Dynamics of Complex Systems

Part 1:

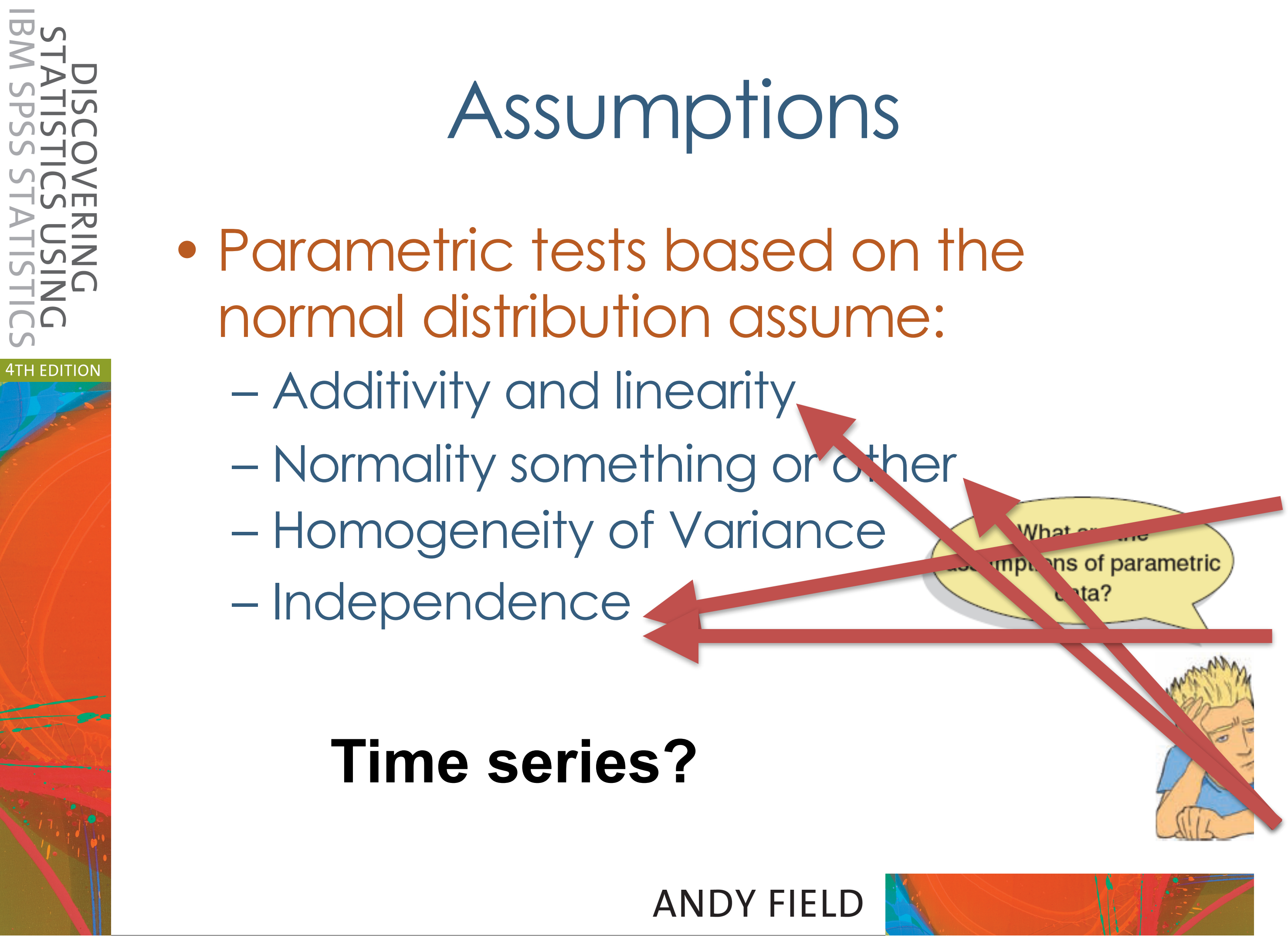
What is Complexity Science?
Story so far + Assignment

Part 2: SIMULATION (cntd.)

Closer look at modelling growth
Multivariate Models

Behavioural Science Institute
Radboud University Nijmegen

Assumptions - Ergodicity



Einstein (1905) on Brownian motion:

(i) the **independence** of individual particles,

(ii) the existence of a **sufficiently small time scale** beyond which *individual displacements are statistically independent*, and

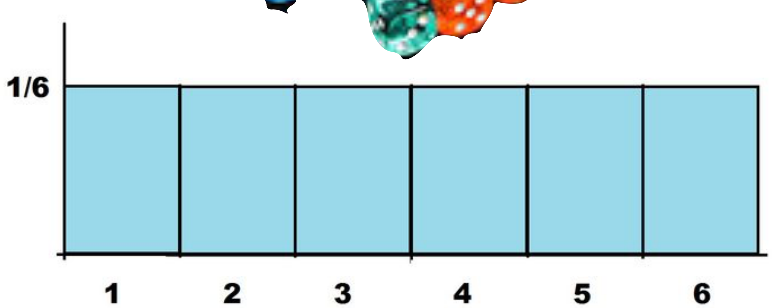
(iii) the property that the particle displacements during this time scale correspond to **a typical mean free path** *distributed symmetrically in positive or negative directions*.

Metzler, R., Jeon, J. H., Cherstvy, A. G., & Barkai, E. (2014). Anomalous diffusion models and their properties: non-stationarity, non-ergodicity, and ageing at the centenary of single particle tracking. *Physical Chemistry Chemical Physics, 16(44)*, 24128-24164

Assumptions - Ergodicity

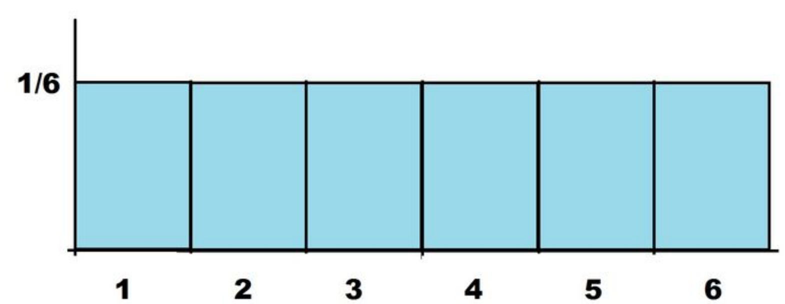
Ergodic process/measure/system

1 time 100 dice
“space-average”



=

100 times 1 die
“time average”



Einstein (1905) on Brownian motion:

(i) the **independence** of individual particles,

(ii) the existence of a **sufficiently small time scale** beyond which *individual displacements are statistically independent*, and

(iii) the property that the particle displacements during this time scale correspond to **a typical mean free path** *distributed symmetrically in positive or negative directions*.

Assumptions - Ergodicity

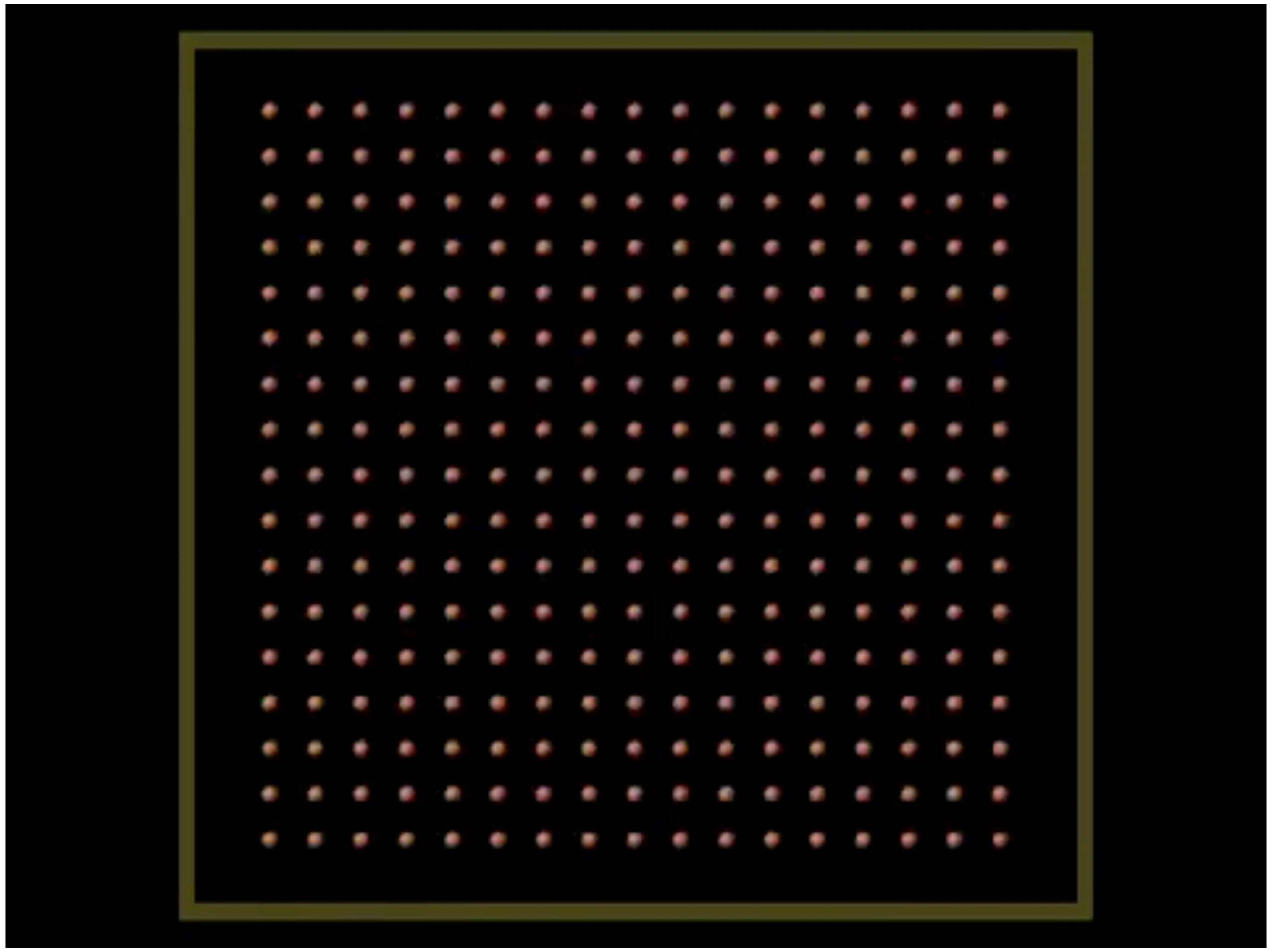
Ergodicity:

Stationarity+Homogeneity+Linear additive effects

= **Memorylessness property**

- no memory for initial conditions
(compare: sensitive dependence on initial conditions)
- no after-effects of interactions

Given infinite time the entire state space will be used:
- All states will have equal probability in the long run
- Maximal entropy = Completely homogeneous



Assumptions - Ergodicity

Ergodicity

- A random process $X(t)$ is ergodic if all of its statistics can be determined from a sample function of the process
- That is, the ensemble averages equal the corresponding time averages with probability one.

5

Thus, you obtain two different results: one statistical analysis over the entire ensemble of people at a certain moment in time, and one statistical analysis for one person over a certain period of time. The first one may not be representative for a longer period of time, while the second one may not be representative for all the people.

The idea is that an ensemble is ergodic if the two types of statistics give the same result. Many ensembles, like the human populations, are not ergodic.

https://larspsyll.wordpress.com/2016/11/23/what-is-ergodicity-2/

Deterministic Chaos & Turbulence

3Brown1Blue Kolmogorov constant 5/3:

https://youtu.be/_UoTTq651dE

Physics Girl vortex rings:

https://youtu.be/N7d_RWyOv20

Dynamics of Complex Systems

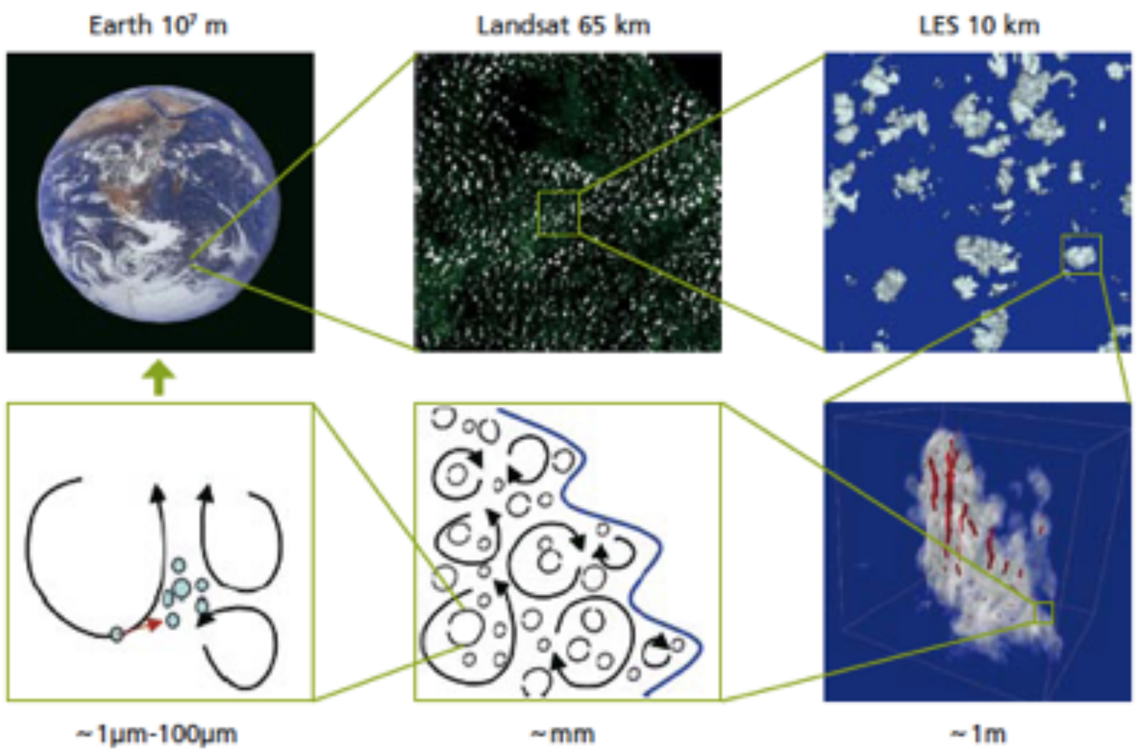
Intro to Complexity Science

II. DYNAMIC PATTERN FORMATION IN PHYSICAL, BIOLOGICAL, AND COMPUTATIONAL SYSTEMS

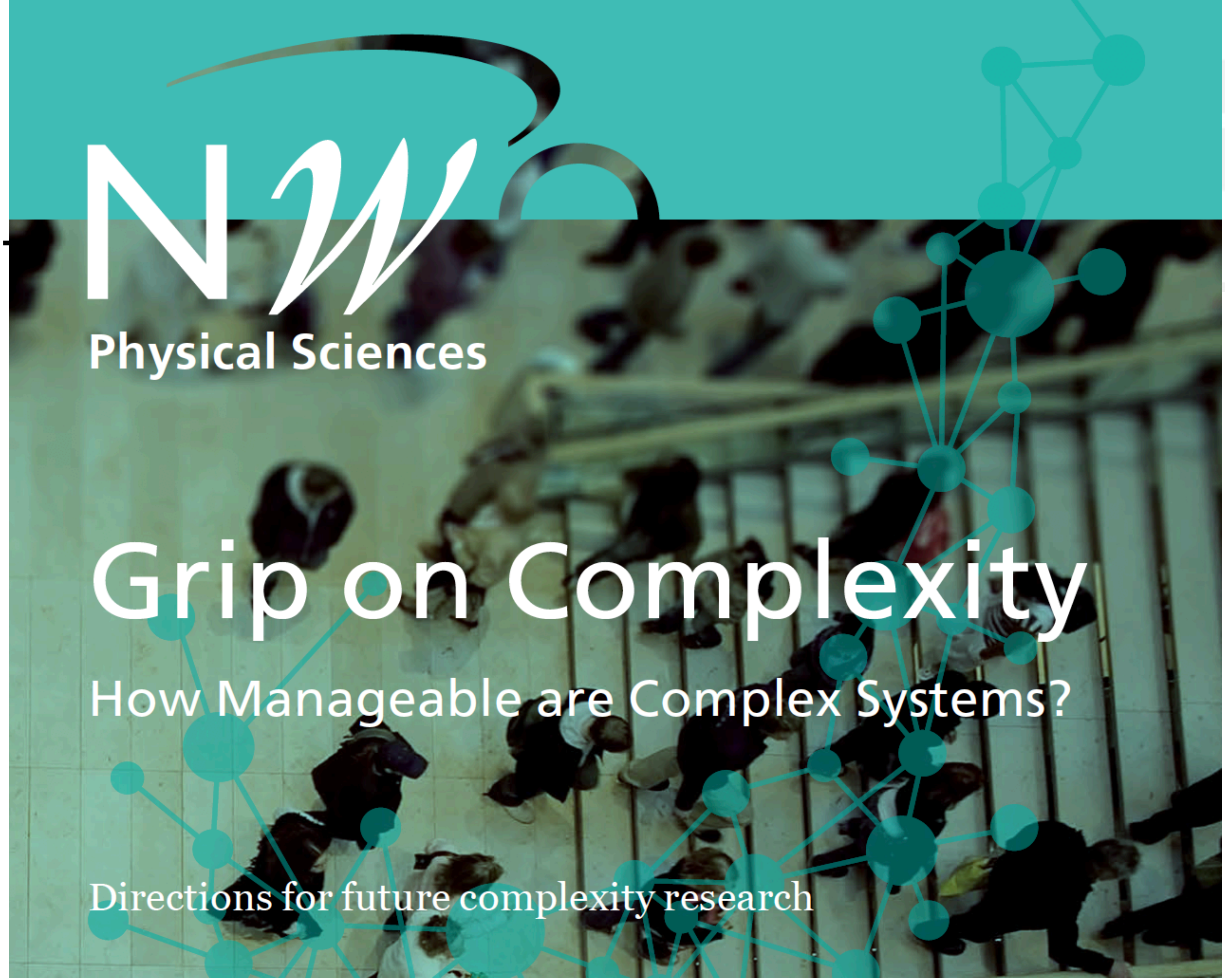
Thelen, E., & Ulrich, B. D. (1991). Hidden skills: A dynamic systems analysis of treadmill stepping during the first year. Monographs of the Society for Research in Child Devevelopment, 56(1), 1-98; discussion 99-104. Retrieved from https://www.ncbi.nlm.nih.gov/pubmed/1922136

Complexity Science

- Time! (Dynamics)
- Self-Organization
- Micro-Macro levels (Emergence)
- Scale invariance



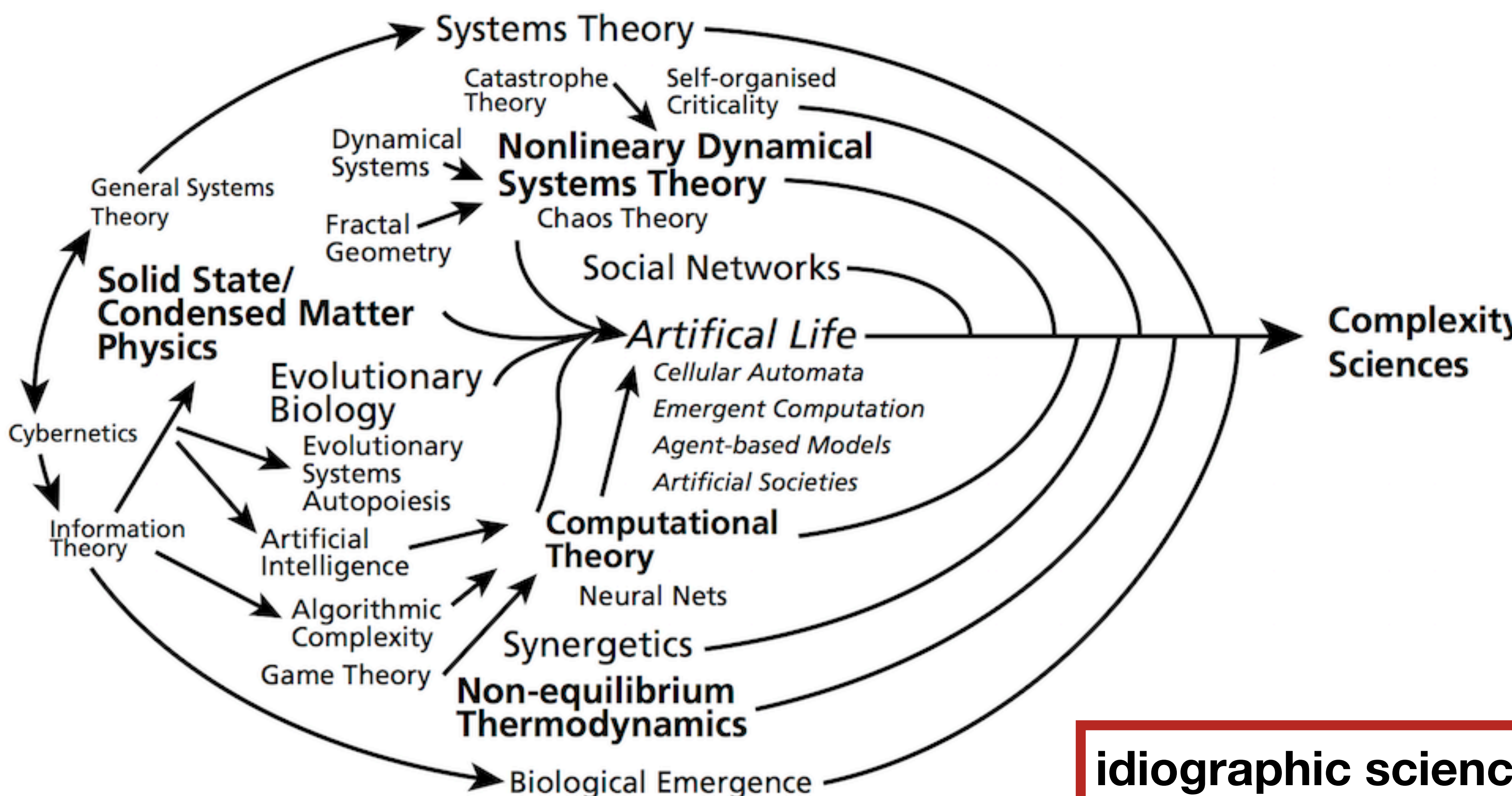
https://pure.uva.nl/ws/files/4259324/63508_302758.pdf



https://www.nwo.nl/en/documents/ew/complexity/paper-grip-on-complexity

Complexity Science

The scientific study of complex dynamical systems and networks



The many foundations of complexity science.

idiographic science!

Our goal is to develop methods for personalised diagnosis and intervention that can actually be used in practice

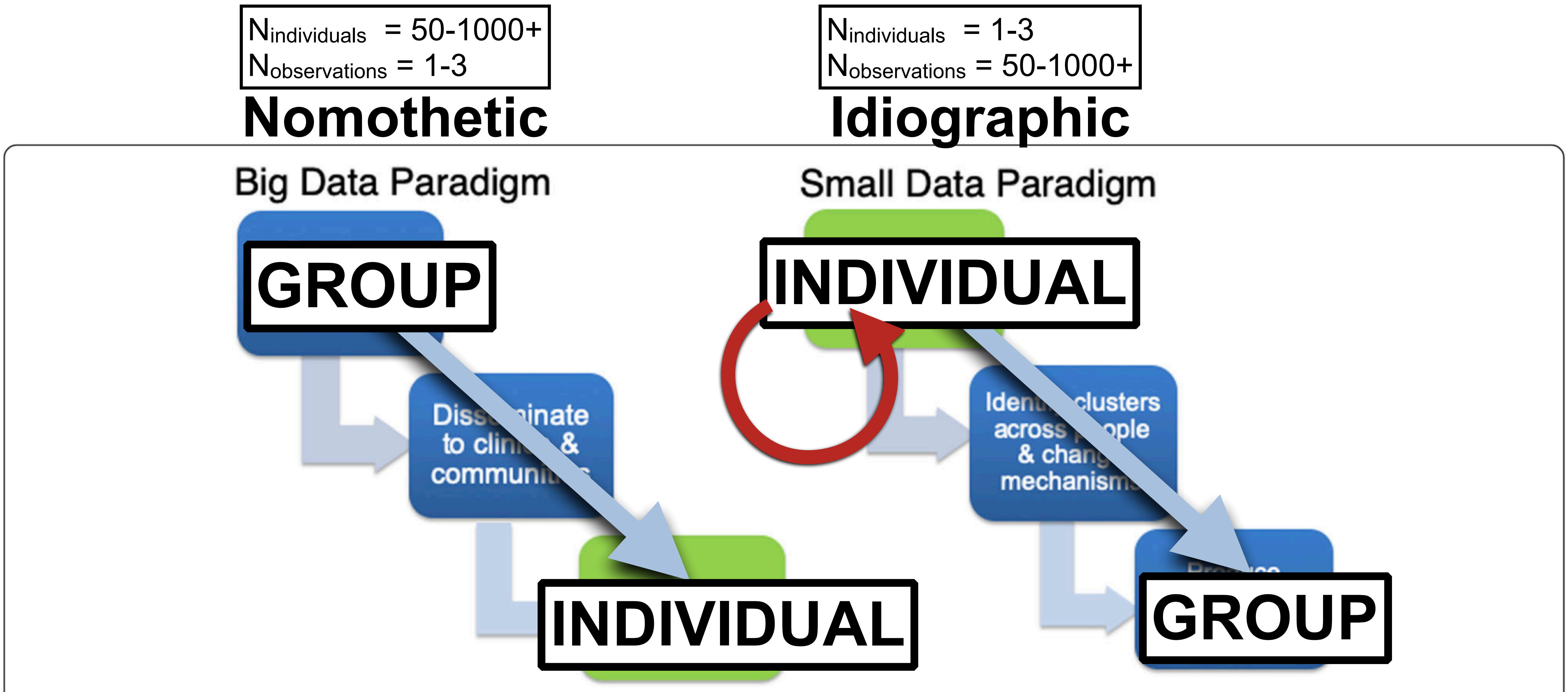


Fig. 1 Small versus big data paradigm pathways to help individuals and transportable knowledge

Hekler, E. B., Klasnja, P., Chevance, G., Golaszewski, N. M., Lewis, D., & Sim, I. (2019). Why we need a small data paradigm. *BMC Med, 17*(1), 133. doi:10.1186/s12916-019-1366-x

Our goal is to develop methods for personalised diagnosis and intervention that can actually be used in practice

Critical Fluctuations as an Early-Warning Signal for Sudden Gains and Losses in Patients Receiving Psychotherapy for Mood Disorders

Merlijn Olthof, Fred Hasselman, Guido Strunk, Marieke van Rooij, Benjamin Aas, Marieke A. Helmich, Günter Schiepek, Anna Lichtwarck-Aschoff

Show less

First Published September 24, 2019 | Research Article | Check for updates
https://doi.org/10.1177/2167702619865969

Article information

Biol Cybern (2010) 102:197–207 201

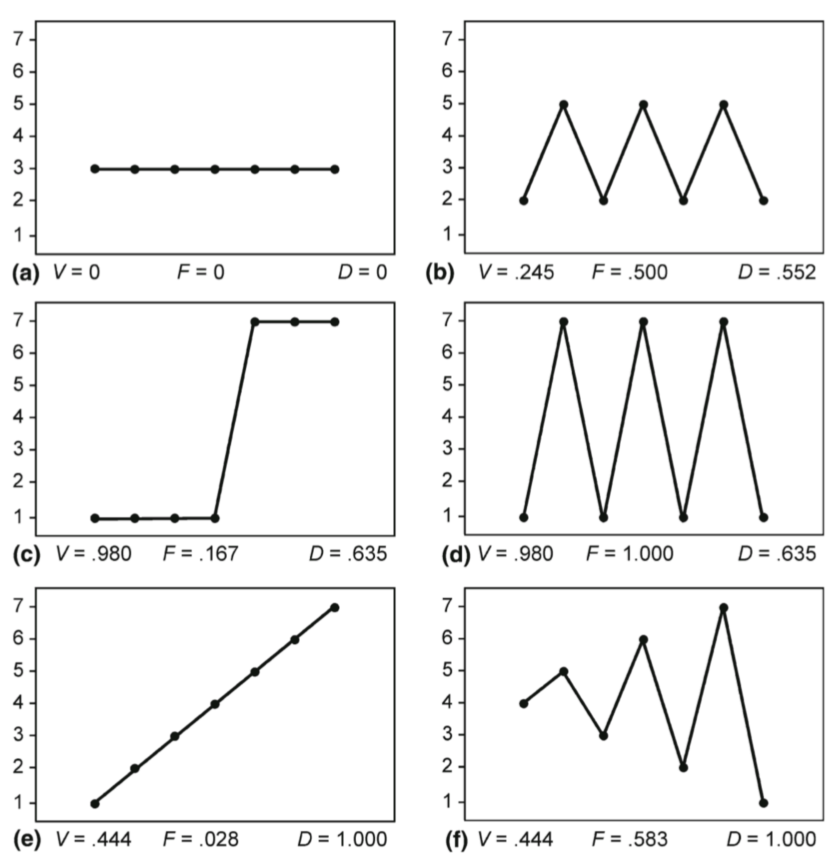
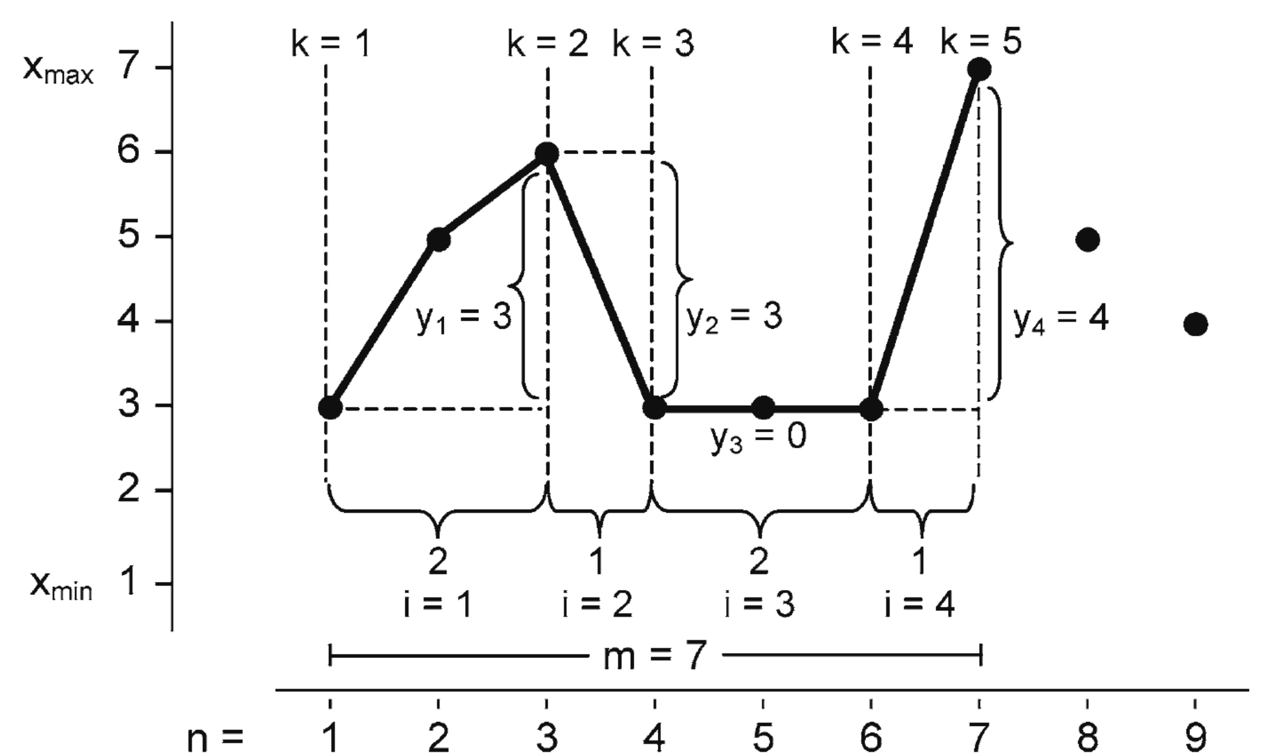


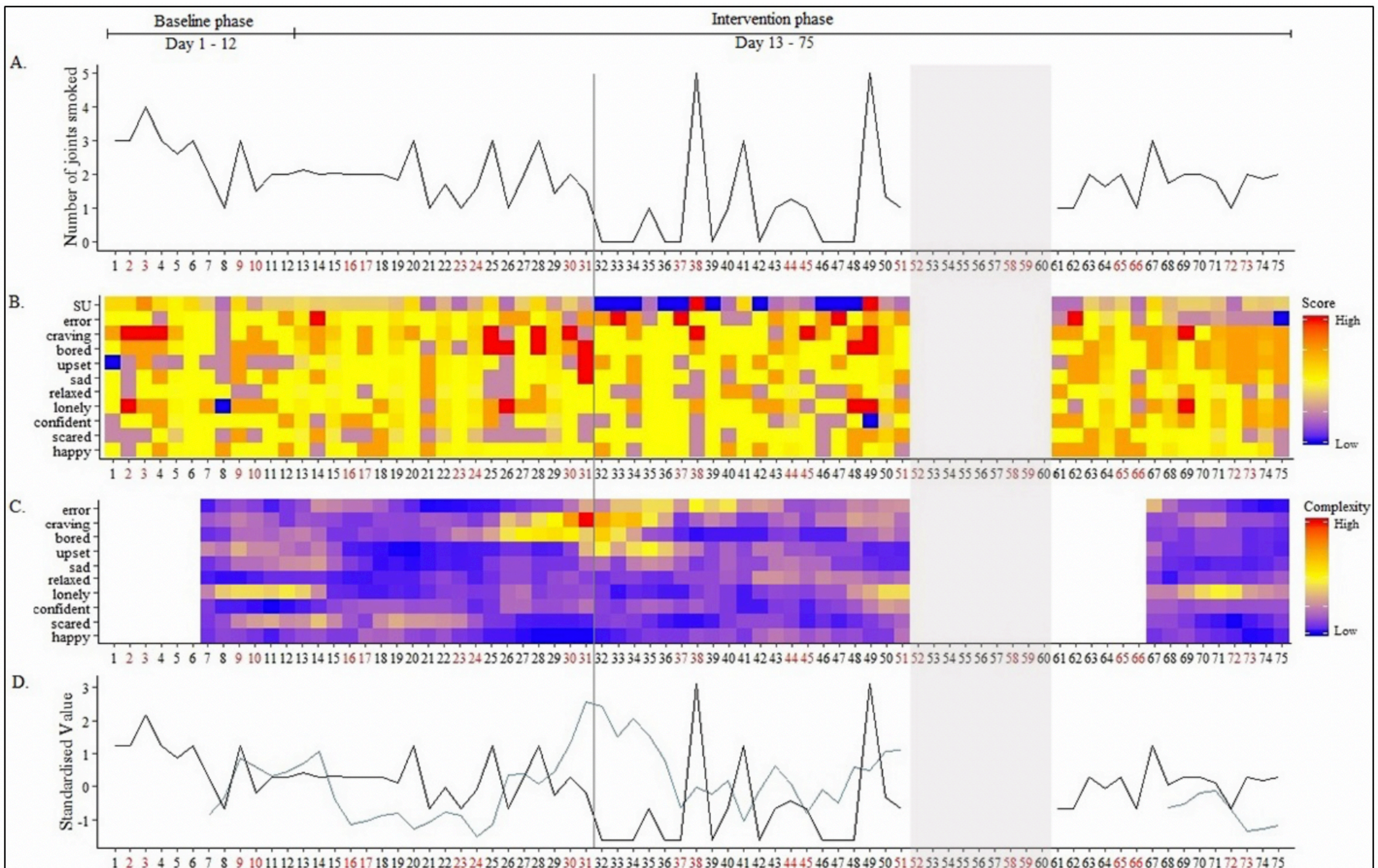
Fig. 2 Variance score (V), degree of fluctuation (F), and degree of distribution (D) of 6 dummy sequences. The ordinate corresponds to a 7-point Likert scale. The variance score V is the ratio between the variance and the greatest possible variance in this case ($s^2 = 10.29$), and thus normalized between [0, 1], as are F and D. **a** In the case of a horizontal line all three scores have the same result: 0. **b** Periodic alternation: F and D are more sensitive than V. **c** The system jumps from one stable state to the other, but without fluctuations. Therefore, F remains small. **d** The sequence realizes the same values as in **c**, but now by manifesting strong fluctuations. F is sensitive to this, V and D do not differ from **c**. **e** and **f** have the same variance, whereas the differences in the shape of the time series are evident. The fluctuation is more accentuated in **f** than in **e**

Biol Cybern (2010) 102:197–207



Our goal is to develop methods for personalised diagnosis and intervention that can actually be used in practice

SUBSTANCE USE IN INTELLECTUAL DISABILITY 30



Our goal is to develop methods for personalised diagnosis and intervention that can actually be used in practice

I Stress and Coping (state-cluster "child", EP, corresponds to factor I of the individual questionnaire)

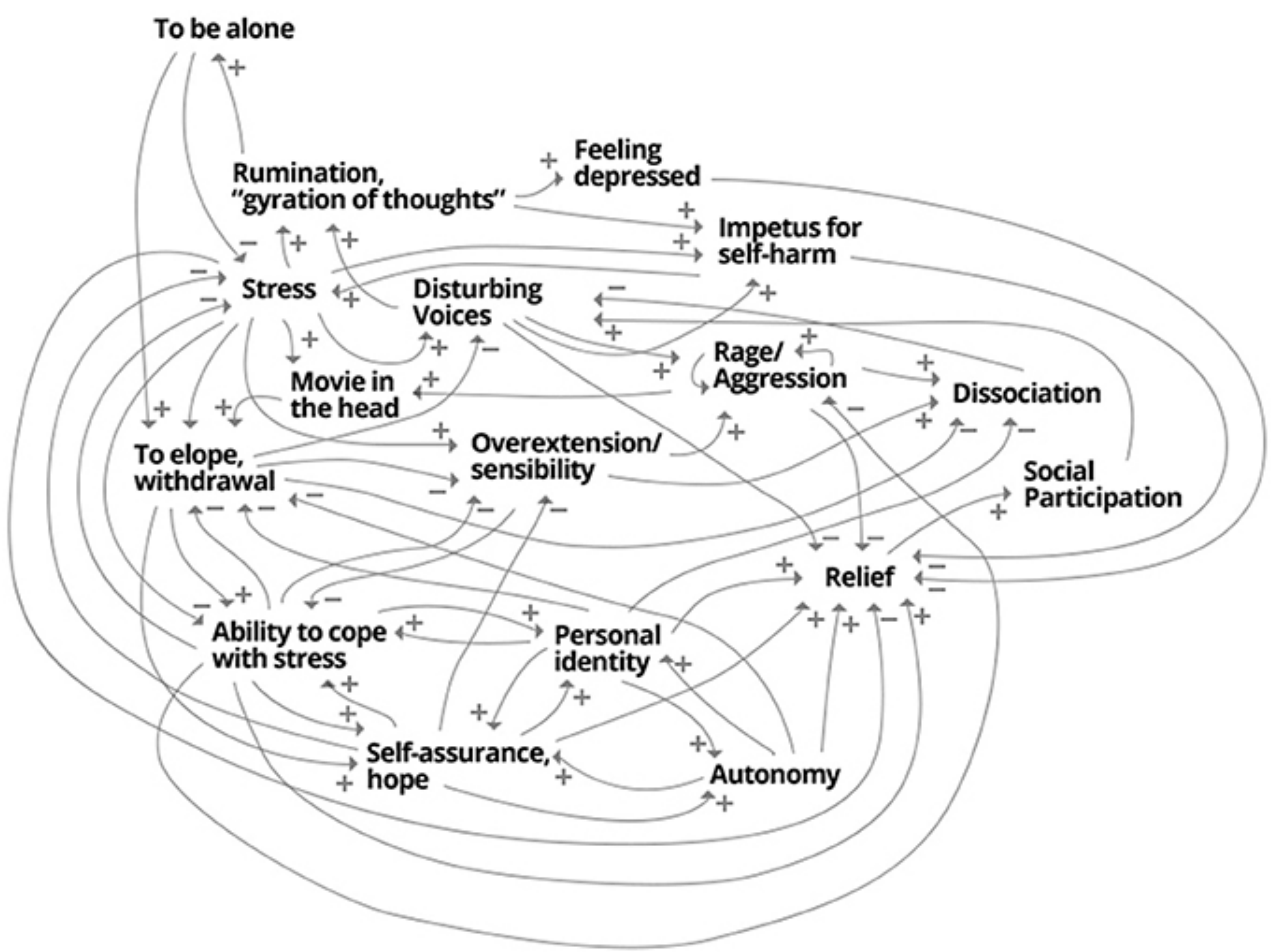
1. Today, I experienced stress …
2. Today, I had to activate my "head-cinema" ("movie in the head") …
3. Today, I zoomed out - dissociated …
4. Today, it was important to me to be alone …
5. Today, the depression carried me away …
6. The impulse to hurt myself was today …
7. Today, I ruminated …
8. The intrusive voices were today …
9. My level of aggression was today …
10. My level of anger was today …
11. Today, I felt overwhelmed …
12. My need for distancing myself from others was today …

II Positive goals and development of identity (state-cluster "adult", ANP, corresponds to factor II of the individual questionnaire)

13. Today, I felt resilient and able to cope with stress …
14. My feelings of inner security were today …
15. My feelings of independence were today …
16. The sense of my own inner identity was today …
17. Today I had a sense of relief …
18. Today, I took part in social life …

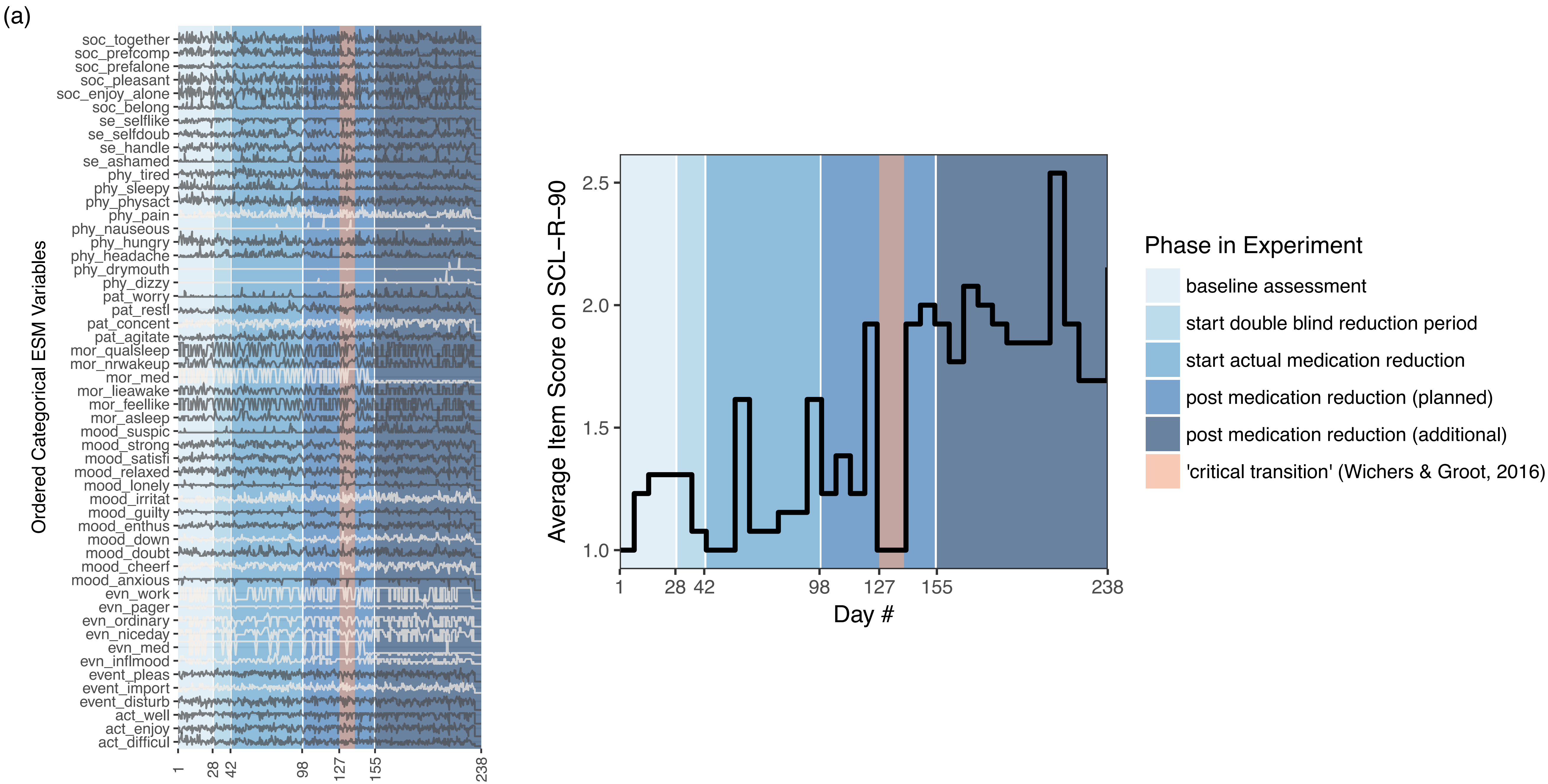
*Match the 18 variables of her ISM, as shown in **Figure 1**, separated into two factors. The client answered these items daily via the online monitoring system SNS. Each question is scored by a visual analog slider (VAS), ranging from 0 to 100 and extrema of "not at all" to "very much" (where applicable).*

Idiographic system modeling



Schiepek, G. K., Stöger-Schmidinger, B., Aichhorn, W., Schöller, H., & Aas, B. (2016). Systemic case formulation, individualized process monitoring, and state dynamics in a case of dissociative identity disorder. *Frontiers in Psychology*, *7*, 1545. https://www.frontiersin.org/articles/10.3389/fpsyg.2016.01545/full

“Critical Slowing Down as a Personalized Early Warning Signal for Depression”



Wichers, M., Groot, P. C., Psychosystems, ESM Grp, & EWS Grp (2016). Critical Slowing Down as a Personalized Early Warning Signal for Depression. Psychotherapy and psychosomatics, 85(2), 114-116. DOI: 10.1159/000441458

Kossakowski, J., Groot, P., Haslbeck, J., Borsboom, D., and Wichers, M. (2017). Data from ‘critical slowing down as a personalized early warning signal for depression’. Journal of Open Psychology Data, 5(1).

What is Complexity Science?
[and why should scientist studying human nature embrace it?]

- **Fundamental problems for main-stream Social & Life Sciences:**
  ➡ Mismatch between research methods and object of measurement
  ➡ Not interdisciplinary (theoretical, empirical, formal, …)

- **Complex behaviour from (physical) principles & laws (bottom-up):**
  ➡ Ecological Psychology / Ecological Physics / Natural Computation

- **Complex behaviour from (physical) principles & laws (top-down):**
  ➡ Complex Systems Approach to Behavioural Science
  ➡ Personalised diagnosis and intervention

What is Complexity Science?

[and why should scientist studying human nature embrace it?]

First
some
basic
(abstract)
concepts

What is a

complex, adaptive, self-organizing, multi-stable, far-from-equilibrium, dissipative, etc.

system?

A system is an entity that can be described as a composition of components, according to one or more organising principles.

The organising principles can take many different forms, but essentially they decide the three important features of systems that have to do with the relationship between **parts** and **wholes**:

1. What are the **relevant scales of observation** of the system?

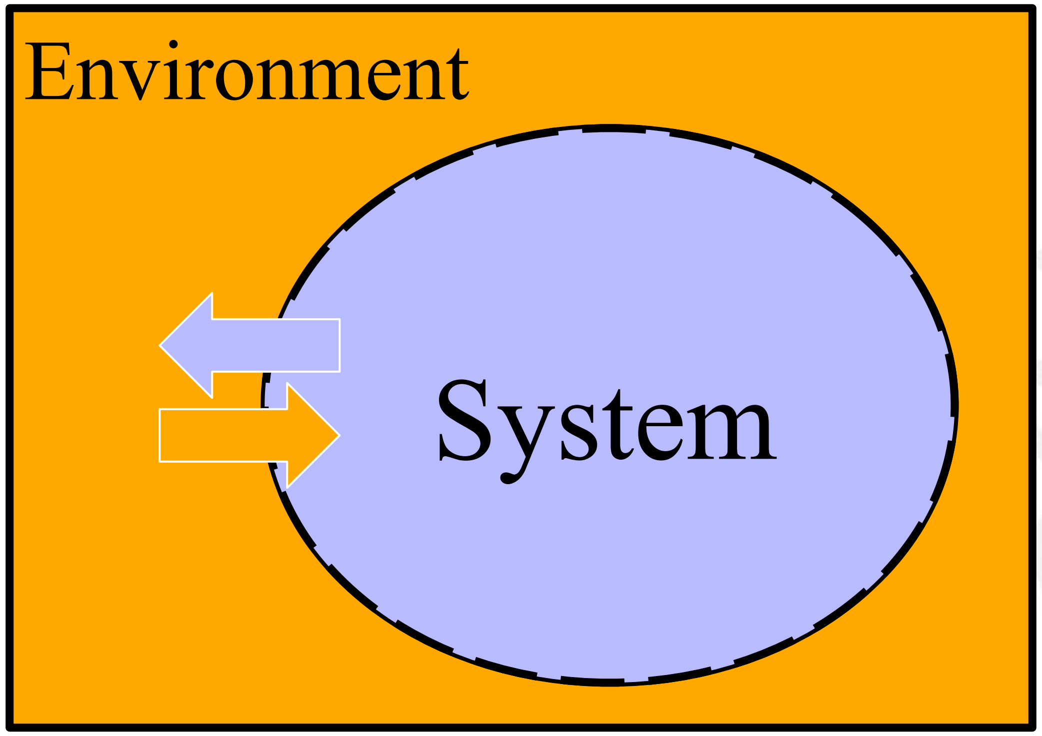
2. What are the **relevant phenomena** that may be observed at the different scales?

3. Can **interactions** with the internal and external environment occur, and if so, do interactions have any effects on the structure and/or behaviour of the system?

What is a system?

A system is an entity that can be described as a composition of components, according to one or more organising principles.

Closed and Open Systems



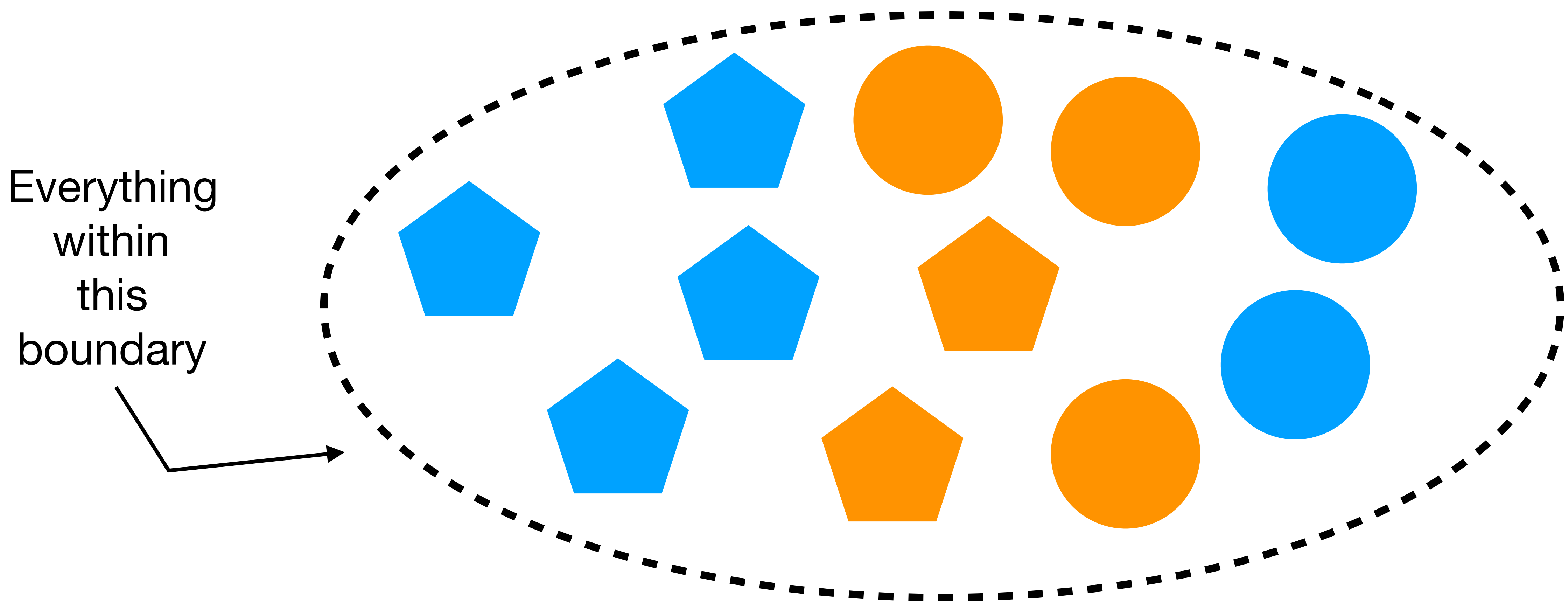
Continuous exchange of matter, energy, and information with the environment.

What is a

complex, adaptive, self-organizing, multi-stable, far-from-equilibrium, dissipative, etc.

system?

A system is an entity that can be described as a composition of components, according to one or more organising principles.



