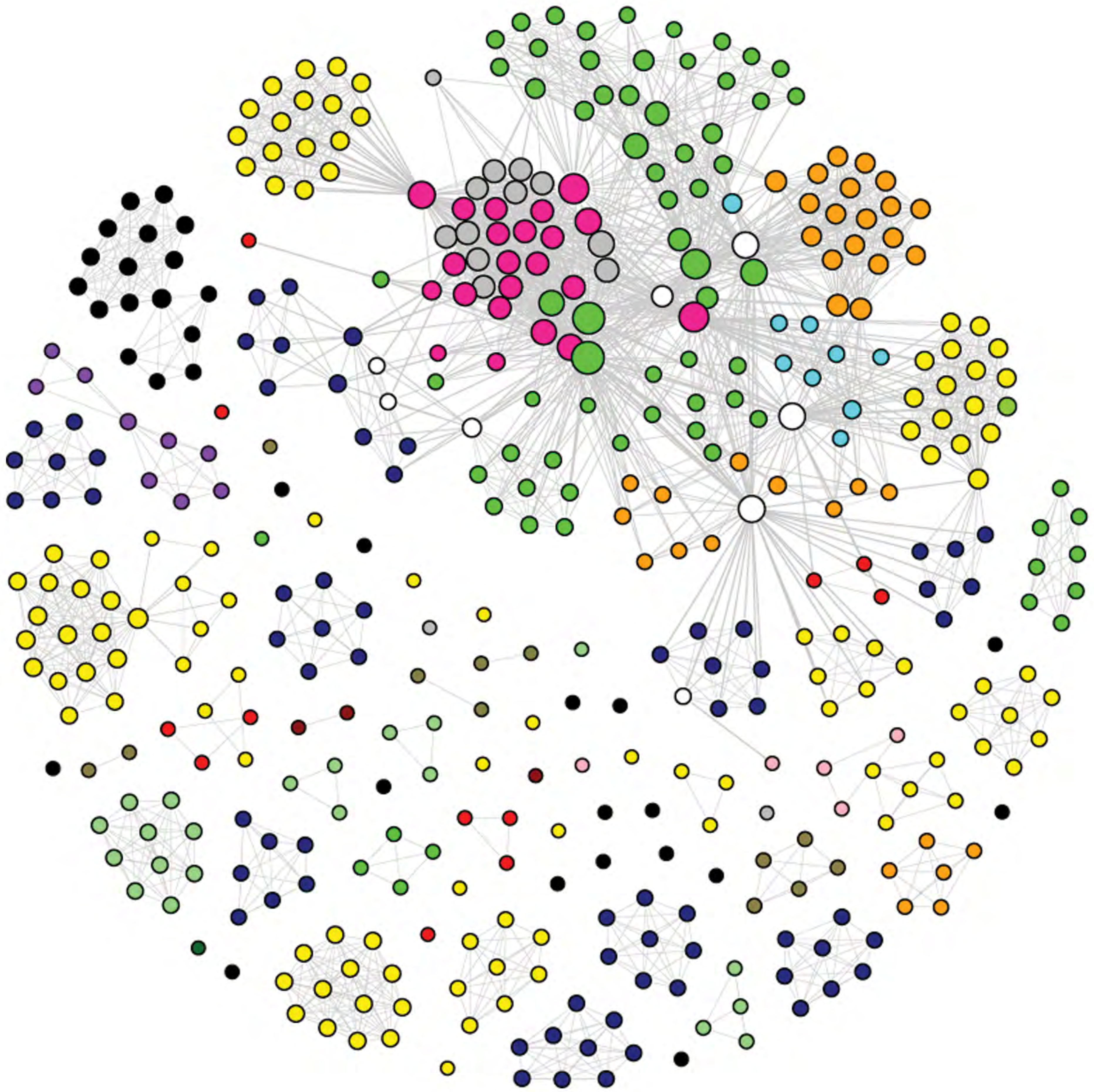
Bartolomei, F., Bosma, I., Klein, M., Baayen, J. C., Reijneveld, J. C., Postma, T. J., et al. (2006). Disturbed functional connectivity in brain tumour patients: evaluation by graph analysis of synchronization matrices. Clinical neurophysiology, 117(9), 2039-49. doi: 10.1016/j.clinph.2006.05.018.

Ponten, S. C., Douw, L., Bartolomei, F., Reijneveld, J. C., & Stam, C. J. (2009). Indications for network regularization during absence seizures: weighted and unweighted graph theoretical analyses. Experimental neurology, 217(1), 197-204. Elsevier Inc. doi: 10.1016/j.expneurol.2009.02.001.

Stam, C. J., Haan, W. de, Daffertshofer, a, Jones, B. F., Manshanden, I., Cappellen van Walsum, a M. van, et al. (2009). Graph theoretical analysis of magnetoencephalographic functional connectivity in Alzheimer's disease. Brain : a journal of neurology, 132(Pt 1), 213-24. doi: 10.1093/brain/awn262.

Stam, C. J. (2010). Use of magnetoencephalography (MEG) to study functional brain networks in neurodegenerative disorders. Journal of the neurological sciences, 289(1-2), 128-34. Elsevier B.V. doi: 10.1016/j.jns.2009.08.028.

Symptom networks
Small-world of DSM-IV



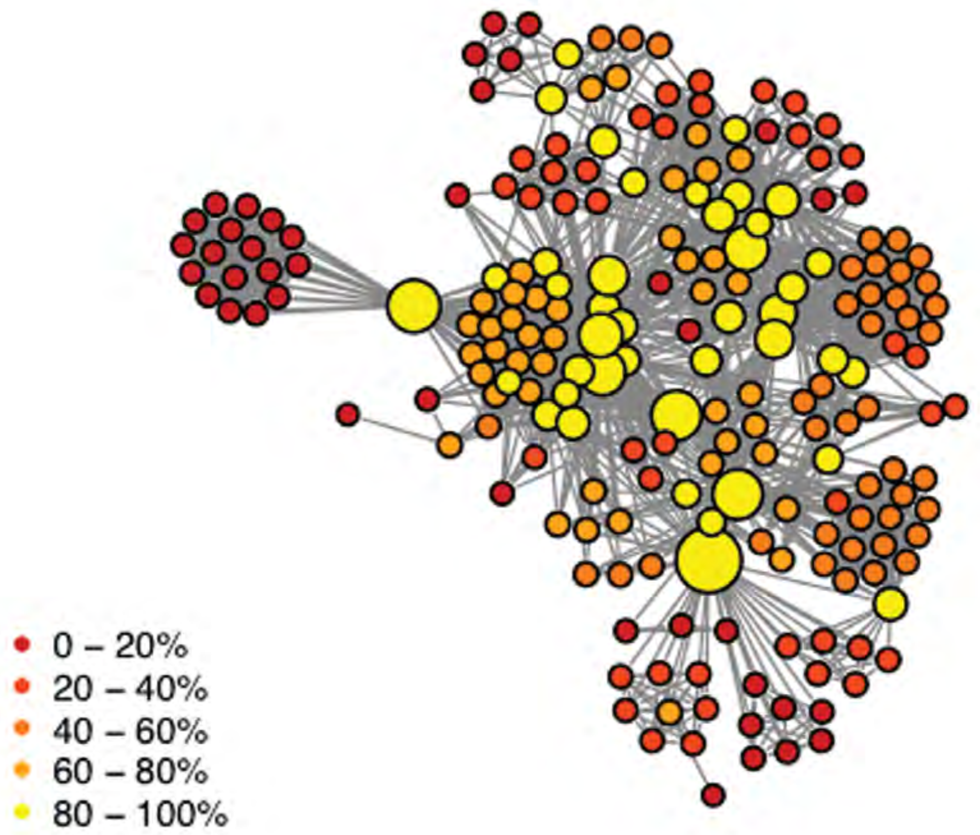
Problemen met het medisch model by psychopathologie:

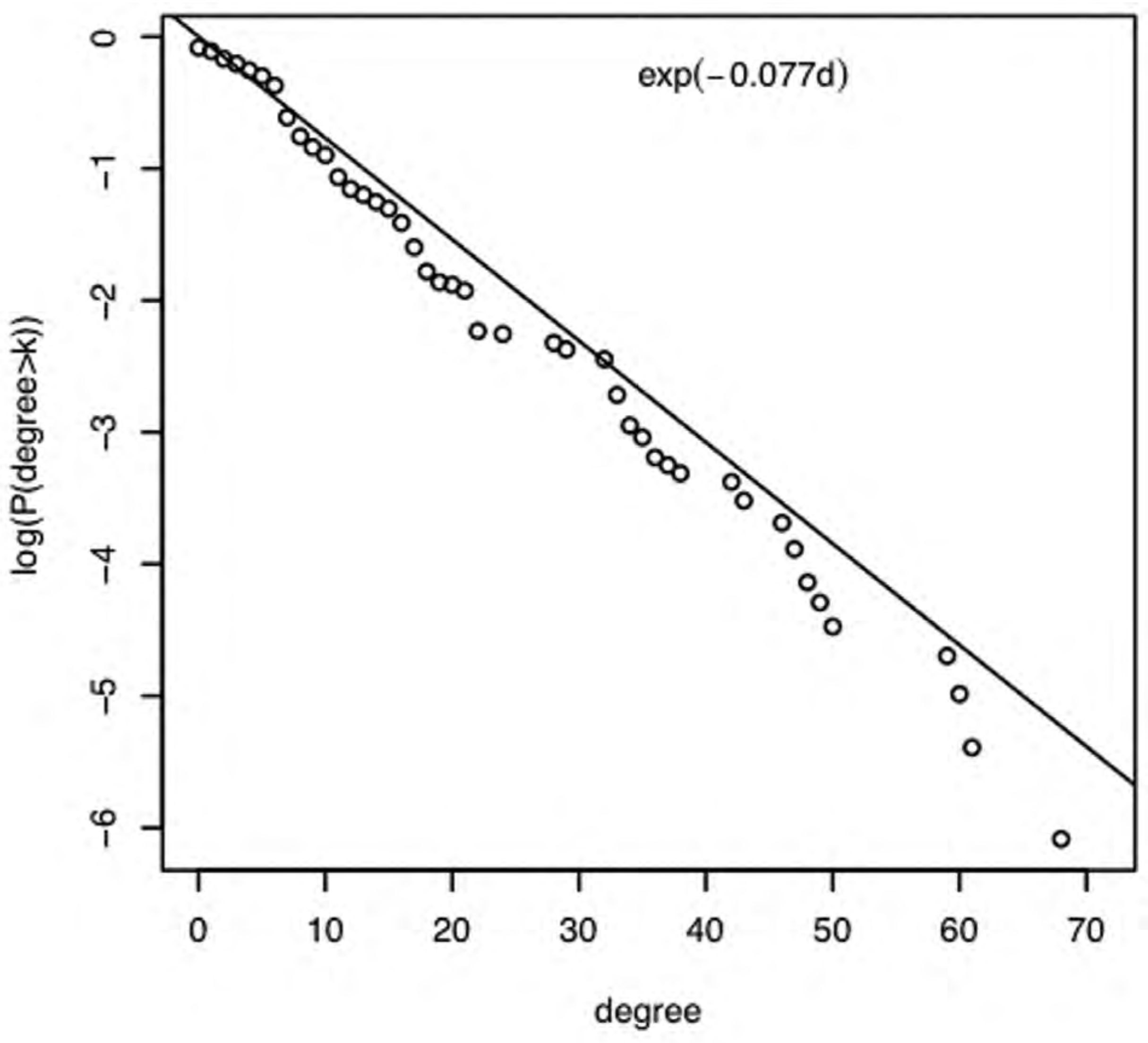
- geen unieke veroorzaker voor symptomen, zoals griepvirus
- symptomen zijn vaak de diagnose en oorzaak tegelijk

- Disorders usually first diagnosed in infancy, childhood or adolescence
- Delirium, dementia, and amnesia and other cognitive disorders
- Mental disorders due to a general medical condition
- Substance-related disorders
- Schizophrenia and other psychotic disorders
- Mood disorders
- Anxiety disorders
- Somatoform disorders
- Facitious disorders
- Dissociative disorders
- Sexual and gender identity disorders
- Eating disorders
- Sleep disorders
- Impulse control disorders not elsewhere classified
- Adjustment disorders
- Personality disorders
- Symptom is featured equally in multiple chapters

Borsboom D, Cramer AOJ, Schmittmann VD, Epskamp S, Waldorp LJ (2011) The Small World of Psychopathology. PLoS ONE 6(11): e27407.
doi:10.1371/journal.pone.0027407

Symptom networks





We show that

a) half of the symptoms in the DSM-IV network are connected,

b) the architecture of these connections conforms to **a small world structure**, featuring a **high degree of clustering** but a **short average path length**, and

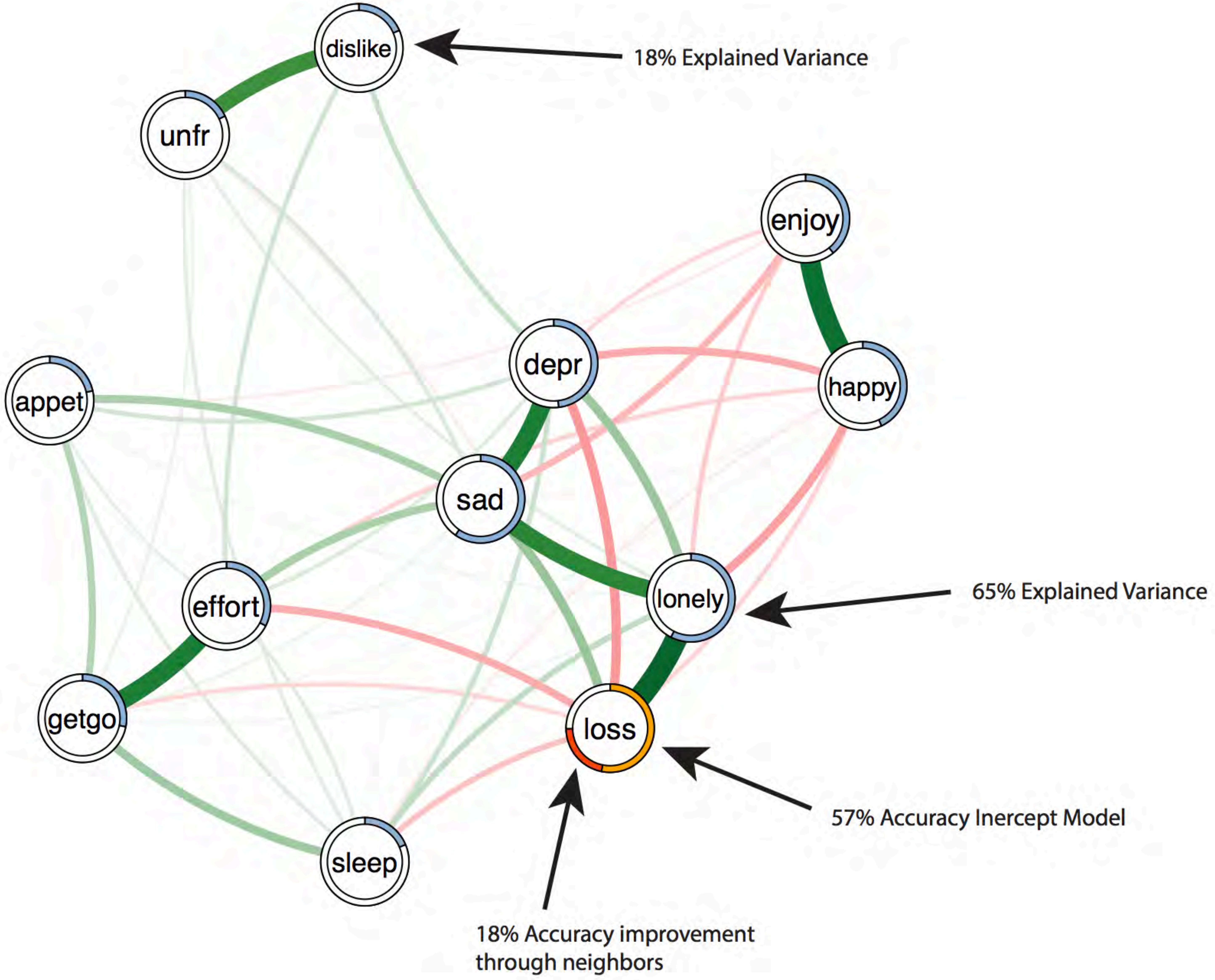
c) distances between disorders in this **structure predict empirical comorbidity rates**. Network simulations of Major Depressive Episode and Generalized Anxiety Disorder show that the model faithfully reproduces empirical population statistics for these disorders.

Borsboom D, Cramer AOJ, Schmittmann VD, Epskamp S, Waldorp LJ (2011) The Small World of Psychopathology. PLoS ONE 6(11): e27407.
doi:10.1371/journal.pone.0027407

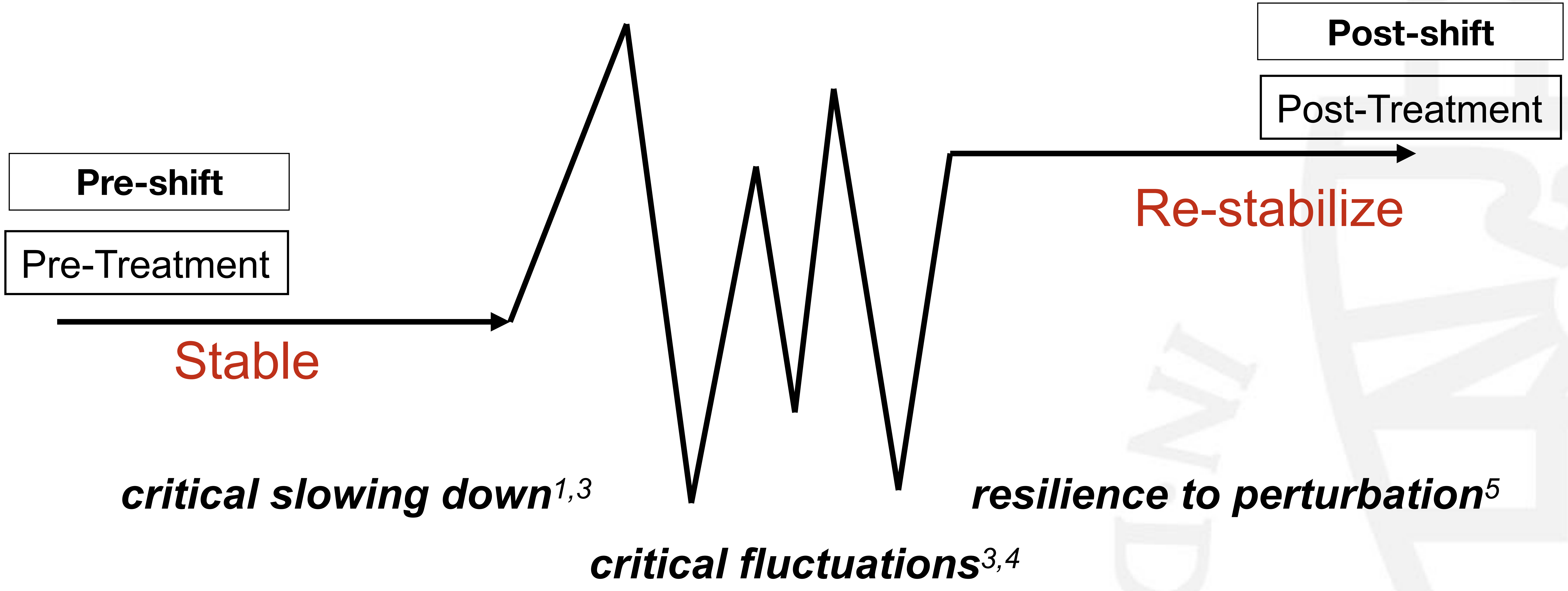
Radboud Universiteit

Symptom netwerken

psychosystems.org



Period of Destabilization



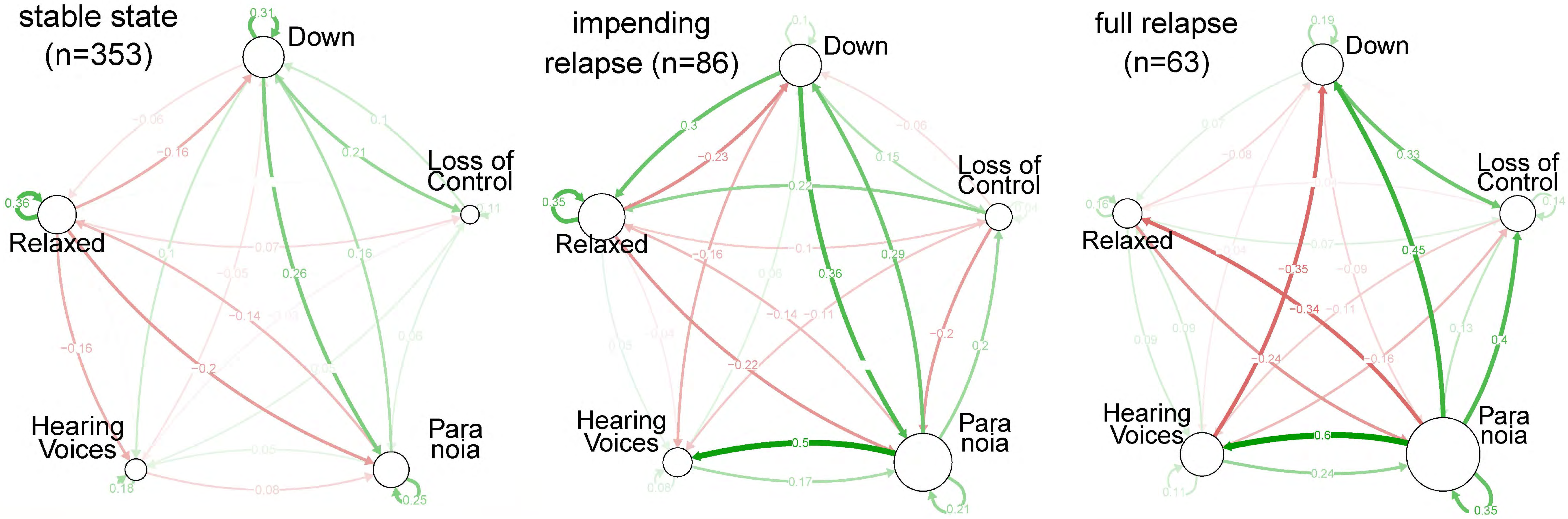
- increase in recovery and switching time after perturbation
- increase in variance, autocorrelation, long-range dependence
- increase in occurrence and diversity of unstable states
- increase in the entropy of the distribution of state occurrences

[1]Scholz JP, Kelso JAS, Schöner G. (1987). Nonequilibrium phase transitions in coordinated biological motion: critical slowing down and switching time. *Physics Letters A 123,* 390–394.
[2]Scheffer M, Bascompte J, Brock W A, Brovkin V, Carpenter SR, Dakos V, Held H, van Nes EH, Rietkerk M, Sugihara G. (2009). Early-warning signals for critical transitions. *Nature 461,* 53–9.
[3]Stephen DG, Dixon JA, Isenhower RW. (2009). Dynamics of representational change: Entropy, Action and Cognition. *JEP: Human Perception and Performance 35,* 1811–1832.
[4]Schiepek G, Strunk G. (2010). The identification of critical fluctuations and phase transitions in short term and coarse-grained time series … *Biological cybernetics 102,*197–207.

