#### ANLP

#### 05 - n-grams (sequences, part I)

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Week	Date	Theme	Focus	Readings	Assignment
1	2019-10-16		Intro	E.1	
	2019-10-17		Probability	https://mml-book.github.io; Sharon Goldwater's tutorial	
2	2019-10-23	Words, Representations	words, relations	JM-3.6	A1 released
	2019-10-24		words, embeddings	JM-3.6, E.3.3.4, E.14.5-6	
3	2019-10-30	Sequences I	n-grams	JM-3.3	
	2019-10-31		/	/	/
4	2019-11-06	Tools / Framings: Classification	binary classification	E.2.0-5, E.4.2-4.4.1, JM-3.4, JM-3.5.0-6	A1 due; A2 released
	2019-11-07		multiclass classification	E.4.2, JM-3.5.6	
5	2019-11-13		discussion of A1		
	2019-11-14	Sequences II	HMMs, POS- Tagging	E.7.0-4, JM-3.8	

Week	Date	Theme	Focus	Readings	Assignment
6	2019-11-20		CRFs	E.7.5, E.8.3	A2 due; A3 released
	2019-11-21	Tools / Framings: NNs	NNs I: FF	E.3.0-3, G.1-4	
7	2019-11-27		discussion of A2		
	2019-11-28		NNs II: RNNs	G.10-11	
8	2019-12-04		NNs III: CNNs, Neural CRFs	E.3.4, E.7.6, G9	A3 due; A4 released
	2019-12-05	Structure	CFGs, CKY, PCFG	E.10.0-5, JM-3.12	
9	2019-12-11		discussion of A3		
	2019-12-12		Dependency parsing I	E.11, JM-3.13	
10	2019-12-18		Dependency parsing II		A4 due
	2019-12-19		discussion of A4		

Week	Date	Theme	Focus	Readings	Assignment
11	2020-01-08		pyTorch practical?	TBA	A5 released
	2020-01-09		pyTorch practical?	TBA	
12	2020-01-15	Semantics	Semantics I	E.12	
	2020-01-16		Semantics II, Seq2Seq		
13	2020-01-22		Seq2Seq II: Attentn & Pointers		A5 due; A6 released
	2020-01-23		discussion of A5		
14	2020-01-29	The Real World	Annotation	TBA	
	2020-01-30		Ethics of doing NLP	TBA	
15	2020-02-05		buffer		
	2020-02-06		final projects		

# today

• well-formed sequences of words — a statistical approach

# Let's play a game

- I will write the start of a sentence on the board.
- Each of you, in turn, gives me a word to continue that sentence, and I will write it down.