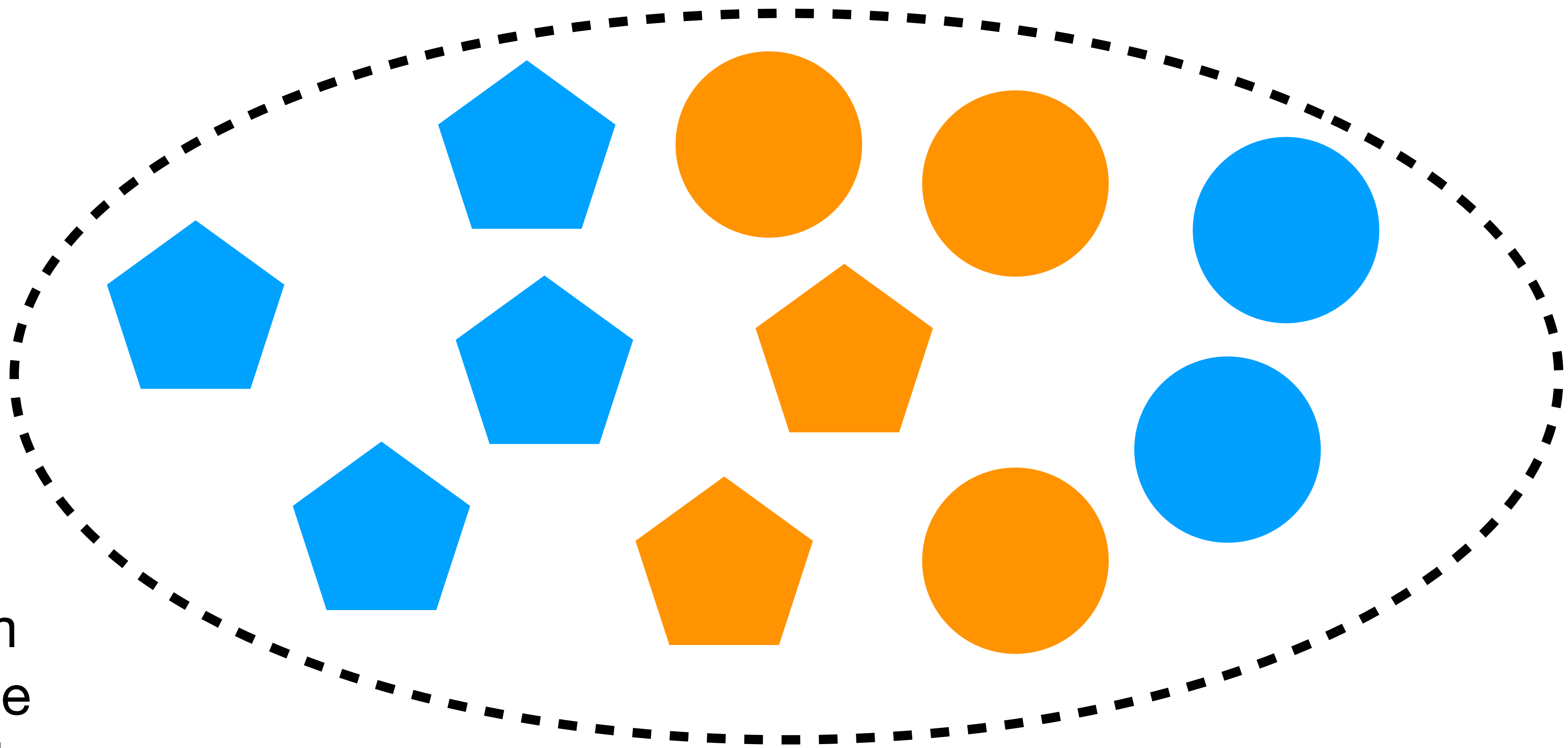
What is a

complex, adaptive, self-organizing, multi-stable, far-from-equilibrium, dissipative, etc.

system?

Degrees of freedom:

The constituent parts of a system whose state configuration at some micro scale, is associated with the behaviour of the system *as a **whole***, the global, or, macro state.



Degrees of freedom available to generate behaviour as a whole

What is a

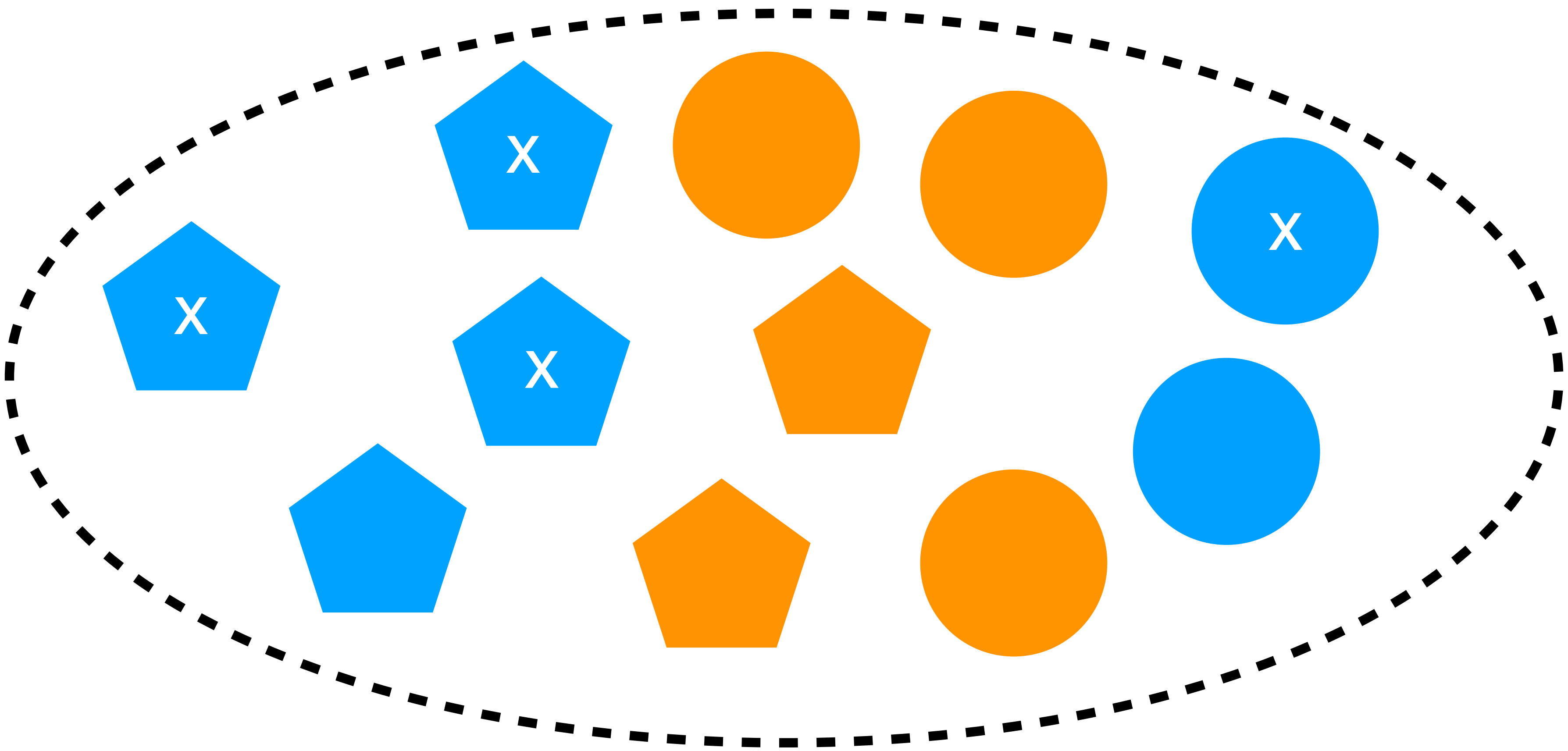
complex, adaptive, self-organizing, multi-stable, far-from-equilibrium, dissipative, etc.

system?

Degrees of freedom:
The constituent parts of a system whose state configuration at some micro scale, is associated with the behaviour of the system *as a **whole**,* the global, or, macro state.

Global state:
Blue

Degrees of freedom can be fixed or free



X = DoF recruited to generate the global state

What is a

complex, adaptive, self-organizing, multi-stable, far-from-equilibrium, dissipative, etc.

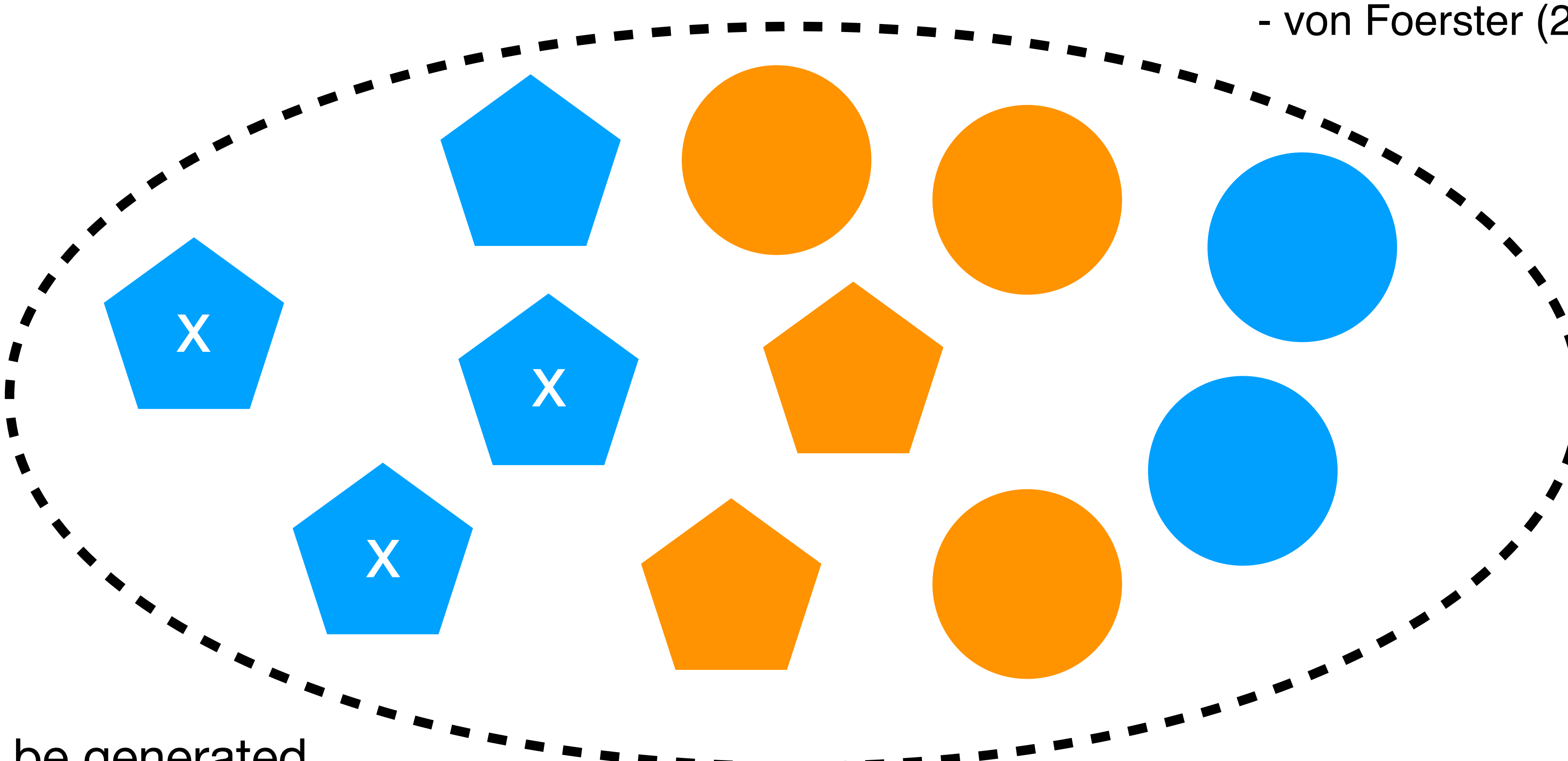
system?

"What is order? Order was usually considered as a wonderful building, a loss of uncertainty. Typically it means that if a system is so constructed that **if you know** the location or **the property of one element, you can make conclusions about the other elements. So order is essentially the arrival of redundancy in a system, a reduction of possibilities.**"

- von Foerster (2001)

Global state:
Blue

Degrees of freedom
can be fixed or free



In systems with many DoF
The same global state can be generated
by many different configurations at the micro-scale:
Uncertainty, disorder, entropy

X = DoF recruited to generate the global state

What is a

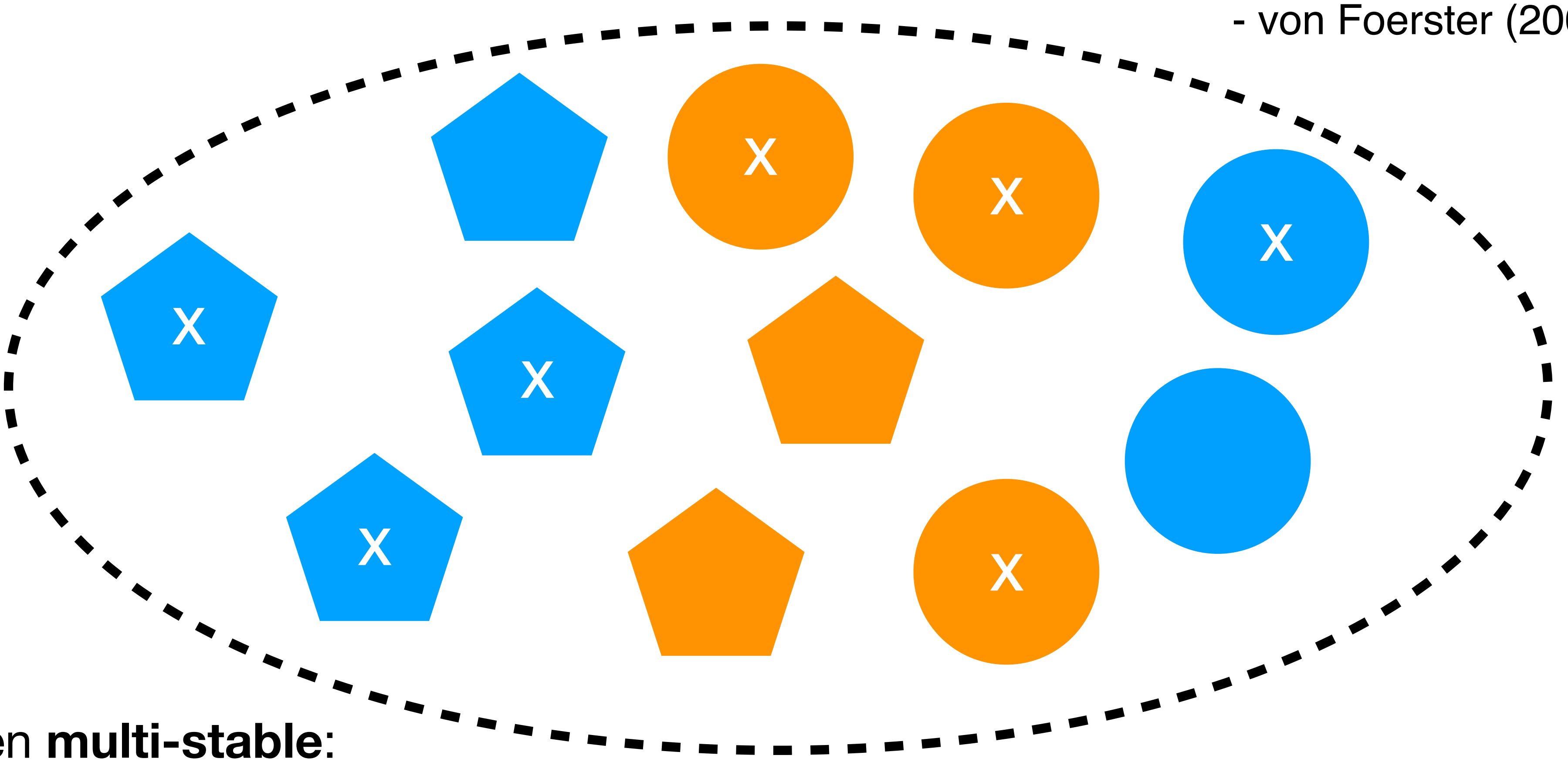
complex, adaptive, self-organizing, multi-stable, far-from-equilibrium, dissipative, etc.

system?

"What is order? Order was usually considered as a wonderful building, a loss of uncertainty. Typically it means that if a system is so constructed that **if you know** the location or **the property of one element, you can make conclusions about the other elements. So order is essentially the arrival of redundancy in a system, a reduction of possibilities.**"

- von Foerster (2001)

Global states:
Blue | **Round**



Degrees of freedom can be fixed or free

Complex systems are often **multi-stable**:
Different macro states can co-exist, or,
a system can quickly switch between states

X = DoF recruited to generate the global state

What is a

complex, adaptive, self-organizing, multi-stable, far-from-equilibrium, dissipative, etc.

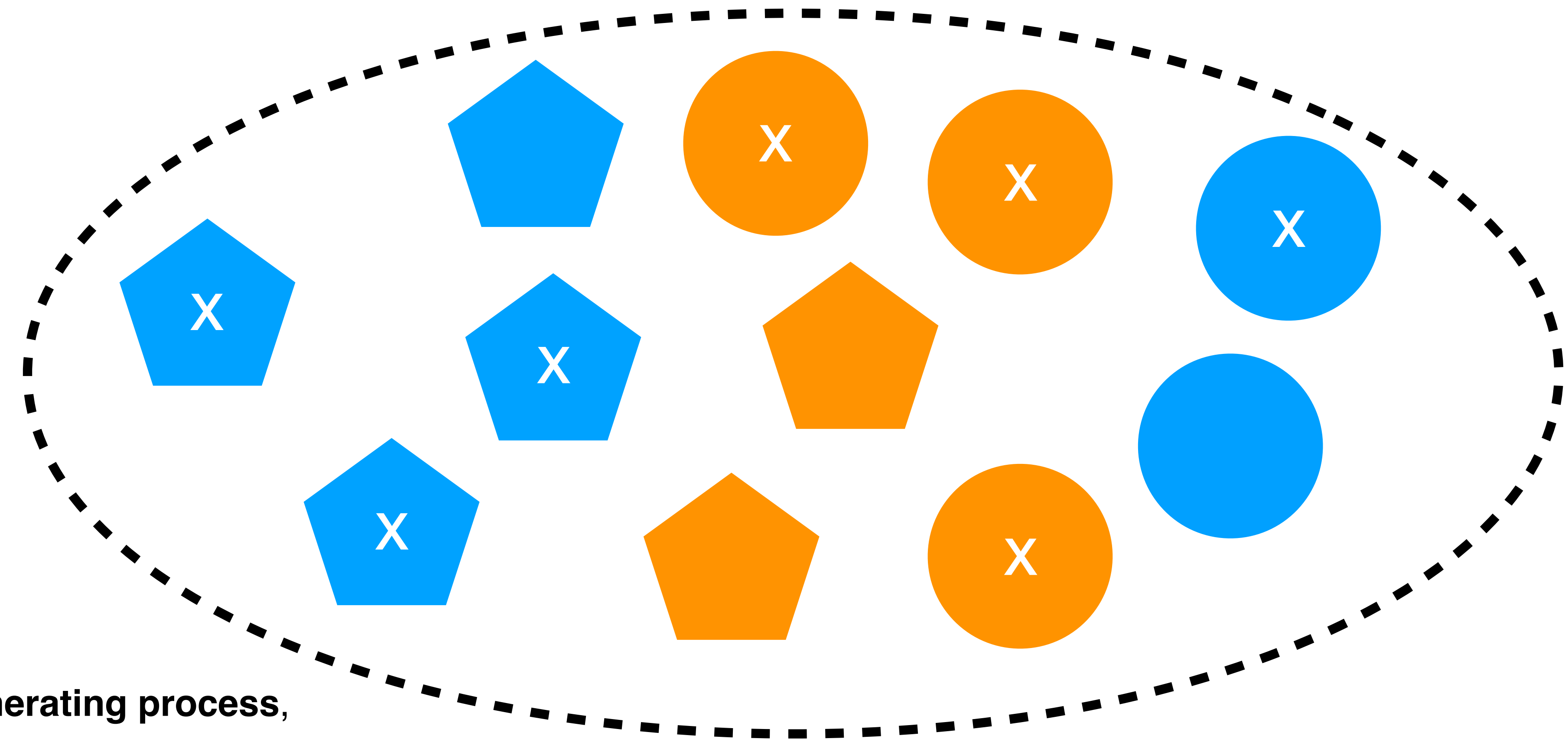
system?

The process of fixing and freeing-up degrees of freedom in is called **self-organisation**:

- In general, the **stability** or **resilience** of a macro state is associated with a reduction, or, constraining of the available DoF
- **Self-Organised Criticality** (SOC) refers a particular state/property that allows easy transition between several different modes of behaviour / dynamic regimes / orders of the system (Complex Adaptive Systems)

Global states:
Blue | **Round**

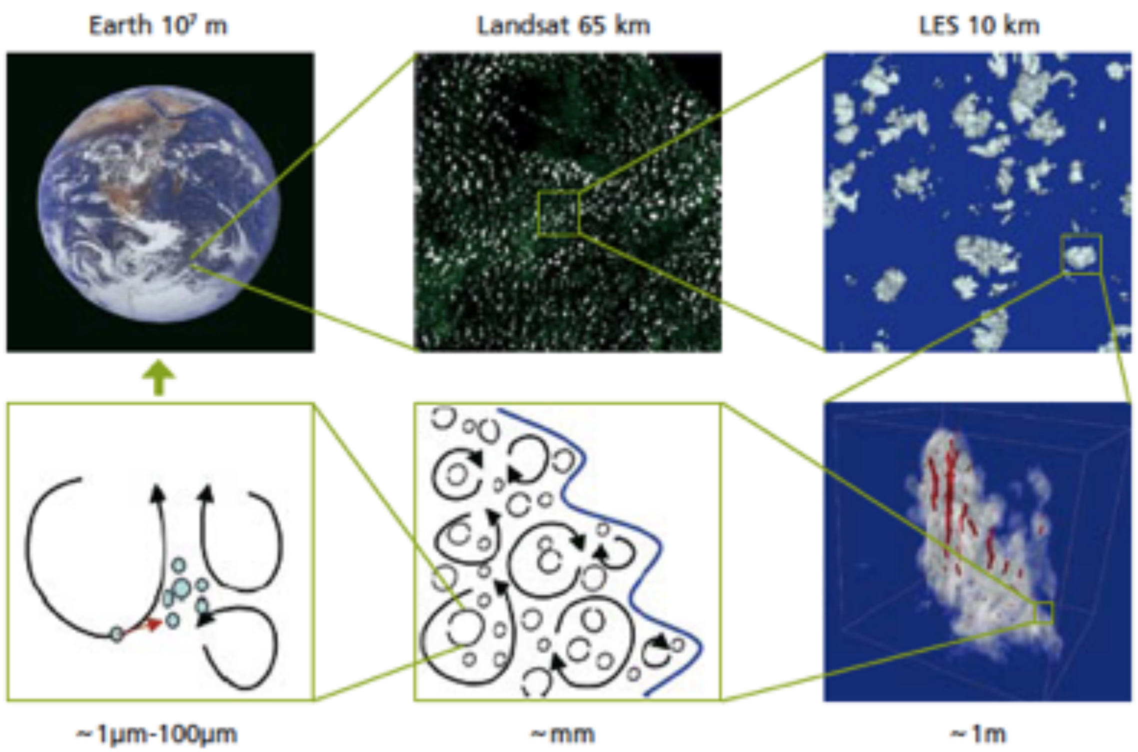
Degrees of freedom can be fixed or free



Self-organisation is an **order-generating process**,
it requires the transformation of free-energy into heat-energy / entropy
Fixing a DoF (generating order) requires the same amount of energy as Freeing up a DoF (= dissipative systems)

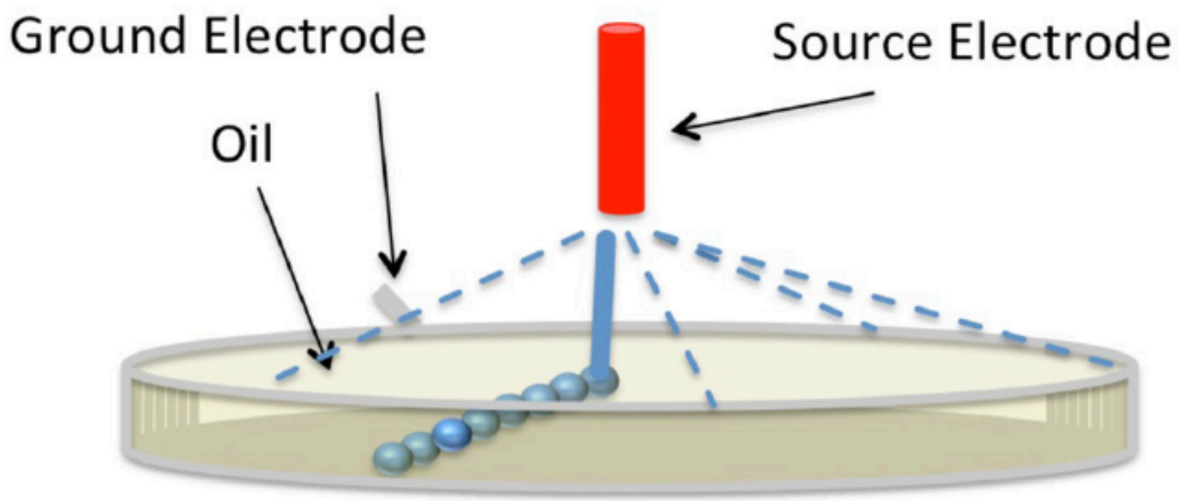
Complexity Science

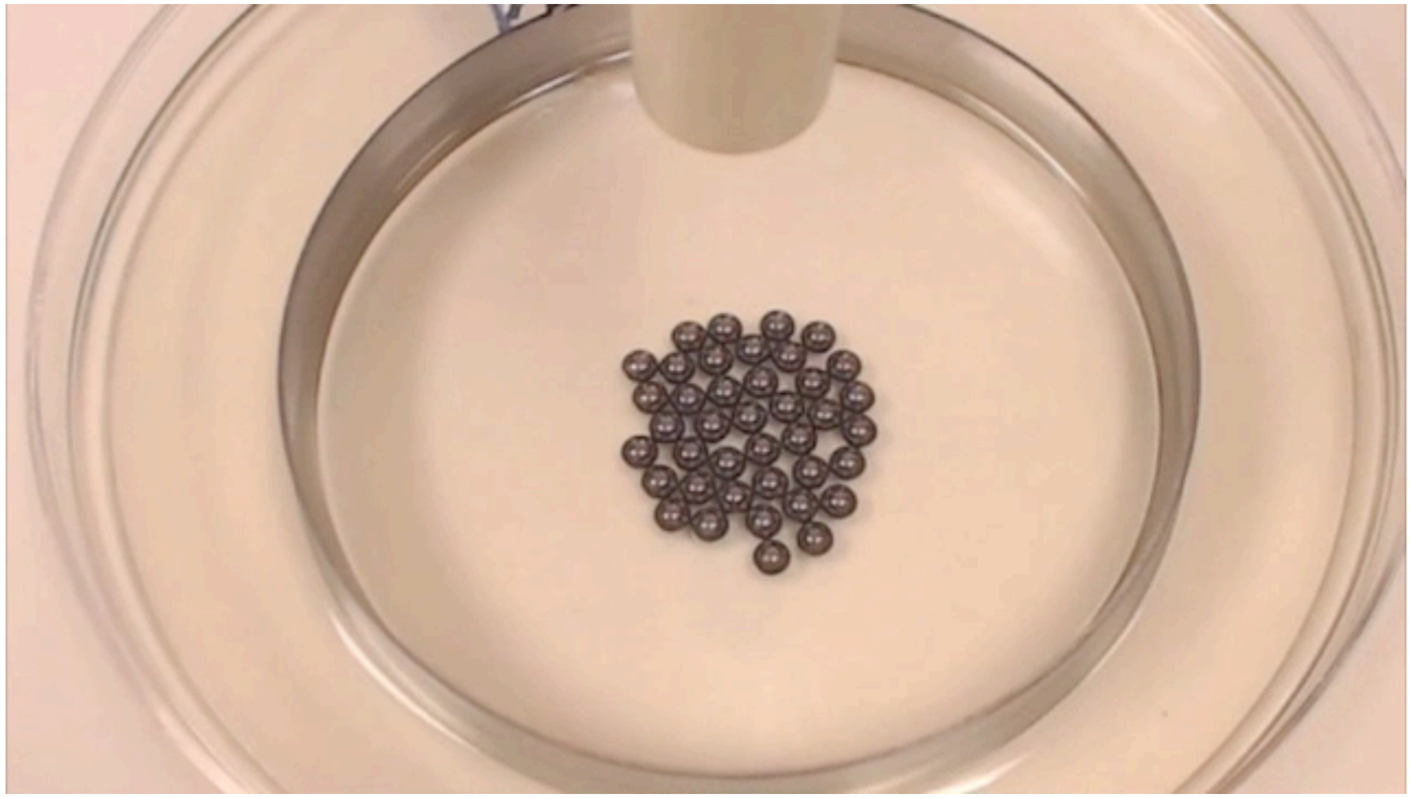
- Time! (Dynamics)
- **Self-Organization**
- Micro-Macro levels (Emergence)
- Scale invariance

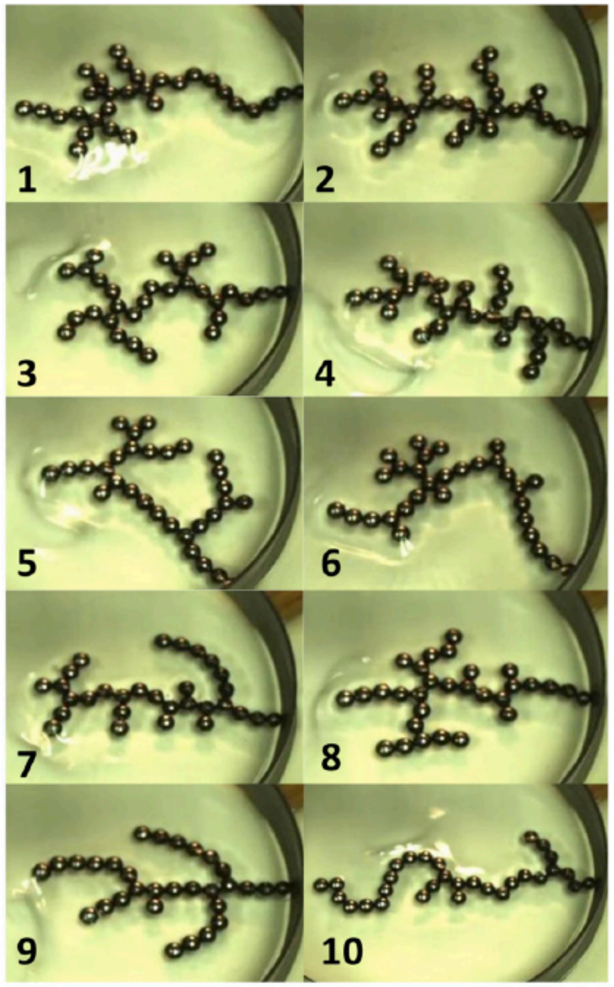


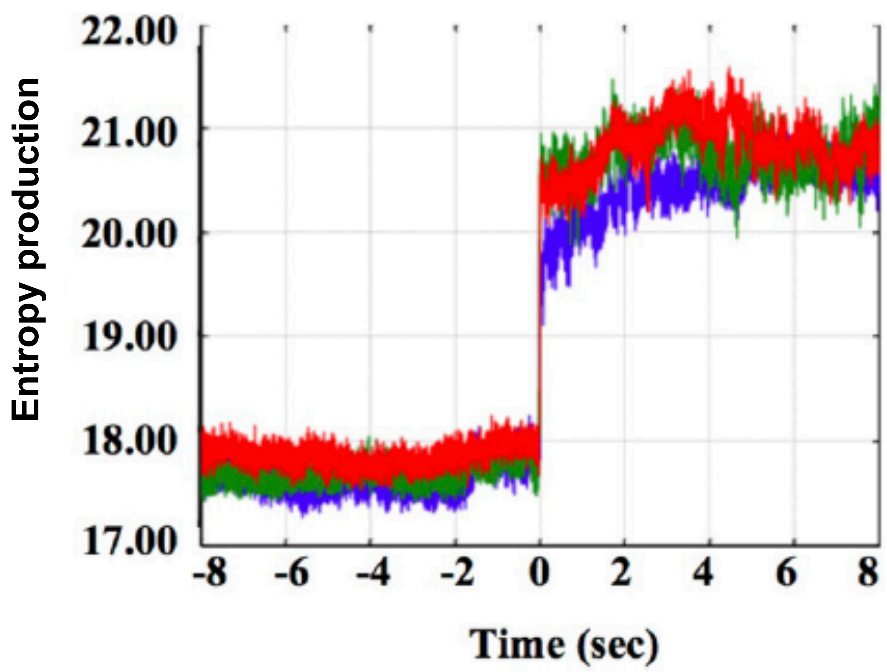
https://pure.uva.nl/ws/files/4259324/63508_302758.pdf

Self-Organisation in Dissipative Systems





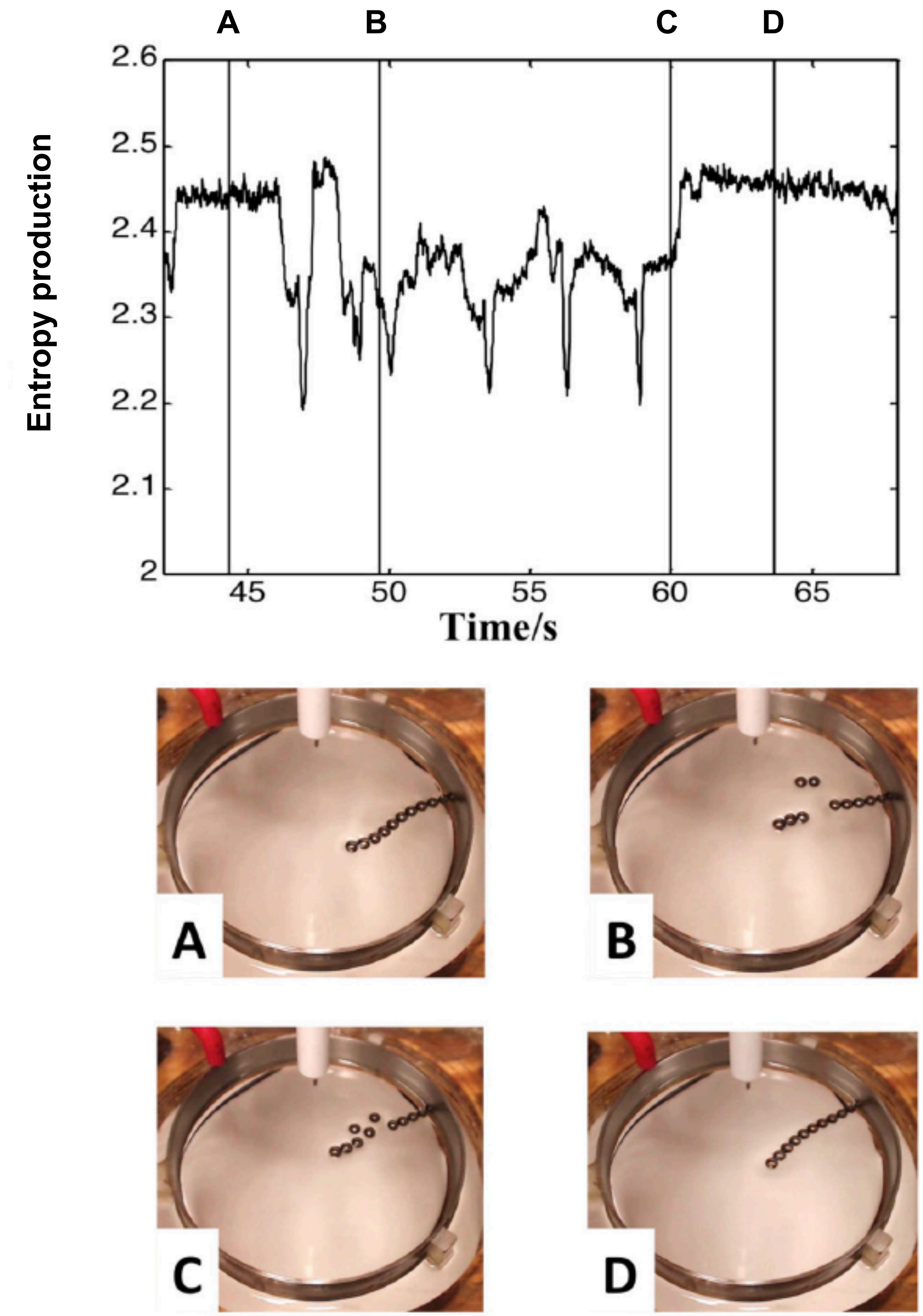




self-organisation:
Tree formation

Kondepudi D, Kay B, Dixon J. (2017). Dissipative structures, machines, and organisms: A perspective. *Chaos, 27(10),* 104607.

Self-Organisation in Dissipative Systems



self-repair:
Resilience to perturbation

Kondepudi D, Kay B, Dixon J. (2017). Dissipative structures, machines, and organisms: A perspective. *Chaos, 27(10),* 104607.

Complex behaviour from (physical) principles & laws (bottom-up)

END DIRECTED EVOLUTION TO STATES OF HIGHER ENTROPY PRODUCTION

More properties:
Memory
Classical conditioning (aversion / preference)

Memristors
[memristor.org]
"memory resistors", are a type of passive circuit elements that maintain a relationship between the time integrals of current and voltage across a two terminal element. Thus, a memristors' resistance varies according to a devices memristance function, allowing, via tiny read charges, access to a "history" of applied voltage

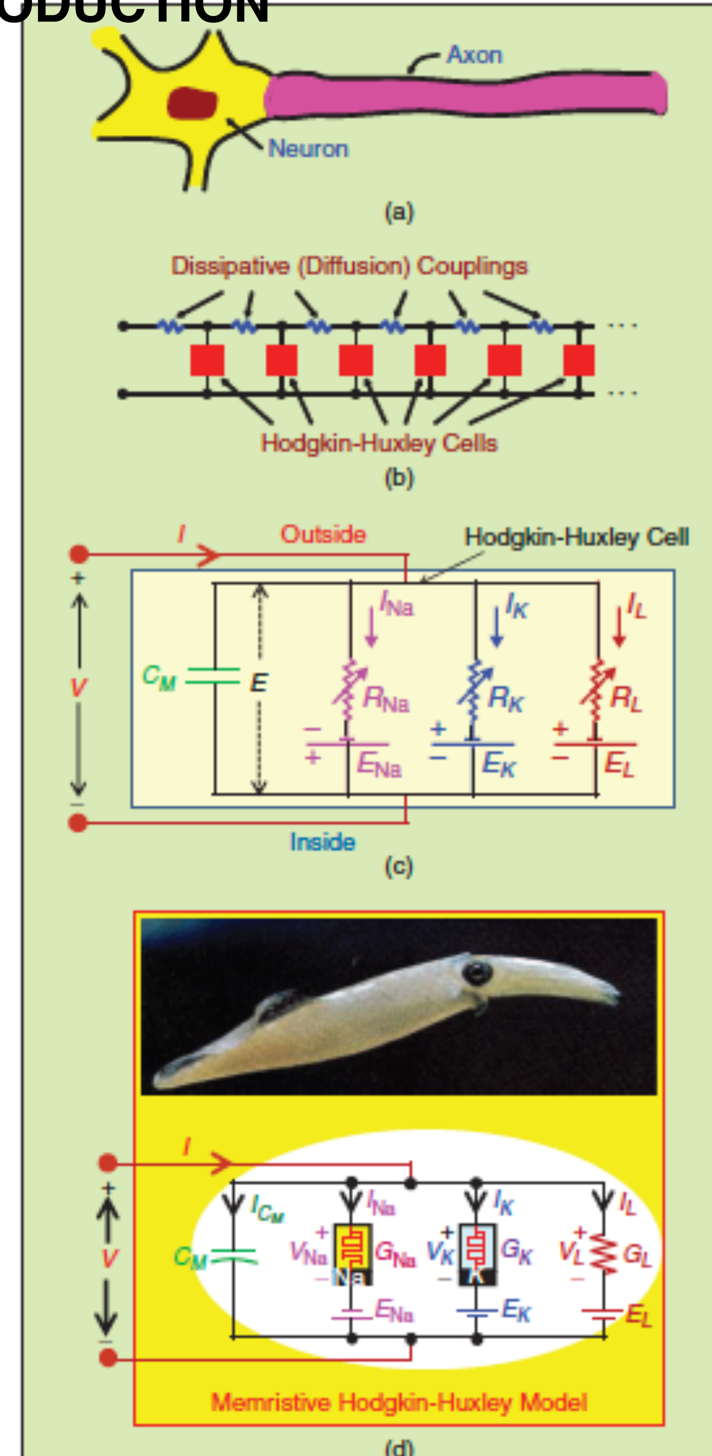
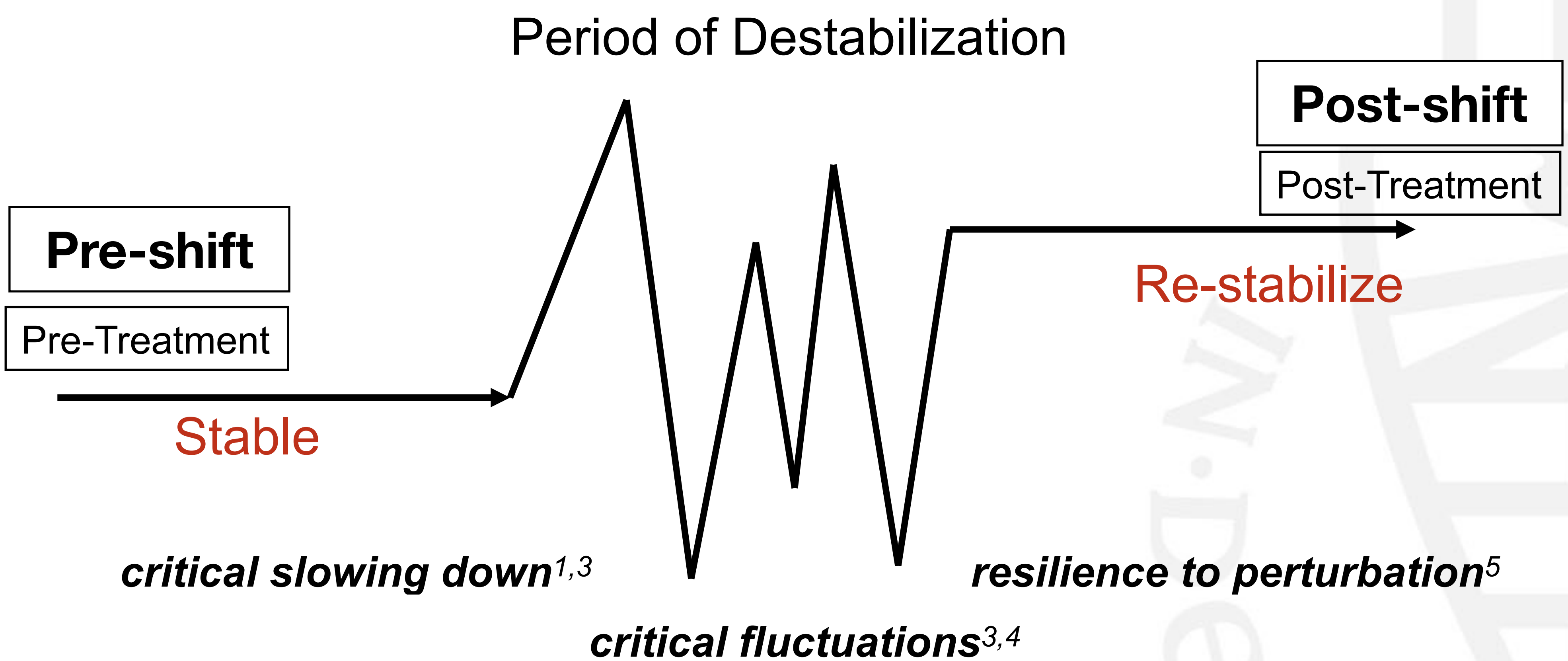


Figure 1. (a) Schematic of a neuron and its axon. (b) One-dimensional axon model made of resistively coupled Hodgkin-Huxley cells. (c) Hodgkin-Huxley circuit model made of a capacitor C_M, a resistor R_L, three batteries E_{Na}, E_K, and E_L, a *time-varying* potassium resistor R_K, and a *time-varying* sodium resistor R_{Na}. (d) Memristive Hodgkin-Huxley axon circuit model.

Sah, M. P., Kim, H., & Chua, L. O. (2014). Brains are made of memristors. *IEEE circuits and systems magazine*, *14*(1), 12-36.

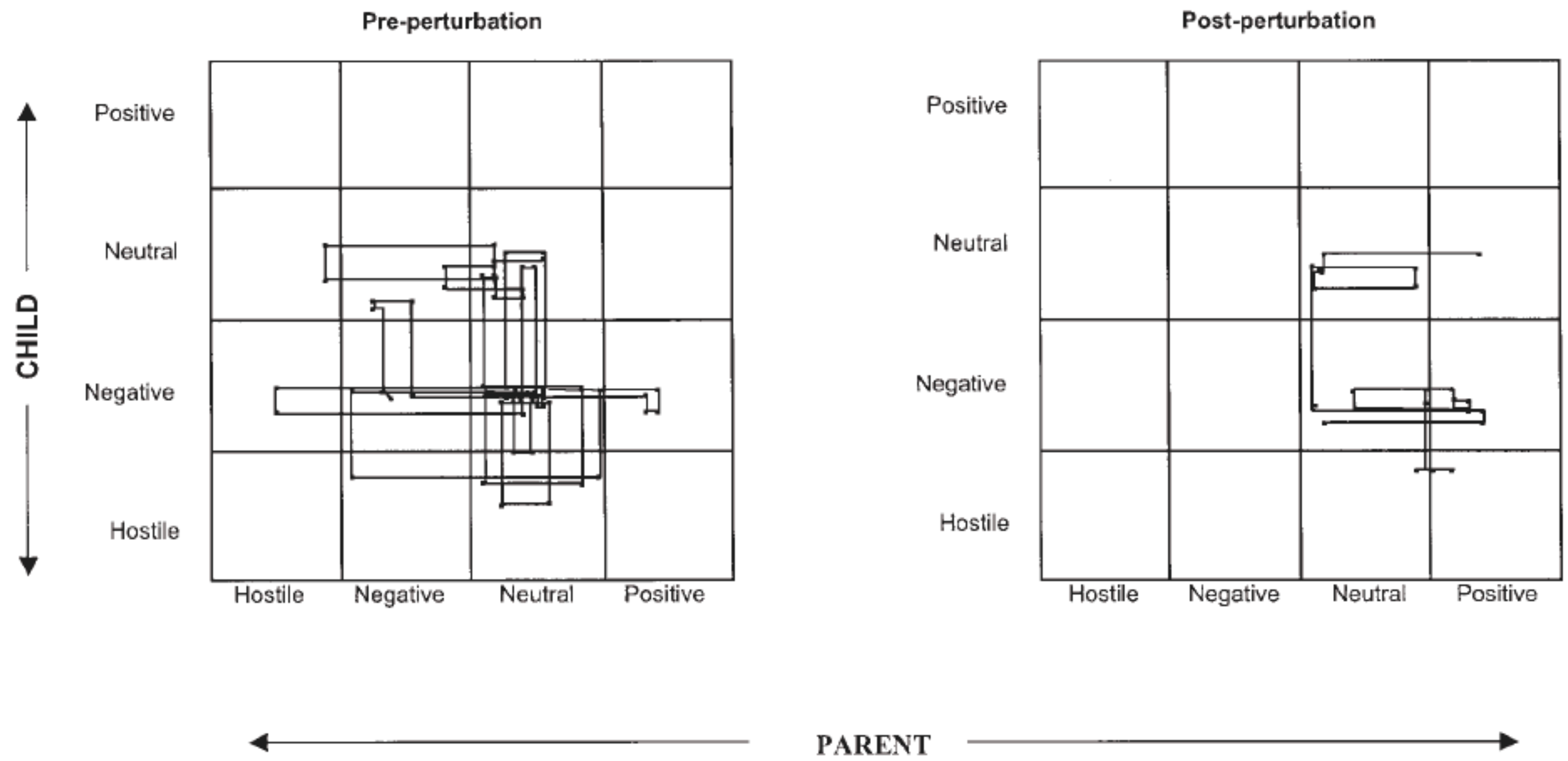
Complex behaviour from (physical) principles & laws (top-down)



- increase in recovery and switching time after perturbation
- increase in variance, autocorrelation, long-range dependence
- increase in occurrence and diversity of unstable states
- **increase in the entropy** of the distribution of state occurrences

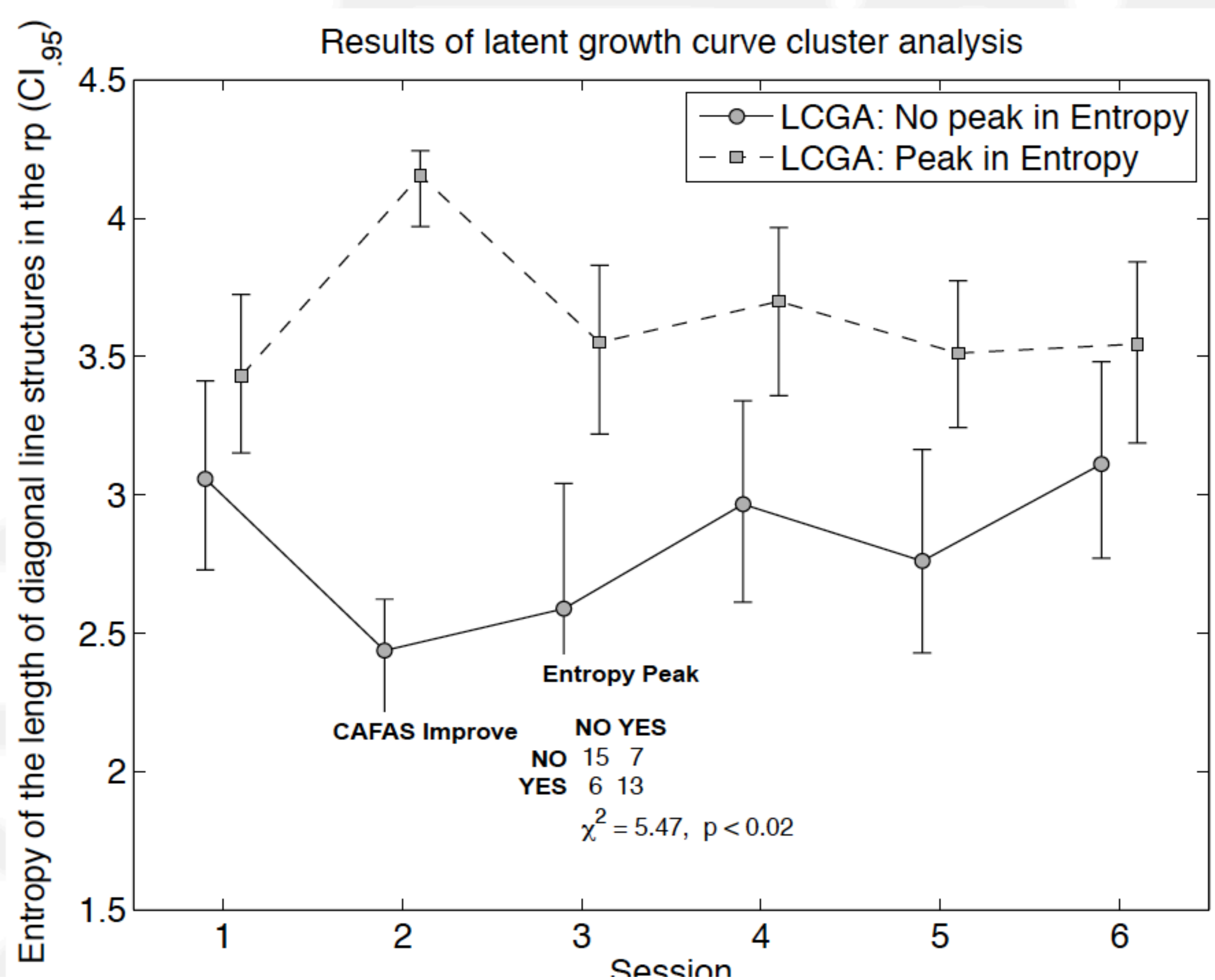
[1]Scholz JP, Kelso JAS, Schöner G. (1987). Nonequilibrium phase transitions in coordinated biological motion: critical slowing down and switching time. *Physics Letters A 123,* 390–394.
[2]Scheffer M, Bascompte J, Brock W A, Brovkin V, Carpenter SR, Dakos V, Held H, van Nes EH, Rietkerk M, Sugihara G. (2009). Early-warning signals for critical transitions. *Nature 461,* 53–9.
[3]Stephen DG, Dixon JA, Isenhower RW. (2009). Dynamics of representational change: Entropy, Action and Cognition. *JEP: Human Perception and Performance 35,* 1811–1832.
[4]Schiepek G, Strunk G. (2010). The identification of critical fluctuations and phase transitions in short term and coarse-grained time series … *Biological cybernetics 102,*197–207.

Complex behaviour from (physical) principles & laws (top-down)



Lichtwarck-Aschoff A, Hasselman F, Cox R, Pepler D, Granic I. (2012). A characteristic destabilization profile in parent-child interactions associated with treatment efficacy for aggressive children. *Nonlinear Dynamics-Psychology and Life Sciences 16*, 353.

