

ANLP

14 - CFGs, CKY (structure, part II)

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where are we?

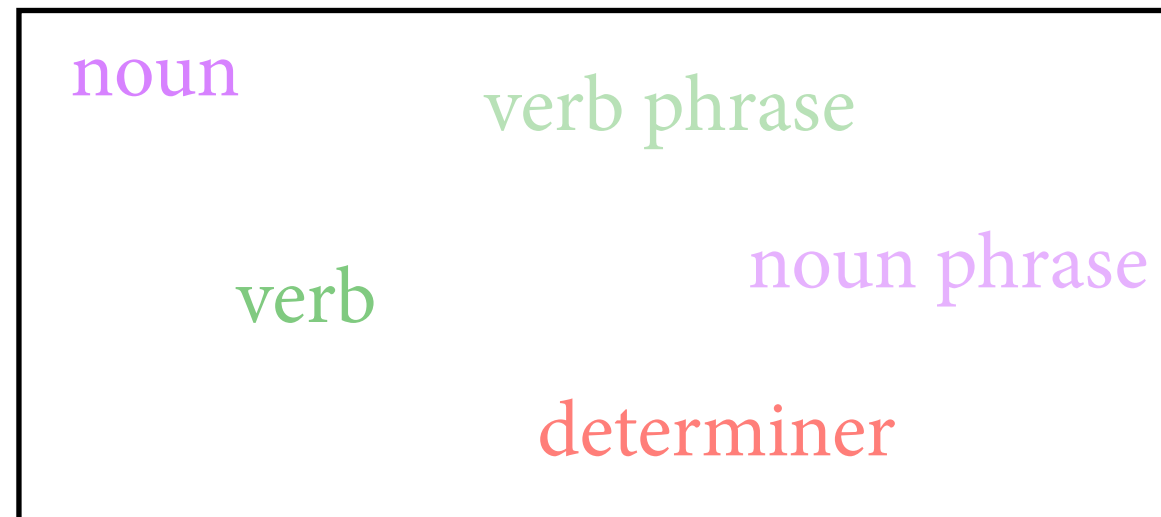
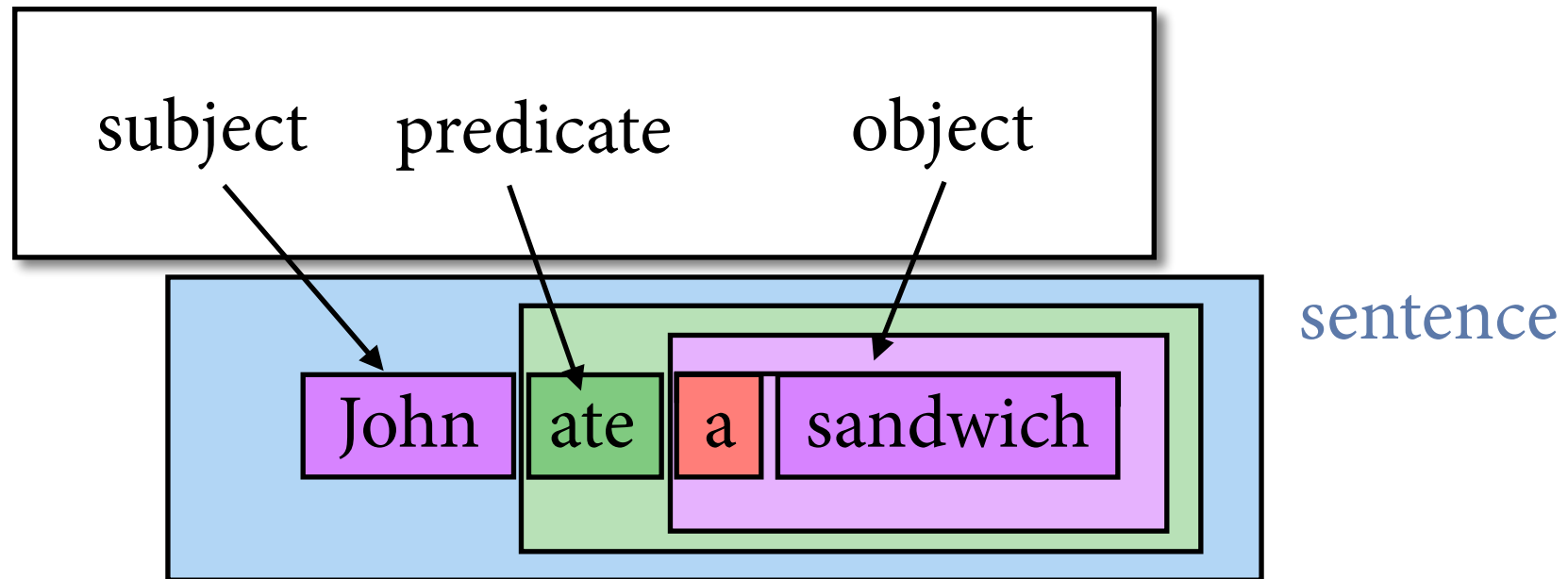
- words, and how to represent them
- sequences of words, modelled with n-gram models (generative model)
- classification of objects, with small label set; weighting of features of object (generative classifier: Naive Bayes; discriminative classifiers: logistic regression, SVMs)
- classification of structured objects, with structured labels (generative: HMMs, discriminative: CRFs)
- yet more methods: NNs, forward and recurrent; pre-training. Gives us continuous representations (= vectors) of words (in context) and sequences.
- today: explicit structural representations of sequences

syntax

- let's assume we want to know who did what to whom
- POS-tags are not enough:
 - ▶ *I ate the spaghetti with chopsticks*
 - ▶ *I ate the spaghetti with meatballs*
 - ▶ *PP VBD DT NN IN NNS*
- We need more structure that tells us about relations btw parts of sentence.
- first: *all* structures; later: best (= ?) structure

Sentences have structure

grammatical functions

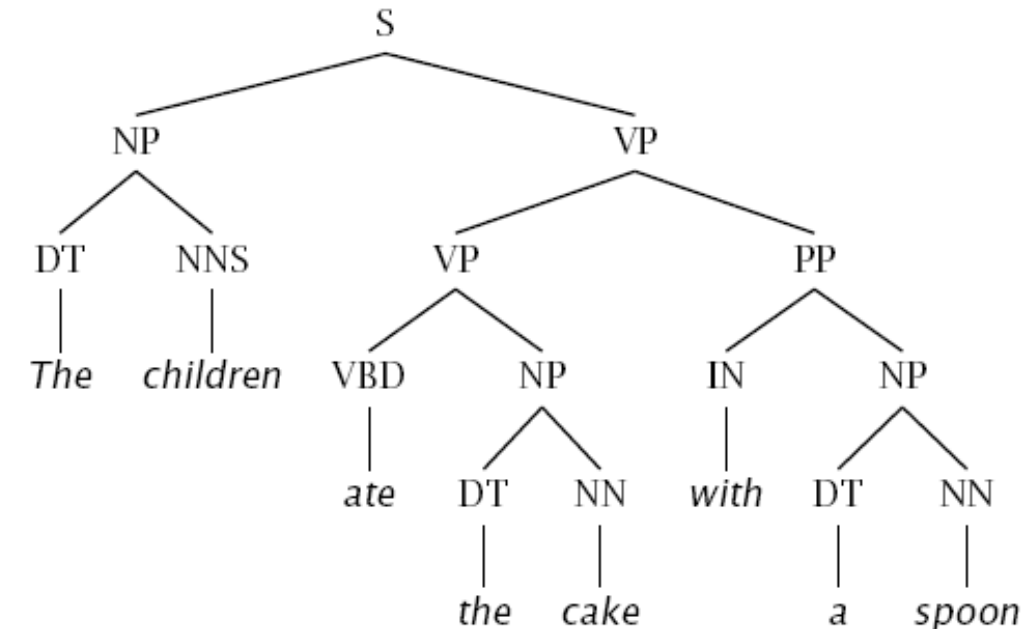


today: constituent structure

- ▶ How do we know what the constituents are?