An n=1 Clinical Network Analysis of Symptoms and Treatment in Psychosis

variation in 'hearing voices', 'down', 'paranoia', 'loss of control' and 'relaxed' (range 1-7) during a year

An n=1 Clinical Network Analysis of Symptoms and Treatment in Psychosis



An n=1 Clinical Network Analysis of Symptoms and Treatment in Psychosis

	Betweenness	Closeness	Inward degree	Outward degree	Node strength
Stable state					
'Down'	5	0.032	0.78	0.92	1.70
'Loss of control'	0	0.015	0.45	0.31	0.76
'Paranoia'	4	0.026	0.87	0.62	1.48
'Hearing Voices'	0	0.015	0.51	0.39	0.90
'Relaxed'	1	0.036	0.61	0.98	1.59
Impending relapse					
'Down'	2	0.056	0.74	1.07	1.81
'Loss of control'	0	0.040	0.51	0.64	1.15
'Paranoia'	7	0.058	1.16	1.34	2.49
'Hearing Voices'	0	0.026	0.90	0.35	1.25
'Relaxed'	0	0.039	1.05	0.95	2.00
Full Relapse state					
'Down'	1	0.025	1.08	0.72	1.80
'Loss of control'	3	0.027	1.12	0.44	1.56
'Paranoia'	7	0.109	1.04	2.18	3.22
'Hearing Voices'	0	0.050	0.95	0.95	1.90
'Relaxed'	0	0.041	0.69	0.59	1.28

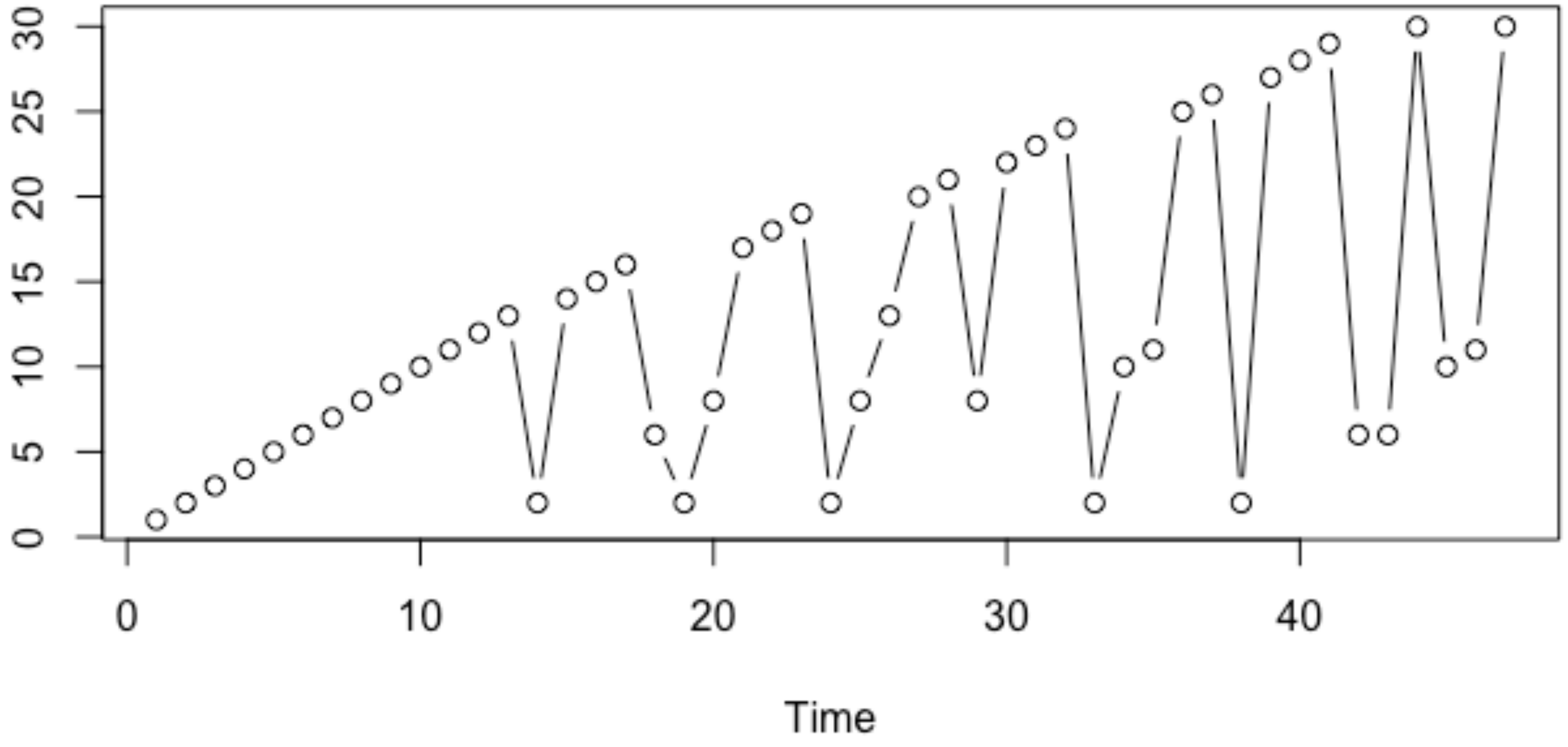
The Spearman correlations as suggested by reviewer #1

doi:10.1371/journal.pone.0162811.t002

Recurrence Quantification Analysis: Nominale Tijdseries

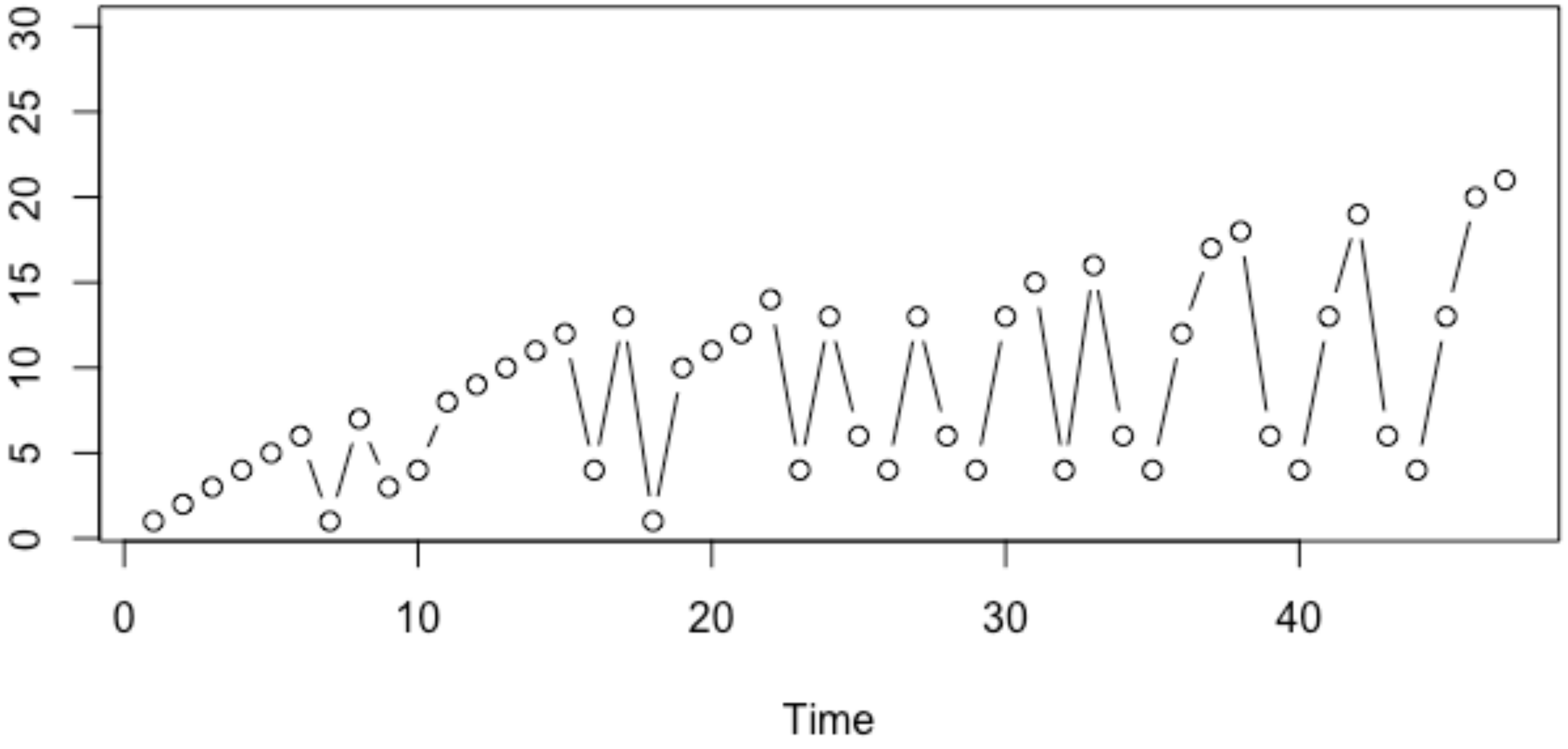
VERHAAL 1

"1 2 3 4 5 6 7 8 9 10 11 12 13 2 14 15 16 6 2 8 17 18 19 2 8 13 20 21 8 22 23 24 2 10 11 25 26 2 27 28 29 6 6 30 10 11 30"



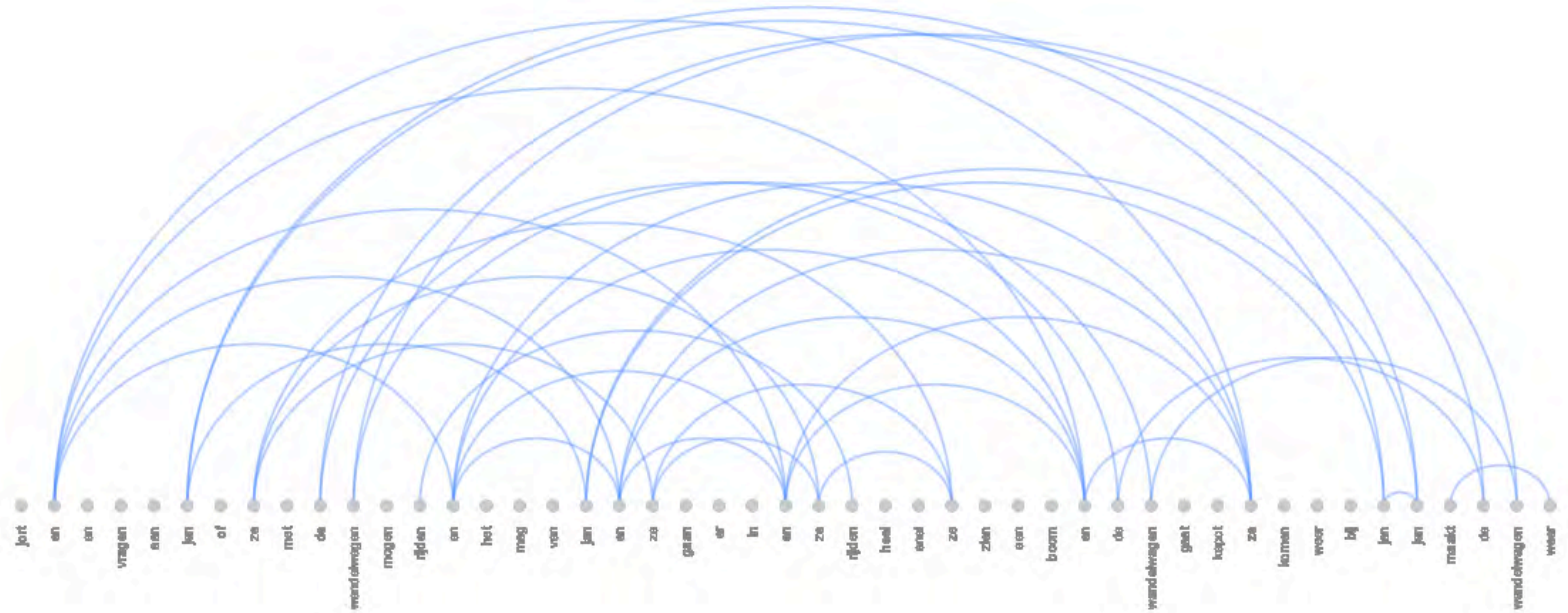
VERHAAL 2

"1 2 3 4 5 6 1 7 3 4 8 9 10 11 12 4 13 1 10 11 12 14 4 13 6 4 13 6 4 13 15 4 16 6 4 12 17 18 6 4 13 19 6 4 13 20 21"

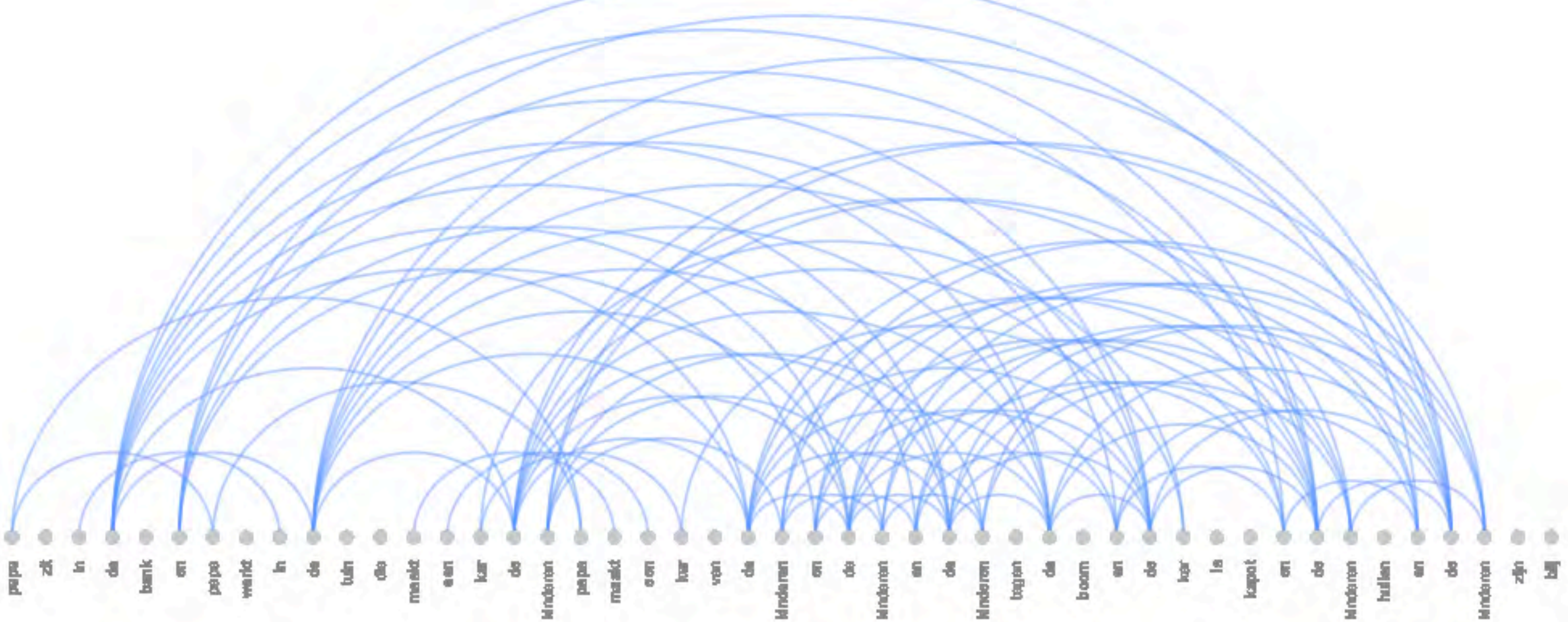


Data from: Huijgevoort, M. A. E. V. (2008). *Improving beginning literacy skills by means of an interactive computer environment* (Doctoral dissertation). http://repository.ubn.ru.nl/bitstream/handle/2066/45170/45170_imprbelis.pdf

Recurrence Network - Verhaal 1

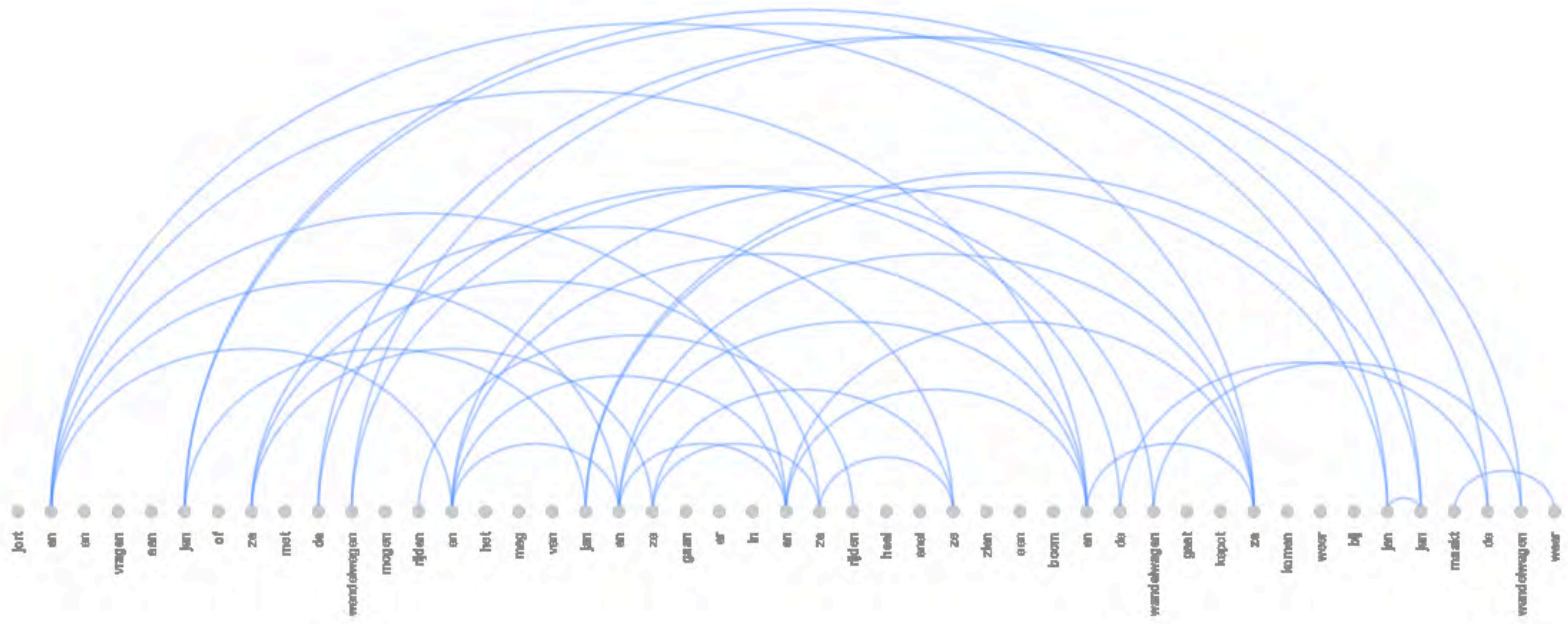


Recurrence Network - Verhaal 2

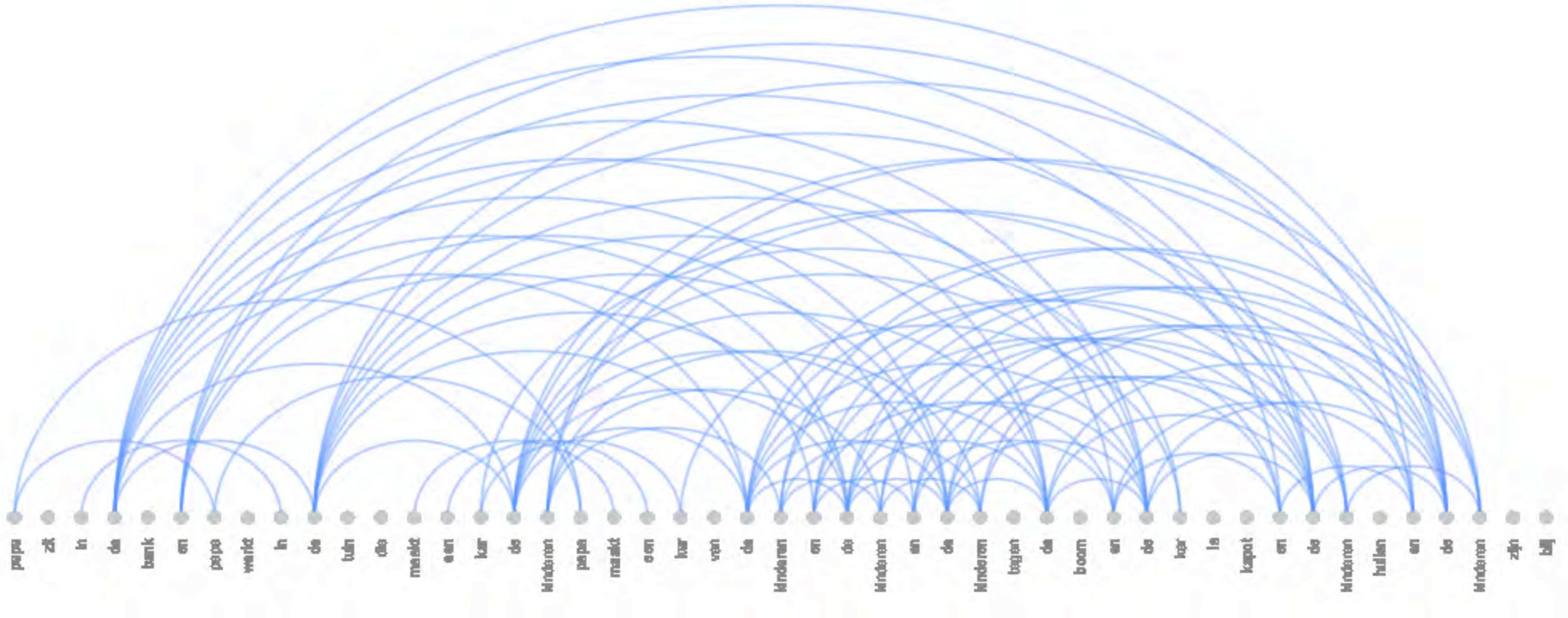


Recurrence Network

Total Degree = 70 *verhaal 1*



Total Degree = 168 *verhaal 2*



Ecological Momentary Assessment

- Measurement is considered to be like classical physical measurement:

➡It is very problematic to interpret measurement outcomes as properties of the theoretical object of measurement (interaction between instrument and object, by merely asking to rate “I feel happy today”)

➡Measurement invariance, monotonicity, etc. do not hold

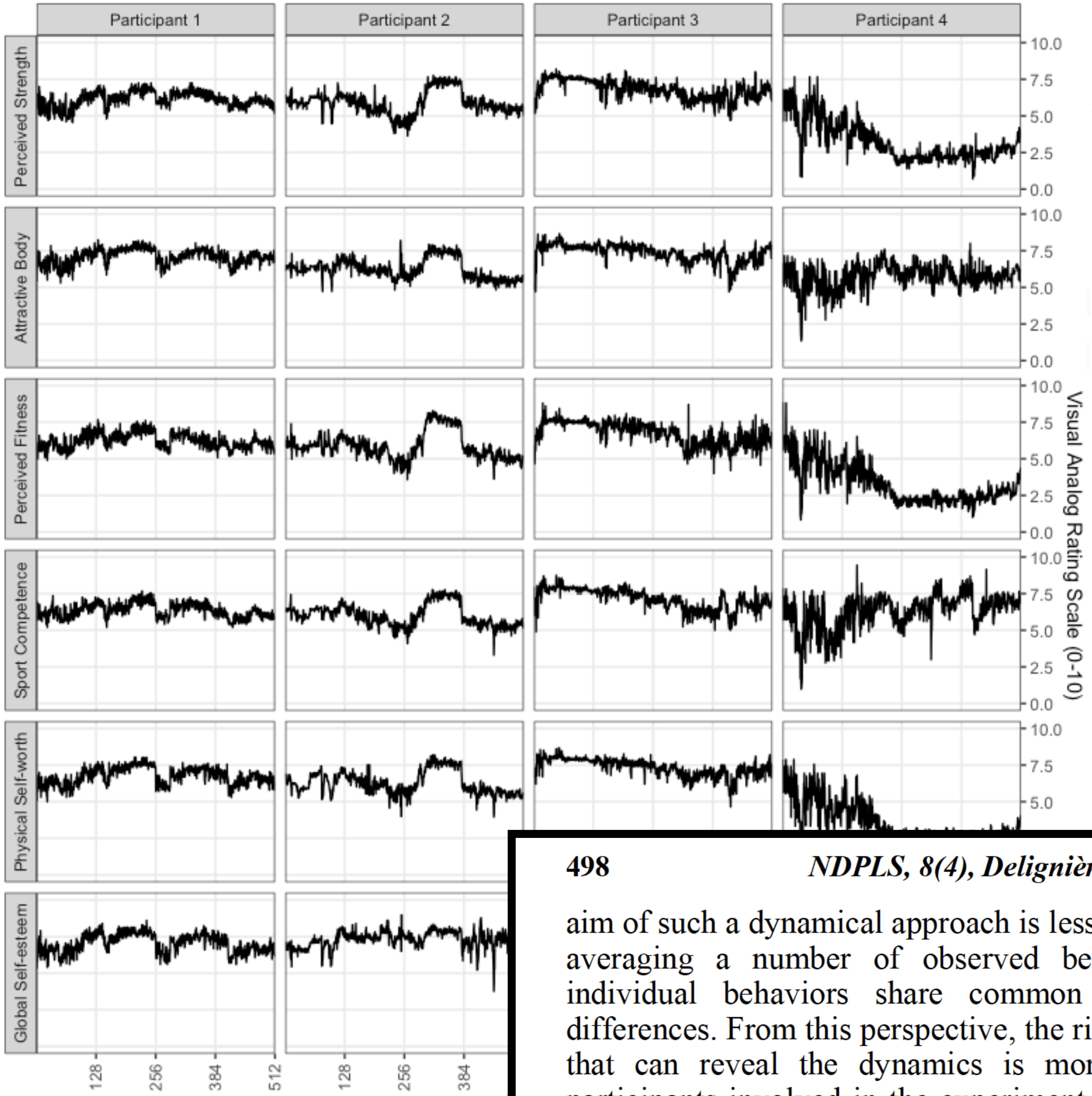
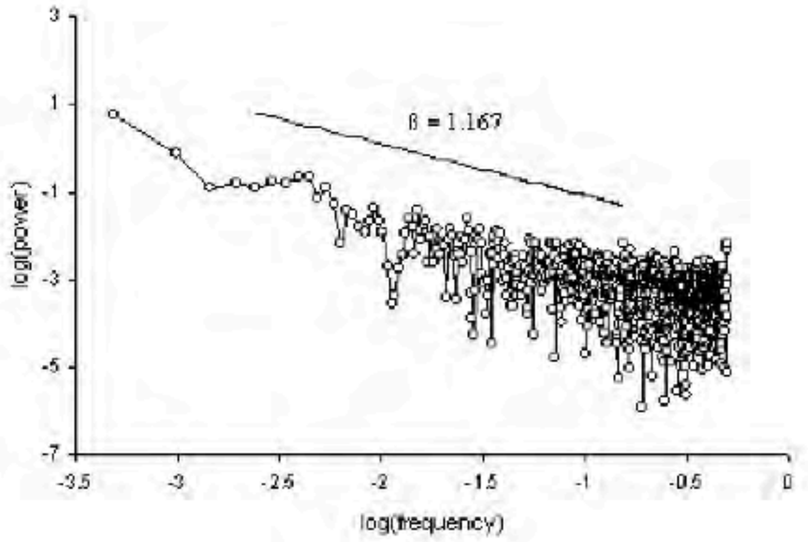


Table 2. Individual Moving Average Coefficients (θ) Obtained through ARIMA Modeling.