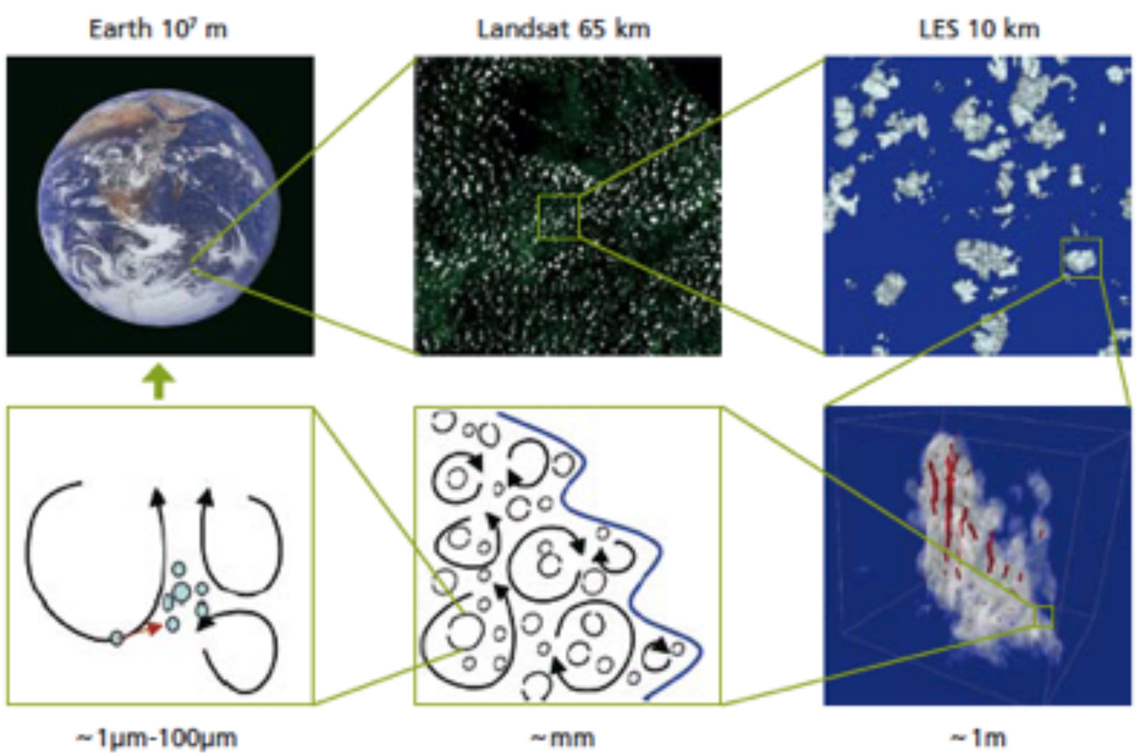
Complex behaviour from (physical) principles & laws (top-down)



Lichtwarck-Aschoff A, Hasselman F, Cox R, Pepler D, Granic I. (2012). A characteristic destabilization profile in parent-child interactions associated with treatment efficacy for aggressive children. *Nonlinear Dynamics-Psychology and Life Sciences 16*, 353.

Complexity Science

- Time! (Dynamics)
- Self-Organization
- **Micro-Macro levels (Emergence)**
- Scale invariance



https://pure.uva.nl/ws/files/4259324/63508_302758.pdf

MICRO-MACRO levels
Emergent patterns... swarms, schools

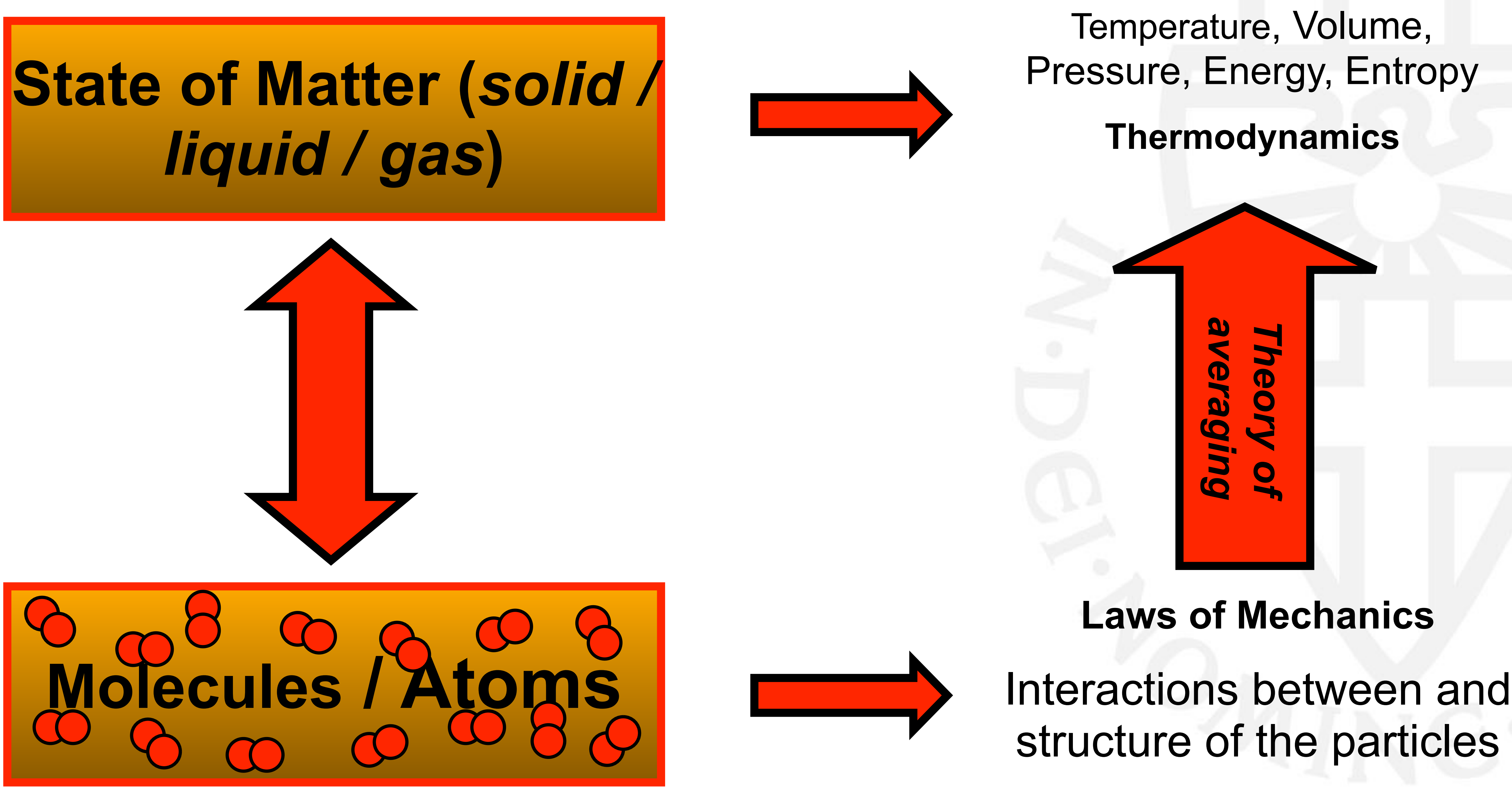
Conway's Game of Life:
Glider gun creating "Gliders"

http://en.wikipedia.org/wiki/Gun_(cellular_automaton)

Levels of Analysis: Micro - Macro

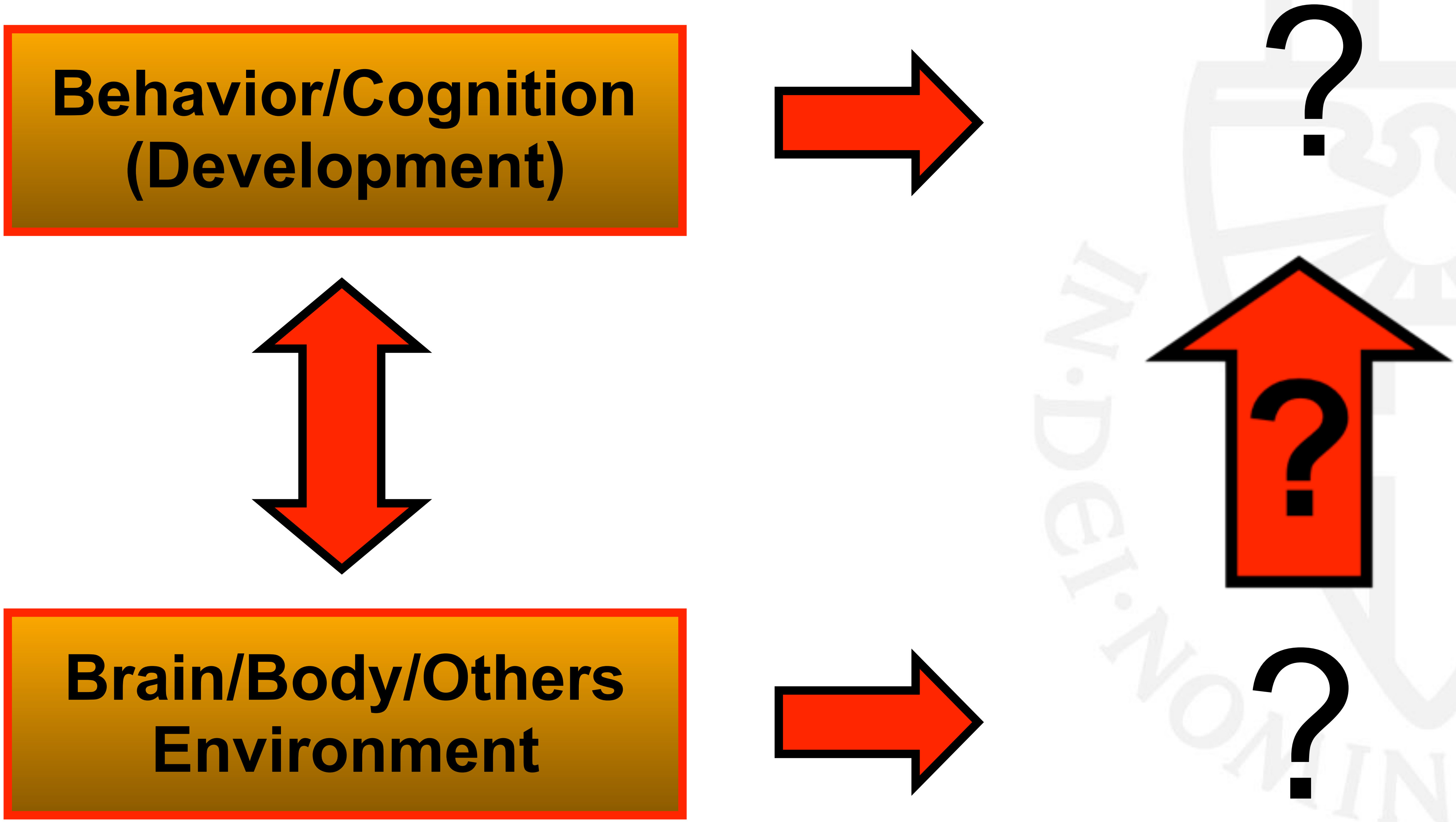
Forms and properties
are emergent,
not expected from
components:
1 watermolecule
does not possess the
property “wet”

Levels of Analysis: Micro - Macro

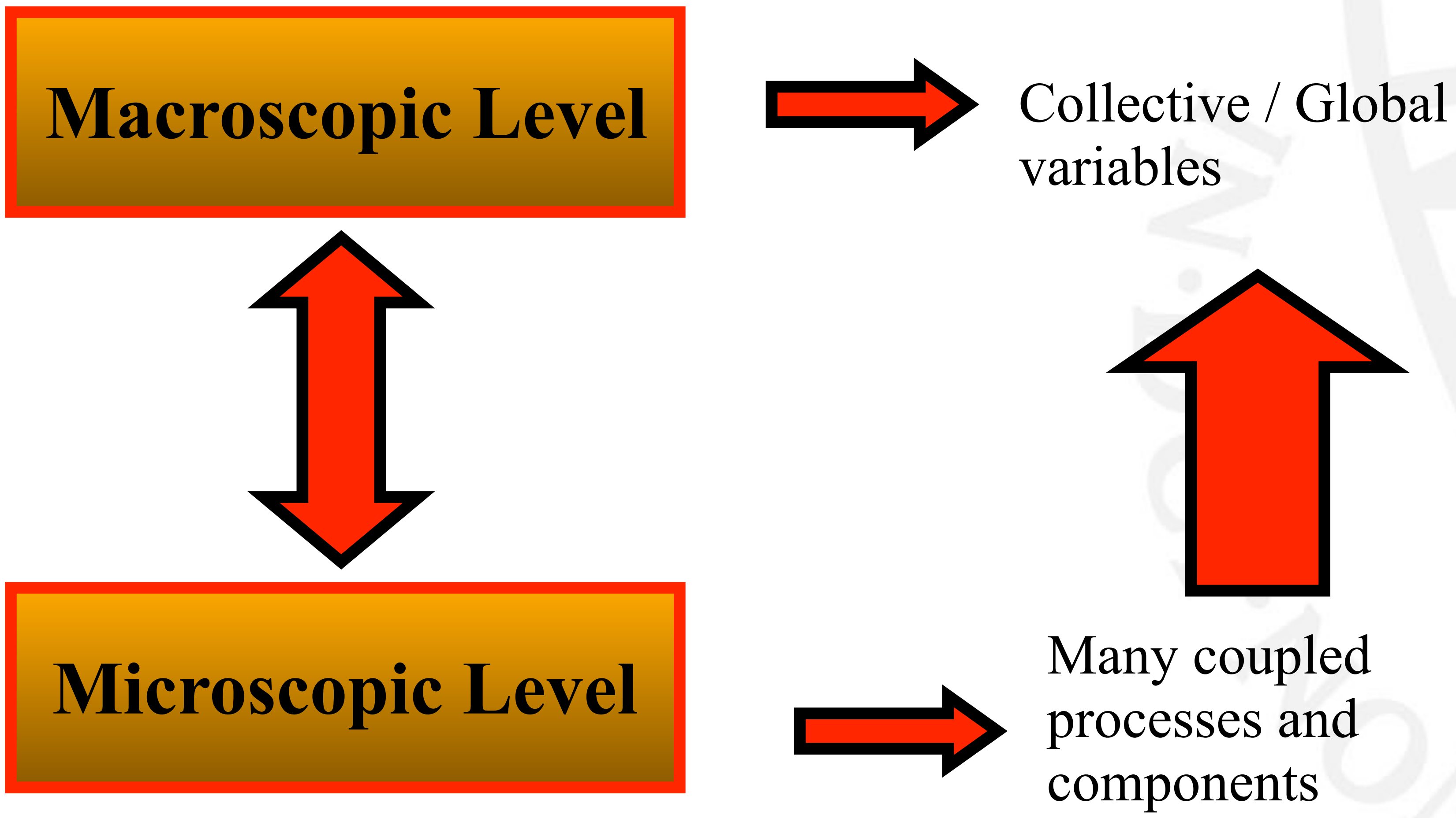


Levels of Analysis: Micro - Macro

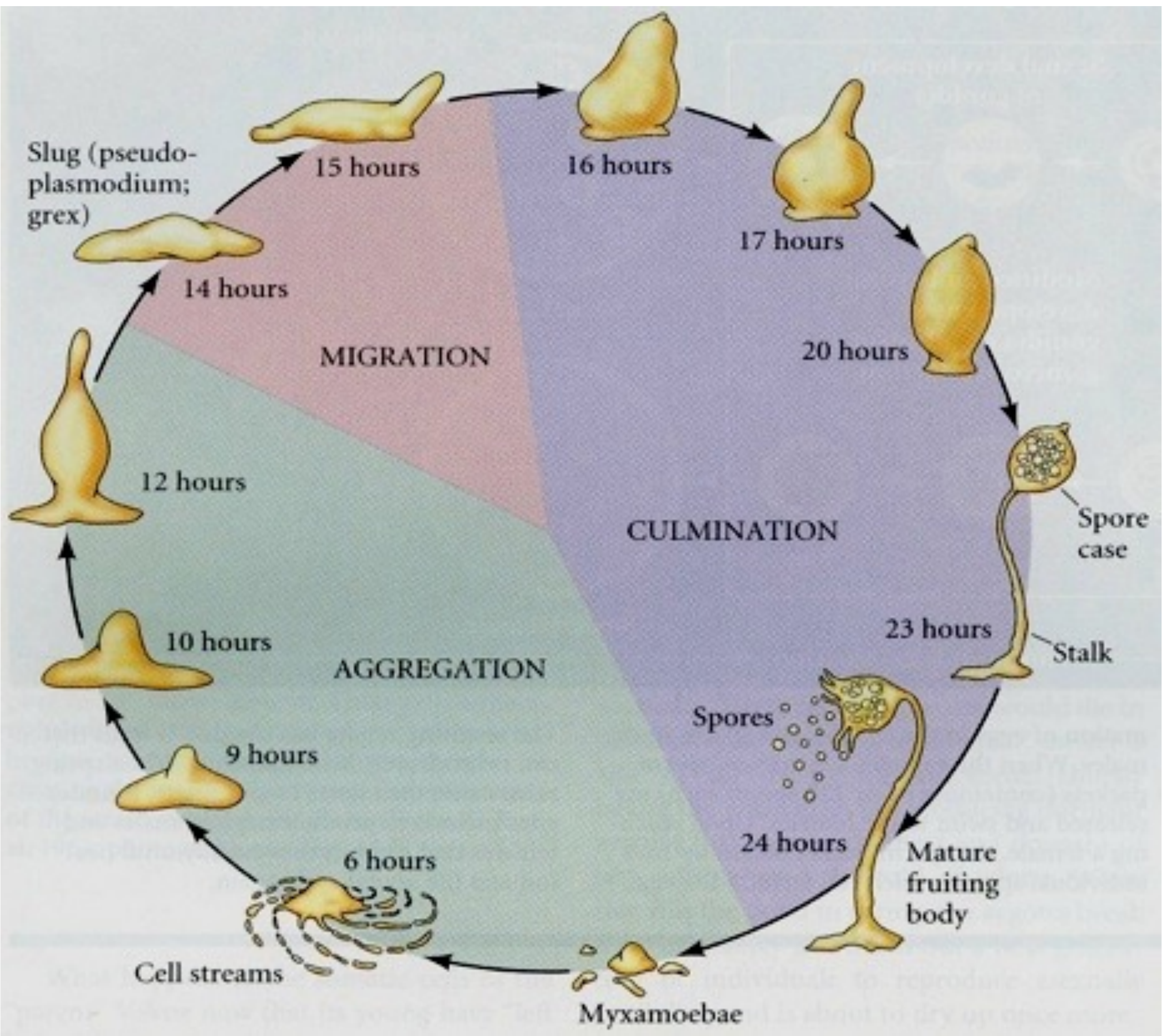
Much to be filled in!



Levels of Analysis: Micro - Macro

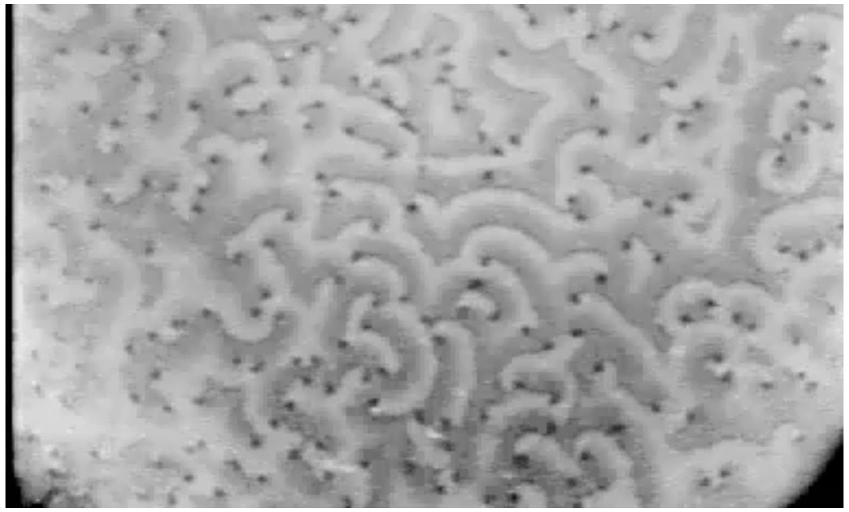


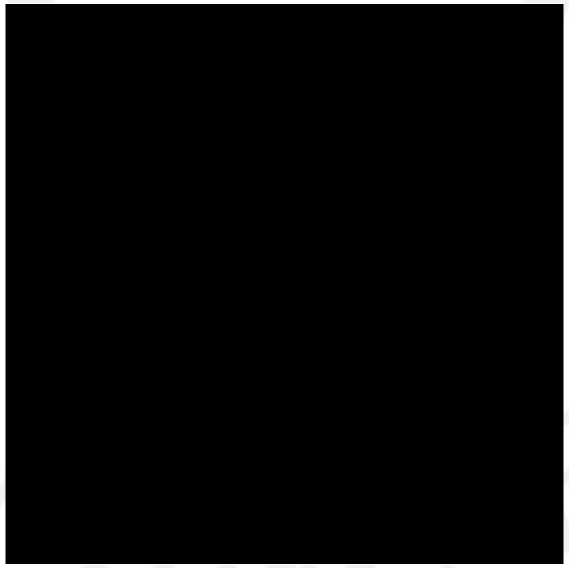
Emergence and Self-Organization: The life-cycle of *Dictyostelium*



1. Free living myxamoebae feed on bacteria and divide by fission.
2. When food is exhausted they aggregate to form a mound, then a multicellular slug.
3. Slug migrates towards heat and light.
4. Differentiation then ensues forming a fruiting body, containing spores.
5. It all takes just 24 hrs.
6. Released spores form new amoebae.

Order parameter: Labelling states of a complex system

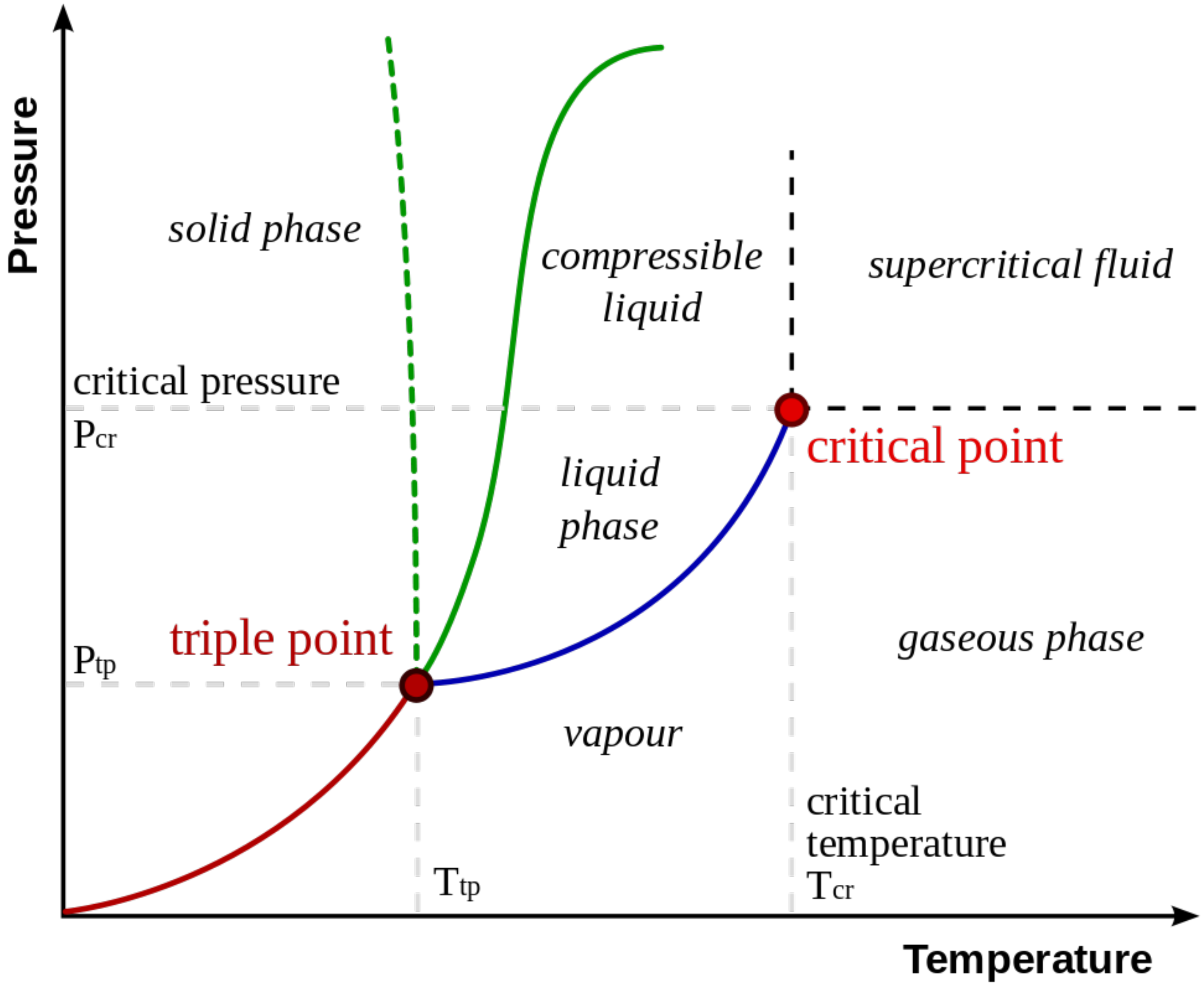


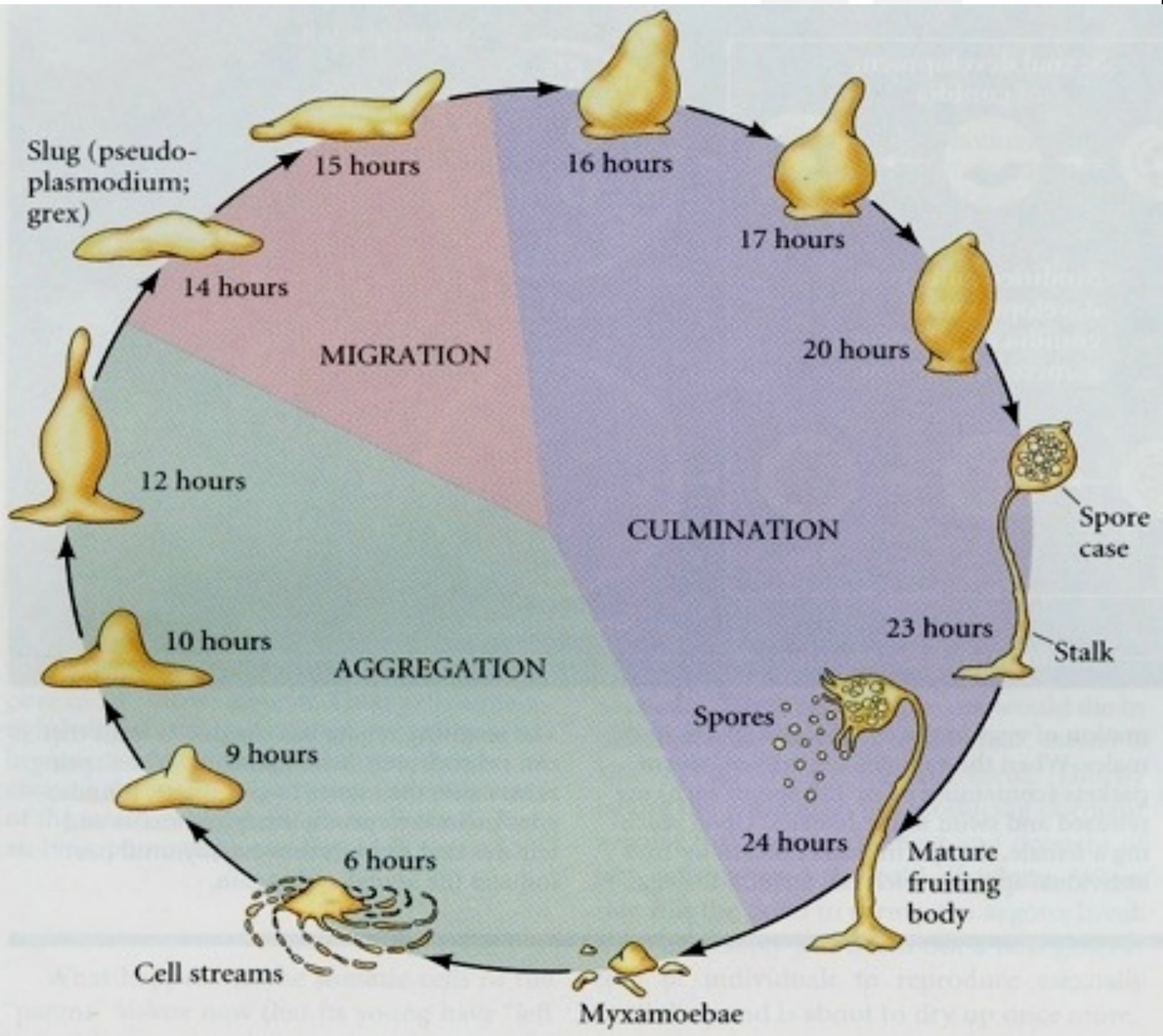


Forms are emergent, self-organised:

Arise from interactions between components → reduction of degrees of freedom

Phase Diagram & Order parameter





The order parameter is often a qualitative description of a macro state / global organisation of the system, conditional on the control parameters:

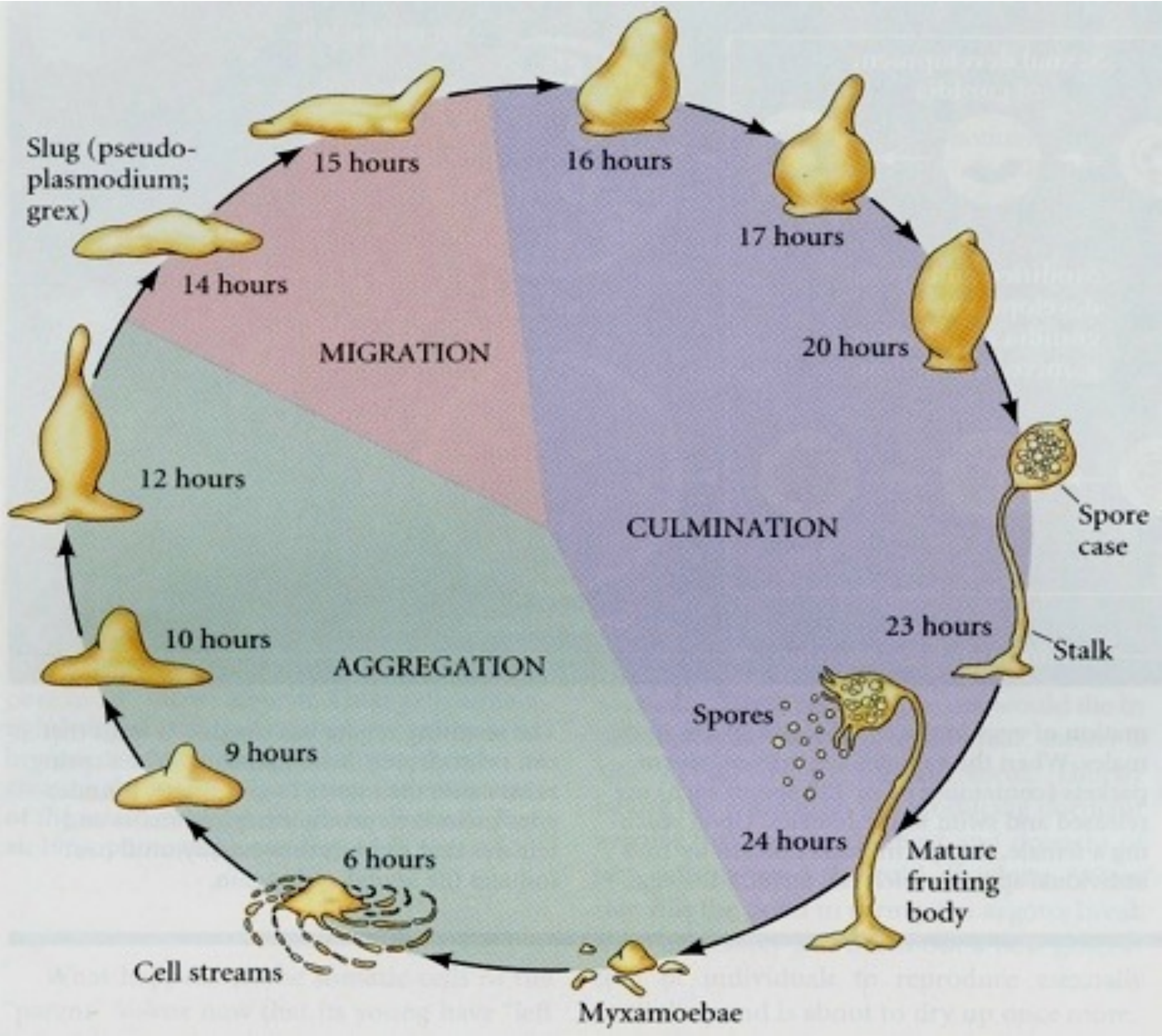
H_2O: Ice (Solid), Water (Liquid), Steam (Vapour)

Disctyostelium: Aggregation (Mound), Migration (Slug), Culmination (Fruiting Body)

https://youtu.be/Juz9pVVsmQQ

Dynamic Metaphor vs. Dynamic Measure

Metaphor: Sate Space / Order Parameter
Measures: Attractor strength / Stability



Order parameter: the qualitatively different states

Control parameter: available food (actually concentration of a chemical that is released if they are starving)

Experiments:
Find out if the process is reversible... add food

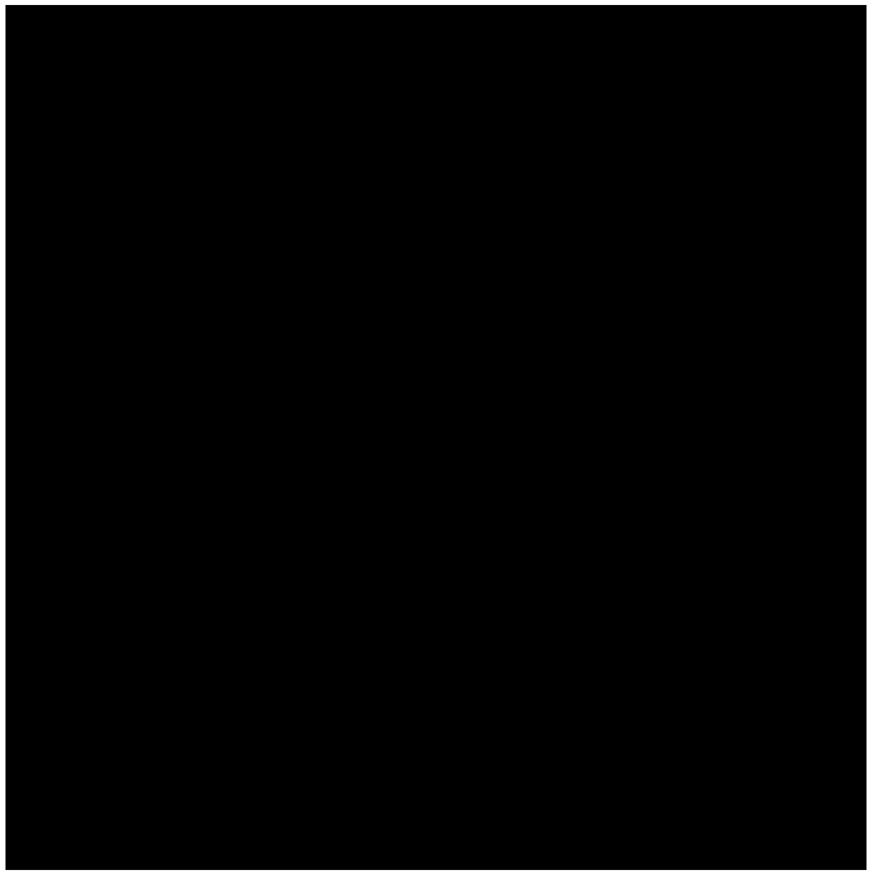
perturb the system during the various phases...

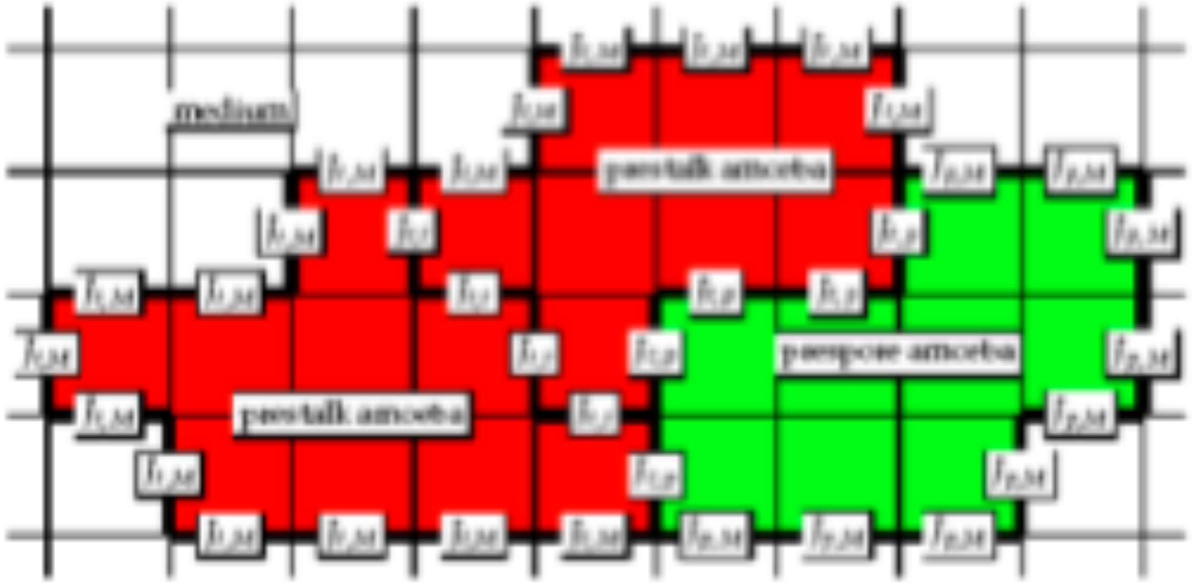
the degrees of freedom of the individual components are increasingly constrained by the interaction:

free living amoebae... slug... immovable sporing pod

nb State space and Phase Space (or: Diagram) are different concepts, but often used interchangeably to describe a State Space… see slide 18

From Pattern Formation to Morphogenesis
Multicellular Coordination in *Dictyostelium Discoideum*
A.F.M. Marée (2000). PhD Thesis, UU.





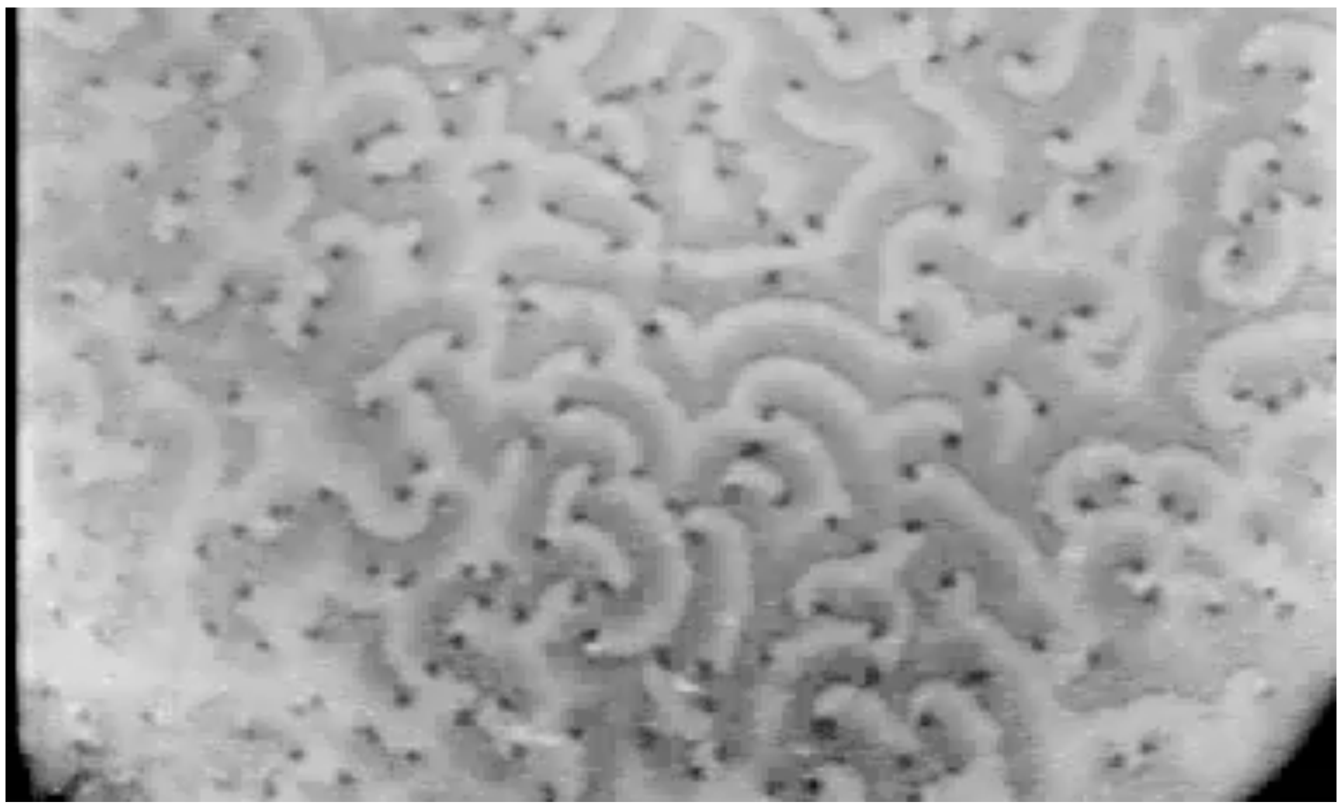
Two-Scale Cellular Automata with Differential Adhesion

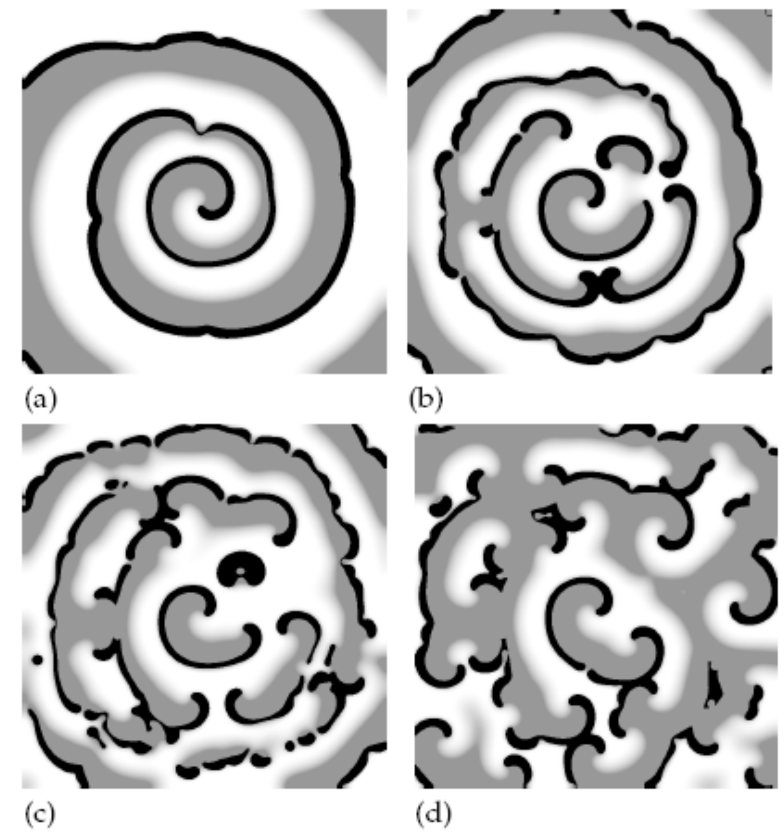
$$H_\sigma = \sum_{\text{all } \sigma, \sigma' \text{ neighbours}} \frac{J_{\tau_\sigma, \tau_{\sigma'}}}{2} + \sum_{\text{all } \sigma, \text{medium neighbours}} J_{\tau_\sigma, \tau_{\text{medium}}} + \lambda (v_\sigma - V)^2, \quad (1.1)$$

Mathematical model of Dictyostelium

Spiral Breakup in Excitable Tissue due to Lateral Instability

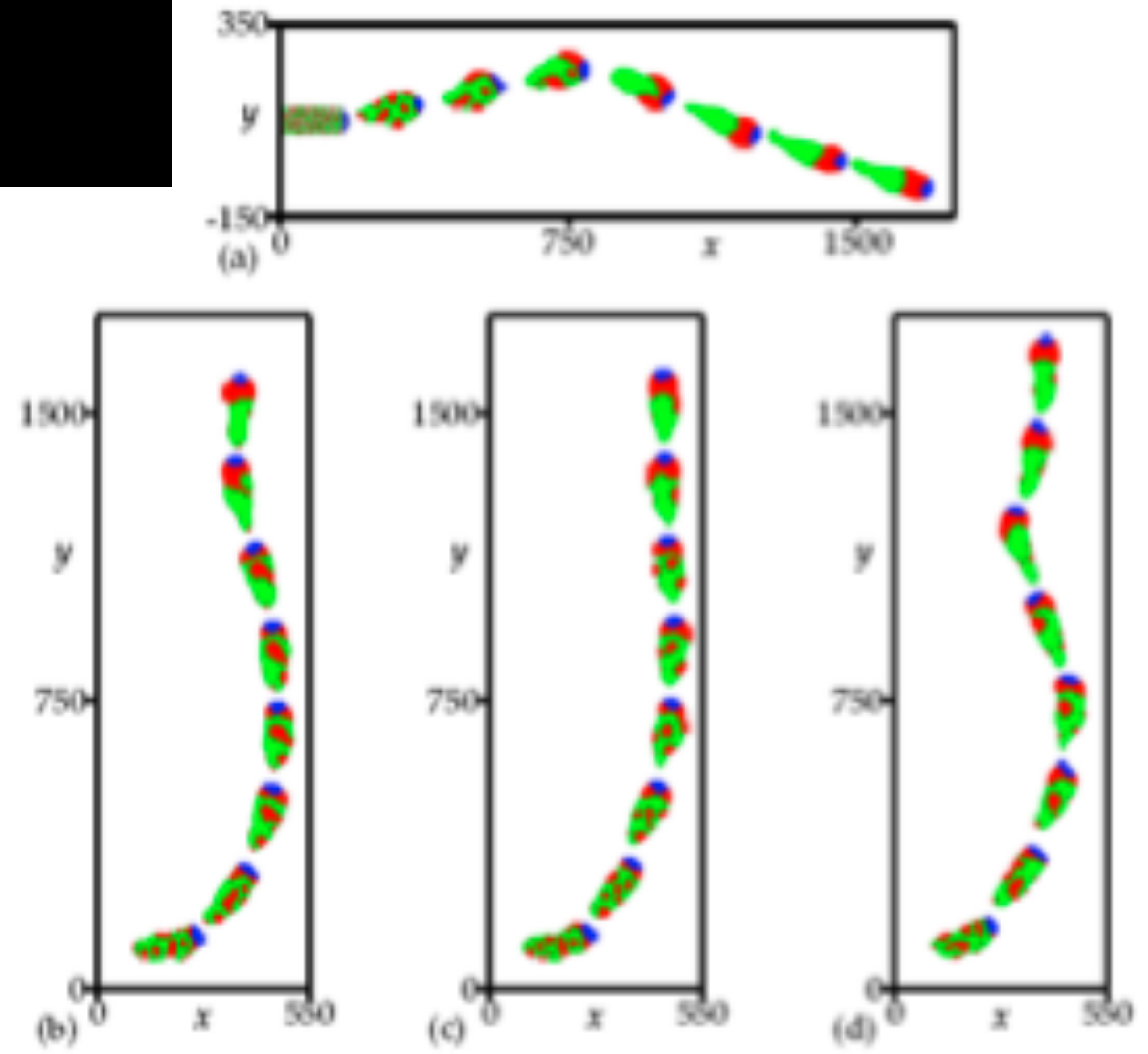
Marée, A. F. M., & Panlov, A.V. (1997). *Physical Review Letters, 78*, 1819-1822.





$$\frac{\partial e}{\partial t} = \Delta e - f(e) - g,$$
$$\frac{\partial g}{\partial t} = D_g \Delta g + \varepsilon(e, g)(ke - g),$$

Mathematical model of Dictyostelium



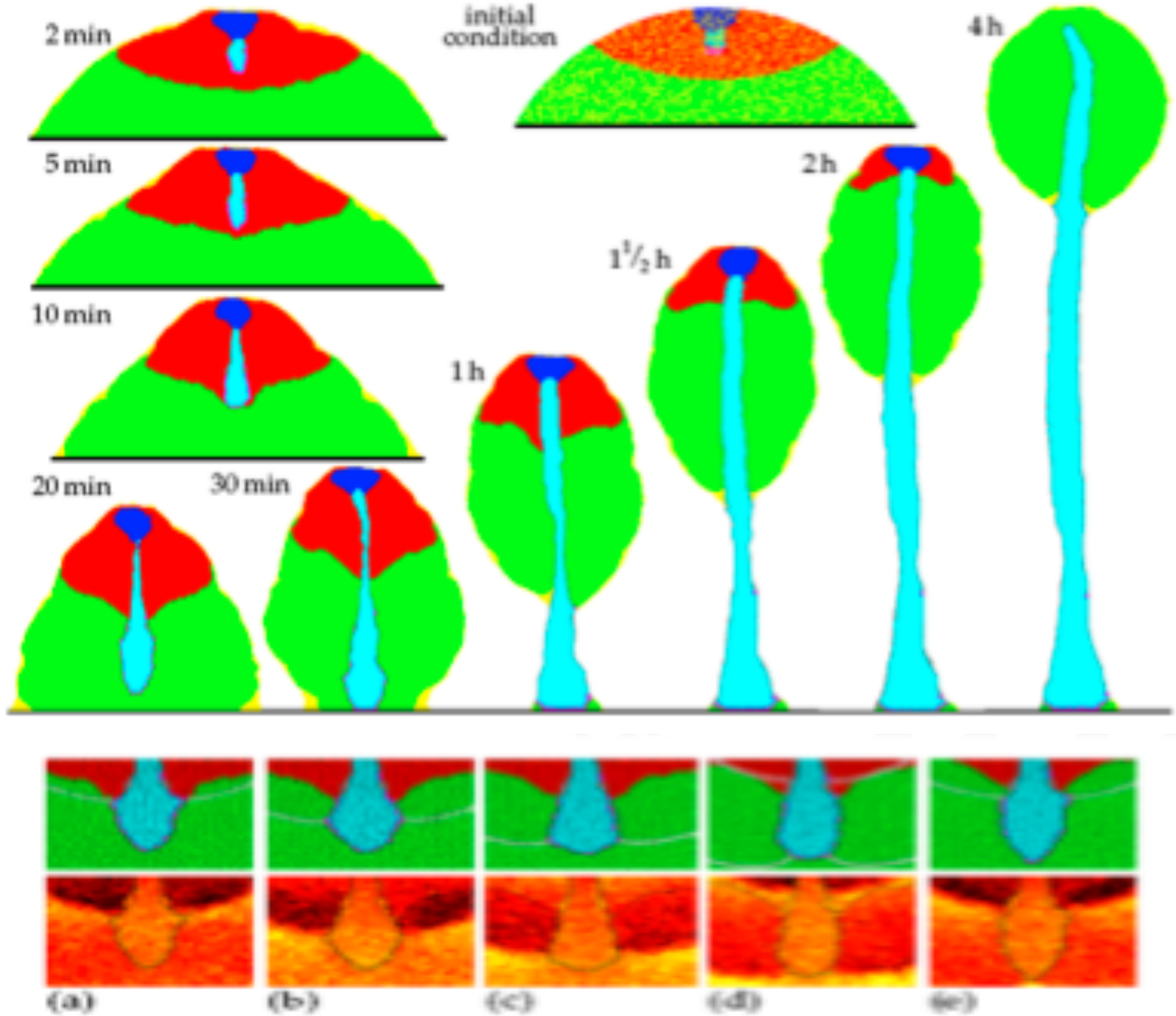
$$H_\sigma = \sum \frac{J_{\text{cell,cell}}}{2} + \sum J_{\text{cell,medium}} + \lambda(v - V)^2,$$

$$\left.\begin{aligned} \frac{\partial c}{\partial t} &= D_c \Delta c - f(c) - r, \\ \frac{\partial r}{\partial t} &= \varepsilon(c)(kc - r), \end{aligned}\right\} \quad \text{inside the amoebae}$$