- ▶ Constituency tests:

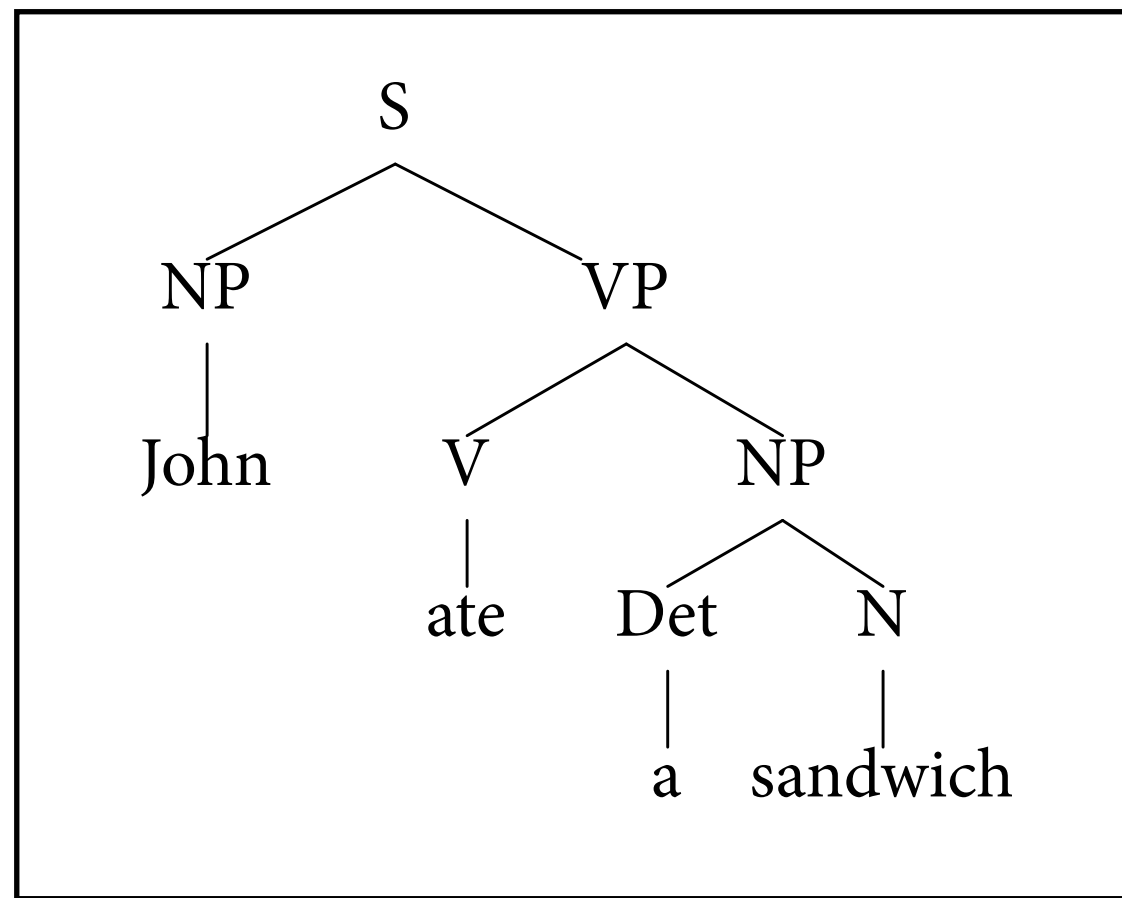
- ▶ Substitution by *proform* (e.g., pronoun)
- ▶ Clefting (*It was with a spoon that...*)
- ▶ Answer ellipsis (What did they eat? *the cake*)
(How? *with a spoon*)



- ▶ Sometimes constituency is not clear, e.g., coordination:
she went to and bought food at the store