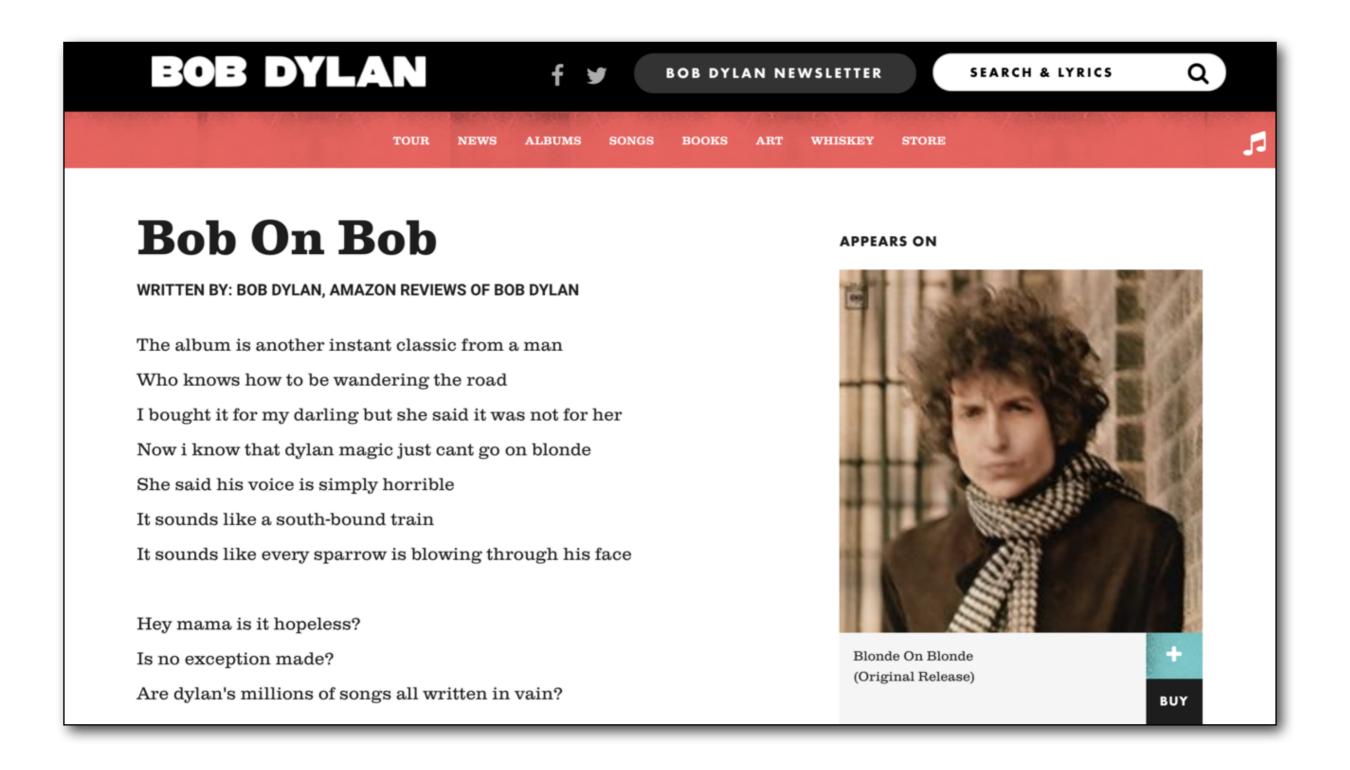
# Let's play another game

- You write a word on a piece of paper.
- You get to see the piece of paper of your neighbor, but none of the earlier words.
- In the end, I will read the sentence you wrote.

#### Statistical models in NLP

- Generative statistical model of language: pd P(w) over NL expressions that we can observe.
  - w may be complete sentences or smaller units
  - will later extend this to pd P(w, t) with hidden random variables t
- Assumption: A corpus of observed sentences w is generated by repeatedly sampling from P(w).
- We try to estimate the parameters of the prob dist from the corpus, so we can make predictions about unseen data.

#### Predictive text models



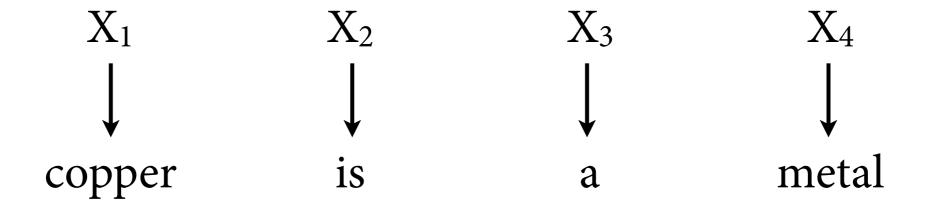
http://objectdreams.tumblr.com/

#### Predictive text models



#### Word-by-word random process

- A language model (LM) is a probability distribution P(w) over sentences.
- Think of it as random process that generates sentences word by word:



# Process from our game

- Each of you = a random variable  $X_t$ ; event " $X_t = w_t$ " means word at position t is  $w_t$ .
- When you chose  $w_t$ , you could see the outcomes of the previous variables:  $X_1 = w_1, ..., X_{t-1} = w_{t-1}$ .
- Thus, X<sub>t</sub> followed a pd

$$P(X_t = w_t \mid X_1 = w_1, \dots, X_{t-1} = w_{t-1})$$

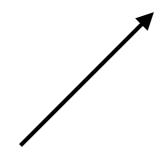
# Process from our game

Assume that X<sub>t</sub> follows some given PD

$$P(X_t = w_t \mid X_1 = w_1, \dots, X_{t-1} = w_{t-1})$$

• Then probability of the entire corpus (or sentence)  $w = w_1 \dots w_n$  is joint probability

$$P(w_1 ... w_n) = P(w_1) \cdot P(w_2 \mid w_1) \cdot P(w_3 \mid w_1, w_2) \\ \cdot ... \cdot P(w_n \mid w_1, ..., w_{n-1})$$



How do we estimate these?

