Exercises for Computational Physics (physik760)
WS 2019/2020

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Exercises for the week from 28th to 31st of October 2019.

3 Random Variable Generation

1: Use the *generalized inverse transform method* to sample $X \sim f_X$ with

$$f_X(x) = \frac{1}{\pi} \frac{1}{\sqrt{x\,(1-x)}}\,,$$

for $0 < x < 1$.

a) derive the c.d.f., F_X, and its inverse

b) confirm numerically that your derivation is correct by computing $F_X^{-1}\left(F_X(x)\right)$

c) use the inverse function method to sample $X \sim f_X$, plot a histogram to confirm visually that the distribution lines up with the target p.d.f.

2: Write a computer programme which implements a linear congruential random number generator given by the following recurrence relation:

$$X_{i+1} = a \cdot X_i \mod m$$

and study its properties for different pairs of

$$(a, m) = \left(7^5, 2^{31}-1\right), (19, 6788793), \left(65539, 2^{31}\right)\,,$$

i.e. calculate its mean value and variance. Do they agree with your expectations? The last (a, m) pair corresponds to an infamous random number generator named RANDU, which dates back to the 1960's
Finally, generate plots of order 10000 points of

$$(X_i/m,\ X_{i+1}/m,\ X_{i+2}/m) \qquad i = 1, 2, ...$$

(3-dimensional scatter plots) using your favorite plotting programme. Can you visually observe correlations? See: G. Marsaglia *Random Numbers Fall Mainly in the Planes.* Proc. National Academy of Sciences **61** (1): 25–28 (1968) for more details. You will see why RANDU became infamous.