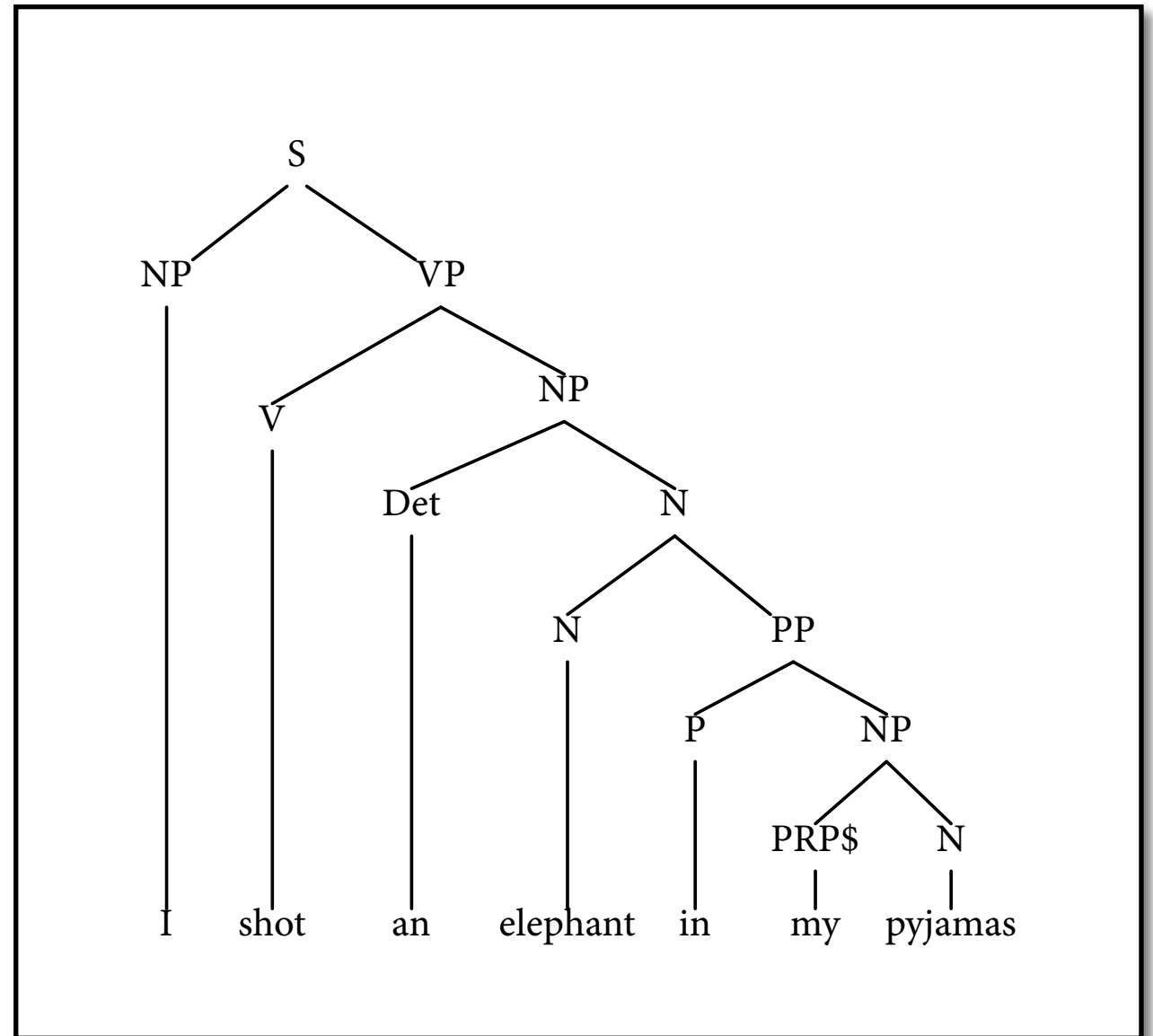
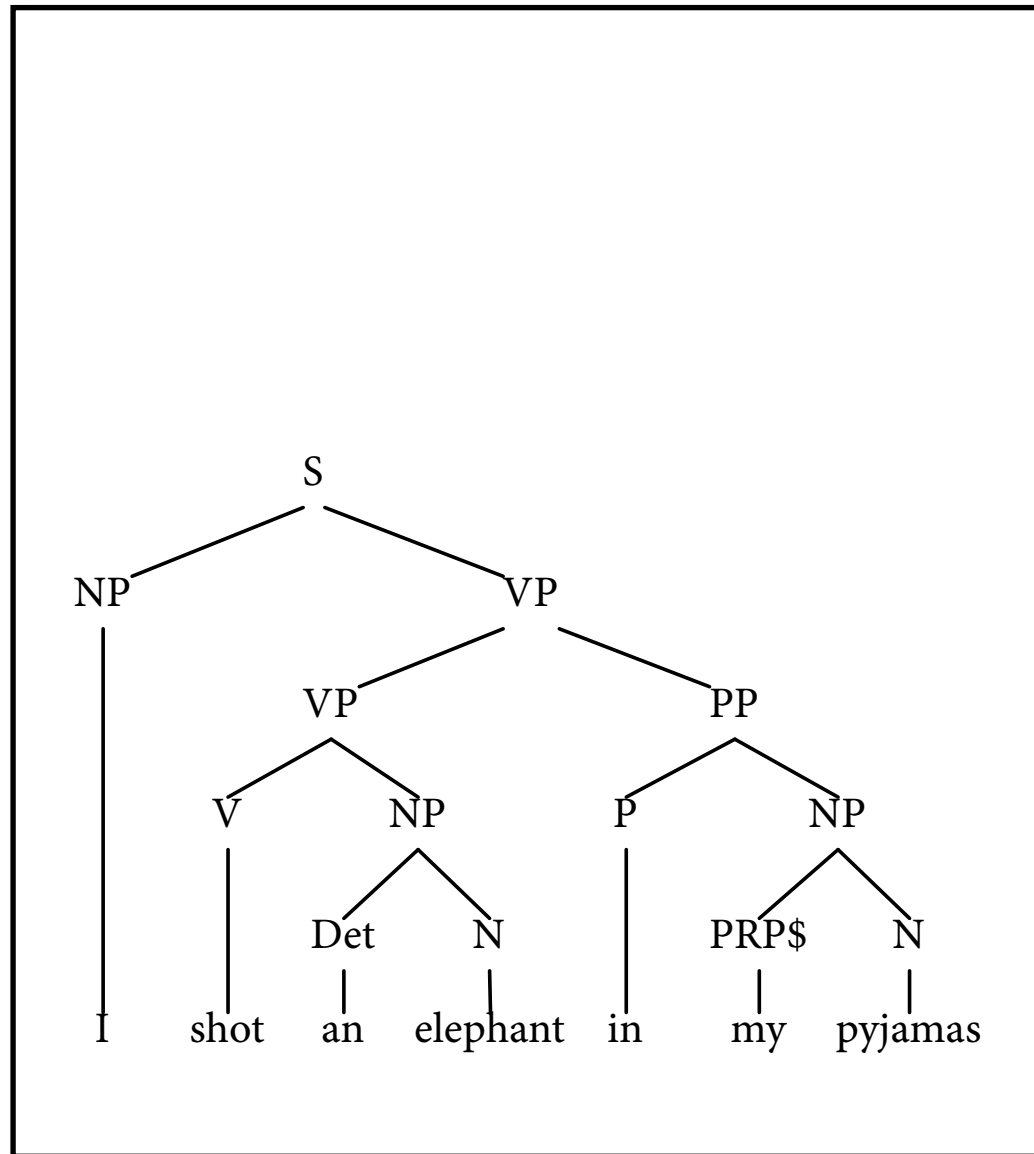
Sentences have structure

Record it conveniently in *phrase structure tree*.



Ambiguity

Special challenge: sentences can have many possible structures.



This sentence is example of *attachment ambiguity*.

Grammars

- A *grammar* is a finite device for describing large (possibly infinite) set of strings.
 - ▶ strings = NL expressions of various types
 - ▶ grammar captures linguistic knowledge about syntactic structure
- There are many different grammar formalisms that are being used in NLP.
- In this course we focus on *context-free grammars*.

Context-free grammars

- Context-free grammar (cfg) G is 4-tuple (N, T, S, P) :
 - ▶ N and T are disjoint finite sets of symbols:
 $T = \textit{terminal}$ symbols; $N = \textit{nonterminal}$ symbols.
 - ▶ $S \in N$ is the *start symbol*.
 - ▶ P is a finite set of *production rules* of the form $A \rightarrow w$,
where A is nonterminal and w is a string from $(N \cup T)^*$.
- Why “context-free”?
 - ▶ Left-hand side of production is a single nonterminal A .
 - ▶ Rule can’t look at context in which A appears.
 - ▶ *Context-sensitive* grammars can do that.

Example

$T = \{\text{John, ate, sandwich, a}\}$

$N = \{S, NP, VP, V, N, Det\}$; start symbol: S

Production rules:

$S \rightarrow NP \ VP$

$V \rightarrow \text{ate}$

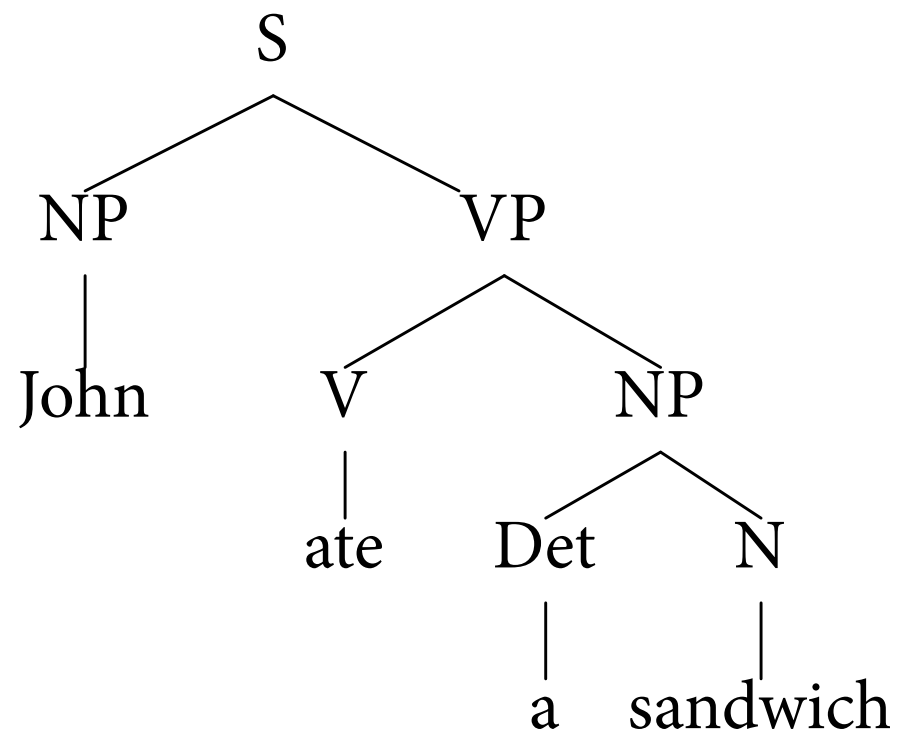
$Det \rightarrow a$

$NP \rightarrow Det \ N$

$NP \rightarrow \text{John}$

$N \rightarrow \text{sandwich}$

$VP \rightarrow V \ NP$



perspectives on grammar

- device for characterising set (sentences of language / the language itself)
- „classifier“: grammatical yes / no
- „classifier“: assigns complex label (tree) to string (or FAIL)
- generative device: generates all sentences of language (eventually)
- today: formal device that can be used in algorithm, to analyse input string

