Exercises for Computational Physics (physik760)
WS 2019/2020

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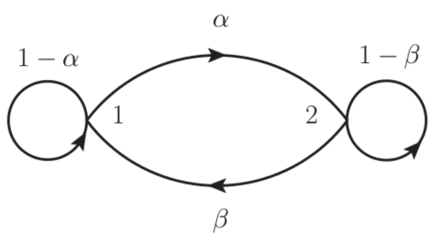
Exercises for the week from 9th to 13th of December 2019.

Simple Markov chain

1: Consider the 2-state Markov chain generated from the stochastic matrix



(cf. Example 6.1.1 in the lecture script).

- For which choices of α, β is the state space reducible / irreducible?
- Calculate the expected return time $\tau = \mathbb{E}_1(T_1)$ to state 1 (starting from state 1) depending on α and β analytically.
- Implement the calculation of τ using repeated application of the transition matrix and summing up to some convergence criterion. (This version is straightforwardly generalizable to more complicated processes).
- Estimate τ by simulating the Markov chain, i.e. implement the random walk on the graph.
- Interpret the results for the two limiting cases (a) $\alpha \to 0$ and (b) $\beta \to 0$ with $\alpha \neq 0$.

2: Ising Model:
Consider the $d = 2$-dimensional Ising model with Hamiltonian

$$H = -\frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h \sum_i s_i$$

where i, j run over all lattice sites and $s_i \in \{-1, 1\}$. In this exercise we will only consider the case of nearest neighbour interactions on a 2-dimensional $N \times N$ lattice with periodic boundary conditions. The first-mentioned restriction amounts to setting $J_{ij} = J = \text{const}$ for neighbouring lattice sites and $J_{ij} = 0$ for all other cases.

- Write a function which computes H for a given spin configuration $s = \{s_i\}$ on a 2-dimensional $N \times N$ lattice with periodic boundary conditions.

- Write a function which computes $\Delta H = H(s) - H(s')$, where $s = \{s_1, \ldots, s_j, \ldots\}$ and $s' = \{s_1, \ldots, s'_j, \ldots\}$, i.e. they differ only at a single lattice site.