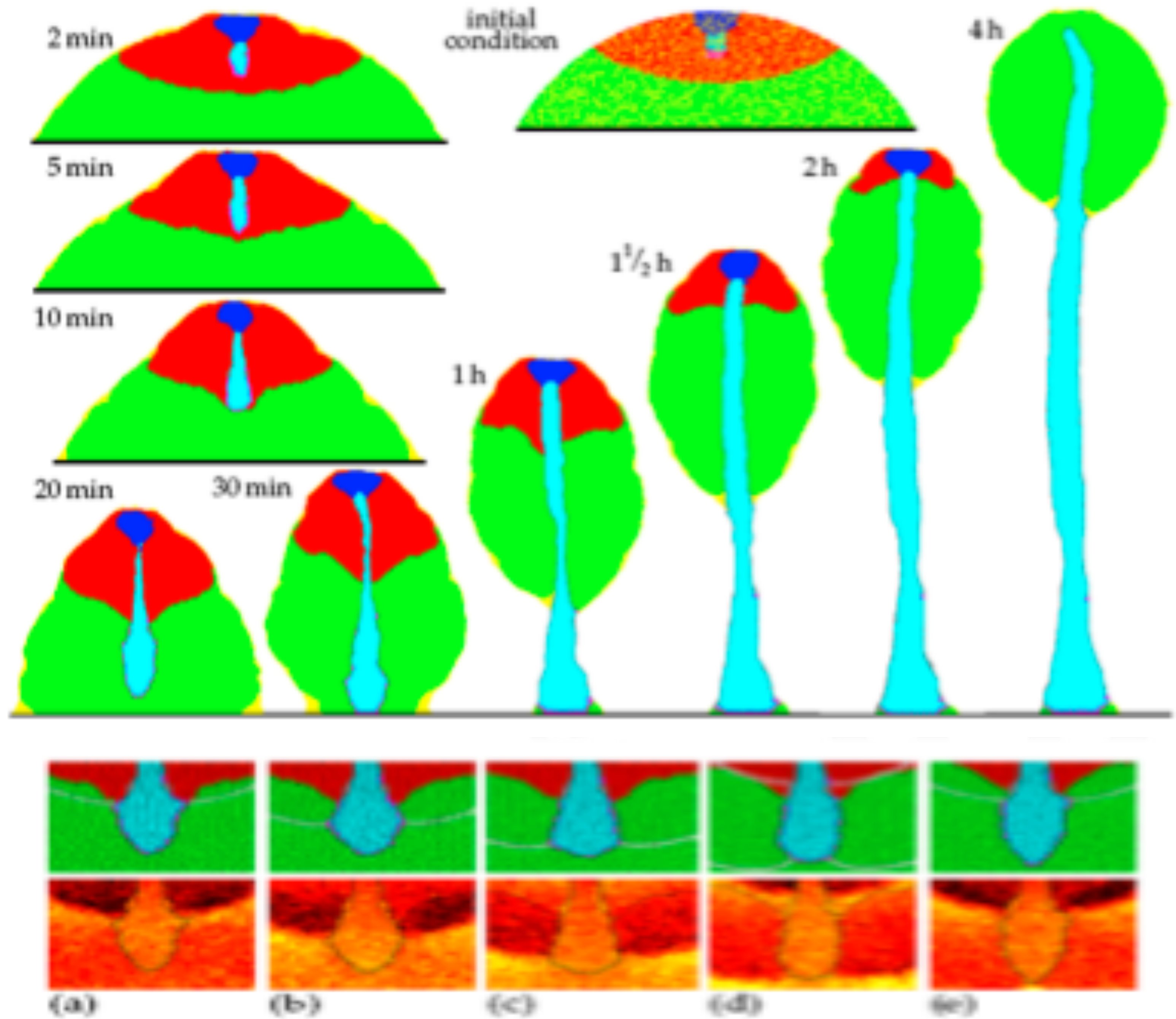
$$\left.\frac{\partial c}{\partial t} = D_c \Delta c - d_c(c - c_0),\right\} \quad \text{outside the amoebae}$$

$$\Delta H' = \Delta H - \mu(c_{\text{automaton}} - c_{\text{neighbour}}),$$

Mathematical model of Dictyostelium



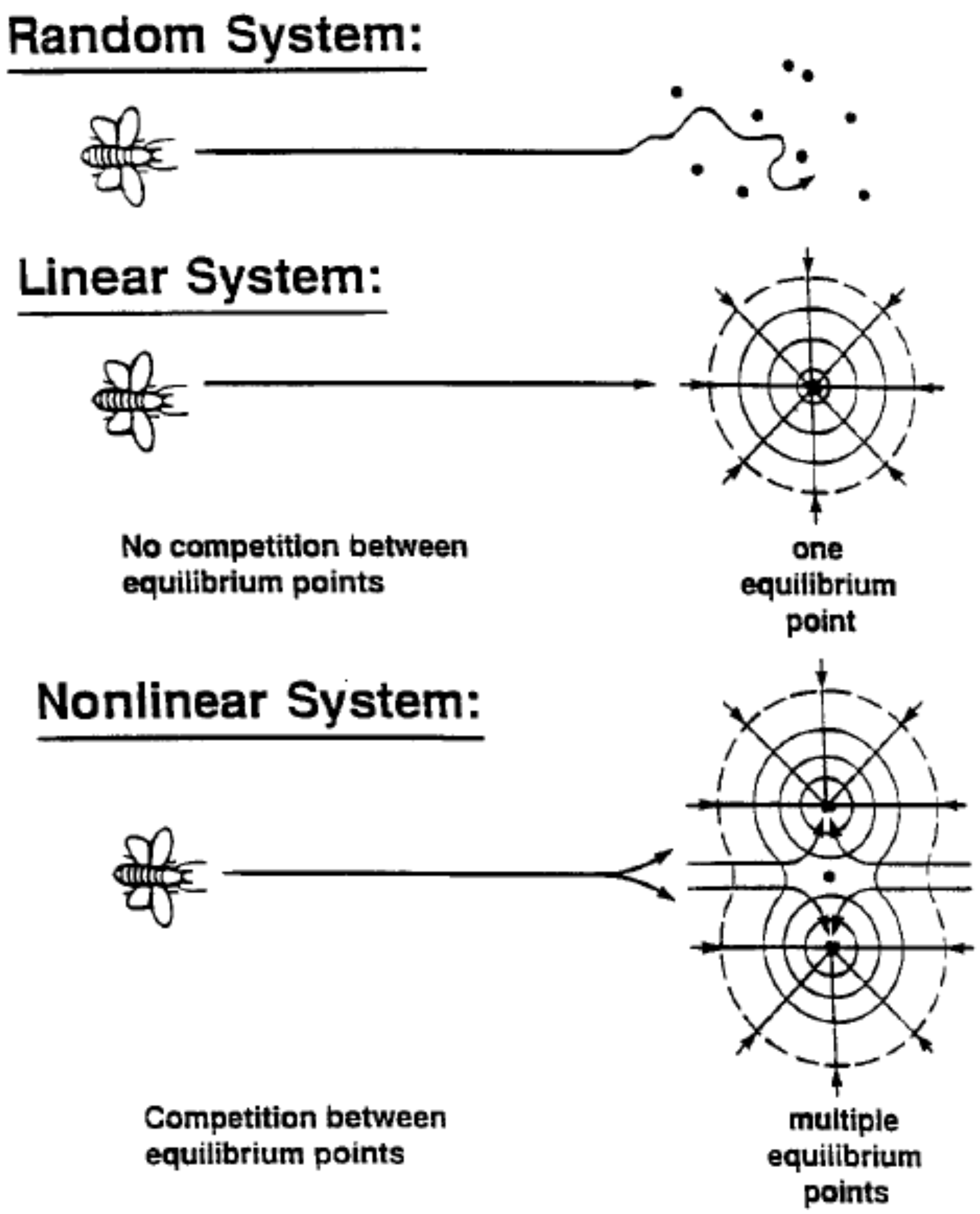
Mathematical model of Dictyostelium



Mathematical model of Dictyostelium

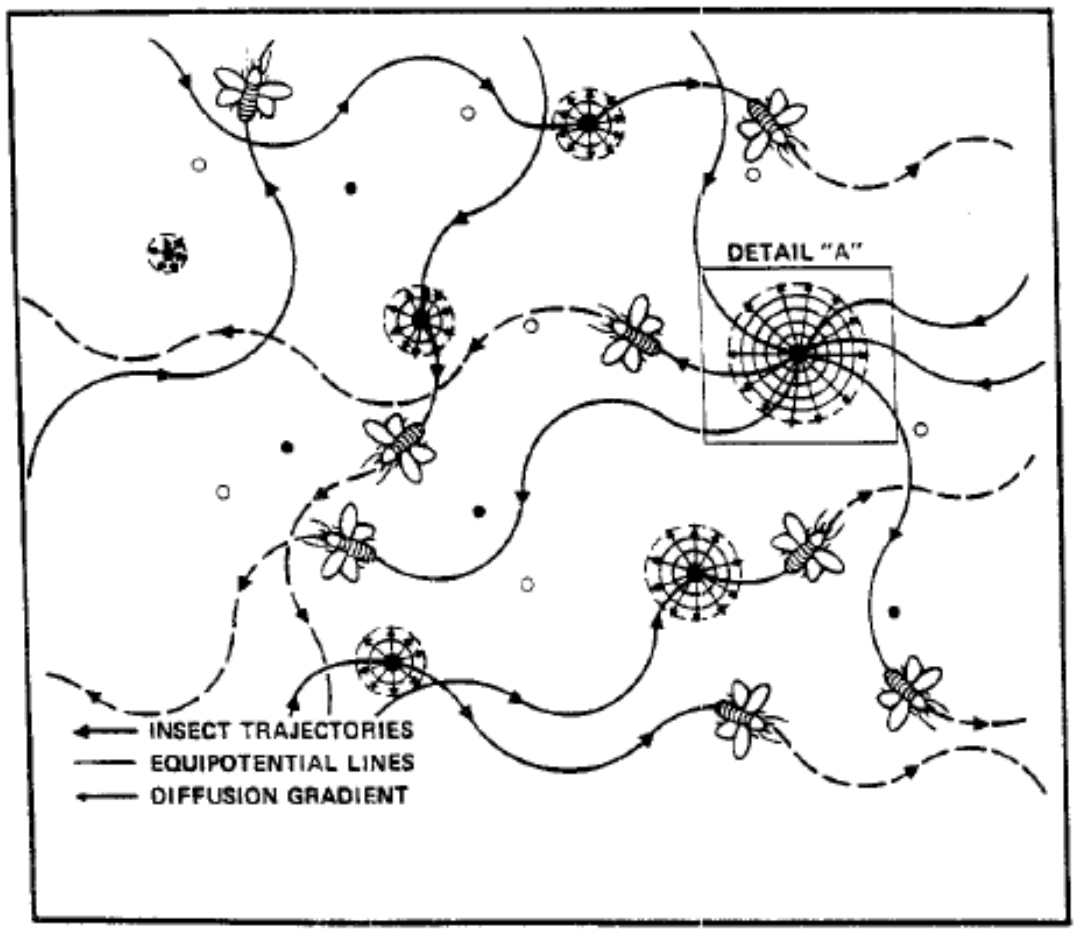
MICRO-MACRO levels

Termite cathedrals: Complex structures from simple rules



MICRO-MACRO levels

Termite cathedrals: Complex structures from simple rules



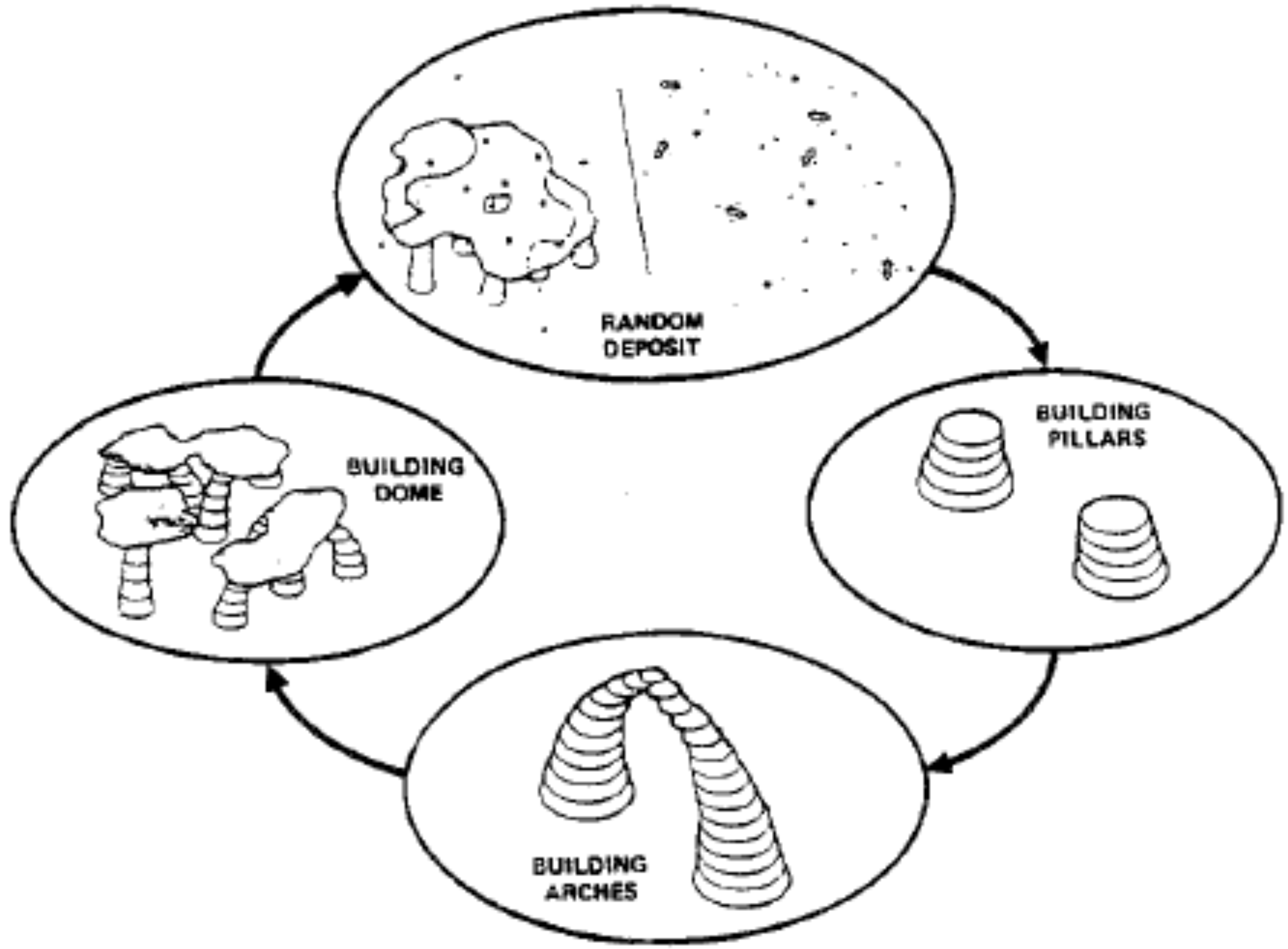


Fig. 14. Circular ring of building phases: each phase is dominated by a

MICRO-MACRO levels

Termite cathedrals: Complex structures from simple rules

Can be “explained” by (local) laws of thermodynamics... termite is a particle in a gradient field...

Dissipative systems: Systems that extract energy from the environment to maintain their internal structure, their internal complexity

Usually: many simple units interact in simple ways to create complex patterns at the global, macro level...

But termites are more complex than classical particles!

Two Metaphors to explain Human Behaviour

Machine Metaphor

- Parts exist for each other, but not by means of each other
- Parts act together to meet the things purpose, but their actions have nothing to do with the thing's construction
- **Open to efficient cause (predicative logic)**
- Human behaviour: **Computation; Information processing**

Organism Metaphor

- Parts are both causes and effects of the thing, both means and end
- Parts act together but also construct and maintain themselves as a whole
- **Closed to efficient cause (impredicative logic)**
- Human Behaviour: **Concinnity; Embodied and Embedded**

Concinnity: Harmony in the arrangement or interarrangement of parts with respect to a whole.

Two types of mathematical formalism for two types of systems

component dominant dynamics

Jakob Bernouiiili (1654-1704): [The application of the Law of large numbers in chance theory] *to predict the weather next month or year, predicting the winner of a game which depends partly on psychological and or physical factors or to the investigation of matters which depend on hidden causes, which can interact in a multitude of ways is* ***completely futile!****"* Vervaet (2004)

A system is ***ergodic*** *iff:*

The averaged behaviour of an observed variable in a substantial ensemble of individuals (space-average) is expected to be equivalent to the average behaviour of an individual observed over a substantial amount of time (time average)

f.i. Throw 100 dice at once, and then throw 1 die 100 times in a row… The expected value will be similar for both measurements

interaction dominant dynamics

Deterministic chaos (Lorenz, 1972)
(complexity, nonlinear dynamics, predictability)

Takens' Theorem (1981)
(phase space reconstruction)

Systems far from thermodynamic equilibrium
(Prigogine, & Stengers, 1984)

SOC / $\frac{1}{f^{\alpha}}$ **noise** (Bak, 1987)

(self-organized criticality, interdependent measurements)

Fractal geometry (Mandelbrot, 1988)
(self-similarity, scale free behaviour, infinite variance)

Aczel's Anti-Foundation Axiom (1988)
(hyperset theory, circular causality, complexity analysis)

Two types of mathematical formalism for two types of systems

component dominant dynamics

The Law of Large Numbers (Bernouiili, 1713) +
The Central Limit Theorem (de Moivre, 1733) +
The Gauss-Markov Theorem (Gauss, 1809) +
Statistics by Intercomparison *(Galton, 1875)* =
Social Physics *(Quetelet, 1840)*

Collectively known as:
The Classical Ergodic Theorems

Molenaar, P.C.M. (2008). On the implications of the classical ergodic theorems: Analysis of developmental processes has to focus on intra individual variation. *Developmental Psychobiology, 50*, 60-69

interaction dominant dynamics

Deterministic chaos (Lorenz, 1972)
(complexity, nonlinear dynamics, predictability)

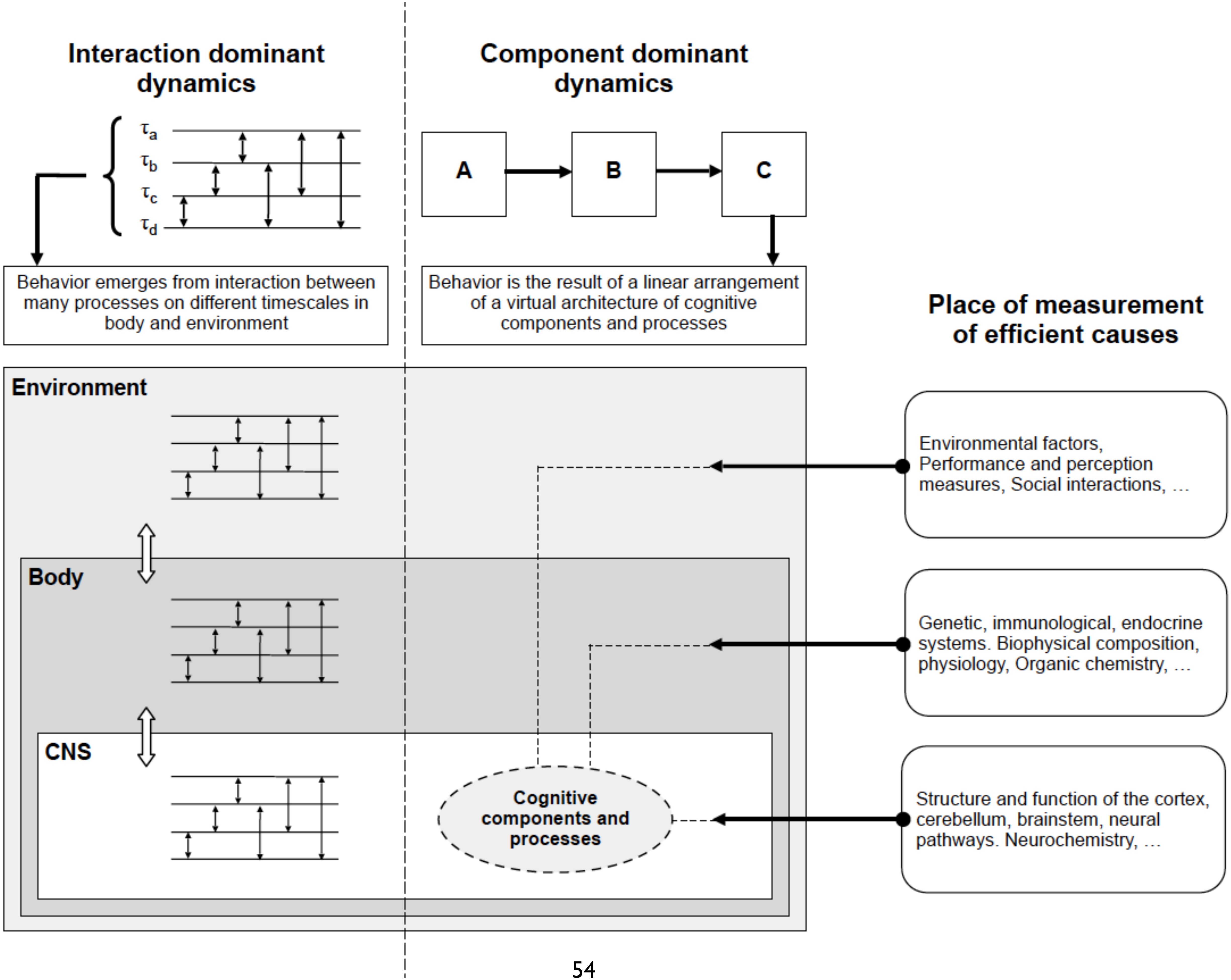
Takens' Theorem (1981)
(phase space reconstruction)

Systems far from thermodynamic equilibrium
(Prigogine, & Stengers, 1984)

SOC / $\frac{1}{f^{\alpha}}$ **noise** (Bak, 1987)
(self-organized criticality, interdependent measurements)

Fractal geometry (Mandelbrot, 1988)
(self-similarity, scale free behaviour, infinite variance)

Aczel's Anti-Foundation Axiom (1988)
(hyperset theory, circular causality, complexity analysis)



Interaction dominant dynamics
τ_a
τ_b
τ_c
τ_d
Behavior emerges from interaction between many processes on different timescales in body and environment
Component dominant dynamics
A
B
C
Behavior is the result of a linear arrangement of a virtual architecture of cognitive components and processes
Place of measurement of efficient causes
Environment
Body
CNS
Cognitive components and processes
Environmental factors, Performance and perception measures, Social interactions, …
Genetic, immunological, endocrine systems. Biophysical composition, physiology, Organic chemistry, …
Structure and function of the cortex, cerebellum, brainstem, neural pathways. Neurochemistry, …

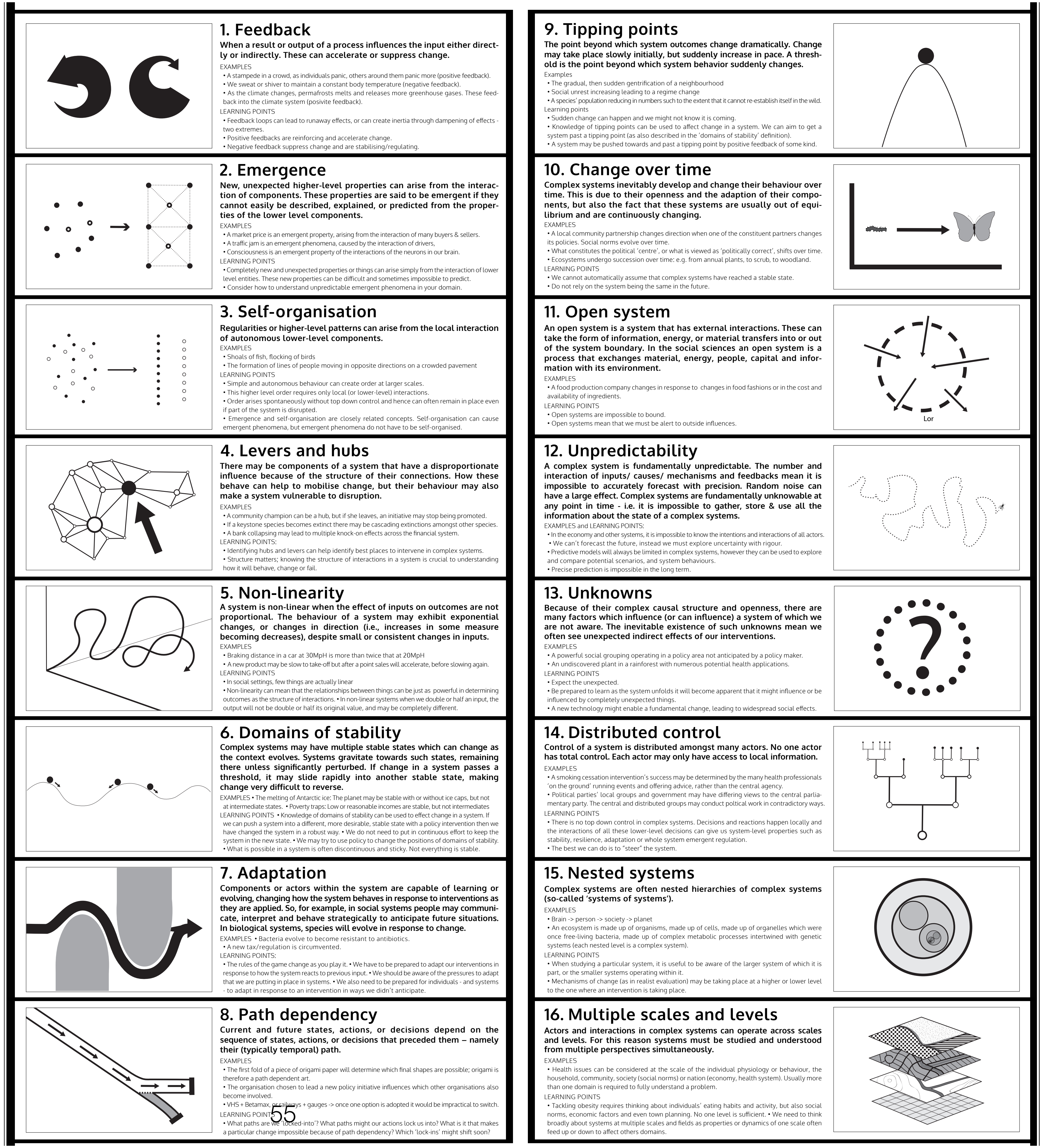
What kind of system?

I call the methods we use: "mostly model-free" "descriptive techniques"

detect / quantify
many characteristic
phenomena observed
in
complex adaptive systems

- Multi-scale fluctuations
- Non-linear dynamics
- Prediction horizons
- Regime changes
- Divergence

There is always a model of course!



1. Feedback

When a result or output of a process influences the input either directly or indirectly. These can accelerate or suppress change.

EXAMPLES
- A stampede in a crowd, as individuals panic, others around them panic more (positive feedback).
- We sweat or shiver to maintain a constant body temperature (negative feedback).
- As the climate changes, permafrosts melts and releases more greenhouse gases. These feedback into the climate system (posivite feedback).

LEARNING POINTS
- Feedback loops can lead to runaway effects, or can create inertia through dampening of effects - two extremes.
- Positive feedbacks are reinforcing and accelerate change.
- Negative feedback suppress change and are stabilising/regulating.

2. Emergence

New, unexpected higher-level properties can arise from the interaction of components. These properties are said to be emergent if they cannot easily be described, explained, or predicted from the properties of the lower level components.

EXAMPLES
- A market price is an emergent property, arising from the interaction of many buyers & sellers.
- A traffic jam is an emergent phenomena, caused by the interaction of drivers,
- Consciousness is an emergent property of the interactions of the neurons in our brain.

LEARNING POINTS
- Completely new and unexpected properties or things can arise simply from the interaction of lower level entities. These new properties can be difficult and sometimes impossible to predict.
- Consider how to understand unpredictable emergent phenomena in your domain.

3. Self-organisation

Regularities or higher-level patterns can arise from the local interaction of autonomous lower-level components.

EXAMPLES
- Shoals of fish, flocking of birds
- The formation of lines of people moving in opposite directions on a crowded pavement

LEARNING POINTS
- Simple and autonomous behaviour can create order at larger scales.
- This higher level order requires only local (or lower-level) interactions.
- Order arises spontaneously without top down control and hence can often remain in place even if part of the system is disrupted.
- Emergence and self-organisation are closely related concepts. Self-organisation can cause emergent phenomena, but emergent phenomena do not have to be self-organised.

4. Levers and hubs

There may be components of a system that have a disproportionate influence because of the structure of their connections. How these behave can help to mobilise change, but their behaviour may also make a system vulnerable to disruption.

EXAMPLES
- A community champion can be a hub, but if she leaves, an initiative may stop being promoted.
- If a keystone species becomes extinct there may be cascading extinctions amongst other species.
- A bank collapsing may lead to multiple knock-on effects across the financial system.

LEARNING POINTS:
- Identifying hubs and levers can help identify best places to intervene in complex systems.
- Structure matters; knowing the structure of interactions in a system is crucial to understanding how it will behave, change or fail.

5. Non-linearity

A system is non-linear when the effect of inputs on outcomes are not proportional. The behaviour of a system may exhibit exponential changes, or changes in direction (i.e., increases in some measure becoming decreases), despite small or consistent changes in inputs.

EXAMPLES
- Braking distance in a car at 30MpH is more than twice that at 20MpH
- A new product may be slow to take-off but after a point sales will accelerate, before slowing again.

LEARNING POINTS
- In social settings, few things are actually linear
- Non-linearity can mean that the relationships between things can be just as powerful in determining outcomes as the structure of interactions. • In non-linear systems when we double or half an input, the output will not be double or half its original value, and may be completely different.

6. Domains of stability

Complex systems may have multiple stable states which can change as the context evolves. Systems gravitate towards such states, remaining there unless significantly perturbed. If change in a system passes a threshold, it may slide rapidly into another stable state, making change very difficult to reverse.

EXAMPLES • The melting of Antarctic ice: The planet may be stable with or without ice caps, but not at intermediate states. • Poverty traps: Low or reasonable incomes are stable, but not intermediates

LEARNING POINTS • Knowledge of domains of stability can be used to effect change in a system. If we can push a system into a different, more desirable, stable state with a policy intervention then we have changed the system in a robust way. • We do not need to put in continuous effort to keep the system in the new state. • We may try to use policy to change the positions of domains of stability. • What is possible in a system is often discontinuous and sticky. Not everything is stable.

7. Adaptation

Components or actors within the system are capable of learning or evolving, changing how the system behaves in response to interventions as they are applied. So, for example, in social systems people may communicate, interpret and behave strategically to anticipate future situations. In biological systems, species will evolve in response to change.

EXAMPLES • Bacteria evolve to become resistant to antibiotics.
- A new tax/regulation is circumvented.

LEARNING POINTS:
- The rules of the game change as you play it. • We have to be prepared to adapt our interventions in response to how the system reacts to previous input. • We should be aware of the pressures to adapt that we are putting in place in systems. • We also need to be prepared for individuals - and systems - to adapt in response to an intervention in ways we didn't anticipate.

8. Path dependency

Current and future states, actions, or decisions depend on the sequence of states, actions, or decisions that preceded them – namely their (typically temporal) path.

EXAMPLES
- The first fold of a piece of origami paper will determine which final shapes are possible; origami is therefore a path dependent art.
- The organisation chosen to lead a new policy initiative influences which other organisations also become involved.
- VHS + Betamax, or railways + gauges -> once one option is adopted it would be impractical to switch.

LEARNING POINTS
- What paths are we 'locked-into'? What paths might our actions lock us into? What is it that makes a particular change impossible because of path dependency? Which 'lock-ins' might shift soon?

9. Tipping points

The point beyond which system outcomes change dramatically. Change may take place slowly initially, but suddenly increase in pace. A threshold is the point beyond which system behavior suddenly changes.

Examples
- The gradual, then sudden gentrification of a neighbourhood
- Social unrest increasing leading to a regime change
- A species' population reducing in numbers such to the extent that it cannot re-establish itself in the wild.

Learning points
- Sudden change can happen and we might not know it is coming.
- Knowledge of tipping points can be used to affect change in a system. We can aim to get a system past a tipping point (as also described in the 'domains of stability' definition).
- A system may be pushed towards and past a tipping point by positive feedback of some kind.

10. Change over time

Complex systems inevitably develop and change their behaviour over time. This is due to their openness and the adaption of their components, but also the fact that these systems are usually out of equilibrium and are continuously changing.

EXAMPLES
- A local community partnership changes direction when one of the constituent partners changes its policies. Social norms evolve over time.
- What constitutes the political 'centre', or what is viewed as 'politically correct', shifts over time.
- Ecosystems undergo succession over time: e.g. from annual plants, to scrub, to woodland.

LEARNING POINTS
- We cannot automatically assume that complex systems have reached a stable state.
- Do not rely on the system being the same in the future.

11. Open system

An open system is a system that has external interactions. These can take the form of information, energy, or material transfers into or out of the system boundary. In the social sciences an open system is a process that exchanges material, energy, people, capital and information with its environment.

EXAMPLES
- A food production company changes in response to changes in food fashions or in the cost and availability of ingredients.

LEARNING POINTS
- Open systems are impossible to bound.
- Open systems mean that we must be alert to outside influences.

12. Unpredictability

A complex system is fundamentally unpredictable. The number and interaction of inputs/ causes/ mechanisms and feedbacks mean it is impossible to accurately forecast with precision. Random noise can have a large effect. Complex systems are fundamentally unknowable at any point in time - i.e. it is impossible to gather, store & use all the information about the state of a complex systems.

EXAMPLES and LEARNING POINTS:
- In the economy and other systems, it is impossible to know the intentions and interactions of all actors.
- We can't forecast the future, instead we must explore uncertainty with rigour.
- Predictive models will always be limited in complex systems, however they can be used to explore and compare potential scenarios, and system behaviours.
- Precise prediction is impossible in the long term.

13. Unknowns

Because of their complex causal structure and openness, there are many factors which influence (or can influence) a system of which we are not aware. The inevitable existence of such unknowns mean we often see unexpected indirect effects of our interventions.

EXAMPLES
- A powerful social grouping operating in a policy area not anticipated by a policy maker.
- An undiscovered plant in a rainforest with numerous potential health applications.

LEARNING POINTS
- Expect the unexpected.
- Be prepared to learn as the system unfolds it will become apparent that it might influence or be influenced by completely unexpected things.
- A new technology might enable a fundamental change, leading to widespread social effects.

14. Distributed control

Control of a system is distributed amongst many actors. No one actor has total control. Each actor may only have access to local information.

EXAMPLES
- A smoking cessation intervention's success may be determined by the many health professionals 'on the ground' running events and offering advice, rather than the central agency.
- Political parties' local groups and government may have differing views to the central parliamentary party. The central and distributed groups may conduct poltical work in contradictory ways.

LEARNING POINTS
- There is no top down control in complex systems. Decisions and reactions happen locally and the interactions of all these lower-level decisions can give us system-level properties such as stability, resilience, adaptation or whole system emergent regulation.
- The best we can do is to "steer" the system.

15. Nested systems

Complex systems are often nested hierarchies of complex systems (so-called 'systems of systems').

EXAMPLES
- Brain -> person -> society -> planet
- An ecosystem is made up of organisms, made up of cells, made up of organelles which were once free-living bacteria, made up of complex metabolic processes intertwined with genetic systems (each nested level is a complex system).

LEARNING POINTS
- When studying a particular system, it is useful to be aware of the larger system of which it is part, or the smaller systems operating within it.
- Mechanisms of change (as in realist evaluation) may be taking place at a higher or lower level to the one where an intervention is taking place.

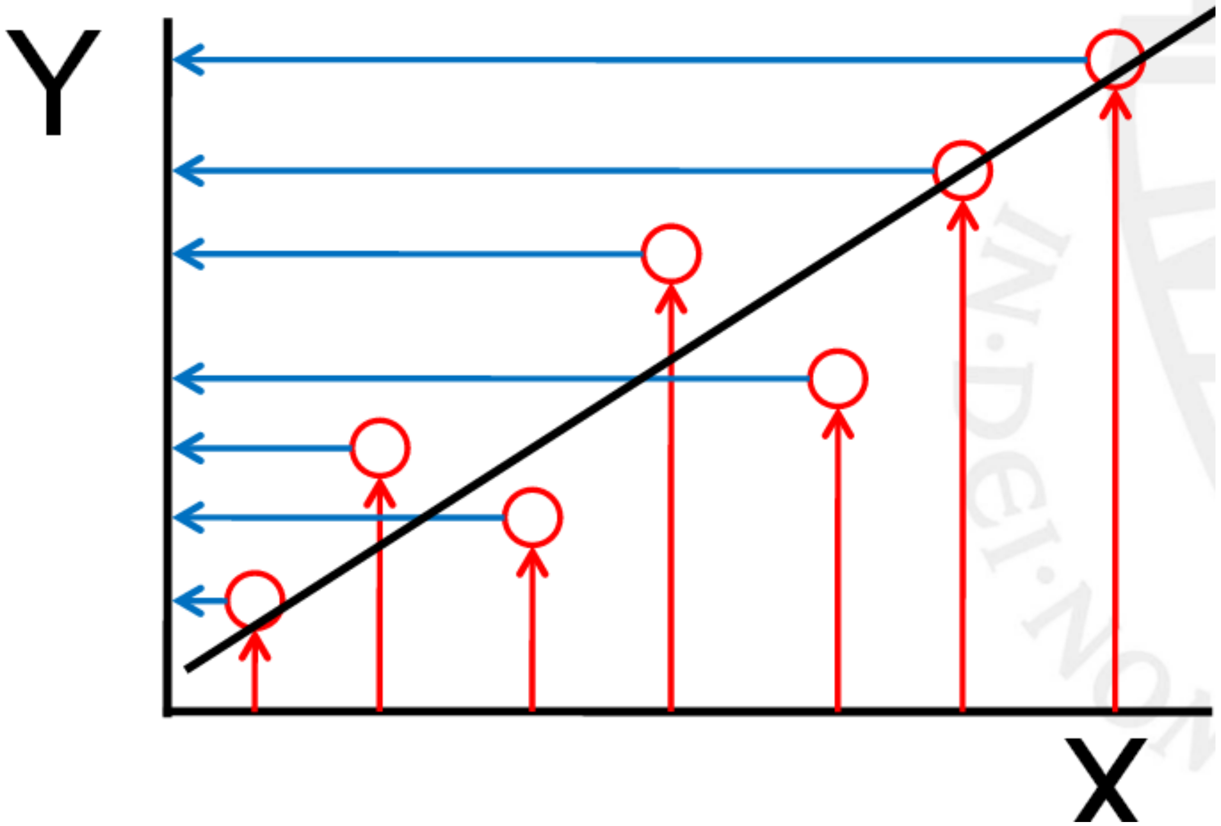
16. Multiple scales and levels

Actors and interactions in complex systems can operate across scales and levels. For this reason systems must be studied and understood from multiple perspectives simultaneously.

EXAMPLES
- Health issues can be considered at the scale of the individual physiology or behaviour, the household, community, society (social norms) or nation (economy, health system). Usually more than one domain is required to fully understand a problem.

LEARNING POINTS
- Tackling obesity requires thinking about individuals' eating habits and activity, but also social norms, economic factors and even town planning. No one level is sufficient. • We need to think broadly about systems at multiple scales and fields as properties or dynamics of one scale often feed up or down to affect others domains.

Mathematics of Change (simulation)



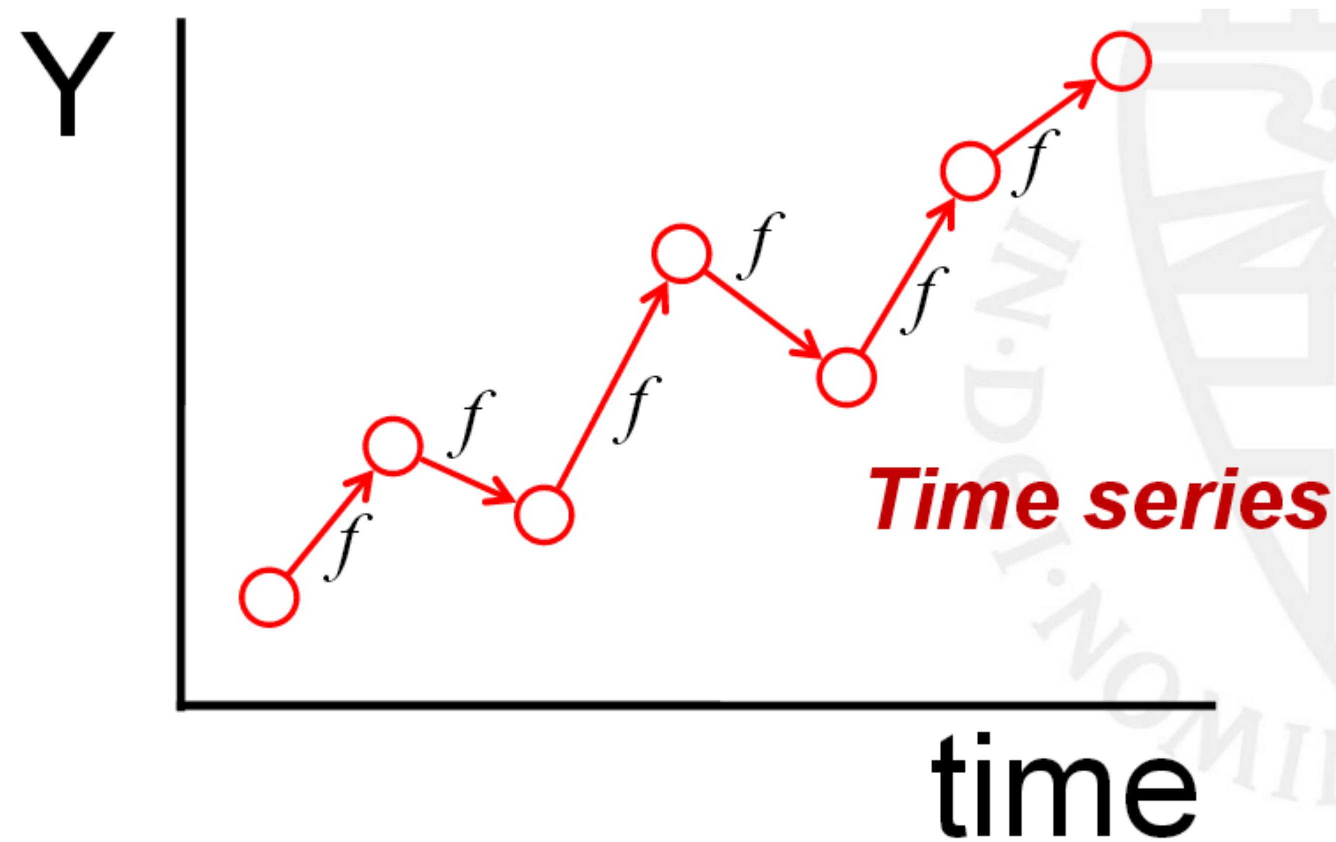


Fig. 30.1 A simple illustration of a reductionist explanation of a child's lexicon size

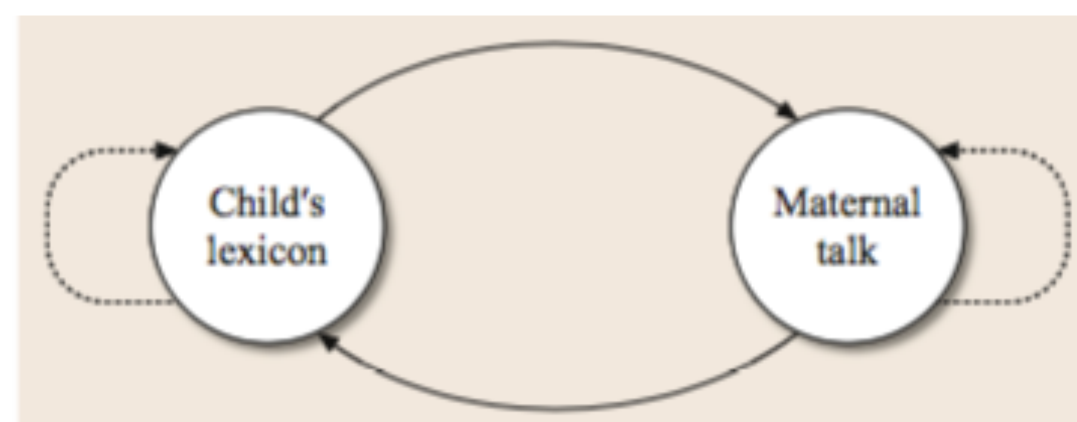


Fig. 30.2 A simple illustration of a dynamic explanation, in which the child's lexicon at a particular moment provides the basis for the lexicon at the next time point(s), while this change is also shaped by the dynamic interactions (literally and metaphorically) with the mother

Den Hartigh, R. J., Cox, R. F., & Van Geert, P. L. (2017). Complex versus complicated models of cognition. In *Springer handbook of model-based science* (pp. 657-669). Springer, Cham.

Story so far - Assignments session 1: Different ways to represent characteristics of change processes

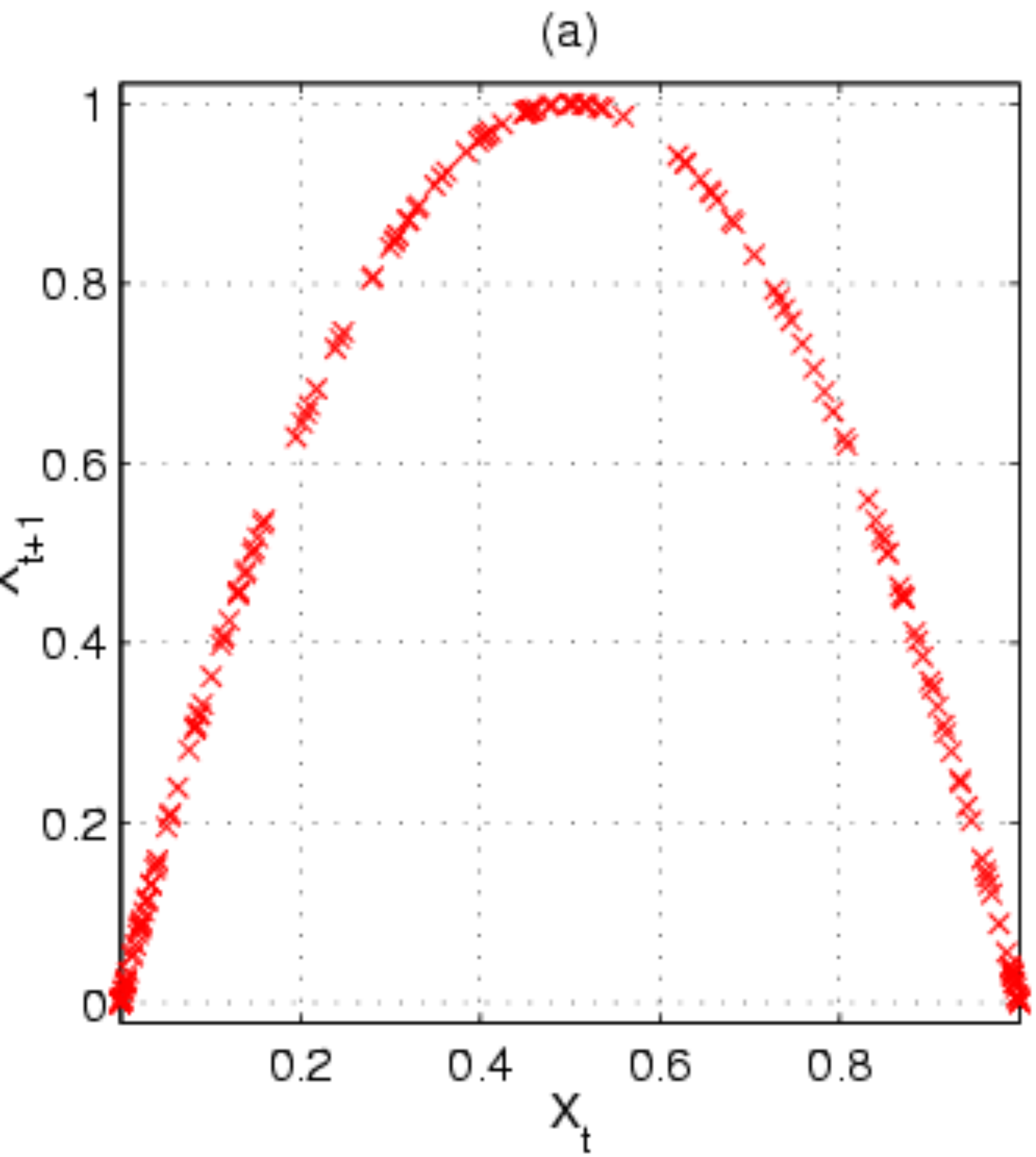
- **Iterative processes -** (coupled) difference / differential equations that represent autocatalytic change processes, the time-evolution of a system observable
- **Timeseries -** a record of values generated by an iterative / change process
- **Solution -** if available, actual iterations of the function are not necessary. Only available for a very limited set of (coupled) equations.
- **The return plot -** a scatterplot of Y_i vs. $Y_{i+1..n}$
- **The state / phase space -** A space spanned by **M** observable **dimensions** of the system.
  - Depending on parameter settings a system can be attracted to just a few states: *Attractors*
  - The cobweb method
- **The phase / bifurcation diagram -** diagram representing the parameter space of a system. Its dimensions represent the possible values of the control parameter(s) of the system. Stable regions are often labelled by an order parameter (solid, liquid, gas).
- **Potential Functions -** A functions describing the relative stability of the 'end-states' of a system…

Return plot

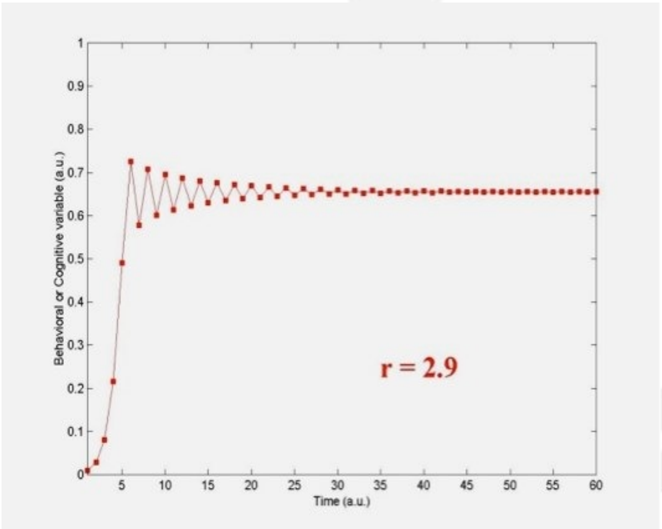
Also:

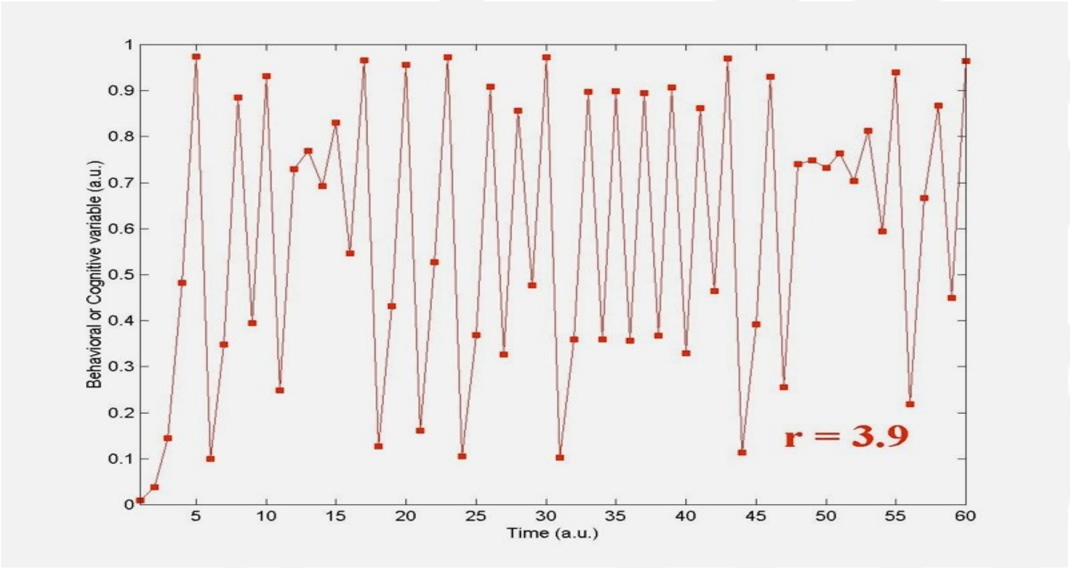
- *Phase plane*
- *Phase diagram*

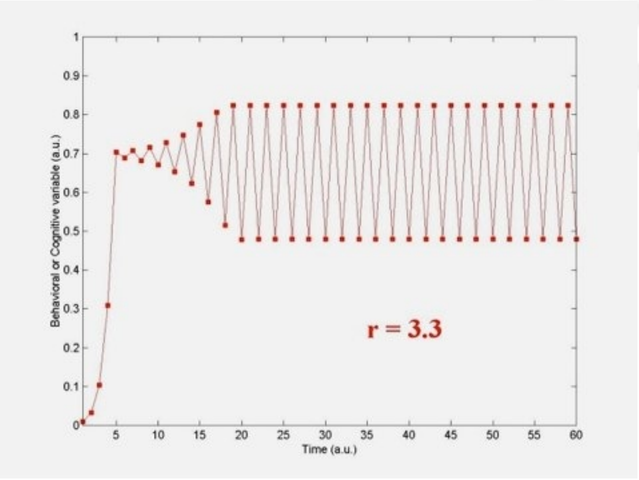
Story so far - Assignments session 1: Return plot of the logistic map



Why the same shape for all these different time series?

