# Exercises for Computational Physics (physik760)
# WS 2019/2020

B. Kostrzewa and C. Urbach

---

Exercises for the week from 25th to 29th of November 2019.

## Autocorrelation

**1:** We want to study autocorrelation in a time series. As a toy model we use the follwing, one-dimensional random walk

$$X_{i+1} = \alpha\, X_i + (1-\alpha)\,\epsilon_i\,, \quad \forall\, i\, \in\, \mathbb{N}$$

and $\epsilon_i \sim \mathcal{U}([0,1])$ is uniformly distributed and $\alpha\, \in\, [0,1]$.
Imagine we run this forever, which gives a random variable $X_\infty$. What are the expectation value and variance of $X_\infty$?
First some theoretical finger exercises

- What are the expectation value and variance of $X_\infty$? As a result for the variance you find $\mathrm{Var}(X_\infty) = \frac{1}{12}\frac{1-\alpha}{1+\alpha}$.
- What is the covariance of any two variables $X_i$, $X_k$, $i,k > 0$, in the sequence analytically?
- What is the integrated autocorrelation time $\tau_\mathrm{int}$ ? What happens for $\alpha \to 0$ and $\alpha \to 1$? As a result you will find

$$\tau_\mathrm{int} = \frac{1}{2}\frac{1+\alpha}{1-\alpha}\,.$$

- We want to estimate the mean value with the good old estimator

$$\bar{X}_N = \frac{1}{N}\sum_{m=1}^{N} X_{\infty+m}$$

  What is the variance (or the error, i.e. square root of the variance) of this estimator? You will find

$$\mathrm{Var}(X_N) = \frac{1}{N}\frac{1}{12}$$

  up to $\mathcal{O}(1/N^2)$.

**2:** Now let's assume that we do not know the detailed construction of the $X_n$, i.e. we are only given the sequence $X_1, X_2, \ldots$ without knowing about the $\epsilon$s etc.. So we need to deduce everything from sampling the sequence.

- Implement the sampling and the estimation of $\mathrm{Var}(X_\infty)$, $\mathrm{Cov}(X_{\infty+m}, X_\infty)$ and $\tau_{\mathrm{int}}$. As a stopping criterion for the summation of the correlation function use the condition $W_{\max} > \{4 \sim 10\}\, \tau_{\mathrm{int}}(W_{\max})$.

- We estimate the variance $\mathrm{Var}(X_\infty)$ and the integrated autocorrelation time $\tau_{\mathrm{int}}$ and get the variance of $\bar{X}_N$ from

$$\mathrm{Var}(\bar{X}_N) = \frac{2}{N}\, \mathrm{Var}(X_\infty)\, \tau_{\mathrm{int}}\,.$$

  Does it work out? Compare also the above derived formulas.