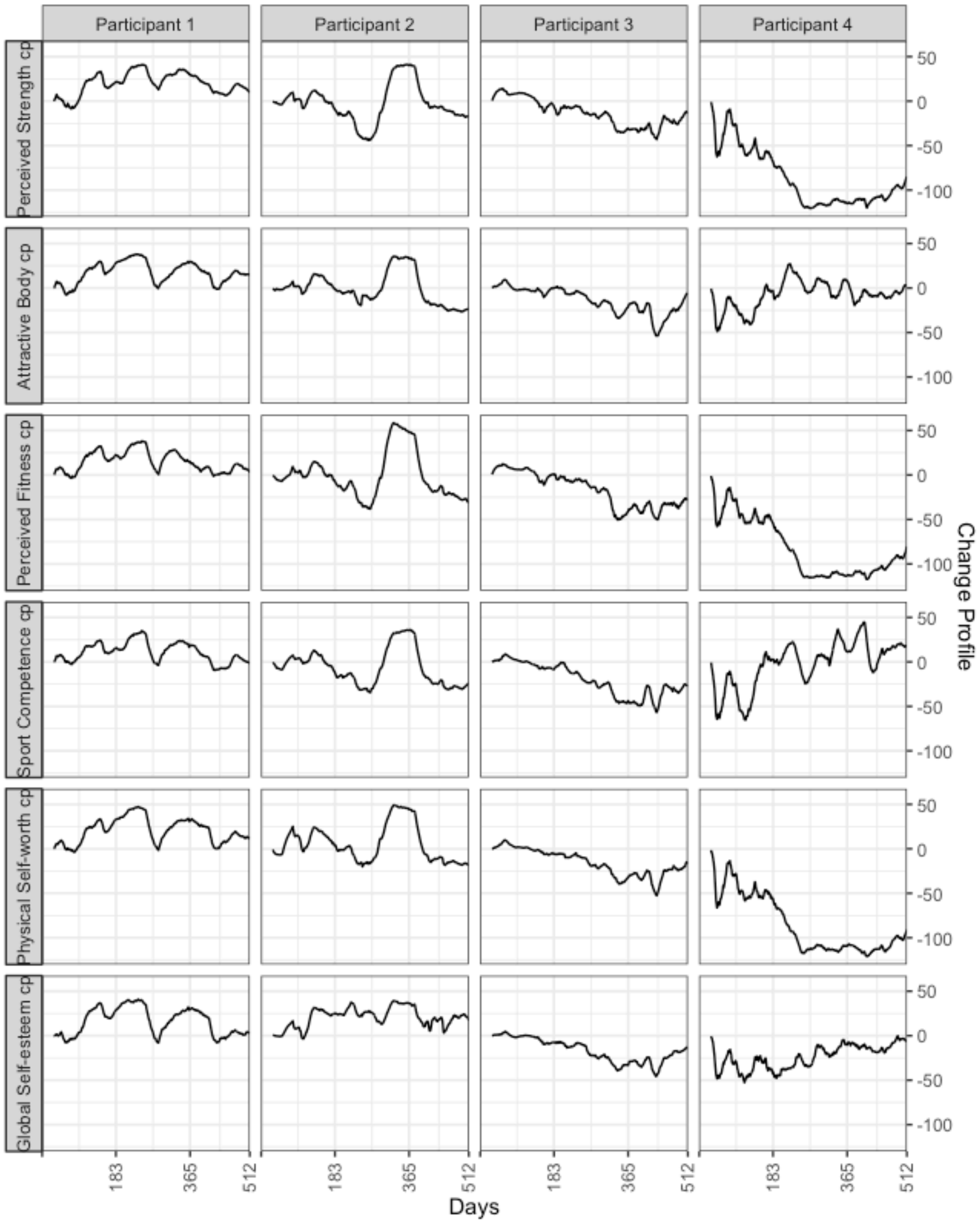
Participant	GSE	PSW	PC	SC	APP	PS
1	0.58	0.65	0.70	0.66	0.63	0.69
2	0.35	0.46	0.48	0.50	0.45	0.46
3	0.58	0.65	0.75	0.63	0.56	0.68
4	0.66	0.56	0.60	0.59	0.64	0.53

Table 3. Individual ß Exponents Obtained with Spectral Analysis.

Participant	GSE	PSW	PC	SC	APP	PS
1	1.17	1.15	0.95	1.00	1.15	0.95
2	1.13	1.39	1.36	1.24	1.27	1.23
3	1.09	1.05	0.96	1.34	1.12	1.11
4	0.96	1.14	1.02	1.18	0.95	1.05



aim of such a dynamical approach is less to derive an epistemic model by averaging a number of observed behaviors than to evidence that individual behaviors share common dynamics, despite superficial differences. From this perspective, the richness of the individual data sets that can reveal the dynamics is more crucial than the number of participants involved in the experiment. Researchers in many fields, for

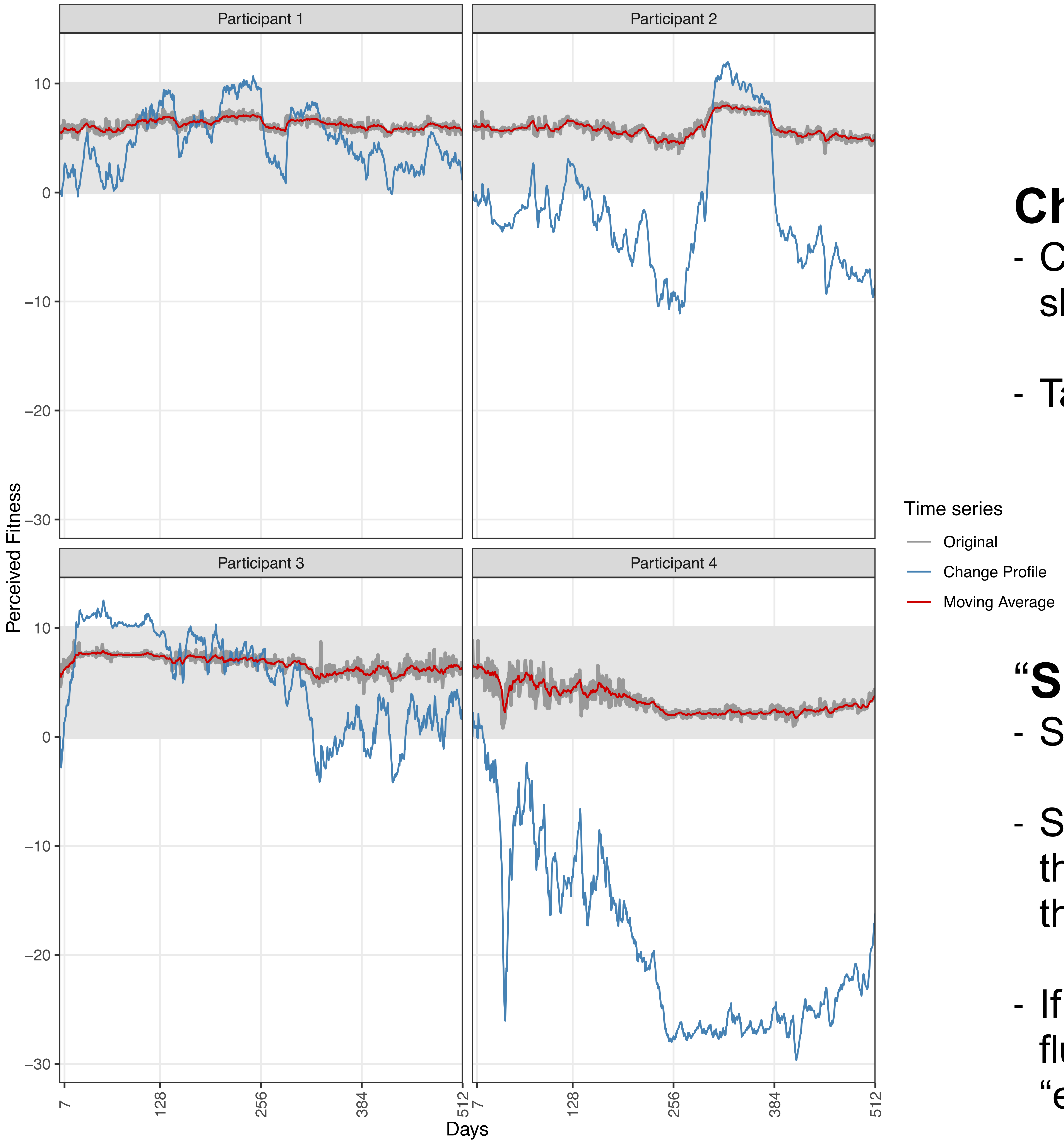


Change Profiles:

- Center on a moving average in a sliding window
- Take the cumulative sum

"Solves" some concerns:

- Scale is irrelevant/relative
- Small fluctuations are added in the cum. sum but, don't impact the shape of the overall profile
- If present, persistent levels & fluctuation patterns can be "exaggerated" (see y-scale)



Change Profiles:

- Center on a moving average in a sliding window
- Take the cumulative sum

"Solves" some concerns:

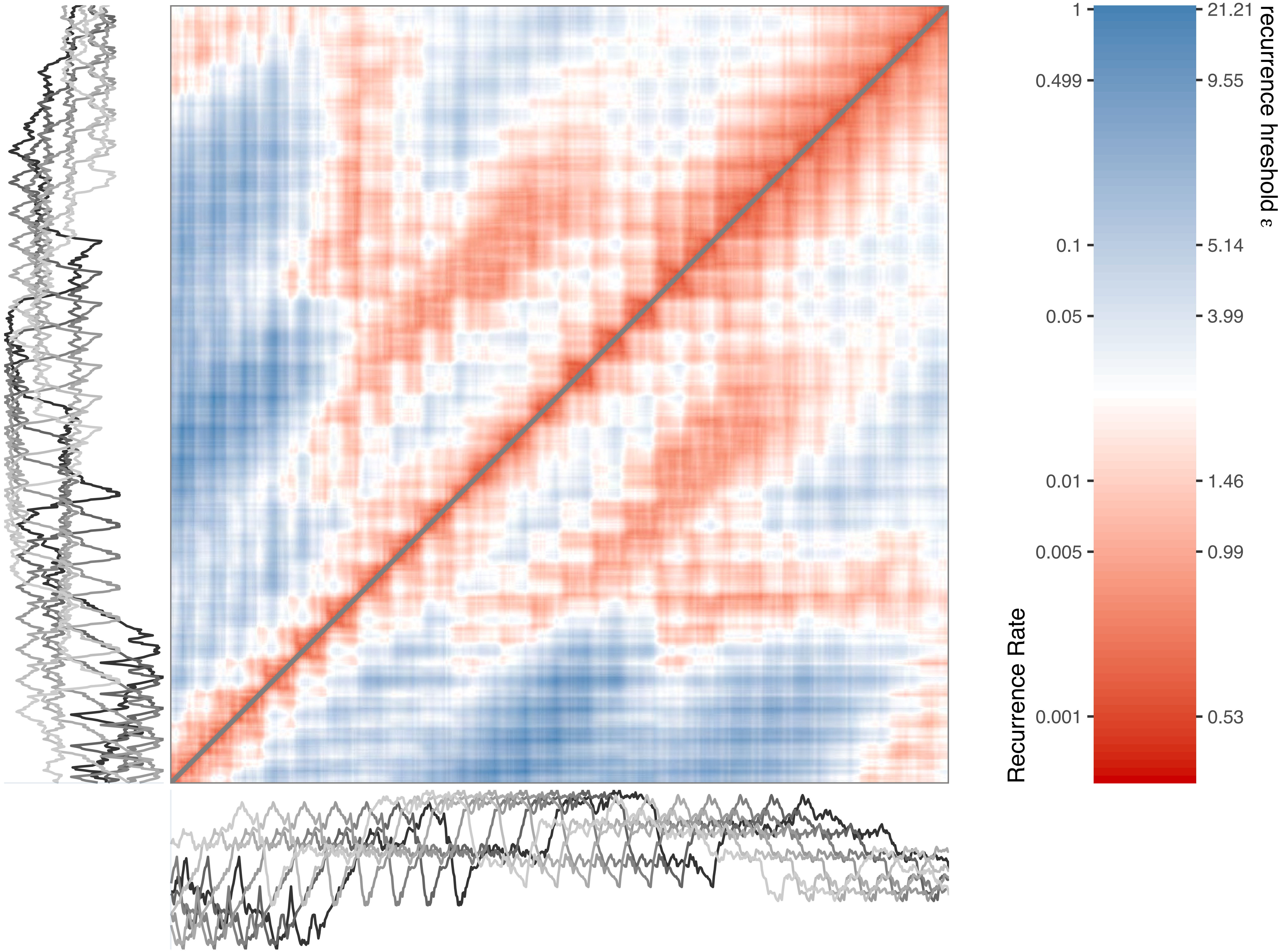
- Scale is irrelevant/relative
- Small fluctuations are added in the cum. sum but, don't impact the shape of the overall profile
- If present, persistent levels & fluctuation patterns can be "exaggerated" (see y-scale)

Recurrence Based Approach

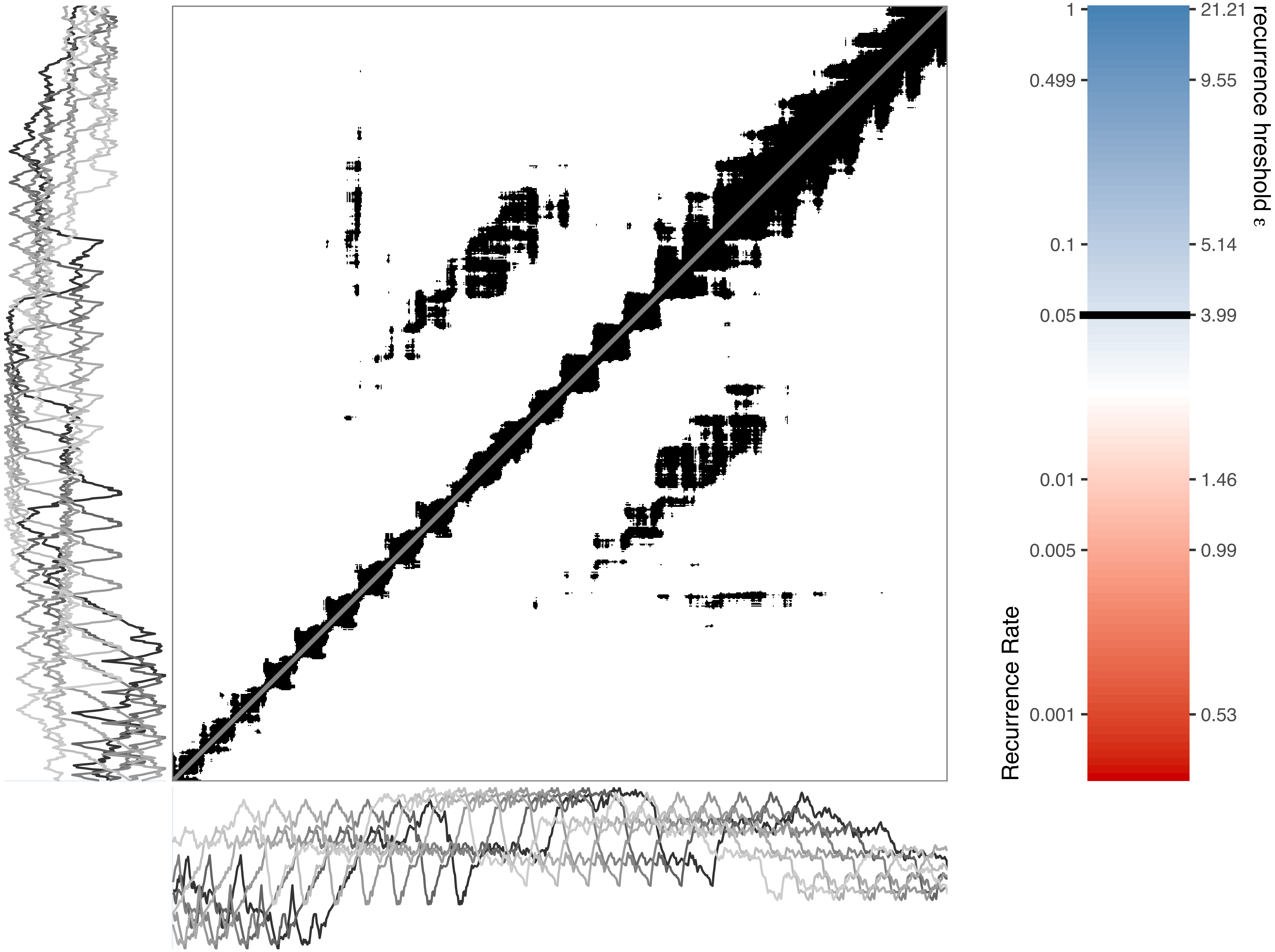
Quantifying State Dynamics

- Essentially “model free”
- Few data assumptions
- Can detect and quantify nonstationarity, etc
- Recurrence Networks can deal with multivariate time series data

Distance Matrix >> Recurrence Matrix



Recurrence/Incidence/Adjacency Matrix



Physics Reports 787 (2019) 1–97

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

Complex network approaches to nonlinear time series analysis

Yong Zou [a,*], Reik V. Donner [b,c,**], Norbert Marwan [c], Jonathan F. Donges [c,d], Jürgen Kurths [c,e,f]

[a] *Department of Physics, East China Normal University, Shanghai 200062, China*
[b] *Department of Water, Environment, Construction and Safety, Magdeburg–Stendal University of Applied Sciences, Breitscheidstraße 2, 39114 Magdeburg, Germany*
[c] *Potsdam Institute for Climate Impact Research (PIK) – Member of the Leibniz Association, Telegrafenberg A31, 14473 Potsdam, Germany*
[d] *Stockholm Resilience Centre, Stockholm University, Kräftriket 2B, 114 19 Stockholm, Sweden*
[e] *Saratov State University, 4410012 Saratov, Russia*
[f] *Department of Physics, Humboldt University Berlin, Newtonstraße 15, 12489 Berlin, Germany*

A R T I C L E I N F O

Article history:
Accepted 22 October 2018
Available online 2 November 2018
Editor: I. Procaccia

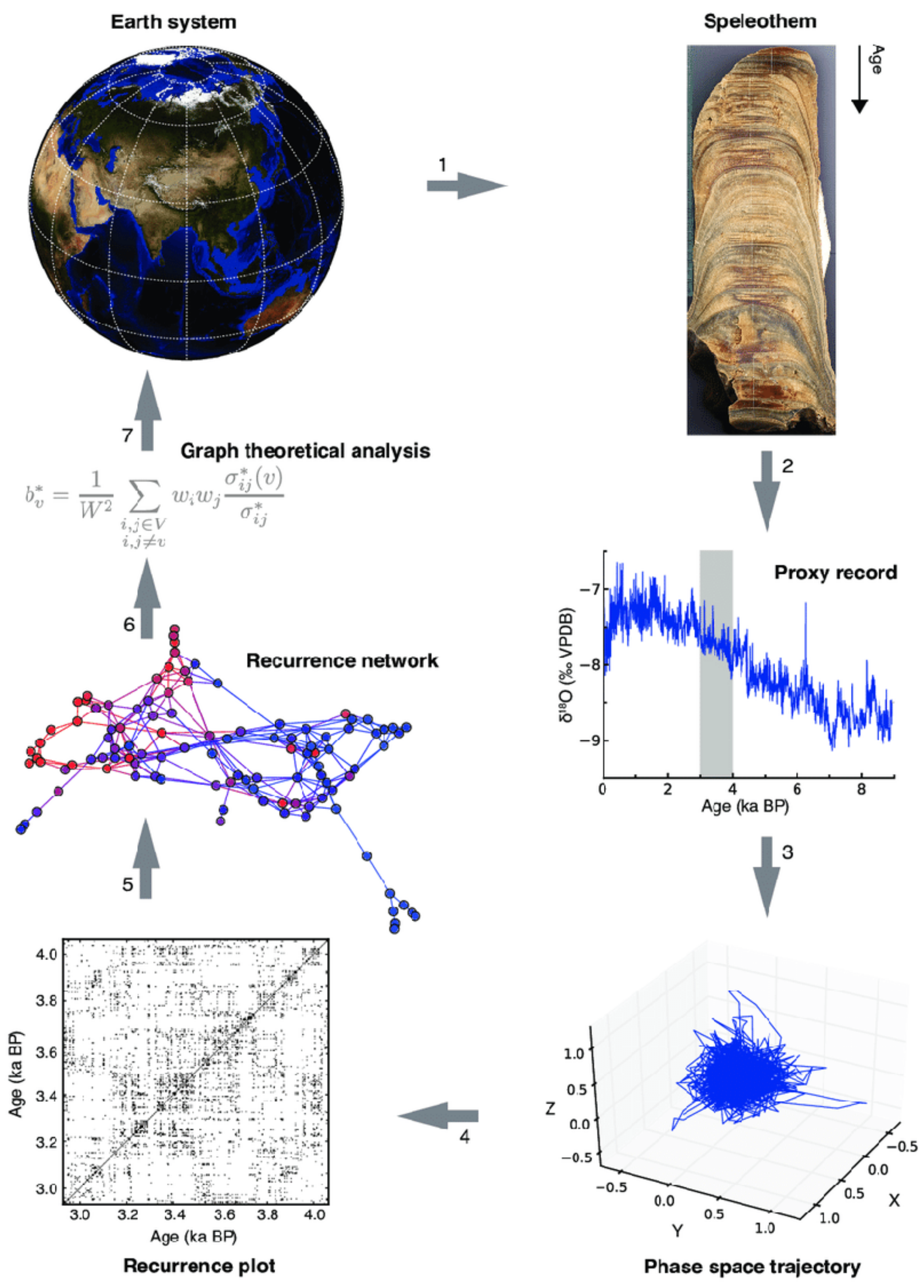
Keywords:
Complex networks
Nonlinear dynamics
Recurrences
Visibility
Transition networks

A B S T R A C T

In the last decade, there has been a growing body of literature addressing the utilization of complex network methods for the characterization of dynamical systems based on time series. While both nonlinear time series analysis and complex network theory are widely considered to be established fields of complex systems sciences with strong links to nonlinear dynamics and statistical physics, the thorough combination of both approaches has become an active field of nonlinear time series analysis, which has allowed addressing fundamental questions regarding the structural organization of nonlinear dynamics as well as the successful treatment of a variety of applications from a broad range of disciplines. In this report, we provide an in-depth review of existing approaches of time series networks, covering their methodological foundations, interpretation and practical considerations with an emphasis on recent developments. After a brief outline of the state-of-the-art of nonlinear time series analysis and the theory of complex networks, we focus on three main network approaches, namely, phase space based recurrence networks, visibility graphs and Markov chain based transition networks, all of which have made their way from abstract concepts to widely used methodologies. These three concepts, as well as several variants thereof will be discussed in great detail regarding their specific properties, potentials and limitations. More importantly, we emphasize which fundamental new insights complex network approaches bring into the field of nonlinear time series analysis. In addition, we summarize examples from the wide range of recent applications of these methods, covering rather diverse fields like climatology, fluid dynamics, neurophysiology, engineering and economics, and demonstrating the great potentials of time series networks for tackling real-world contemporary scientific problems. The overall aim of this report is to provide the readers with the knowledge how the complex network approaches can be applied to their own field of real-world time series analysis.