Some important concepts

- *One-step derivation* relation \Rightarrow :
 $w_1 A w_2 \Rightarrow w_1 w w_2$ iff $A \rightarrow w$ is in P
(w_1, w_2, w are strings from $(N \cup T)^*$)
- *Derivation* relation \Rightarrow^* is reflexive, transitive closure:
 $w \Rightarrow^* w_n$ if $w \Rightarrow w_1 \Rightarrow \dots \Rightarrow w_n$ (for some $n \geq 0$)
- *Language* $L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$

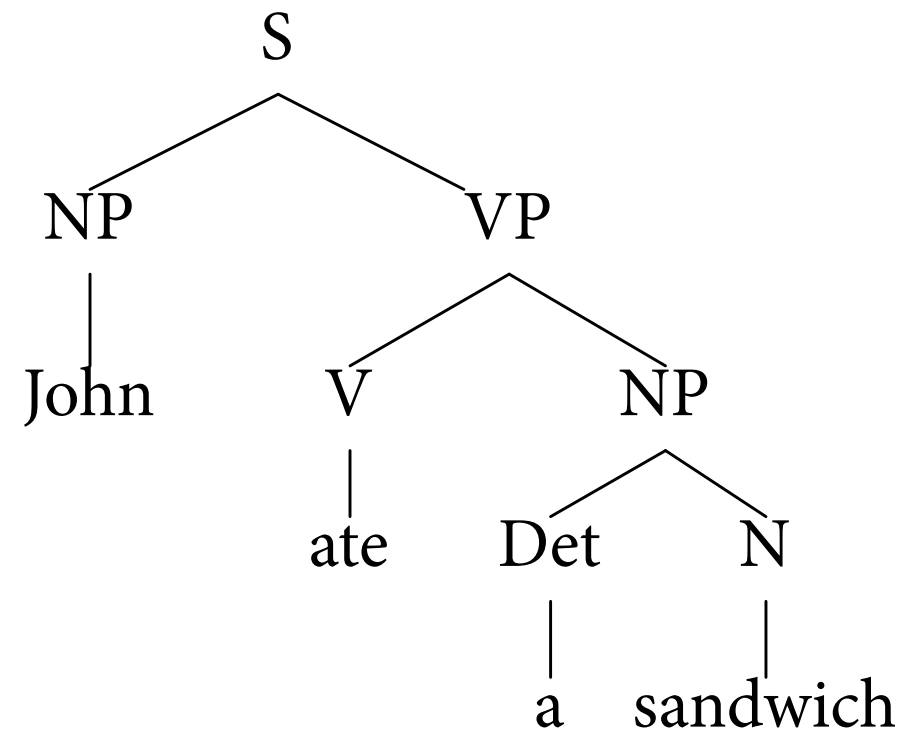
Derivations and parse trees

Parse tree provides readable, high-level view of derivation.

derivation

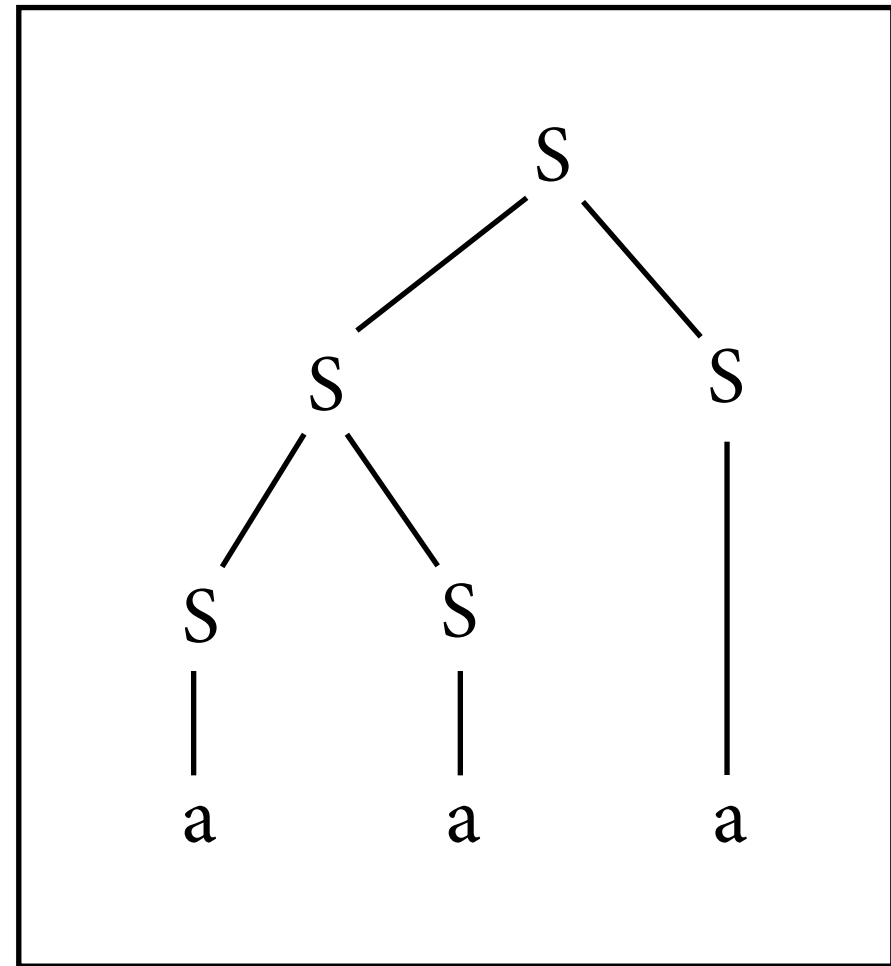
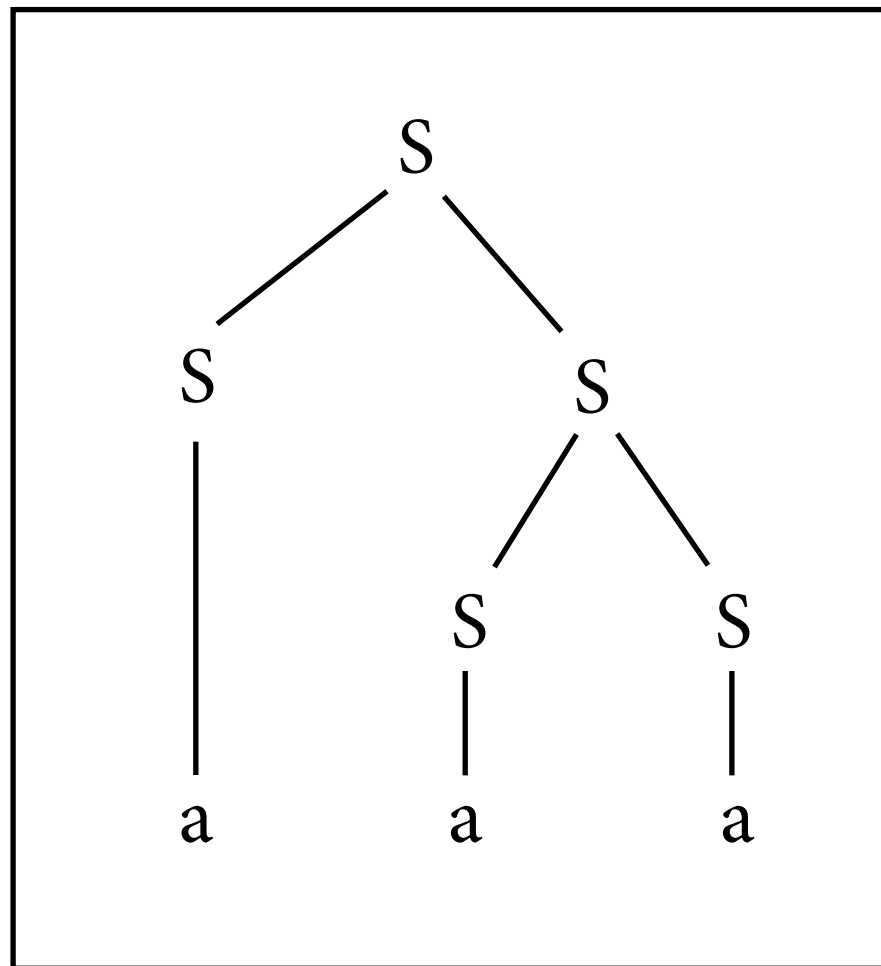
$S \Rightarrow NP \ VP \Rightarrow \text{John} \ VP$
 $\Rightarrow \text{John} \ V \ NP \Rightarrow \text{John ate} \ NP$
 $\Rightarrow \text{John ate Det} \ N$
 $\Rightarrow \text{John ate a} \ N$
 $\Rightarrow \text{John ate a sandwich}$