#### Statistical models

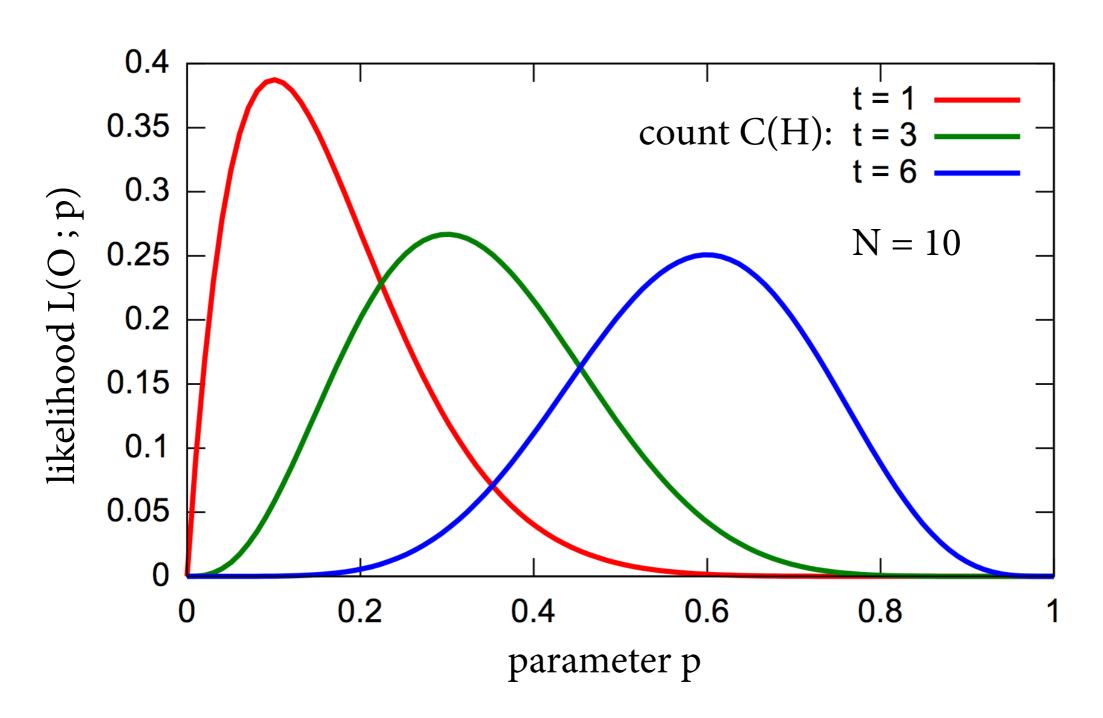
- We want to use prob theory to estimate a model of a generating process from observations about its outcomes.
- Simpler case: we flip a coin 100 times and observe H 61 times. Should we believe that it is a fair coin?
  - observation: absolute freq C(H) = 61, C(T) = 39; thus relative freq f(H) = 0.61, f(T) = 0.39
  - ▶ model: assume rv X follows a Bernoulli distribution, i.e. X has two outcomes, and there is a value p such that P(X = H) = p and P(X = T) = 1 p.
  - want to estimate the parameter p of this model

#### Fit of model and observations

- How do we quantify how well a model fits with the observations we made?
- Out of the many possibilities, easiest is to look at the likelihood: probability P(O; p) of the observations O given the values p for the model parameters.
- Maximum likelihood estimation: find parameter values for which the likelihood of O is maximal.