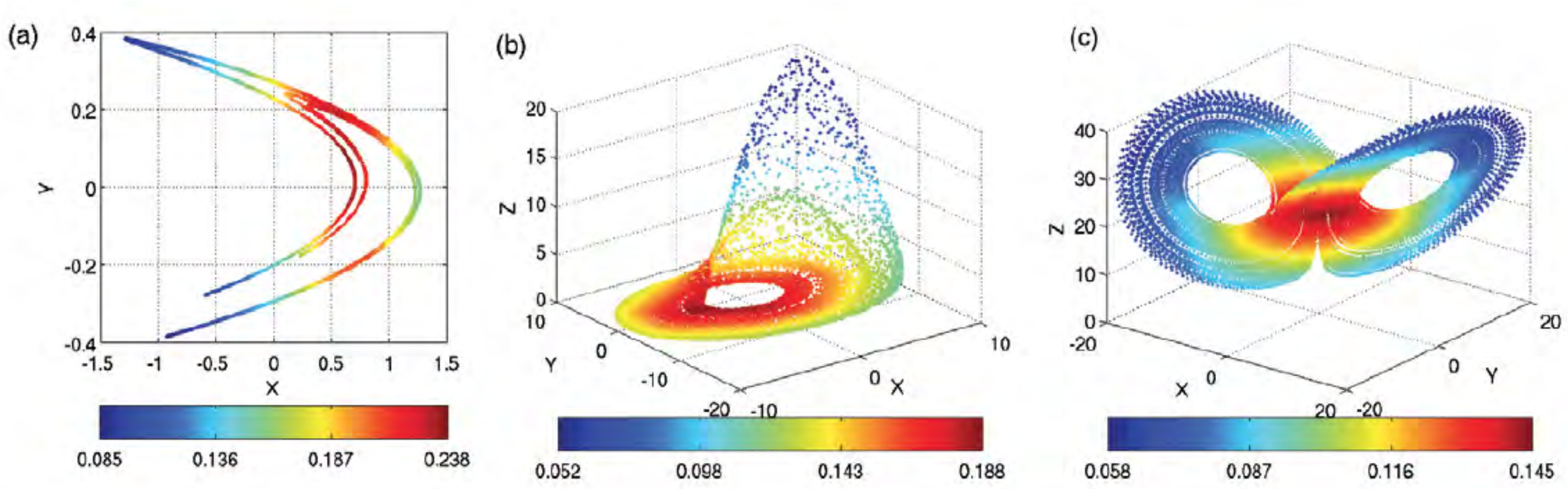
Recurrence/Incidence/Adjacency Matrix

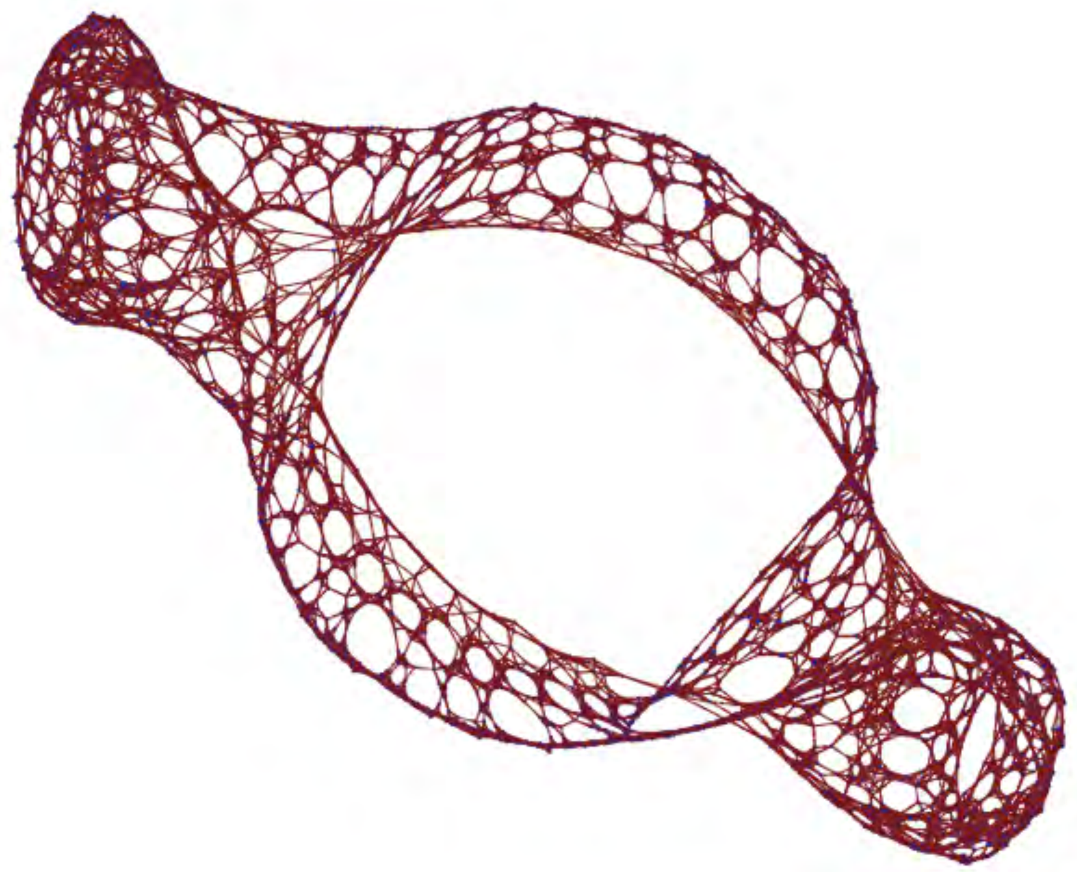
- Time points are nodes (vertices)
- Nodes are connected by lines (edges) if they recur at some later point in time
- Network measures can be interpreted as quantifying aspects of the temporal dynamics, some are equivalent to RQA measures… some are different (e.g. weighted and/or directed network measures)

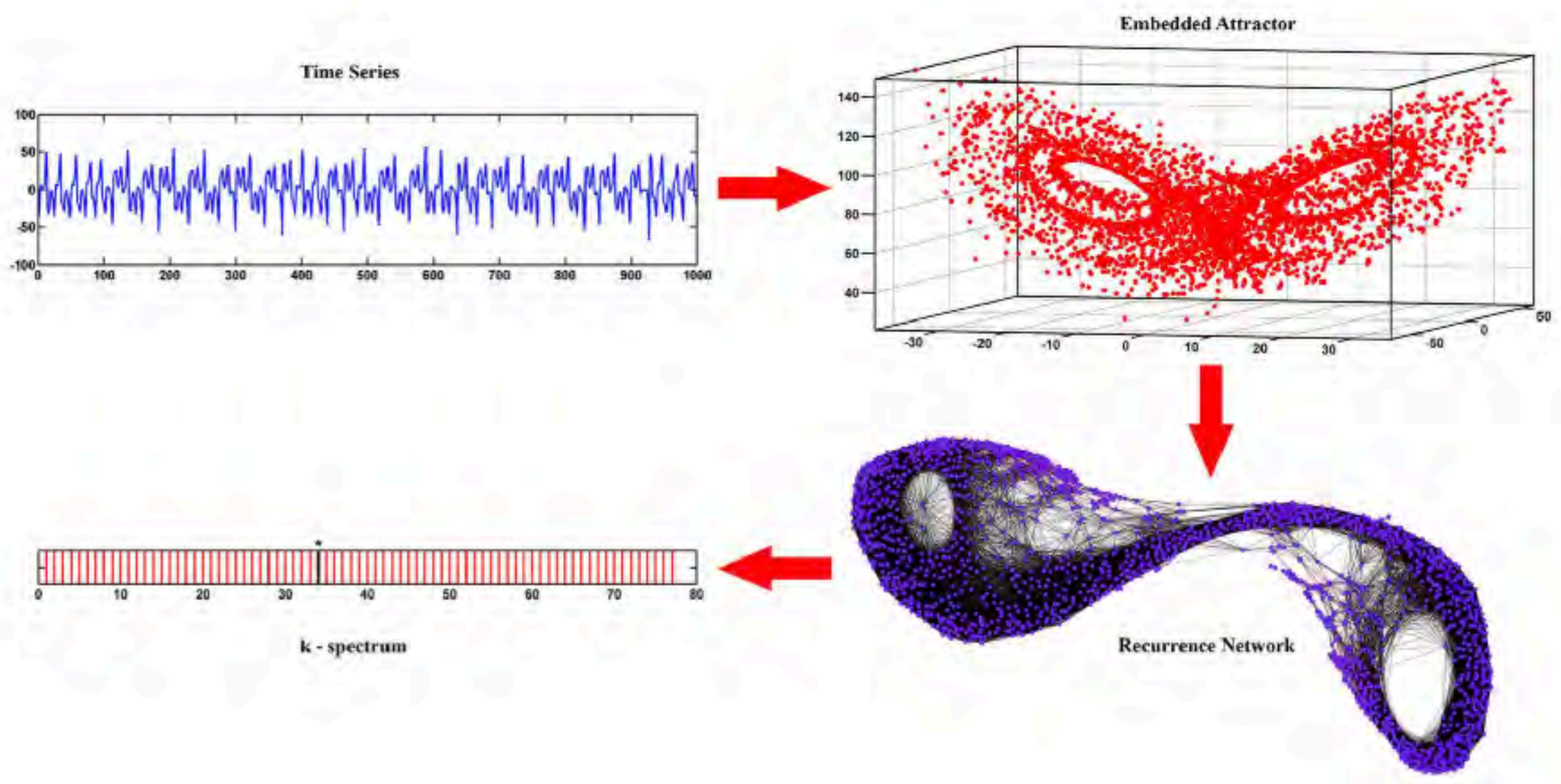


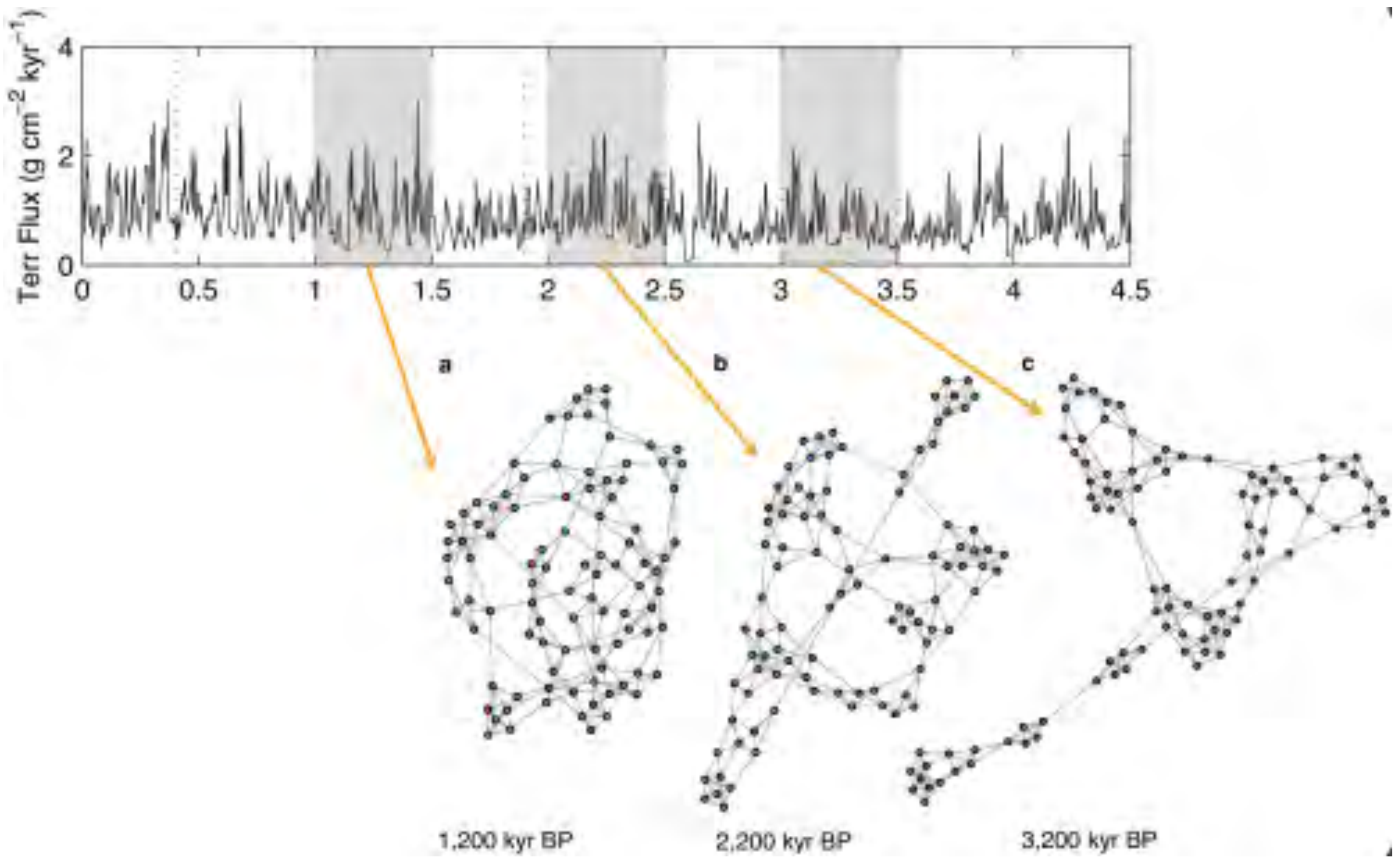
Donges, J. F., Donner, R., Marwan, N., Breitenbach, S. F., Rehfeld, K., & Kurths, J. (2015). Non-linear regime shifts in Holocene Asian monsoon variability: potential impacts on cultural change and migratory patterns. *Climate of the Past*, *11*(5), 709-741.

Recurrence/Incidence/Adjacency Matrix









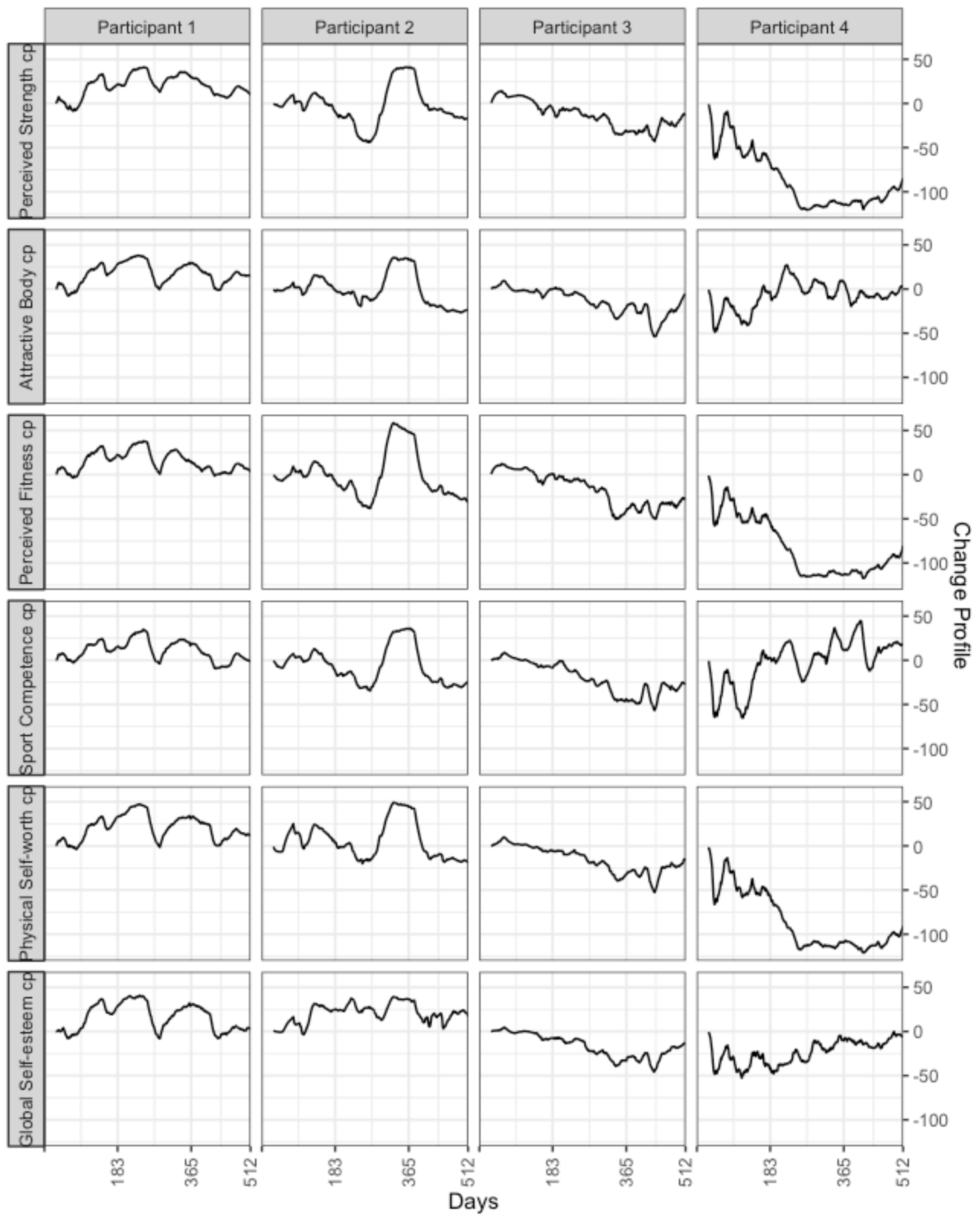
Small, M., Zhang, J., & Xu, X. (2009). Transforming time series into complex networks. Complex Sciences, 2078–2089.

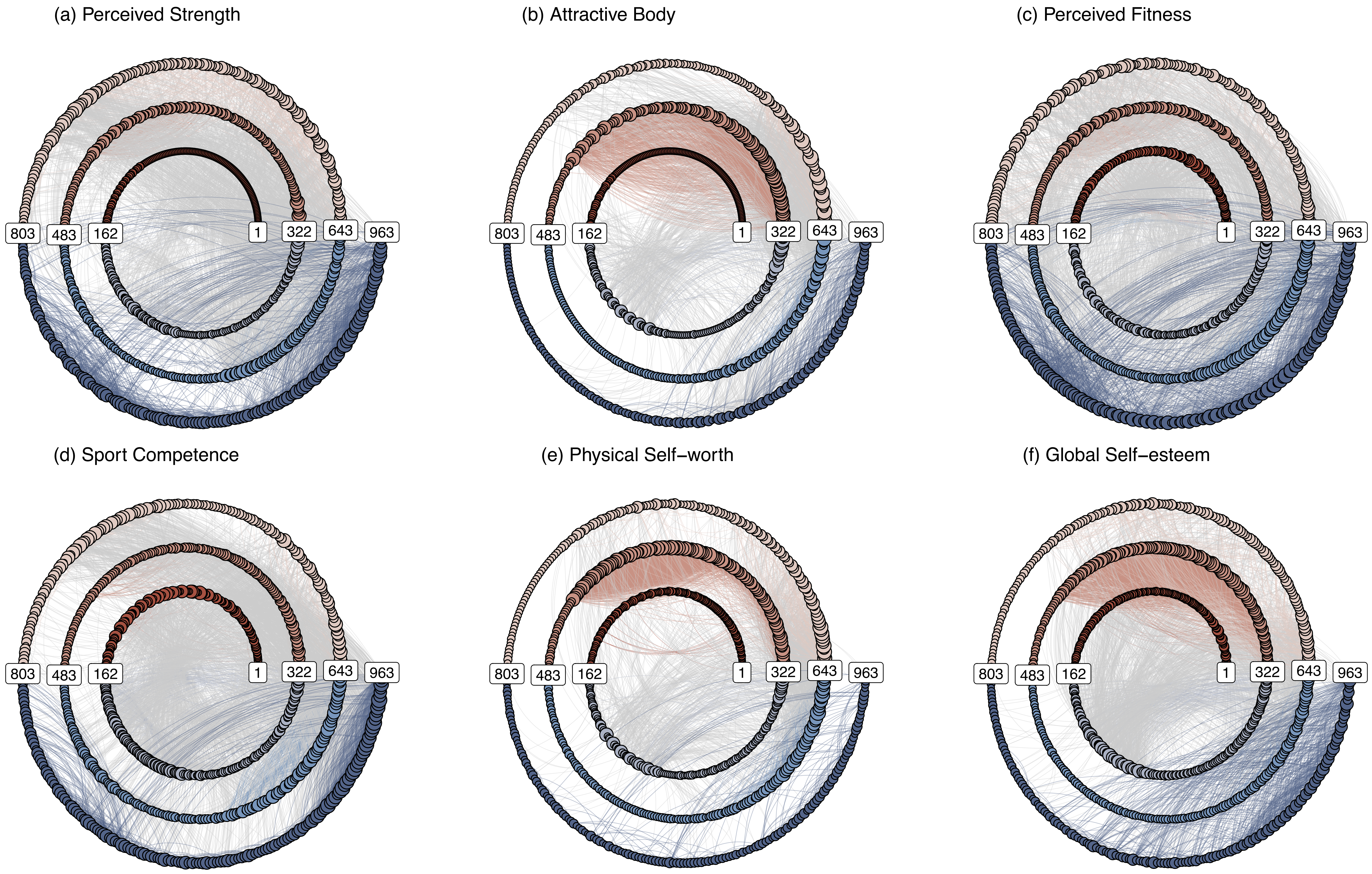
Jacob, R., Harikrishnan, K. P., Misra, R., & Ambika, G. (2015). How does noise affect the structure of a chaotic attractor: A recurrence network perspective. *arXiv preprint arXiv:1508.02724*.

https://www.uu.nl/en/events/clue-training-7-pyunicorn-complex-network-and-recurrence-analysis-toolbox

Recurrence/Incidence/Adjacency Matrix

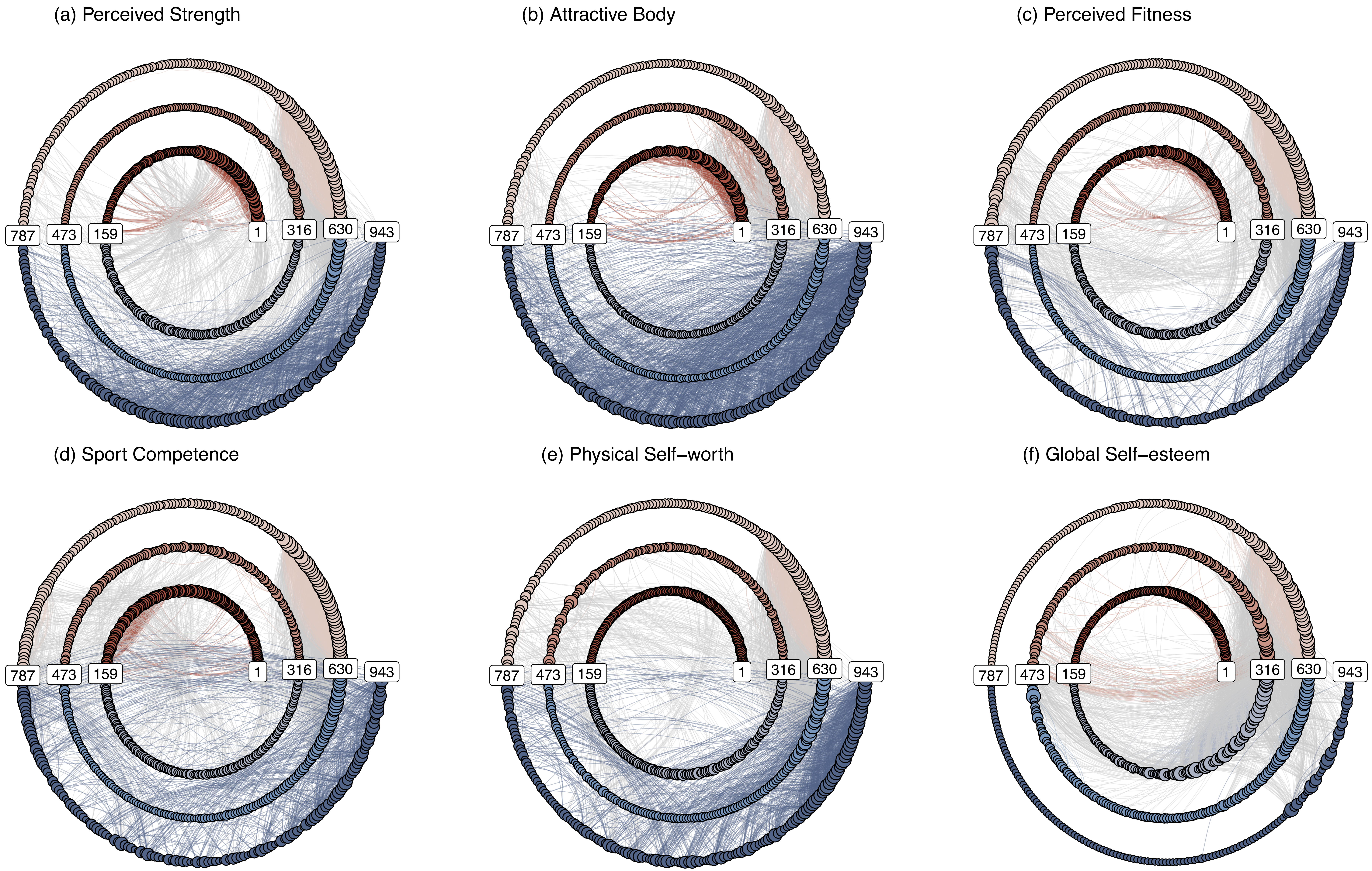
- Find a graph layout that makes a bit more sense





