Exercises for Computational Physics (physik760)
WS 2019/2020

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Exercises for the week from 11th to 15th of November 2019.

Linear Regressions

1: Download the global temperature anomaly data set

`http://data.giss.nasa.gov/gistemp/tabledata_v3/GLB.Ts+dSST.csv`
or
`https://ecampus.uni-bonn.de/goto_ecampus_file_1141356_download.html`

and understand its content.

a) Plot the average annual temperature anomaly versus the years, making the assumption that for each year, the anomaly in each month is an independent sample. (use this to compute estimates of errors on the year-by-year means)

b) Assume further that the data for the anomaly is not correllated between the years.

c) Determine which polynomial degree $n = 0, 1, 2, 3$ best fits this data.

The accept-reject method

As we have seen in the previous exercises, transform methods can be used to sample from arbitrary distributions if we are able to compute the Jacobian or the inverse of the c.d.f. of the target distribution. However, when neither of these is possible or if the evaluation of the inverse of the c.d.f. is costly, the *accept-reject* method offers a powerful alternative.

2: Consider the one-parameter family of probability density functions

$$f_\alpha(x) = b_\alpha \sqrt{1-x^2}\cos(\alpha x)^2$$

defined on the set $x \in [-1, 1]$ and with normalization b_α

$$b_\alpha = \left[\int_{-1}^{1} \sqrt{1-x^2}\,\cos(\alpha x)^2 \right]^{-1} .$$

Tasks:

a) Take first $\alpha = 0$. Sample from $X \sim f_0$ using the accept-reject method with uniform instrumental density $g \sim \mathcal{U}(-1, 1)$

$$g(x) = \frac{1}{2}\, \mathbb{1}_{[-1,1]}(x)\,.$$

Determine b_0 analytically and compare your observed distribution with the true p.d.f.

b) Now generalize to $\alpha \neq 0$. Sample $X \sim f_\alpha$ with the accept-reject method and use f_0 as your instrumental density. Implement the accept-reject method and explore it for $\alpha \in (0, 16]$ in a rough stepping.

c) Why does the accept-reject algorithm not require explicit knowledge of b_α?

d) An important measure of efficiency is the *acceptance ratio* (AR), which is the ratio of accepted x divided by the number of tests. How does it vary with α and why?

e) Based on the previous two tasks, can you determine a numerical estimate for b_α from your sampling?

f) *Fun question:* For those who are interested, what is the limiting AR for $\alpha \to \infty$?

g) The generalization of the accept-reject method to multiple dimensions is easy. We can sample from the distribution with p.d.f.

$$\begin{aligned} f_\alpha^D &= \sqrt{1 - |x|^2} \prod_{i=1}^{D} \cos(\alpha^i\, x_i^i)^2\, \mathbb{1}_{|x|^2<1} \\ |x|^2 &= x_1^2 + \cdots + x_D^2\,, \end{aligned}$$

by generating D-tuples $[x_1, x_2, \ldots, x_D]$ and performing the same test as above, accepting or rejecting the entire tuple.

i) Implement the accept-reject method for $D \in \{2, 3, \ldots, 5\}$.

ii) For $\alpha = 2$, what can you say about the acceptance ratio as a function of D?