parse tree



Big languages

Number of parse trees can grow exponentially in string length.

$$S \rightarrow S S$$
$$S \rightarrow a$$


Recognition and parsing

- Let G be a cfg and w be a string.
- *Word problem*: is $w \in L(G)$?
 - ▶ Algorithms that solve it are called *recognizers*.
- *Parsing problem*: enumerate all parse trees of w .
 - ▶ Algorithms that solve it are called *parsers*.
- Every parser also solves the word problem.

Parsing algorithms

- How can we solve the word and parsing problem so systematically that we can implement it?
- One simple approach: shift-reduce algorithm (here: only for the word problem).
- Then: Analyze efficiency of SR and replace it with faster algorithm: CKY.

demo

```
In [1]: import nltk
```

```
In [2]: nltk.app.srparser()
```

Try to get to a complete parse (a tree spanning the whole input) by repeated applications of the „shift“ and the „reduce“ operation.

Shift-Reduce Parsing

$T = \{\text{John, ate, sandwich, a}\}$

$N = \{S, NP, VP, V, N, Det\}$; start symbol: S

Production rules:

$S \rightarrow NP \ VP$

$VP \rightarrow V \ NP$

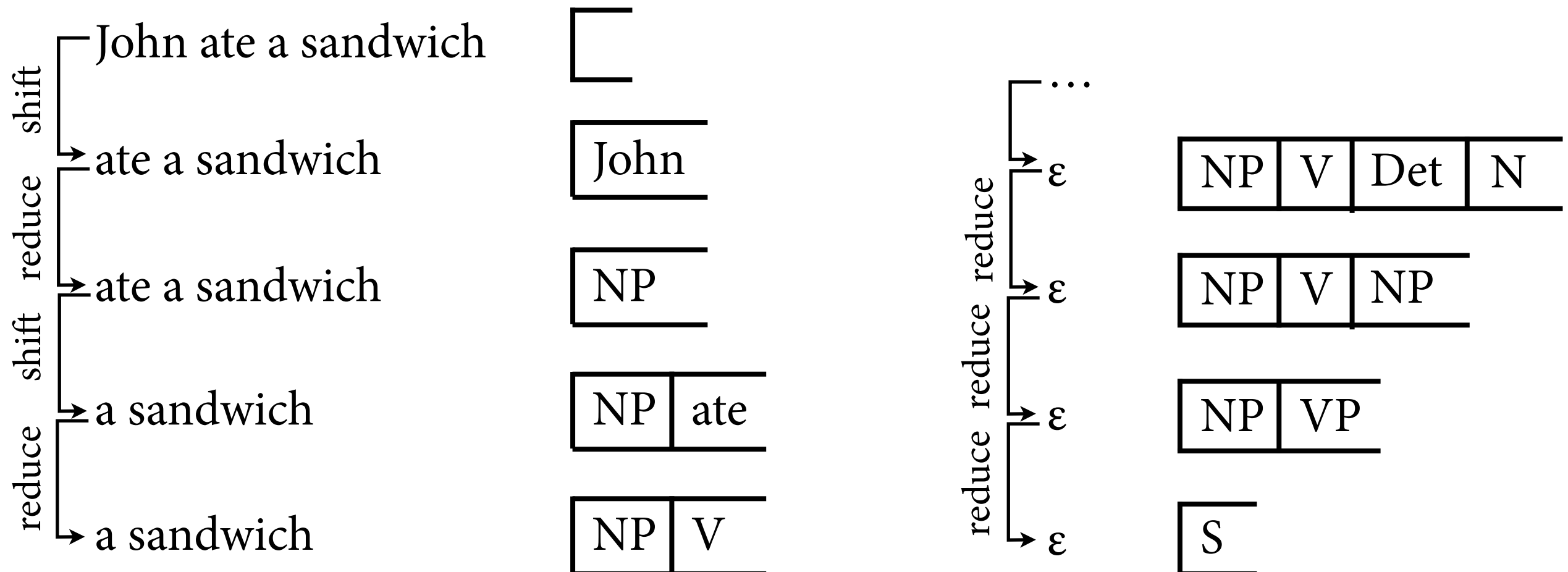
$V \rightarrow \text{ate}$

$Det \rightarrow a$

$NP \rightarrow Det \ N$

$NP \rightarrow \text{John}$

$N \rightarrow \text{sandwich}$



Shift-Reduce Parsing

- Read input string step by step. In each step, we have
 - ▶ the remaining input words we have not shifted yet
 - ▶ a *stack* of terminal and nonterminal symbols
- In each step, apply a rule:
 - ▶ Shift: moves the next input word to the top of the stack
 - ▶ Reduce: applies a production rule to replace top of stack with the nonterminal on the left-hand side
- Sentence is in language of cfg iff we can read the whole string and stack contains only start symbol.

Shift-Reduce Parsing

- Shift rule:
 $(s, a \cdot w) \rightarrow (s \cdot a, w)$
- Reduce rule:
 $(s \cdot w', w) \rightarrow (s \cdot A, w)$ if $A \rightarrow w'$ in P
- Start: (ε, w)
- Apply rules *nondeterministically*:
Claim $w \in L(G)$ if there *exists* some sequence of steps that derive (S, ε) from (ε, w) .