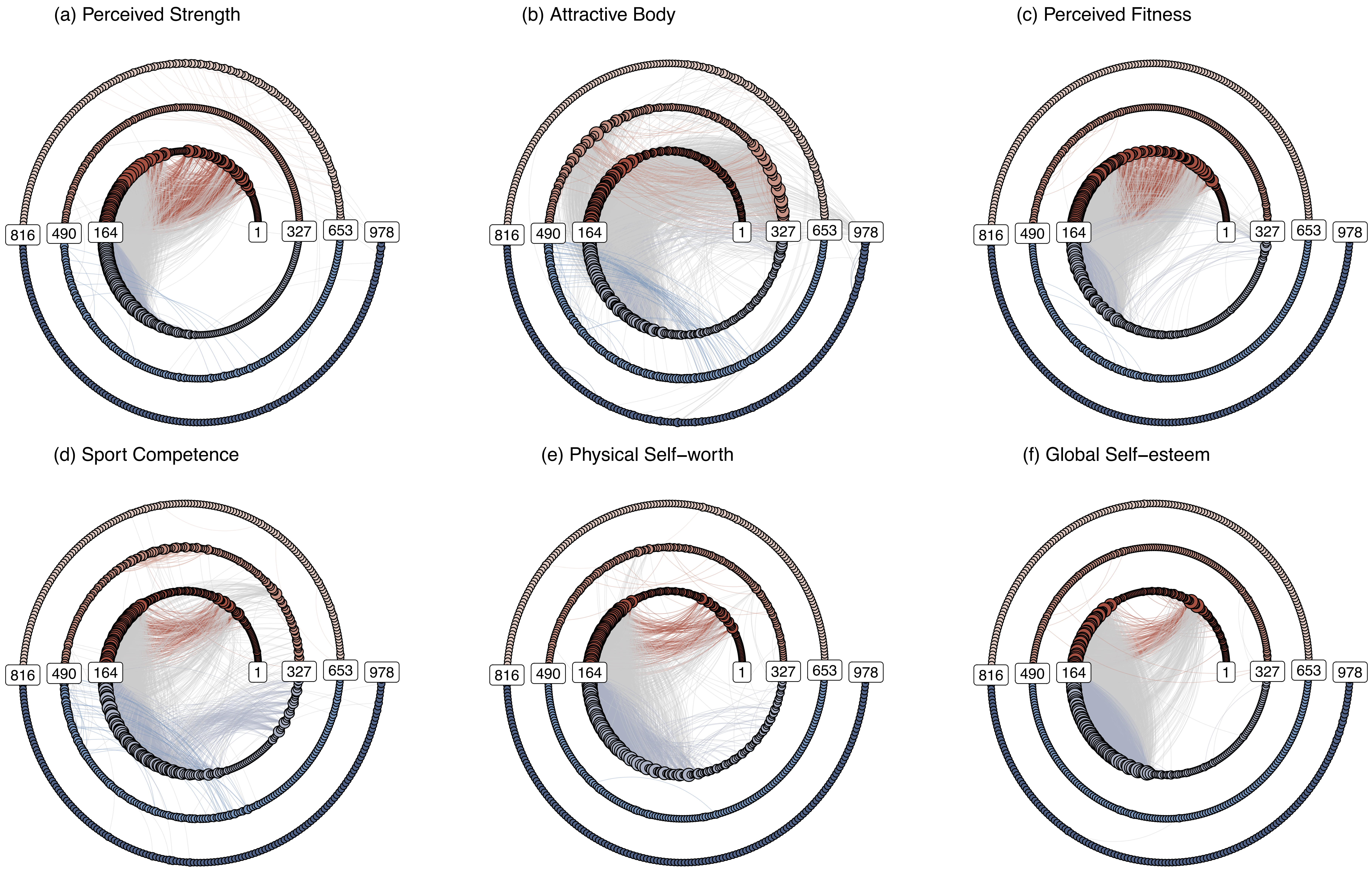
Participant 1

Hasselman, F. & Bosman, A.M.T. (submitted). Studying Complex Adaptive Systems with Internal States: A Recurrence-Based Analysis Strategy for Multivariate Time Series Data Representing Self-Reports of Human Experience. *Frontiers in Applied Mathematics and Statistics.*



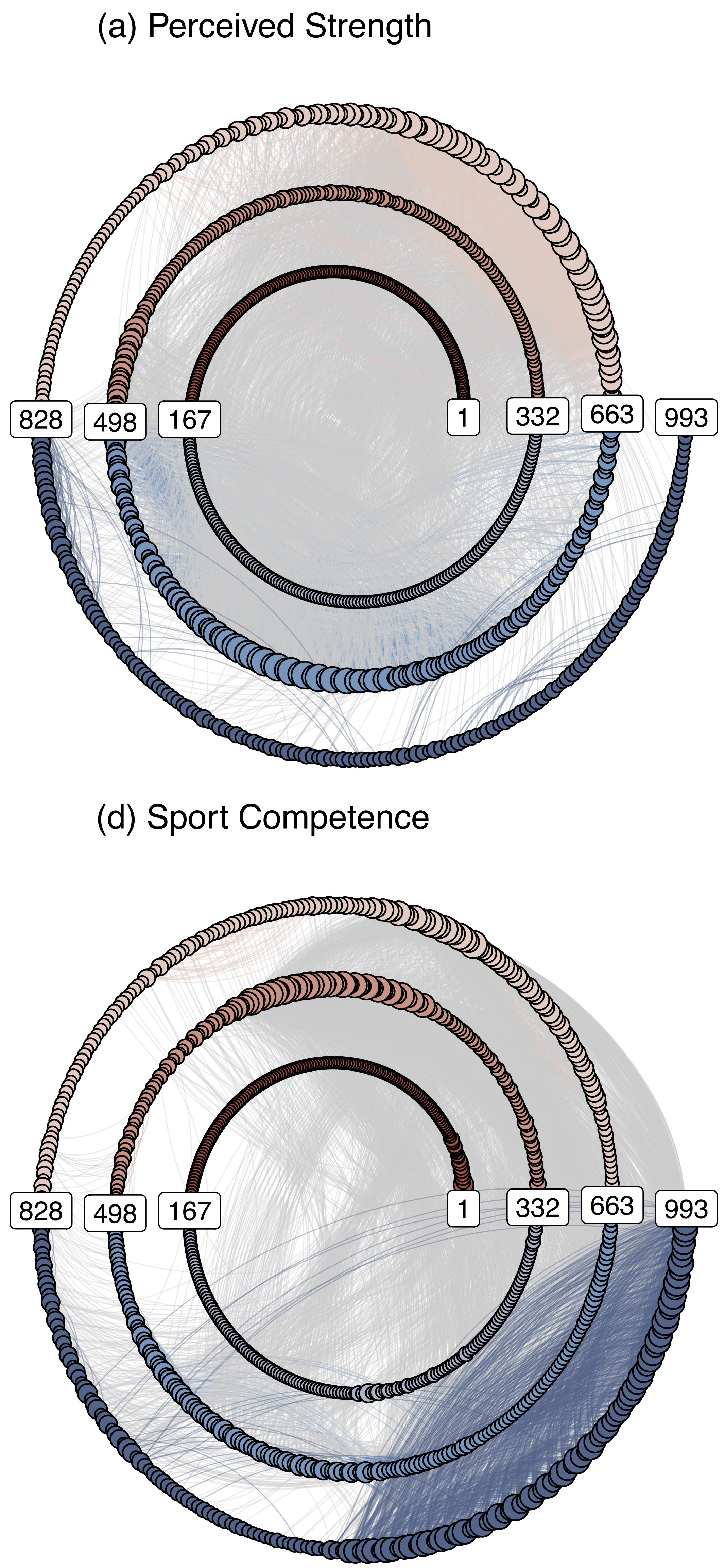
Participant 2

Hasselman, F. & Bosman, A.M.T. (submitted). Studying Complex Adaptive Systems with Internal States: A Recurrence-Based Analysis Strategy for Multivariate Time Series Data Representing Self-Reports of Human Experience. *Frontiers in Applied Mathematics and Statistics.*



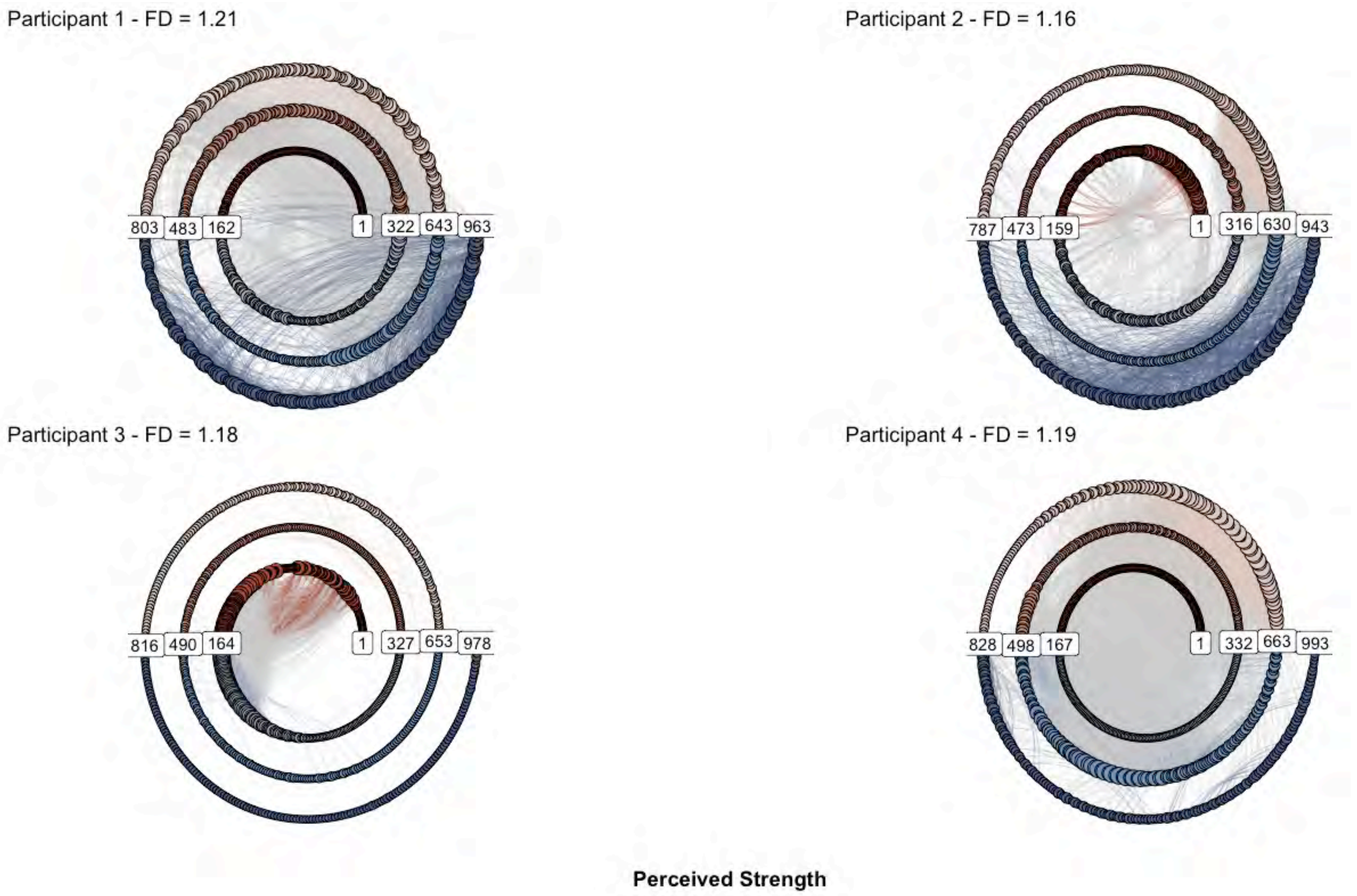
Participant 3

Hasselman, F. & Bosman, A.M.T. (submitted). Studying Complex Adaptive Systems with Internal States: A Recurrence-Based Analysis Strategy for Multivariate Time Series Data Representing Self-Reports of Human Experience. *Frontiers in Applied Mathematics and Statistics.*



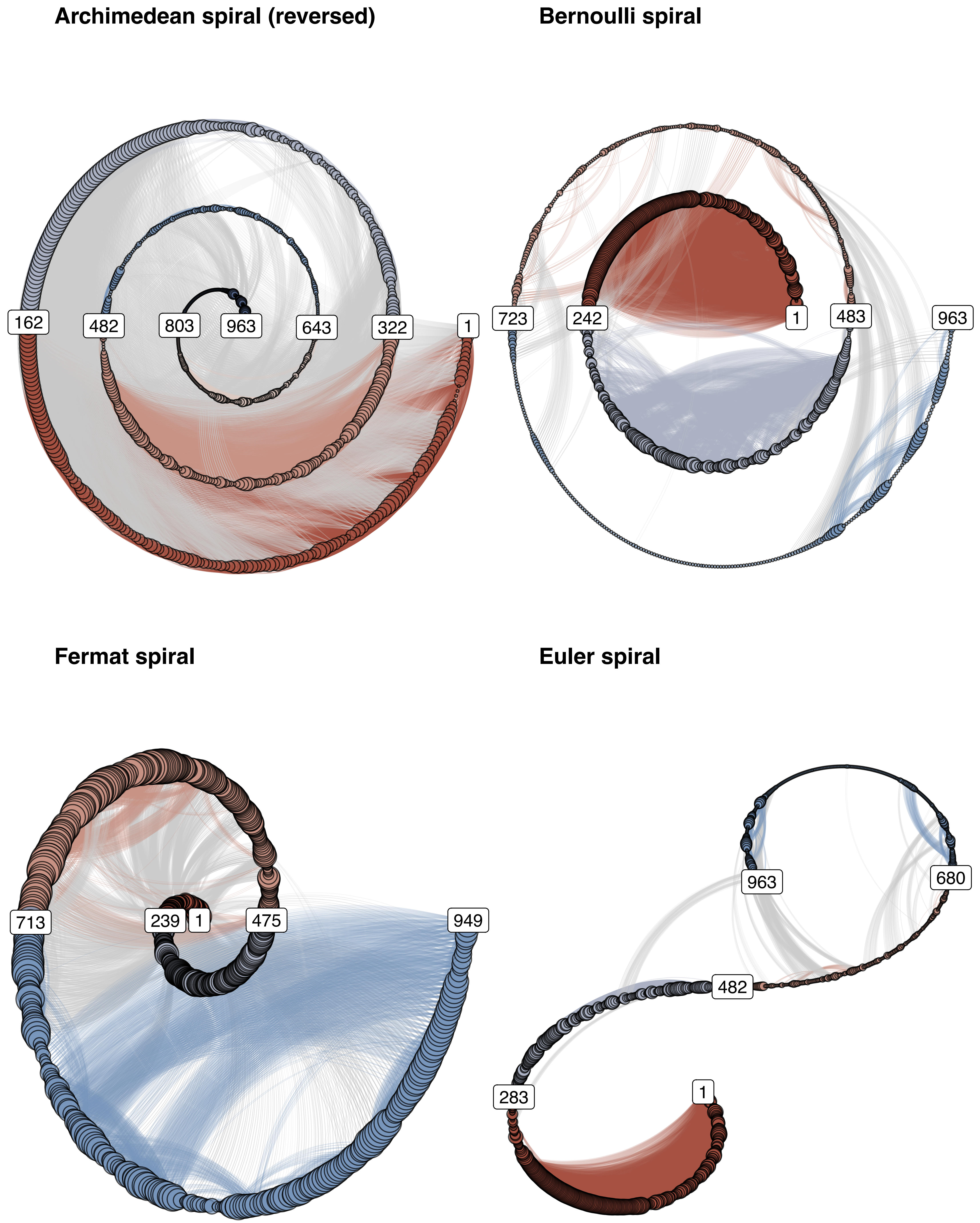
Participant 4

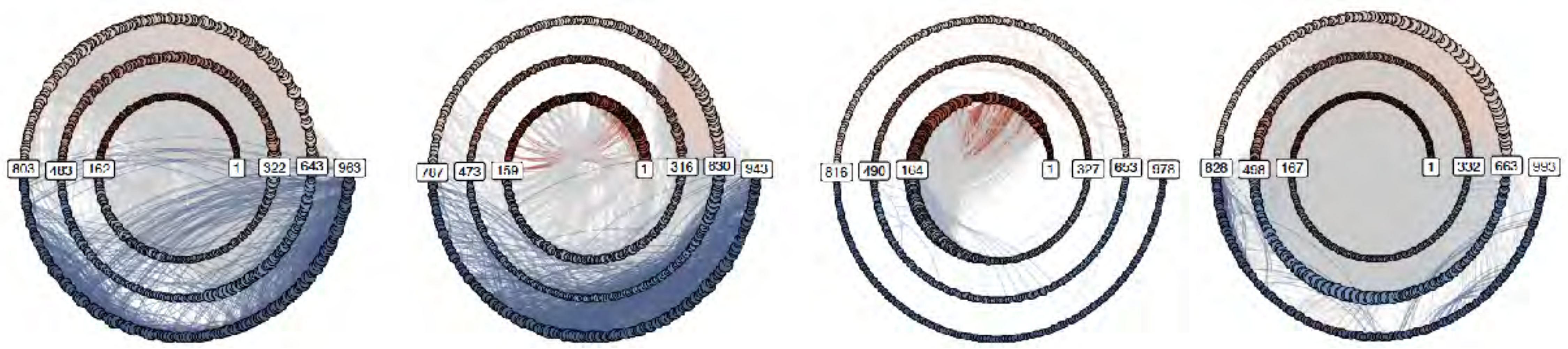
Hasselman, F. & Bosman, A.M.T. (submitted). Studying Complex Adaptive Systems with Internal States: A Recurrence-Based Analysis Strategy for Multivariate Time Series Data Representing Self-Reports of Human Experience. *Frontiers in Applied Mathematics and Statistics.*



Hasselman, F. & Bosman, A.M.T. (submitted). Studying Complex Adaptive Systems with Internal States: A Recurrence-Based Analysis Strategy for Multivariate Time Series Data Representing Self-Reports of Human Experience. *Frontiers in Applied Mathematics and Statistics.*

Participant 3
Global Self-Esteem

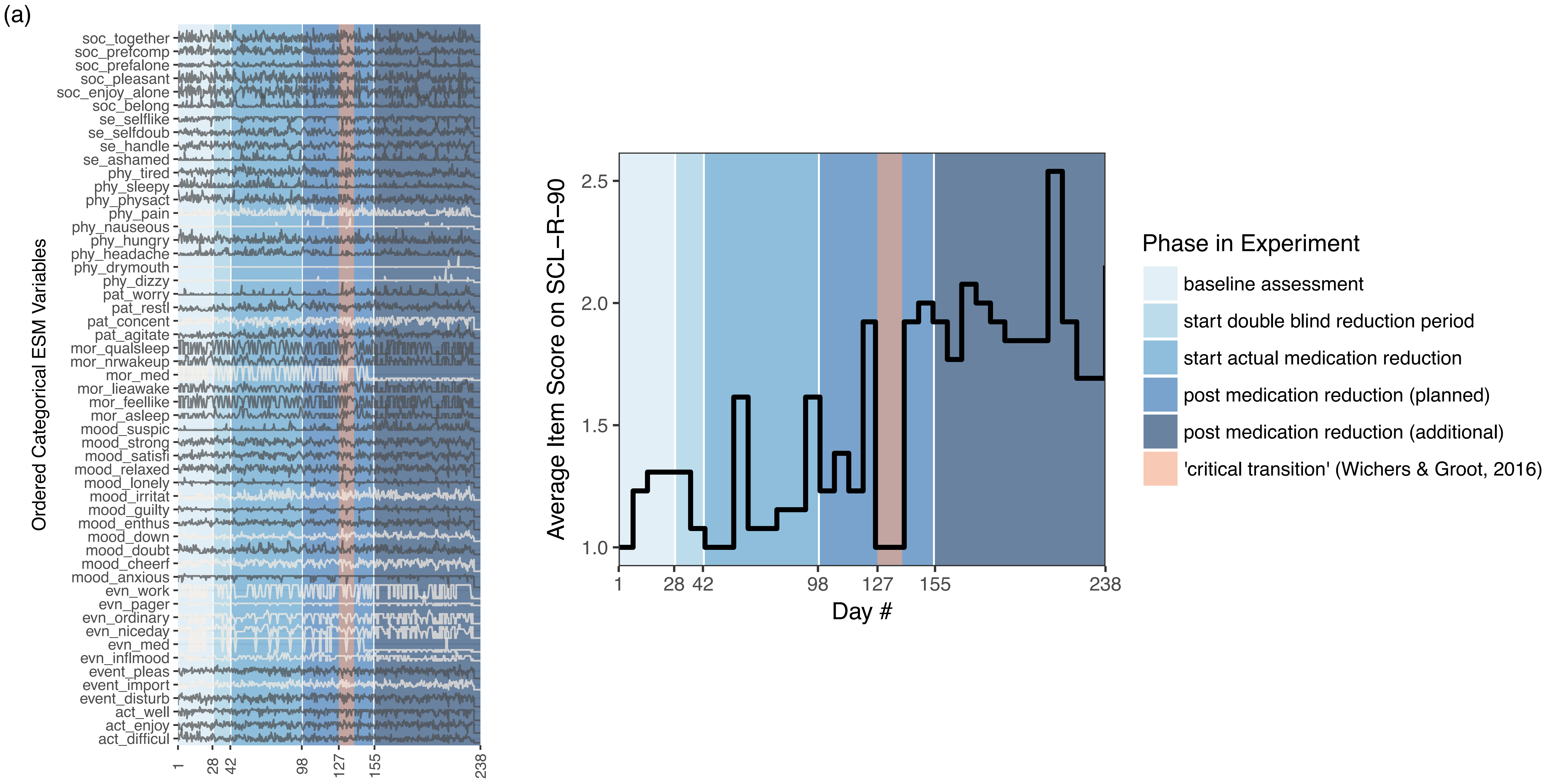




Perceived Strength	P1	P2	P3	P4
Centralization Degree	0.056	0.068	0.093	0.091
Centralization Betweenness	0.036	0.028	0.002	0.076
Centralization Closeness	0.002	0.001	0	0.001
Transitivity	0.387	0.528	0.783	0.51
Assortativity Degree	0.332	0.436	0.479	0.361
average Path Length	4.98	4.69	2.41	6.82
SWI	28.50	36.48	112.61	27.87
Diameter	12	36	9	22
Fractal Dimension	1.21	1.16	1.18	1.19

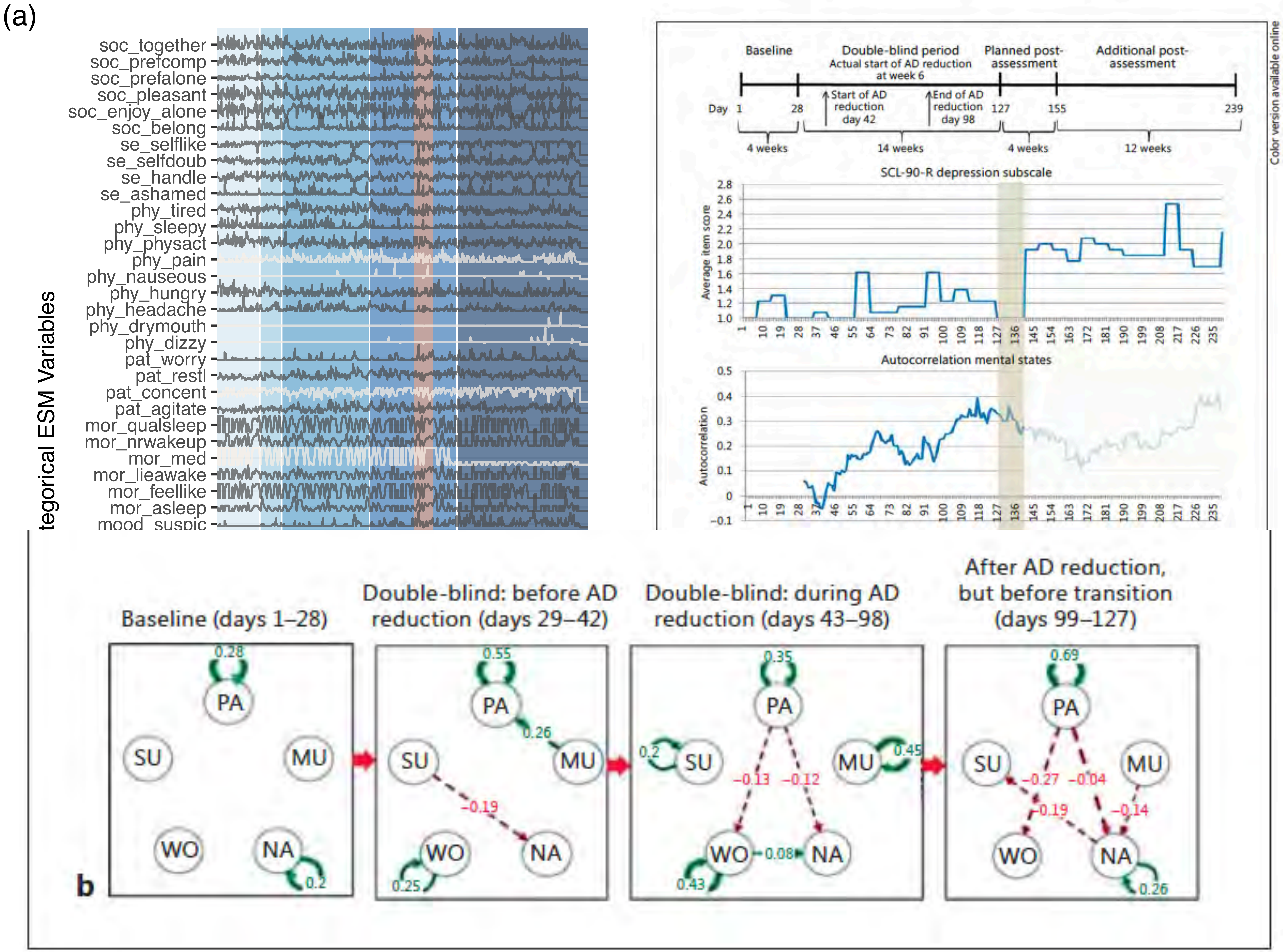
Hasselman, F. & Bosman, A.M.T. (submitted). Studying Complex Adaptive Systems with Internal States: A Recurrence-Based Analysis Strategy for Multivariate Time Series Data Representing Self-Reports of Human Experience. *Frontiers in Applied Mathematics and Statistics.*

“Critical Slowing Down as a Personalized Early Warning Signal for Depression”



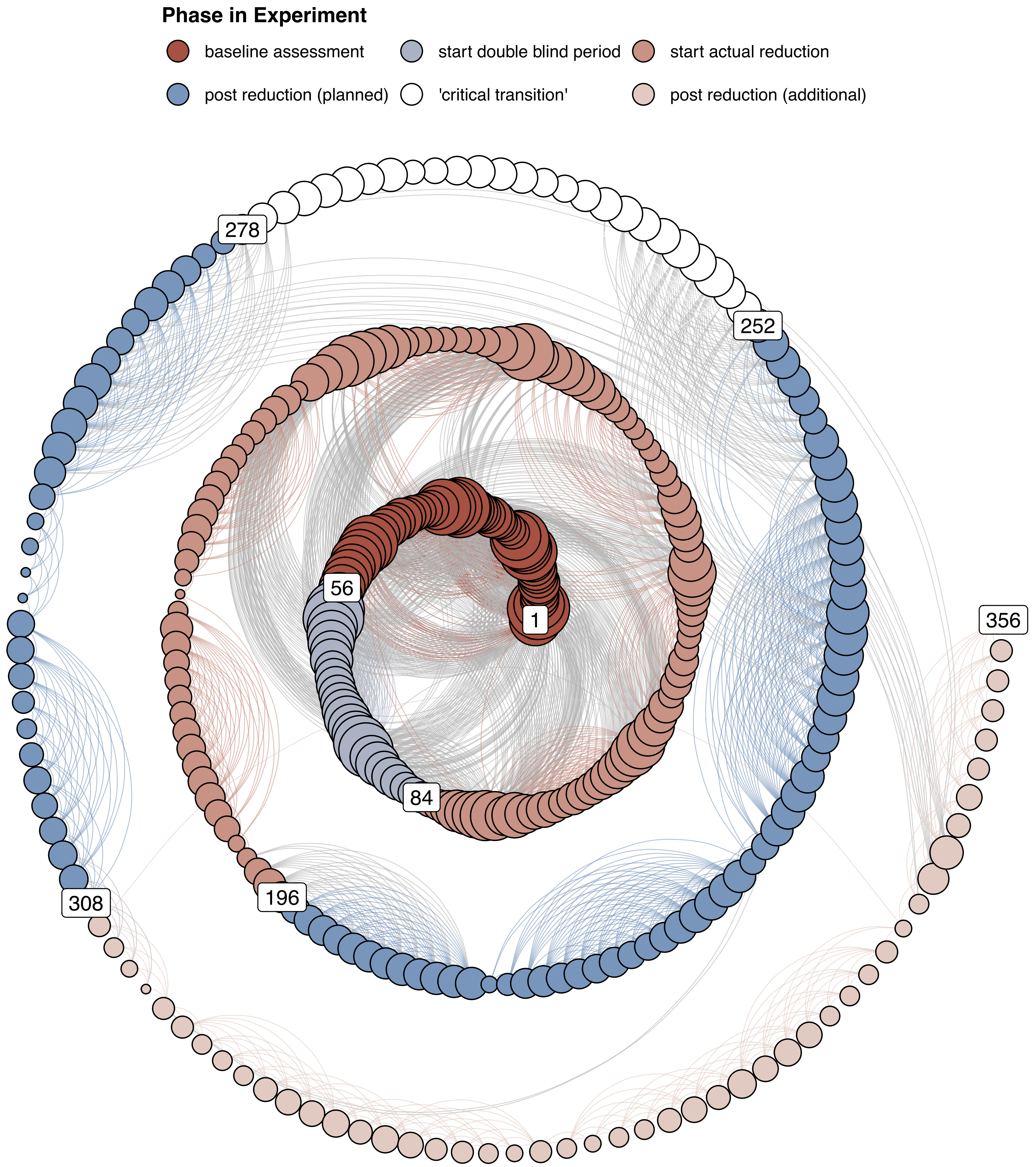
Wichers, M., Groot, P. C., Psychosystems, ESM Grp, & EWS Grp (2016). Critical Slowing Down as a Personalized Early Warning Signal for Depression. Psychotherapy and psychosomatics, 85(2), 114-116. DOI: 10.1159/000441458

Kossakowski, J., Groot, P., Haslbeck, J., Borsboom, D., and Wichers, M. (2017). Data from ‘critical slowing down as a personalized early warning signal for depression’. Journal of Open Psychology Data, 5(1).

