

Approximate Blockwise Likelihood Estimation

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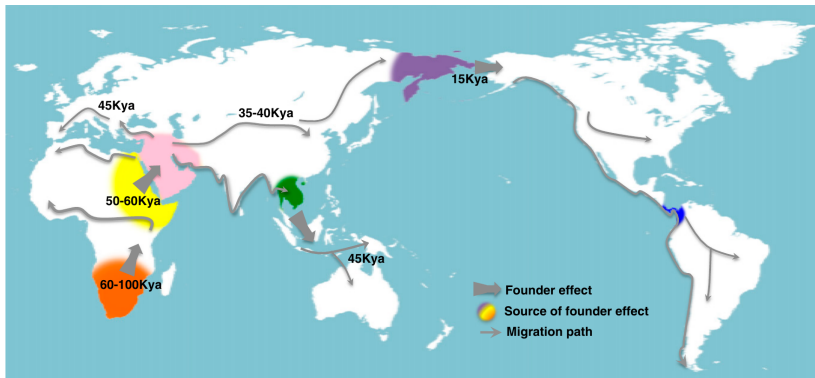
August 29, 2019

We move, and have been moving!



Wikimedia Commons

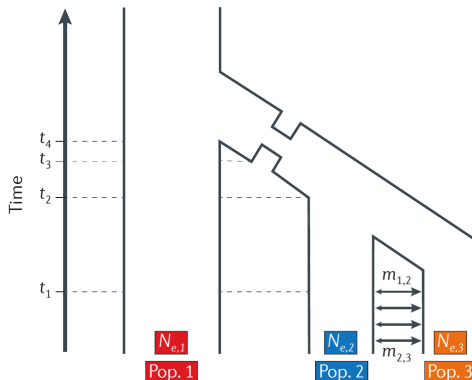
Picturing modern human migration



Henn *et al.* 2012

Simplifying a complex demography

modelling interacting panmictic units



Demographic inference using genomic data

The two major approaches

- Based on the SFS
 - No modeling of linkage
 - Usually no recombination
- Based on the haplotype structure
 - No intra-locus recombination
 - Tracts of IBD/IBS sharing
 - Recombination via the SMC

Aim : Find common ground between the two approaches

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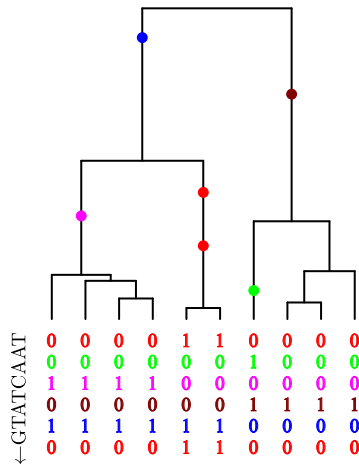
Demographic inference using genomic data

The two major approaches

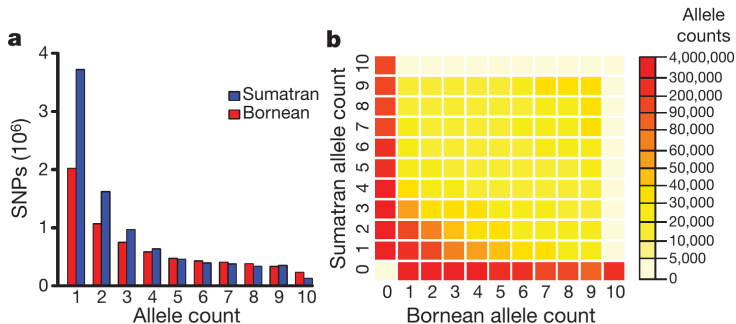
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Gene genealogies and polymorphisms



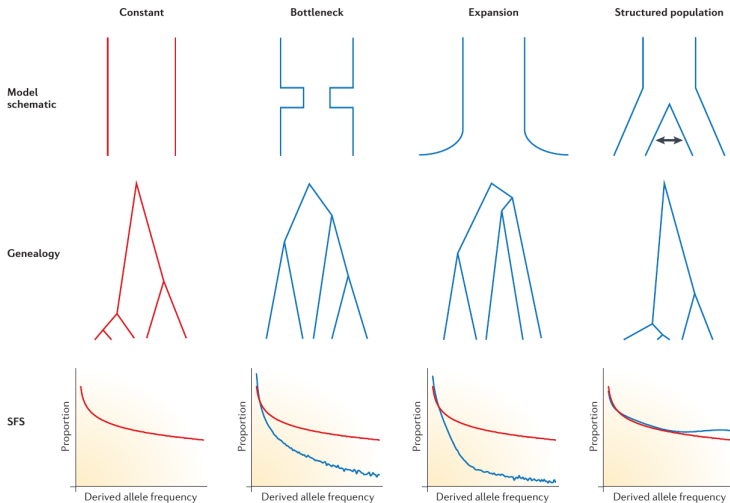
The Site Frequency Spectrum (SFS)



a. SFS

b. Joint SFS

Statistical identifiability and the SFS

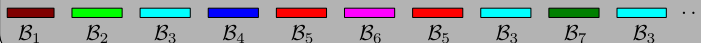


Schraiber and Akey 2015

Extending the SFS

The Blockwise SFS (bSFS)