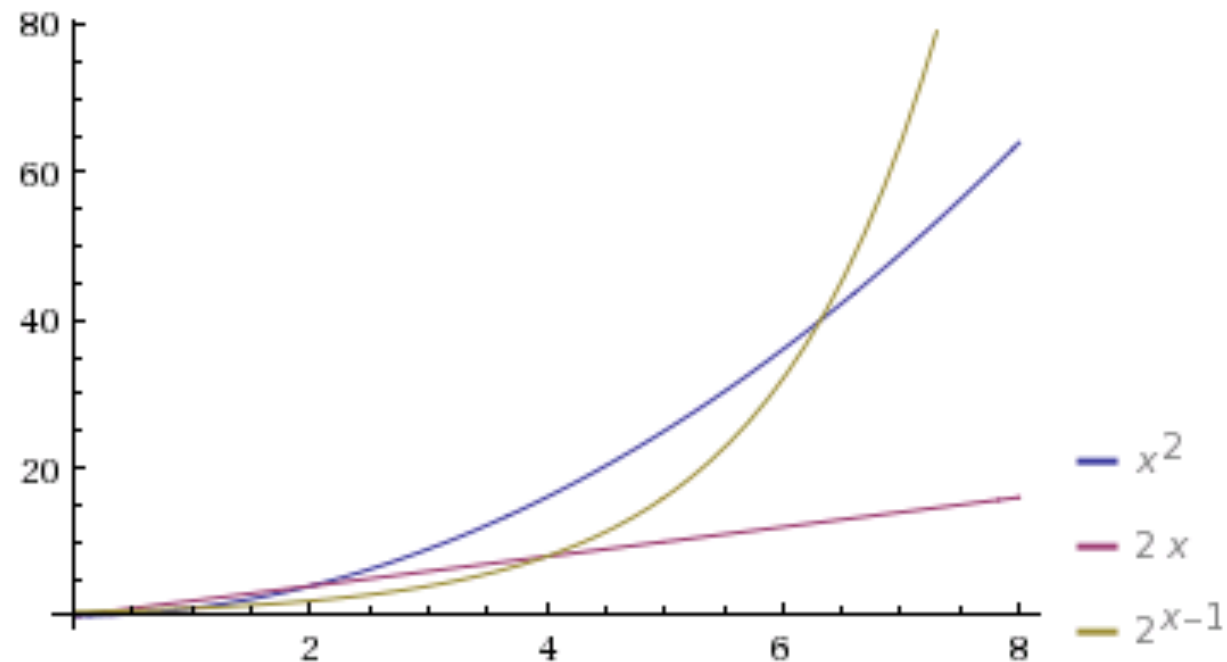
Nondeterminism

- Claim that string is in language of cfg iff (S, ϵ) can be derived by *any one* sequence of shift and reduce steps.
- This is very important because there are many stack-string pairs where multiple rules can be applied:
 - ▶ shift-reduce conflict
 - ▶ reduce-reduce conflict
- In practice, we need to try all sequences out.
 - ▶ Compilers for programming languages avoid this by careful language design: no ambiguity in grammar.

Analyzing Shift-Reduce

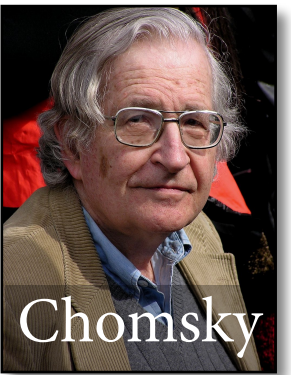
- If string has length n and grammar has k nonterminals, then there are $O(k^n)$ ways of assigning strings of nonterminals to words.
 - These can all be explored, especially when the string is *not* in the language.
- Big O Notation
 - Complexity of algorithm
 - Behaviour as function of input size can be described as this function (here: exponential)
 - Bad news!

Polynomial vs. exponential



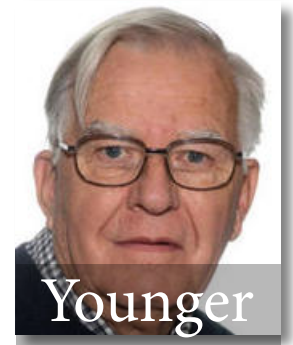
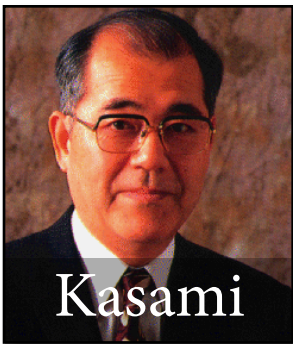
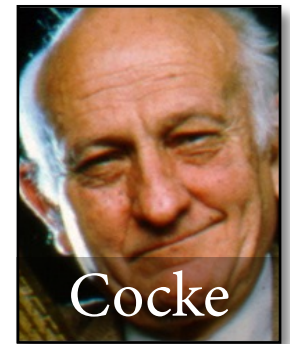
- We often distinguish between *polynomial* and *exponential* runtime. Rule of thumb: exponential = too slow for practical use.
- Is there a polynomial algorithm for the word problem?

Chomsky Normal Form



- A cfg is *in Chomsky normal form (CNF)* if each of its production rules has one of these two forms:
 - ▶ $A \rightarrow BC$: right-hand side is exactly two nonterminals
 - ▶ $A \rightarrow c$: right-hand side is exactly one terminal
- For every cfg G , there is a weakly equivalent cfg G' which is in CNF.
 - ▶ that is, $L(G) = L(G')$

The CKY Algorithm



- Simplest and most-used chart parser for cfgs in CNF.
- Developed independently in the 1960s by John Cocke, Daniel Younger, and Tadao Kasami.
 - ▶ sometimes also called CYK algorithm
- Bottom-up algorithm for discovering statements of the form “ $A \Rightarrow^* w_1 \dots w_{k-1} ?$ ”

The CKY Recognizer

$S \rightarrow NP \ VP$
 $NP \rightarrow Det \ N$
 $VP \rightarrow V \ NP$

$V \rightarrow ate$
 $NP \rightarrow John$

$Det \rightarrow a$
 $N \rightarrow sandwich$

Chart

$S \Rightarrow^* w$

	$i = 1$	2	3	4
5	S	VP	NP	N
4			Det	...
3		V	...	
$k = 2$	NP	...		

... sandwich
 sandwich
 ate
 a
 ate
 John

Cell at column i , row k :
 $\{ A \mid A \Rightarrow^* w_i \dots w_{k-1} \}$

The CKY Recognizer

$S \rightarrow NP \ VP$

$V \rightarrow \text{ate}$

$\text{Det} \rightarrow \text{a}$

$NP \rightarrow \text{Det} \ N$

$NP \rightarrow \text{John}$

$N \rightarrow \text{sandwich}$

$VP \rightarrow V \ NP$

	$i = 1$	2	3	4	
5	1,5	2,5	3,5	4,5	... sandwich
4	1,4	2,4	3,4	... a	sandwich
3	1,3	2,3	... ate	a	
$k = 2$	1,2	... John	ate		
	John				