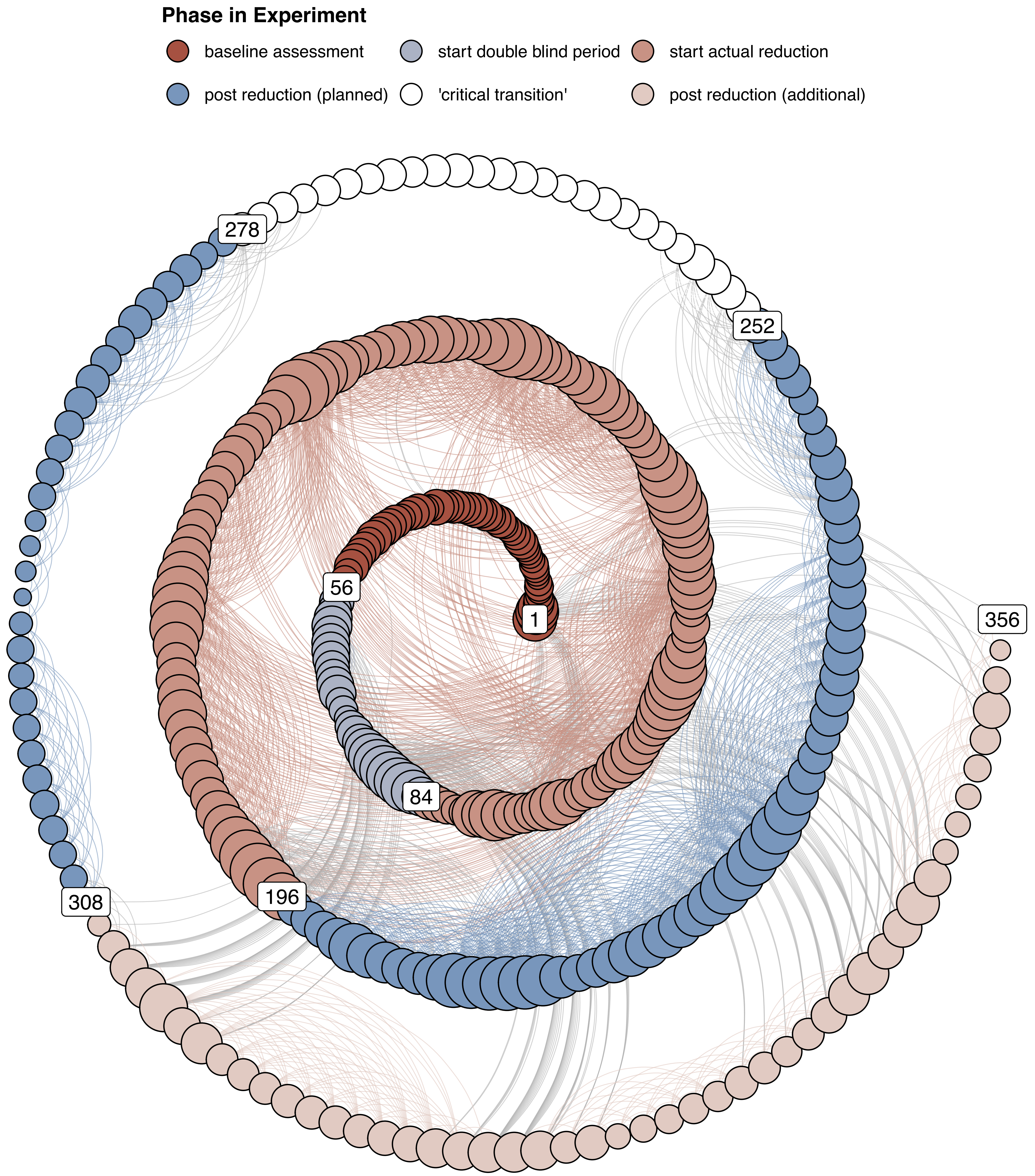


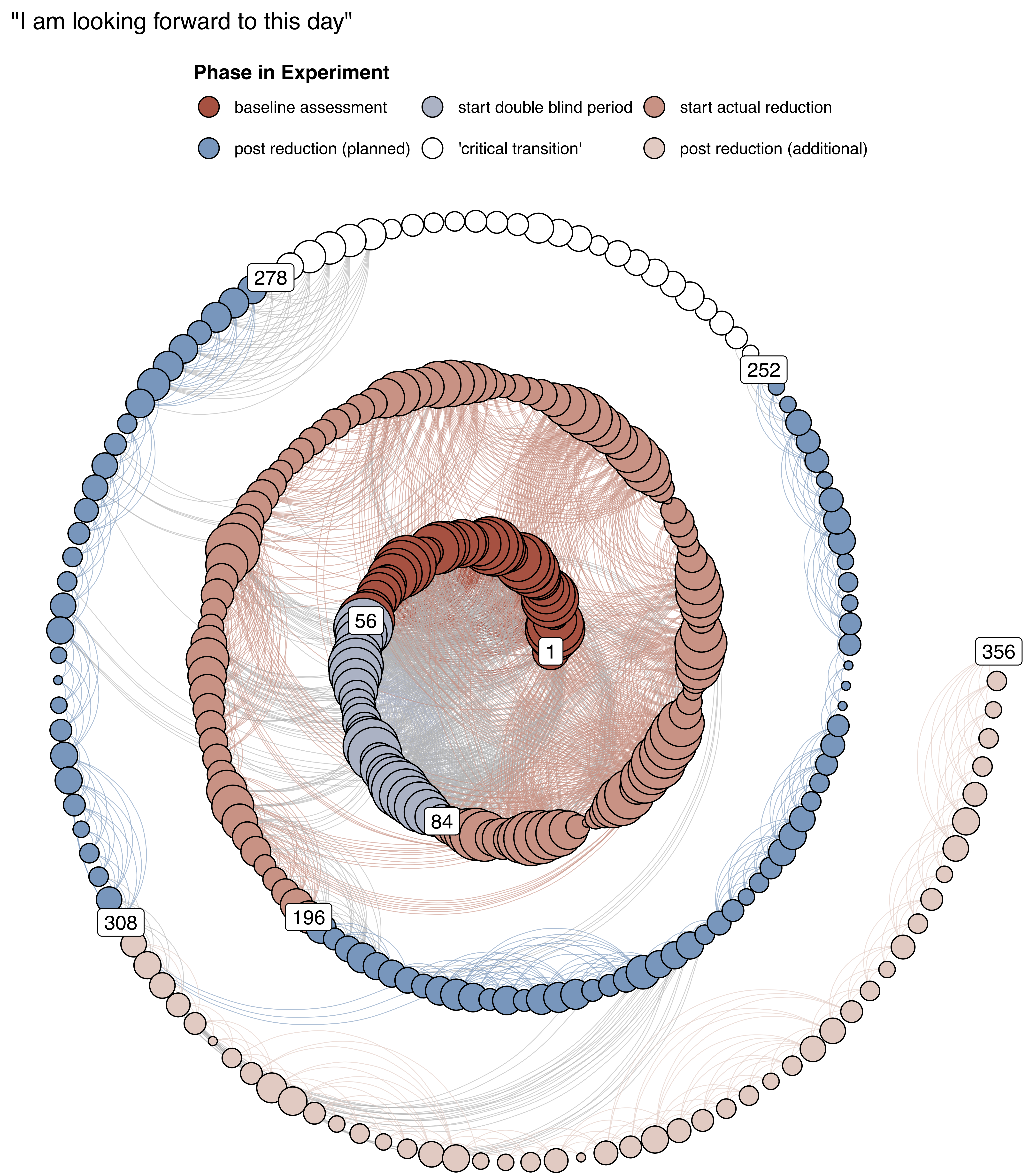
(a)
Categorical ESM Variables
Baseline
Double-blind period
Actual start of AD reduction at week 6
Planned post-assessment
Additional post-assessment
Start of AD reduction day 42
End of AD reduction day 98
Day
4 weeks
14 weeks
4 weeks
12 weeks
SCL-90-R depression subscale
Average item score
Autocorrelation mental states
Autocorrelation
Color version available online
Baseline (days 1–28)
Double-blind: before AD reduction (days 29–42)
Double-blind: during AD reduction (days 43–98)
After AD reduction, but before transition (days 99–127)
PA
SU
MU
WO
NA
b

"I feel down"

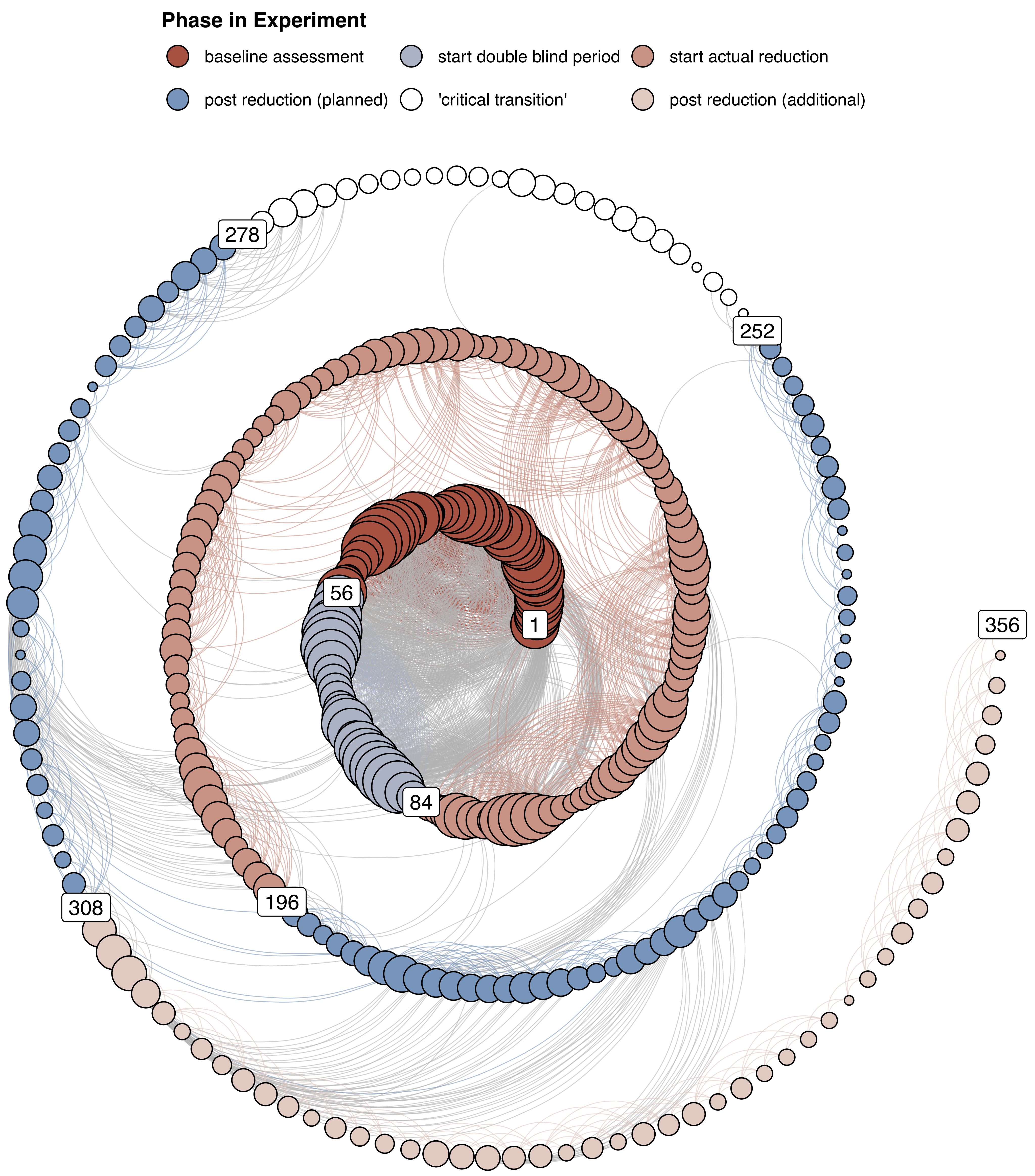

Phase in Experiment
baseline assessment
start double blind period
start actual reduction
post reduction (planned)
'critical transition'
post reduction (additional)
1
56
84
196
252
278
308
356

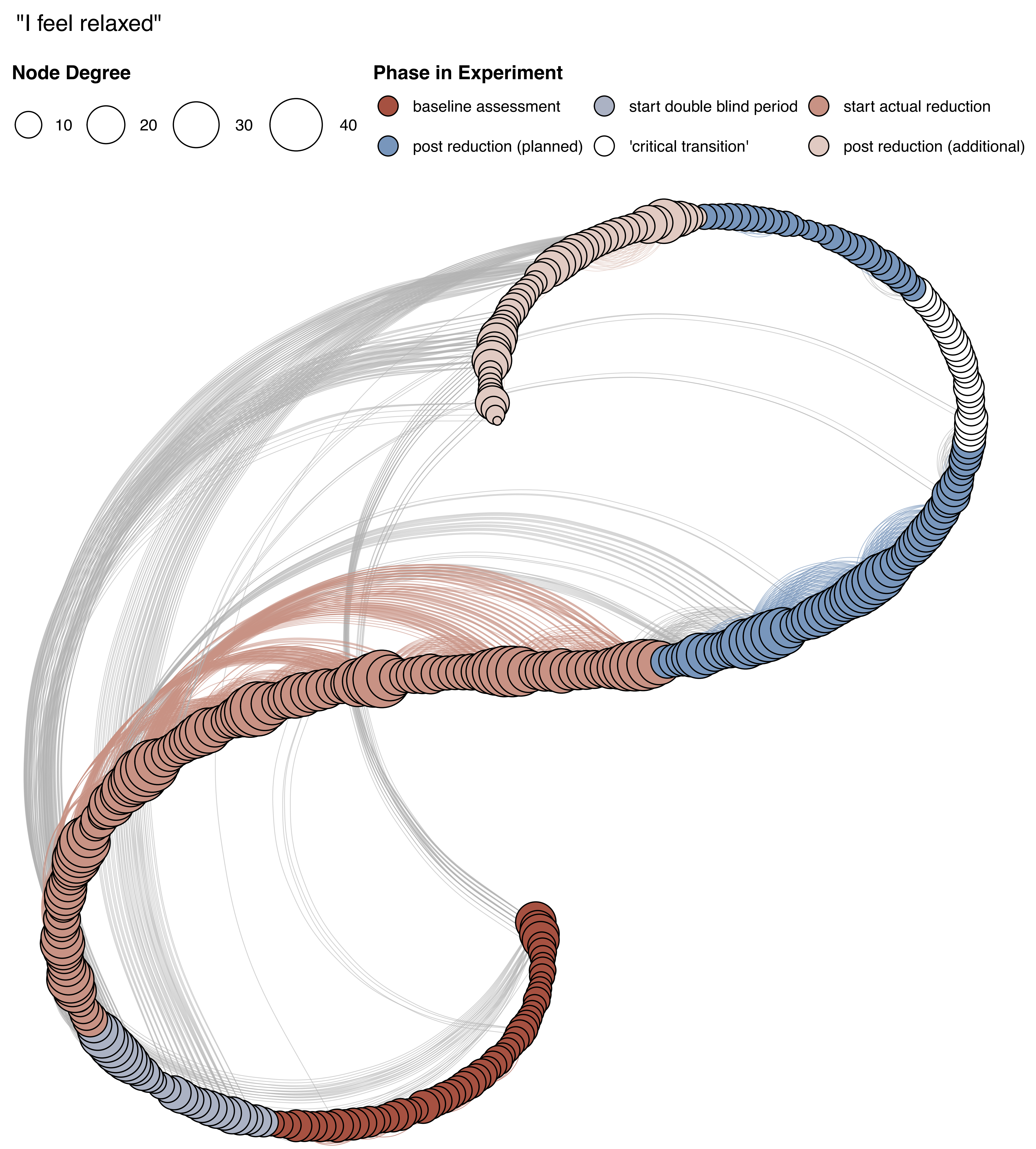
"I feel relaxed"




"I am looking forward to this day"
Phase in Experiment
baseline assessment
start double blind period
start actual reduction
post reduction (planned)
'critical transition'
post reduction (additional)
278
252
356
56
1
84
308
196

"I slept well"


Phase in Experiment
baseline assessment
start double blind period
start actual reduction
post reduction (planned)
'critical transition'
post reduction (additional)
1
56
84
196
252
278
308
356


"I feel relaxed"
Node Degree
10
20
30
40
Phase in Experiment
baseline assessment
start double blind period
start actual reduction
post reduction (planned)
'critical transition'
post reduction (additional)

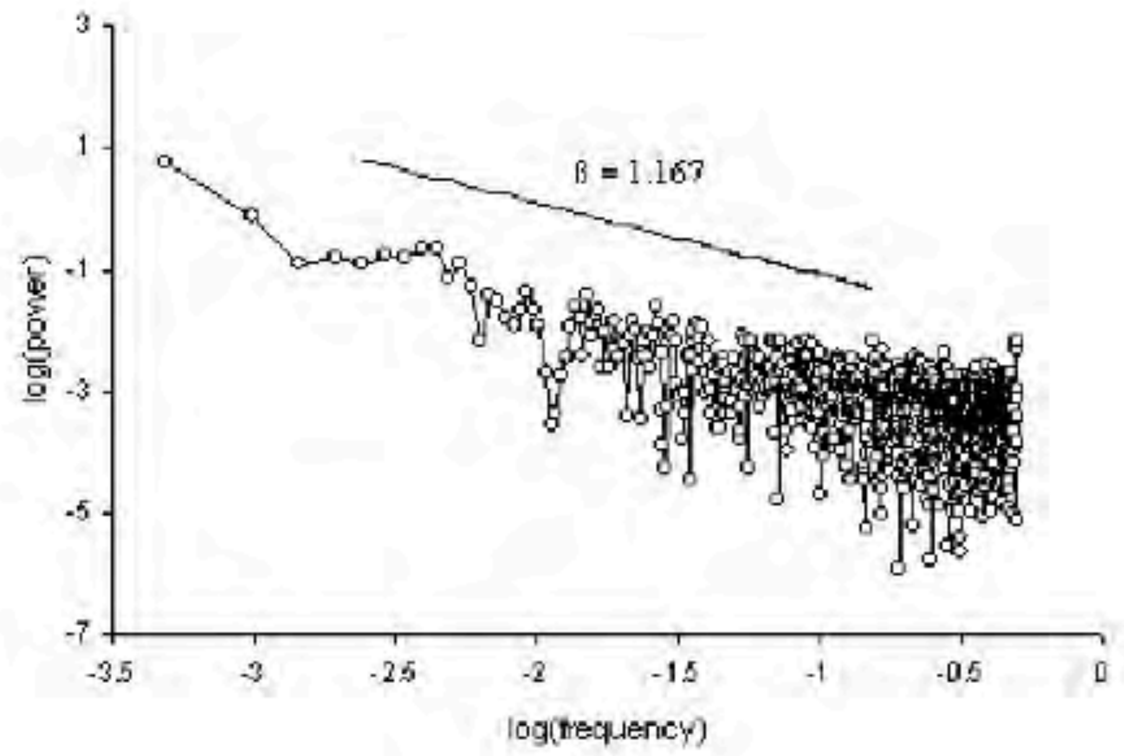
Scaling of Network Strength vs. Scaling in Timeseries

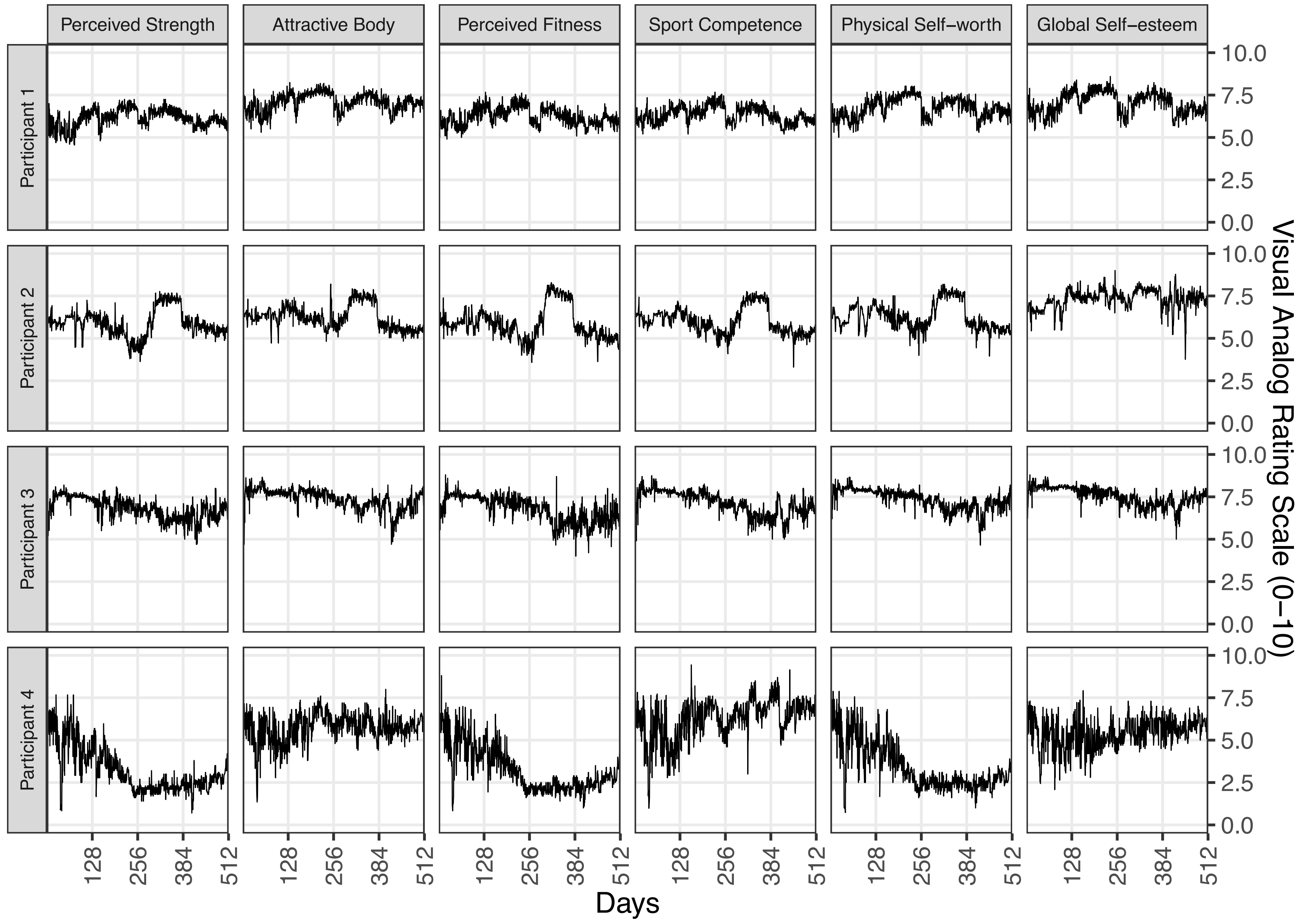
Table 2. Individual Moving Average Coefficients (θ) Obtained through ARIMA Modeling.