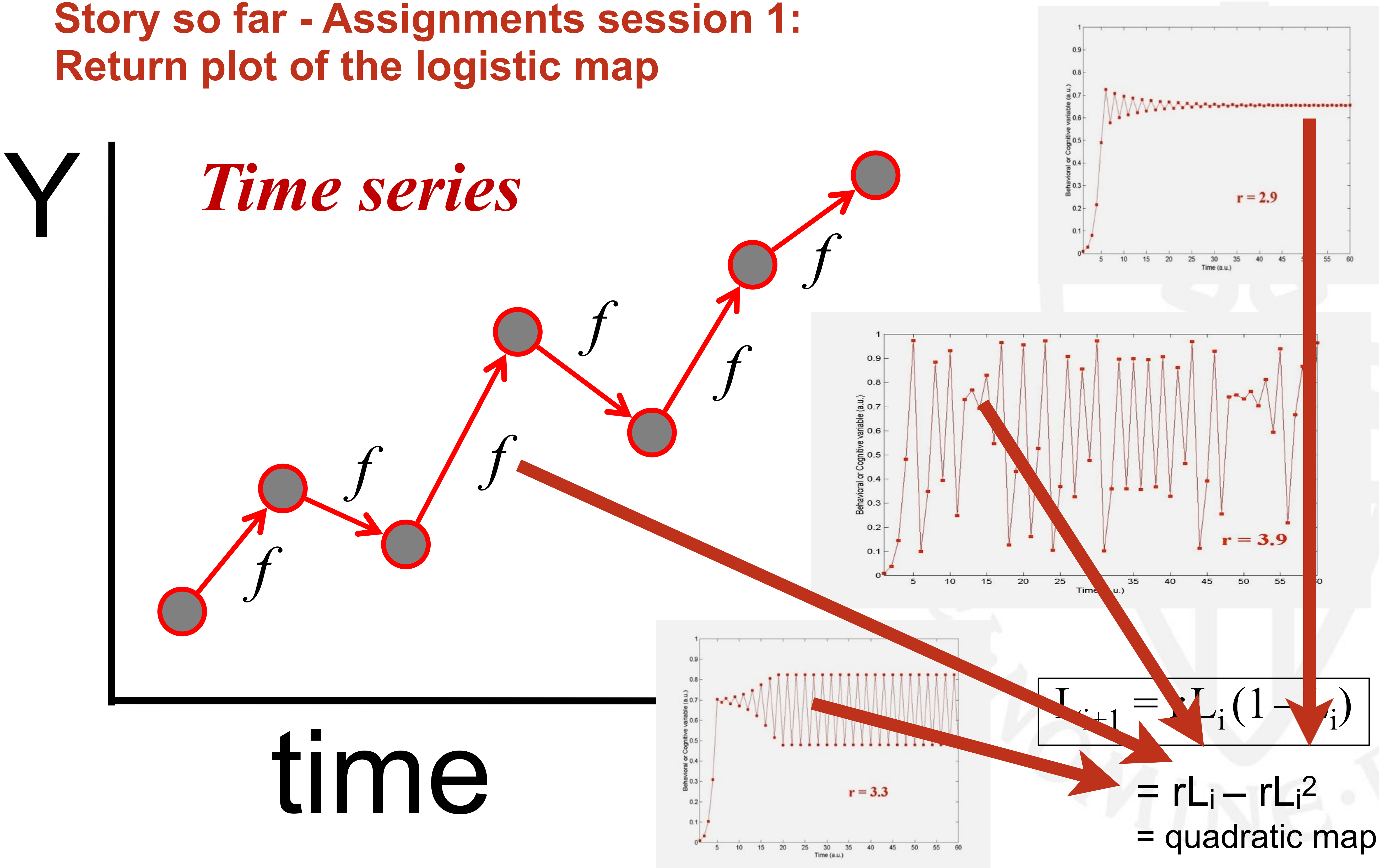






$$\boxed{L_{i+1} = r\,L_i\,(1 - L_i)}$$

Story so far - Assignments session 1: Return plot of the logistic map



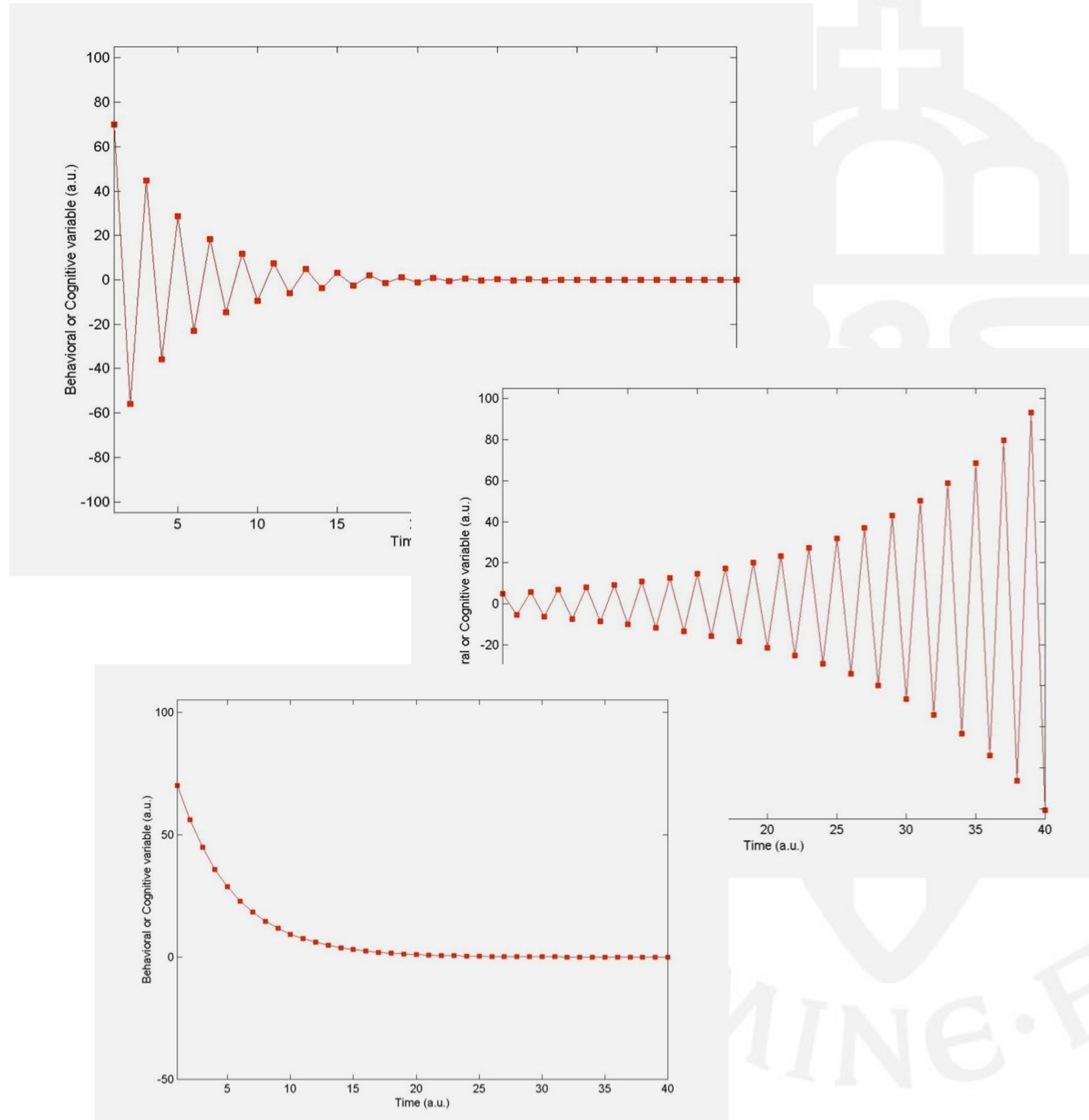
$$L_{i+1} = rL_i(1 - L_i)$$

$= rL_i - rL_i^2$

= quadratic map

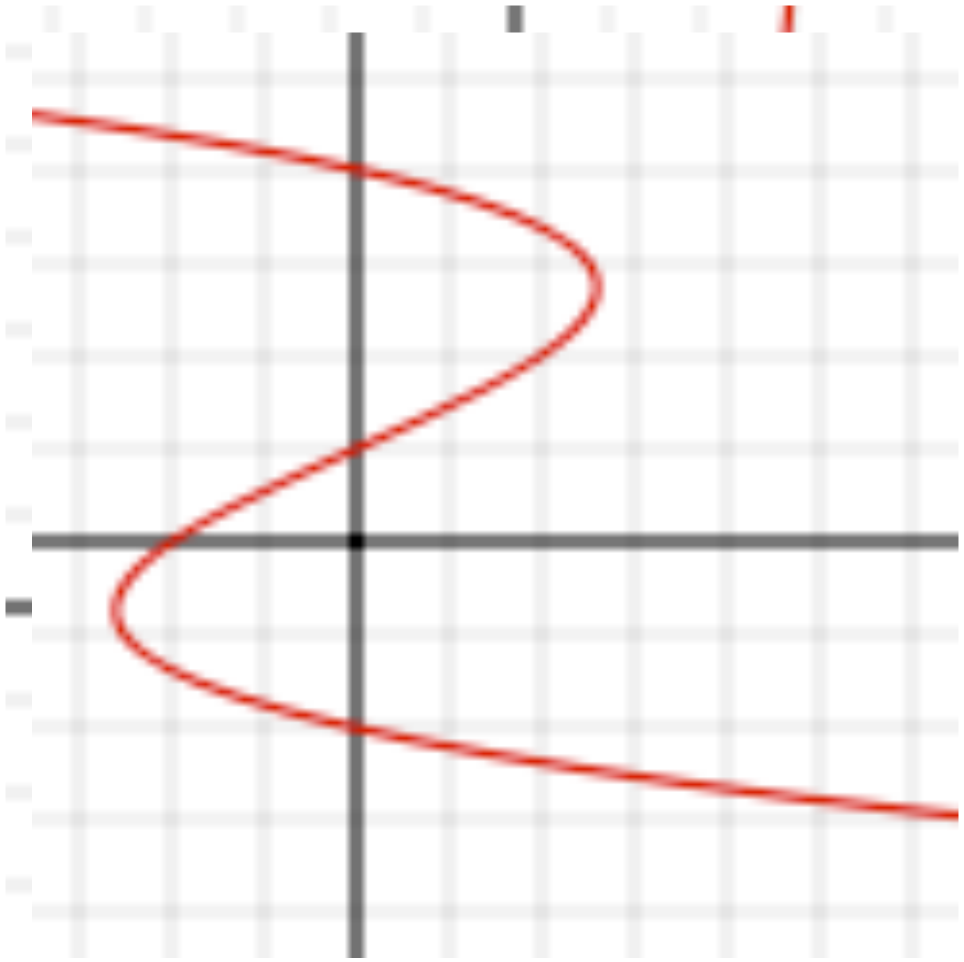
Return plot quiz

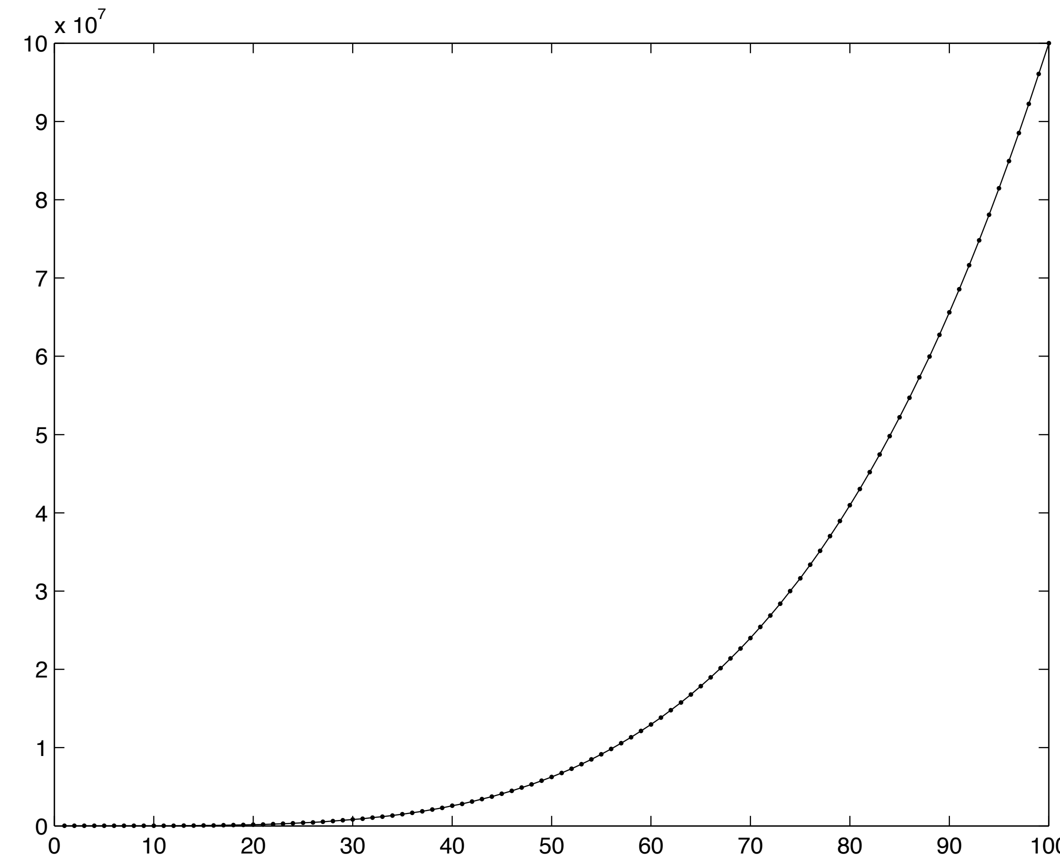
$$\boxed{Y_{i+1} = a \cdot Y_i}$$

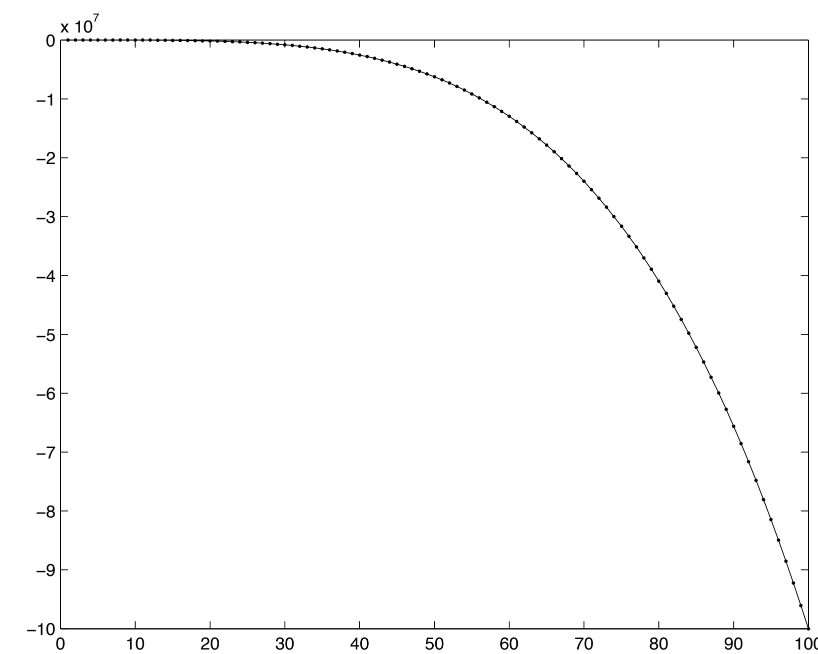


Return plot quiz

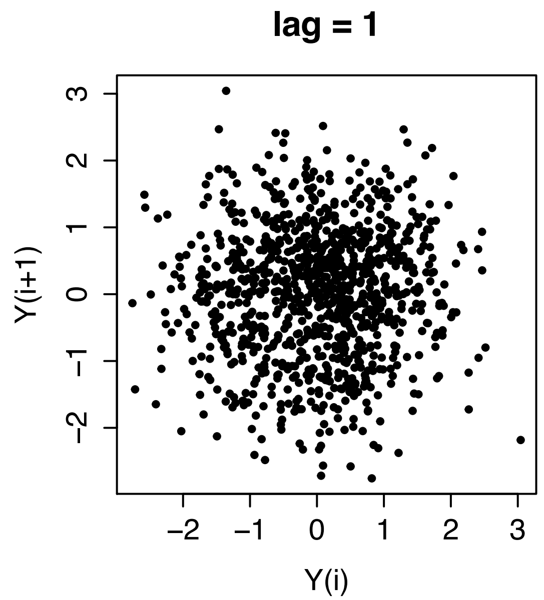
$$Y_{i+1} = a \cdot Y_i^3 + b \cdot Y_i^2 + \ldots$$

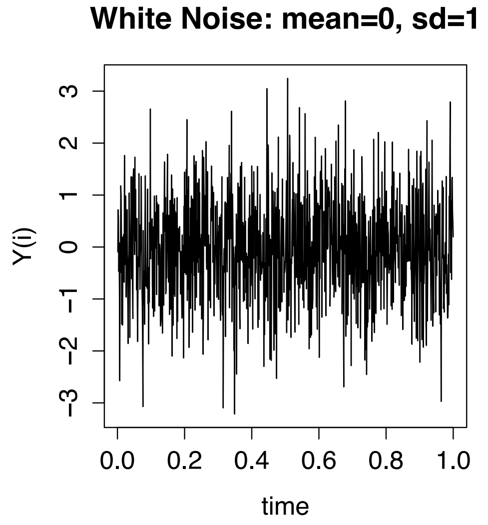






Return plot quiz



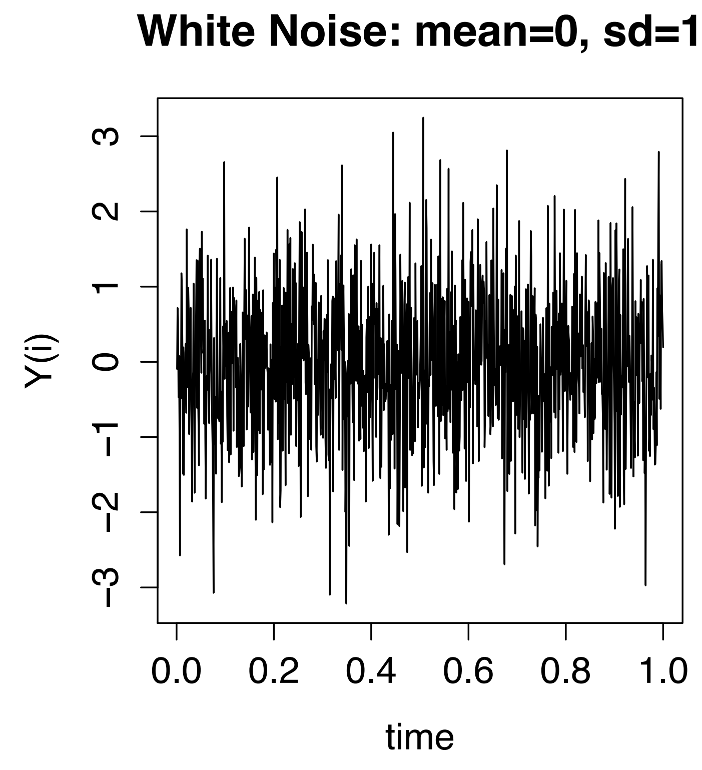


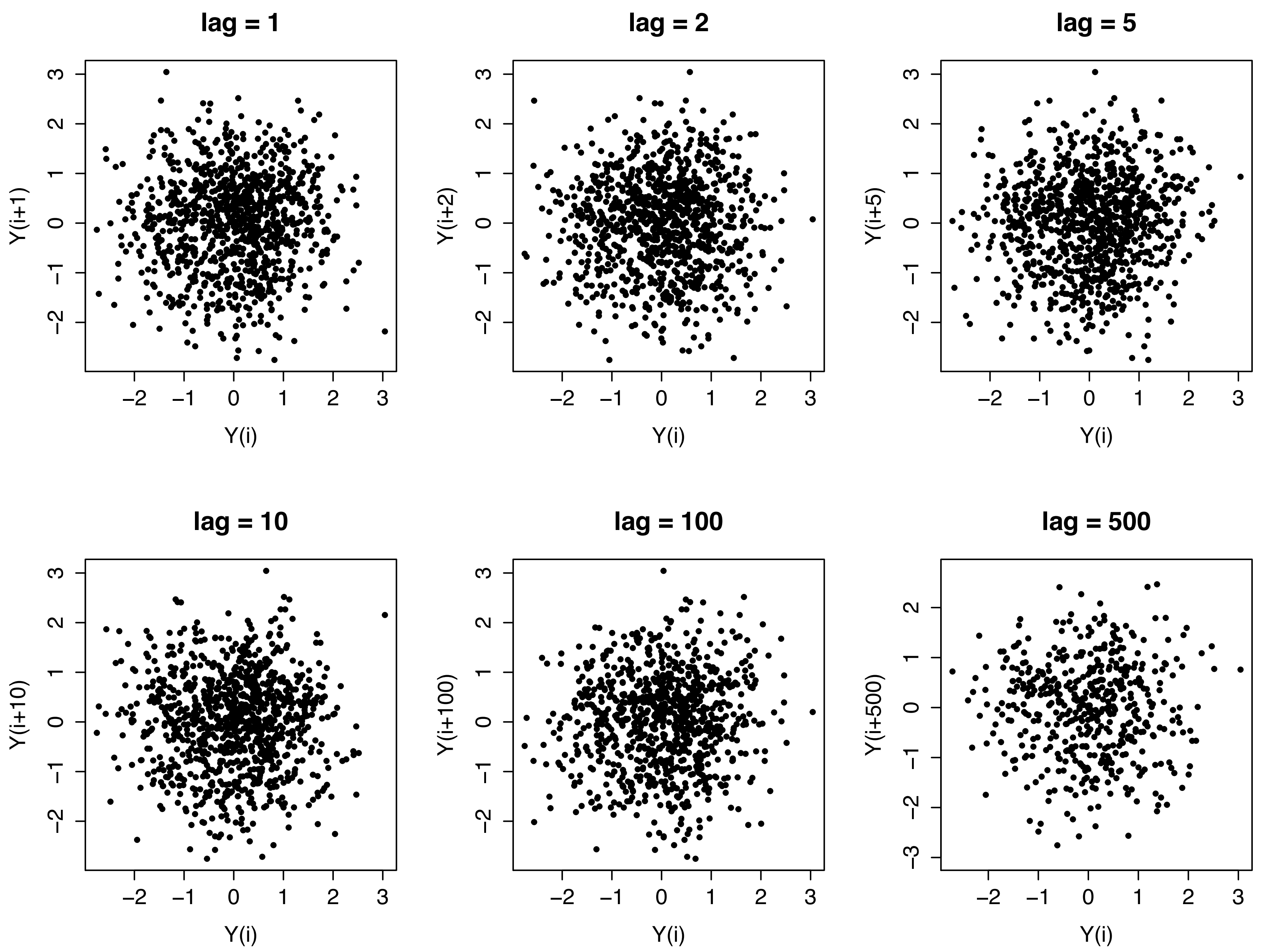
White noise
Completely random
(Gaussian distribution)

What happens at different lags?

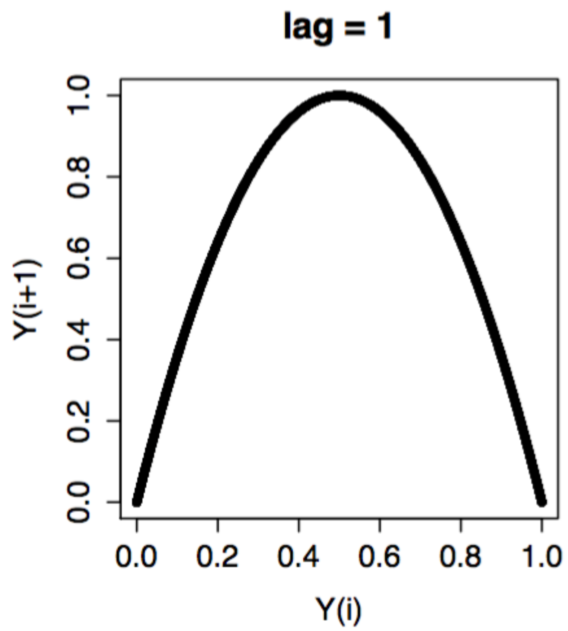
Return plot quiz

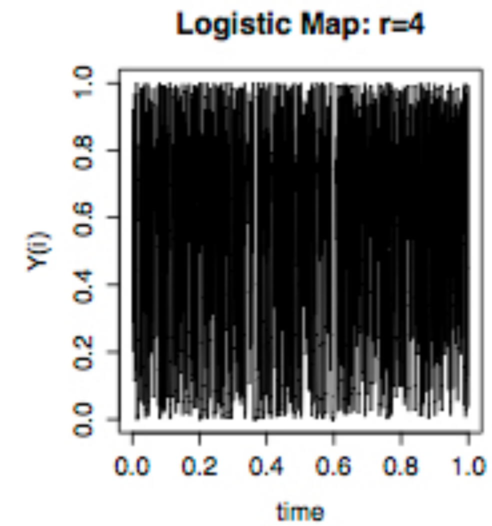
White noise
Completely random
(Gaussian distribution)





Return plot quiz



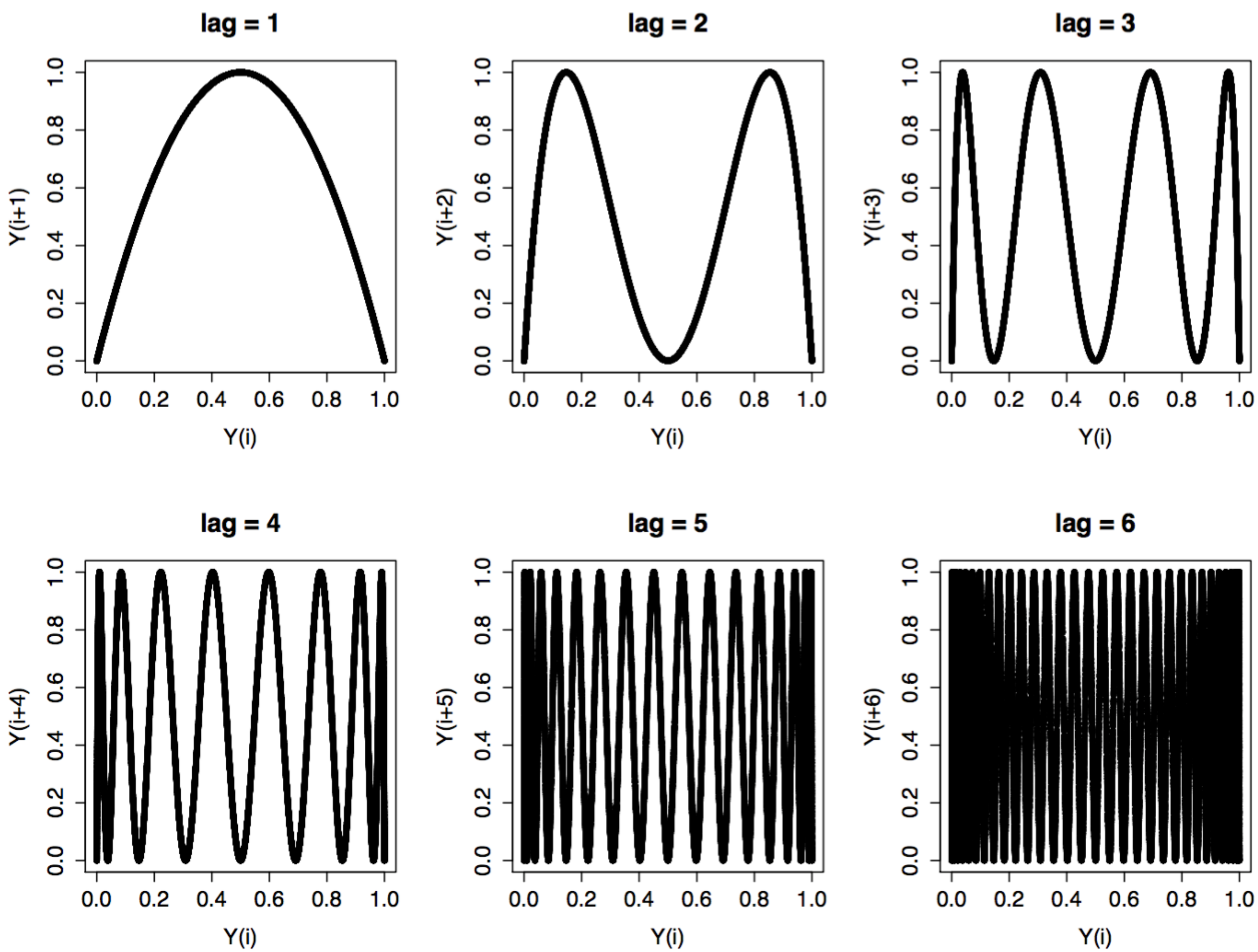


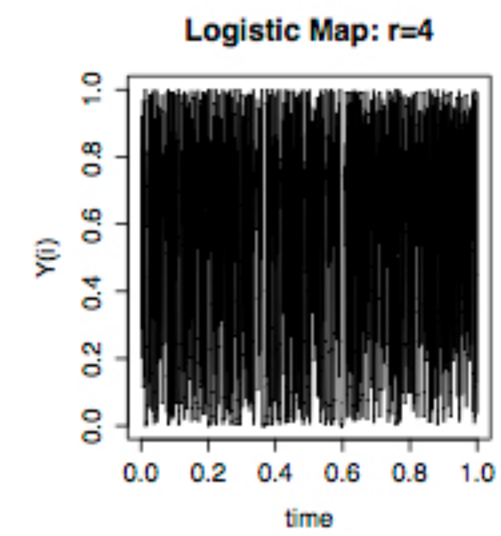
Iterative Process
Completely deterministic
(deterministic chaos)

What happens at different lags?

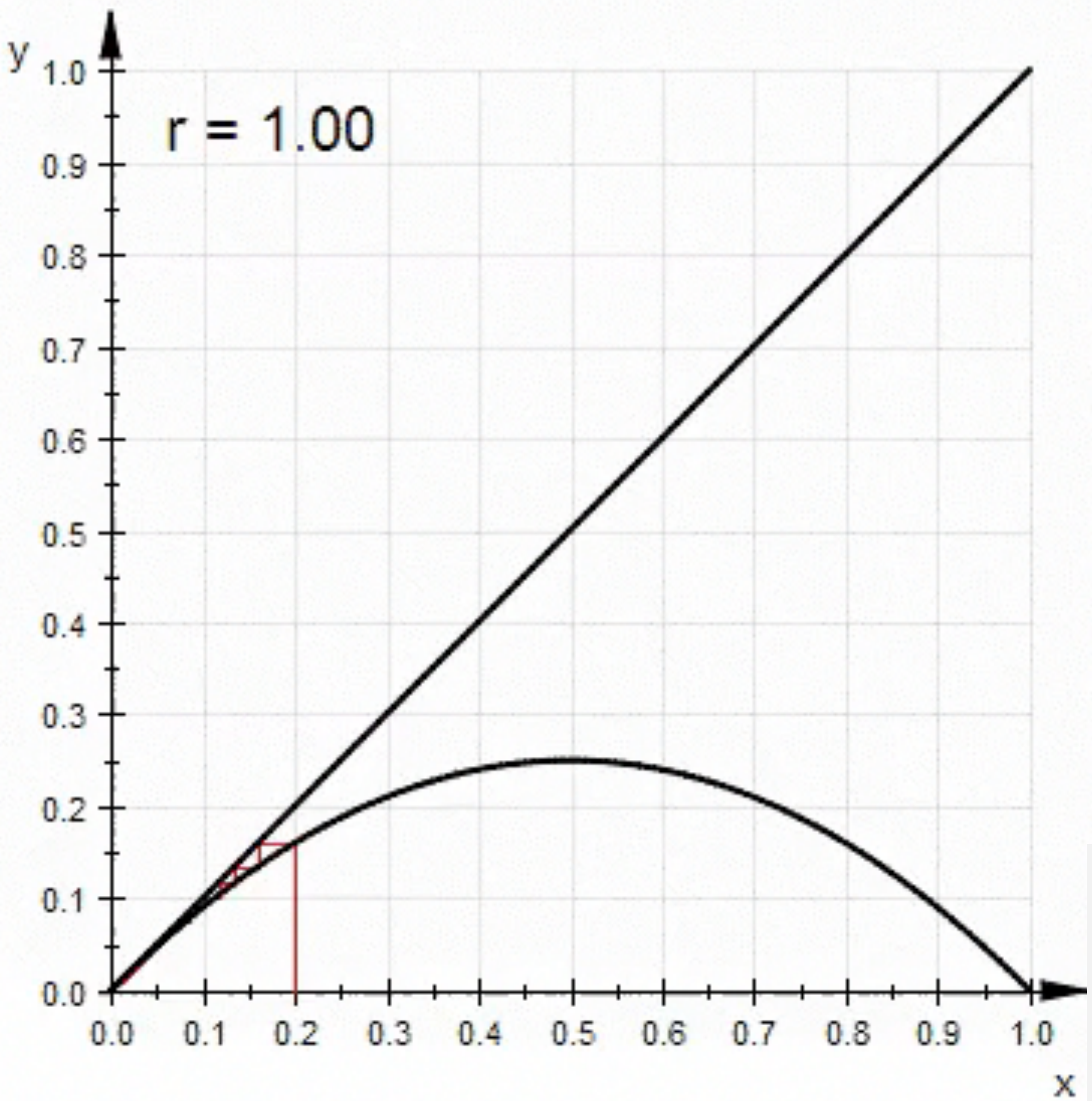
Return plot quiz

Iterative Process
Completely deterministic
(deterministic chaos)

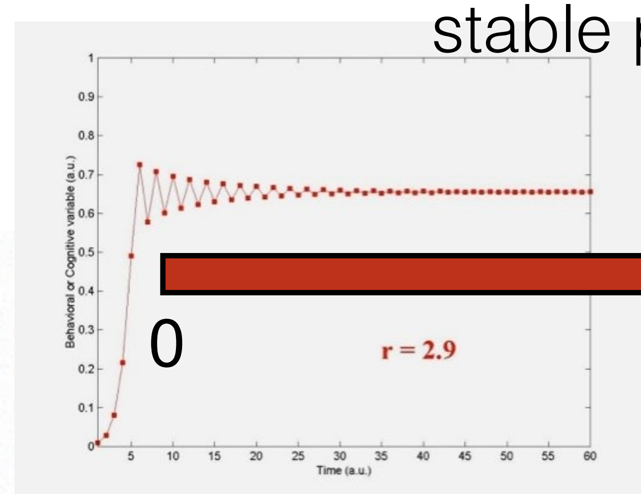


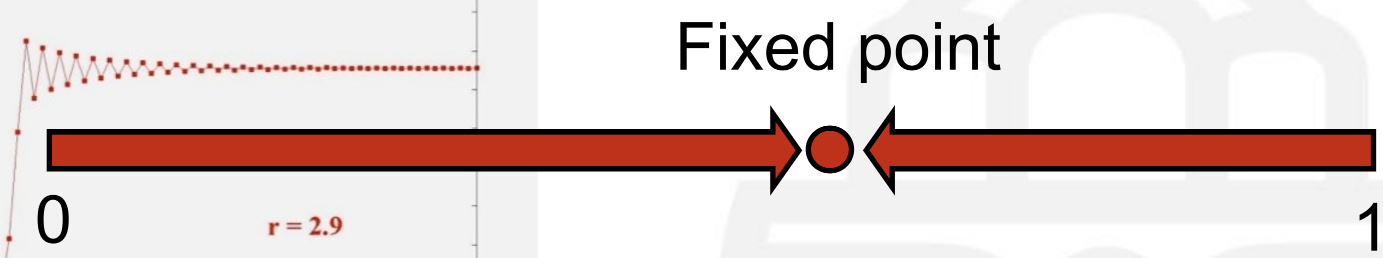


EXTRA: Cobweb method

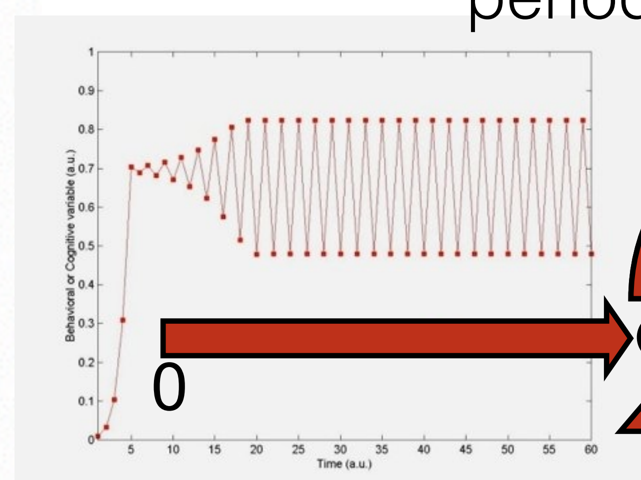


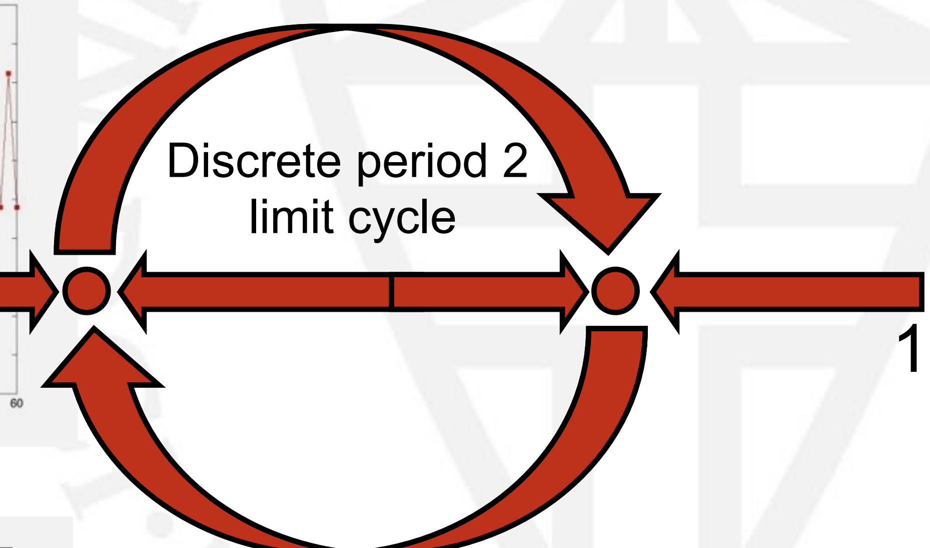
stable points (point attractors)

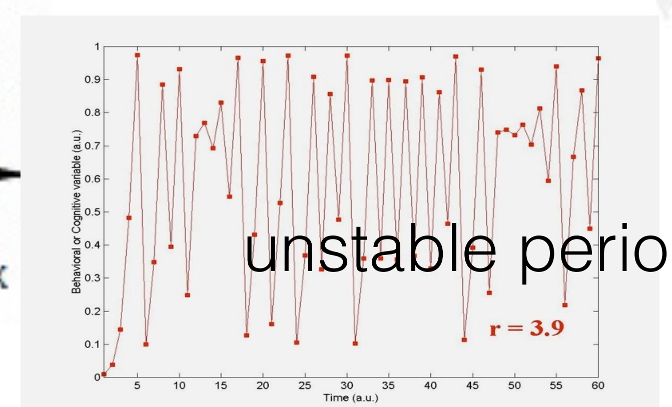




periodic orbits (limit cycles)

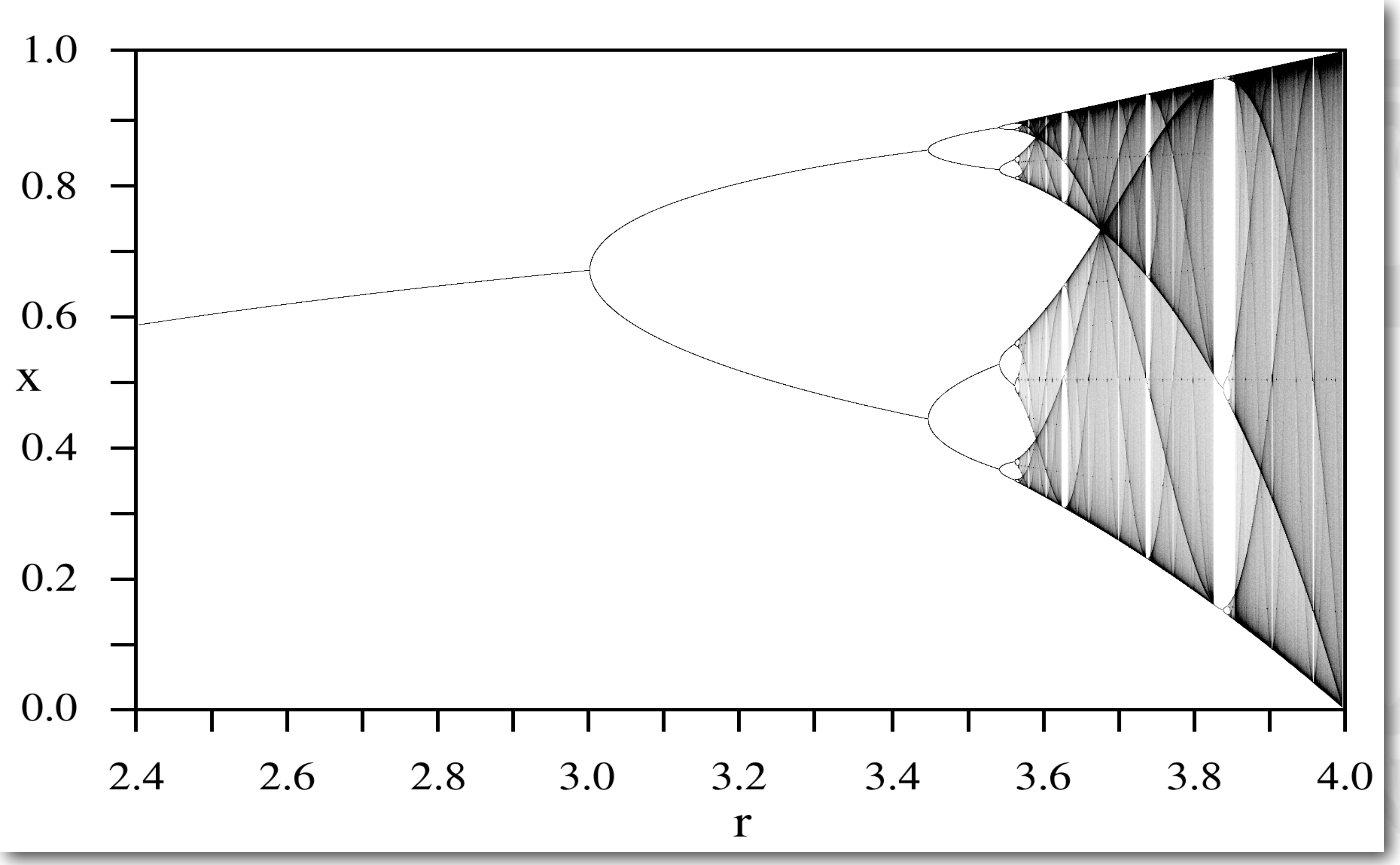






unstable periodic orbits (strange attractors)

Bifurcation diagram



This equation will change how you see the world

https://youtu.be/ovJcsL7vyrk

Bifurcation diagram

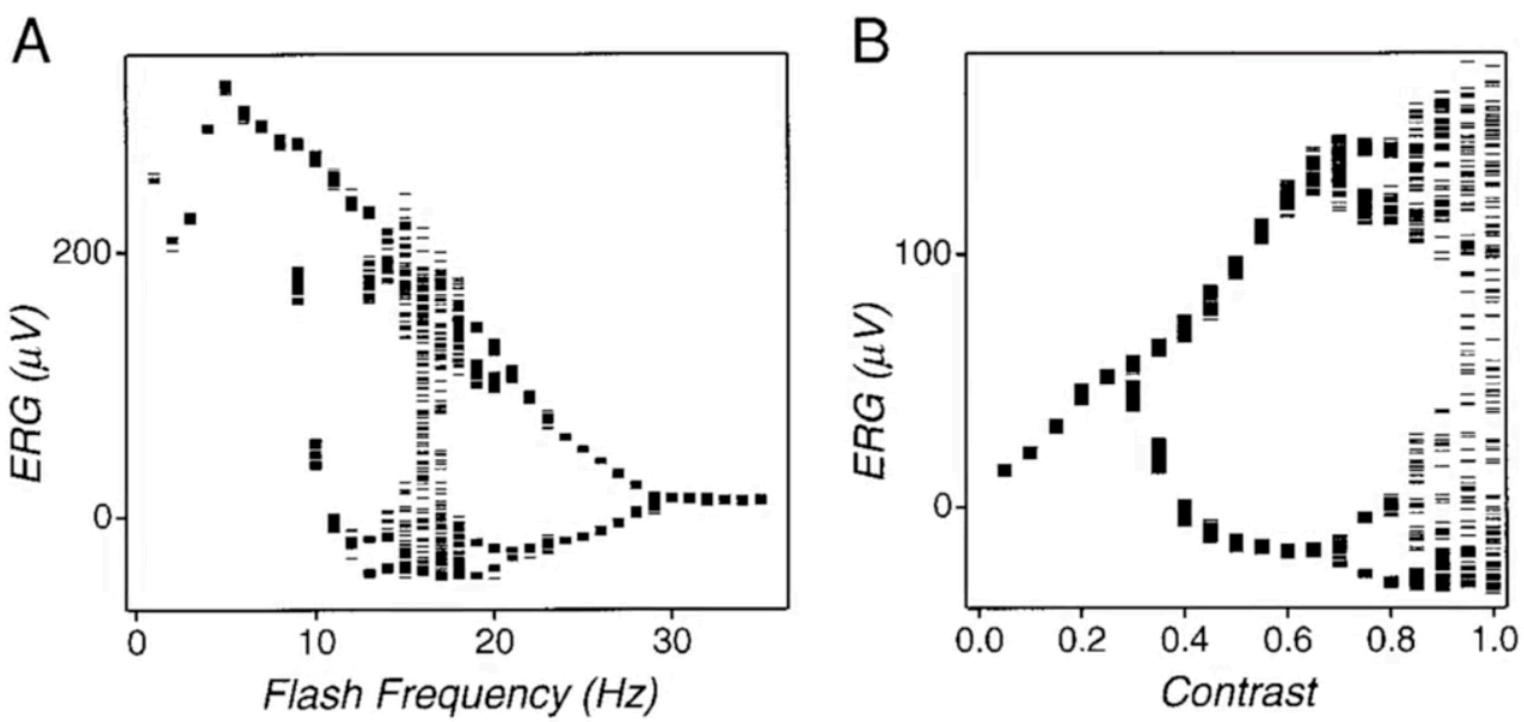
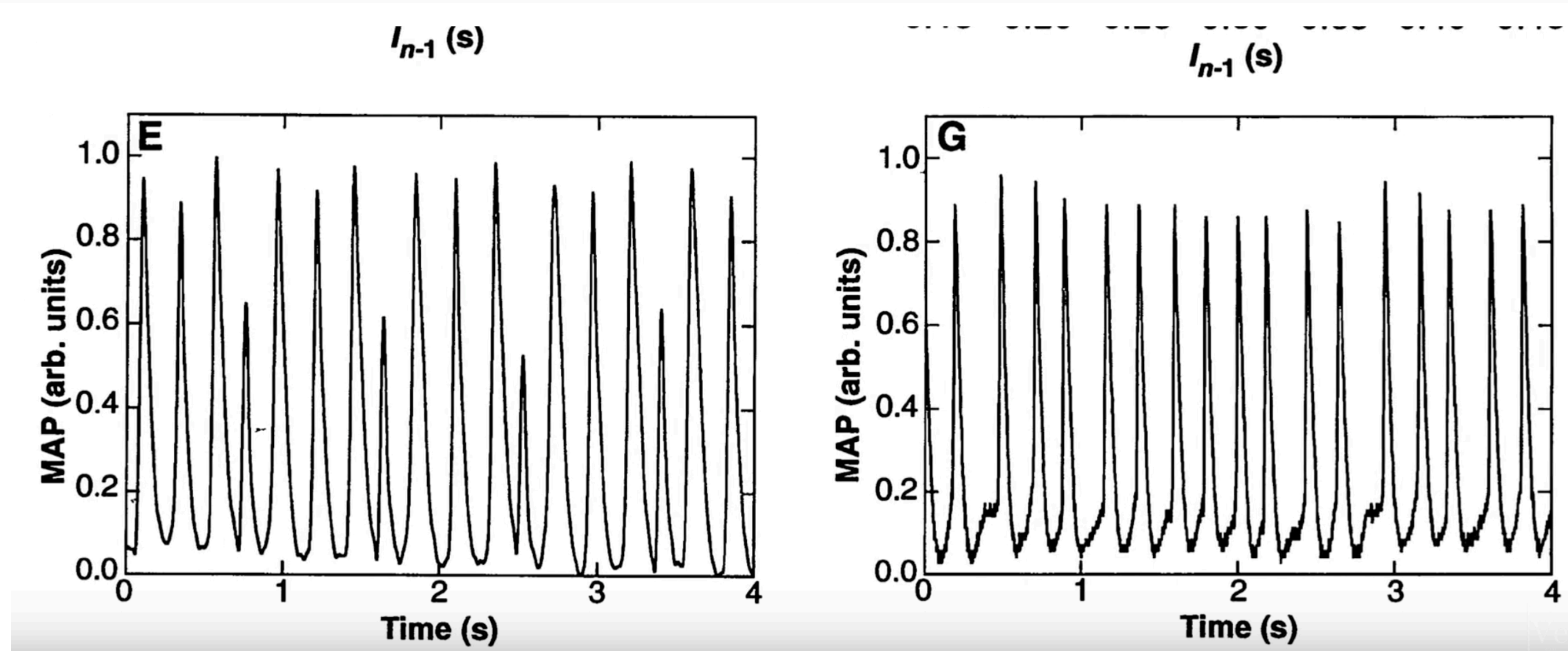
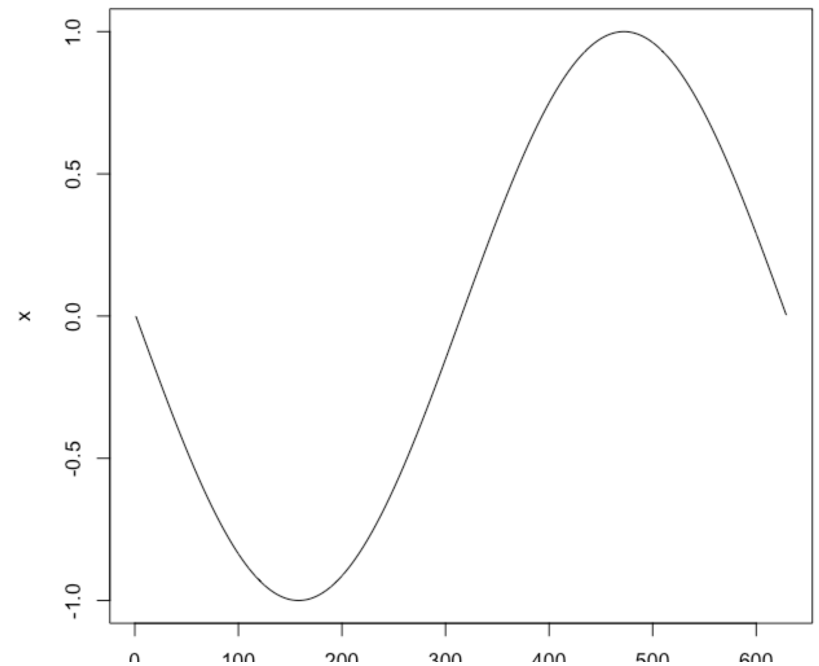


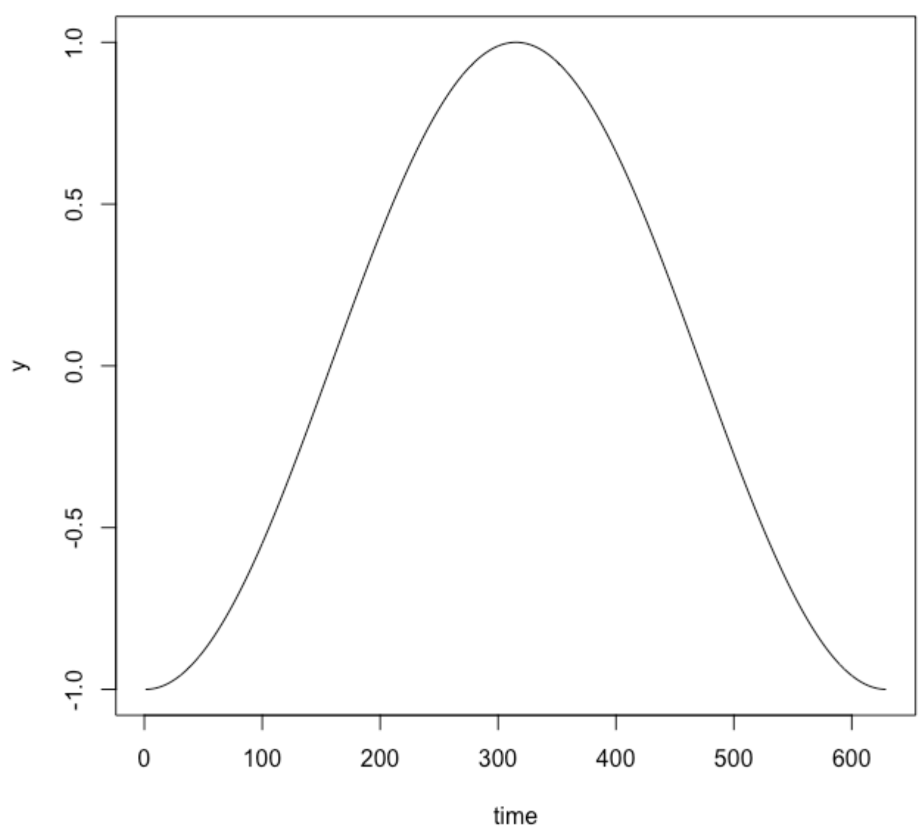
FIG. 4. Bifurcation plot of the salamander ERG. *A*: ERG amplitude as a function of flash frequency at contrast $C = 1.0$. For each frequency value on the abscissa, the ERG was recorded for 12 s. The maximum of this waveform was located, and the values at the corresponding phase in all other flash intervals were plotted on the ordinate. These correspond to the markers on the ERG trace in Fig. 1, *B–E*. *B*: ERG amplitude as a function of contrast at flash frequency $f = 16$ Hz, displayed as in *A*.