$1,5 = 1,2 + 2,5$
 or $1,3 + 3,5$
 or $1,4 + 4,5$

The CKY Recognizer

$S \rightarrow NP \ VP$

$V \rightarrow \text{ate}$

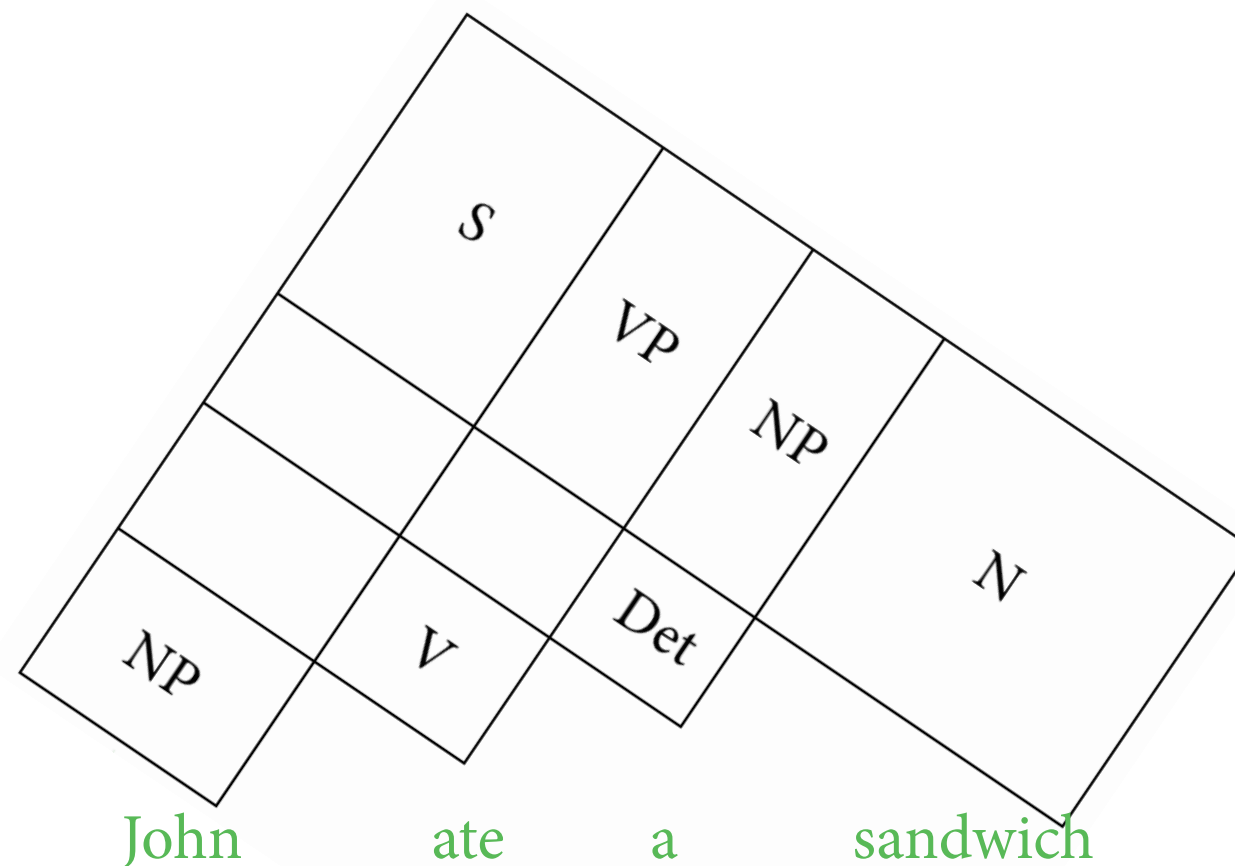
$\text{Det} \rightarrow \text{a}$

$NP \rightarrow \text{Det} \ N$

$NP \rightarrow \text{John}$

$N \rightarrow \text{sandwich}$

$VP \rightarrow V \ NP$

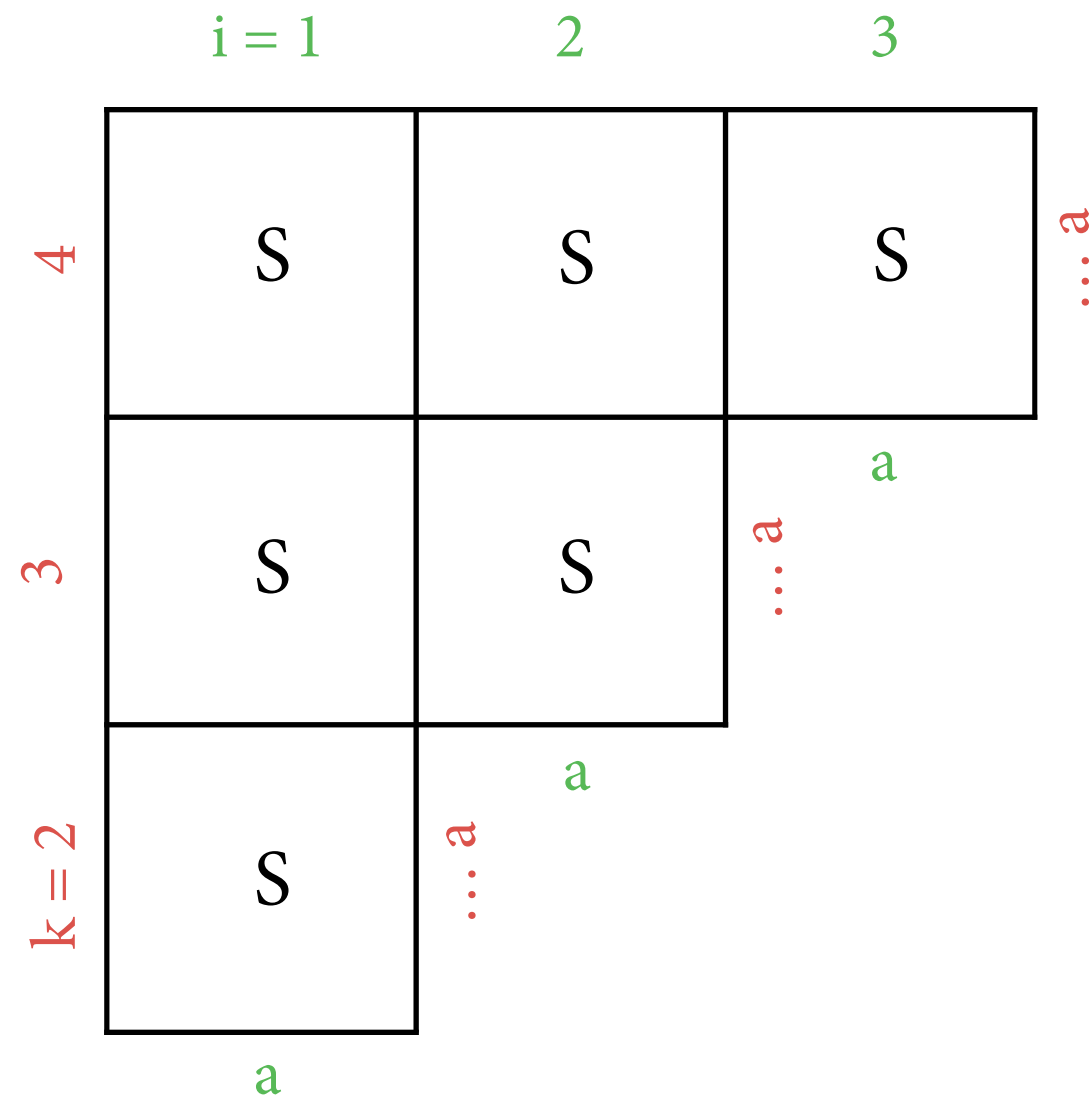


perhaps easier to see
the trees this way...

The CKY Recognizer

$S \rightarrow S S$

$S \rightarrow a$



CKY recognizer: pseudocode

Data structure: $\text{Ch}(i,k)$ eventually contains $\{A \mid A \Rightarrow^* w_i \dots w_{k-1}\}$
(initially all empty).

for each i from 1 to n :

 for each production rule $A \rightarrow w_i$:

 add A to $\text{Ch}(i, i+1)$

for each *width* b from 2 to n :

 for each *start position* i from 1 to $n-b+1$:

 for each *left width* k from 1 to $b-1$:

 for each $B \in \text{Ch}(i, i+k)$ and $C \in \text{Ch}(i+k, i+b)$:

 for each production rule $A \rightarrow B C$:

 add A to $\text{Ch}(i, i+b)$

claim that $w \in L(G)$ iff $S \in \text{Ch}(1, n+1)$

Complexity

- *Time* complexity of CKY recognizer is $O(n^3)$, although number of parse trees grows exponentially.
- *Space* complexity of CKY recognizer is $O(n^2)$ (one cell for each substring).
- Efficiency depends crucially on CNF.
Naive generalization of CKY to rules $A \rightarrow B_1 \dots B_r$ raises time complexity to $O(n^{r+1})$.