Short reads (*e.g.* RADSeq)



"Chopped" Genome



$$\mathcal{B}_{SFS} = (n_{B_1}, n_{B_2}, n_{B_3}, n_{B_4}, n_{B_5}, n_{B_6}, n_{B_7}, \dots) = (1, 1, 3, 1, 2, 1, 1, \dots)$$

An exact analytical method makes use of the Generating Function of branch lengths

A General Method for Calculating Likelihoods Under the Coalescent Process

K. Lohse,* R. J. Harrison,[†] and N. H. Barton*^{*,†}

*Institute of Evolutionary Biology, University of Edinburgh, Edinburgh EH9 3JT, United Kingdom, [†]East Malling Research, East Malling ME19 6BJ, United Kingdom, and [‡]Institute of Science and Technology, A-3400 Klosterneuburg, Austria

Neandertal Admixture in Eurasia Confirmed by Maximum-Likelihood Analysis of Three Genomes

Konrad Lohse*^{*,†} and Laurent A. F. Frantz[‡]

*Institute of Evolutionary Biology, University of Edinburgh, Edinburgh, EH9 3JT, United Kingdom, and [†]Animal Breeding and Genomics Group, Wageningen University, De Elst 1, Wageningen, WD 6708, The Netherlands

Inferring Bottlenecks from Genome-Wide Samples of Short Sequence Blocks

Lynsey Bunnefeld*^{*,†} Laurent A. F. Frantz,^{†,‡} and Konrad Lohse*

*Institute of Evolutionary Biology, University of Edinburgh, Edinburgh EH9 3FL, United Kingdom, and [†]Animal Breeding and Genomics Centre, Wageningen University, Wageningen 6708 PB, The Netherlands

Lohse *et al.* 2011, Lohse & Frantz 2014, Bunnefeld *et al.* 2015

Approximating the bSFS

Approximate Blockwise Likelihood Estimation (ABLE)

Felsenstein equation (discretized Chapman-Kolmogorov)

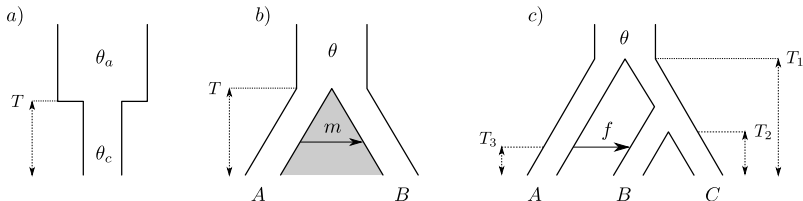
$$\mathcal{L}(\Theta) \propto p(\mathcal{D} \mid \Theta) = \sum_{\mathcal{G}} p(\mathcal{D} \mid \mathcal{G}, \Theta) p(\mathcal{G} \mid \Theta)$$

Sampling genealogies $\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_M$ from $p(\mathcal{G} \mid \Theta)$ yields a Monte Carlo estimator of the bSFS likelihood

$$p(\mathcal{B}_{SFS} \mid \Theta) \approx \frac{1}{M} \sum_{i=1}^M p(\mathcal{B}_{SFS} \mid \mathcal{G}_i, \Theta)$$