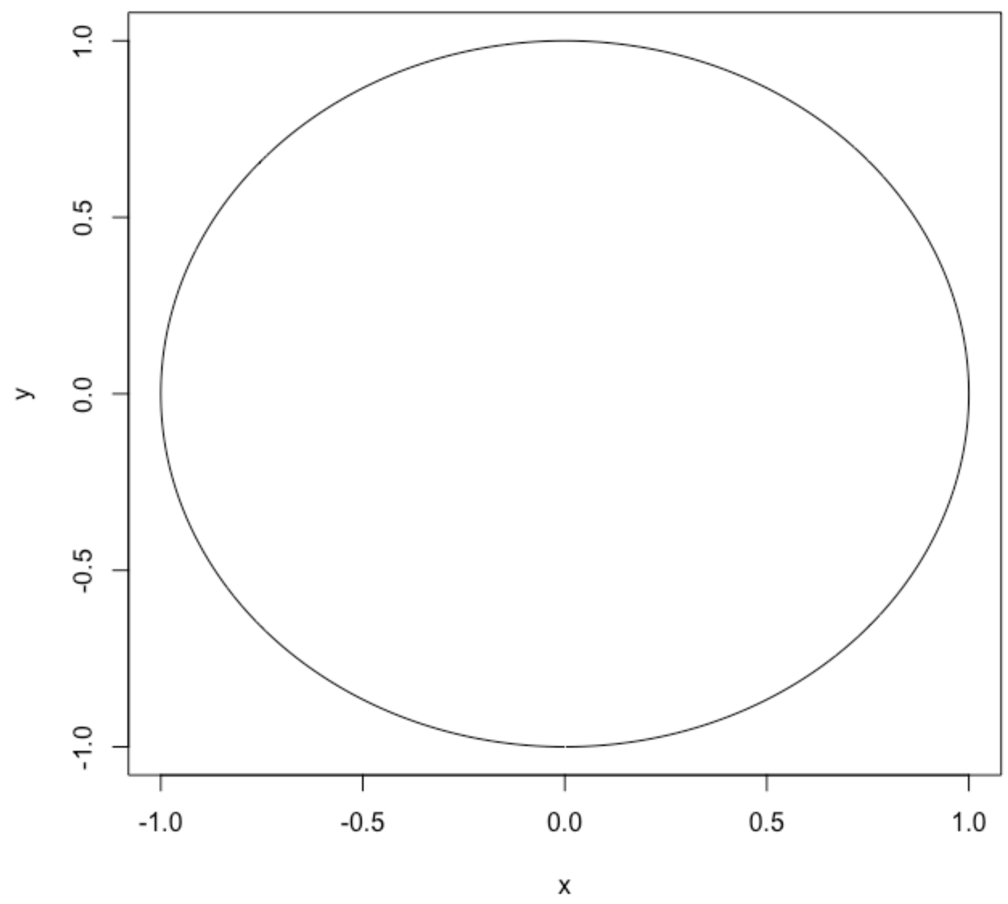
Heart rate (of rabbits)



Coupled Systems:
Plot system dimensions
(Phase space, state space







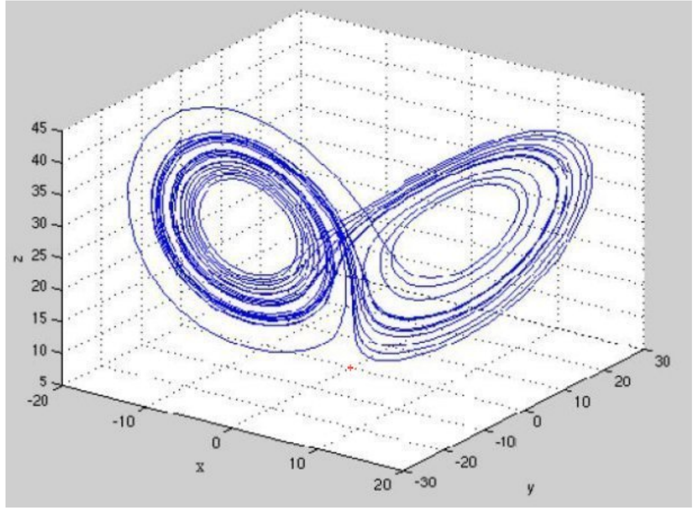
State Space / Phase Space

A state space is spanned by a system's state variables (dimensions), these are often iterative processes.

Every potential micro-scale configuration of a system is represented by a coordinate in state space. Each point is a state, a degree of freedom. *Change of states over time = Trajectory through state space, an orbit.*

Generally, state space is not completely filled by trajectories = not all d.o.f available = attracted to specific states / trajectories

$$\frac{dx}{dt} = a(y-x)$$
$$\frac{dy}{dt} = x(b-z) - y$$
$$\frac{dz}{dt} = xy - cz$$



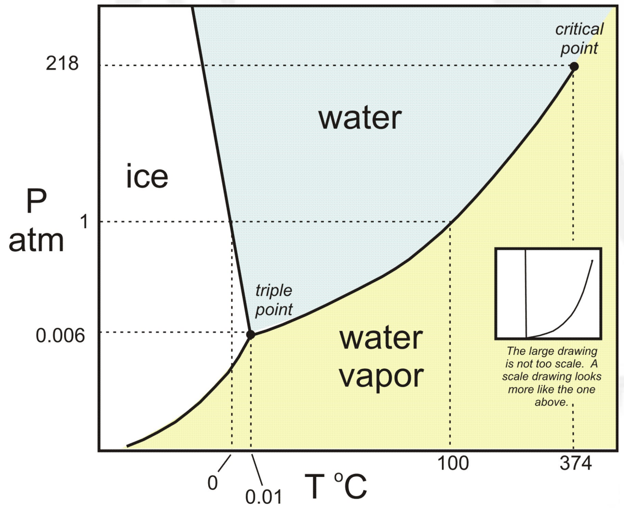
http://universe-review.ca/I01-18-LorenzEqs2.jpg

Phase Diagram / Phase Portrait

A phase diagram labels the order parameter of a system for different values of control parameter(s).

Regions represent qualitatively different states at the *global-scale*, the coordinates represent control parameter values associated with a global state. *Time does not have to be represented in the space.*

Generally, a phase diagram is completely filled with labelled regions of qualitatively different states, also called: phases / orders / regimes / modes of the system



http://cft.fis.uc.pt/eef/Fisica01/fluids/h2o_phase_diagram.jpg

Dynamics of Complex Systems

A Closer look at modelling growth
Multivariate Models
Simulation of continuous time

Van Geert's framework for cognitive growth

Paul van Geert (1991) introduced an '*ecology of species of cognitive growers*'
(These terms are not accidentally chosen, these models also have applications in biology)

- Cognitive growth is an **autocatalytic** process (of discrete units)

- Cognitive growth is **limited** by available resources

- Cognitive growth may be a **delayed** process

- Cognitive growth rate may **vary** depending on certain events

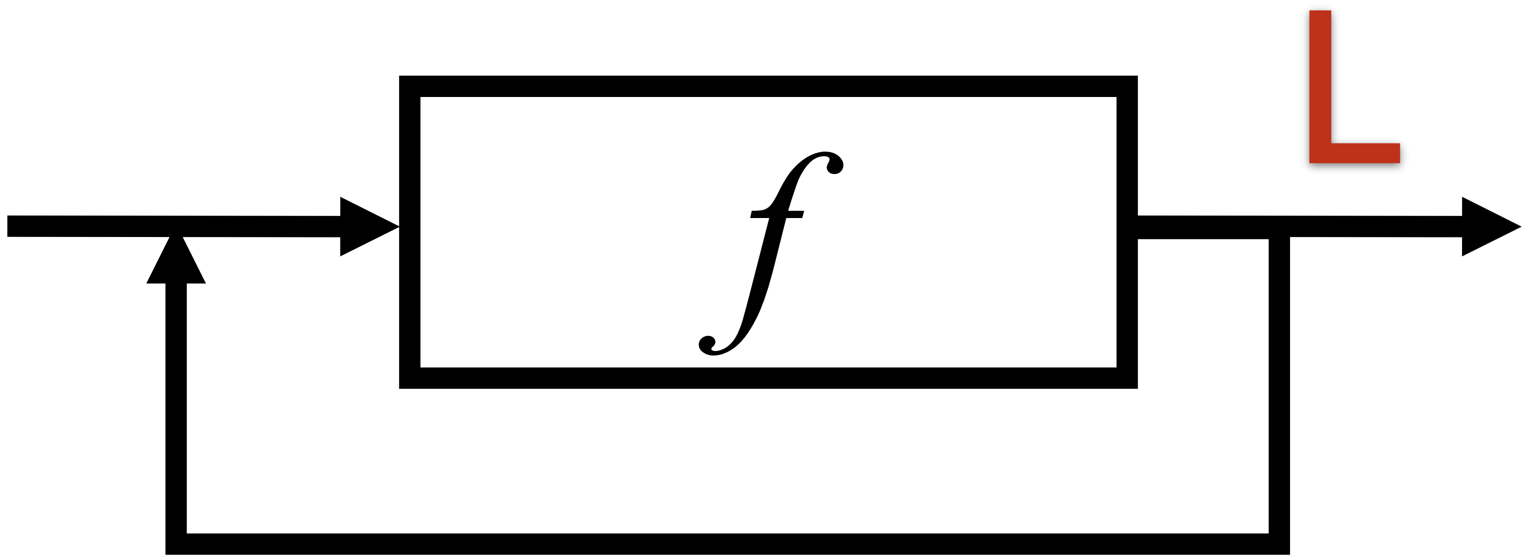
- Cognitive growth may be in **competition** with, or receive **support** from other growth processes

van Geert, P. (1991). A dynamic systems model of cognitive and language growth. *Psychological Review, 98*(1), 3–53.
https://doi.org/10.1037/0033-295X.98.1.3

Cognitive growth is an **autocatalytic** process

Autocatalytic means the current growth level (L) is dependent on the level already reached, by now you should know what that means:

$$L_{i+1} = f(L_i) \quad \text{or} \quad \frac{dL}{dt} = f(L)$$



Exponential unlimited growth
A linear map … and a linear flow ~

The growth rate is proportional to the current growth level:

$$\boxed{L_{i+1} = r \cdot L_i}$$

$$\boxed{\frac{dL}{dt} = r \cdot L}$$

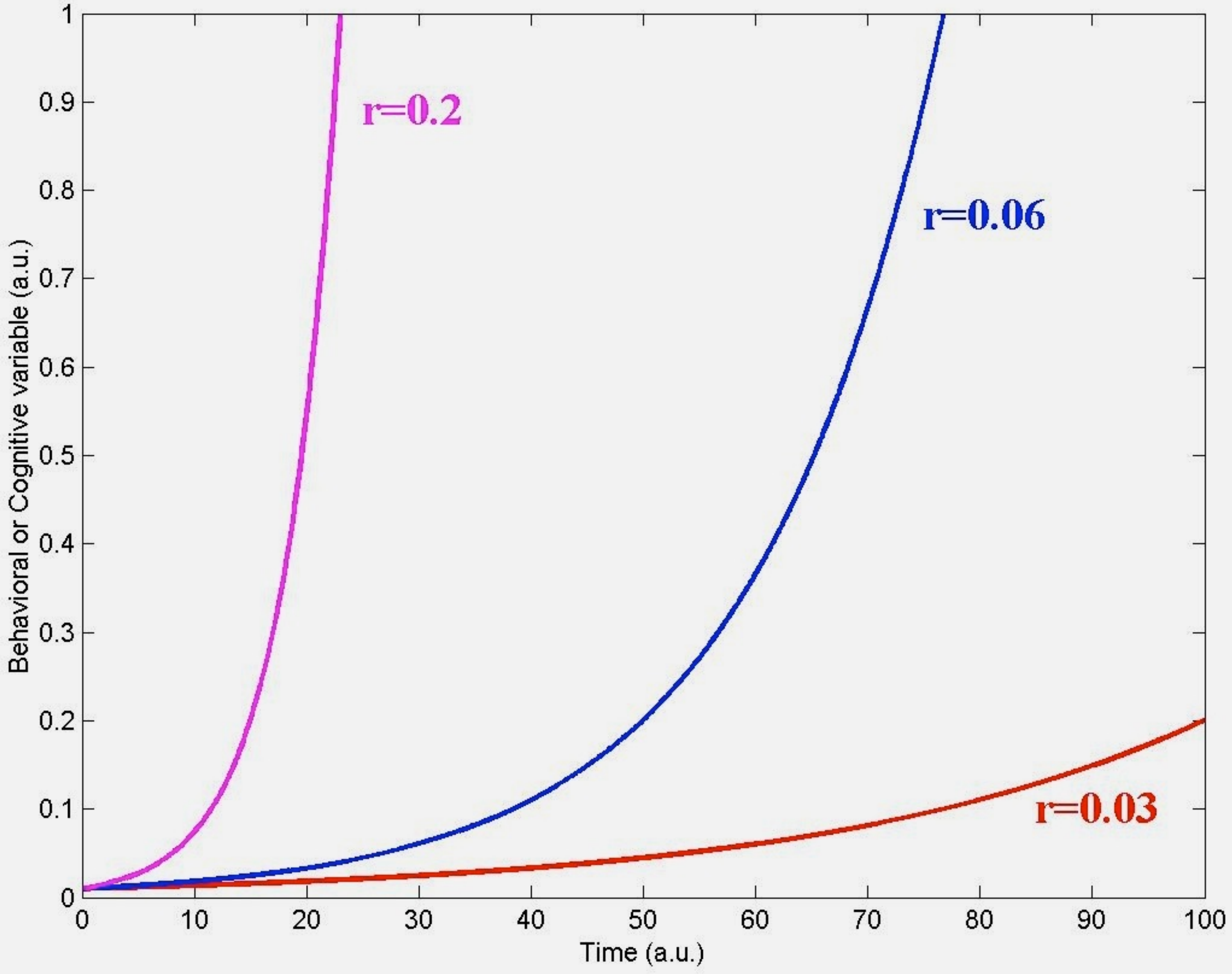
Analytic Solution

$$\boxed{L_i = r^i \cdot L_0}$$

Analytic Solution

$$\boxed{L(t) = L_0 \cdot e^{-r \cdot t}}$$

Exponential unlimited growth (Flow ~)



Real world examples:

- Interest rate ('Rente')
- Radioactive decay (r<0)

Cognitive growth is **limited** by available resources

Learning and development are restricted by the limited availability of resources. Maximum possible level of growth is the **Carrying Capacity**, **K**.

The carrying capacity reflects '*the maximum number of words the child can acquire and maintain, given the present conditions of internal and external cognitive support*'.

The growth rate is now proportional to what is left to grow (K-L):

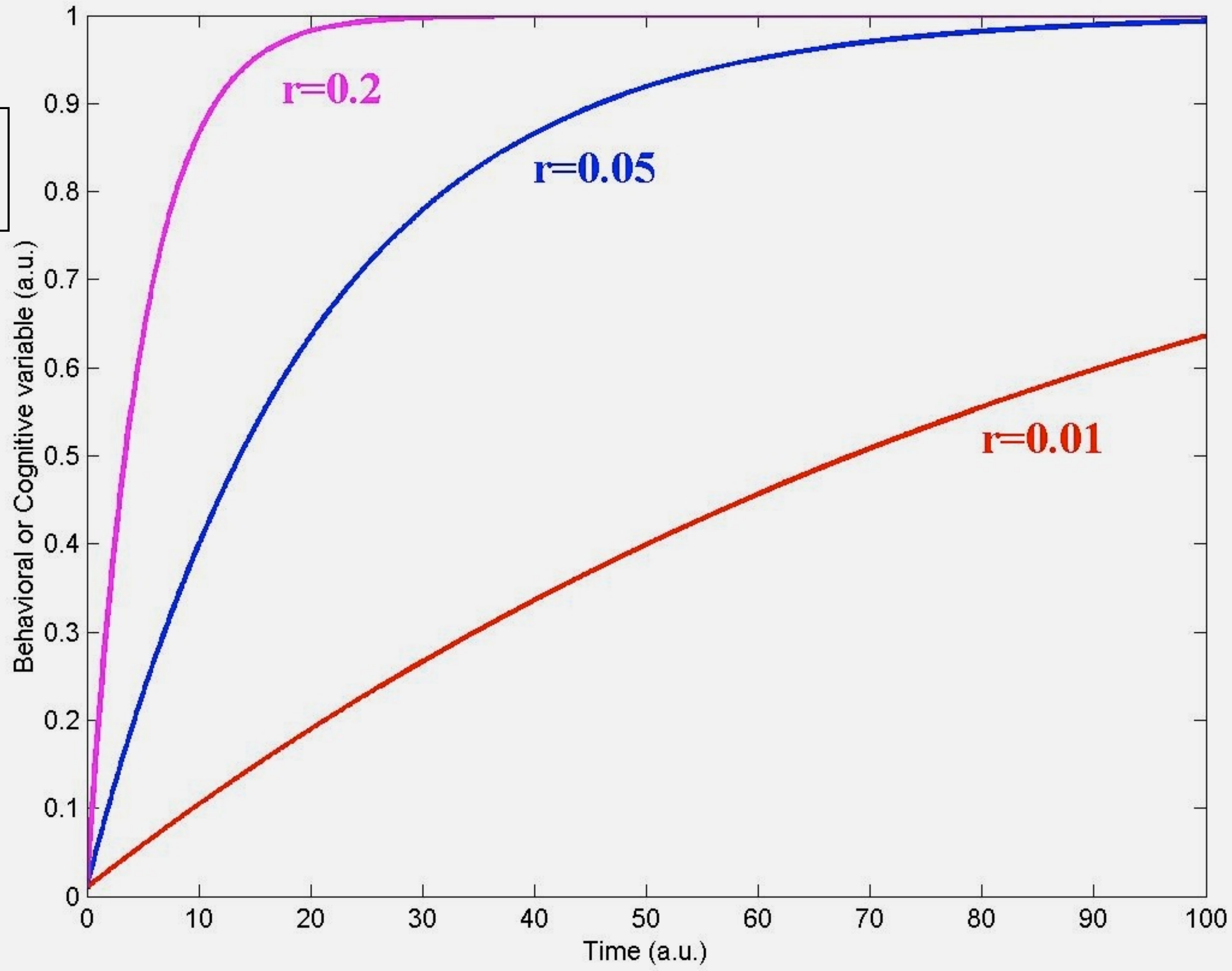
$$\frac{dL}{dt} = r \cdot (K - L)$$

<u>Restricted growth:</u>

$$L(t) = K - (K - L_0) \cdot e^{-r \cdot t}$$

Restricted Growth (Flow ~)

K=1



Real world examples:

-Noun-concept problem solving

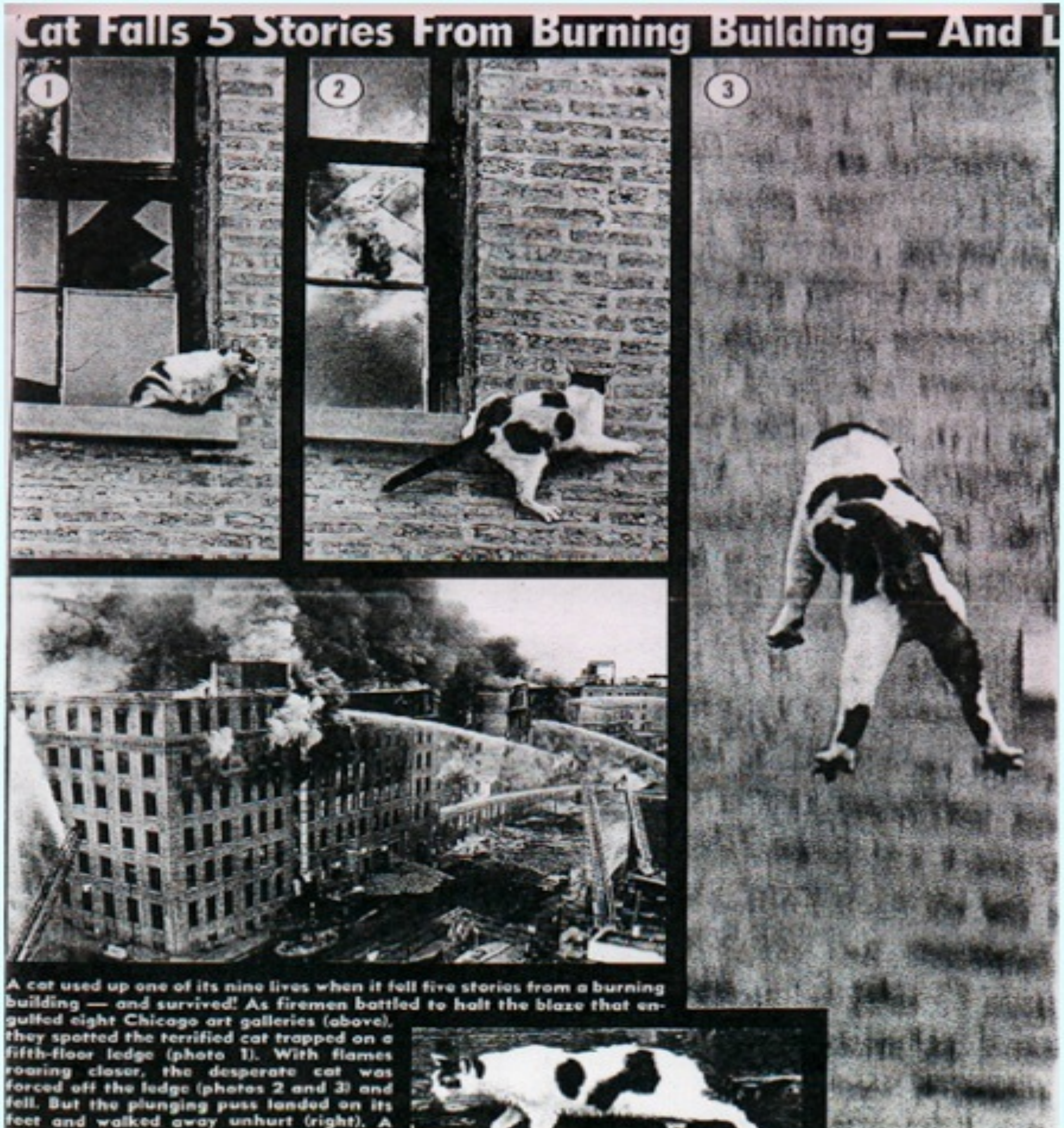
-Speed of raindrops (2 – 5 m/s)

- Speed of a skydiver (200 km/h)

→ *Terminal velocity…*

Is not so terminal for cats (till 32 stories)

San Francisco Cat


Cat Falls 5 Stories From Burning Building — And L

A cat used up one of its nine lives when it fell five stories from a burning building — and survived! As firemen battled to halt the blaze that engulfed eight Chicago art galleries (above), they spotted the terrified cat trapped on a fifth-floor ledge (photo 1). With flames roaring closer, the desperate cat was forced off the ledge (photos 2 and 3) and fell. But the plunging puss landed on its feet and walked away unhurt (right). A

Real world examples:

-Noun-concept problem solving

-Speed of raindrops (2 – 5 m/s)

- Speed of a skydiver (200 km/h)

→ *Terminal velocity…*

Is not so terminal for cats (till 32 stories)

Logistic growth

If we combine these 2 **linear** models we get **nonlinear** restricted (logistic) growth

$$Y_{i+1} = rY_i(K - Y_i)$$

$$Y_{i+1} = rY_i(K - Y_i)$$

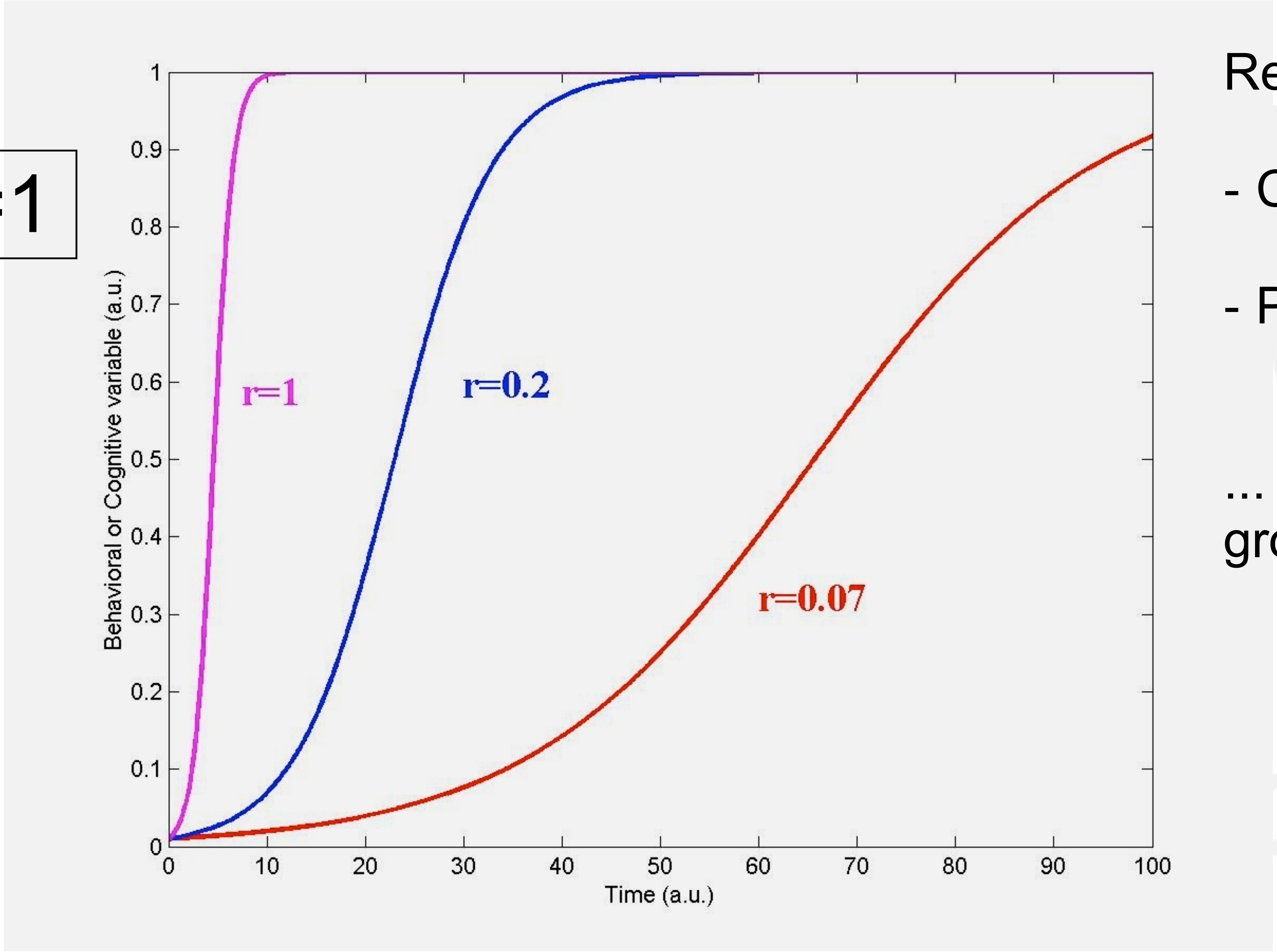
no analytic solution

$$\frac{dY}{dt} = rY(K - Y)$$

$$Y(t) = \frac{KY_0}{Y_0 + (K - Y_0)e^{-Krt}}$$

Logistic Growth (Flow ~)

K=1



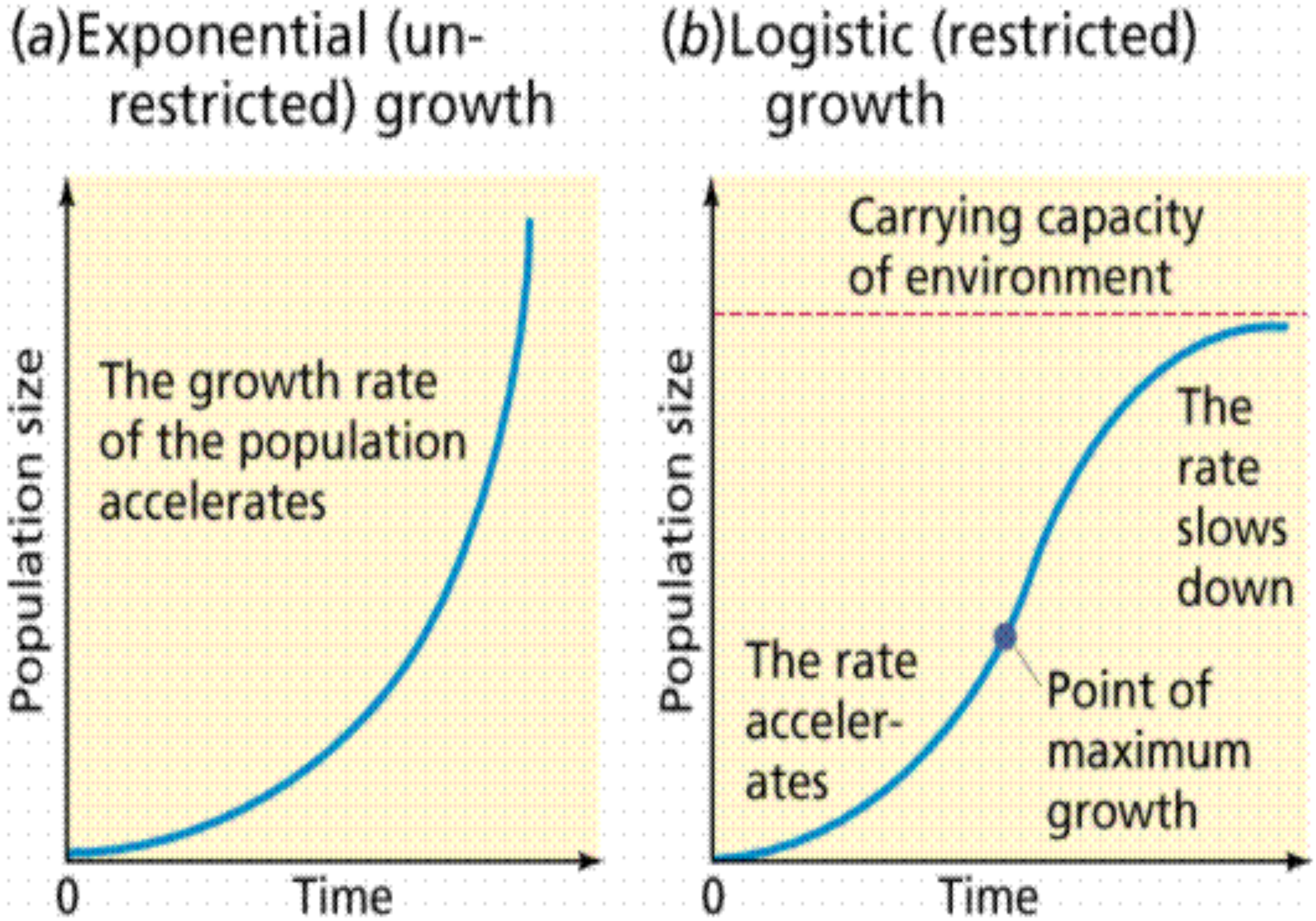
Real world examples:

- Cell splitting

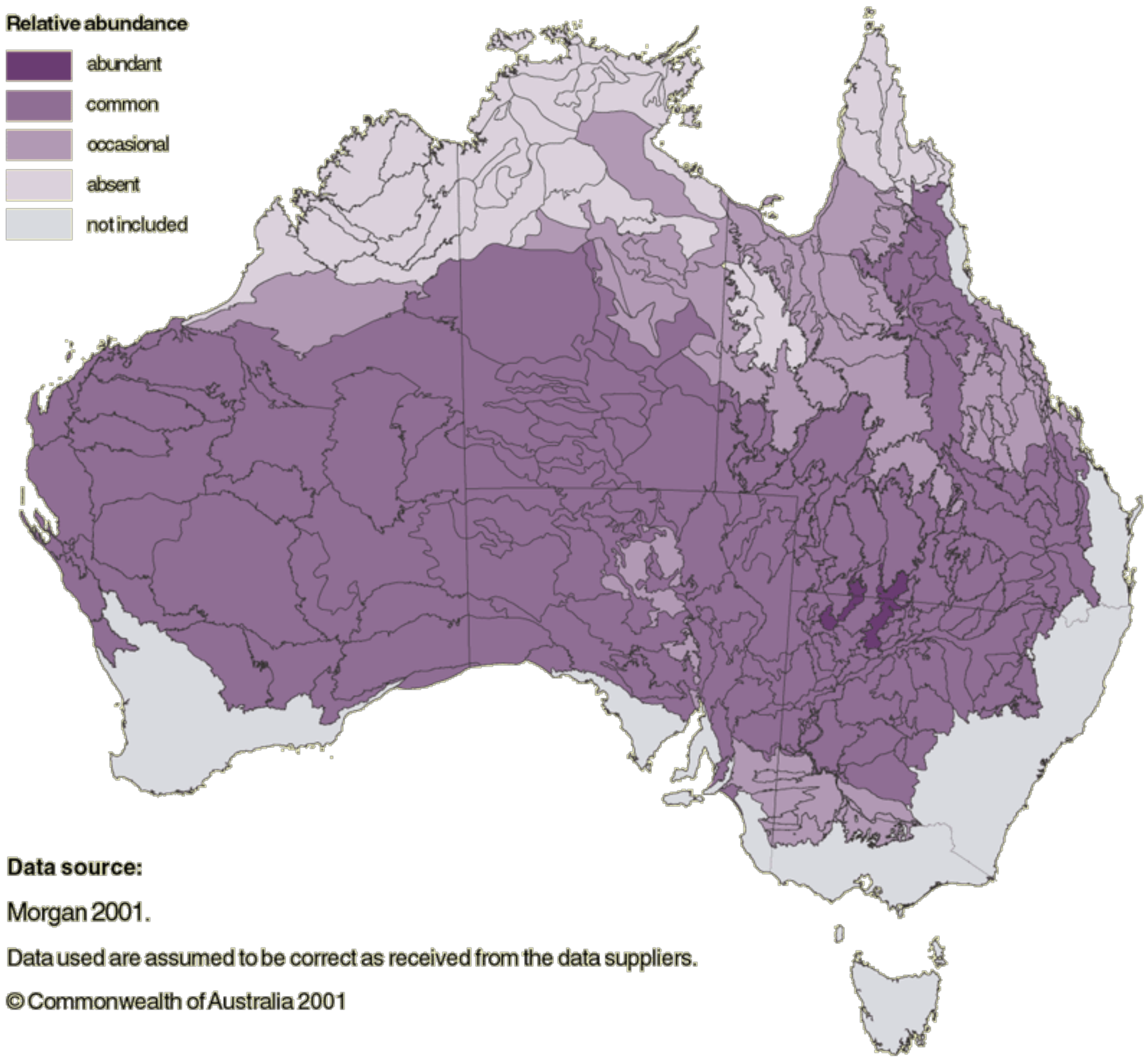
- Population growth

... anything that grows....

Logistic Growth (Flow ~)



Logistic Growth (Flow ~)



Data source:

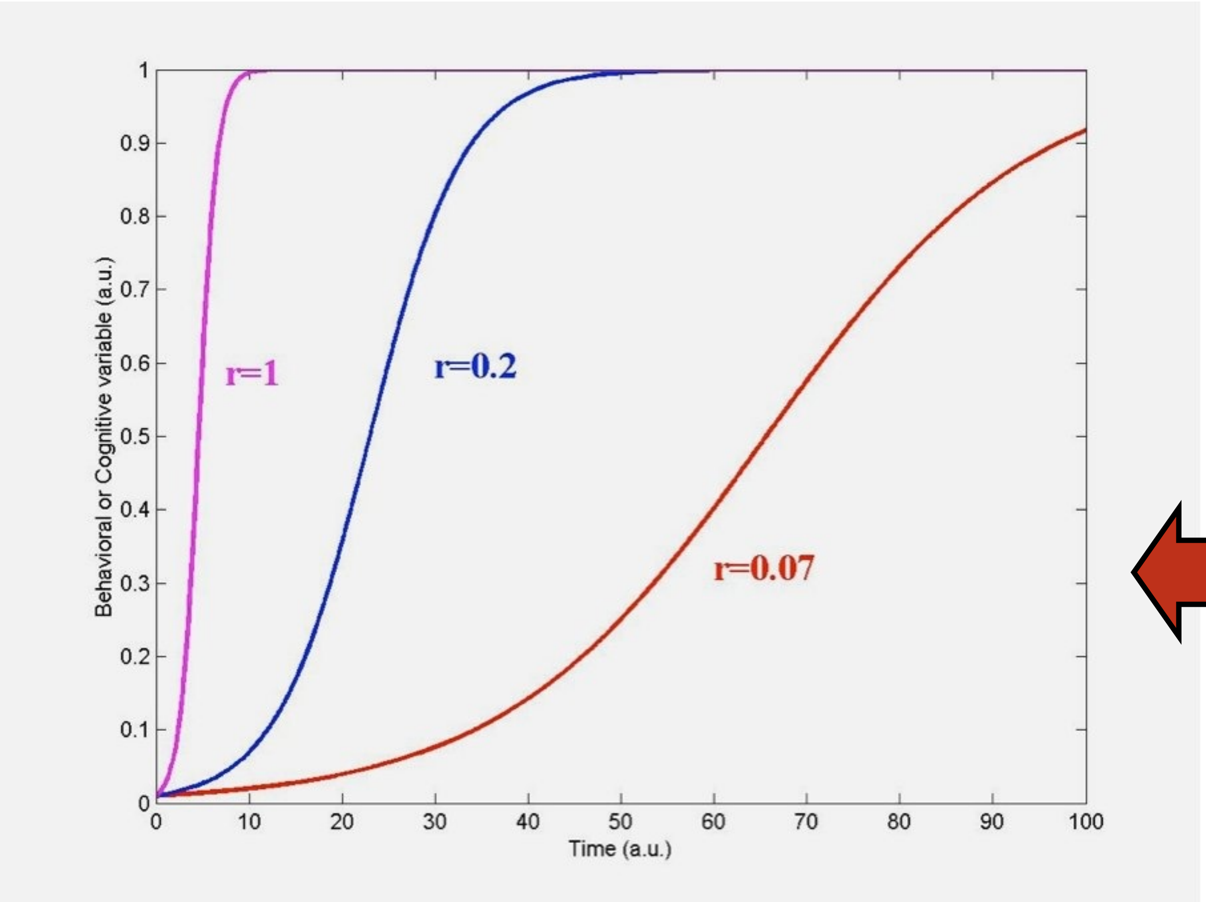
Morgan 2001.

Data used are assumed to be correct as received from the data suppliers.

Exponential population growth:
Start with 16 rabbits and
no restriction on growth
(or an extremely high
carrying capacity)

EXTRA: Working with known solutions of dynamic systems

If you have real data which you think can be described by a known system which has a solution, proceeding is straightforward -> fit the solution to the data to get the parameters...



$$\frac{dL}{dt} = rL(K - L)$$

$$L(t) = \frac{KL_0}{L_0 + (K - L_0)e^{-Krt}}$$

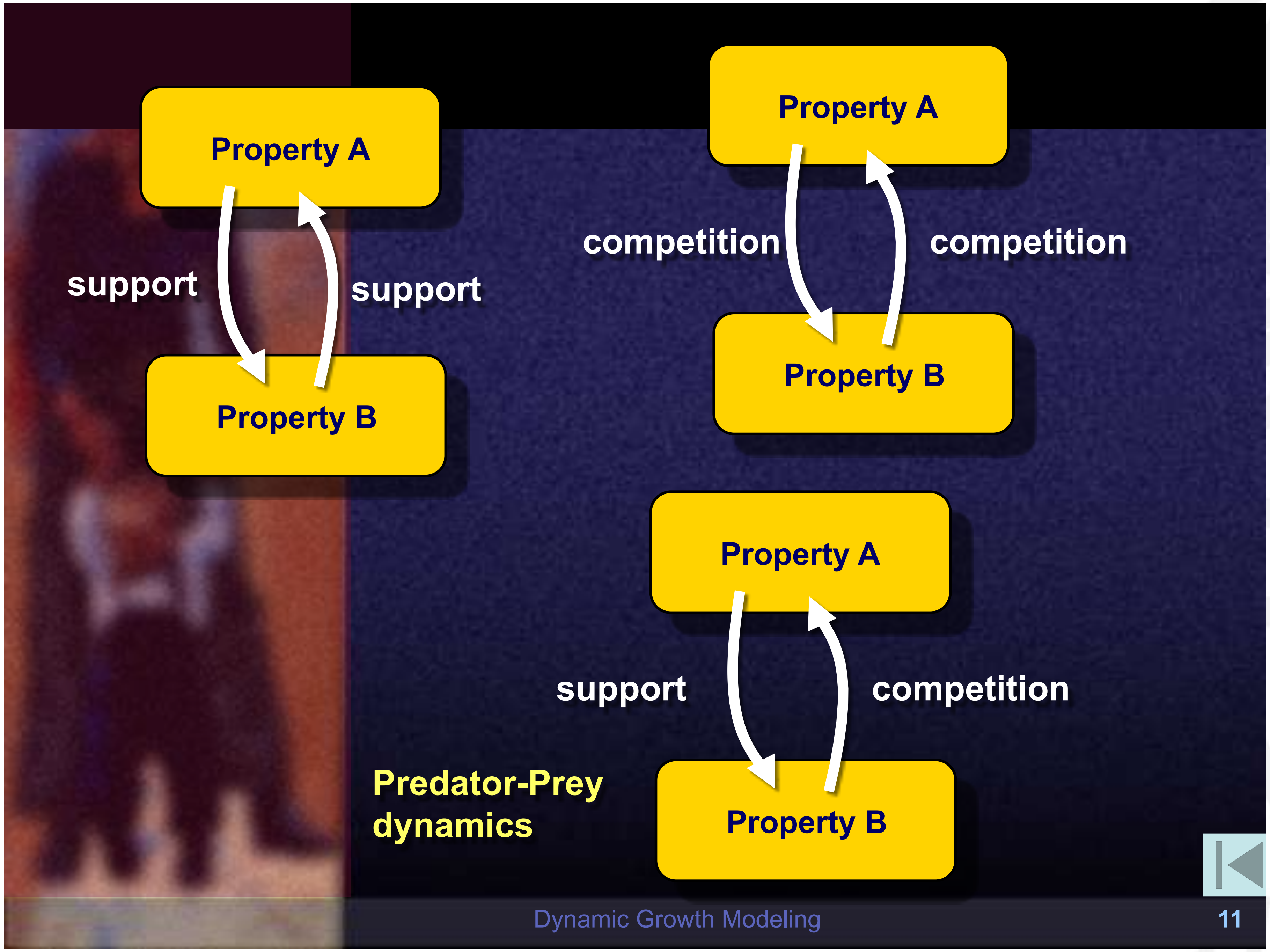
Dynamics of Complex Systems

A Closer look at modelling growth
Multivariate Models
Simulation of continuous time

f.hasselman@bsi.ru.nl

Behavioural Science Institute
Radboud University Nijmegen

Simple Interaction dynamics



Multivariate Models... Multivariate State Space

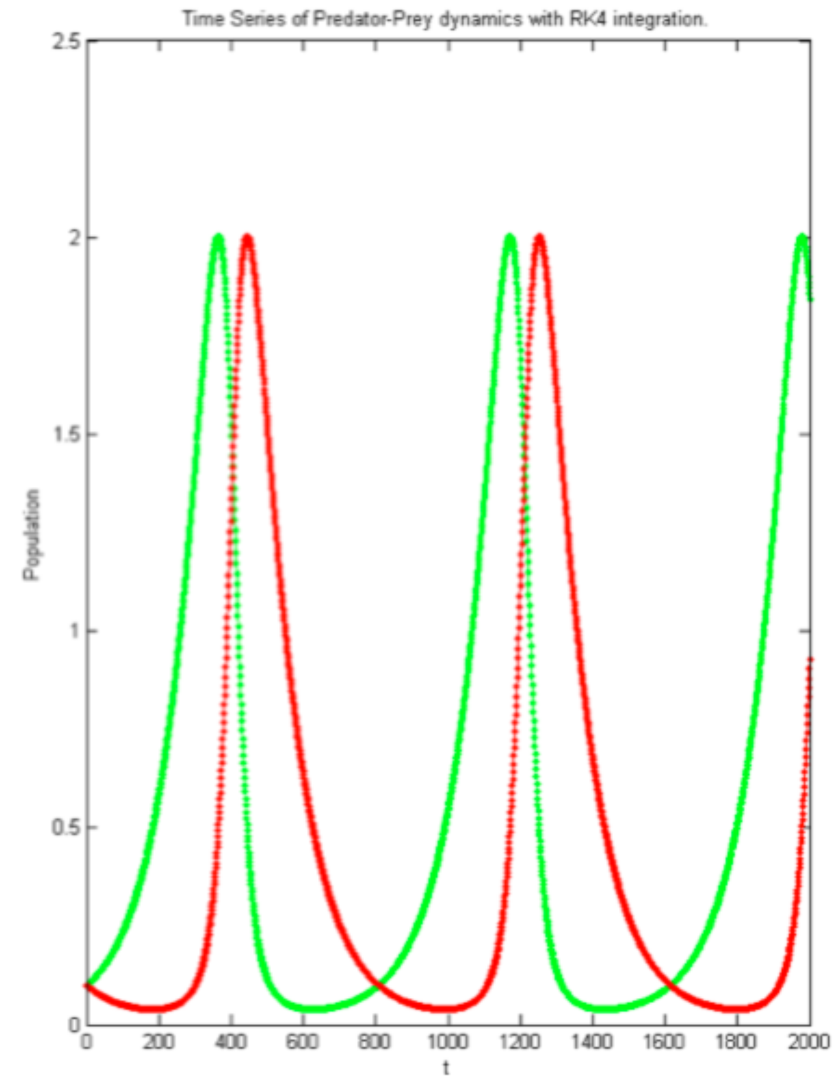
Predator-Prey model (Lotka-Volterra)

$$\frac{dR}{dt} = (a - b \times F) \times R,$$
$$\frac{dF}{dt} = (c \times R - d) \times F.$$

A 2-D state space
2 coupled flows ~

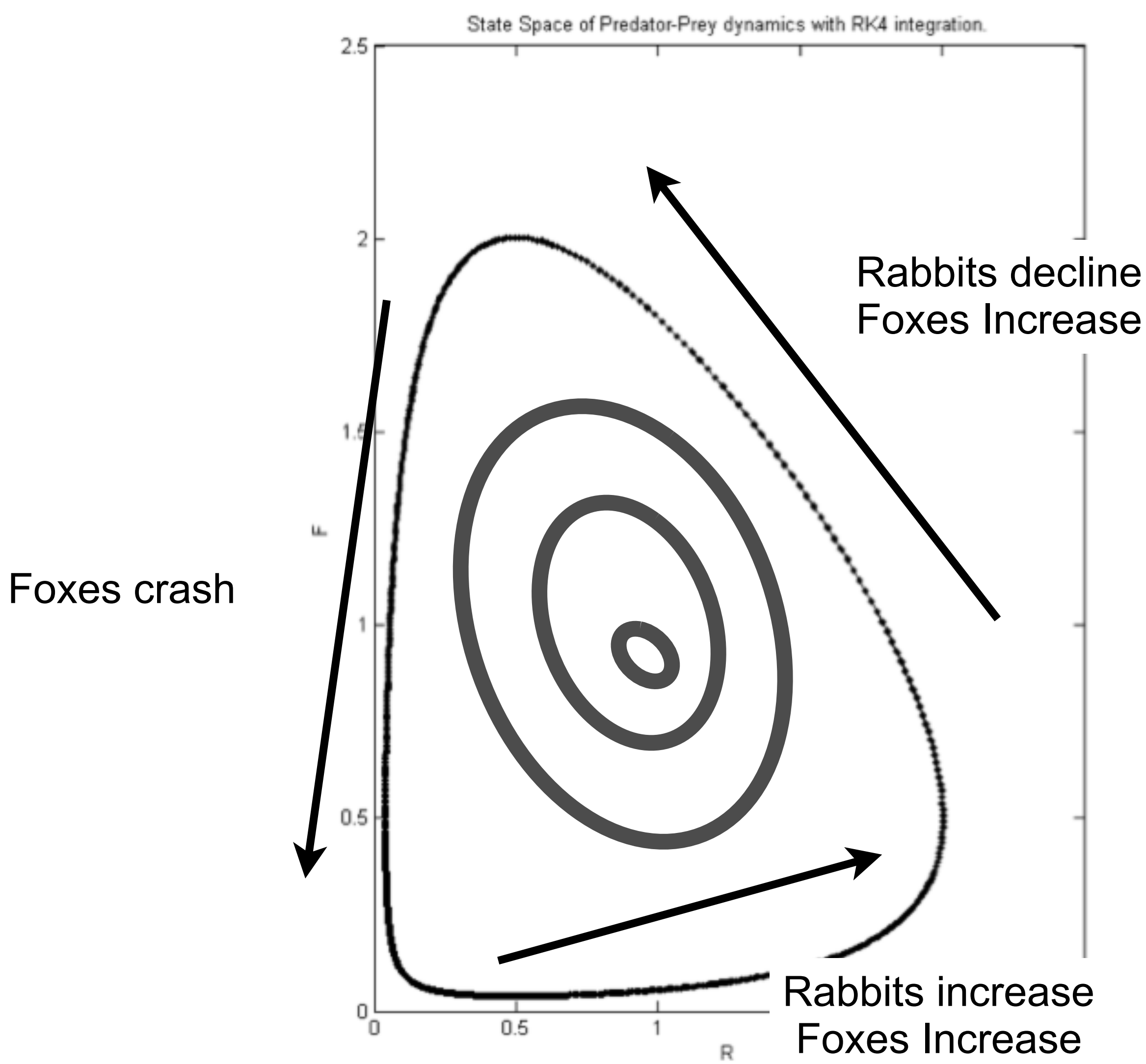
- *R* is the ***number of rabbits*** in a year
- *F* is the ***number of foxes*** in a year