Participant	GSE	PSW	PC	SC	APP	PS
1	0.58	0.65	0.70	0.66	0.63	0.69
2	0.35	0.46	0.48	0.50	0.45	0.46
3	0.58	0.65	0.75	0.63	0.56	0.68
4	0.66	0.56	0.60	0.59	0.64	0.53

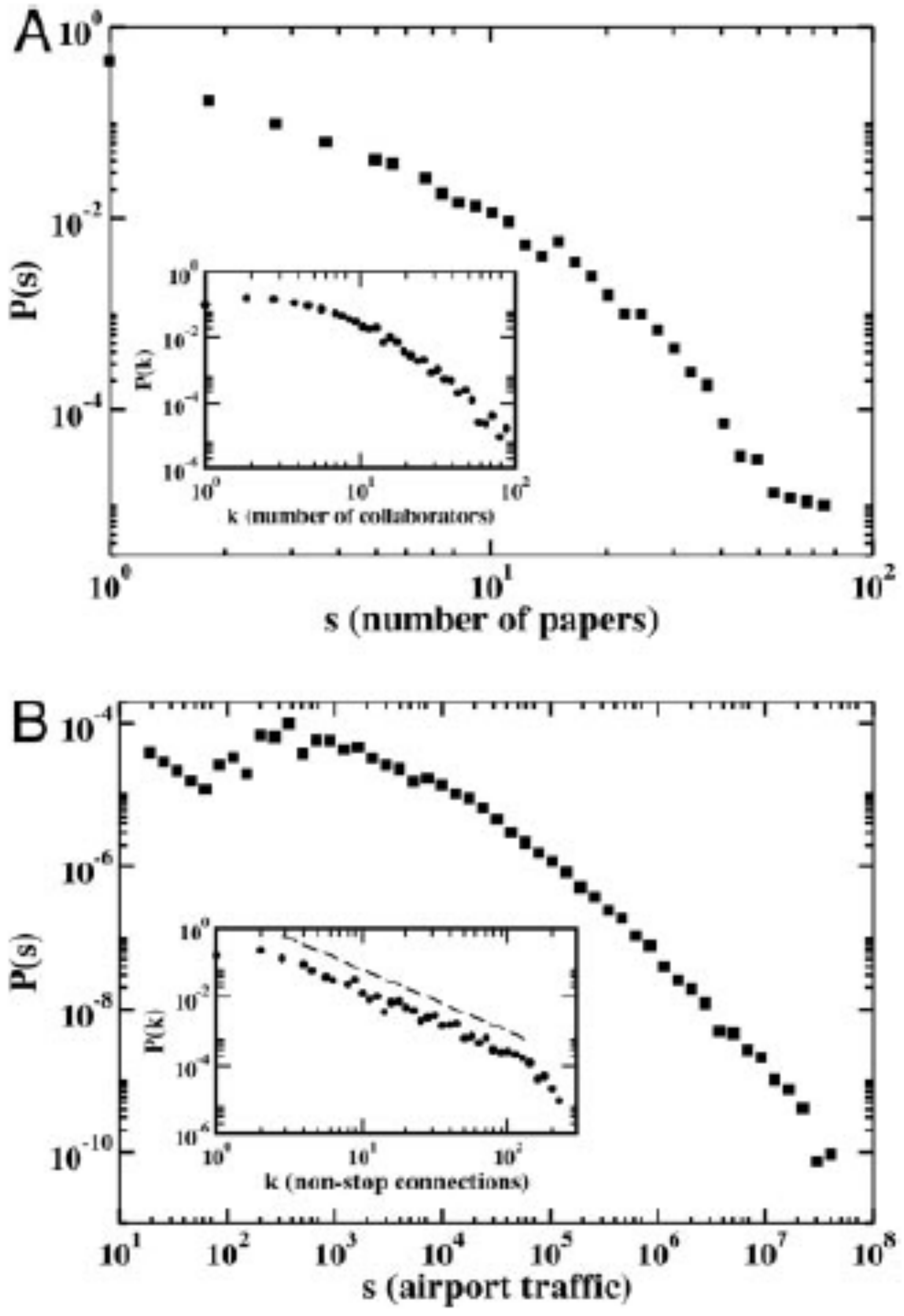
Table 3. Individual ß Exponents Obtained with Spectral Analysis.

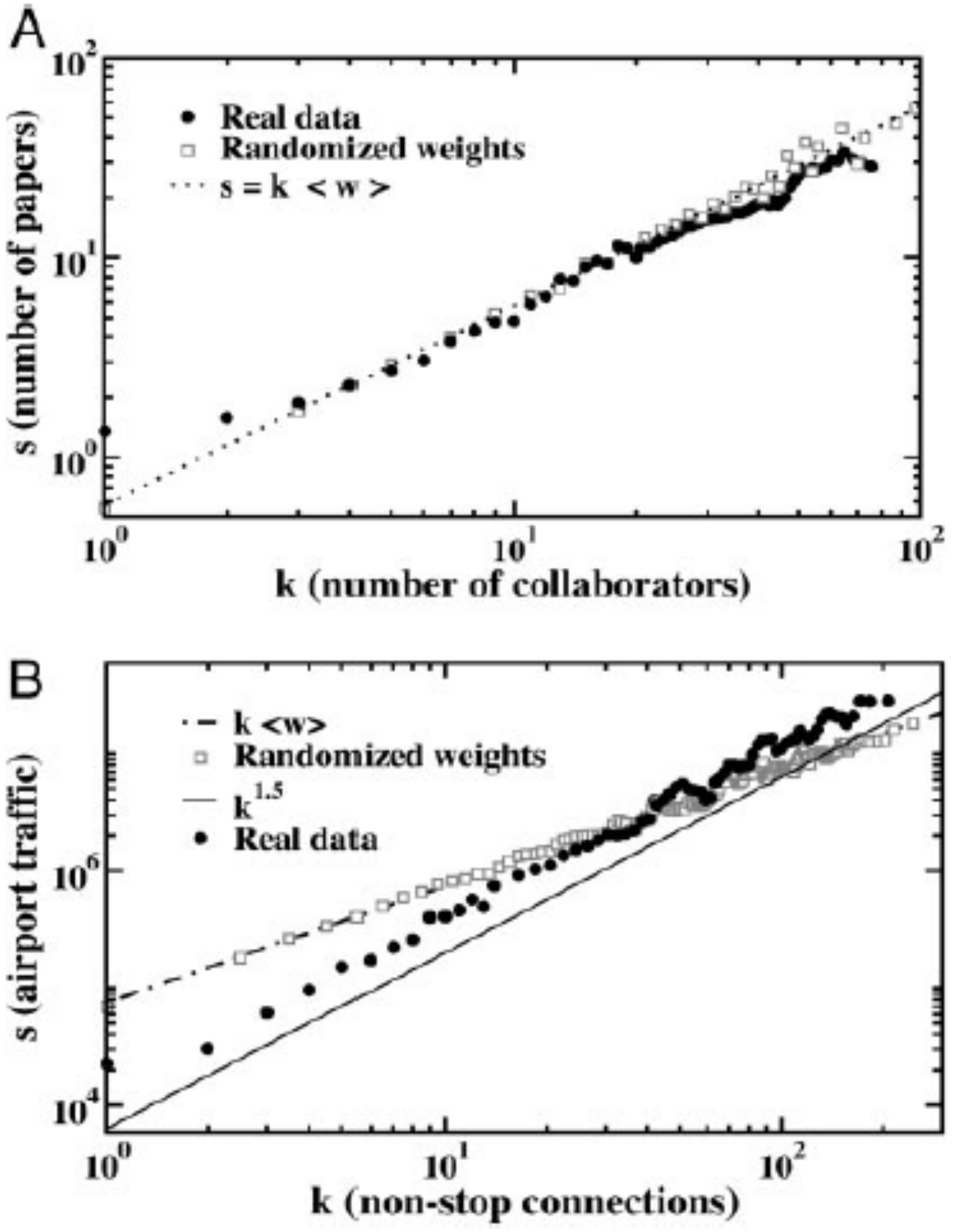
Participant	GSE	PSW	PC	SC	APP	PS
1	1.17	1.15	0.95	1.00	1.15	0.95
2	1.13	1.39	1.36	1.24	1.27	1.23
3	1.09	1.05	0.96	1.34	1.12	1.11
4	0.96	1.14	1.02	1.18	0.95	1.05





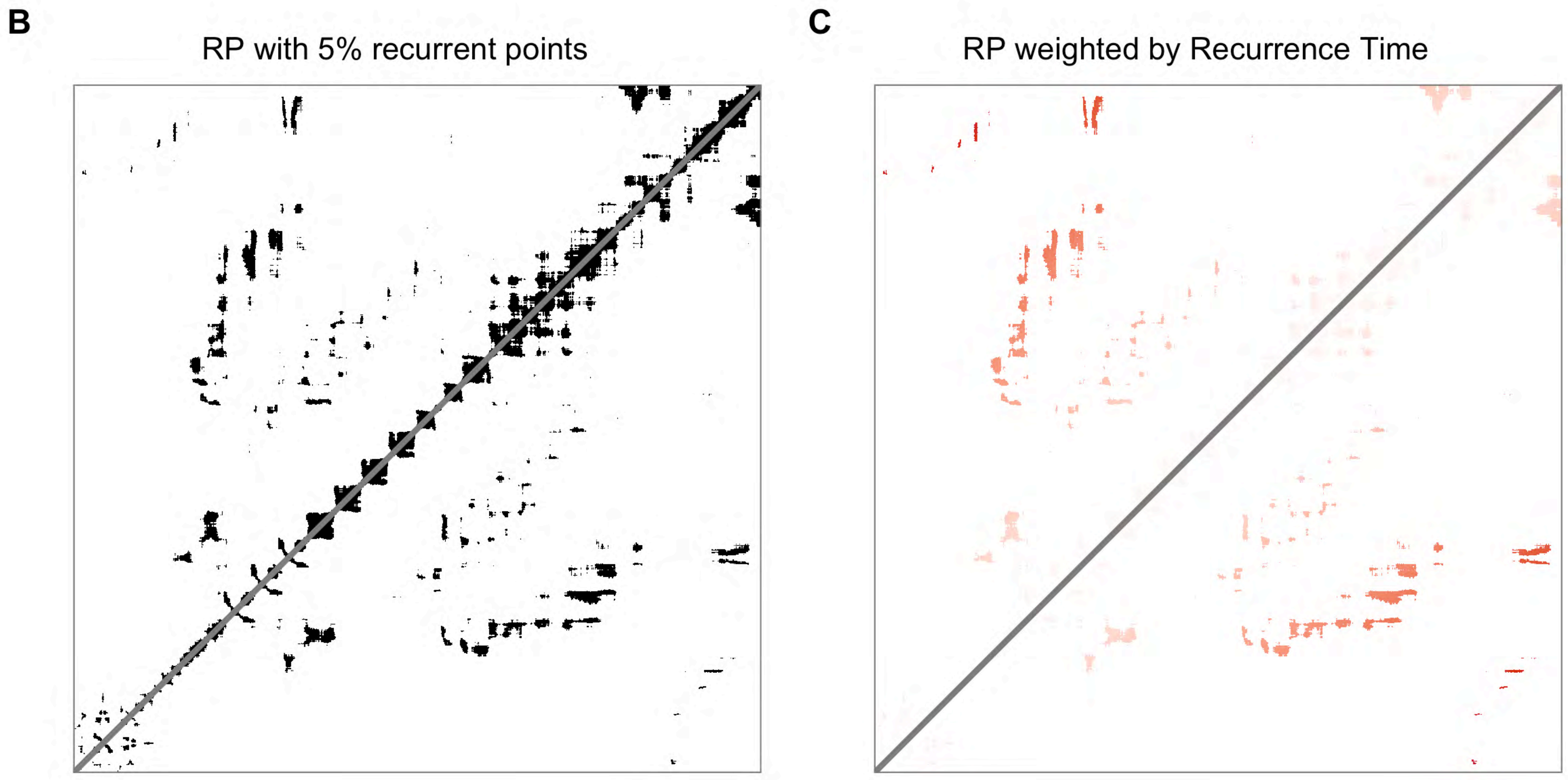
Weighted Adjacency Matrix

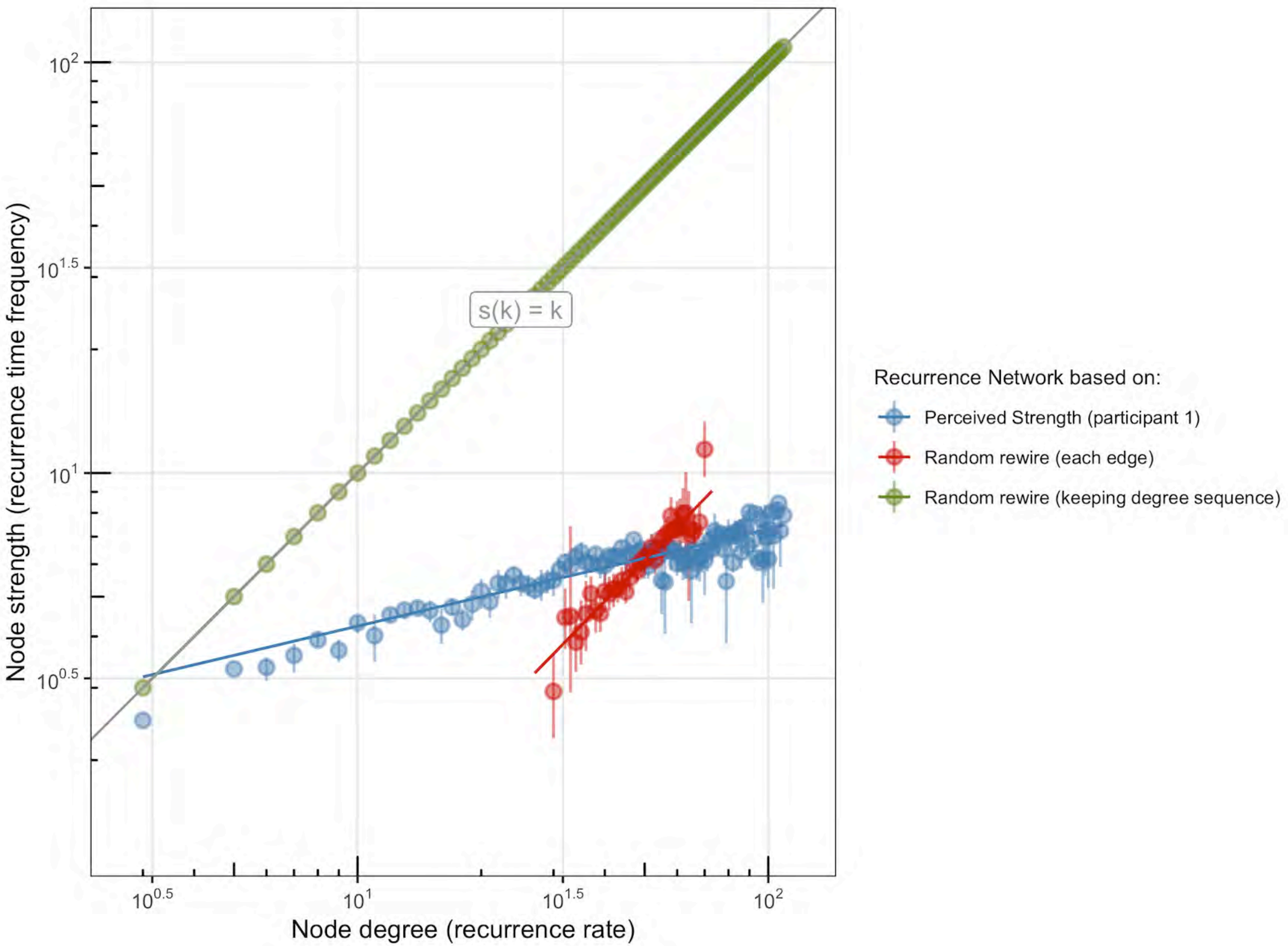




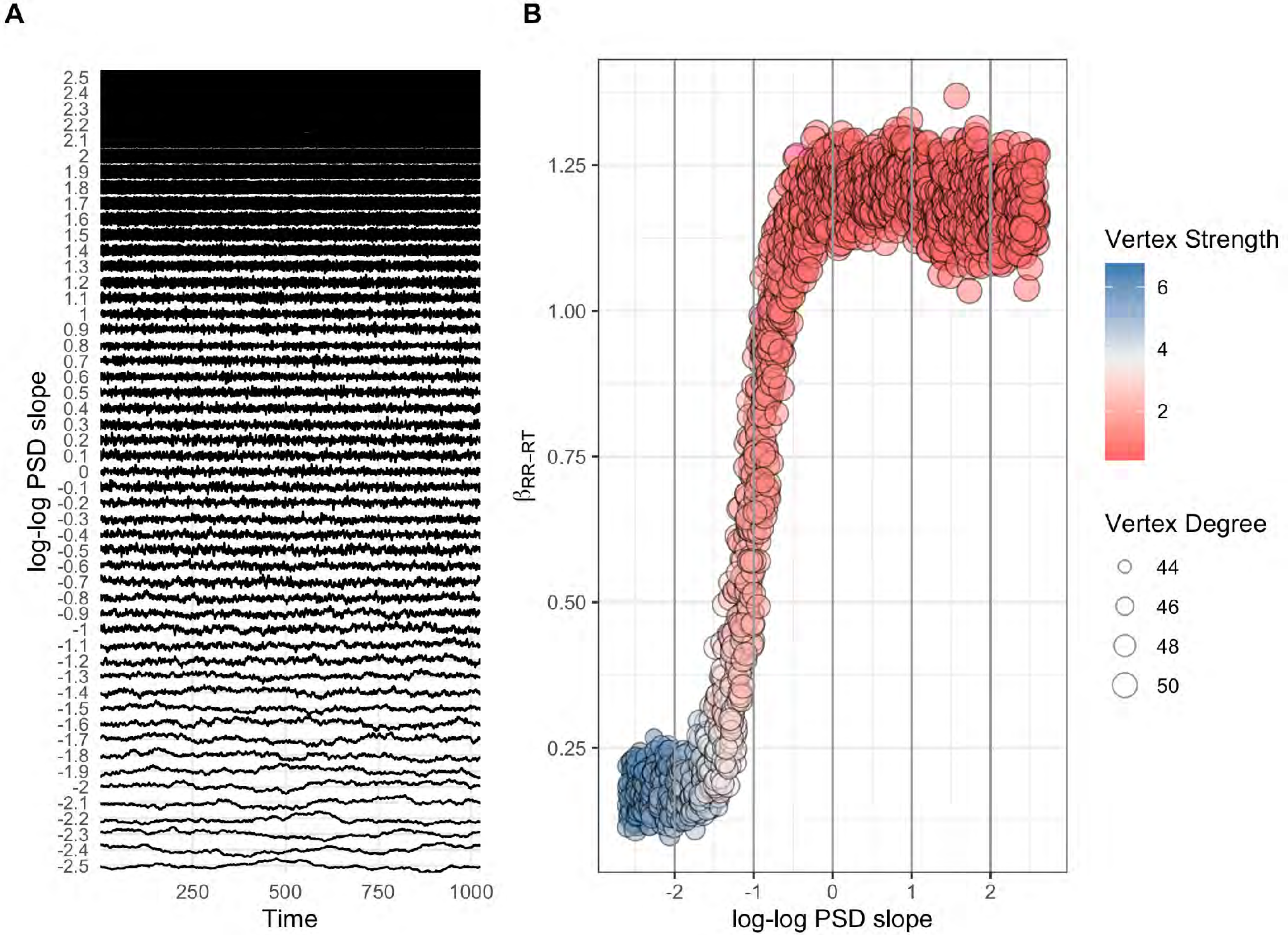
Barrat, A., Barthelemy, M., Pastor-Satorras, R., & Vespignani, A. (2004). The architecture of complex weighted networks. *Proc Natl Acad Sci U S A, 101*(11), 3747-3752. doi:10.1073/pnas.0400087101

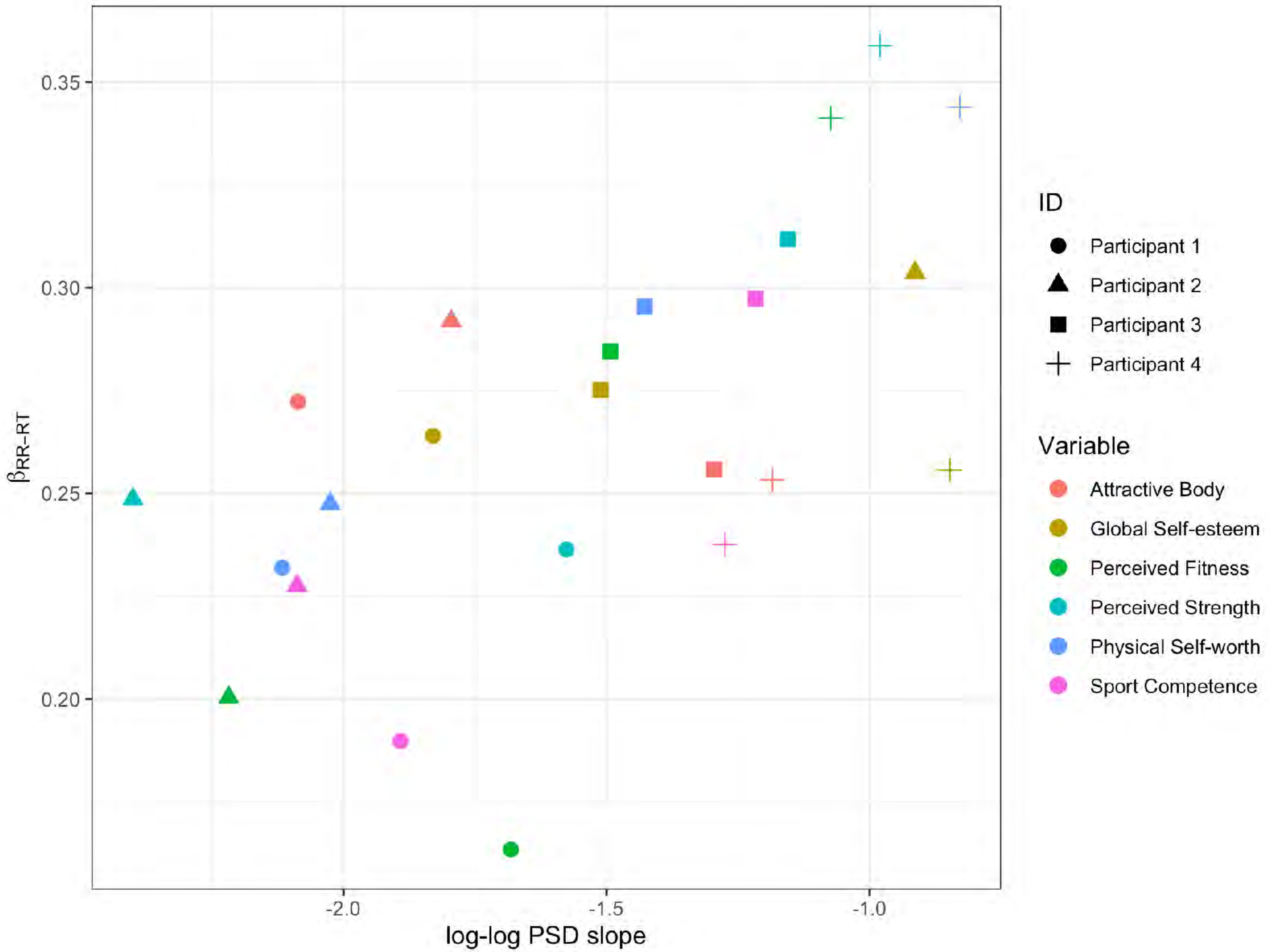
Weighted Adjacency Matrix





s(k) = k
Node strength (recurrence time frequency)
Node degree (recurrence rate)
$10^{0.5}$
10^{1}
$10^{1.5}$
10^{2}
Recurrence Network based on:
Perceived Strength (participant 1)
Random rewire (each edge)
Random rewire (keeping degree sequence)