#### Likelihood functions

likelihood L(O; p) =  $p^{C(H)} * (1-p)^{C(T)} * binom(N, C(H))$ 



(Wikipedia page on MLE; licensed from Casp11 under CC BY-SA 3.0)

#### **ML Estimation**

- Goal: Find value for p that maximizes the likelihood of the observations.
- For Bernoulli models, it is extremely easy to estimate the parameters that maximize the likelihood:
  - P(X = a) = f(a)
  - in the coin example above, just take p = f(H)
- Can prove that relative frequency is an ML estimator for a lot of different statistical models (Bernoulli, multinomial, etc.; see link on course page).

#### Parameters of the model

- Our model has one parameter for  $P(X_t = w_t \mid w_1, ..., w_{t-1})$  for each t and  $w_1, ..., w_t$ .
- Can use maximum likelihood estimation:

$$P(w_t \mid w_1, \dots, w_{t-1}) = \frac{C(w_1 \dots w_{t-1} w_t)}{C(w_1 \dots w_{t-1})}$$

- Let's say a natural language has  $10^5$  different words. How many tuples  $w_1, \dots w_t$  of length t?
  - $t = 1: 10^5$
  - $t = 2: 10^{10}$  different contexts
  - $t = 3: 10^{15}$ ; etc.

#### Sparse data problem

- Typical corpus sizes:
  - ▶ Brown corpus: about 10<sup>6</sup> tokens
  - ▶ Gigaword corpus: about 10<sup>9</sup> tokens
- Problem exacerbated by Zipf's Law:
  - Order all words by their absolute frequency in corpus (rank 1 = most frequent word).
  - ▶ Then log(absolute frequency) falls linearly with log(rank); i.e., most words are really rare.
  - Zipf's Law is very robust across languages and corpora.

## Independence assumptions

- Let's pretend that word at position t depends only on the words at positions t-1, t-2, ..., t-k for some fixed k (Markov assumption of degree k).
- Then we get an n-gram model, with n = k+1:

$$P(X_t \mid X_1, \dots, X_{t-1}) = P(X_t \mid X_{t-k}, \dots, X_{t-1})$$
 for all t.

- Special names for unigram models (n = 1), bigram models (n = 2), trigram models (n = 3).
  - ▶ Thus our second game was a bigram model.

# Independence assumptions

- We assume statistical independence of X<sub>t</sub> from events that are too far in the past, although we know that this assumption is incorrect.
- Typical tradeoff in statistical NLP:
  - if model is too shallow, it won't represent important linguistic dependencies
  - ▶ if model is too complex, its parameters can't be estimated accurately from the available data

low n

→
modeling errors

high n

estimation errors

## Bigrams: an example

JOHN READ MOBY DICK MARY READ A DIFFERENT BOOK SHE READ A BOOK BY CHER

p(JOHN READ A BOOK)

$$= p(\mathsf{JOHN}|\bullet) \ p(\mathsf{READ}|\mathsf{JOHN}) \ p(\mathsf{A}|\mathsf{READ}) \ p(\mathsf{BOOK}|\mathsf{A}) \ p(\bullet|\mathsf{BOOK})$$

$$= \frac{c(\bullet \; \mathsf{JOHN})}{\sum_{w} c(\bullet \; w)} \ \frac{c(\mathsf{JOHN} \; \mathsf{READ})}{\sum_{w} c(\mathsf{JOHN} \; w)} \ \frac{c(\mathsf{READ} \; \mathsf{A})}{\sum_{w} c(\mathsf{READ} \; w)} \ \frac{c(\mathsf{A} \; \mathsf{BOOK})}{\sum_{w} c(\mathsf{A} \; w)} \ \frac{c(\mathsf{BOOK} \; \bullet)}{\sum_{w} c(\mathsf{BOOK} \; w)}$$

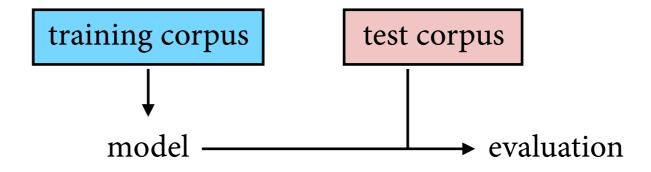
$$= \frac{1}{3} \qquad \frac{1}{1} \qquad \frac{2}{3} \qquad \frac{1}{2} \qquad \frac{1}{2}$$

$$\approx 0.06$$

(. is special sentence start and end token)

#### n-grams: Evaluation

- Measure quality of n-gram model using perplexity  $PP(w) = P(w_1 ... w_N)^{-1/N}$  of test data  $w = w_1 ... w_N$ .
- To get honest picture of model's performance, evaluate it on test data that was not used for training.