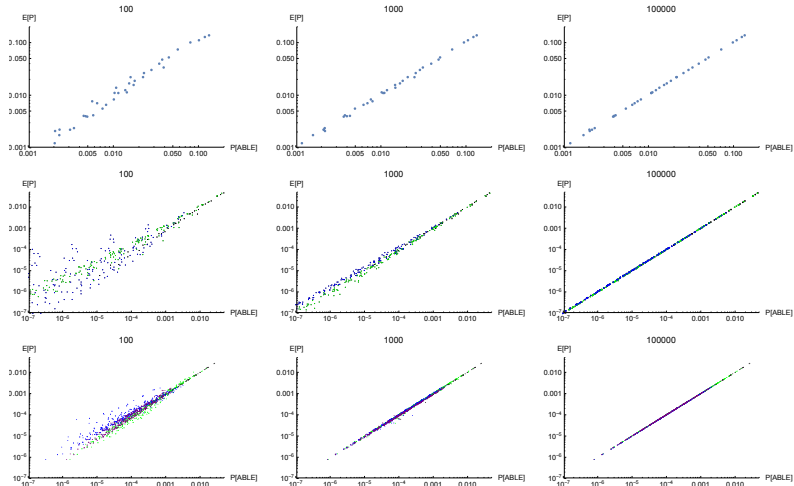
checkABLE

Expected bSFS : approximate vs. analytical



Asymptotic convergence of the bSFS

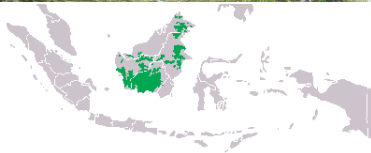
100, 1K & 100K genealogies



Every point represents a bSFS category

Orangutans : a tale of two islands

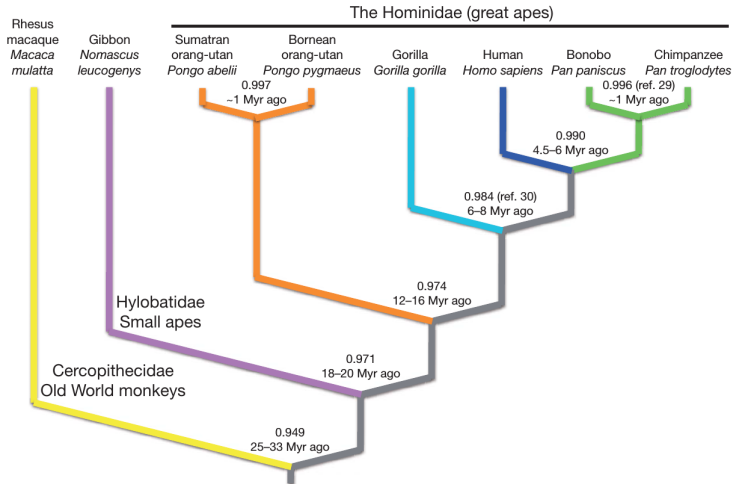
Pongo pygmaeus



Pongo abelii



Orangutans : a tale of two islands



Orangutans : a tale of two islands

LETTER

doi:10.1038/nature09687

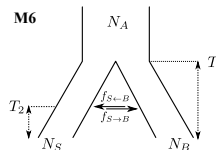
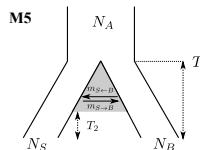
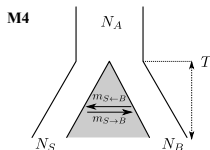
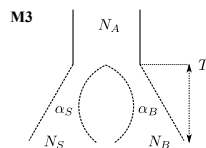
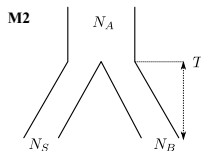
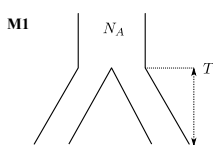
Comparative and demographic analysis of orang-utan genomes