A
B
log-log PSD slope
Time
250
500
750
1000
β_{RR-RT}
Vertex Strength
6
4
2
Vertex Degree
44
46
48
50



0.35
0.30
0.25
0.20
β_{RR-RT}
-2.0
-1.5
-1.0
log-log PSD slope
ID
Participant 1
Participant 2
Participant 3
Participant 4
Variable
Attractive Body
Global Self-esteem
Perceived Fitness
Perceived Strength
Physical Self-worth
Sport Competence

Multiplex Recurrence Networks

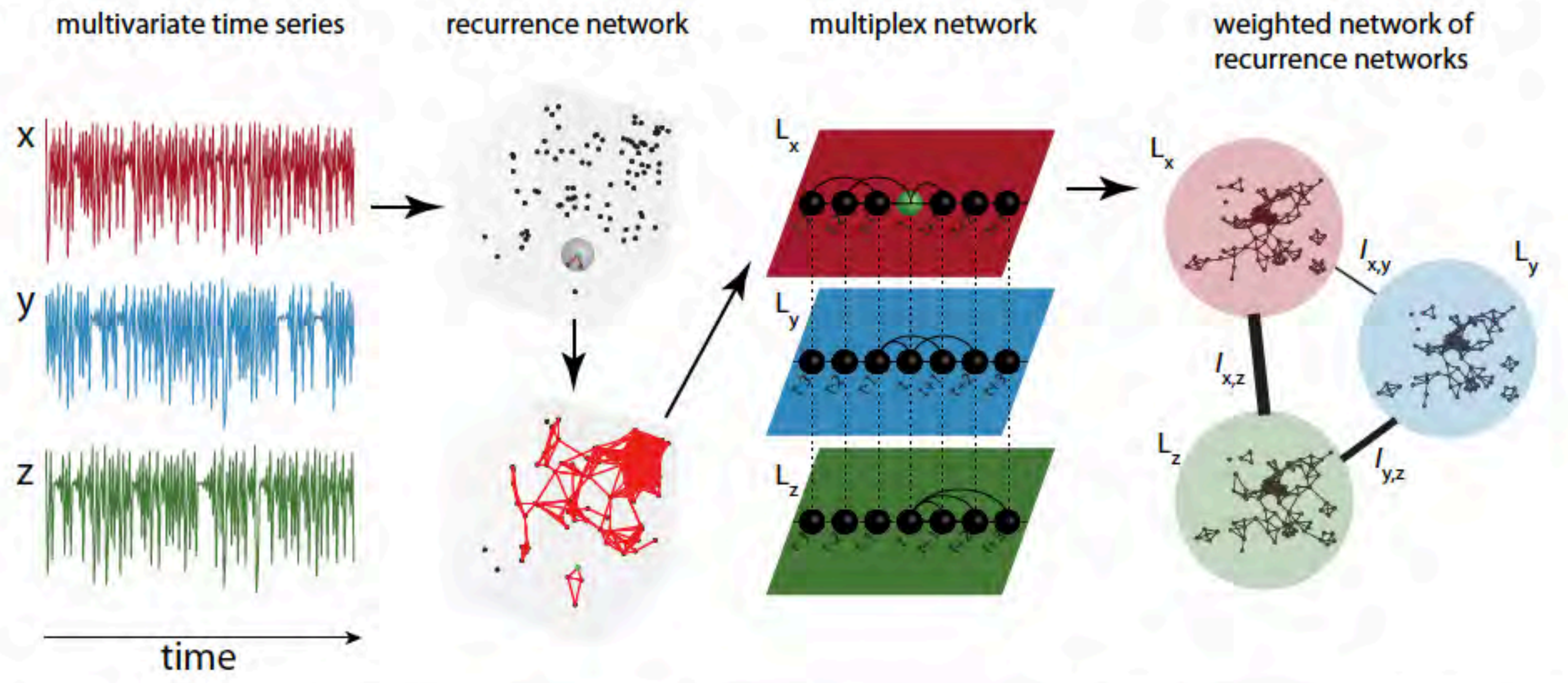


FIG. 1. Illustration of the procedure for generating network structures from a multivariate time series: multivariate time series → recurrence networks → multiplex recurrence network → weighted network of recurrence networks. Dashed lines between multiplex networks' layers connect all layers to all.

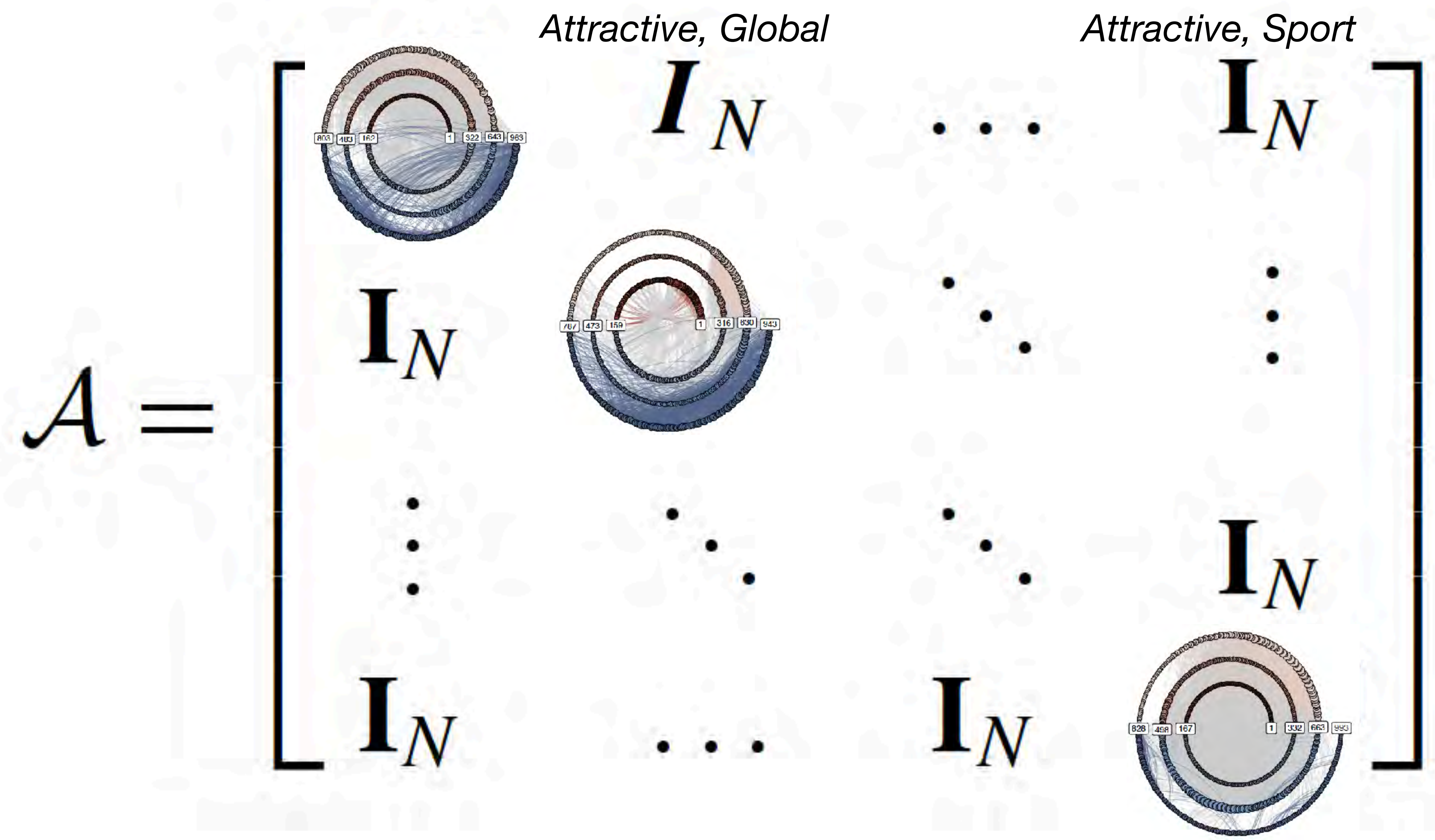
Interlayer Mutual Information

$$\boxed{I_{\alpha,\beta} = \sum_{\kappa^{[\alpha]}} \sum_{\kappa^{[\beta]}} P\left(\kappa^{[\alpha]}, \kappa^{[\beta]}\right) \log \frac{P\left(\kappa^{[\alpha]}, \kappa^{[\beta]}\right)}{P\left(\kappa^{[\alpha]}\right), P\left(\kappa^{[\beta]}\right)}}$$

Same number of nodes (time points)
Different degree, etc.

Interlayer Mutual Information

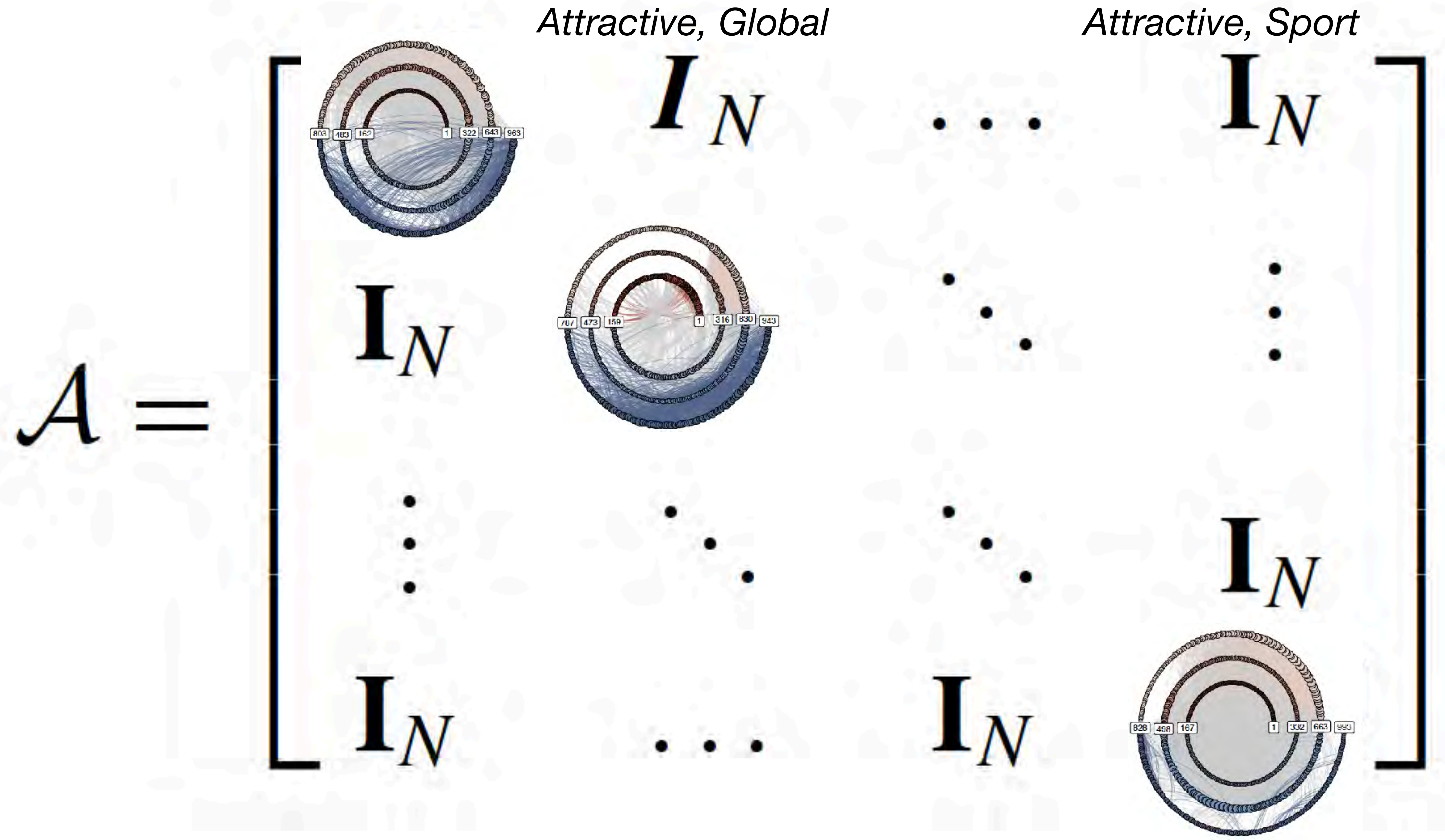
$$\boxed{I_{\alpha,\beta} = \sum_{\kappa^{[\alpha]}} \sum_{\kappa^{[\beta]}} P\left(\kappa^{[\alpha]}, \kappa^{[\beta]}\right) \log \frac{P\left(\kappa^{[\alpha]}, \kappa^{[\beta]}\right)}{P\left(\kappa^{[\alpha]}\right), P\left(\kappa^{[\beta]}\right)}}$$

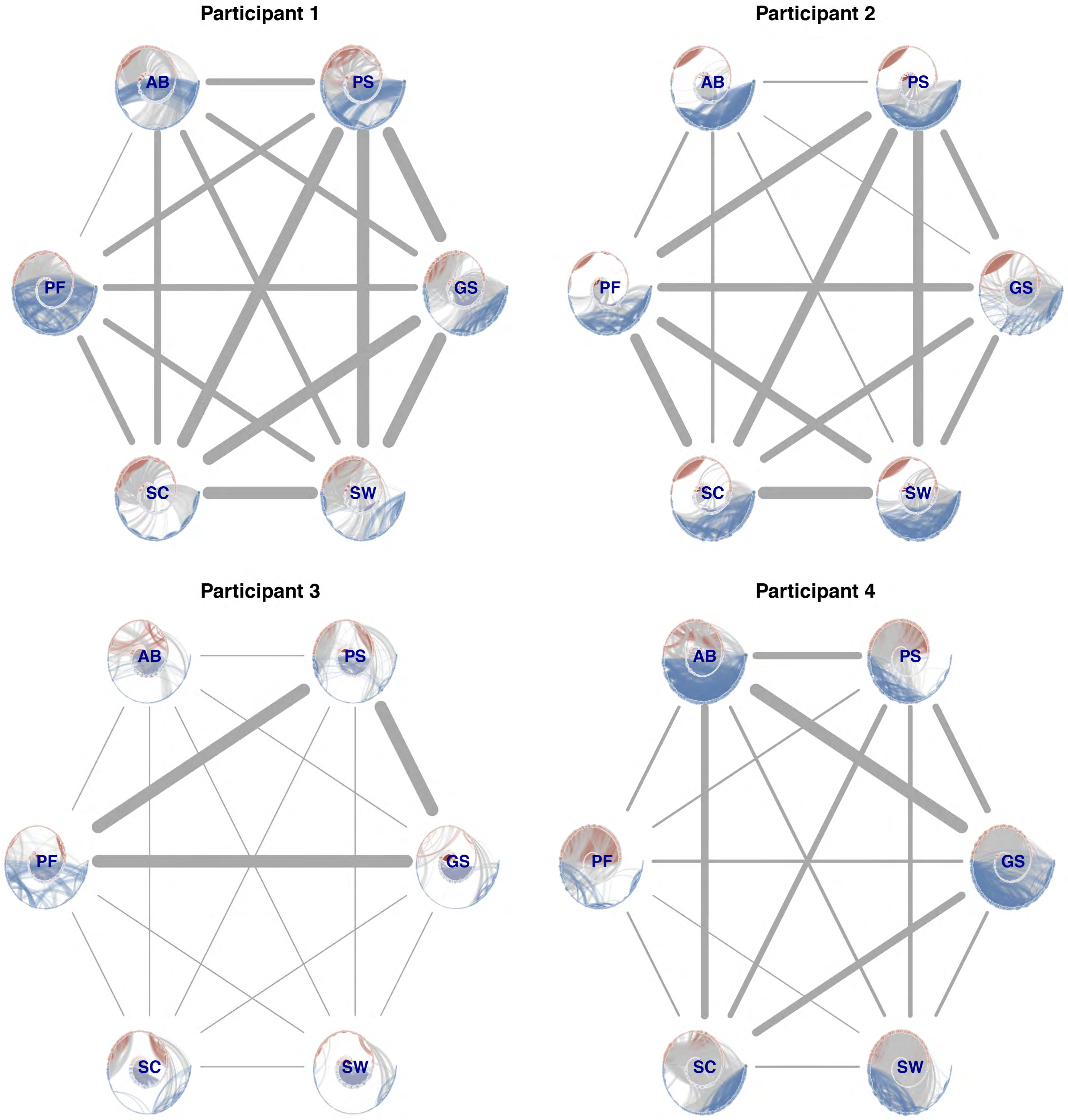


Interlayer Mutual Information based on Strength

$$\boxed{I_{\alpha,\beta} = \sum_{\kappa^{[\alpha]}} \sum_{\kappa^{[\beta]}} P\left(\kappa^{[\alpha]}, \kappa^{[\beta]}\right) \log \frac{P\left(\kappa^{[\alpha]}, \kappa^{[\beta]}\right)}{P\left(\kappa^{[\alpha]}\right), P\left(\kappa^{[\beta]}\right)}}$$

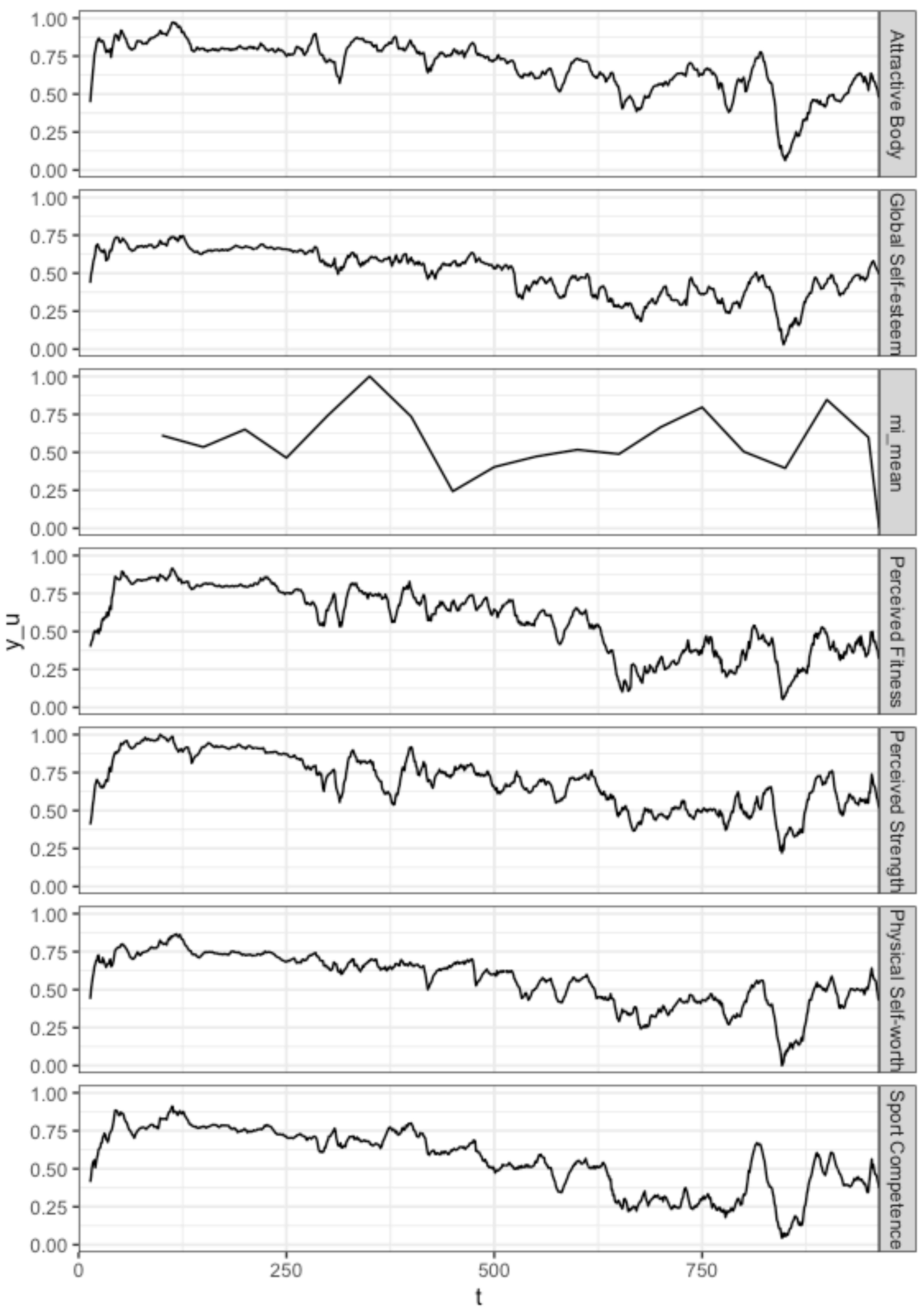
$$I_{\alpha,\beta} = \sum_{s(\kappa)^{[\alpha]}} \sum_{s(\kappa)^{[\beta]}} P\left(s(\kappa)^{[\alpha]}, s(\kappa)^{[\beta]}\right) \cdot log \frac{P\left(s(\kappa)^{[\alpha]}, s(\kappa)^{[\beta]}\right)}{P\left(s(\kappa)^{[\alpha]}\right) P\left(s(\kappa)^{[\beta]}\right)}$$



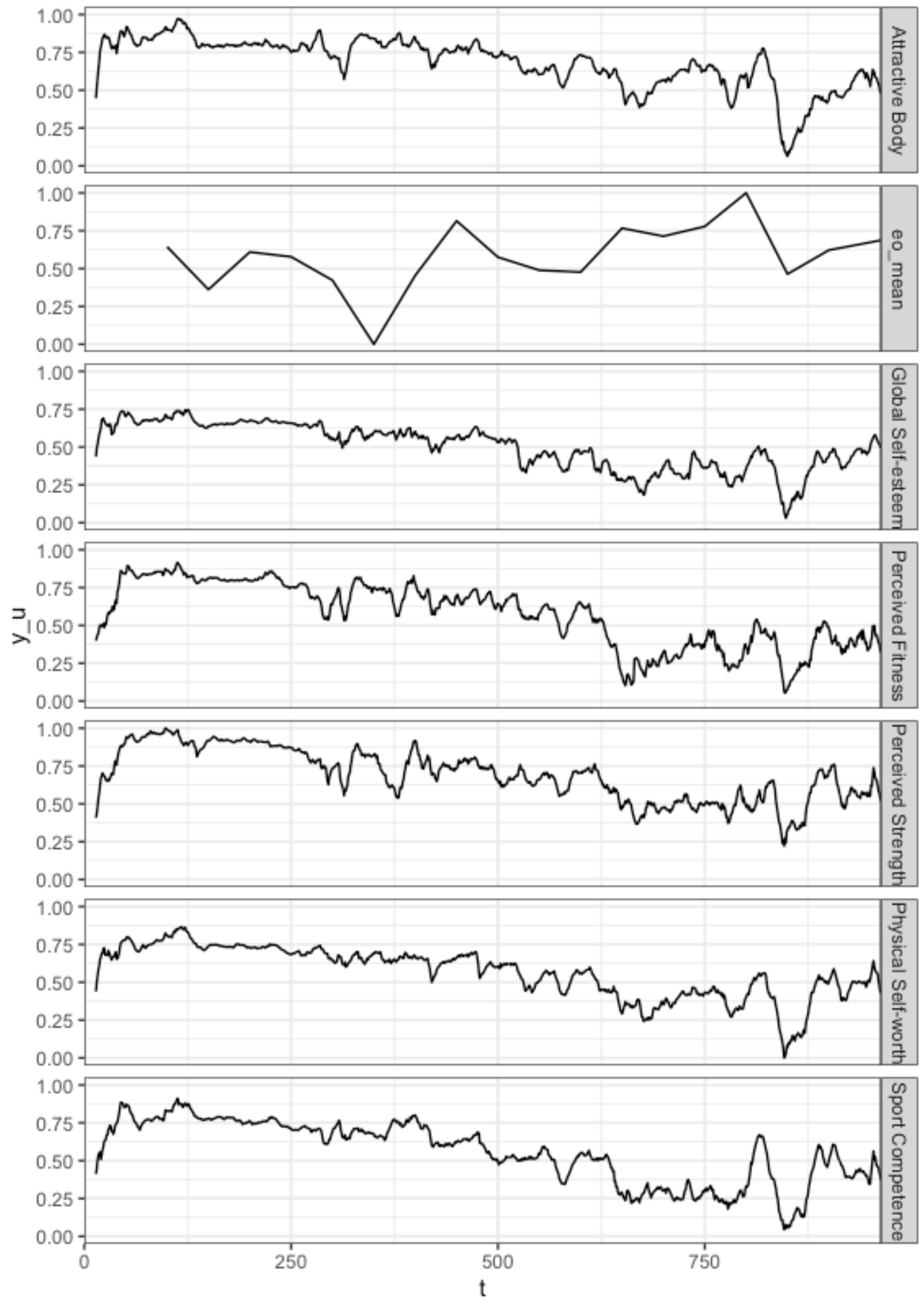

Participant 1
AB
PS
PF
GS
SC
SW
Participant 2
AB
PS
PF
GS
SC
SW
Participant 3
AB
PS
PF
GS
SC
SW
Participant 4
AB
PS
PF
GS
SC
SW

window: 01 | start: 1 | stop: 100

Interlayer Mutual Information (subject 3)



Edge Overlap (subject 2)



Take Home Message

NO NEED TO REDUCE DIMENSION OF MULTIVARIATE TIME SERIES DATA

USE MULTIPLEX RECURRENCE NETWORKS