• Maximum likelihood model for training corpus is not necessarily good for test corpus (overfitting).

## Bigrams: a problem

JOHN READ MOBY DICK MARY READ A DIFFERENT BOOK SHE READ A BOOK BY CHER

p(CHER READ A BOOK)

$$= p(\mathsf{CHER}|\bullet) \ p(\mathsf{READ}|\mathsf{CHER}) \ p(\mathsf{A}|\mathsf{READ}) \ p(\mathsf{BOOK}|\mathsf{A}) \ p(\bullet|\mathsf{BOOK})$$

$$= \frac{c(\bullet \ \mathsf{CHER})}{\sum_{w} c(\bullet \ w)} \ \frac{c(\mathsf{CHER} \ \mathsf{READ})}{\sum_{w} c(\mathsf{CHER} \ w)} \ \frac{c(\mathsf{READ} \ \mathsf{A})}{\sum_{w} c(\mathsf{READ} \ w)} \ \frac{c(\mathsf{A} \ \mathsf{BOOK})}{\sum_{w} c(\mathsf{A} \ w)} \ \frac{c(\mathsf{BOOK} \ \bullet)}{\sum_{w} c(\mathsf{BOOK} \ w)}$$

$$= \frac{0}{3} \qquad \frac{0}{1} \qquad \frac{2}{3} \qquad \frac{1}{2} \qquad \frac{1}{2}$$

$$= 0$$

#### Unseen data

- ML estimate is "optimal" only for the corpus from which we computed it.
- Usually does not generalize directly to new data.
  - ▶ Ok for unigrams, but there are so many bigrams.
- ML estimate predicts probability of 0 for n-grams that were not observed in training. This is a disaster because product with 0 is always 0.

# Smoothing techniques

• Basic idea: Replace ML estimate

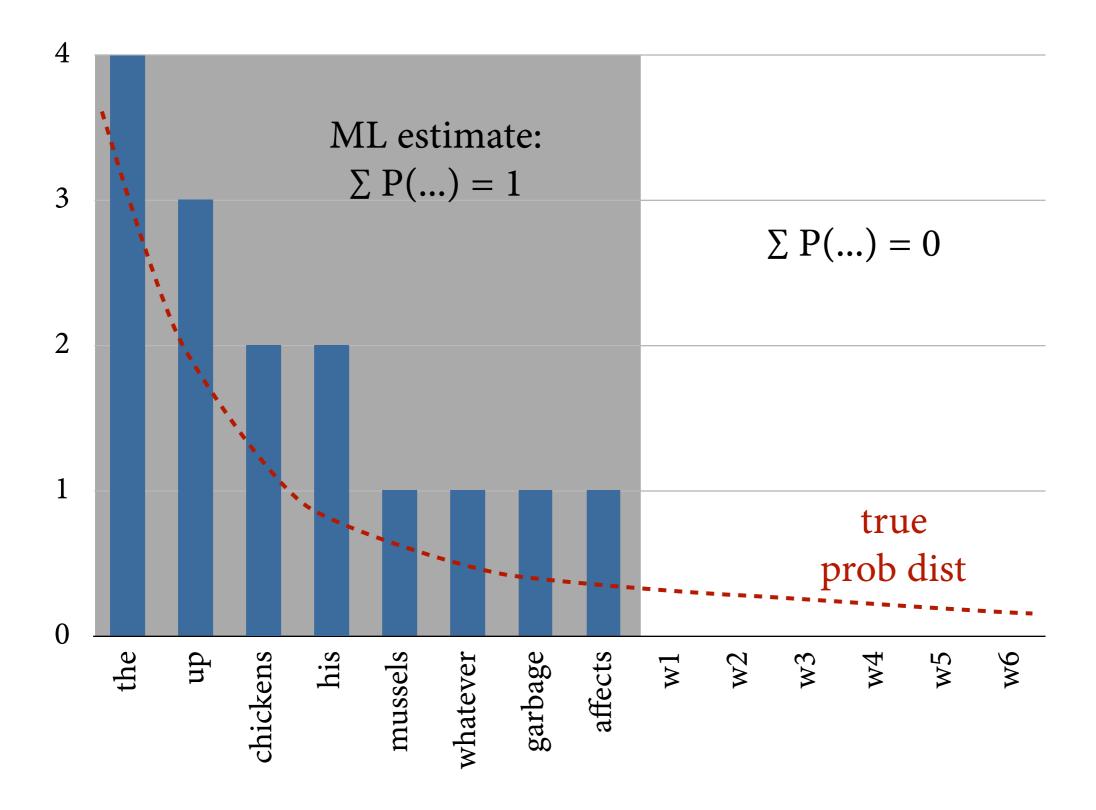
$$P_{\text{ML}}(w_i \mid w_{i-1}) = \frac{C(w_{i-1}w_i)}{C(w_{i-1})}$$