Multivariate Models... Multivariate State Space

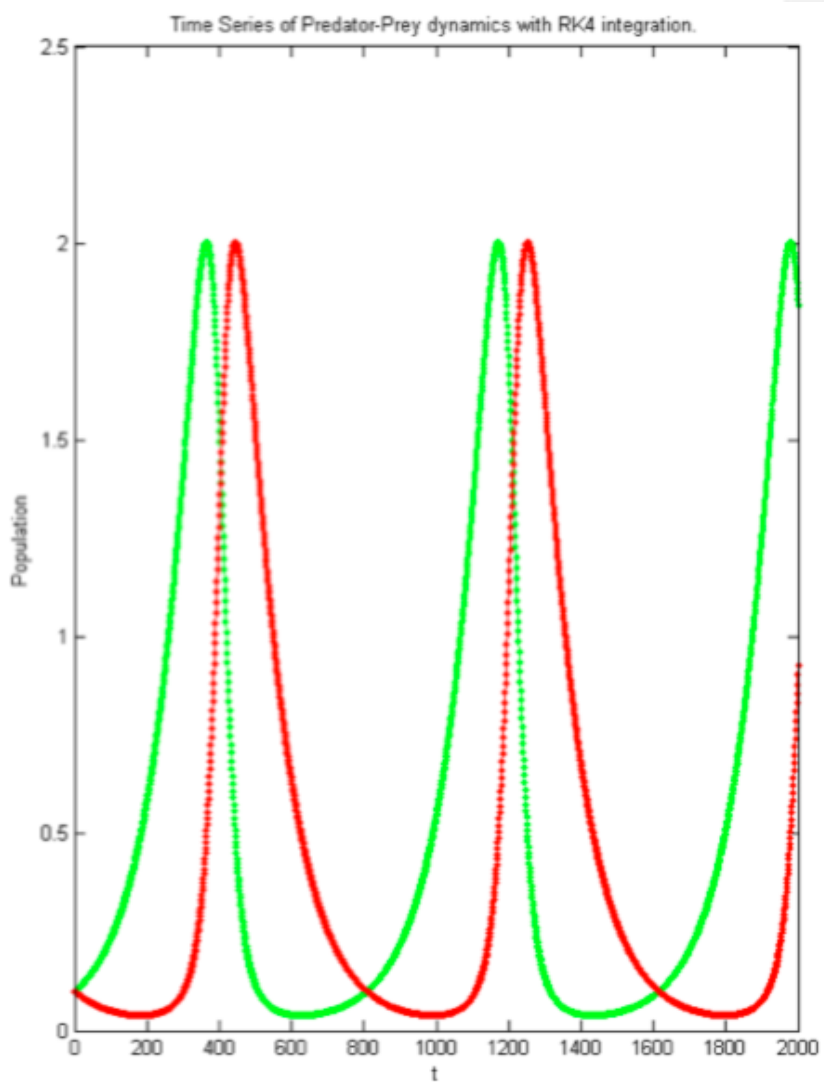


Time Series

Multivariate Models... Multivariate State Space



State Space



Time Series

Lorenz System

$$\frac{dx}{dt} = a(y - x),$$

$$\frac{dy}{dt} = x(b - z) - y,$$

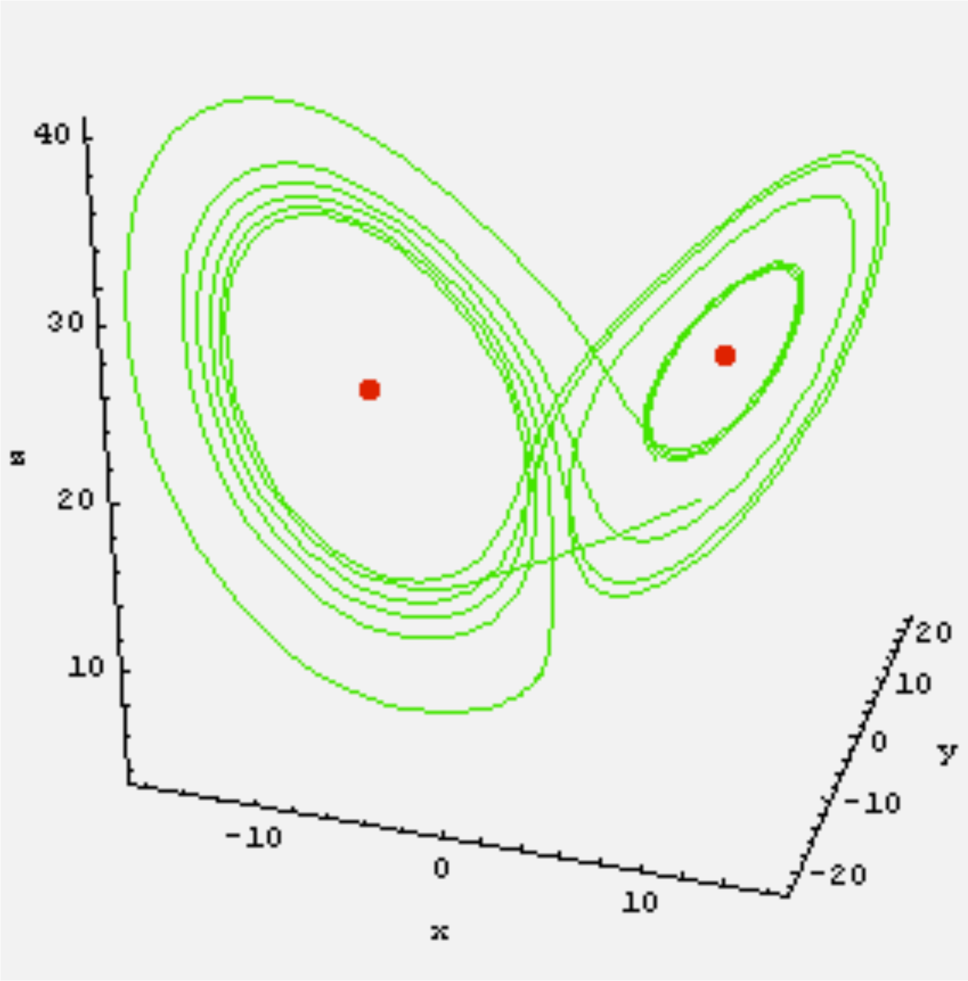
$$\frac{dz}{dt} = xy - cz.$$

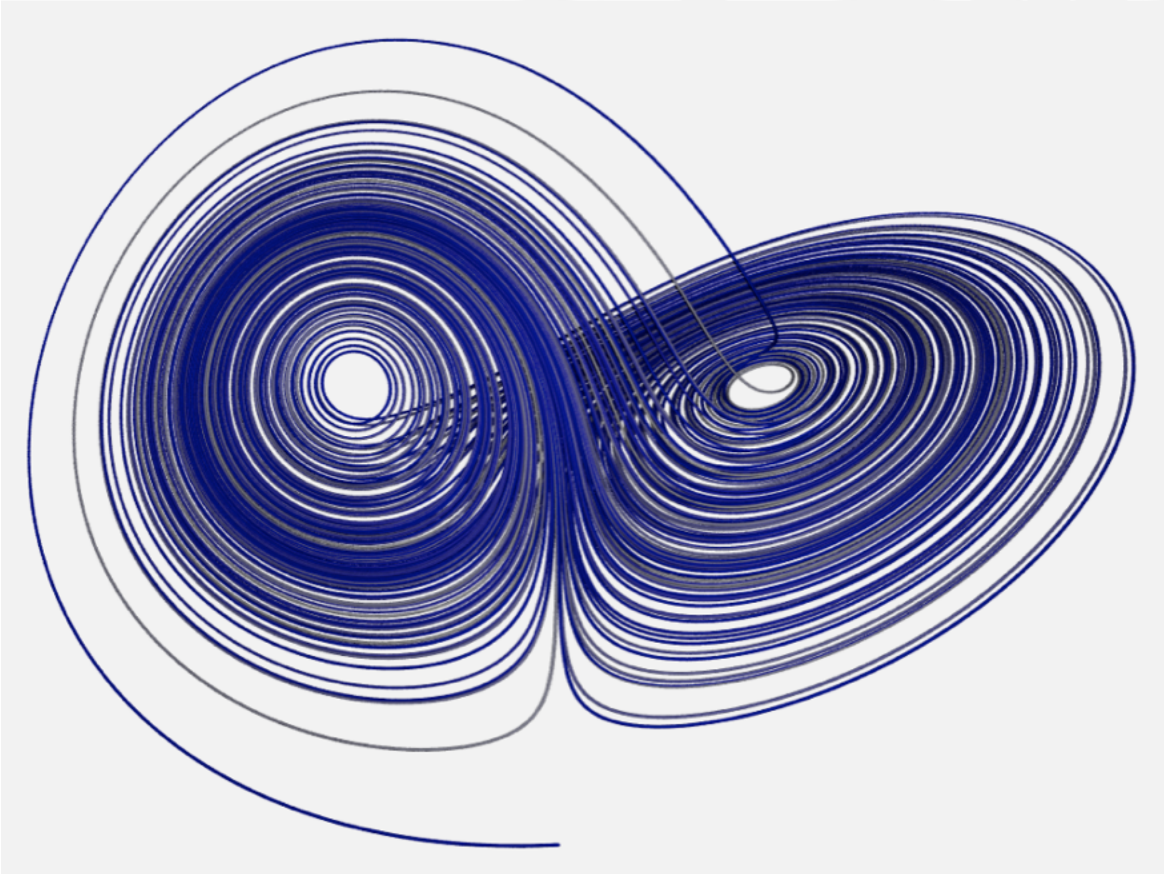
Interaction dominant dynamics
Multiple processes (3)
Multiple Scales (time)

x depends on y and x
y depends on x, y and z
z depends on x, y and z

A 3-D state space
3 coupled flows ~

Lorenz Attractor





A note on simulating differential equations: ~flows ~

Differential equations are **continuous**…

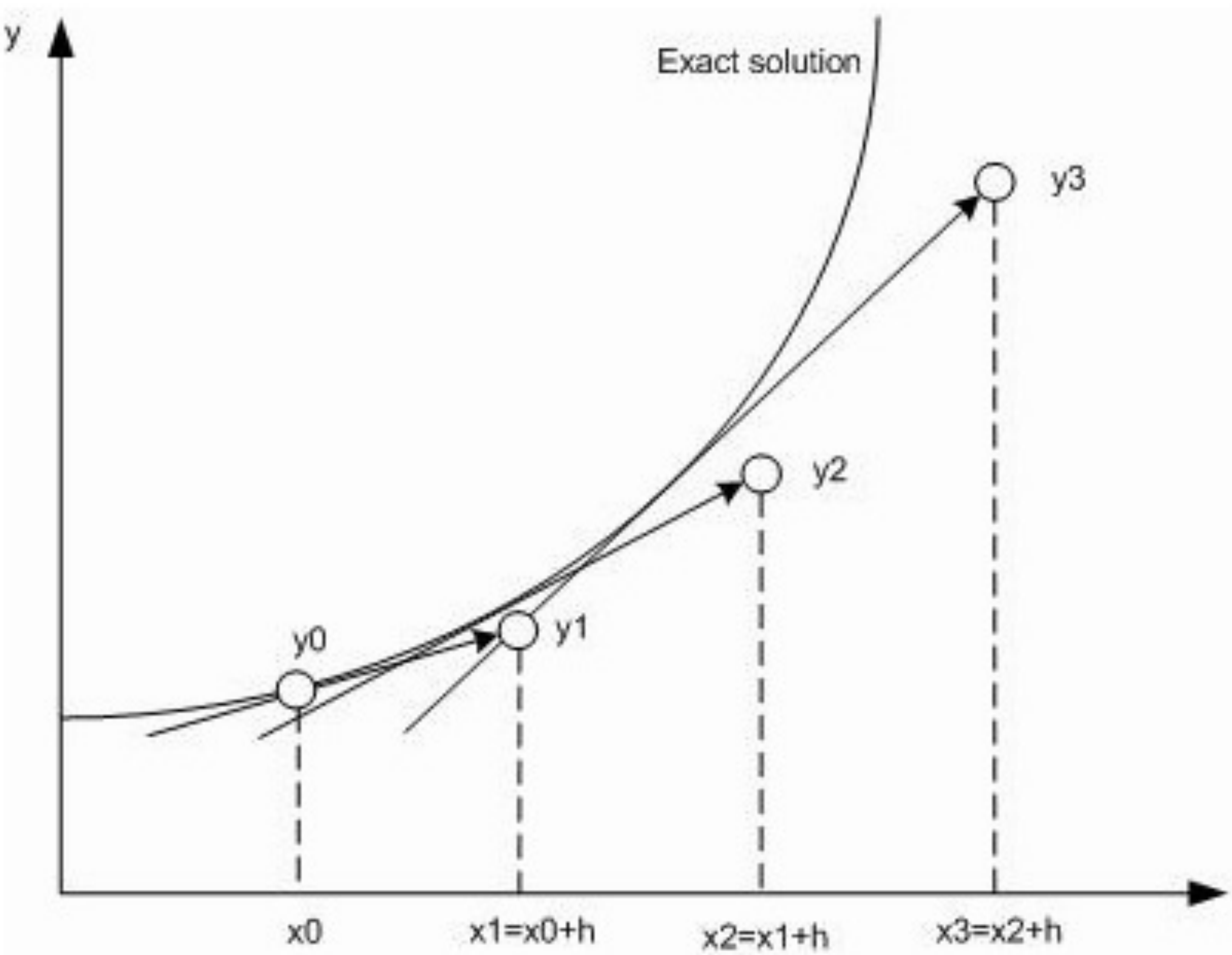
To find out how they behave when there is no solution we need to ‘discretize’ them and approximate the solution with a difference equation: Numerical integration

The easiest (but most error prone) method is Euler’s method (18th century):

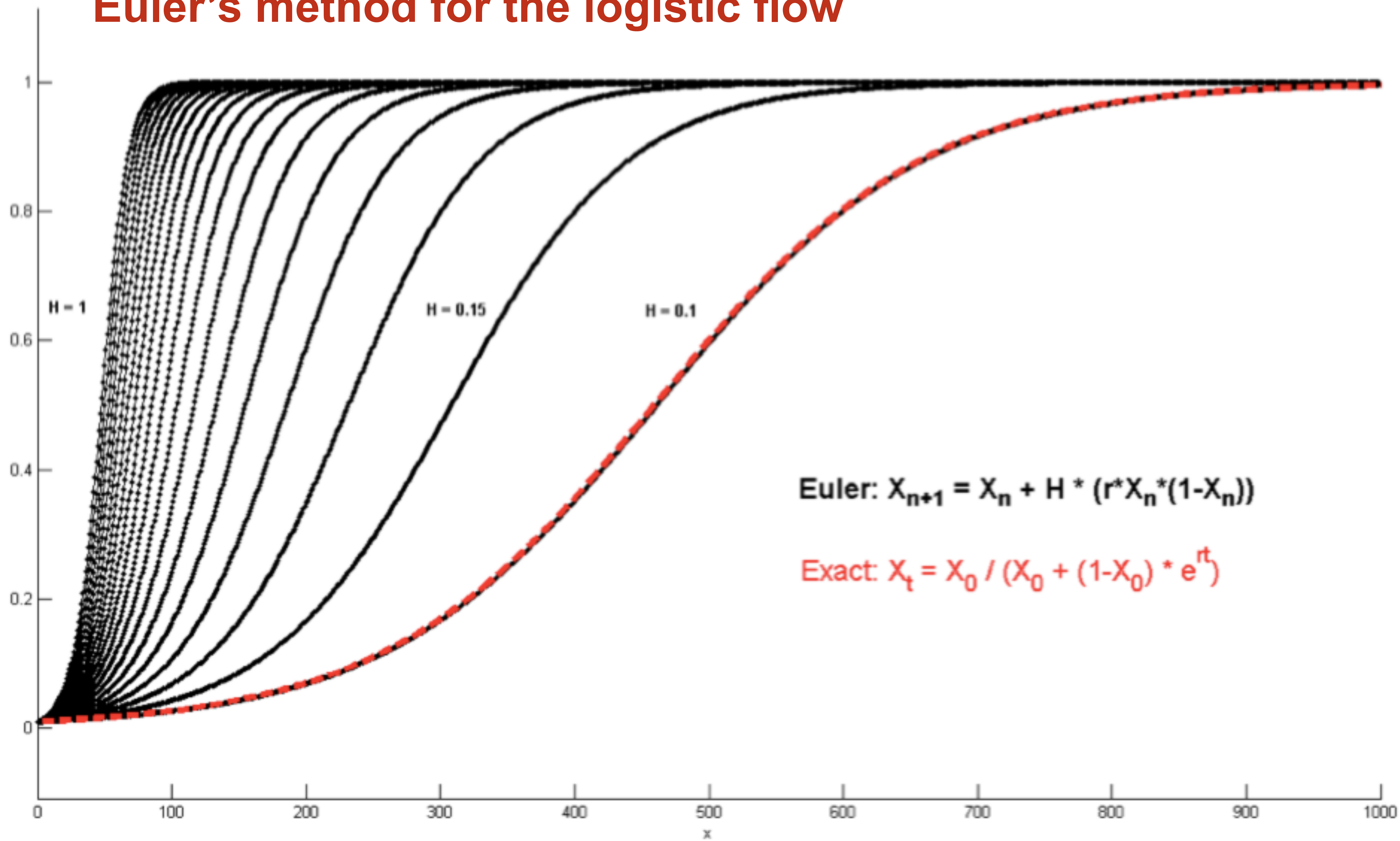
$\mathbf{X_{n+1} = X_{n+H} * f(X_n)}$ where **H** = step length

Checking how well the approximation is, can easily be done if we know an analytic algebraic solution

A note on simulating differential equations: ~flows ~ Euler's method



Euler's method for the logistic flow



Runge-Kutta 4th Order Method

$$\mathbf{k}_1 = h \cdot f(\mathbf{y}_n)$$

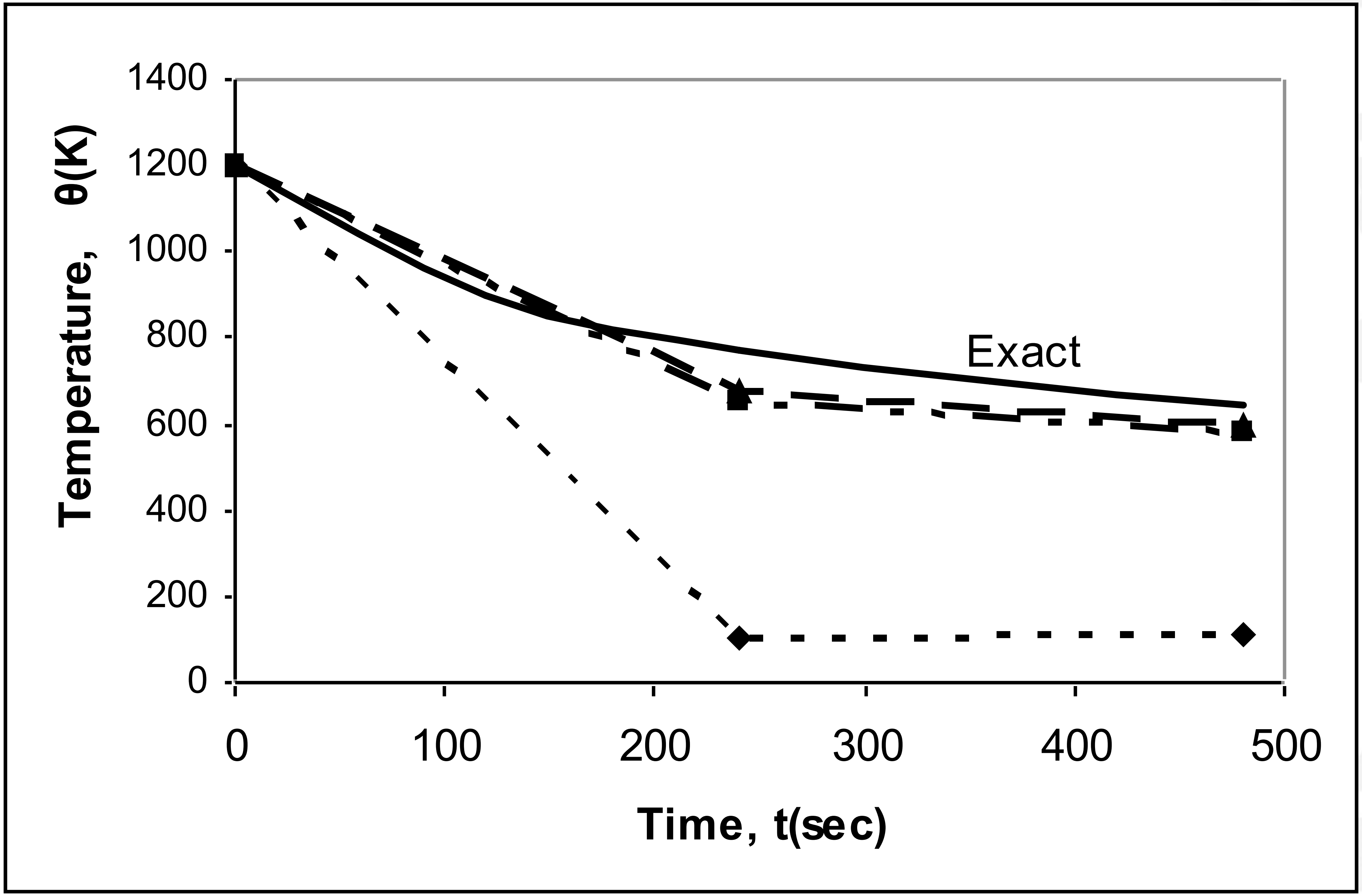
$$\mathbf{k}_2 = h \cdot f\left(\mathbf{y}_n + \frac{\mathbf{k}_1}{2}\right)$$

$$\mathbf{k}_3 = h \cdot f\left(\mathbf{y}_n + \frac{\mathbf{k}_2}{2}\right)$$

$$\mathbf{k}_4 = h \cdot f(\mathbf{y}_n + \mathbf{k}_3)$$

$$\Rightarrow \mathbf{y}_{n+1} = \mathbf{y}_n + \frac{\mathbf{k}_1}{6} + \frac{\mathbf{k}_2}{3} + \frac{\mathbf{k}_3}{3} + \frac{\mathbf{k}_4}{6}$$

Comparison of methods



Dynamics of Complex Systems

EXTRA:

Cobweb method
Working with analytic solutions

f.hasselman@bsi.ru.nl

Behavioural Science Institute
Radboud University Nijmegen

Cobweb method

Cobweb Method

An insightful and easy way to visualize the dynamics of a one-dimensional map.

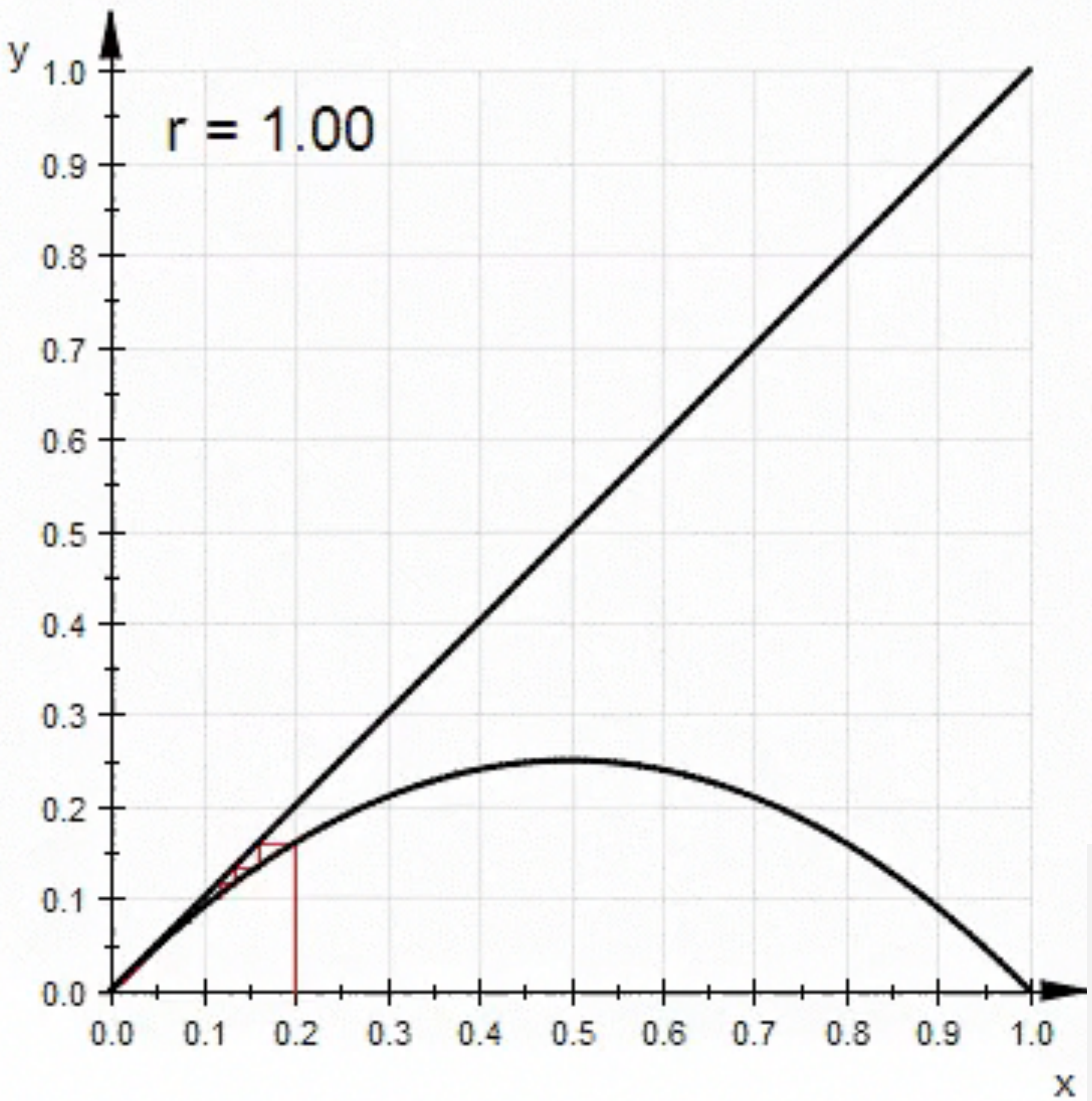
Useful when an algebraic solution can no longer be found.

Preparations (General):

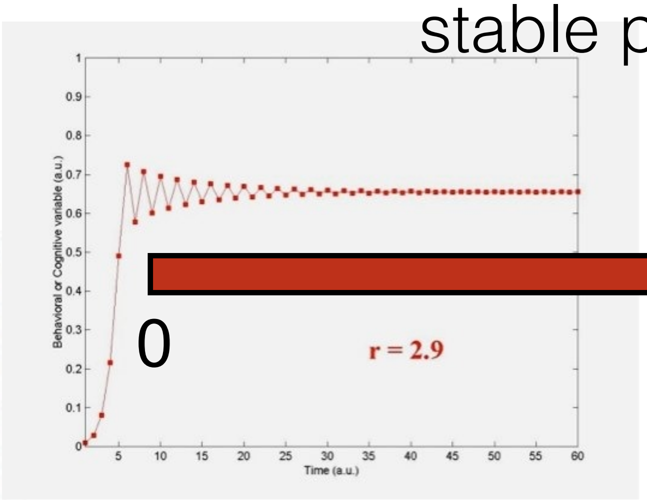
→ Draw the graph of $f(Y_i)$ (= Y_{i+1}) in the Y_{i+1} - Y_i (phase) plane;

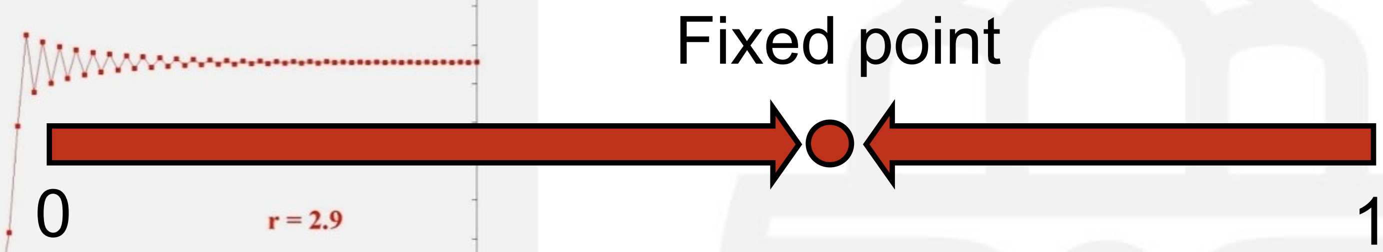
→ Also draw the line $Y_i = Y_{i+1}$ (45^o).

Cobweb method

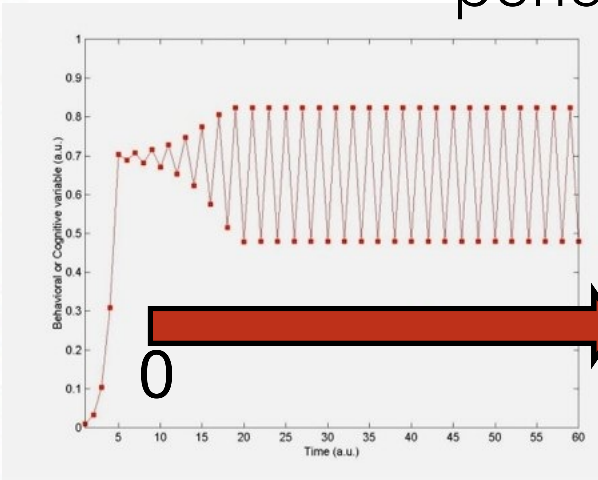


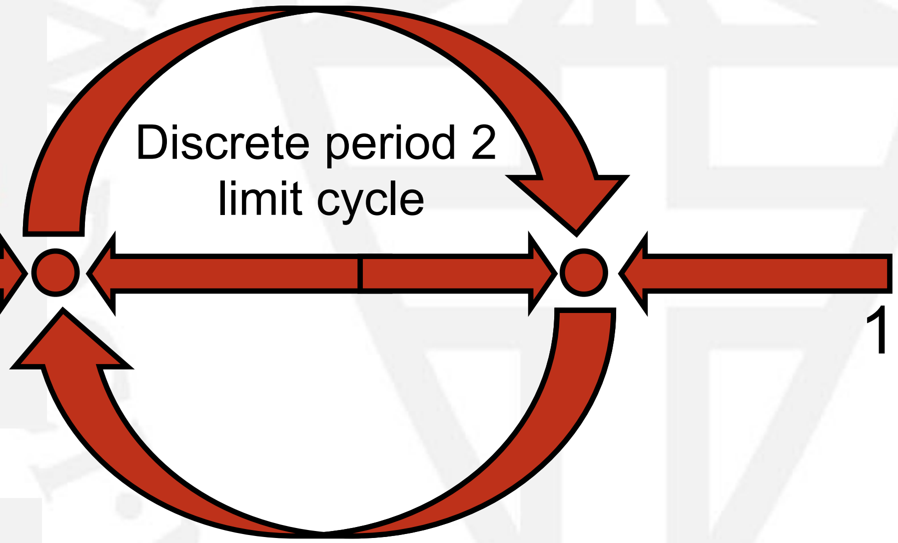
stable points (point attractors)

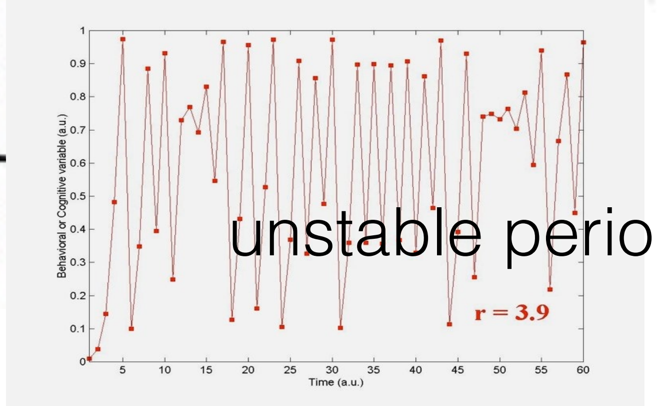




periodic orbits (limit cycles)





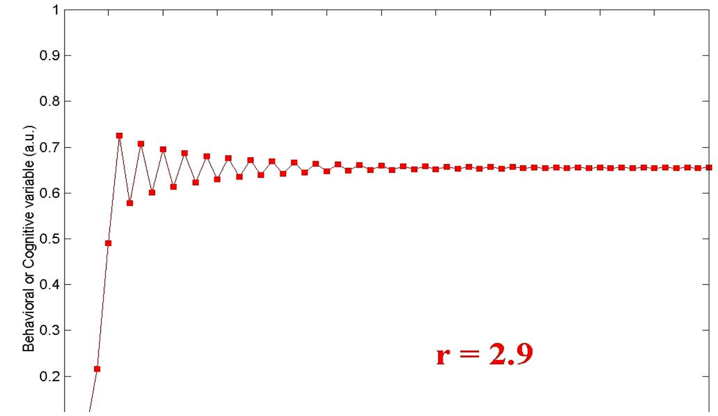


unstable periodic orbits (strange attractors)

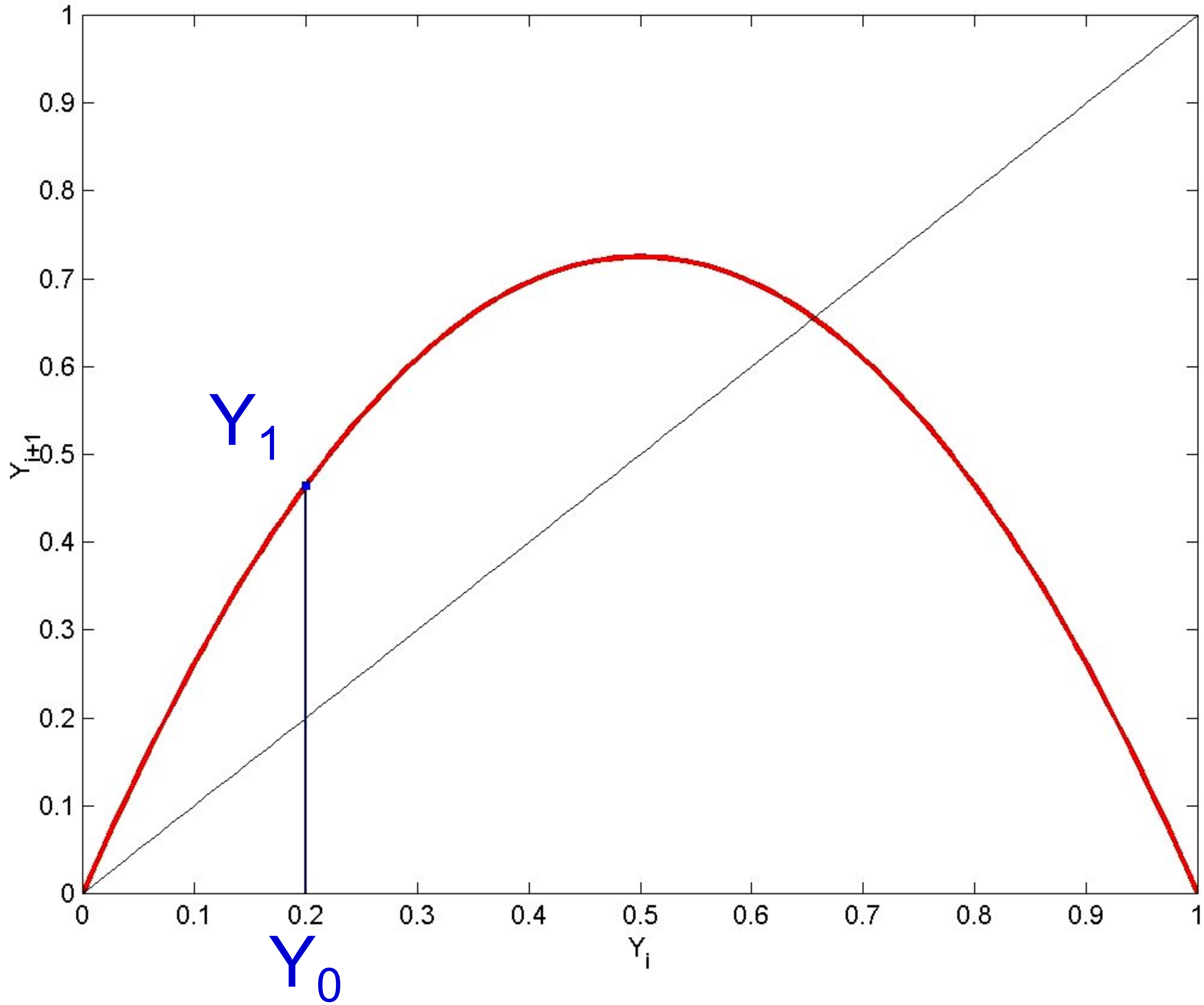
Cobweb Method

Start with Y_0 on horizontal axis:

Go vertical to f: Gives you Y_1;



r=2.9

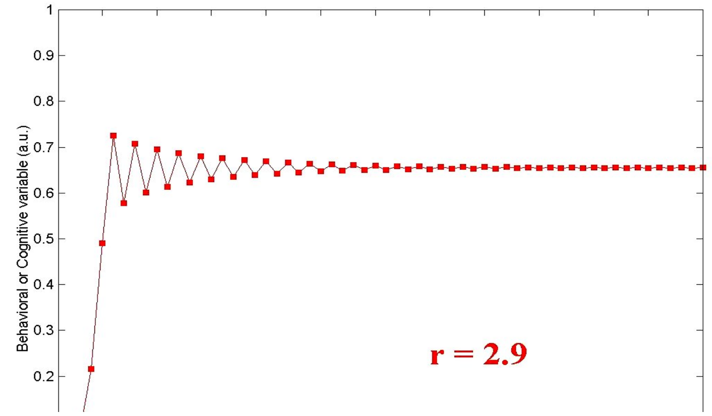


Cobweb Method

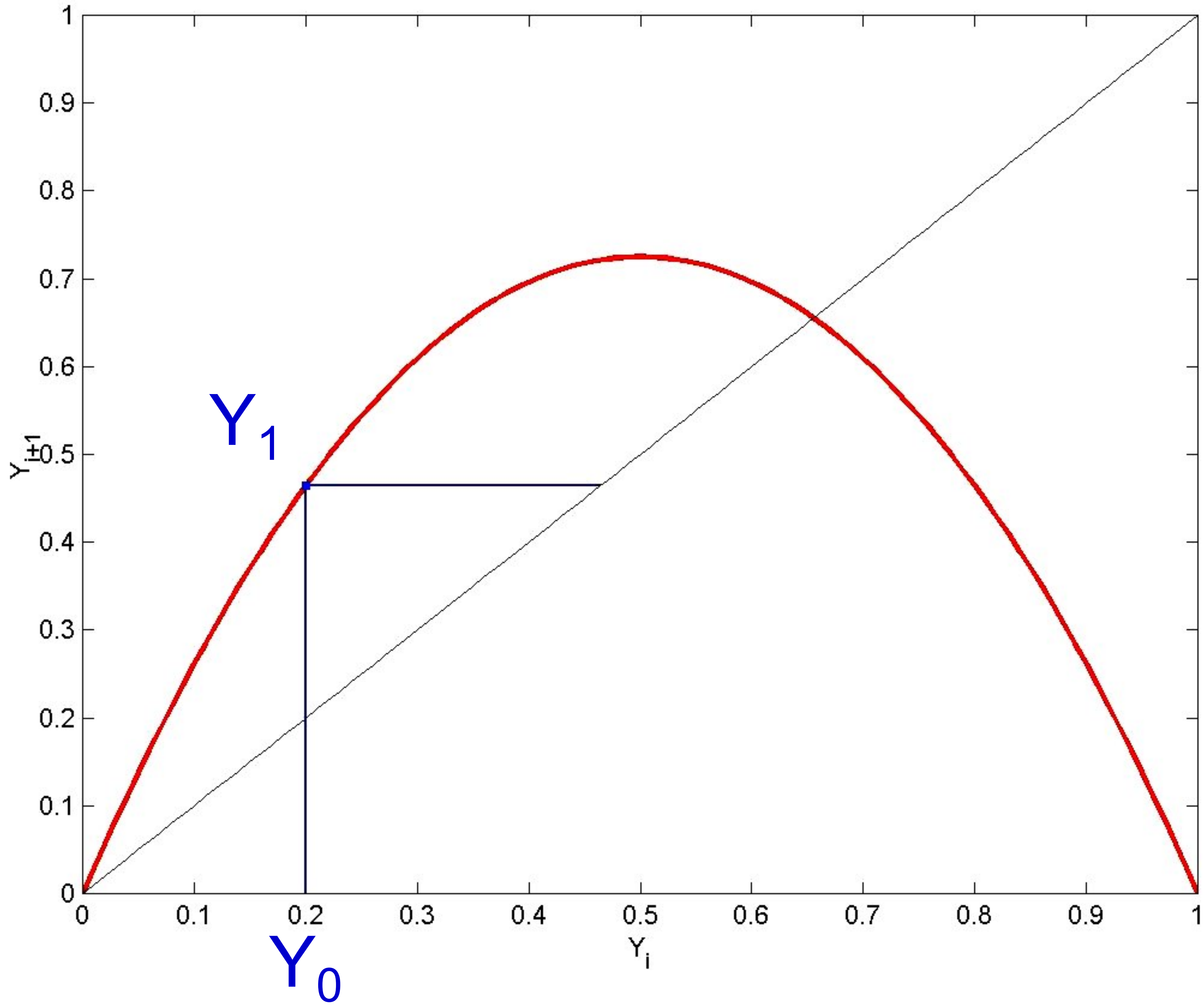
Start with Y_0 on horizontal axis:

Go vertical to *f*: Gives you Y_1;

→ Go horizontal to the 45^o line;



r=2.9



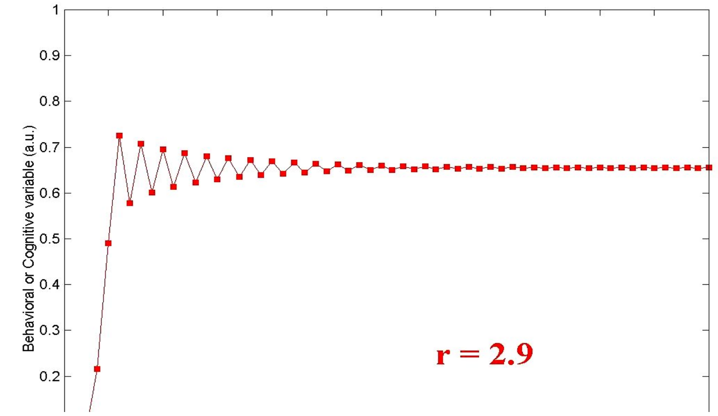
Cobweb Method

Start with Y_0 on horizontal axis:

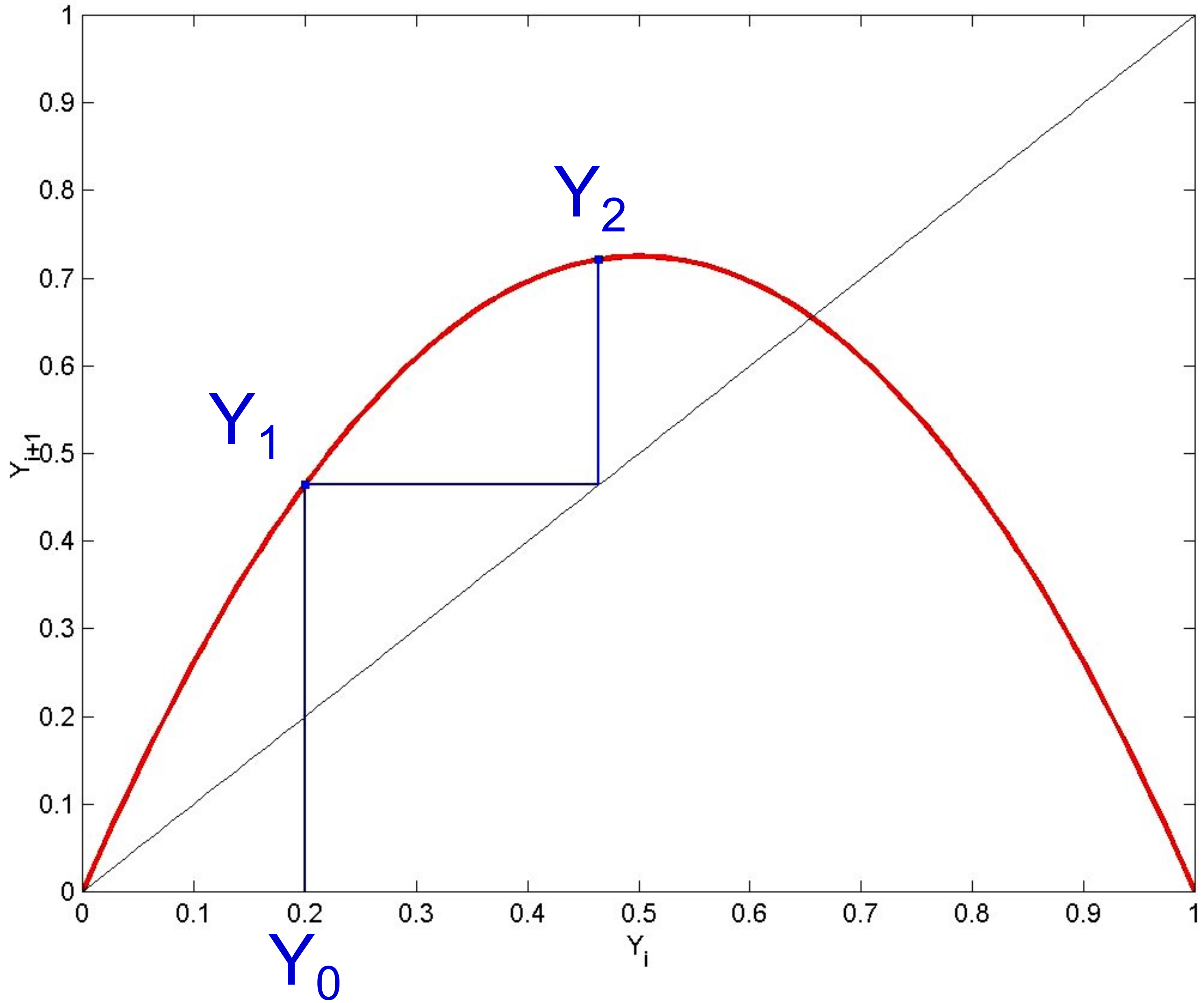
Go vertical to f: Gives you Y_1;

→ Go horizontal to the 45^o line;

→ Go vertical to f: Gives you Y_2;



r=2.9



Cobweb Method

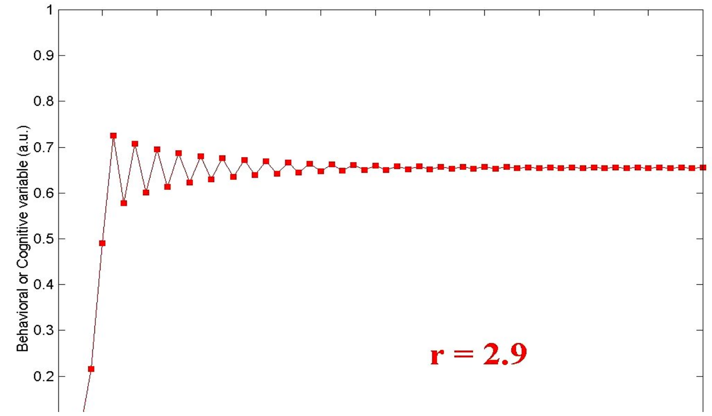
Start with Y_0 on horizontal axis:

Go vertical to *f*: Gives you Y_1;

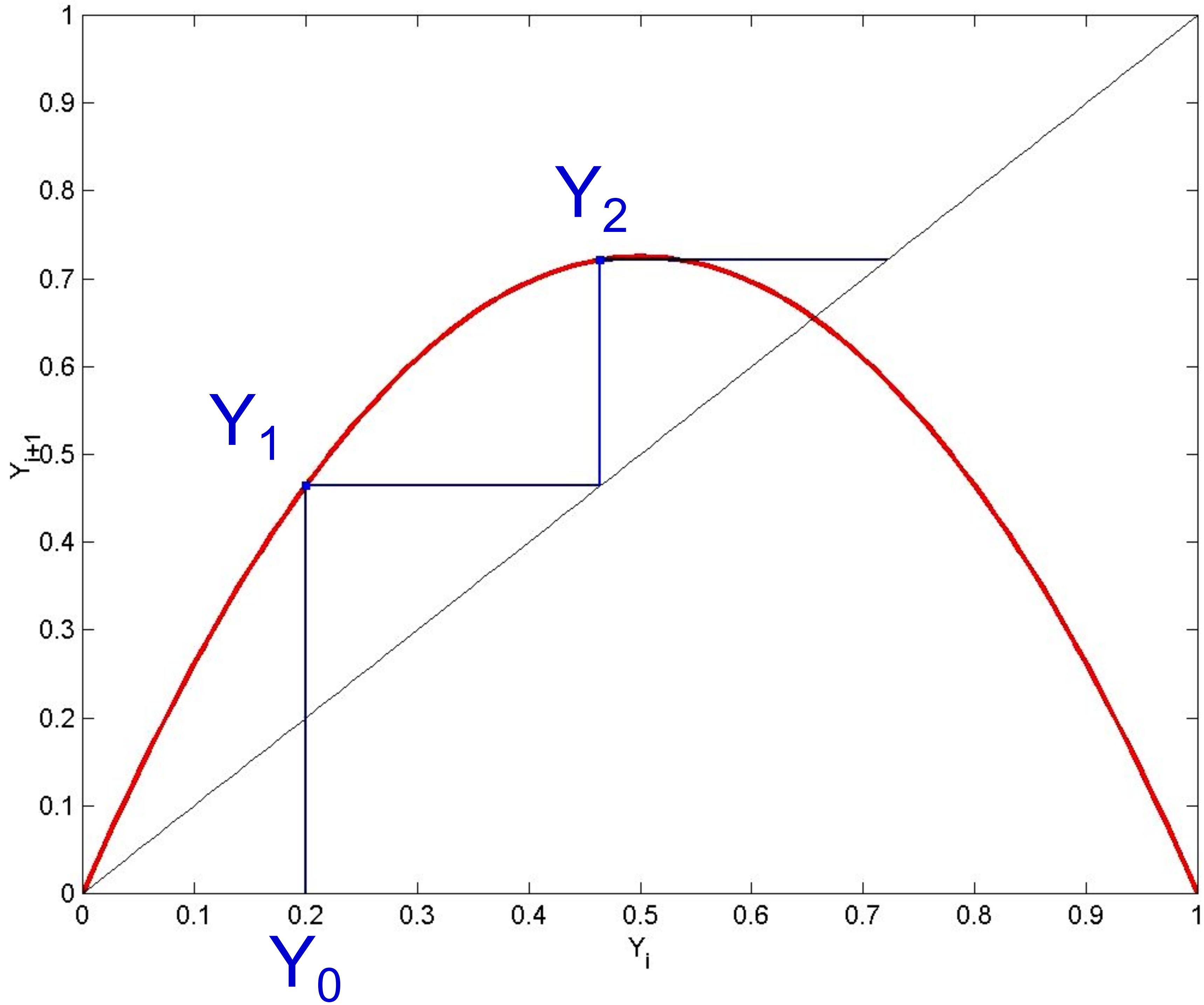
→ Go horizontal to the 45^o line;

→ Go vertical to *f*: Gives you Y_2;

→ Go horizontal to the 45^o line;



r=2.9



Cobweb Method

Start with Y_0 on horizontal axis:

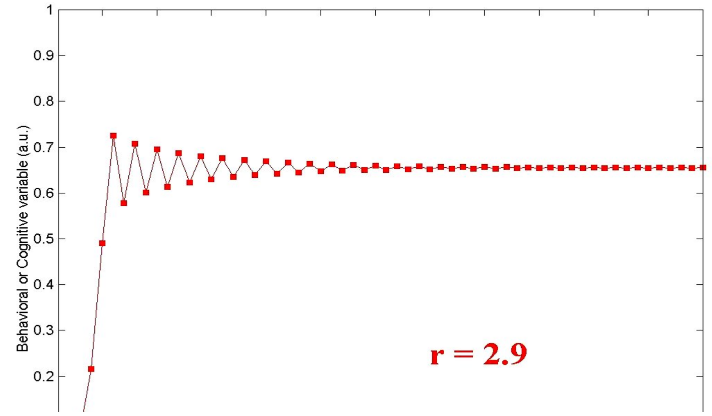
Go vertical to *f*: Gives you Y_1;

→ Go horizontal to the 45^o line;

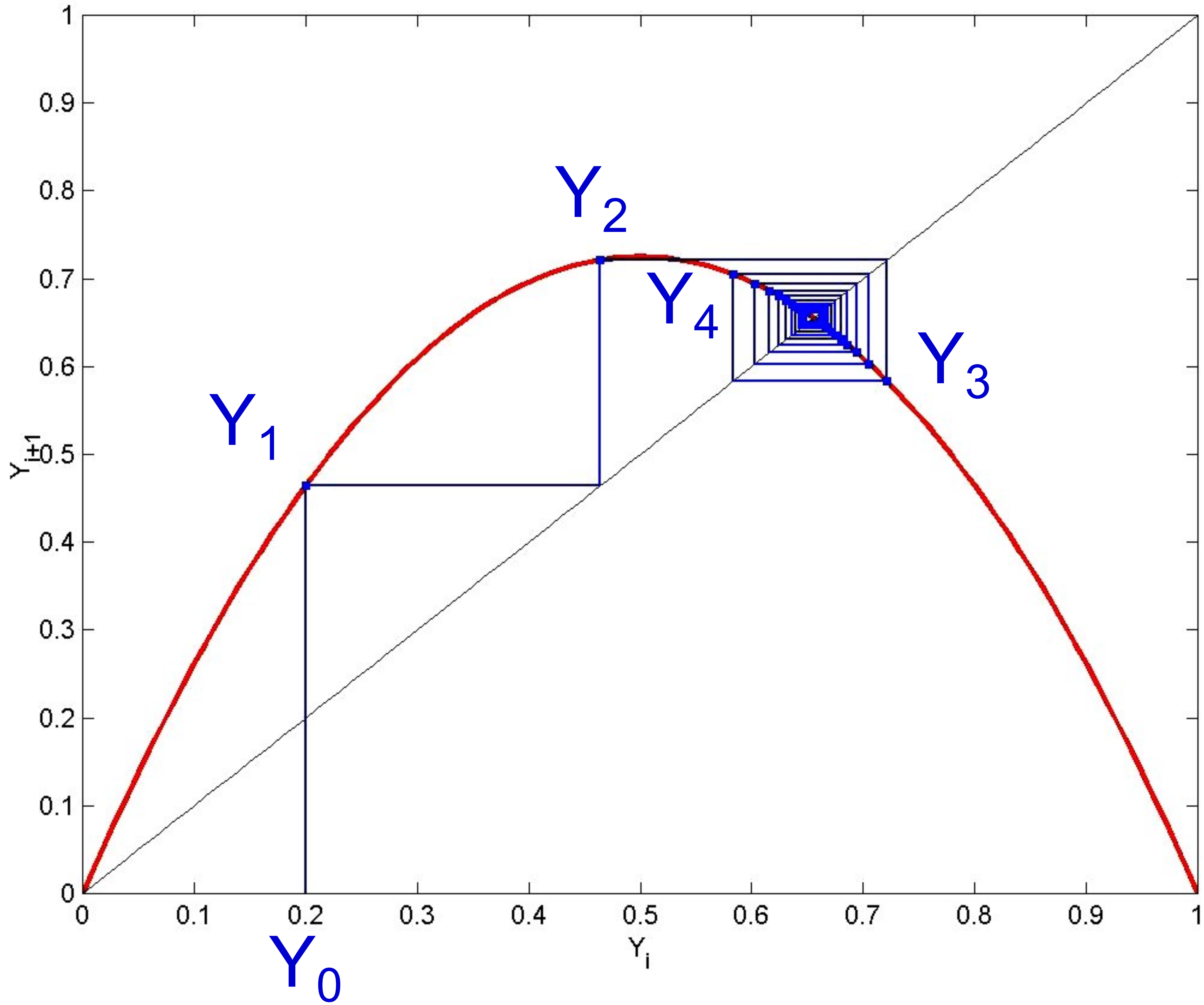
→ Go vertical to *f*: Gives you Y_2;

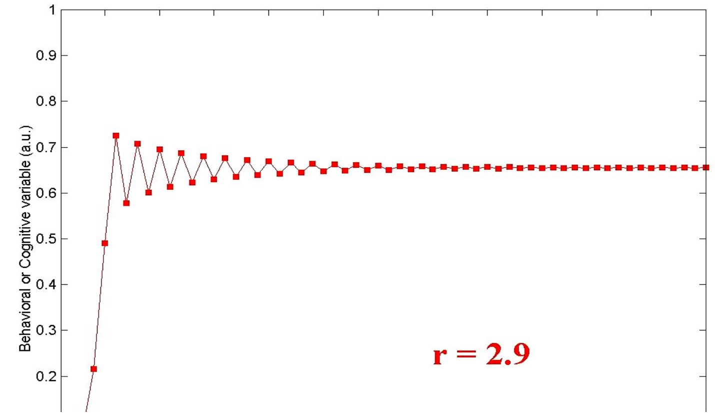
→ Go horizontal to the 45^o line;

→ Etc. etc.