Correctness

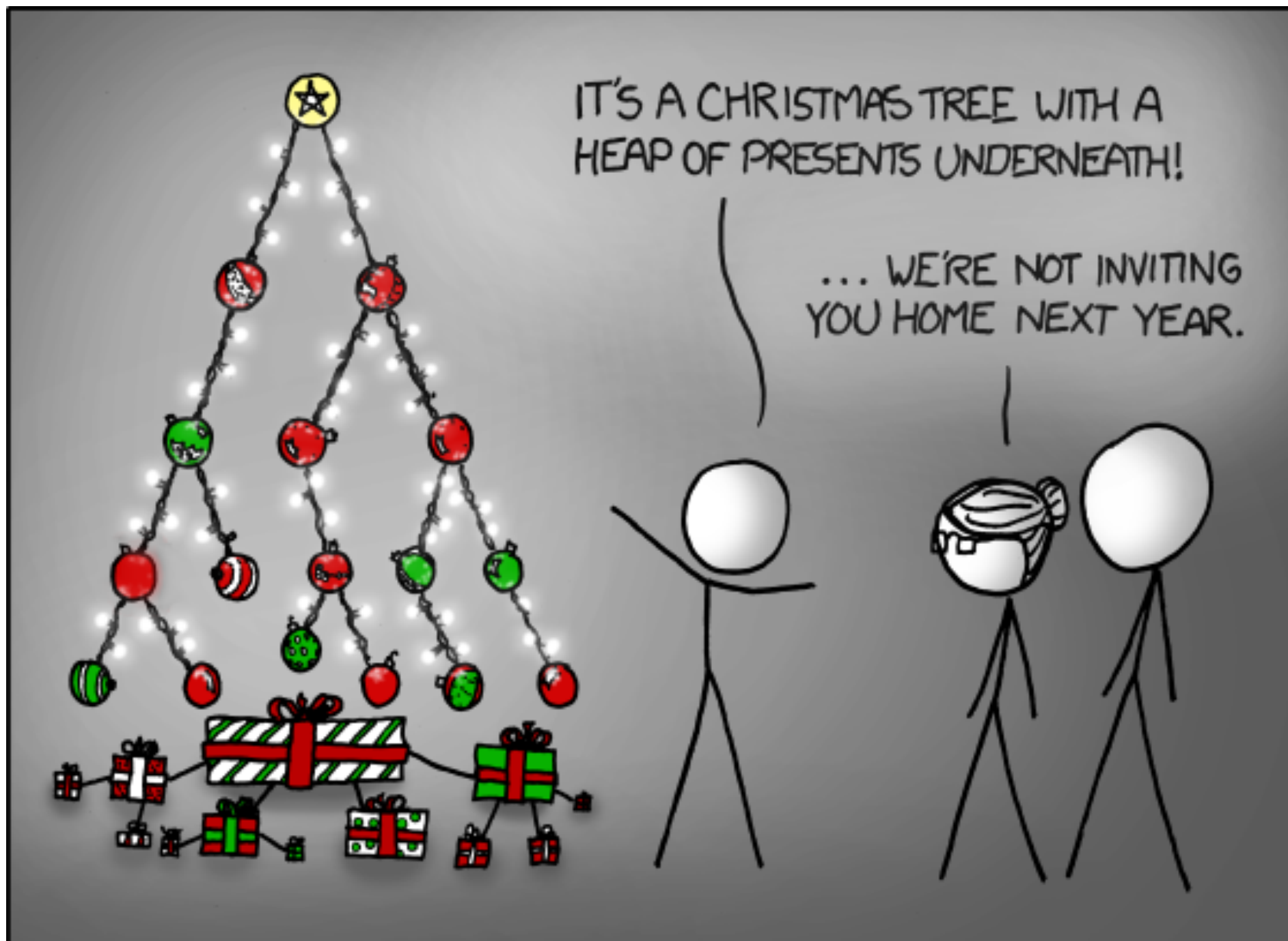
- Soundness: CKY *only* derives true statements.
 - ▶ If CKY puts A into $\text{Ch}(i,k)$, then there is rule $A \rightarrow BC$ and some j with $B \in \text{Ch}(i,j)$ and $C \in \text{Ch}(j,k)$.
 - ▶ Induction hypothesis: for shorter spans, have $B \Rightarrow^* w_i \dots w_{j-1}$.
Thus $A \Rightarrow B C \Rightarrow^* w_i \dots w_{j-1} C \Rightarrow^* w_i \dots w_{k-1}$
- Completeness: CKY derives *all* true statements.
 - ▶ Each derivation $A \Rightarrow^* w_i \dots w_{k-1}$ starts with a first step;
say $A \Rightarrow B C \Rightarrow^* w_i \dots w_{j-1} C \Rightarrow^* w_i \dots w_{k-1}$
 - ▶ Important: ensure that all nonterminals for shorter spans are known before filling $\text{Ch}(i,k)$.

Recognizer to Parser

- Parser: need to construct parse trees from chart.
- Do this by memorizing how each $A \in \text{Ch}(i,k)$ can be constructed from smaller parts.
 - ▶ built from $B \in \text{Ch}(i,j)$ and $C \in \text{Ch}(j,k)$ using $A \rightarrow B C$: store (B,C,j) in *backpointer* for A in $\text{Ch}(i,k)$.
 - ▶ analogous to backpointers in HMMs
- Once chart has been filled, enumerate trees recursively by following backpointers, starting at $S \in \text{Ch}(1,n+1)$.

Conclusion

- Context-free grammars: most popular grammar formalism in NLP.
 - ▶ there are also other, more expressive grammar formalisms
- CKY: most popular parser for cfgs.
 - ▶ very simple polynomial algorithm, works well in practice
 - ▶ there are also other, more complicated algorithms
- Next time: put parsing and statistics together.



slide credits

slides that look like this

come from

Question 2: Tagging

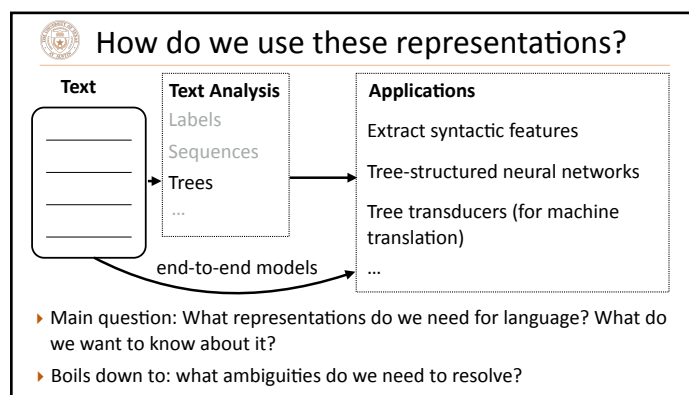
- Given observations y_1, \dots, y_T , what is the most probable sequence x_1, \dots, x_T of hidden states?
- Maximum probability:

$$\max_{x_1, \dots, x_T} P(x_1, \dots, x_T \mid y_1, \dots, y_T)$$

- We are primarily interested in $\arg \max$:

$$\begin{aligned} & \arg \max_{x_1, \dots, x_T} P(x_1, \dots, x_T \mid y_1, \dots, y_T) \\ &= \arg \max_{x_1, \dots, x_T} \frac{P(x_1, \dots, x_T, y_1, \dots, y_T)}{P(y_1, \dots, y_T)} \\ &= \arg \max_{x_1, \dots, x_T} P(x_1, \dots, x_T, y_1, \dots, y_T) \end{aligned}$$

earlier editions of this class (ANLP), given by Alexander Koller



CS388 given by Greg Durrett at U Texas, Austin

and their use is gratefully acknowledged. I try to make any modifications obvious, but if there are errors on a slide, assume that I added them.