by estimate with adjusted bigram count

$$P^*(w_i \mid w_{i-1}) = \frac{C^*(w_{i-1}w_i)}{C(w_{i-1})}$$

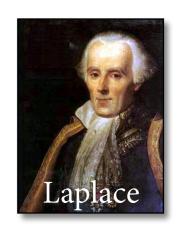
- Redistribute counts from seen to unseen bigrams.
- Generalizes easily to n-gram models with n > 2.

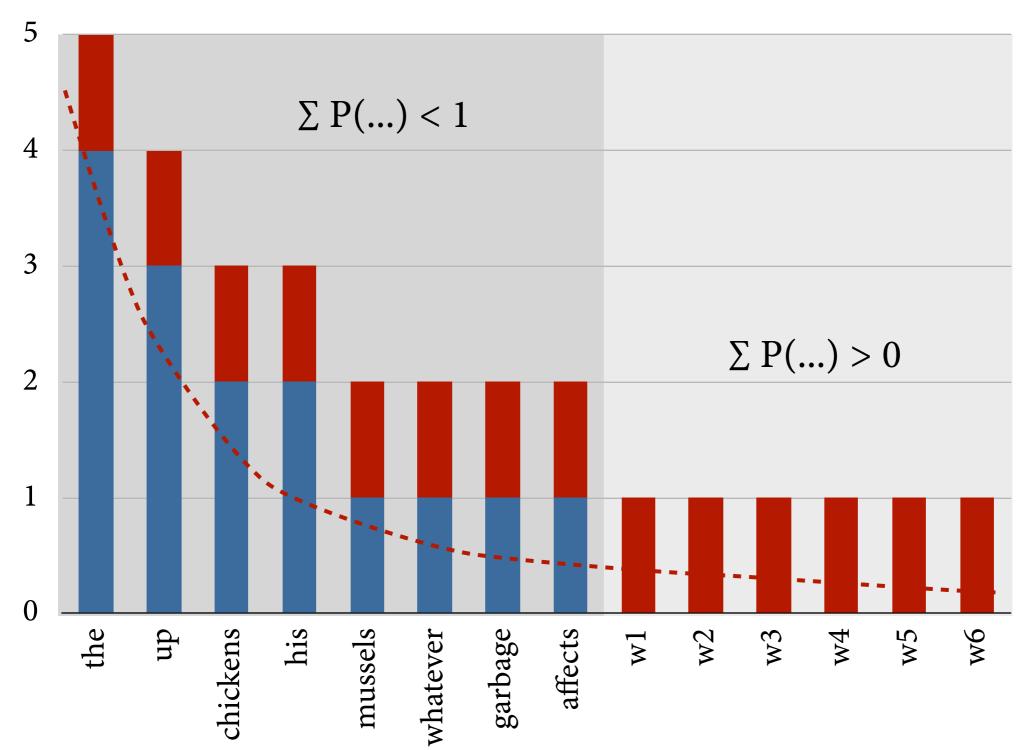
## Smoothing



C(eat X) in Brown corpus

#### Add-one Smoothing





# Add-one Smoothing

• Count every bigram (seen or unseen) one more time than in corpus and normalize:

$$P_{\text{lap}}(w_i \mid w_{i-1}) = \frac{C(w_{i-1}w_i) + 1}{\sum_{w} (C(w_{i-1}w) + 1)} = \frac{C(w_{i-1}w_i) + 1}{C(w_{i-1}) + |V|}$$

JOHN READ MOBY DICK MARY READ A DIFFERENT BOOK SHE READ A BOOK BY CHER

|V| = 11, |seen bigram types| = 11  $\Rightarrow$  110 unseen bigrams  $p(\mathsf{JOHN}\ \mathsf{READ}\ \mathsf{A}\ \mathsf{BOOK})$ 

$$=$$
  $\frac{1+1}{11+3}$   $\frac{1+1}{11+1}$   $\frac{1+2}{11+3}$   $\frac{1+1}{11+2}$   $\frac{1+1}{11+2}$ 

$$\approx 0.0001$$

p(CHER READ A BOOK)

$$=$$
  $\frac{1+0}{11+3}$   $\frac{1+0}{11+1}$   $\frac{1+2}{11+3}$   $\frac{1+1}{11+2}$   $\frac{1+1}{11+2}$ 

$$\approx 0.00003$$

# Add-one Smoothing

- Easy to implement, but dramatically overestimates probability of unseen events.
  - In the Cher example:  $P_{lap}(unseen \mid w_{i-1}) \ge 1/14$ ; thus "count"  $(w_{i-1} unseen) \approx 110 * 1/14 = 7.8$ .
  - ▶ Compare against 12 bigram tokens in training corpus.
- This has been a very (<u>very</u>) active area of research for many years, and many very sophisticated solutions have been proposed, e.g. using second-order information about the corpus (how expectable are rare events).
- Importance of this reduced thanks to recent different methods for estimating pd. (neural networks).

# why do language modelling?

- predictive text input
- important component of many applications:
  - machine translation
  - speech recognition
- common structure: generate many candidates, rank them according to "plausibility" as sentence

#### Conclusion

- Statistical models of natural language.
- Language models with n-grams.
- The problem of data sparseness.
- Smoothing.

# Collaboration on Assignments

#### Acceptable:

- discussing alternatives on how to do something
- asking someone for a description on how their algorithm works
- explaining on a conceptual level how you overcame an error message
- using a blog post/website for info on how an algorithm works/making your code more efficient

#### Unacceptable:

- working together on code
- dividing the assignment into parts
- using previous or existing solutions as a starting point
- copying source code from the web (& editing it)
- copying definitions/answers to discussion questions from a textbook or the web

# slide credits

slides that look like this

come from

#### **Question 2: Tagging**

- Given observations y<sub>1</sub>, ..., y<sub>T</sub>, what is the most probable sequence x<sub>1</sub>, ..., x<sub>T</sub> of hidden states?
- Maximum probability:

$$\max_{x_1} P(x_1, \dots, x_T \mid y_1, \dots, y_T)$$

• We are primarily interested in arg max:

$$\arg \max_{x_1, \dots, x_T} P(x_1, \dots, x_T \mid y_1, \dots, y_T) 
= \arg \max_{x_1, \dots, x_T} \frac{P(x_1, \dots, x_T, y_1, \dots, y_T)}{P(y_1, \dots, y_T)} 
= \arg \max_{x_1, \dots, x_T} P(x_1, \dots, x_T, y_1, \dots, y_T)$$

earlier editions of this class (ANLP), given by Alexander Koller

and their use is gratefully acknowledged. I try to make any modifications obvious, but if there are errors on a slide, assume that I added them.