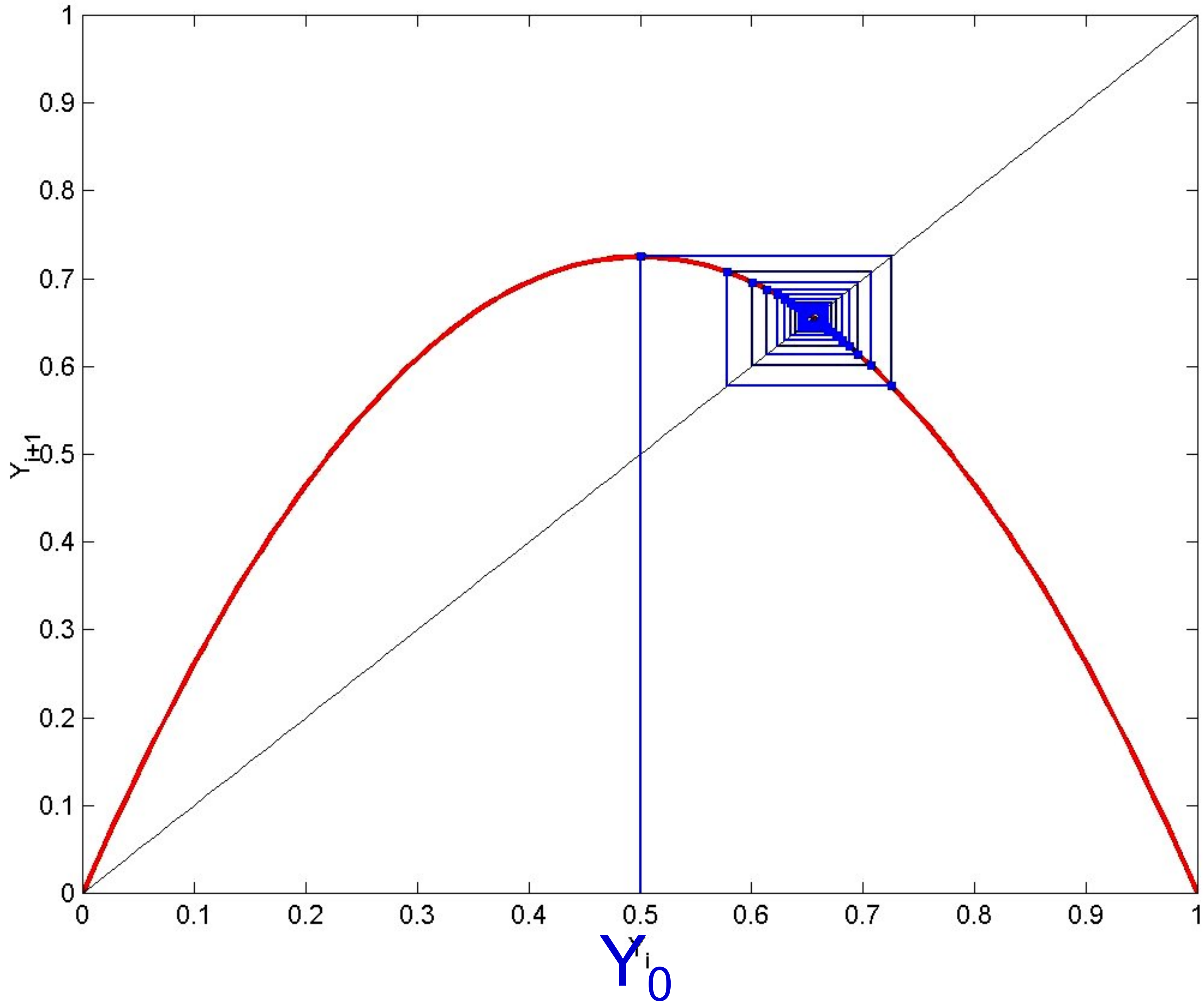


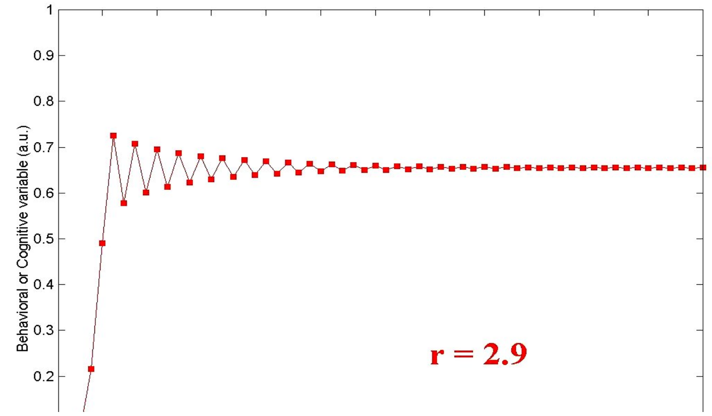
r=2.9



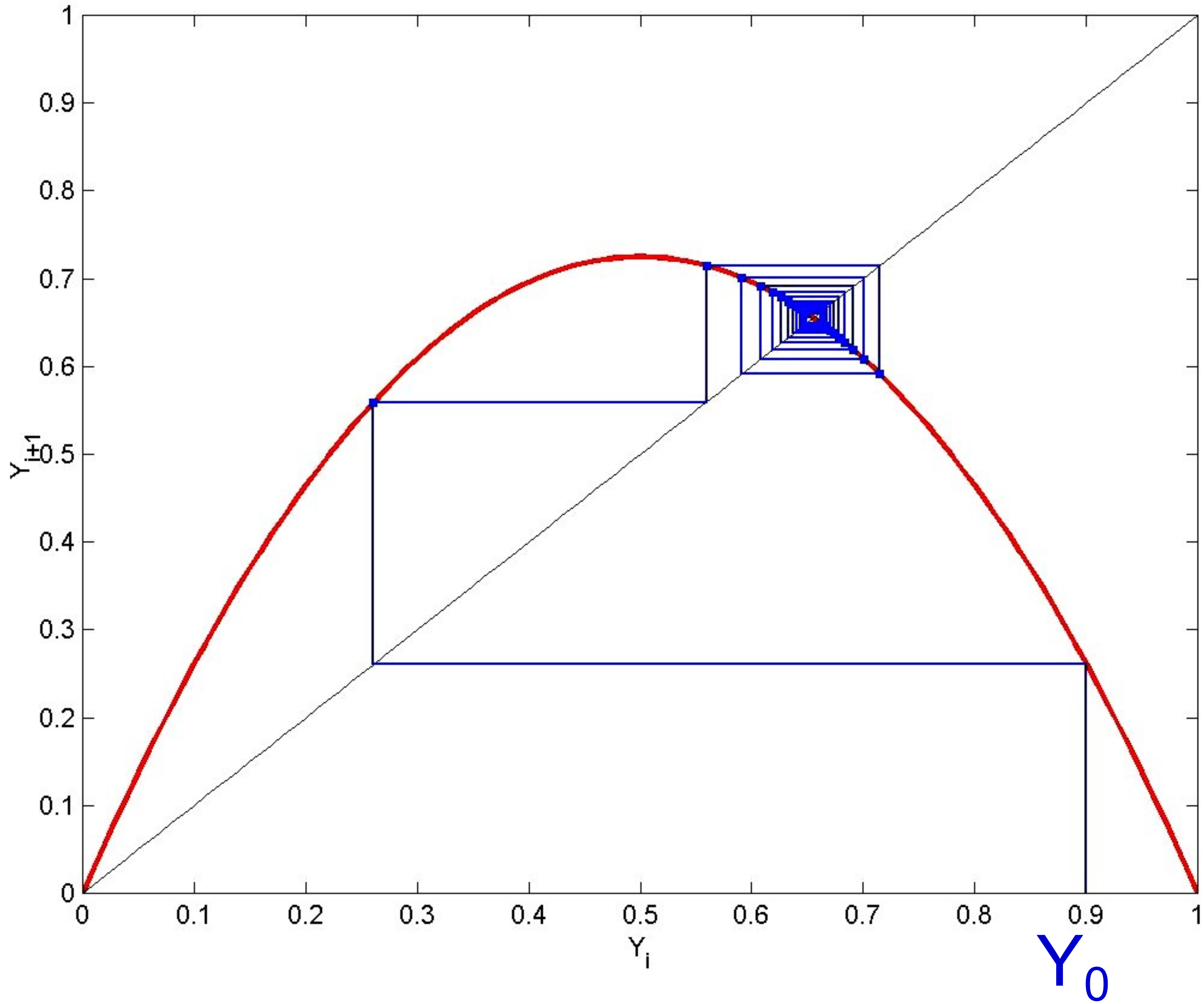


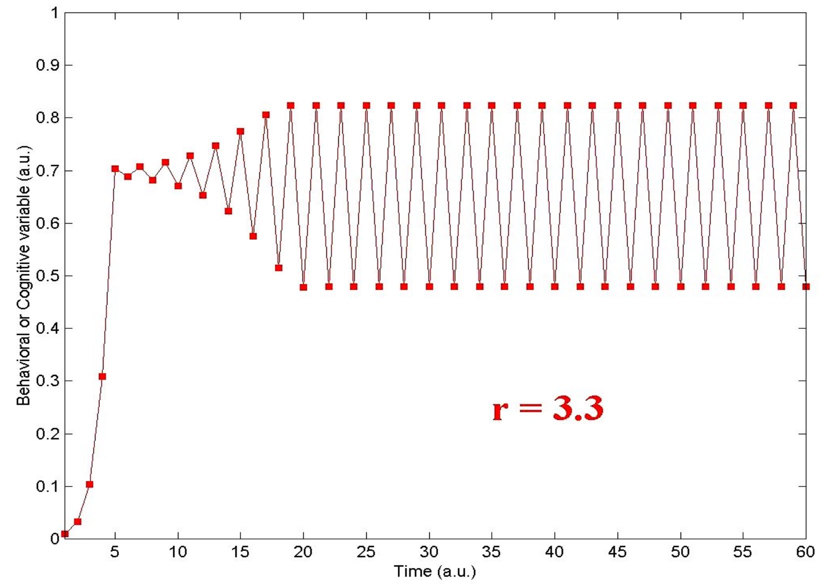
r=2.9



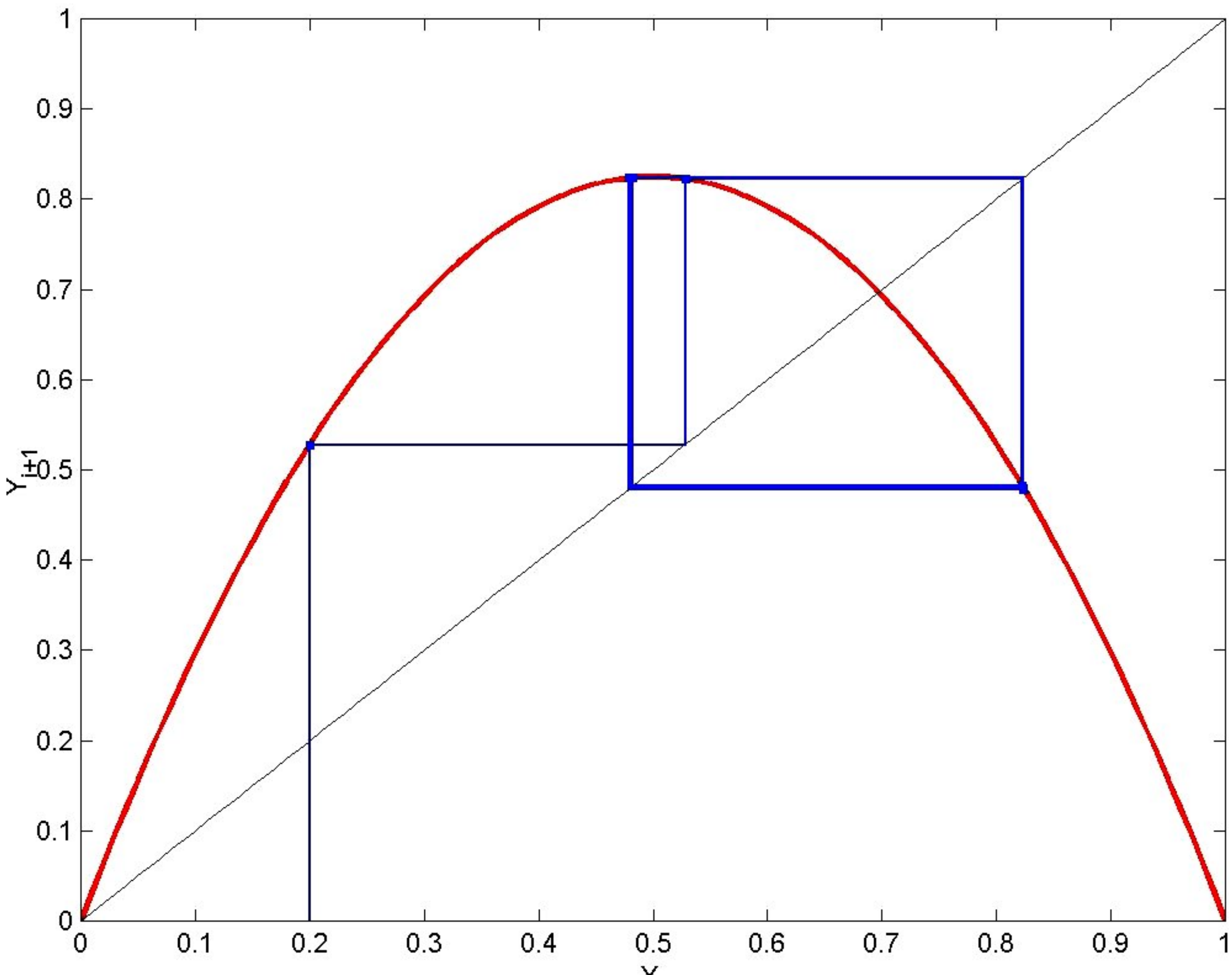


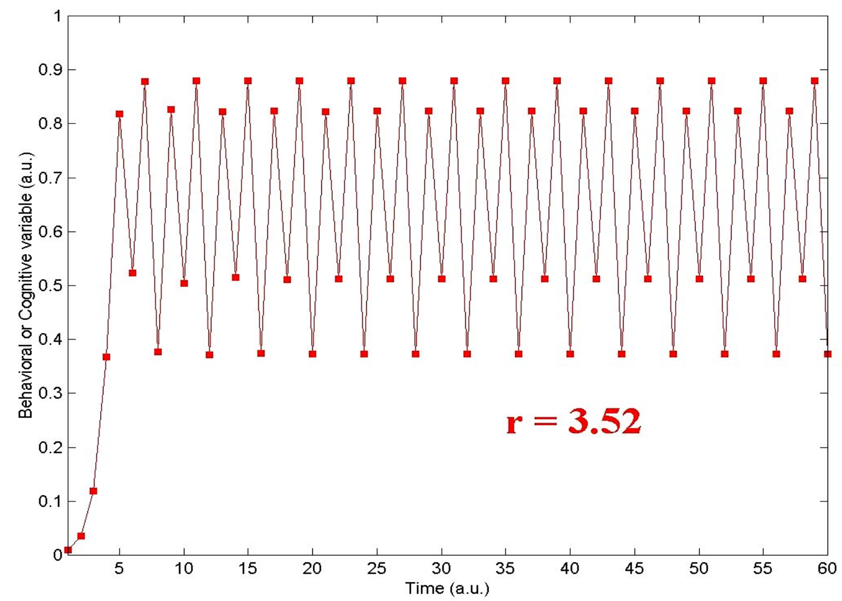
r=2.9



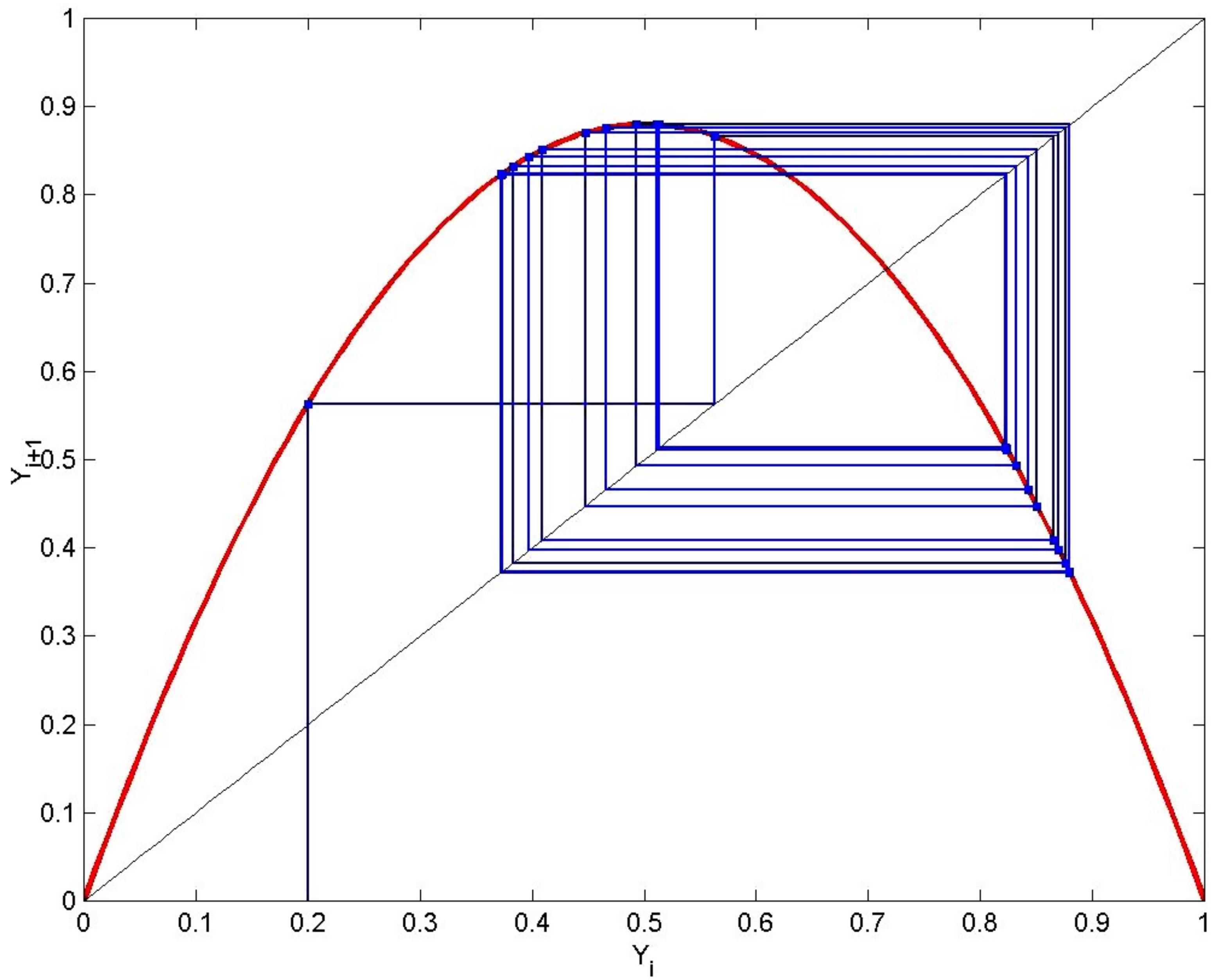


r=3.3





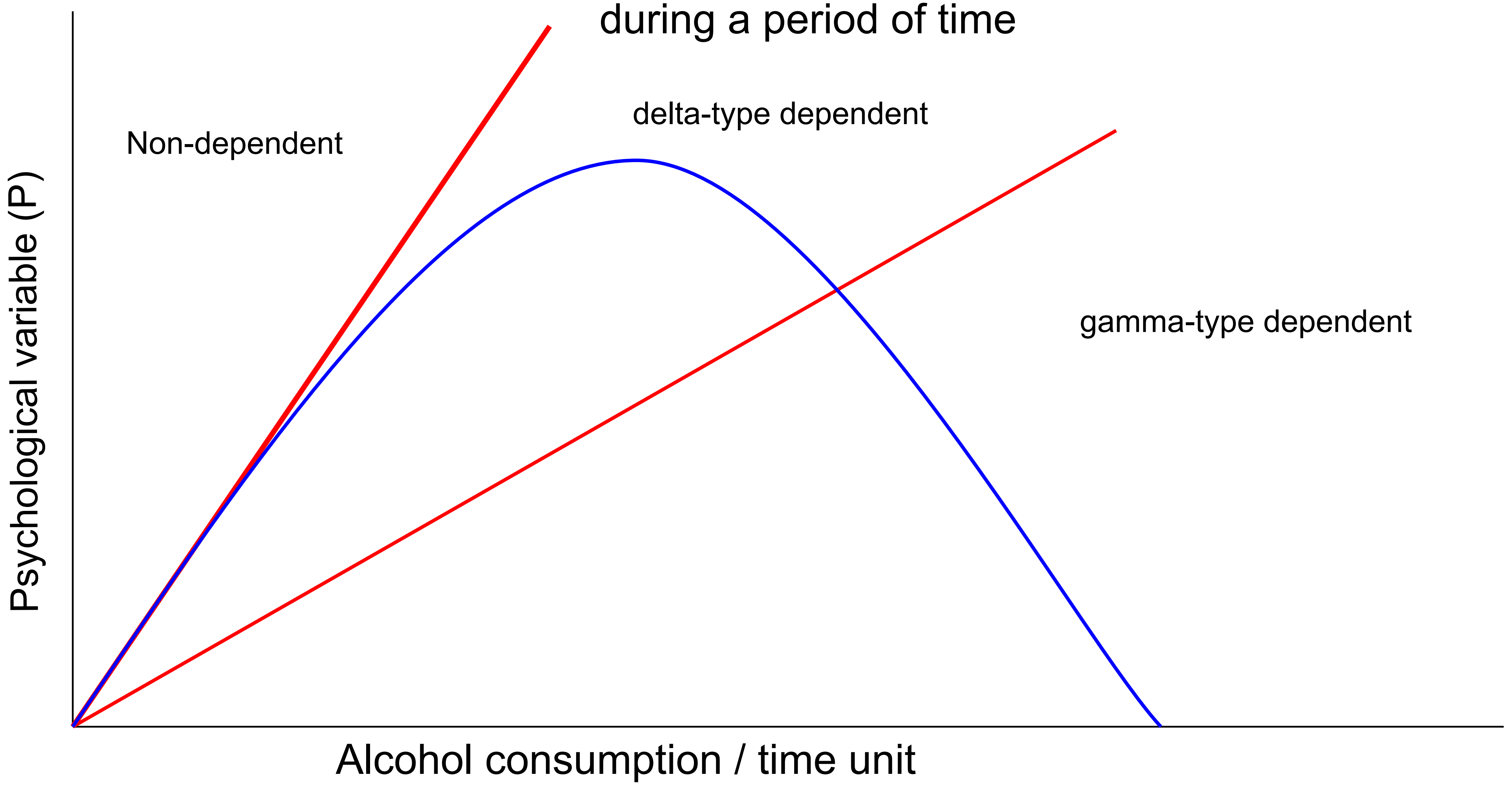
r=3.52

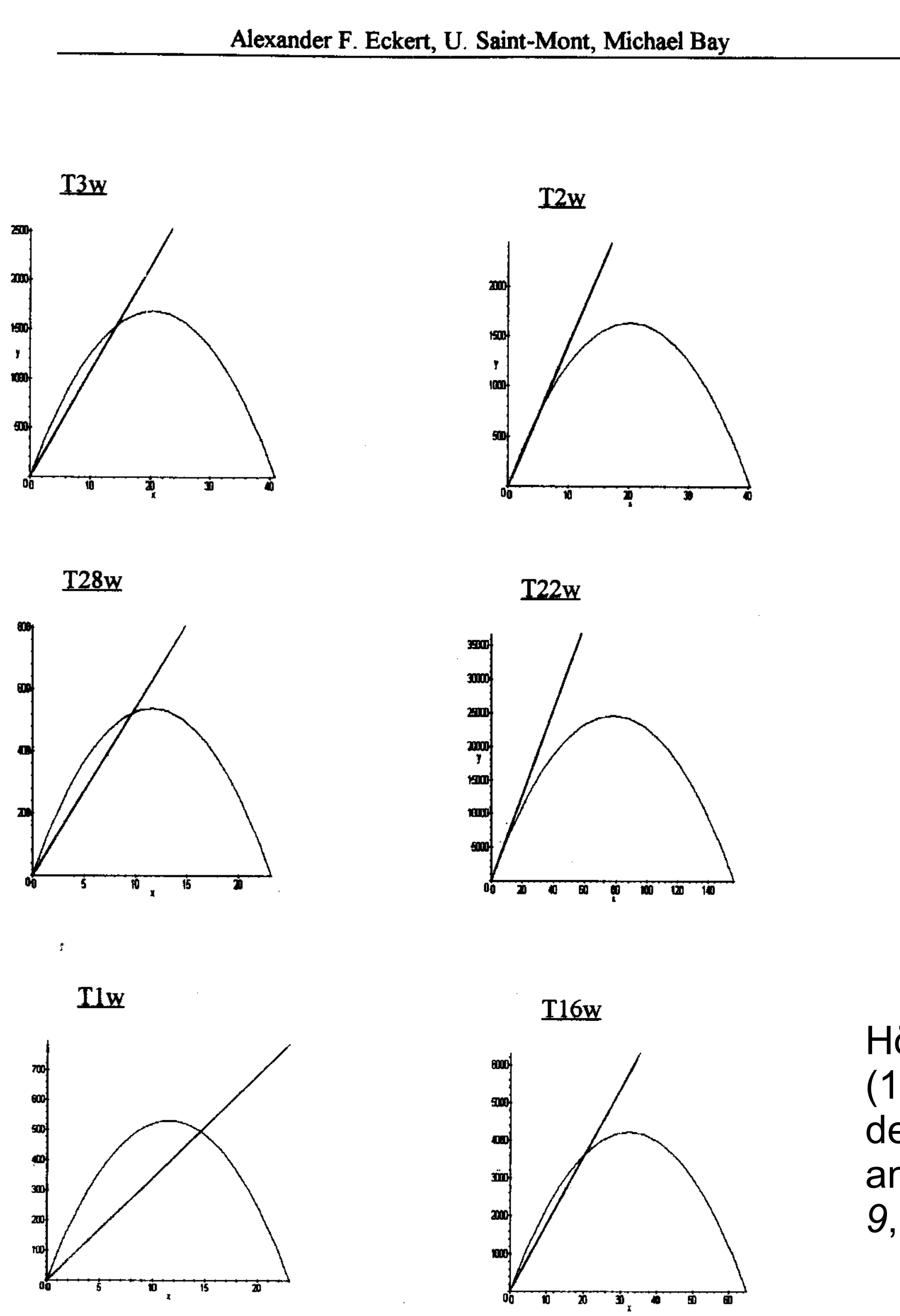


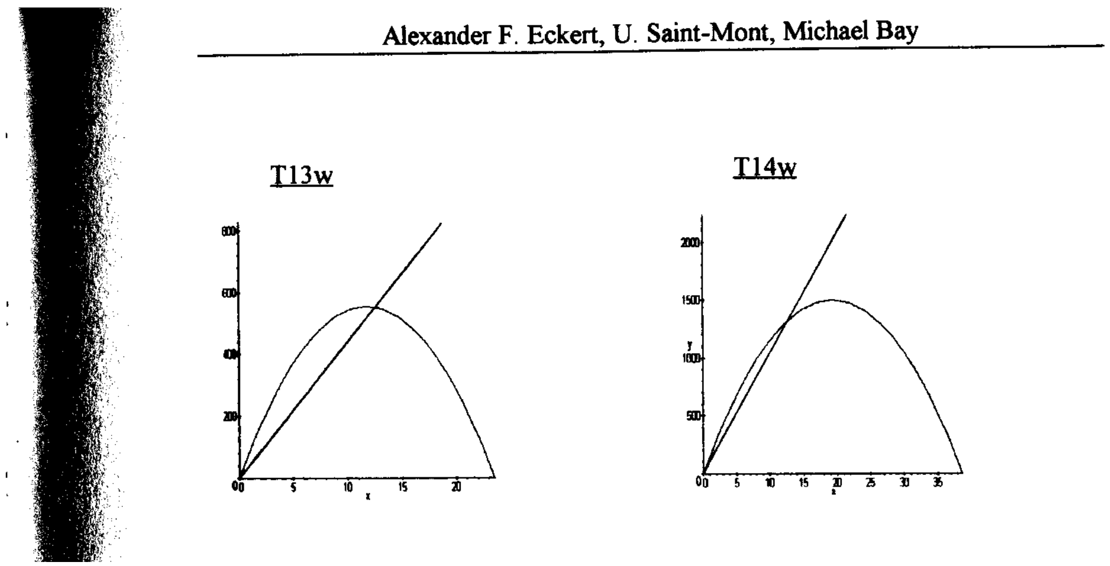
Teufelskreis: Alcohol addiction

A. Eckert

Model Alcohol dependency by just looking at the amount of alcohol consumed during a period of time







These are timeseries of rats drinking alcohol. Different types of dependency on alcohol can be described by the logistic map!!

Hölter, S.M., Engelmann, M., Kirschke, C., Liebsch, G., Landgraf R., & Spanagel, R. (1998). Long-term ethanol self-administration with repeated ethanol deprivation episodes changes ethanol drinking patterns and increases anxiety related behavuiour during ethanol deprivation in rats. *Biological Pharmacology, 9*, 41-48.

Using Analytic Solutions

Structural portion, *which embodies our hypothesis about the shape of each person's true trajectory of change over time*

Stochastic portion, *which allows for the effects of random error from the measurement of person i on occasion j. Usually assume* $\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$

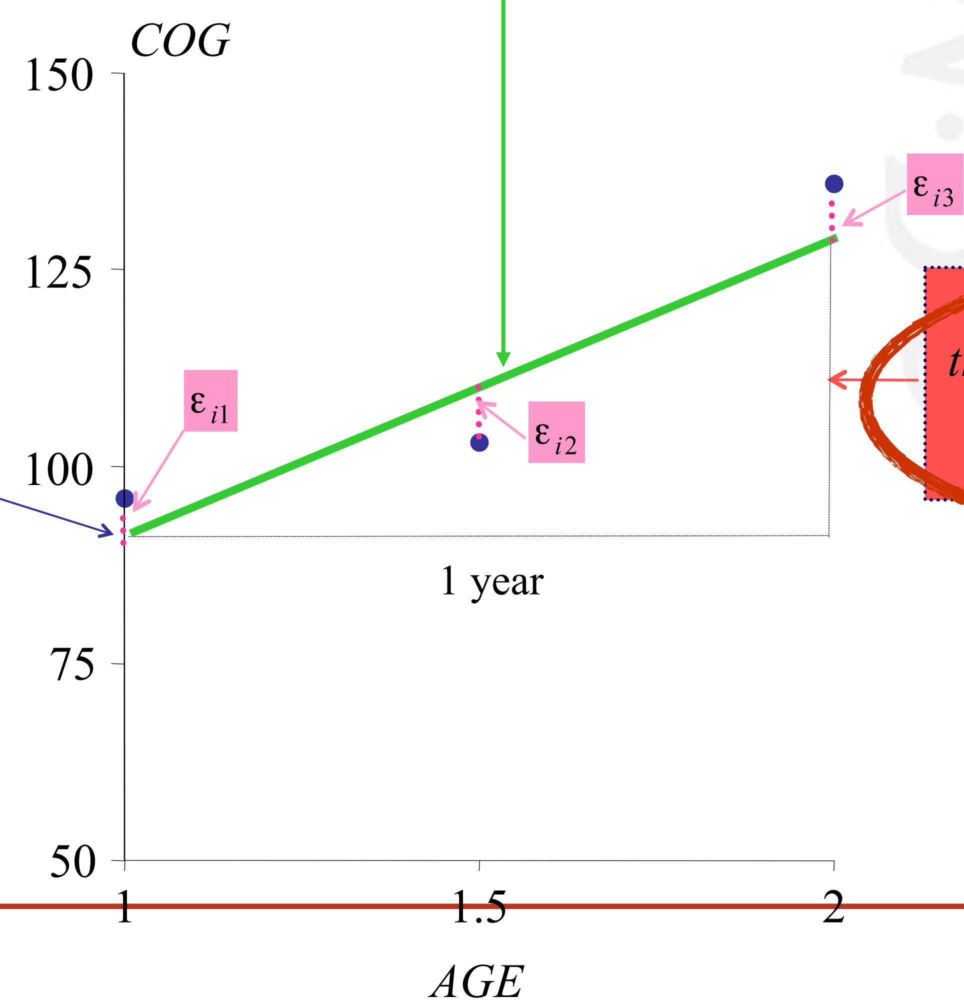
Key assumption: In the population, COG_{ij} is a linear function of child i's AGE on occasion j

$$COG_{ij} = \left[\pi_{0i} + \pi_{1i}(AGE_{ij} - 1)\right] + \left[\varepsilon_{ij}\right]$$

- ***i indexes persons (i=1 to 103)***
- ***j indexes occasions (j=1 to 3)***

*Individual i's hypothesized **true change trajectory***

*ε_{i1}, ε_{i2}, and ε_{i3} are deviations of i's true change trajectory **from linearity** on each occasion (measurement error)*



*π_{0i} is the **intercept** of i's true change trajectory, his true value of COG at AGE=1, his **"true initial status"***

*π_{1i} is the **slope** of i's true change trajectory, his yearly rate of change in true COG, his true "annual **rate of change**"*

(ALDA, Section 3.2, pp. 49-51)

For a line:
2 parameters
(slope + intercept)

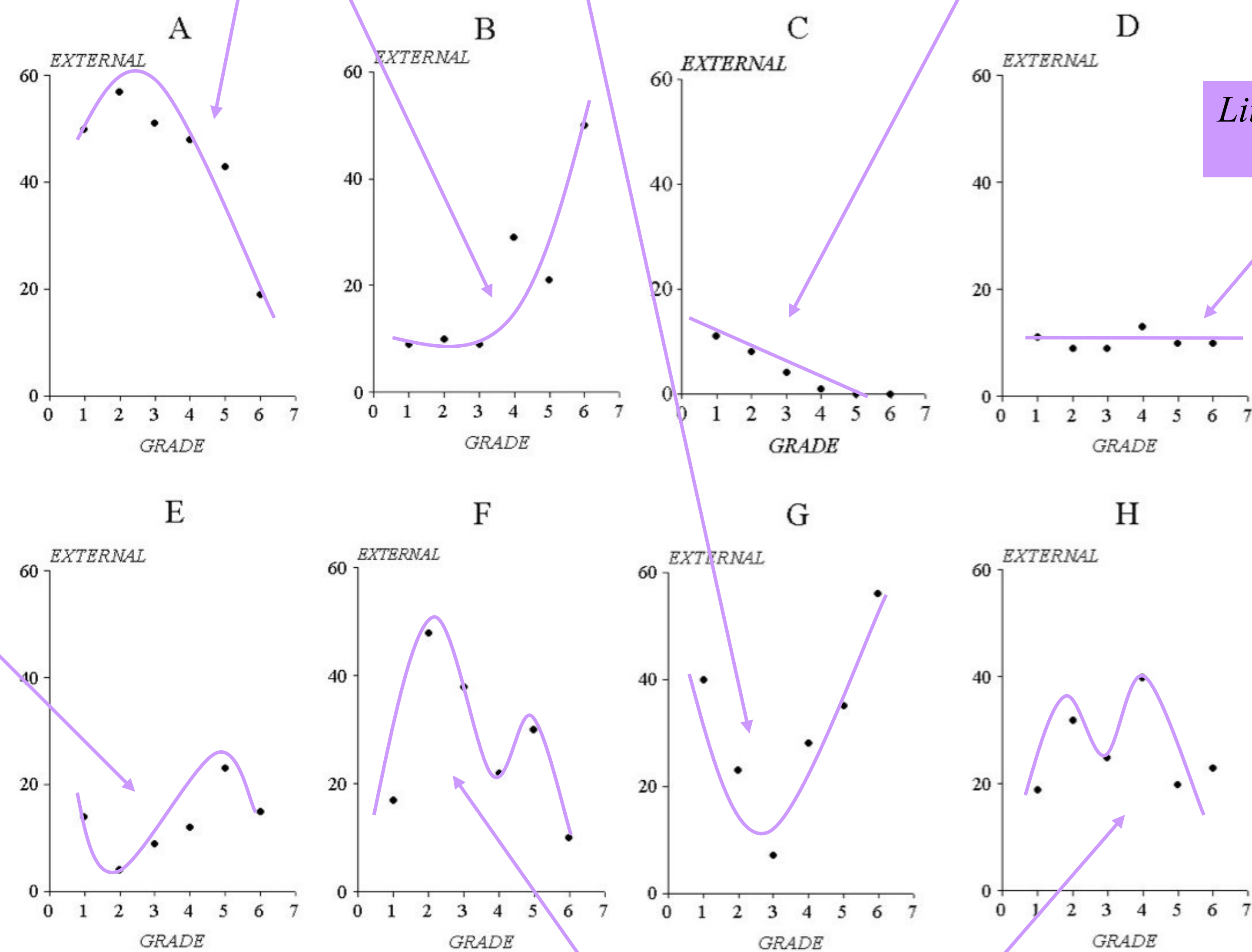
Other shapes?

Quadratic change (but with varying curvatures)

Linear decline (at least until 4th grade)

Little change over time (flat line?)

Two stationary points? (suggests a cubic)



Assumptions…

the true change trajectory is a polynomial function of time of unknown order in the population

Three stationary points? (suggests a quartic!!!)

When faced with so many different patterns, how do you select a <u>common</u> polynomial for analysis?

http://gseacademic.harvard.edu/~alda/Chapter%20presentations.htm

(ALDA, Section 6.3.2, pp 217-220)

Looking for explained variance of polynomial components, NOT based on a theoretical process

A logistic S-shape would require 4 fixed polynomial parameters

		Parameter	Model A No change	Model B Linear change	Model C Quadratic change	Model D Cubic change
Fixed Effects						
Composite model	Intercept (1st grade status)	$\boxed{\gamma_{00}}$	12.96***	13.29***	13.97***	13.79***
	TIME (linear term)	γ_{10}		−0.13	−1.15	−0.35
	TIME^2 (quadratic term)	γ_{20}			0.20	−0.23
	TIME^3 (cubic term)	γ_{30}				0.06
Variance Components						
Level-1:	Within-person	σ_ε^2	70.20***	53.72***	41.98***	40.10***
Level-2:	In 1st grade status	σ_0^2	87.42***	123.52***	107.08***	126.09***
	Linear term					
	variance	σ_1^2		4.69**	24.60*	88.71
	covar with 1st grade status	σ_{01}		−12.54*	−3.69	−51.73
	Quadratic term					
	variance	σ_2^2			1.22*	11.35
	covar with 1st grade status	σ_{02}			−1.36	22.83~
	covar with linear term	σ_{12}			−4.96*	−31.62
	Cubic term					
	variance	σ_3^2				0.08
	covar with 1st grade status	σ_{03}				−3.06~
	covar with linear term	σ_{13}				2.85
	covar with quadratic term	σ_{23}				−0.97
Goodness-of-fit						
	Deviance statistic		2010.3	1991.8	1975.8	1967.0
	AIC		2016.3	2003.8	1995.8	1997.0
	BIC		2021.9	2015.0	2014.5	2025.1

~$p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$.

(ALDA, Section 6.3.3, pp 220-223)

Change according to a process: Exponential growth

The growth rate is proportional to the current growth level:

$$\boxed{Y_{i+1} = r \cdot Y_i}$$

$$\boxed{\frac{dY}{dt} = r \cdot Y}$$

Analytic Solution

Analytic Solution

$$\boxed{Y_i = r^i \cdot Y_0}$$

$$\boxed{Y(t) = Y_0 \cdot e^{rt}}$$

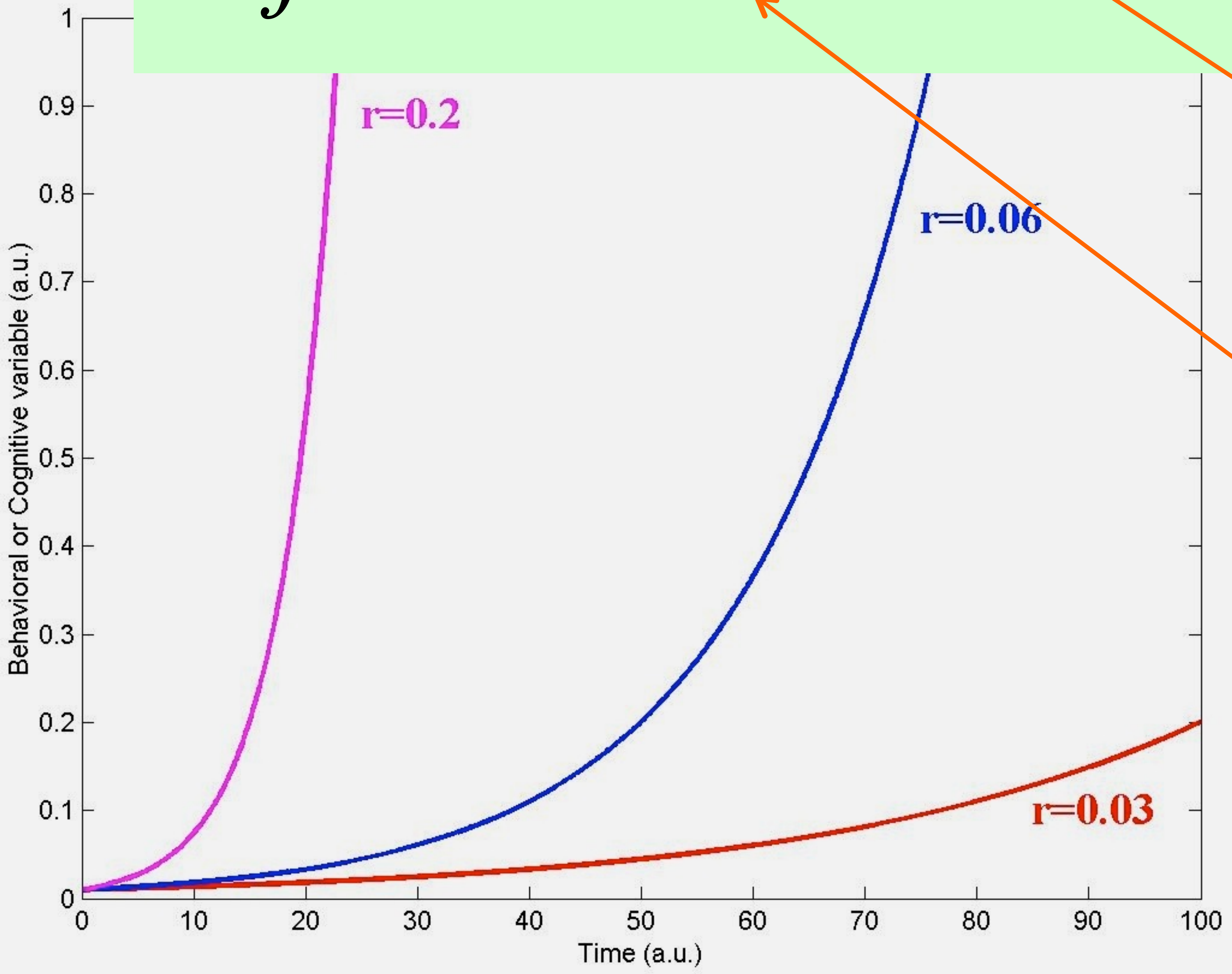
Difference equation: Map …

Differential equation: Flow ~

Exponential growth (Flow ~)

© Singer & Willett

$$Y_{ij} = \pi_{0i} e^{\pi_{1i} TIME_{ij}} + \varepsilon_{ij}$$



$$\frac{dY}{dt} = r \cdot Y$$

$$Y(t) = Y_0 \cdot e^{rt}$$

Y_0= Initial condition

Radboud University Nijmegen

Restricted growth

Introduce a carrying capacity K: Upper limit for growth

The growth rate is now proportional to what is left to grow (K-Y):

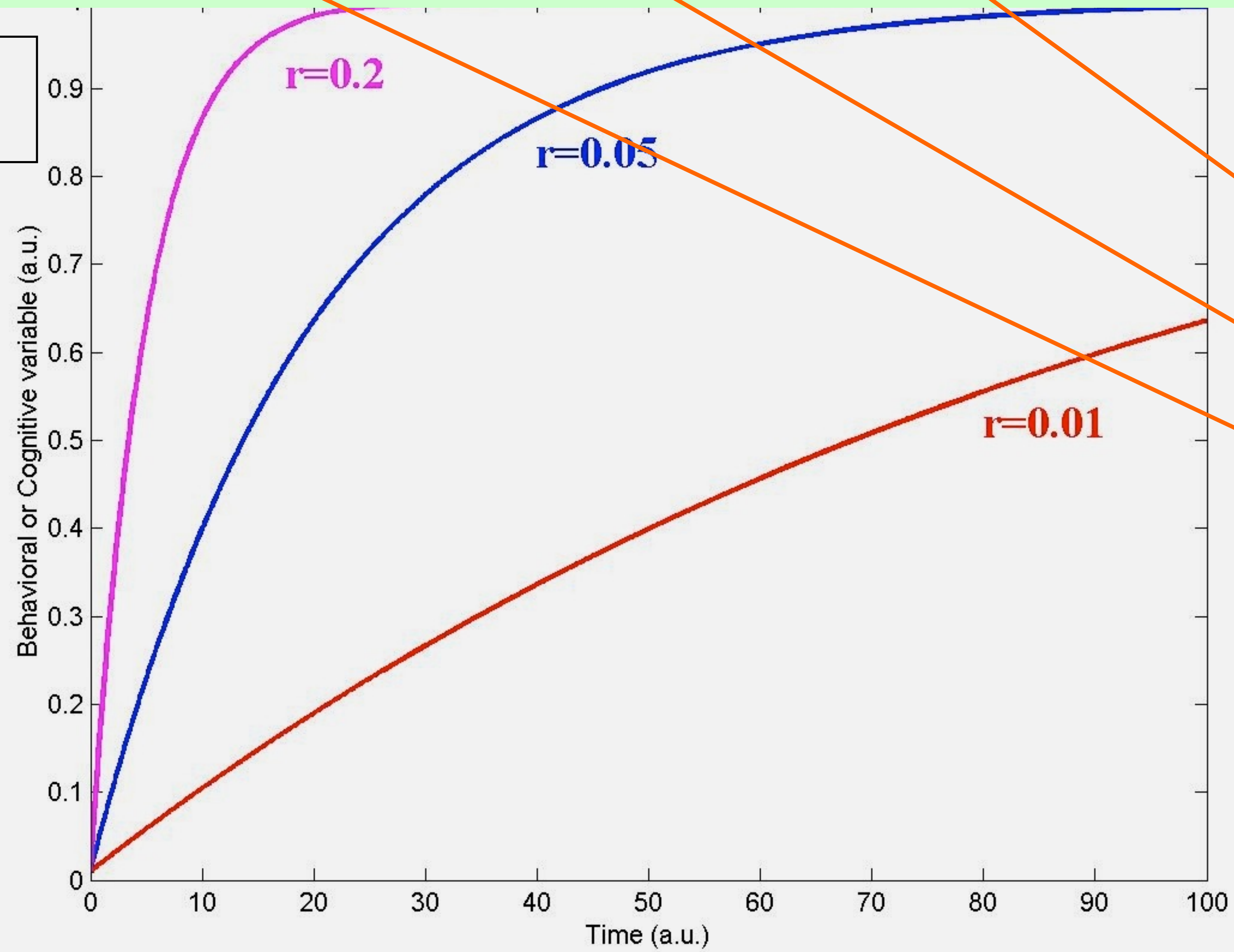
$$\boxed{\frac{dY}{dt} = r \cdot (K - Y)}$$

$$\boxed{Y(t) = K - (K - Y_0) \cdot e^{-rt}}$$

Restricted growth

$$Y_{ij} = \alpha_i - (\alpha_i - \pi_{0i})e^{-\pi_{1i} TIME_{ij}} + \varepsilon_{ij}$$

K=1



$$\frac{dY}{dt} = r \cdot (K - Y)$$

$$Y(t) = K - (K - Y_0) \cdot e^{-rt}$$

Logistic growth

If we combine these **linear** models we get **nonlinear** restricted (logistic) growth

$$Y_{i+1} = rY_i(K - Y_i)$$

$$Y_{i+1} = rY_i(K - Y_i)$$

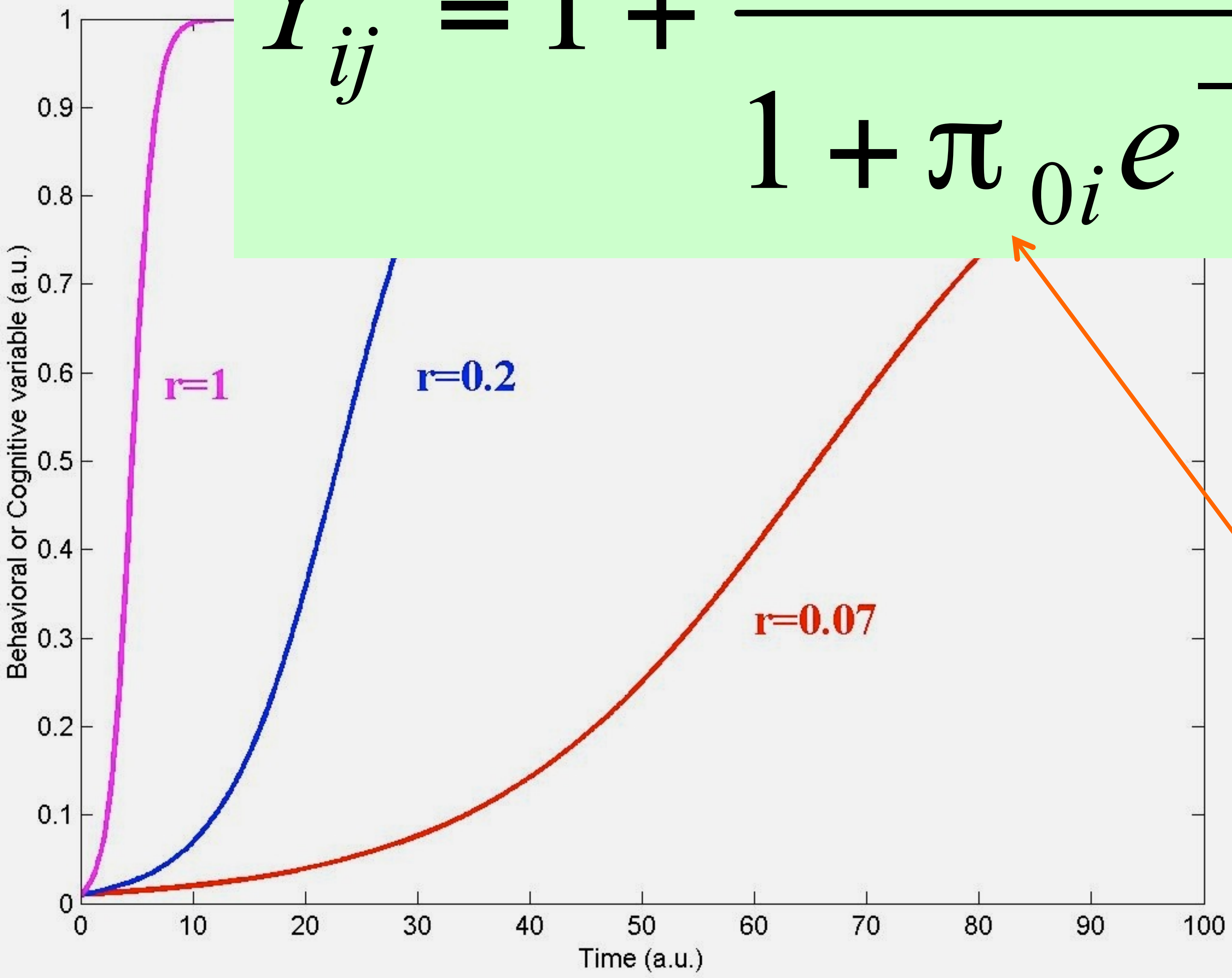
no analytic solution

$$\frac{dY}{dt} = rY(K - Y)$$

$$Y(t) = \frac{KY_0}{Y_0 + (K - Y_0)e^{-Krt}}$$

Logistic Growth (Flow ~)

$$Y_{ij} = 1 + \frac{19}{1 + \pi_{0i} e^{-(\pi_{1i} TIME_{ij})}} + \varepsilon_{ij}$$



$$\boxed{\frac{dY}{dt} = rY(K - Y)}$$

$$\boxed{Y(t) = \frac{KY_0}{Y_0 + (K - Y_0)e^{-Krt}}}$$