Devin P. Locke¹, LaDeana W. Hillier¹, Wesley C. Warren¹, Kim C. Worley², Lynne V. Nazareth², Donna M. Muzny², Shiw-Pyng Yang¹, Zhengyuan Wang¹, Asif T. Chinwalla¹, Pat Minx¹, Makedonka Mitreva¹, Lisa Cook¹, Kim D. Delehaunty¹, Catrina Fronick¹, Heather Schmidt¹, Lucinda A. Fulton¹, Robert S. Fulton¹, Joanne O. Nelson¹, Vincent Magrini¹, Craig Pohl¹, Tina A. Graves¹, Chris Markovic¹, Andy Cree², Huyen H. Dinh², Jennifer Hume², Christie L. Kovar², Gerald R. Fowler², Gerton Lunter^{3,4}, Stephen Meader³, Andreas Heger³, Chris P. Ponting³, Tomas Marques-Bonet^{5,6}, Can Alkan⁵, Lin Chen⁵, Ze Cheng⁵, Jeffrey M. Kidd⁵, Evan E. Eichler^{5,7}, Simon White⁸, Stephen Searle⁸, Albert J. Vilella⁹, Yuan Chen⁹, Paul Flicek⁹, Jian Ma¹⁰, Brian Raney¹⁰, Bernard Suh¹⁰, Richard Burhans¹¹, Javier Herrero⁹, David Haussler¹⁰, Rui Faria^{6,12}, Olga Fernando^{6,13}, Fleur Darré⁶, Domènec Farré⁶, Elodie Gazave⁶, Meritxell Oliva⁶, Arcadi Navarro^{6,14}, Roberta Roberto¹⁵, Oronzo Capozzi¹⁵, Nicoletta Archidiacono¹⁵, Giuliano Della Valle¹⁶, Stefania Purgato¹⁶, Mariano Rocchi¹⁵, Miriam K. Konkel¹⁷, Jerilyn A. Walker¹⁷, Brygg Ullmer¹⁸, Mark A. Batzer¹⁷, Arian F. A. Smit¹⁹, Robert Hubley¹⁹, Claudio Casola²⁰, Daniel R. Schrider²⁰, Matthew W. Hahn²⁰, Victor Quesada²¹, Xose S. Puente²¹, Gonzalo R. Ordóñez²¹, Carlos López-Otín²¹, Tomas Vinar²², Brana Brejova²², Aakrosh Ratan¹¹, Robert S. Harris¹¹, Webb Miller¹¹, Carolin Kosiol²³, Heather A. Lawson²⁴, Vikas Taliwal²⁵, André L. Martins²⁵, Adam Siepel²⁵, Arindam RoyChoudhury²⁶, Xin Ma²⁵, Jeremiah Degenhardt²⁵, Carlos D. Bustamante²⁷, Ryan N. Gutenkunst²⁸, Thomas Mailund²⁹, Julien Y. Dutheil²⁹, Asger Hobolth²⁹, Mikkel H. Schierup²⁹, Oliver A. Ryder³⁰, Yuko Yoshinaga³¹, Pieter J. de Jong³¹, George M. Weinstock¹, Jeffrey Rogers², Elaine R. Mardis¹, Richard A. Gibbs² & Richard K. Wilson¹

Locke *et al.* 2011

Inferring demography AND recombination rates

The model choice pipeline



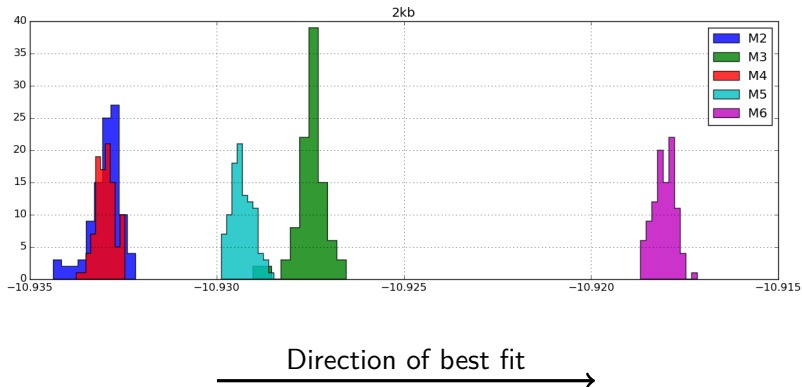
Results from 2kb blocks

Total spliced length : 163 Mbp

Model	N_A	c	T	N_S	N_B	α_S	α_B	$4N_A m_{S \rightarrow B}$	$4N_A m_{S \leftarrow B}$	T_2	$f_{S \rightarrow B}$	$f_{S \leftarrow B}$	$\ln L$
M1	18 200	1.58×10^{-8}	387 000										-907 477
M2	1 380	2.06×10^{-8}	294 000	22 100	8 610								-891 341
M3	2 180	2.09×10^{-8}	306 000	21 800	5 490	-0.003	-0.728						-891 308
M4	1 260	2.11×10^{-8}	320 000	22 300	8 210			0.025	0.000				-892 423
M5	1 280	1.87×10^{-8}	1 807 000	21 600	8 850			1.568	2.202	274 000			-892 225
M6	1 420	2.73×10^{-8}	816 000	22 400	8 910					295 000	0.121	0.267	-891 139

$\mu : 1 \times 10^{-8} / bp / generation$
 20 yrs/generation
 2 diploid genomes per pop.

Relative model fit for 2kb blocks distribution of 100 LnLs using 1M ARGs



ABLE : a quick summary

- Uses the **bSFS**, a very rich summary of genomic data
- **Does not require polarized data** (*i.e.* no outgroups)
- **Does not require phased data** and accounts for linkage
- Can **infer recombination rates** along with demography
- Is **computationally efficient** (coded in C/C++)
- Uses ***ms*** for sampling from $p(\mathcal{G}_i | \theta) / p(\mathcal{A} | \Theta)$
- Runs on **parallel threads** using OpenMP

Download **v0.1** from <https://github.com/champost/ABLE>

Collaborators

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